# HOUSING COLLATERAL, CONSUMPTION INSURANCE
## AND RISK PREMIA:
## AN EMPIRICAL PERSPECTIVE

Hanno Lustig
Stijn Van Nieuwerburgh

Housing Collateral, Consumption Insurance and Risk Premia: An Empirical Perspective
Hanno Lustig and Stijn Van Nieuwerburgh
NBER Working Paper No. 9959
September 2003
JEL No. G1

## **ABSTRACT**

In a model with housing collateral, the ratio of housing wealth to human wealth shifts the conditional

distribution of asset prices and consumption growth. A decrease in house prices reduces the

collateral value of housing, increases household exposure to idiosyncratic risk, and increases the

conditional market price of risk. Using aggregate data for the US, we find that a decrease in the ratio

of housing wealth to human wealth predicts higher returns on stocks. Conditional on this ratio, the

covariance of returns with aggregate risk factors explains eighty percent of the cross-sectional

variation in annual size and book-to-market portfolio returns.

An electronic data appendix for this paper is available at www.nber.org/data-appendix/w9959.

Hanno Lustig
Department of Economics
University of Chicago
1126 East 59th Street
Chicago, IL 60657
and NBER
hlustig@uchicago.edu

Stijn Van Nieuwerburgh
Stern School of Business
Department of Finance
Suite 9-190
44 West 4th Street
New York, NY 10012
svnieuwe@stern.nyu.edu

# Introduction

We identify a novel collateral channel that transmits shocks in the housing market to risk premia. In a model with collateralized borrowing and lending, the ratio of housing wealth to human wealth, *the housing collateral ratio*, changes the conditional distribution of consumption growth across households. When the collateral ratio is low, the dispersion of consumption growth across households is more sensitive to aggregate consumption growth shocks and this raises the market price of aggregate risk.

The model predicts that risk premia fluctuate predictably over time and in the cross-section. These predictions are confirmed by US equity return data. Investors demand a larger compensation for a given amount of aggregate risk in times when the housing collateral ratio is low. This implies that the housing collateral ratio predicts aggregate stock returns over time. In the cross-section, the model predicts that assets whose returns are more tightly correlated with aggregate consumption growth shocks when collateral is scarce trade at a discount. Conditional on the housing collateral ratio, the covariance of returns with aggregate consumption growth shocks explains about eighty percent of the cross-sectional variation in US stock returns, because the returns of value stocks are more correlated with aggregate consumption growth shocks during low collateral periods than are growth stocks. In Lustig & VanNieuwerburgh (2003) we calibrate this model and we show that the collateral effect can match the time-series and cross-sectional variation in risk premia in US data at moderate levels of risk aversion.

There are two main channels that transmit shocks originating in the housing market to the risk premia in asset markets. First, households want to hedge against rental price shocks or consumption basket composition shocks when the utility function is non-separable in non-durable consumption and housing services. This introduces a new risk factor which is the focus of recent work by Piazzesi, Schneider, & Tuzel (2002) and Yogo (2003). In particular, if housing services and consumption are complements then households command a larger risk premium if returns and rental price growth are positively correlated. This approach relies on a particular type of non-separability to generate large risk premia, but does not rely on imperfect consumption insurance. We argue that the market price of composition risk is small and roughly constant for plausible parameter values, in line with earlier work by Dunn & Singleton (1986) and Eichenbaum & Hansen (1990). They report substantial evidence against the null of separability in a representative agent model with non-durable consumption and durables, but they conclude that introducing durables does not help in reducing the pricing errors for stocks.

Second, a drop in the housing collateral ratio adversely affects the risk sharing technology

that enables households to insulate consumption from labor income shocks, and the distribution of consumption growth fans out as this ratio decreases. When housing prices decrease, collateral is destroyed and households are more exposed to idiosyncratic labor income risk. The risk associated with these collateral constraints contributes a liquidity factor to the stochastic discount factor. The collateral effect does not hinge on whether and how preferences over nondurable and housing consumption are non-separable. Instead, it relies on imperfect consumption insurance among households induced by occasionally binding collateral constraints. The housing market delivers shocks to the risk-sharing technology.

Without these collateral constraints our model collapses to the standard consumption-based capital-asset-pricing model of Lucas (1978) and Breeden (1979). That model prices only aggregate consumption growth risk and it has been rejected by the data (e.g. Hansen & Singleton (1983)). Our paper addresses two empirical failures of the consumption-based capital-asset-pricing model (CCAPM).

First, because US aggregate consumption growth is approximately i.i.d., the CCAPM implies a market price of risk that is approximately constant. However, in the data, stock market returns are predictable and this suggests that the market price of aggregate risk varies over time (e.g. Fama & French (1988), Campbell & Shiller (1988), Ferson, Kandel, & Stambaugh (1987), Whitelaw (1997), Lamont (1998), Lettau & Ludvigson (2003) and Campbell (2000) for an overview). Our model delivers time variation in the market price endogenously through the housing market. As the housing collateral ratio decreases, the conditional volatility of the liquidity factor increases. In the data, the housing collateral ratio *does* predict the aggregate US stock market return, mainly at lower frequencies.

Second, the covariance of asset returns with consumption growth explains only a small fraction of the variation in the cross-section of stock returns of firms sorted in portfolios according to size (market capitalization) and value (book-value to market-value ratio) characteristics (Fama & French (1992)). In response to this failure, Fama & French (1993) directly specify the stochastic discount factor as a linear function of the market return, the return on a small minus big firm portfolio, and a high minus low book-to-market firm portfolio. The empirical success of this three-factor model has motivated quite some more recent research on the underlying macroeconomic sources of risk for which their factors proxy (e.g. Bansal, Dittmar, & Lundblad (2002), Lettau & Ludvigson (2001b), Santos & Veronesi (2001) and Cochrane (2001) for an overview). Our model generates large value premia for stocks whose dividends are more sensitive to the housing collateral ratio. In the data, the collateral model explains eighty percent of the variability in annual returns of the Fama-French size and book-to-market portfolios. For annual

3

returns, this matches the empirical success of the Fama & French (1993) three-factor model and recent conditional consumption-based asset pricing models (e.g. Lettau & Ludvigson (2001b)).

We measure the aggregate stock of housing collateral in three different ways: the value of outstanding mortgages, the value of residential real estate (structures and land) and the value of residential fixed assets (structures). The housing collateral ratio, which we label $my$, is measured as the deviation from the cointegration relationship between the value of the aggregate housing stock and aggregate labor income.

The collateral constraints are motivated by the empirical importance of housing as a collateral asset. In the US, two-thirds of households own their house. For the median-wealth homeowner, home equity represents seventy percent of household net worth (Survey of Consumer Finance, 1998). Residential real estate wealth accounts for twenty-eight percent of total household net worth and sixty-eight percent of non-financial assets, while home mortgages make up sixty-four percent of household liabilities (Flow of Funds, Federal Reserve, averages for 1952-2002). Currently, the value of residential wealth exceeds the total household stock market wealth ($13.6 trillion) and the mortgage market is the largest credit market in the US ($6.1 trillion).

The model that underlies these empirical results contains the following essential ingredients. It is an endowment economy with a continuum of agents who are subject to labor income shocks. As in Lustig (2001), we allow households to forget their debts. The new feature of our model is that each household owns part of the housing stock. Housing provides utility services and collateral services. When a household chooses to forget its debts, it loses all its housing wealth but its labor income is protected from creditors, and the household is not excluded from trading. This gives rise to collateral constraints whose tightness depends on the abundance of housing collateral. We measure this by the housing collateral ratio: the ratio of collateralizable housing wealth to non-collateralizable human wealth.

The stochastic discount factor contains a new component which we label the *aggregate liquidity shock* (see Lustig (2001)). It is the growth rate of a cross-sectional moment of the consumption share distribution. The household's consumption share of the total endowment, both for non-durables and housing services, increases whenever the household switches to a state with a binding constraint. When a large fraction of households is constrained this growth rate is high. We call this a liquidity shock.

The housing collateral ratio changes the conditional moments of the aggregate liquidity shock. When the housing collateral ratio is low, households run into binding collateral constraints more frequently. This increases the conditional standard deviation of the aggregate weight shock, and, by the same token, the market price of risk. Thus, endogenous movements in the housing

4

collateral ratio turn the liquidity shocks in the stochastic discount factor on and off, and this induces heteroskedasticity and counter-cyclicality in the Sharpe ratio. This collateral mechanism is a novel feature of the model.

In Lustig & VanNieuwerburgh (2003) we fully calibrate and solve the model. The equilibrium aggregate liquidity shock is a function of the primitives of the model: the preferences, the household endowment process, the aggregate endowment process and the aggregate rental price process. The model generates large, time-varying risk premia and a value premium for stocks whose dividends are more responsive to collateral ratio shocks.

The empirical strategy here is to estimate the stochastic process for the aggregate weight shocks. In a first step we allow this process to depend in a non-linear fashion on the aggregate pricing factors. We estimate the parameters from the moment conditions implied by the Euler equations for the aggregate market return, the risk-free rate, a long term bond and a limited number of size and book-to-market portfolios. The Euler inequalities for the representative agent allow us to precisely estimate the coefficient of risk aversion. The estimated coefficients of relative risk aversion are plausible (between 2 and 5) and much lower than those for the other models we consider, as shown by Figure 1. In addition, the parameters in the aggregate weight shock specification are estimated precisely and have the sign predicted by the collateral channel. The pricing errors are small and the model cannot be rejected.

The linear specification for the aggregate weight shock fits the data best and it allows us to make contact with the linear factor models in empirical finance. This specification delivers a conditional version of the CCAPM with the housing collateral ratio as the conditioning variable. The housing collateral ratio summarizes the investor's time-varying information set. This model prices the 25 Fama-French size and book-to-market portfolios surprisingly well. It assigns the value premium to risk compensation for a higher correlation of value returns with aggregate risk factors in times when collateral is scarce. We provide evidence on dividend dynamics that potentially explains why value returns respond differently to aggregate shocks when the collateral ratio is small.

We organize the paper as follows. In section 1, we briefly discuss other related literature. Section 2 describes the essence of the model. The third section discusses the composition channel and collateral channel in more depth. Section 4 contains the time-series predictability results and sections 5 and 6 contain the empirical results for the cross-section. Section 7 concludes. Appendix A contains technical remarks about the estimation and a more detailed description of the data. The most important figures and tables appear in the main text, all others in Appendix B.

Figure 1: CCAPM, HCAPM and Collateral-CAPM - Moments of the estimated SDF. The Hansen-Jagannathan bounds are computed using annual data from 1926-2002 for the real value-weighted market return, the risk-free rate, a 10 year bond and $R^{HML}$, the return on the high value minus low value stock portfolio. The model parameters are estimated by GMM for these 4 test assets. The plot shows the moments of the implied SDF.
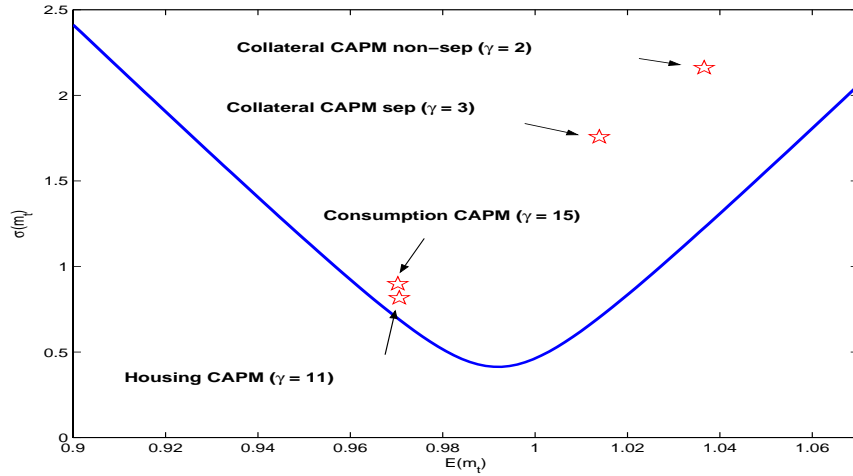


## 1    Related Literature

Our paper is closest to the work of Lettau & Ludvigson (2001b). We also develop a scaled version of the CCAPM. Our state variable $my$ summarizes information about future returns on housing *relative* to human capital while the scaling variable in Lettau & Ludvigson (2001b) is the consumption-wealth ratio, which summarizes household expectations about future returns on the entire market portfolio.

In a different class of models, Cogley (2002) and Brav, Constantinides, & Gezcy (2002) find that including higher moments like the standard deviation and skewness of the consumption growth distribution reduces the size of Euler equation errors for stock returns. This evidence is consistent with our model. In case of a large aggregate weight shock, the dispersion of the consumption growth distribution increases, while its skewness decreases. This provides indirect evidence for the consumption growth distribution shocks that drive our results. We provide a theory of what governs these shocks.

Cochrane (1996) explores the explanatory power of residential and non-residential investment for equity returns in the context of his production-based asset pricing framework (Cochrane (1991a)). Li, Vassalou, & Xing (2002) find that investment growth, including household sector

investment which is largely residential, can help account for a large fraction of the cross-sectional variation in equity returns. Similarly, Kullmann (2002) uses returns on residential and commercial real estate to improve the performance of the capital asset pricing model. We do not offer a theory of what distinguishes value firms from growth firms, but we report empirical evidence that the dividend process of value firms is more sensitive to the housing collateral ratio than the dividend process for growth firms. Our model generates significantly larger premia for claims to dividend processes that are highly sensitive to collateral shocks (Lustig & VanNieuwerburgh (2003)).

Life-cycle and portfolio choice models with housing such as Krueger & Villaverde (2001), Cocco (2000), Yao & Zhang (2002), Flavin & Yamashita (2002) posit an exogenous price process for housing. We endogenize the price of the asset but we abstract from life-cycle considerations.

## 2 Model

Our economy's risk sharing technology is subject to shocks originating in the housing market and these shocks determine the size of the wedge between the market's valuation of payoffs and the representative agent's IMRS. The stochastic discount factor in our model is

$$m_{t+1} = m_{t+1}^a g_{t+1}^\gamma,$$

where $m_{t+1}^a$ is the IMRS of a representative agent who consumes non-durable consumption and housing services, and $g_{t+1}^\gamma$ is the *liquidity factor* contributed by the solvency constraints. This factor can be interpreted as the aggregate cost of the solvency constraints. When these solvency constraints do not bind, the liquidity factor disappears and payoffs can be priced off the representative agent's IMRS $m_{t+1}^a$. We show that this liquidity factor can explain some of the variation in US stock returns over time and in the cross-section.

### 2.1 Collateral Channel

In our economy a continuum of agents are endowed with claims to stochastic labor income streams. These agents consume non-durable consumption and housing services. The markets for housing services are frictionless in that the ownership of the asset and use of its services are completely distinct.

We use $s^t$ to denote the history of events $s^t = (y^t, z^t)$, where $y^t$ denotes the history of idiosyncratic events and $z^t$ denotes the history of aggregate events. Households can purchase

housing services $\{h_t(s^t)\}$ in the spot markets at spot prices $\{\rho_t(z^t)\}$ as well as non-durable consumption $\{c_t(s^t)\}$.

Although agents cannot sell claims to their labor income stream $\{\eta_t(s^t)\}$, they can trade a complete set of contingent claims to insure against idiosyncratic labor income risk, but these trades are subject to solvency constraints. The solvency constraints can be stated as a restrictions on the value of a household's consumption claim, net of its labor income claim:

$$\Pi_{s^t}\left[\left\{c_t(s^t) + \rho_t(z^t)h_t(s^t)\right\}\right] \geq \Pi_{s^t}\left[\left\{\eta_t(s^t)\right\}\right],$$

where $\Pi_{s^t}\left[\left\{d_t(s^t)\right\}\right]$ denotes the price of a claim to $\left\{d_t(s^t)\right\}$.

The supply of housing wealth relative to human wealth governs the tightness of the solvency constraints. $\left\{c_t^a(z^t)\right\}$ denotes the aggregate endowment stream of non-durable consumption and $\left\{h_t^a(z^t)\right\}$ denotes the aggregate endowment of housing services. The effectiveness of the risk sharing technology our economy is endowed with depends on the ratio of the housing wealth to total wealth. We call this ratio the housing collateral ratio $my$:

$$my_t(z^t) = \frac{\Pi_{z^t}\left[\left\{\rho h^a\right\}\right]}{\Pi_{z^t}\left[\left\{c^a\right\}\right]}$$

Suppose the households in this economy derive no utility from housing services, then there is no collateral in this economy and $my$ is zero. All the solvency constraints necessarily bind at all nodes and households are in autarchy. As $my$ increases, perfect risk sharing becomes feasible.

The aggregate housing collateral ratio $my$ is a quasi-sufficient statistic for the risk sharing capacity of this economy. The stock of collateral is allocated efficiently across households in a stationary equilibrium and this absolves us from the need to track the entire distribution of collateral across households.

Shocks to $my$ change the conditional distribution of consumption across households and asset prices. The next section explains exactly how shocks to $my$ impinge on allocations and prices.

**Consumption**   We use Pareto-Negishi weights to characterize the equilibrium prices and allocations. $\xi_t(\ell, s^t)$ denotes the stochastic Pareto-Negishi weight of a household that starts off with initial weight $\ell$ and $\xi_t^a(z^t)$ denotes the $1/\gamma$-th cross-sectional moment of these household weights at aggregate node $z^t$ : $\int \xi_t(y^t, z^t)^{\frac{1}{\gamma}} d\Phi_t(z^t)$. $\Phi_0$ is the initial distribution over $\ell(\theta_0, s_0)$, implied by the initial wealth distribution $\Theta_0$. $\Phi_t(z^t)$ is the distribution over weights after aggregate history $z^t$. These weights are constant as long as the agent does no switch to a state with a binding constraint. The consumption share of an agent equals the ratio of his individual

8

stochastic Pareto-Negishi weight to the aggregate Pareto-Negishi weight:

$$c_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} c_t^a(z^t) \text{ and } h_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} h_t^a(z^t), \tag{1}$$

$\xi_t^a(z^t)$ is a non-decreasing stochastic process. An agent's consumption share decreases until she switches to a state with a binding constraint. These Pareto-Negishi weights are raised to the cutoff level $l_t^c(y_t, z^t)$ whenever a constraint binds. The cutoff levels for the weights increase as the housing collateral ratio decreases. When there is no housing collateral, the cutoff level for the consumption share equals the household's labor income share:

$$\frac{l_t^c(y_t, z^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} = \underline{\omega}_t(y_t, z^t) \nearrow \widehat{\eta}(y_t, z_t) \text{ as } my \searrow,$$

where $\widehat{\eta}(y_t, z_t)$ is the labor income share relative to the total non-durable endowment. The lower the collateral ratio, the larger the increase in its consumption share when it switches to a state with a binding solvency constraint. Household consumption becomes increasingly sensitive to income shocks as the housing collateral ratio decreases (see Figure 2.)[1]

In a stationary equilibrium, each household's consumption share is drifting downwards as long as it does not switch to a state with a binding constraint. The rate at which these shares decrease depends on the housing collateral ratio. When this ratio is low, the solvency constraints are tight, many households are highly constrained and the remainder experience large consumption share drops. The risk-free rate is low, inducing households to decumulate assets at a high rate. When this ratio is high enough, none of the households are constrained and interest rates are high. The growth rate of aggregate weight process $\xi_t^a(z^t)$ determines the consumption growth of the unconstrained households and these households price payoffs in each state of the world.

## 3  The Market Price of Aggregate Risk

Using the risk sharing rules in (1), the following expression for the IMRS of the unconstrained household emerges: $m_{t+1} = m_{t+1}^a g_{t+1}^\gamma$. This is the SDF in the sense of (Hansen & Jagannathan (1991)) that prices payoffs. The first section focusses on $m^a$ and it examines the data through the lens of a representative agent model. The new risk factor contributed by the non-separability in the utility function is referred to as composition risk. We show that the market price of

---

[1]The calibration is discussed in Lustig & VanNieuwerburgh (2003).

Figure 2: The Consumption Share Cutoff of One Household and $my$ The dotted line is the collateral
ratio and the full line is $\omega_t$: $\beta$ is .95, $\gamma$ is 8 and $\varepsilon$ is .15. The graph shows a 200 period simulation of the model at an annual
frequency.
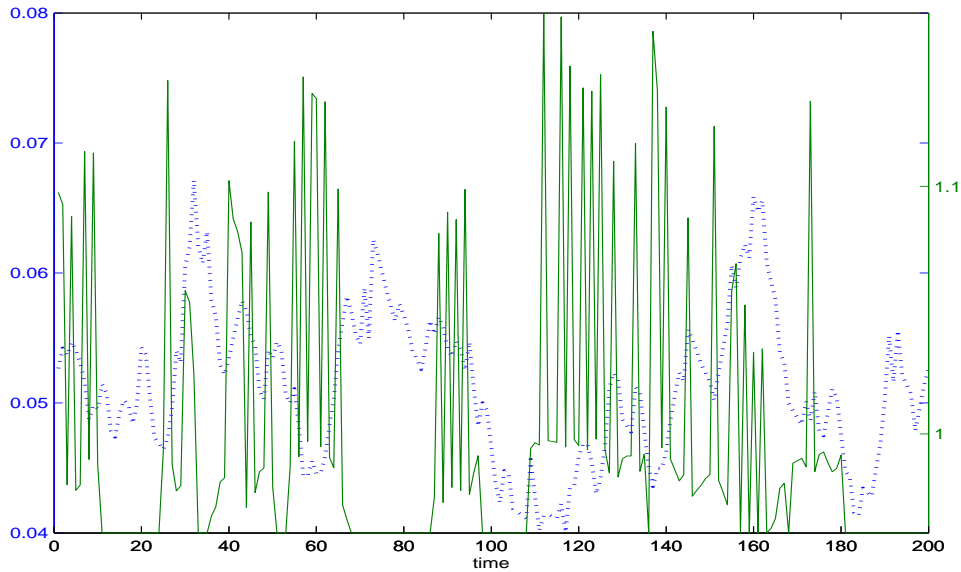


composition risk is likely to be small. The second section focusses on $g_{t+1}$ and it examines the
liquidity risk factor contributed by the collateral channel more carefully.

## 3.1   Composition Effect

Without the collateral constraints, ours is a representative agent economy. If utility is non-
separable, the housing market introduces a novel risk factor: shocks to the non-housing expen-
diture share. The representative agent's marginal utility growth is determined by aggregate
consumption growth and non-housing expenditure share growth:

$$m_{t+1}^a = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\gamma} \left( \frac{\alpha_{t+1}}{\alpha_t} \right)^{\frac{\varepsilon - \frac{1}{\gamma}}{\frac{1}{\gamma}(\varepsilon - 1)}}$$

where $\alpha_t$ is the non-housing expenditure share, $\gamma$ is the coefficient of relative risk aversion
and $\varepsilon$ is the intratemporal elasticity of substitution between housing services and non-durable

10

consumption.

We simply apply this stochastic discount factor to the data. We use the annual personal consumption expenditures data from the BEA (1929-2002) to compute the expenditure shares. Consumption is non-durable consumption measured by total expenditures on non-durable goods and services minus apparel and minus rent and imputed rent. The non-housing expenditure share is the ratio of non-durable consumption to rent expenditures and non-durable consumption.

Composition risk is small. The standard deviation of aggregate expenditure share growth is .7 percent between 1929 and 2002, much smaller than the standard deviation of aggregate consumption growth. A large power coefficient is needed on the expenditure share growth term for composition risk to matter. Piazzesi et al. (2002) (henceforth PST) show that only values for the elasticity of intratemporal substitution $\varepsilon$ close to but slightly larger than one, and low values for the intertemporal elasticity deliver a volatile SDF. The power coefficient on the expenditure share growth rate reaches values of 85 and higher. In this range, the model delivers a large market price of risk of about 1.3, at the cost of overstating the volatility of rental price growth by a factor ranging from 3 when $\varepsilon$ is 1.05 to 15 when $\varepsilon$ is 1.01.[2] In addition, our estimates of $\varepsilon$ are always smaller than 1, implying a much smaller market price of composition risk (see Table 10 in the Appendix).[3]

Second, in this region of the parameter space, composition risk is "good times" risk and its market price is negative. The SDF is volatile, but it is not the right kind of volatility, because expenditure share growth and consumption growth are positively correlated -the correlation is .46 over the entire sample. Marginal utility growth increases in expansions, when aggregate consumption growth is high, rather than in recessions, because the composition risk effects swamps the consumption risk effect. The model implies a large but *negative* equity premium of 4 percent over the entire sample, because equity provides an effective hedge against this "good times" risk and the market imputes a negative price to composition risk.[4]

Finally, this composition effect generates little or no variation in the conditional Sharpe ratio's on risky assets. The conditional volatility of the SDF is roughly constant. PST introduce heteroskedasticity in the expenditure shares to address this defect of the model, but is hard

---

[2]Put differently, if we would have restated the SDF in terms of rental price growth and applied the actual rental price data to this calibration, the new risk factor would have contributed very little because the implied expenditure shares would have been roughly constant. In fact, when $\varepsilon$ is 1, the expenditure shares are constant and there is no composition risk.

[3]The value of $\varepsilon$ implied by the PST estimates for the cross-sectional regressions is also smaller than one.

[4]The equity premium is computed as $\frac{-cov(m,R^e)}{E(m)} = \frac{-0.0463}{1.1314}$ using the sample moments of the SDF and the annual value-weighted return on the market less the annualized return on 1-month T-bill , where $\varepsilon = 1.05$, $\beta = .99$ and $\gamma = 5$. For $\varepsilon = 1.01$ the moments cannot be computed.

to conceive of preferences and/or a technology that would deliver this feature in equilibrium. Our collateral channel generates endogenous time-variation in the conditional moments of the pricing kernel.

We conclude that the share of risk premia that is due to composition risk is likely to be small for plausible preferences, because (1) volatile SDF's can only be generated by preferences that generate too much rental price volatility and because (2) composition risk is "good times" risk, at least in the region of the parameter space that delivers volatile SDF's.

## 3.2   Liquidity Effect and the Collateral Channel

We now focus on the case of separable utility. The representative agent's IMRS $m^a$ is the aggregate consumption growth rate raised to the power $-\gamma$ and the SDF reduces to:

$$m_{t+1} = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\gamma} g_{t+1}^\gamma$$

For the liquidity effect to increase the volatility of the SDF, the liquidity factor needs to be negatively correlated with aggregate consumption growth. There are two features that deliver a negative correlation between aggregate consumption growth and the aggregate liquidity shock: (1) an increase in the cross-sectional dispersion of labor income shocks, and (2) a decrease in the amount of collateral, both when aggregate consumption growth is low. The first one relies on the time series properties of labor income in the US, the second one on the time series properties of rental prices in the housing market. Both of these channels amplify the effect of aggregate consumption growth shocks on the SDF.

**Dispersion of Labor Income Shocks**   Constantinides & Duffie (1996) build a negative correlation between the dispersion of consumption growth across households and aggregate stock returns in their model to generate large risk premia, drawing on earlier work by Mankiw (1986). The first channel in our model is a different version of this (see Lustig (2001)). It delivers a negative correlation between the standard deviation of the consumption growth distribution and stock returns, but this correlation is the equilibrium outcome of the interaction between the solvency constraints and the time series properties of the labor income process. A larger fraction of agents draws higher labor income shares $\widehat{\eta}(y, z)$ when aggregate consumption growth is low and, as a result of the persistence of labor income shocks, higher cutoff levels $l^c(y, z^t)$. This increases the size of the aggregate weight shock in low aggregate consumption growth states. There is

12

some empirical support for this channel. Storesletten, Telmer, & Yaron (2003) conclude that the volatility of idiosyncratic labor income shocks in the US more than doubles in recessions.

**Collateral Supply Shocks**   If the rental price of housing services declines in response to a negative aggregate consumption growth shocks, liquidity shocks will tend to be larger when aggregate consumption growth is low, because the destruction of collateral tightens the solvency constraints.  In the US rental prices increase in response to a positive aggregate consumption growth in the post-war sample shocks while the expenditure ratio $z$, increases as well, both in the post-war sample and over the entire sample.  Table 9 in the Appendix list the regression results.

**Time-Varying Market Price of Risk**   The housing collateral ratio governs the amount of risk sharing that can be sustained and variations in the ratio endogenously generate heteroskedasticity in the SDF. Low housing collateral ratios coincide with a high conditional volatility of the SDF, because a large fraction of households will be severely constrained in case of an adverse aggregate consumption growth shock.  This mechanism leaves a huge footprint in the cross-sectional standard deviation of consumption growth, plotted in Figure 3.

The next section concentrates on measuring the US housing collateral ratio directly.  Our measure reveals a surprising amount of historical variation that is consistent with the variation in US stock returns.

## 4   Time Series Evidence

In testing the model, we chose to measure the housing collateral ratio $my$ directly, simply because forces outside our model probably influence housing prices.  In the model, both the housing collateral ratio and the rental price or the expenditure share are valid state variables. The first section concentrates on its measurement.  The second section confronts our measure with stock returns.

### 4.1   Measuring the Housing Collateral Ratio

$my$ is defined as the ratio of collateralizable housing wealth to non-collateralizable human wealth. Human wealth is unobserved. Following Lettau & Ludvigson (2001a), we assume that the non-stationary component of human wealth $H$ is well approximated by the non-stationary component of labor income $Y$. In particular, $\log(H_t) = \log(Y_t) + \epsilon_t$, where $\epsilon_t$ is a stationary random process.

Figure 3: The Standard Deviation of Consumption Growth, Liquidity Shocks and $my$ The dotted line is the collateral ratio: $\beta$ is .95, $\gamma$ is 8 and $\varepsilon$ is .15l ratio: $\beta$ is .95. The left panel plots $my$ against the cross-sectional std. of consumption growth (right axis). The right panel plots $my$ against the aggregate liquidity shock $g$ (right axis).
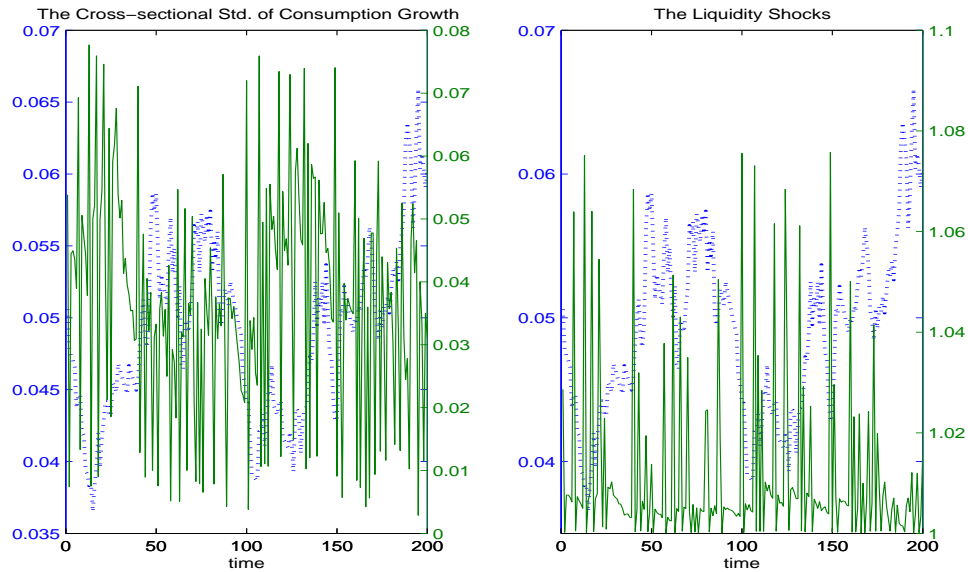


The assumption is valid in a model in which the expected return on human capital is stationary (see Jagannathan & Wang (1996) and Campbell (1996)).

**Housing Collateral** We use three distinct measures of the housing collateral stock $HV$: the value of outstanding home mortgages ($mo$), the market value of residential real estate wealth ($rw$) and the net stock current cost value of owner-occupied and tenant occupied residential fixed assets ($fa$). The first two time series are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds data (Federal Board of Governors) for 1945-2001. The last series is from the Fixed Asset Tables (Bureau of Economic Analysis) for 1925-2001.

We use both the value of mortgages $HV^{mo}$ and the total value of residential fixed assets $HV^{rw}$ to allow for changes in the extent to which housing can be used as a collateral asset, and we use both $HV^{rw}$, which is a measure of the value of housing owned by households, and $HV^{fa}$ which is a measure of the value of housing households live in, to allow for changes in the

14

home-ownership rate over time. Appendix A.3 provides detailed sources. Real per household variables are denoted by lower case letters. The real, per household housing collateral series $hv^{mo}, hv^{rw}, hv^{fa}$ are constructed using the all items CPI from the BLS, $p^a$, and the total number of households, $N$, from the Bureau of the Census.

**Income**  Aggregate income is labor income plus net transfer income. Nominal data are from the Historical Statistics of the US for 1926-1930 and from the National Income and Product Accounts for 1930-2001. Consumption and income are deflated by $p^c$ and $p^a$ and divided by the number of households $N$.

**Cointegration**  Log, real, per household real estate wealth $(\log hv)$ and labor income plus transfers $(\log y)$ are non-stationary. According to an augmented Dickey-Fuller test, the null hypothesis of a unit root cannot be rejected at the 1 percent level. This is true for all three measures of housing wealth $(hv = mo, rw, fa)$.

If a linear combination of $\log hv$ and $\log y$, $\log(hv_t) + \varpi \log(y_t) + \chi$ , is trend stationary, the components $\log hv$ and $\log y$ are said to be stochastically cointegrated with cointegrating vector $[1, \varpi, \chi]$. We additionally impose the restriction that the cointegrating vector eliminates the deterministic trends, so that $\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi$ is stationary. A likelihood-ratio test (Johansen & Juselius (1990)) shows that the null hypothesis of no cointegration relationship can be rejected, whereas the null hypothesis of one cointegration relationship cannot. This is evidence for one cointegration relationship between housing collateral and labor income plus transfers. Table 12 reports the results of this test and of the vector error correction estimation of the cointegration coefficients:

$$\begin{bmatrix} \Delta \log(hv_t) \\ \Delta \log(y_t) \end{bmatrix} = \alpha \left[ \log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi \right] + \sum_{k=1}^{K} D_k \begin{bmatrix} \Delta \log(hv_{t-k}) \\ \Delta \log(y_{t-k}) \end{bmatrix} + \varepsilon_t. \qquad (2)$$

The $K$ error correction terms are included to eliminate the effect of regressor endogeneity on the distribution of the least squares estimators of $[1, \varpi, \vartheta, \chi]$. The housing collateral ratio $my$ is measured as the deviation from the cointegration relationship:

$$my_t = \log(hv_t) + \hat{\varpi} \log(y_t) + \hat{\vartheta} t + \hat{\chi}.$$

The OLS estimators of the cointegration parameters are superconsistent: They converge to their true value at rate $1/T$ (rather than $1/\sqrt{T}$). The superconsistency allows us to use the

15

housing collateral ratio $my$ as a regressor without need for an errors-in-variables standard error correction.

We also estimate the constant and trend in the cointegrating relationship while imposing the restriction $\varpi = -1$. This is the second block of each panel in table 12. For $mo$ and $fa$, we find strong evidence for one cointegrating relationship. The coefficient on $\log y_t$ is precisely estimated (significant at the 1 percent level, not reported), varies little between subperiods, and the 95 percent confidence interval contains -1. The resulting time-series are stationary. The null hypothesis of a unit root is rejected for $mymo$ and $myfa$. For each subperiod, the correlation between the residual estimated assuming $\varpi = -1$ and the one with $\varpi$ freely estimated is higher than 0.95. For $rw$, the evidence for a cointegrating relationship is weaker, except for the 1925-2002 period. Furthermore, the slope coefficient in the cointegration relationship varies considerably between subperiods and does not contain -1 in its 95 percent confidence interval. The correlation between the residual estimated assuming $\varpi = -1$ and the one with $\varpi$ freely estimated is 0.81 for the entire sample, 0.88 for 1925-2002 and 0.89 for the post-war period.

For consistency we impose $\varpi = -1$ on all three of these series. The housing collateral ratios are labelled $mymo$, $myrw$ and $myfa$. For the common sample period 1925-2001, the correlation between $mymo$ and $myrw$ is 0.89, 0.76 between $mymo$ and $myfa$ and 0.86 between $myrw$ and $myfa$. Figure 4 displays $my$ between 1889 and 2002. All three series exhibit large persistent swings. They reach a maximum deviation in 1932-33. Residential wealth and residential fixed assets are 30 and 34 percent above their respective joint trends with human wealth. Mortgage debt is 53 percent above its trend. The series reach a minimum in 1944-45, when $mymo$ is $-.92$, $myrw$ is $-.57$ and $myfa$ is $-.38$. $mymo$ and $myrw$ have increased considerably since the year 2000: from .24 to .36 and from 0.19 to 0.30 respectively. Figure 5 shows the cointegration residuals $my$ for that post-war period. Housing collateral wealth fluctuates within 30 percent below and above the long-run trend with human wealth.[5]

In section 4.2 we provide evidence that the housing collateral ratio predicts stock returns. This suggests that the market price of risk is indeed a function of $my$.

---

[5]When housing wealth deviates from its long-run ratio with labor income, the equilibrium relationship is restored by transitory movements in both housing wealth and labor income. Table 13 (in appendix B) shows the estimation results of a bivariate vector autoregression of changes in housing wealth and labor income. The lagged housing collateral ratio, $my_{t-1}$, is an exogenous regressor. The coefficients on $my_{t-1}$ in both equations have about the same size (and opposite signs). This suggests that the transitory return to the common trend is achieved by both variables.

16

Figure 4: Housing Collateral Ratio 1889-2002. Measured by outstanding home mortgages ($mo$), non-farm residential wealth ($rw$) and residential fixed asset wealth ($fa$) relative to human wealth ($y$)
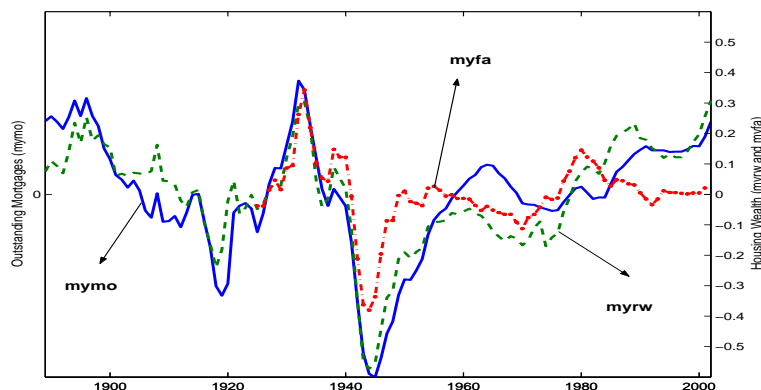


Figure 5: Housing Collateral Ratio 1945-2002. Measured by outstanding home mortgages ($mo$), non-farm residential wealth ($rw$) and residential fixed asset wealth ($fa$) relative to human wealth ($y$)
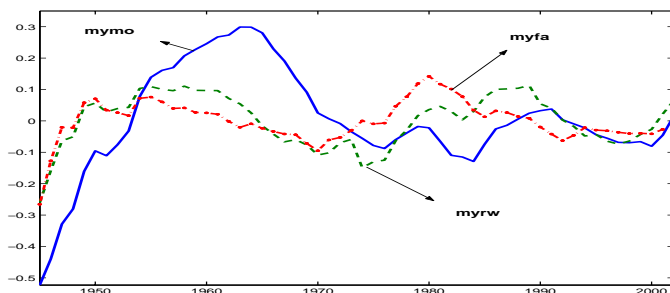


## 4.2 Time-Series Predictability

The model predicts that the reward for aggregate risk is higher when housing collateral is scarce. We document this negative relationship for the US market return.

**Market Return** Our market return is the cum-dividend return on the Standard and Poor's composite stock price index. The market return is expressed in excess of a risk-free rate, the annual return on six-month prime commercial paper. The returns are available for the period 1889-2001 from Robert Shiller's web site.

17

**VAR**   A bivariate vector autoregression of one-year excess returns on the aggregate stock market and the housing collateral ratio provides a first look at the predictability question. We study the response of excess returns to an innovation in $my$. Figure 10 plots the impulse-reponse of the excess return to a one-standard-deviation innovation in $myfa$, the $my$ measure for fixed assets. The initial drop in the equity risk premium is followed by a further decrease which persists for multiple years.[6] The effect is large: a 4 percentage point innovation to $myfa$ causes a 2.4 percentage point decrease in the equity risk premium. Between 1941 and 1942, $myfa$ declined by 20 percentage points in one year. Our estimates reveal a 12 percentage point increase in the risk premium! [7]

**Long Horizon Predictability**   We also project long-horizon excess returns on the housing collateral ratio. The $K$-year continuously compounded excess return is defined as $r_{t+K}^{vw,K} = (r_{t+1}^{vw,1} + ... + r_{t+K}^{vw,1})$ where $r_t^{vw,1}$ equals $\log(1 + R_t^{vw,e})$. Figure 6 shows the housing collateral ratio ($mymo$) and the annualized ten-year excess return. The series exhibit a negative correlation of $-0.52$. Regressions of the one- to ten-year cumulative stock returns on the housing collateral ratio ($mymo$) provide further evidence of predictability.

Figure 6: 10-year Excess Market Return and the Collateral ratio $mymo$.
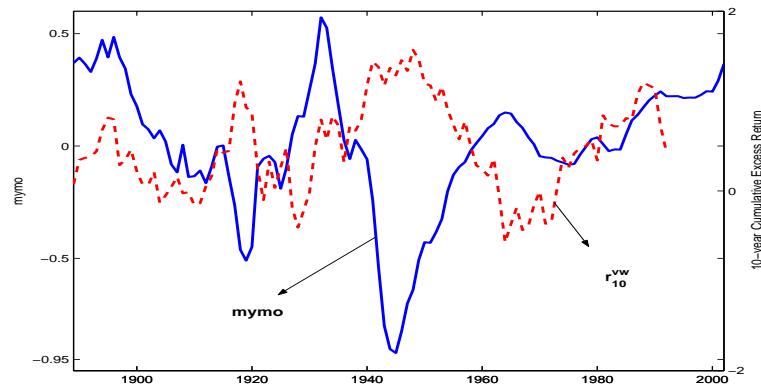


Row 1 of table 1 reports the least squares coefficient estimate on the housing collateral ratio

---

[6]The optimal lag length for the VAR is two years according to the Akaike Information criterion. The covariance matrix of innovations has small off-diagonal elements, i.e. their innovations have a small common component. Therefore, changing the ordering of the variables $R^{vw,e}$ and $my$ in the VAR does not affect the impulse-responses.

[7]The impulse responses for the two housing collateral measures $mymo$ and $myrw$ show a similar pattern. They are available upon request.

| 89-02 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $b_{my}^{LS}$ | .09 | .18 | .26 | .33 | .42 | .54* | .70** | .87** | 1.04*** | 1.14*** |
| $R^2$ | .01 | .02 | .03 | .03 | .04 | .07 | .10 | .14 | .18 | .20 |
| $b_{my}^{ss}$ | .08 | .16 | .23 | .29 | .38 | .49 | .64 | .80 | .97 | 1.07 |
| $p$ | [.14] | [.14] | [.15] | [.16] | [.15] | [.13] | [.11] | [.08] | [.07] | [.06] |
| 45-02 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $b_{my}^{LS}$ | -.01 | .08 | .23 | .41** | .62*** | .81*** | 1.00*** | 1.22*** | 1.49*** | 1.71*** |
| $R^2$ | .00 | .00 | .03 | .08 | .13 | .18 | .23 | .29 | .37 | .43 |
| $b_{my}^{ss}$ | .00 | .11 | .28 | .46 | .68 | .88 | 1.08 | 1.32 | 1.59 | 1.83 |
| $p$ | [.58] | [.43] | [.32] | [.25] | [.19] | [.16] | [.14] | [.11] | [.09] | [.07] |

Table 1: Long-Horizon Predictability Regressions. The results are for the regression $R_{t+K}^{vw,e} = b_0 + b_{my}\left(\frac{my^{max}-my_t}{my^{max}-my^{min}}\right) + \epsilon_{t+K}$, where $R_{t+K}^{vw,e}$ are cumulative excess returns on the S&P Composite Index over a $K$-year horizon. The housing collateral ratio $my$ is $mymo$, estimated on the entire sample in the first panel and on the postwar sample in the second panel. $my^{max}$ and $my^{min}$ are the maximum and minimum observation on $mymo$ in the respective samples. The first row reports least squares estimates for $b_{my}$. Newey-West HAC standard errors are used to denote significance at the 1 percent (***), 5 percent (**) and 10 percent level (*). The second row reports the $R^2$ for this OLS regression. The third row reports small sample coefficient estimates (see A.1). The fourth row gives the p-value of the null hypothesis of no predictability. For 1889-2002, the sample size decreases from 113 observations for K=1 to 104 years for K=10. For 1945-2002, it decreases from 57 observations for K=1 to 48 years for K=10.

for the period 1889-2001 and row 5 contains the estimates for the postwar period. All of the slope coefficients are positive. A low housing collateral ratio predicts a high future risk premium as predicted by the model . The $R^2$ of the least-squares regression increases with the horizon, to 20 percent for the entire sample and 43 percent in the postwar period (row 5, $k = 10$). There are two econometric problems with the ordinary least squares regression:

$$r_{t+K}^{vw,K} = b_0 + b_{my}my_t + e_{t+1}. \tag{3}$$

First, because the forecasting variable $my$ is a slow-moving process, the least squares estimator of the coefficient on $my$, $b_{my}^{LS}$, suffers from persistent regressor bias in small samples (Stambaugh (1999)). Second, because $r_{t+K}^{vw,K}$ contains overlapping observations, the standard errors on $b_{my}^{LS}$ need to be corrected for serial correlation in the residuals $e$. [8] To address the persistent regressor bias and the serial correlation issues we conduct a bootstrap exercise, detailed in appendix A.1. The small-sample coefficient estimates generated by bootstrapping -rows 3 and 7 in table 1- are positive at every horizon, slightly lower than the least squares estimates in the entire sample, but slightly higher in the post-war sample. Rows 4 and 8 show the $p$-value of a two-sided test statistic of the no-predictability null, generated by bootstrap. The statistic measures one minus the likelihood of observing the least squares coefficient estimate when returns are in fact unpredictable. For $K \geq 5$, there is evidence against the null-hypothesis at the 15 percent level.

---

[8] The asymptotic corrections advocated by Hansen & Hodrick (1980) have poor small sample properties. Ang & Bekaert (2001) find that use of those standard errors leads to over-rejection of the no-predictability null.

**Comparison Across Models** Many financial and macroeconomic variables have forecasting power for the market return. The dividend-price and dividend-earnings ratio, the treasury bill rate, the term spread between long-term government bonds and treasury bills and the default spread between low- and high-grade corporate bonds are financial variables with forecasting power for excess returns. A subset of those variables, such as the investment-capital ratio (Cochrane (1991a)), the consumption-wealth ratio, $cay$, (Lettau & Ludvigson (2001a)) and the labor income - consumption ratio, $lc$, (Santos & Veronesi (2001)), are macroeconomic variables.

We compare the return-forecasting ability of the housing collateral ratio with that of the dividend yield ($\log dp$), the consumption-wealth ratio ($cay$), and the labor income-consumption ratio ($lc$). The two samples we consider are (1) the longest available (1926-2002) and (2) the postwar sample (1945-2002). Table 14 displays the results for the long-horizon regressions. The dividend yield is a strong predictor of excess returns in both samples (lines 2 and 9). For $K \geq 5$, the forecasting power of the housing collateral ratio is almost as strong (lines 1 and 8). Lines 5 and 12 show that the housing collateral ratio contains information that is relevant for predicting returns beyond what is included in the dividend yield. Both coefficients remain jointly significant for $K > 5$. The $R^2$ goes up to 57 percent in the postwar period for 10-year returns, 13 percent more than in each of the individual regressions.

Lettau & Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption to wealth ratio as conditioning variable. The ratio is measured as the deviation from the common trend in consumption, labor income and financial wealth ($cay$).[9] Periods with high $cay$ indicate high expected future returns, thereby rationalizing a high propensity to consume out of wealth. In the representative agent economy of Santos & Veronesi (2001), the ratio of labor income to consumption $lc$ predicts stock returns.[10] Times in which investors finance a large fraction of consumption out of labor income rather than out of stock dividend income ($lc$ is high), are less risky. For the entire sample, $cay$ and $lc$ have no forecasting ability (lines 3 and 4). Moreover, the coefficients have the wrong sign. This problem goes away in the postwar period (lines 10 and 11). When both the housing collateral ratio and the labor income to consumption ratio are included, $lc$ has the right sign for $K \geq 5$ (line 6). The coefficient on the

---

[9]We construct the $cay$ variable for the period 1926-2001 using log real per household total consumption expenditures ($c$), log real per household labor income plus transfers ($ylt$) and log real per household financial wealth ($fw$). We find evidence for one cointegration relationship between the three variables. The estimated relationship we find with annual data is $cay = \log c - .259 \log fw - .682 \log y + 0.035$, estimated with one lag. For the postwar period we use the cay variable constructed by Lettau and Ludvigson for the period 1948-2001: $cay = c - 0.292 fw - 0.597 y$.

[10]We construct their scaling variable as the ratio of annual labor income to total consumption expenditures, for the period 1926-2001.

housing collateral ratio remains significant and the $R^2$ of the regression increases to 47 percent. In the postwar period, $my$ and $lc$ enter jointly significantly and explain up to 54 percent of the time-series variation in excess returns (line 13). The highest predictive power is obtained when all three variables $my$, $lc$, and $dp$ are included (lines 7 and 14). In the postwar period the $R^2$ goes up to 64 percent for $K = 10$, and all three coefficients are measured precisely.

# 5 Cross-sectional Evidence: Non-linear Factor Model

We use returns on stock portfolios sorted by size and value characteristics, bond returns and the return on a risk-free asset to test our model. Size and book-to-market value are asset characteristics that challenge the standard CCAPM. Historically, small firm stocks and high book-to-market firm stocks have had much higher returns. In the post-war period, the size premium has largely disappeared, but the value premium is still prominent. The value premium -the average return difference between the lowest and the highest book-to-market decile- is 5.7 percent over the entire sample.

**Size and Book-to-Market Portfolios**   A total of twenty-five portfolios of NYSE, NASDAQ and AMEX stocks are grouped each year into five size bins and five value (book-to-market ratio) bins. Size is market capitalization at the end of June. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. Portfolio returns are value-weighted. We also include the market return $R^{vw}$, the value-weighted return on all NYSE, AMEX and NASDAQ stocks. All returns are expressed in excess of an annual return on a one-month Treasury bill rate (from Ibbotson Associates). The returns are available for the period 1926-2001 from Kenneth French's web site and are described in more detail in Fama & French (1992). The first column of table 11 shows mean and standard deviation for the 26 excess returns.

## 5.1 Measuring the Liquidity Factor

In the model, the aggregate weight shock depends on the entire history of aggregate shocks $z^\infty$ and $my_0$. To solve the model numerically, we rely on an approximation of $g(z^\infty, my_0)$, the

growth rate of the aggregate weight process, using a truncated history of aggregate shocks and the current $my_t$.[11]

To bring the model to the data, we take a similar approach and use a flexible, non-linear function of the relevant state variables to parametrize the investor's forecast of aggregate weight growth:

$$log(g_t(z_t^\infty, my_0)) \simeq \phi(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a; my_t),$$

where $F_t^a$ denotes the vector of aggregate factors $F_{t+1}^a = (\Delta \log(c_{t+1}^a), \Delta \log(\alpha_{t+1}))'$, consisting of aggregate consumption growth and expenditure share growth. We use GMM to identify the function $\phi$ in addition to the structural parameters from the moment conditions:

$$E_t[m_{t+1}^a exp(\gamma * \phi(F_{t+1}^a, F_t^a, \ldots, F_{t-k}^a; my_{t+1}))R_{t+1}^j] = 1,$$

where $R_{t+1}^j, j = 1, \ldots, n$ are the returns on the test assets. In addition, the theory imposes additional inequality constraints (1) $\phi(F_{t+1}^a, F_t^a, \ldots, F_{t-k}^a; my_{t+1}) \geq 0$ and (2) $E_t[m_{t+1}^a R_{t+1}^j] \leq 1$ that further restrict the set of feasible parameters. (1) follows from the fact that $\xi_t^a(z^t)$ is a non-decreasing process and this immediately implies (2). The second set of inequality constraints will prove useful in identifying the structural parameter $\gamma$.[12]

## 5.2 Non-Linear Factor Model and GMM

We use Chebyshev orthogonal polynomials to approximate the non-linear $\phi$-function. The advantage is that the different terms of the polynomial have the usual interpretation as linear pricing factors (Chapman (1997)).

**Moment Restrictions** The first moments are the average pricing errors for the test asset returns and the risk-free rate:

$$E[m_{t+1}R_{t+1}^j - 1] = 0$$

The theory tells us that the aggregate weight shock $g_{t+1}$ is exactly equal to one when the constraints do not bind and strictly greater than one in all other periods. This delivers two additional restrictions. First, we impose parametric restrictions on the polynomial such that

---

[11]This is discussed in Lustig & VanNieuwerburgh (2003). These approximations work well, because the supply of collateral is distributed efficiently across households. The percentage forecast error has a natural interpretation: it equals the percentage deviation between aggregate consumption and the aggregate endowment.

[12]Luttmer (1991) exploits this restriction to derive Hansen-Jagannathan bounds in an environment with solvency constraints .

$\phi(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a; my_t) = 0$ when $my$ equals $my^{max}$. In particular, we restrict ourselves to functions of the form $\phi(\cdot) = (my^{max} - my_{t-1}) * \widetilde{\phi}(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a)$.[13]

Second, it is useful to impose the inequality restrictions $E[m_{t+1}^a R_{t+1}^i - 1] \leq 0$ on the representative agent's Euler equations by adding the Kuhn-Tucker moment conditions to the standard moment conditions:

$$\lambda(\theta) E[m_{t+1}^a R_{t+1}^j - 1] = 0$$

We adopt the penalty function approach by parametrizing the Lagrangian multiplier $\lambda$ as $exp(cE[m_{t+1}^a R_{t+1}^j - 1])$ for a positive scalar $c$. [14] Without these inequality constraints, $\gamma$, which features multiplicatively in both components of the SDF, cannot be separately identified. In a first pass, we estimate the model using four different assets. In a second stage, we expand the set of test assets.

**Results with Four Test assets** First, we estimate our model using only four test assets: the risk-free rate, the value weighted market return, the 10 year bond return and the return on a portfolio that goes long in value and short in growth (the Fama-French benchmark portfolio $R^{HML}$). We use annual real, gross holding period returns from 1926 until 2002 (77 observations).[15] There are four standard moment conditions and four Kuhn-Tucker moment conditions, adding up to a total of eight moment conditions. We use the identity matrix as a weighting matrix. The Chebyshev polynomial is restricted to be first order. This restriction is tested in the next subsection.

Table 2 shows how the coefficient estimates converge to low risk aversion estimates and significantly negative $\theta_2$ estimates as the penalty parameter is increased. The representative agent's Euler inequalities rule out high $\gamma$ estimates.

The coefficient estimates lend support to the collateral channel. The estimate for $\hat{\theta}_1$ implies that periods with less collateral coincide with a high value for the aggregate weight shock and SDF. The estimate for $\hat{\theta}_2$ implies that, when aggregate consumption growth is below average (the rescaled consumption growth is negative), the aggregate weight shock is large. This effect increases as collateral becomes scarcer (higher $my^{max} - my_t$) as predicted by the model. The average pricing errors are small and the null hypothesis that all pricing errors are zero *cannot*

---

[13]We cannot allow the aggregate weight shock to be a function of aggregate consumption growth in isolation, because that would preclude identification of $\gamma$.

[14]To solve saddle point problems numerically, the shadow price of a binding constraint is usually approximated by the product of the penalty parameter and the constraint violation, see Judd (1998) theorem 4.7.1. The algorithm involves increasing the penalty parameter until convergence is achieved.

[15]Results with quarterly data for 1952.1:2002.4 are also available.

be rejected.

| $c$ | 10 | 15 | 20 | 30 | 40 | 60 | 80 | 100 | 150 | 300 | 500 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\gamma}$ | 23.48 | 13.23 | 8.83 | 5.62 | 4.13 | 2.84 | 2.23 | 1.84 | 1.24 | 1.24 | 1.24 |
| s.e. | (16.32) | (9.34) | (5.79) | (4.40) | (3.46) | (2.67) | (2.52) | (5.52) | (.78) | (.75) | (.72) |
| $\hat{\theta}_1$ | .13 | .18 | .24 | .34 | .44 | .62 | .77 | .92 | 1.34 | 1.34 | 1.34 |
| s.e. | (.05) | (.09) | (.14) | (.25) | (.36) | (.59) | (.880) | (2.66) | (.74) | (.71) | (.68) |
| $\hat{\theta}_2$ | -.34 | -.60 | -.90 | -1.42 | -1.93 | -2.80 | -3.56 | -4.30 | -6.41 | -6.40 | -6.39 |
| s.e. | (.39) | (.74) | (1.09) | (1.89) | (2.68) | (4.20) | (6.00) | (15.16) | (4.14) | (4.05) | (3.96) |
| $J$ | 31.56 | 9.68 | 3.83 | 1.61 | 1.16 | .94 | .89 | .89 | .79 | .40 | .27 |
| $p$ | .000 | .0847 | .5748 | .8999 | .9487 | .9671 | .9729 | .9708 | .9779 | .9953 | .9982 |

Table 2: GMM Coefficient Estimates for separable Coll-CAPM. These estimates were obtained using 4 moment conditions and 4 inequality conditions. The Chebyshev polynomial is first order. $\beta$ was fixed at .95. The identity weighting matrix was used in estimating the model. We use the Newey West procedure with 3 lags which corrects for serial correlation and heteroskedasticity. We use a repeated GMM procedure, allowing for ten iterations.

**Results with Seven Test Assets**  Next, we use a more extensive set of seven test assets: a three month T-bill, a 10 year government bond, the value weighted aggregate stock market and the four extreme size and value portfolios, in particular S1B1, S1B5, S5B1, and S5B5. This adds up to a total of fourteen moment conditions. The order of the Chebyshev polynomial varies from 1 to 3.

The left panel of Table 3 reports the estimates for the collateral model with separable preferences. In the linear specification of the aggregate weight shock all coefficients are estimated precisely and the point estimates are not very different from the ones we reported in the case of four test assets. This model dominates the ones with higher order terms in the polynomial on the basis of a likelihood ratio test, a Wald test and a Lagrange multiplier test; the null hypothesis that all higher order polynomial coefficients are zero cannot be rejected. [16]

**Comparing Pricing Errors**  With seven test assets, the null that pricing errors are zero is rejected on the basis of its J-statistic, except in the non-separable case with a first order polynomial. This model *cannot* be rejected. The Collateral-CAPM does much better than the other models we consider for all of the test assets (see Table 4). Only the small value portfolio (S1B5) is still priced poorly by the collateral model with separability (Coll-CAPM 1), although the collateral model without separability (Coll-CAPM 2) reduces this error to 2.5 percent. Coll-CAPM 3 allows for history dependence by including lagged consumption growth in $\phi$. This does

---

[16]When the unrestricted model is the second order and the restricted model is the first order polynomial model, the p-values are : *polynomial order 1 vs 2*: LHR 1.00, Wald .93, LM .91 *polynomial order 1 vs 3*: LHR 1.00, Wald .70, LM .76. We conclude that the first order polynomial specification for the aggregate weight shock is the best fitting one.

| Model | Separability | | | Non-Separability | | |
|---|---|---|---|---|---|---|
| Polynomial Order | 1 | 2 | 3 | 1 | 2 | 3 |
| $\hat{\gamma}$ | 4.78 | 4.96 | 4.67 | 2.07 | 1.35 | 1.35 |
| s.e. | [1.72] | [1.68] | [1.58] | [.32] | [.19] | [.20] |
| $\hat{\theta}_1$ | .37 | .63 | -.31 | -.15 | 7.31 | .44 |
| s.e. | [.31] | [3.15] | [2.77] | [2.43] | [3.96] | [6.78] |
| $\hat{\theta}_2$ | -2.00 | -1.93 | -6.61 | -5.16 | -8.14 | 6.44 |
| s.e. | [.92] | [.85] | [13.46] | [1.78] | [4.87] | [7.40] |
| $\hat{\theta}_3$ | | .33 | -.77 | -3.67 | 3.63 | .23 |
| s.e. | | [4.05] | [3.16] | [8.31] | [28.37] | [10.14] |
| $\hat{\theta}_4$ | | | -1.69 | | -.92 | -6.59 |
| s.e. | | | [5.26] | | [19.73] | [5.83] |
| $\hat{\theta}_5$ | | | | | 7.56 | 6.53 |
| s.e. | | | | | [ 14.21] | [3.13] |
| $\hat{\theta}_6$ | | | | | | 7.53 |
| s.e. | | | | | | [7.95] |
| $\hat{\theta}_7$ | | | | | | -.70 |
| s.e. | | | | | | [13.37] |
| $J$ | 33.68 | 110.37 | 56.31 | 14.18 | 60.36 | 54.15 |
| $p$ | .0000 | .0000 | .0000 | .1671 | .0000 | .0000 |

Table 3: GMM Coefficient Estimates for Coll-CAPM. These estimates were obtained using 7 moment conditions and 7 inequality conditions. $\beta$ was fixed at .95. The left panel imposes separability by fixing $\epsilon$ at 0, wile the left panel fixes $\epsilon$ at .85. In the separable case, the penalty parameter $c$ is 100 in columns 1, 12 in column 2 and 11 in column 3, while in the non-separable case, the penalty parameter $c$ is 300 in column 1, and 2 in columns 2 and 3. The identity weighting matrix was used in estimating the model. We use the Newey West procedure with 3 lags which corrects for serial correlation and heteroskedasticity. We use a repeated GMM procedure, allowing for ten iterations.

not produce significant improvements.

By contrast, the CCAPM does a poor job at pricing the risk free rate and the long-term bond, while massively overpricing growth portfolios and underpricing value portfolios.[17]

Since the higher order terms in the polynomials are not significant, we choose to impose linearity on $\widetilde{\phi}$. This allows us to make contact with the linear factor model literature, and it also allows us to increase the number of test assets in the estimation stage.

# 6 Cross-sectional Evidence: Linear Factor Model

First, we assume $\widetilde{\phi}$ is linear in $(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a)$, and, second, we assume that the housing collateral ratio follows an autoregressive process $my_{t+1}(my_t, F_{t+1}^a)$ that interacts linearly with the aggregate factors. The innovations to the aggregate factors are the structural innovations

---
[17]Table 10 in the Appendix reports the coefficient estimates for the CCPAM and the HCAPM

|  | C-CAPM | H-CAPM | Coll-CAPM 1 | Coll-CAPM 2 | Coll-CAPM 3 |
|---|---|---|---|---|---|
| $R^f$ | -.069 | -.057 | .016 | .006 | .015 |
| $(t-stat)$ | (-3.11) | (-3.78) | (.68) | (.30) | (3.38) |
| $R^{vw}$ | -.016 | -.000 | -.010 | -.002 | -.010 |
| $(t-stat)$ | (-0.85) | (-.04) | (-.73) | (-.25) | (-1.38) |
| $R^{S1B1}$ | -.060 | -.042 | -.017 | .009 | -.011 |
| $(t-stat)$ | (-1.53) | (-1.68) | (-.19) | (.23) | (-.78) |
| $R^{S1B5}$ | .070 | .078 | .042 | .024 | .043 |
| $(t-stat)$ | (1.83) | (-3.44) | (1.71) | (.39) | (1.42) |
| $R^{S5B1}$ | -.021 | -.001 | -.021 | -.007 | -.021 |
| $(t-stat)$ | (-1.00) | (-.09) | (-1.58) | (-.25) | (-1.56) |
| $R^{S5B5}$ | .071 | .070 | .001 | -.003 | -.003 |
| $(t-stat)$ | (2.20) | (3.76) | (.01) | (-.14) | (-.18) |
| $R^{bond}$ | -.059 | -.045 | -.013 | -.029 | -.017 |
| $(t-stat)$ | (-2.72) | (-2.73) | (-.25) | (-.88) | (-2.98) |

Table 4: **Pricing Errors**. Average pricing errors implied by the previously reported model estimations. GMM t-stats are in parentheses. The Coll-CAPM pricing errors pertain to the model with first order polynomials only. Coll-CAPM 1 is the model with separability, Coll-CAPM 2 does not impose separability, while Coll-CAPM 3 adds a lagged consumption growth term as factor in $\phi$. The coefficient estimates for 2 and 3 are not reported. The errors for the inequality restrictions are not reported. The penalty parameter $c$ is 100, 300 and 12 in columns 3,4 and 5 respectively.

in our model.

**Aggregate Weight Shocks**  We propose a *linear* expression for $\phi(\cdot)$:

$$\phi(F_t^a, F_{t-1}^a, \ldots, F_{t-k}^a; my_t) = (my^{max} - my_t)B(L)(F_t^a - \Upsilon) \tag{4}$$

where $B(L)$ is a polynomial of order $k$ in the lag operator and $\Upsilon$ is the unconditional mean of the aggregate factors $F_t^a$. For now, we set $k = 0$, but we test for additional history dependence in the estimation exercise by including up to four lags of the factors, $F_{t-k}^a$ for $k = 1, 2, 3, 4$, in $B(L)$. $B_{ij}$ denotes the coefficient on factor $i$ in lag $j$.

## 6.1  Linear Factor Model and Fama-MacBeth

The factor model for the weight shocks and the autoregressive process for $my$ provide a complete description of the pricing model. By combining $\phi_{t+1}\left(my_{t+1}, F_{t+1}^a\right)$ and $my_{t+1}(my_t, F_{t+1}^a)$, the stochastic discount factor can be stated in terms of the aggregate factors $F_{t+1}^a$ and the state variable $my_t$. A first-order Taylor approximation of this expression delivers our linear factor model:

$$m_{t+1} \approx \tilde{\delta}(const - \theta^a F_{t+1}^a - \theta^c F_{t+1}^c + \gamma\varepsilon_{t+1}), \tag{5}$$

26

where the constraint factors $F_{t+1}^c$ are[18]:

$$F_{t+1}^c = (my^{max} - my_t)(1, F_{t+1}^a)$$

When the utility kernel is separable, the equity risk premium is determined by the conditional covariance of its returns with consumption growth and a state-varying market price of risk:

$$E_t\left[R_{t+1}^{e,j}\right] \approx \tilde{\delta} R_t^f \gamma \left[(1 + \Upsilon_c)^{-1} - B_{10}\gamma(1-\rho)my^{max} - B_{10}\rho\left(my^{max} - my_t\right)\right]$$
$$Cov_t\left(\Delta \log c_{t+1}^a, R_{t+1}^{e,j}\right)$$

where $R_t^f$ is the risk-free rate at time t. If $B_{10}$ is zero, the expression collapses to the standard CCAPM of Lucas (1978) and Breeden (1979). The market price of consumption risk is determined by the coefficient of relative risk aversion $\gamma$. In contrast, our theory predicts an increase in the size of the aggregate weight shock when aggregate consumption growth is low, driven by an increase in idiosyncratic risk. Consumption growth has an effect on the liquidity shock: $B_{10} < 0$. When housing collateral is scarce ($my^{max} - my_t$ is large), the market price of consumption risk is high.

Non-separability introduces a second covariance in the risk premium equation: the covariance with expenditure share changes. If $B_{20}$ is zero, the market price of composition risk is constant. In contrast, if $B_{20} < 0$, the market price of composition risk is high when housing collateral is scarce ($my^{max} - my_t$ is large).

**Unconditional $\beta$-Representation** The discount factor consists of a representative agent and a constraint component:

$$m_{t+1} = -\theta F_{t+1}, \tag{6}$$

where $\theta$ is a vector of constants, $\theta = \left(const, \tilde{\theta}\right)$ and $\tilde{\theta} = (\theta^a, \theta^c)$ and $F_{t+1} = \left(1, \tilde{F}_{t+1}\right)$. $\tilde{F}_{t+1} = \left(F_{t+1}^{a\prime}, F_{t+1}^{c\prime}\right)'$ is a vector of representative agent and constraint risk pricing factors. The conditioning information is embedded in the scaled constraint factors while $\theta$ itself is constant.

---

[18]The associated factor loadings are:

$$\theta^a = \left(\gamma - \gamma(1-\rho)B_{10}my^{max}, \frac{-\varepsilon + \frac{1}{\gamma}}{\frac{1}{\gamma}(\varepsilon - 1)} - \gamma(1-\rho)B_{20}my^{max}\right)$$
$$\theta^c = (\gamma\rho(B_{10}\Upsilon_c + B_{20}\Upsilon_\rho), -\gamma\rho B_{10}, -\gamma\rho B_{20})$$

27

These constraint factors contain the original aggregate factors scaled by the housing collateral ratio $my_t$. $my_t$ is the conditioning variable that summarizes the investor's information set. The model can be tested using the unconditional orthogonality conditions of the discount factor and excess asset returns $j$:

$$E\left[m_{t,t+1}R_{t+1}^{e,j}\right] = 0. \tag{7}$$

Using the definition of the risk-free rate and the covariance, the unconditional factor model in (6) implies an unconditional $\beta$-representation:

$$E\left[R_{t+1}^{e,j}\right] = \tilde{\delta}\bar{R}^f\tilde{\theta}Cov\left(\tilde{F}_{t+1}, R_{t+1}^{e,j}\right) = \tilde{\lambda}\tilde{\beta}^j$$

where $\bar{R}^f$ is the average risk-free rate, $\tilde{\beta}^j$ is asset $j$'s risk exposure and $\tilde{\lambda}$ is a transformation of the parameter vector $\tilde{\theta}$[19]:

$$\begin{aligned}
\tilde{\beta}^j &= Cov\left(\tilde{F}, \tilde{F}'\right)^{-1}Cov\left(\tilde{F}, R^{e,j}\right) \\
\tilde{\lambda} &= \tilde{\delta}\bar{R}^f\tilde{\theta}Cov\left(\tilde{F}, \tilde{F}'\right)
\end{aligned}$$

This unconditional $\beta$-representation is the equation we estimate using the Fama-MacBeth procedure.

**Computational Procedure**   We apply the two-stage Fama-MacBeth procedure and estimate the unconditional $\beta$-representation $E\left[R_{t+1}^{e,j}\right] = \tilde{\lambda}\tilde{\beta}^j$. In a first time-series stage, for each asset $j$ separately, excess returns are regressed on factors to uncover the $\tilde{\beta}$'s. In a second cross-sectional stage, average excess returns are regressed on the $\tilde{\beta}$'s from the first stage to obtain the market prices of risk $\tilde{\lambda}$. Appendix A.2 describes the procedure in more detail.

**Results with 26 Test Assets**   We use all 25 size and book-to-market portfolios and the value weighted market return as test assets. Only the results for annual data from 1926 until 2002 (77 observations) are presented in the paper, but the results with quarterly data for 1952.1:2002.4 are also available.

Table 5 reports the estimates for the market price of risk $\tilde{\lambda}$ obtained from the second-stage of the Fama-MacBeth procedure. Below the estimates for $\tilde{\lambda}$, we report conventional standard

---

[19]Lettau & Ludvigson (2001b) point out that $\tilde{\lambda}$ does not have a straightforward interpretation as the vector of market prices of risk. The market prices of risk $\lambda$ depend on the conditional covariance matrix of factors which is unobserved.

errors and Shanken (1992) standard errors, which correct for the fact that the $\tilde{\beta}$'s are generated regressors from the first time-series step.

Row 1 shows the standard CCAPM. It explains 9 percent of the cross-sectional variation in excess returns of the size and book-to-market portfolios between 1926 and 2002. Unsurprisingly, the coefficient of relative risk aversion $\gamma$ implied by the market price of consumption risk $\tilde{\lambda}_c$ is very high (22, not reported). With non-separable preferences but perfect commitment, the change in relative rental prices scaled by the housing expenditure share is an additional asset pricing factor. This is the HCAPM of PST. The non-separability effect increases the $R^2$ to 50 percent (row 2). Rows 3 through 8 investigate the collateral effect. With separable preferences, the new asset pricing factors are the housing collateral ratio $my$ and consumption growth scaled by $my$. The fit improves to 73 - 88 percent for the respective measures of the housing collateral ratio (rows 3-5). The coefficients on the interaction terms are positive and significant. With non-separable preferences, the interaction term of $my$ with expenditure share growth is an additional asset pricing factor (rows 6-8). The new interaction term has a positive factor loading, but does not enter statistically significantly. Except for the conditioning variable $myfa$, non-separability does not add much to the explanatory power of the collateral CAPM.

The intercept in the cross-sectional regression, $\tilde{\lambda}_0$ should be zero. Its estimate is positive and significant in rows 1 and 2, but becomes insignificant for the collateral CCAPM.

The coefficient estimates for $\tilde{\lambda}$ can be related to the structural parameters of the model, and we can infer that a decrease in the housing collateral ratio $my_t$ increases the market price of consumption risk. This follows because the estimated $\tilde{\lambda}_{my.c}$ is positive. Why? We know that $\tilde{\lambda} = \tilde{\theta} \left[ \tilde{\delta} \bar{R}^f Cov \left( \tilde{F}, \tilde{F}' \right) \right]$ and the loadings $\tilde{\theta}$ can be backed out of the $\tilde{\lambda}$ estimates. These factor loadings are listed in Table 6. The first two loadings on the constraint factors $\theta_1^c$ and $\theta_2^c$ are positive. Assuming the AR-coefficient $\rho$ for $my$ is positive, this implies that $B_{10}$ is negative -$B_{10}$ is the coefficient on aggregate consumption growth in the aggregate weight growth function. An adverse aggregate consumption growth shock increases the the aggregate weight shock and hence the risk premium. This is exactly the effect predicted by the theory. [20] The implied $B_{20}$ estimates are negative as well (see Table 6).

---

[20] A negative consumption growth shocks has two effects. First, a recession decreases $my$ which makes the risk-sharing bounds narrower. Second, a recession coincides with an increase in the income dispersion, which makes the bounds narrower as well. In either case, the extent to which a recession narrows the bounds depends on the level of $my$ or, equivalently, the housing collateral ratio. When the risk-sharing bounds are narrower, agents run more frequently into them and the aggregate weight growth is high. When housing collateral is scarce, $my^{max} - my_{t+1}$

|  | $\lambda_0$ | $\lambda_c$ | $\lambda_\rho$ | $\lambda_{my}$ | $\lambda_{my.c}$ | $\lambda_{my.\alpha}$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| 1 | 8.87 | 1.61 |  |  |  |  | **9.4** |
| CCAPM | (2.55) | (1.01) |  |  |  |  | 5.6 |
|  | [2.77] | [1.18] |  |  |  |  |  |
| 2 | 6.90 | .51 | .55 |  |  |  | **37.5** |
| HCAPM | (2.31) | (.88) | (.24) |  |  |  | 32.0 |
|  | [2.60] | [1.08] | [.30] |  |  |  |  |
| 3 | 4.22 | 1.94 |  | -.03 | 2.23 |  | **86.5** |
| Separable Prefs. | (2.29) | (1.05) |  | (.06) | (.79) |  | 84.6 |
| mymo | [3.31] | [1.58] |  | [.09] | [1.17] |  |  |
| 4 | 3.52 | 2.12 |  | -.03 | 1.36 |  | **87.8** |
| Separable Prefs. | (2.25) | (1.02) |  | (.03) | (.47) |  | 86.1 |
| myrw | [3.33] | [1.58] |  | [.05] | [.72] |  |  |
| 5 | 2.81 | .97 |  | -.00 | .66 |  | **73.3** |
| Separable Prefs. | (2.27) | (.94) |  | (.02) | (.35) |  | 69.7 |
| myfa | [2.93] | [1.28] |  | [.03] | [.47] |  |  |
| 6 | 2.87 | 2.59 | .11 | -.02 | 2.77 | .05 | **87.4** |
| Non-Sep. Prefs. | (2.73) | (.81) | (.26) | (.06) | (.64) | (.18) | 84.3 |
| mymo | [4.45] | [1.38] | [.44] | [.10] | [1.09] | [.30] |  |
| 7 | 3.62 | 2.30 | .25 | -.03 | 1.45 | .11 | **88.1** |
| Non-Sep. Prefs. | (2.48) | (.92) | (.20) | (.03) | (.41) | (.08) | 85.1 |
| myrw | [3.81] | [1.47] | [.33] | [.06] | [.65] | [.13] |  |
| 8 | 3.20 | 1.56 | -.05 | -.02 | .94 | .05 | **85.4** |
| Non-Sep. Prefs. | (2.44) | (1.01) | (.21) | (.02) | (.38) | (.06) | 81.7 |
| myfa | [4.11] | [1.75] | [.38] | [.04] | [.66] | [.11] |  |

Table 5: Cross-Sectional Results with Aggregate Pricing Factors. The sample period is 1926-2002. The asset pricing factors are $\Delta \log(c_{t+1})$ in row 1, $\Delta \log(c_{t+1})$ and $A_t \Delta \log(\rho_{t+1})$ in row 2, $\Delta \log(c_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t) \Delta \log(c_{t+1})$ in rows 3-5 and $\Delta \log(c_{t+1})$, $\Delta \log(\alpha_{t+1})$, $my^{max} - my_t$ ,$(my^{max} - my_t) \Delta \log(c_{t+1})$ and $(my^{max} - my_t) \Delta \log(\alpha_{t+1})$ in rows 6-8. The housing collateral variable is $mymo$ in rows 3 and 6, $myrw$ in row 4 and 7 and $myfa$ in row 5 and 8. $my$ is estimated with data from 1925-2002. The variables used are $\Delta c_1$ and $\Delta \alpha_1$ described in the data appendix. The estimation is done using the Fama-MacBeth procedure. The set of test assets is $T1$. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the $R^2$ and the adjusted $R^2$ just below it.

Figure 7 compares the CCAPM and the collateral-CAPM under separability. The left panel plots the sample average excess return on each of the 26 portfolios against the return predicted by the standard CCAPM. The CCAPM hardly explains any of the variation in excess returns across portfolios. The right panel, which corresponds to the estimates in row 4 of table 5, shows the returns predicted by the collateral-CAPM. Most of the size and value portfolios line up along the 45 degree line.

Table 7 reports the sample average pricing errors on each of the 26 portfolios. Relative to the CCAPM, the collateral-CAPM largely eliminates the overpricing of growth stocks and the underpricing of value stocks. The average pricing error across portfolios is 3.27 percentage points for the CCAPM (first column, second to last row) but less than half as large for the

---

is large. A negative consumption growth shock increases $\phi_{t+1}$ for $B_{10} < 0$. When $my_{t+1} = my^{max}$, there is no effect of innovations to aggregate consumption and rental price growth on the expression for the aggregate weights: $\phi_{t+1}$ is one.

| Model | Separability | | | Non-Separability | | | | |
|---|---|---|---|---|---|---|---|---|
| Parameter | $\theta_1^a$ | $\theta_1^c$ | $\theta_2^c$ | $\theta_1^a$ | $\theta_2^a$ | $\theta_1^c$ | $\theta_2^c$ | $\theta_3^c$ |
| $mymo$ | 44.17 | 0.91 | 29.42 | 57.67 | 8.10 | 1.17 | 38.09 | 4.43 |
| $myrw$ | 35.59 | 41 | 12.68 | 39.04 | 5.91 | 0.44 | 13.85 | 1.66 |
| $myfa$ | 16.19 | 0.11 | 5.02 | 25.39 | 3.74 | 0.17 | 7.85 | 1.06 |

Table 6: Coll-CAPM Factor Loadings. Factor Loadings implied by the Fama-MacBeth coefficient estimates. The implied standard errors not reported.

Figure 7: CCAPM and Collateral-CAPM - Aggregate Pricing Factors. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against predicted excess returns by standard Consumption-CAPM. Right Panel: against predicted returns by Collateral-CAPM (under separability).
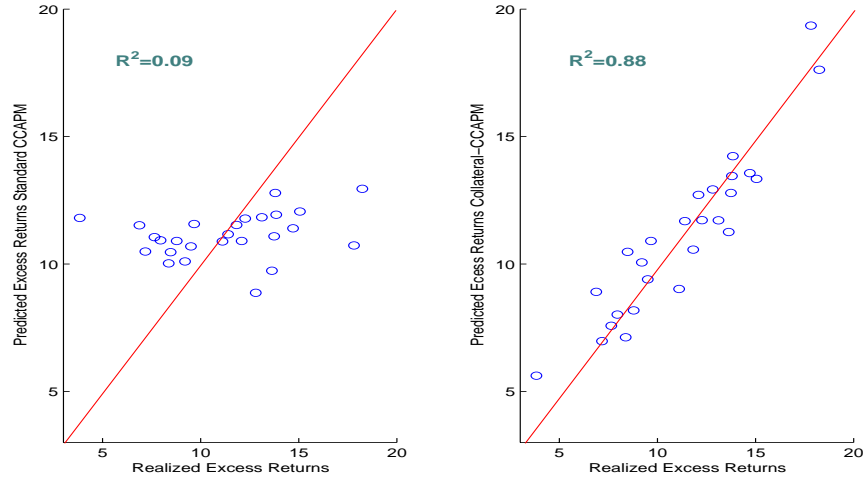


collateral-CAPM (1.21 percent, last column). The errors are comparable in size and sign to the Fama & French (1993) three-factor model (second column of table 7, see also section 6.2). Especially the pricing errors on the small growth firms (S1B1 and S1B2) and large growth and value firms (S5B1, S5B4, S5B5) are lower than for the three-factor model. The last row of the table shows a $\chi^2$-distributed test statistic for the null hypothesis that all pricing errors are zero. The collateral-CAPM is the only model for which the hypothesis of zero pricing errors cannot be rejected at the 5 percent level.[21]

---

[21]Because of the sampling error in the regressors the Shanken correction for the $\chi^2$ test statistics is large. This is because the macro-economic factors have a low sample variance and the size of the standard-error correction is inversely related to this variability. While increasing the standard errors, this correction reduces the $\chi^2$ test statistic (see A.2). The result that the collateral-CAPM fails to reject the null hypothesis of zero pricing errors should be interpreted in this light.

| Portfolio | CCAPM | Fama-French | Collateral-CCAPM |
|---|---|---|---|
| RVW | 2.97 | -.20 | .07 |
| S1B1 | 7.97 | 3.96 | 1.79 |
| S1B2 | 1.89 | 2.51 | 1.23 |
| S1B3 | -1.01 | -1.08 | -.34 |
| S1B4 | -7.10 | -2.07 | 1.53 |
| S1B5 | -5.29 | -2.78 | -.62 |
| S2B1 | 4.64 | 1.94 | 2.03 |
| S2B2 | -.30 | -1.15 | -1.27 |
| S2B3 | -2.65 | -1.23 | -.96 |
| S2B4 | -3.31 | -.79 | -1.14 |
| S2B5 | -3.00 | -.08 | -1.72 |
| S3B1 | 1.99 | -1.41 | 2.00 |
| S3B2 | -.24 | -1.12 | .28 |
| S3B3 | -.50 | -.62 | -.55 |
| S3B4 | -1.28 | .03 | -1.40 |
| S3B5 | -1.92 | 1.58 | .38 |
| S4B1 | 1.65 | -2.54 | -1.25 |
| S4B2 | .90 | .70 | .85 |
| S4B3 | -.22 | .24 | -2.08 |
| S4B4 | -1.18 | .64 | .62 |
| S4B5 | -3.90 | -.11 | -2.38 |
| S5B1 | 3.41 | -2.52 | -.07 |
| S5B2 | 3.31 | .73 | -.21 |
| S5B3 | 2.13 | .46 | -.59 |
| S5B4 | 1.19 | 1.96 | -.10 |
| S5B5 | -3.95 | -.86 | .10 |
| Average | 3.27 | 1.61 | 1.21 |
| $\chi^2$ | 72.1*** | 61.1*** | 35.1* |

Table 7: Average Pricing Errors. Pricing errors from the Fama-MacBeth Regressions with aggregate pricing factors. The set of returns is the value weighted market return and the 25 size and book-to-market portfolio returns. The sample is 1926-2002. The second column reports errors from the consumption CAPM, the third from the three-factor Fama-French model and the last column reports average errors from the collateral CAPM with scaling variable $myrw$ and separability in preferences (line 4 in table 5). The last two rows report the square root of the average squared pricing errors and the $\chi^2$ statistic for the null hypothesis that all pricing errors are zero. The degrees of freedom are 25, 23 and 23 respectively. Three stars denote rejection of the null hypothesis at the 1 percent level, 2 stars at the 5 percent level and 1 star at the 10 percent level.

As a robustness check, we relax the Markov assumption that we imposed on the aggregate weight shock by including additional lags of the aggregate factors in the empirical specification of the aggregate weight process. This introduces additional asset pricing factors in the unconditional $\beta$-representation. Table 15 in the Appendix reports the estimation results for $k \in 1, 2, 3, 4$ for the sample 1929-2002. They show that the fit of the cross-sectional estimation does not improve significantly and the extra factors enter insignificantly. Only for $k = 4$ is there some additional explanatory power. We conclude that the Markov assumption fits the data well.

As second robustness exercise, we estimate the cross-sectional regression for the postwar period (1945-2002). The results are in table 16 in the Appendix. The benchmark consumption

CAPM performs much better in the postwar sample (49 percent $R^2$). The collateral-CAPM with separable preferences explains between 70 and 83 percent and the collateral-CAPM with non-separable preferences between 76 and 84 percent of the cross-sectional variation in the 26 portfolios. The implied coefficient estimates for $B_1$ are still negative, so that the post-war results confirm the presence of the collateral effect.

**Time-Varying Betas**  Why does the collateral-CAPM help explain the value premium? In the model, a stock's riskiness is determined by the covariance of its returns with aggregate risk factors *conditional* on the state variable $my$. The conditional covariance reflects time-variation in risk premia. If time variation in risk premia is important for explaining the value premium, then stocks with high book-to-market ratios should have a larger covariance with aggregate risk factors in risky times, when $my$ is low ($my^{max} - my_t$ is high), than in less risky times, when $my$ is high ($my^{max} - my_t$ is low). This is the pattern we find in the data.

We estimate the risk exposure (the $\beta$'s) for each of the twenty-five size and book-to-market portfolios and the value weighted market return. This is the first step of the Fama-MacBeth two-step procedure. To make the point more forcefully we impose separability on the preferences over housing and non-durable consumption:

$$R_{t+1}^{e,j} = \tilde{\beta}_0^j + \tilde{\beta}_c^j \Delta \log c_{t+1} + \tilde{\beta}_{my}^j (my^{max} - my_t) + \tilde{\beta}_{my.c}^j (my^{max} - my_t) \Delta \log c_{t+1}. \qquad (8)$$

Equation (8) allows the covariance of returns with consumption growth to vary with $my$. For each asset $j$, we define the conditional consumption beta as $\beta_t^j = \tilde{\beta}_c^j + (my^{max} - my_t)\,\tilde{\beta}_{my.c}^j$. We estimate equation (8) and compute the average consumption beta in good states, defined as times in which $my$ is one standard deviation above zero, and in bad states (risky times) when $my$ is one standard deviation below zero. Table 17 shows that the high book-to-market portfolios (B4 and B5) have a consumption $\beta$ that is large when housing collateral is scarce and small in times of collateral abundance. The opposite is true for growth portfolios (B1 and B2). Moreover, the value stocks have higher consumption betas than the growth stocks in bad states, and vice versa for the good states. This is the sense in which value portfolios are riskier than growth portfolios.

The left panel of figure 8 shows that the value portfolios (B4, B5) have a high return and the growth portfolios (B1, B2) have a low return. The right panel plots realized excess returns against $\tilde{\beta}_{my.c}^j$, the exposure to the interaction term of the housing collateral ratio with aggregate consumption growth. Growth stocks in the lower left corner have a low exposure to collateral

constraint risk whereas value stocks have a large exposure. So, value stocks, are riskier than growth stocks because their returns are more highly correlated with the aggregate factors when risk is high ($my^{max} - my_t$ is high) than when risk is low ($my^{max} - my_t$ is low). Because both the estimates of $\tilde{\lambda}_{my.c}$ and of $\tilde{\beta}_{my.c}^{j}$ are positive, value stocks are predicted to have a higher risk premium. The value premium is the compensation for the fact that high book-to-market firms pay low returns when housing collateral is scarce and constraints bind more frequently.

Figure 8: Collateral CAPM: The Value Premium. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against excess returns predicted by the collateral-CAPM with $myrw$. Right Panel: Realized average excess returns against $\tilde{\beta}_{my.c}$ exposure to interaction term of $my_t$ and $\Delta \log c_{t+1}$.
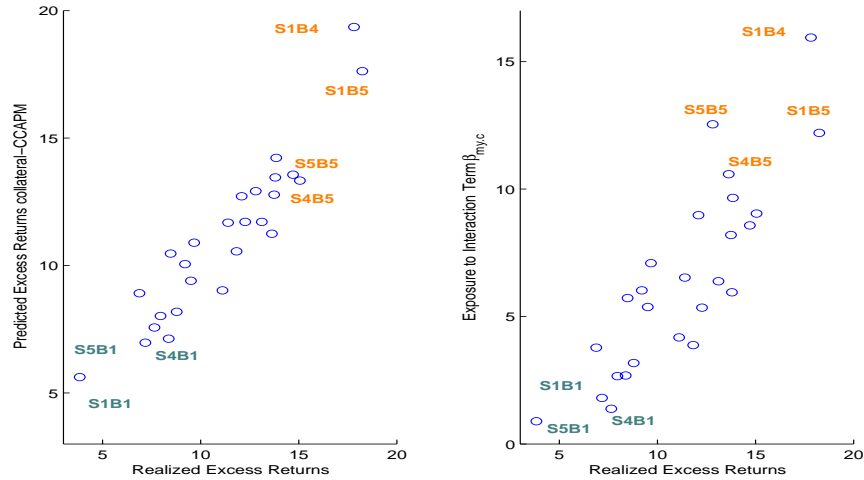


## 6.2 Comparison Across Models

The cross-sectional explanatory power of the collateral-CAPM proposed in this paper compares favorably to other asset pricing models. Table 8 compares return-based asset pricing models in rows 1-3 with consumption-based models in rows 4-6.

The capital asset pricing model relates the returns on stocks to their correlation with the return on the wealth portfolio. In the standard CAPM of Lintner (1965), the return on the wealth portfolio is proxied by the market return $R^{vw}$ (row 1). It explains 28 percent of annual returns. Because stock market wealth is an incomplete total wealth measure, Jagannathan & Wang (1996) include the return on human wealth in the return on the wealth portfolio. That return is measured by the growth rate in labor income (plus transfers). The $R^2$ in row 2

34

| Model | $\tilde{\lambda}_0$ | $\tilde{\lambda}_1$ | $\tilde{\lambda}_2$ | $\tilde{\lambda}_3$ | $R^2$ |
|---|---|---|---|---|---|
| 1 | −.30 | 9.35 | | | 28.3 |
| Static CAPM | (4.02) | (4.68) | | | 25.3 |
| Sharpe-Lintner | [4.40] | [5.65] | | | |
| 2 | 2.24 | 7.13 | 3.91 | | 37.2 |
| Human Capital-CAPM | (3.93) | (4.58) | (1.28) | | 31.7 |
| Jagannathan-Wang | [4.87] | [6.16] | [1.71] | | |
| 3 | 1.47 | 7.48 | −.00 | 5.57 | 49.3 |
| $lc$-conditional CAPM | (4.17) | (4.82) | (0.02) | (4.99) | 42.4 |
| Santos-Veronesi | [5.49] | [6.42] | [0.03] | [6.62] | |
| 4 | 8.88 | 1.61 | | | 9.4 |
| Static CCAPM | (2.55) | (1.01) | | | 5.6 |
| Breeden-Lucas | [2.77] | [1.18] | | | |
| 5 | 3.88 | 2.68 | .02 | .42 | 86.2 |
| $cay$-conditional CCAPM | (2.32) | (1.05) | (.02) | (.30) | 84.3 |
| Lettau-Ludvigson | [3.84] | [1.80] | [.03] | [.53] | |
| 6 | 3.52 | 2.12 | −.03 | 1.36 | 87.8 |
| Collateral-CAPM | (2.25) | (1.02) | (.03) | (.47) | 86.1 |
| this paper | [3.33] | [1.58] | [.05] | [.71] | |
| 7 | 10.21 | −2.46 | 2.71 | 6.30 | 78.1 |
| Three-factor model | (4.63) | (5.17) | (1.68) | (1.74) | 75.1 |
| Fama-French | [5.24] | [6.32] | [2.52] | [2.56] | |

Table 8: Model Comparison: 7 models, 1926-2002. Row 1: factor is $R_{t+1}^{vw,e}$. Row 2: factors are $R_{t+1}^{vw,e}$ and $R_{t+1}^{hc,e}$. Row 3 factors: $R_{t+1}^{vw,e}$, $lc_t$ and $lc_t R_{t+1}^{vw,e}$. Row 4: $\Delta \log(c_{t+1})$. Row 5: $\Delta \log(c_{t+1})$, $cay_t$, $cay_t \Delta \log(c_{t+1})$. Row 6 is the collateral model under separability: $\Delta \log(c_{t+1})$, $myrw^{max} - myfa_t$, and $(myrw^{max} - myfa_t)\Delta \log(c_{t+1})$. Row 7: $R_{t+1}^{vw,e}$, $R_{t+1}^{smb,e}$, and $R_{t+1}^{hml,e}$. The second column gives the zero-$\beta$ return $\tilde{\lambda}_0$. OLS standard errors are in parenthesis, Shanken corrected standard errors are in brackets.

increases slightly to 37 percent. Kullmann (2002) investigates the improvements to the CAPM when residential housing wealth is incorporated into the definition of wealth. In our model housing wealth affects returns only through the collateral ratio. In the economy of Santos & Veronesi (2001), times in which investors finance a large fraction of consumption out of labor income ($lc$ is low), are less risky. Their conditional CAPM explains 50 percent of the annual returns (row 3).[22]

The Fama & French (1993) three-factor model adds a size and a book-to-market factor to the standard CAPM. The size factor is the return on a hedge portfolio that goes long in small firms and short in big firms ($smb$). The value factor is the return on a hedge portfolio that goes long in high book-to-market firms and short in low book-to-market firms ($hml$). This model accounts for 78 percent of the cross-sectional variation in annual returns (row 7). Given its good fit, this model serves as the empirical benchmark.

In contrast to the previous models, consumption-based asset pricing models measure the riskiness of an asset directly off the covariance with marginal utility growth. One of the objectives

---

[22]The authors also investigate a scaled version of the CCAPM, as we do, but their results for the scaled CCAPM are not as strong as for the scaled CAPM.

of this literature has been to identify macroeconomic sources of risk that can explain the empirical success of the Fama & French (1993) size and book-to-market factors. The fourth row reports the standard CCAPM of Breeden (1979). Lettau & Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption-wealth ratio as scaling variable. The market price of consumption risk increases in times with low *cay* (recessions). The Lettau-Ludvigson model explains 86 percent of the annual cross-sectional variation.

Model 6 is our collateral-CCAPM. The model goes a long way in accounting for the cross-sectional differences in returns on the 25 Fama-French portfolios and the market return. The $R^2$ of 88 percent improves upon the fit of the Fama-French model.[23]

Sofar we have documented that the returns of value firms are more correlated with aggregate risk in times when the housing collateral ratio is low. The next section takes the next step by identifying one potential source of this pattern by examining the response of dividends to collateral shocks.

## 6.3 Dividends on Value Portfolios

We use annual dividend data on each of the 10 book-to-market decile portfolios. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. We follow Bansal et al. (2002) by constructing dividends from value-weighted total returns and price appreciation rates on the decile value portfolios (both from Kenneth French). We construct nominal annual dividends by summing up monthly nominal dividends. The data are for 1952-1999.

Table 19 shows how dividends of the high book-to-market portfolios (normalized by the aggregate labor income plus transfers) are strongly positively correlated with the housing collateral ratio (e.g. the tenth decile portfolio B10). The normalized dividend process for the low book-to-market ratio portfolios is strongly negatively correlated with $my$ (e.g. the first decile portfolio B1). Using a bivariate VAR, we study the response of the normalized dividend process on the growth (B1) and value (B10) portfolios to an impulse in the housing collateral ratio. Figure 9 shows the impulse response graphs. The responses of the dividend shares to an innovation to myfa are mirror images of each other. In Lustig & VanNieuwerburgh (2003) we show that claims to dividend processes with this type of "value "response to $my$ shocks trade at a substantial

---

[23]Table 18 in the Appendix investigates the residual explanatory power of the idiosyncratic portfolio characteristics, size and value. The top panel includes the log market capitalization into the regression, the bottom panel the log value weighted book-to-market ratio of the portfolio. In contrast with the return-based models and the static CAPM, there are no residual size nor value effects in the collateral-CCAPM. The same conclusion is found for the post-war sample.

discount in equilibrium.

Figure 9: Response of the the log Dividend Share on Portfolio B1 to Impulse in Collateral Ratio $myfa$. Sample is 1952-2002, standard errors are computed from 5,000 Monte Carlo simulations.
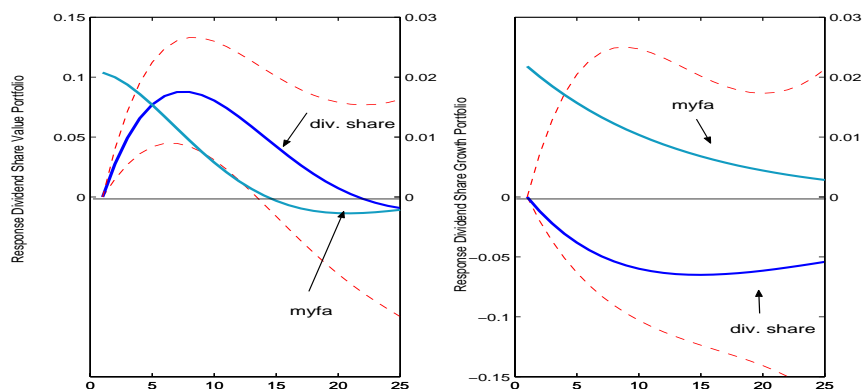


# 7  Conclusion

House price fluctuations play an important role in explaining the time-series and cross-sectional variation in asset returns. Given the magnitude of the housing market this is unsurprising. This paper shows that the way in which housing affects asset returns is through the role of housing as a collateral asset. The housing collateral mechanism endogenously generates heteroskedasticity and counter-cyclicality in the market price of risk. In Lustig & VanNieuwerburgh (2003) we solve for the equilibrium of the model numerically, while this paper focusses on connecting the model to the data. We specify the liquidity factor in the SDF as a semi-parametric function of the housing collateral ratio and the aggregate pricing factors.

The Euler equation restrictions for the stock market return, a short-term bond, a long-term bond and a few size and book-to-market portfolios, in addition to the Euler inequality restrictions for the representative agent, yield precise, low risk aversion estimates. The estimated liquidity shocks are larger in times of low aggregate consumption growth when housing collateral is scarce, as predicted by the model. A linear version of this model prices the 25 size and book-to-market portfolios remarkably well.

We attribute the failure of the CCAPM to shocks to the economy's risk sharing technology. There is a wealth of empirical evidence against full consumption insurance at different levels

of aggregation: at the household level (e.g. Attanasio & Davis (1996) and Cochrane (1991b)), the regional level (e.g. Hess & Shin (1998)) and the international level (e.g. Backus, Kehoe, & Kydland (1992)). Blundell, Pistaferri, & Preston (2002) find evidence for shocks to the economy's risk sharing technology. Lustig & VanNieuwerburgh (2002) provide direct empirical support for the underlying time-variation in risk-sharing. Using US metropolitan area data, we find that the degree of insurance between regions decreases when the housing collateral ratio is low. Our theory only predicts strong consumption growth correlations when housing collateral is abundant. The data seem to support this qualification; conditioning on $my$ weakens the consumption growth puzzle for US regions.

# References

Ang, Andrew, & Geert Bekaert, 2001, Stock return predictability: Is it there?, nBER Working Paper No.w8207.

Attanasio, Orazio P., & Steven J. Davis, 1996, Relative wage movements and the distribution of consumption, *The Journal of Political Economy* 104, 1127–1262.

Backus, David, Patrick Kehoe, & Finn Kydland, 1992, International real business cycles, *Journal of Political Economy* 100, 745–75.

Bansal, Ravi, Robert Dittmar, & Chris Lundblad, 2002, Consumption, dividends, and the cross-section of expected returns, mimeo.

Blundell, Richard, Luigi Pistaferri, & Ian Preston, 2002, Partial insurance, information and consumption dynamics, mimeo.

Brav, Alon, George M. Constantinides, & Christopher C. Gezcy, 2002, Asset pricing with heterogeneous consumers and limited participation: Empirical evidence, *Journal of Political Economy* 110(4), 793–824.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

Campbell, John Y., 1996, Understanding risk and return, *The Journal of Political Economy* 104, 298–345.

Campbell, John Y., 2000, Asset pricing at the millennium, *Journal of Finance* 55, 1515–1567.

Campbell, John Y., & Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Census, Bureau, 1976, *Historical Statistics of the United States, Colonial Times to 1970* (U.S. Department of Commerce).

Chapman, David, 1997, Approximating the asset pricing kernel, *The Journal of Finance* 52(4), 1383–1410.

Cocco, Joao, 2000, Hedging house price risk with incomplete markets, working Paper London Business School.

Cochrane, John H., 1991a, Production based asset pricing and the link between stock returns and macroeconomic fluctuations, *Journal of Finance* 46, 209–238.

Cochrane, John H., 1991b, A simple test of consumption insurance, *The Journal of Political Economy* 99, 957–976.

Cochrane, John H., 1996, A cross-sectional test of an investment based asset pricing model, *The Journal of Political Economy* 104, 572–621.

Cochrane, John H., 2001, *Asset Pricing* (Princeton University Press, Princeton, N.J.).

Cogley, Tim, 2002, Idiosyncratic risk and the equity premium: Evidence from the consumer expenditure survey, *Journal of Monetary Economics* 49, 309–334.

Constantinides, George M., & Darrell Duffie, 1996, Asset pricing with heterogeneous consumers, *Journal of Political Economy* 104, 219–240.

Dunn, K., & K. Singleton, 1986, Modeling the term structure of interest rates under nonseparable utility and durability of goods, *Journal of Financial Economics* 17, 769–799.

Eichenbaum, Martin, & Lars Peter Hansen, 1990, Estimating models with intertemporal substitution using aggregate time series data, *Journal of Business and Economic Statistics* 8, 53–69.

Fama, Eugene F., & Kenneth R. French, 1988, Dividend yields and expected stock returns., *Journal of Financial Economics* 25, 23–49.

Fama, Eugene F., & Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F., & Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Ferson, Wayne E., Shmuel Kandel, & Robert F. Stambaugh, 1987, Tests of asset pricing with time-varying expected risk-premiums and market betas, *Journal of Finance* 42, 201–220.

Flavin, Marjorie, & Takashi Yamashita, 2002, Owner-occupied housing and the composition of the household portfolio, *American Economic Review* 79, 345–362.

Hansen, Lars Peter, & Robert J. Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, *The Journal of Political Economy* 88, 829–853.

Hansen, Lars Peter, & R. Jagannathan, 1991, Implications of security markets data for models of dynamic economies, *Journal of Policitical Economy* 99, 252–262.

Hansen, Lars Peter, & Kenneth J. Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249–265.

Hess, Gregory D., & Kwanho Shin, 1998, Intranational business cycles in the united states, *Journal of International Economics* 44, 289–313.

Jagannathan, Ravi, & Zhenyu Wang, 1996, The conditional capm and the cross-section of expected returns, *The Journal of Finance* 51, 3–53.

Johansen, Soren, & Katarina Juselius, 1990, Maximum likelihood estimation and inference on cointegration with applications to the demand for money, *Oxford Bulletin of Economics and Statistics* 52, 169–210.

Judd, Kenneth, 1998, *Numerical Methods in Economics.* (The MIT Press, Cambridge, Massachusetts), first edition.

Krueger, Dirk, & Jesus Villaverde, 2001, Consumption and saving over the life cycle; how important are consumer durables?, working Paper Stanford University.

Kullmann, Cornelia, 2002, Real estate and its role in asset pricing, working Paper Columbia GSB.

Lamont, Owen, 1998, Earnings and expected returns, *Journal of Finance* 53, 1563–1587.

Lettau, Martin, & Sydney C. Ludvigson, 2003, Measuring and modeling variation in the risk-return trade-off, in Yacine Ait-Sahalia & Lars Peter Hansen, (eds.) *Handbook of Financial Econometrics.*

Lettau, Martin, & Syndey Ludvigson, 2001a, Consumption, aggregate wealth and expected stock returns, *The Journal of Finance* 56, 815–849.

Lettau, Martin, & Syndey Ludvigson, 2001b, Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying, *The Journal of Political Economy* 109, 1238–1287.

Li, Qing, Maria Vassalou, & Yuhang Xing, 2002, An investment-growth asset pricing model, working Paper Columbia GSB.

Lintner, John, 1965, Security prices, risk, and maximal gains from diversification, *The Journal of Finance* 20, 587–615.

Lucas, Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–54.

Lustig, Hanno, 2001, The wealth distribution and aggregate risk, mimeo Stanford University.

Lustig, Hanno, & Stijn VanNieuwerburgh, 2002, Housing collateral and consumption insurance across us regions, mimeo.

Lustig, Hanno, & Stijn VanNieuwerburgh, 2003, A theory of housing collateral and risk premia, working Paper NYU Stern and University of Chicago.

Luttmer, Erzo, 1991, *Asset Pricing in Economies with Frictions*, Ph.D. thesis.

Maddison, Angus, 2001, *The World Economy: A Millennial Perspective*, Economic History Services (OECD, Paris).

Mankiw, N. G., 1986, The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics* 17, 211–219.

Mehra, & Edward Prescott, 1985, The equity premium: A puzzle., *Journal of Monetary Economics* 15, 145–161.

Piazzesi, Monika, Martin Schneider, & Selale Tuzel, 2002, Housing, consumption and asset pricing, mimeo.

Santos, Tano, & Pietro Veronesi, 2001, Labor income and predictable stock returns, mimeo.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *The Review of Financial Studies* 5, 1–33.

Stambaugh, Robert F., 1999, Predictive regressions, *Journal of Financial Economics* 54, 375–421.

Storesletten, K., C. Telmer, & A. Yaron, 2003, Cyclical dynamics of idiosyncratic labor income risk, *Journal of Political Economy* forthcoming.

Whitelaw, Robert F., 1997, Time-varying sharpe ratios and market timing, mimeo, NYU Stern School of Business.

Yao, Rui, & Harold H. Zhang, 2002, Optimal consumption and portfolio choices with risky housing and borrowing constraint, mimeo.

Yogo, Motohiro, 2003, A consumption-based explanation of the cross-section of expected stock returns, working Paper, Harvard University.

# A  Appendix

## A.1  Bootstrap Procedure

The bootstrap procedure addresses the persistent-regressor bias of the OLS coefficient estimators and serial correlation in the OLS residuals in the estimation of equation (3). We compute small-sample coefficient estimates and small-sample p-values for the null hypothesis of no predictability.

A univariate specification test shows that $my$ is best described by an AR(2) process.

$$\left(\frac{my^{max} - my_{t+1}}{my^{max} - my^{min}}\right) = c_0 + c_1 \left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + c_2 \left(\frac{my^{max} - my_{t-1}}{my^{max} - my^{min}}\right) + v_{t+1}, \tag{9}$$

where $v$ is i.i.d. mean zero.

If annual returns are truly unforecastable, the data generating process for $\left(r_{t+1}^{vw,e}\right)$ is

$$
\begin{aligned}
r_{t+1}^{vw,1} &= b_{0,1}^{np} + e_{t+1}^1 \\
r_{t+2}^{vw,2} &= b_{0,2}^{np} + e_{t+1}^1 + e_{t+2}^1 \\
r_{t+K}^{vw,K} &= b_{0,K}^{np} + e_{t+1}^1 + e_{t+2}^1 + ... + e_{t+K}^1
\end{aligned}
\tag{10}
$$

where $e^1$ is an i.i.d. mean zero process of innovations to the 1-year log excess return. Under the no-predictability null, $K$-period returns have a MA($K$) error structure because of overlapping observations.

Similarly, with predictability

$$
\begin{aligned}
r_{t+1}^{vw,1} &= b_{0,1}^{wp} + b_{1,1}^{wp} \left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u_{t+1}^1. \\
r_{t+2}^{vw,2} &= b_{0,2}^{wp} + b_{1,2}^{wp} \left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u_{t+1}^1 + u_{t+2}^1. \\
r_{t+K}^{vw,K} &= b_{0,K}^{wp} + b_{1,K}^{wp} \left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u_{t+1}^1 + u_{t+2}^1 + ... + u_{t+K}^1
\end{aligned}
\tag{11}
$$

where $u^1$ is an i.i.d. mean zero process of innovations to the 1-year log excess return.

In a preliminary step, we estimate the no-predictability coefficients $b_{0,1}^{np}$ through $b_{0,K}^{np}$ in $K$ OLS regressions of the form of equation (10). We do the same for the with-predictability coefficients $(b_{0,1}^{wp}, b_{1,1}^{wp})$ through $(b_{0,K}^{wp}, b_{1,K}^{wp})$ in $K$ OLS regressions of the form of equation 11. We store the residuals $\{e^1, u^1\}$.

The bootstrap exercise for the small sample bias consists of the following steps.

**step 1** Draw a sample of length $T$ with replacement from the residuals $\{u^1, v\}$ obtained in preliminary step.

**step 2** For given $my_0, my_1$ (which we set equal to the sample values) and parameter estimates from preliminary step, build up time series for $r^{e,K}$ and $my_{t+1}$ recursively from equations (11) and (9), for $K \in \{1, ..., 10\}$. This takes into account the MA(K) structure of the innovations.

**step 3** Estimate by OLS the intercept and the coefficient on the rescaled housing collateral ratio in the return equation. Let the coefficient on $my$ be $b_{1,K}^{n,*}$, for $K \in \{1, ..., 10\}$.

**step 4** Repeat steps 1 through 4 $N = 20,000$ times.

The small sample coefficient estimate is $\frac{1}{N} \sum b_{1,K}^{n,*}$. The bias equals $\frac{1}{N} \sum b_{1,K}^{n,*} - b_{1,K}^{LS}$.

The second bootstrap exercise (for the no-predictability p-value) proceeds as the first, except it imposes the null hypothesis of no predictability in step 1. It consists of the following steps.

**step 1** Draw a sample of length $T$ with replacement from the residuals $\{e^1, v\}$ obtained in preliminary step.

**step 2** For given $my_0, my_1$ (which we set equal to the sample values) and parameter estimates from preliminary step, build up time series for $r^{e,K}$ and $my_{t+1}$ recursively from equations (10) and (9), for $K \in \{1, ..., 10\}$. This takes into account the MA(K) structure of the innovations.

**step 3** Estimate by OLS the intercept and the coefficient on the rescaled housing collateral ratio in the return equation. Let the coefficient on $my$ be $b_{1,K}^{n,**}$, for $K \in \{1, ..., 10\}$.

**step 4** Repeat steps 1 through 4 $N = 20,000$ times.

The p-value is the frequency of observing estimates $b_{1,K}^{n,**}$ smaller than the least-squares estimate $b_{1,K}^{LS}$ :

$$p = 1 - \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}_{b_{1,K}^{n,**} > b_{1,K}^{LS}}.$$

It's the p-value of a two-sided test of no predictability.

## A.2    Fama-MacBeth Procedure

The estimation problem is

$$E\left[R_{t+1}^{e,j}\right] = \tilde{\lambda} \tilde{\beta}^{j\prime},$$

with

$$
\begin{aligned}
\tilde{\lambda} &= \theta Cov\left(F_{t+1}, F_{t+1}'\right), \\
\tilde{\beta}^j &= Cov\left(F_{t+1}, F_{t+1}'\right)^{-1} Cov\left(F_{t+1}, R_{t+1}^{e,j}\right).
\end{aligned}
$$

First, for each $j$, the vector $\tilde{\beta}^j$ is obtained from the time-series regression of returns on the factors. Given the limited length of the time series, the $\beta$ are estimated using 1 regression over the entire sample instead of a rolling regression.

$$R_t^{e,j} = \beta_0^j + \tilde{\beta}_a^{j'} F_t^a + \tilde{\beta}_c^{j'} F_t^c + \epsilon_t^j \qquad t = 1, 2, ..., T. \tag{12}$$

Let $\Sigma = E[\epsilon_t \epsilon_t']$ be the $N \times N$ covariance matrix of the errors $\epsilon_t = \left[\epsilon_t^1, ..., \epsilon_t^N\right]$. Second, for each $t$, a cross-sectional regression of returns on the estimated $\tilde{\beta}^i$ uncovers estimates for $\left(\tilde{\lambda}_a, \tilde{\lambda}_c\right)$ and the zero-$\beta$ return $\tilde{\lambda}_0$:

$$R_t^{e,j} = \tilde{\lambda}_0 + \tilde{\beta}_{a,t}^{j'} \tilde{\lambda}_{a,t} + \tilde{\beta}_{c,t}^j \tilde{\lambda}_{c,t} + \alpha_t^j \qquad j = 1, 2, ..., J.$$

The estimator for the price of risk is the time series average of the estimated second stage coefficients: $\tilde{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \tilde{\lambda}_t$. The $\alpha_t^j$ are the pricing errors, $\hat{\alpha}^j = \frac{1}{T} \sum_{t=1}^{T} \hat{\alpha}_t^j$. The sampling error for $\hat{\lambda}$ and the covariance matrix for $\hat{\alpha}$ are given by:

$$\begin{aligned} var\left(\hat{\lambda}_{FMB}\right) &= \frac{1}{T^2} \left[ \sum_{t=1}^{T} \left(\hat{\lambda}_t - \hat{\lambda}\right)^2 \right] \\ cov\left(\hat{\alpha}_{FMB}\right) &= \frac{1}{T^2} \left[ \sum_{t=1}^{T} \left(\hat{\alpha}_t - \hat{\alpha}\right)\left(\hat{\alpha}_t - \hat{\alpha}\right)' \right] \end{aligned}$$

The pricing errors are the basis for a goodness of fit statistic $\hat{\alpha} cov(\hat{\alpha})^{-1} \hat{\alpha}$. In a regression with $K$ factors the statistic has a $\chi^2$ distribution with $J - K$ degrees of freedom. A second measure of fit is the $R^2$ constructed from the cross-sectional variance of the time-averages (denoted by a bar) of errors and returns for each of the portfolios:

$$R_{FMB}^2 = 1 - \frac{var\left(\overline{\hat{\alpha}_t^i}\right)}{var(\overline{R_{t+1}^i - R_t^f})}$$

The $\beta's$ are generated regressors from a first stage time-series analysis. Generally, this error in variables problem gives rise to an underestimation of the asymptotic variance-covariance matrix. The standard errors for $\lambda$ and $\alpha$ are corrected following Shanken (1992). Let the matrix $\Sigma_f$ be the covariance matrix of the factors $F$. The Shanken correction to the variance of the estimator $\hat{\lambda}$ and the covariance matrix of pricing errors $\alpha$ is:

$$\begin{aligned} Var\left(\hat{\lambda}_{corr}\right) &= Var\left(\hat{\lambda}_{uncorr}\right)\left(1 + \hat{\lambda}' \Sigma_F^{-1} \hat{\lambda}\right) + \frac{1}{T} \Sigma_F \\ cov\left(\hat{\alpha}_{corr}\right) &= \left(1 + \hat{\lambda}' \Sigma_F^{-1} \hat{\lambda}\right) cov\left(\hat{\alpha}_{uncorr}\right). \end{aligned}$$

Cochrane (2001) (pp.241-242) describes a GMM procedure that carries out the time series and cross-sectional estimation jointly. It corrects for serial correlation and conditional heteroskedasticity in the residuals $\epsilon_t^i$ and for correlation of $\alpha_t^i$ across assets.

## A.3   Data Appendix

**Labor Income plus Transfers**   1929-2002: Bureau of economic Analysis, NIPA Table 2.1, Aggregate labor income is the sum of wage and salary disbursements (line 2), other labor income (line 9), and proprietors' income with inventory valuation and capital consumption adjustments (line 10). Transfers is transfer payments

to persons (line 16) minus personal contributions for social insurance (line 23). Prior to 1929, labor income plus transfers is 0.65 times nominal GDP. Nominal GDP Between 1929 and 2002, the ratio of labor income plus transfers to nominal GDP stays between .65 and .70 and equals .65 in 1929 and 1930. Nominal GDP for 1889-1928 is from Maddison (2001).

**Number of Households**  For 1889-1945: Census (1976), series A335, A2, and A7. Household data are for 1880, 1890 1900, 1910, 1920, 1930, and 1940, while the population data are annual. In constructing an annual series for the number of households, we assume that the number of persons per household declines linearly in between the decade observations. For 1945-2002: U.S. Bureau of the Census, table HH-1, Households by Type: 1940: Present.

**Price Indices**  All Items ($p^a$) 1890-1912: Census (1976), Cost of Living Index (series L38). 1913-2002: CPI (BLS), base year is 1982-84. In parenthesis are the last letters of the BLS code. All codes start by CUUR0000S. Total price index ($p^a$): All items (code A0). Shelter ($p^h$): Item rent of primary residence (code EHA). Food ($p^c$) 1913-2002: Item food (code AF1). Apparel ($p^{app}$) 1913-2002: Item apparel (code AA).

## Aggregate Consumption

**Total Consumption Expenditures** $C$  1909-1928: Census (1976), Total Consumption Expenditures (series G470). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929-2002: Bureau of economic Analysis, NIPA table 2.2. Total Consumption expenditures is personal consumption expenditures (line 1).

**Housing Services Consumption** $C_{rent}$  1909-1928: Census (1976), Rent and Imputed Rent (series G477). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929-2002: Bureau of economic Analysis, NIPA table 2.2. Housing services consumption $H$ is nominal consumption on housing services (line 14).

**Food Consumption** $C_{food}$  1909-1928: Census (1976), Food (series G471 + G472 + G473). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929-2002: Bureau of economic Analysis, NIPA table 2.2. Nominal consumption of food (line 7).

**Apparel Consumption** $C_{cloth}$  For 1909-1928: Census (1976), Apparel (series G474). The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. For 1929-2002: Bureau of economic Analysis, NIPA table 2.2. Nominal consumption of clothing and shoes (line 8).

**Housing Expenditure share** $A$  It is computed in two ways. The nondurable consumption share $\alpha = 1 - A$

First, for 1909-2002, the housing expenditure share is computed as rent and imputed rent divided by total consumption expenditures minus rent and imputed rent and minus apparel. The observations are for 1909, 1914, 1919, 1921, 1923, 1925, and 1927. The cell entries for 1920, 22, 24, 26, and 28 are the average of the adjacent cells. The corresponding measure for the nondurable consumption share is $\alpha_1 = 1 - A_1$.

Second, for 1929-2002: The housing expenditure share is $A$ is nominal consumption on housing services (line 14) divided by nominal consumption of non-durables (line 6) and services (line 13) minus clothing and shoes (line 8). The corresponding measure for the nondurable consumption share is $\alpha_2 = 1 - A_2$.

**Real Per Household Consumption Growth** $dc$  It is computed in two ways. First, for 1922-2002, we construct *real* nondurable consumption, as total consumption deflated by the all items CPI minus rent deflated by the rent component of the CPI minus clothes and shoes deflated by the apparel CPI component. Per household variables are obtained by dividing by the number of households. The missing data for 1924, 26, and 28 are interpolated using Mehra & Prescott (1985) real per capita consumption growth. The growth rate $dc_1$ is the log difference multiplied by 100. Second, for 1930-2002, we define *real* nondurable and services consumption (NDS), as nondurable consumption deflated by the NIPA nondurable price index plus services deflated by the NIPA services price index minus housing services deflated by the NIPA housing services price index minus clothes and shoes deflated by the NIPA clothes and shoes price index. The basis of all NIPA price deflators is 1996=100. They are not the same as the corresponding CPI components from the BLS. Per household variables are obtained by dividing by the number of households. The growth rate $dc_2$ is the log difference of $NDS$ multiplied by 100.

**Rental Price Growth** $d\rho$  It is computed in two ways. First, for 1913-2002 we use the ratio of CPI rent component to the CPI food component: $\rho = \frac{p^h}{p^c}$. The growth rate $d\rho_1$ is the log difference multiplied by 100. Second, for 1930-2002, we construct nominal non-durable consumption (non-durables plus services excluding housing services and excluding clothes and shoes) and real non-durable consumption (where each item is separately deflated by its own NIPA price deflator, basis 1996=100). The deflator for nondurable consumption is then the ratio of the nominal to the real non-durable consumption series. The relative rental price is then the ratio of the price deflator for housing services to the price deflator for nondurable consumption. The growth rate $d\rho_2$ is the log difference multiplied by 100.

**Financial Wealth**  In order to construct the consumption wealth ratio at annual frequency (Lettau and Ludvigson (2001)) we need a measure of financial wealth. For 1945-2002: Flow of Funds, Federal Reserve Board, Balance sheet of households and non-profit organizations (B.100). Line 8: Financial Asses (FL154090005.Q). For 1926-1945: Total deposits, all commercial banks, NBER Macro-history database (series 14145). We assume that the ratio of deposits to total wealth decreases slowly from .205 to .185, its level in 1945 (FoF deposits series).

# B   Tables and Figures

| $log(z_t)$ | $\theta$ | $\lambda$ | $log(\rho_t)$ | $\theta$ | $\lambda$ |
|---|---|---|---|---|---|
| $log(z_t^1)$ | .925 (.039) | | $log(\rho_t^1)$ | .953 (.027) | |
| $log(z_t^2)$ | .940 (.037) | | $log(\rho_t^2)$ | .941 (.023) | |
| $log(z_t^1)$ | .890 (.033) | .824 (.141) | $log(\rho_t^1)$ | .955 (.027) | .102 (.181) |
| $log(z_t^2)$ | .940 (.032) | .816 (.159) | $log(\rho_t^2)$ | .932 (.023) | -.321 (.158) |
| $log(z_t^1)$ | .950 (.033) | | $log(\rho_t^1)$ | .851 (.056) | |
| $log(z_t^2)$ | .936 (.026) | | $log(\rho_t^2)$ | .911 (.046) | |
| $log(z_t^1)$ | .957 (.033) | .824 (.180) | $log(\rho_t^1)$ | .817 (.054) | .261 (.240) |
| $log(z_t^2)$ | .952 (.027) | .816 (.181) | $log(\rho_t^2)$ | .896 (.047) | .259 (.172) |

Table 9: Expenditure Share and Rental Price Regression Results Regression results for the log expenditure share $log(z_t)$ and the rental prices $log(\rho_t)$ in an AR(1)-specification with or without consumption growth as additional regressor. $\theta$ is the AR(1) coefficient and $\lambda$ is the consumption growth coefficient. The upper panel reports the results for the entire sample, while the lower panel reports the results for the post-war sample. The variables with subscript 1 are available for 1926-2002. The variables with subscript 2 are available for 1929-2002. The Data Appendix contains the definitions of all the variables .

| Model | $CCAPM$ | $HCAPM$ |
|---|---|---|
| $\hat{\gamma}$ | 10.74 | 4.30 |
| s.e. | [11.94] | [4.82] |
| $\hat{\epsilon}$ | | .81 |
| s.e. | | [.31] |
| $J$ | 26.15 | 24.25 |
| $p$ | 0.0002 | .0002 |

Table 10: GMM Coefficient Estimates CCAPM and HCAPM. These estimates were obtained using the 7 moment conditions. $\beta$ was fixed at .98. The identity weighting matrix was used in estimating the model. We use the Newey West procedure with 3 lags which corrects for serial correlation and heteroskedasticity. We use a repeated GMM procedure, allowing for ten iterations.

47

| VW | Mean | St. Dev | Mean | St. Dev. | B/M |
|------|------|---------|------|----------|------|
| RVW | 7.9 | 20.9 | | | |
| S1B1 | 3.8 | 38.0 | 7.3 | 40.4 | 0.35 |
| S1B2 | 9.7 | 37.4 | 15.6 | 45.2 | 0.70 |
| S1B3 | 13.8 | 35.9 | 17.6 | 40.1 | 1.03 |
| S1B4 | 17.8 | 44.6 | 22.1 | 53.4 | 1.55 |
| S1B5 | 18.2 | 37.6 | 26.2 | 48.6 | 5.52 |
| S2B1 | 6.9 | 32.3 | 7.1 | 35.5 | 0.38 |
| S2B2 | 11.8 | 30.3 | 12.6 | 32.6 | 0.70 |
| S2B3 | 13.7 | 30.5 | 15.0 | 33.6 | 1.03 |
| S2B4 | 14.7 | 32.8 | 15.3 | 35.1 | 1.52 |
| S2B5 | 15.1 | 33.0 | 16.5 | 36.2 | 3.76 |
| S3B1 | 8.5 | 30.5 | 8.0 | 30.2 | 0.38 |
| S3B2 | 11.4 | 28.0 | 11.7 | 29.8 | 0.69 |
| S3B3 | 12.3 | 27.2 | 12.8 | 28.2 | 1.02 |
| S3B4 | 13.1 | 27.8 | 13.7 | 28.1 | 1.51 |
| S3B5 | 13.9 | 32.6 | 14.9 | 32.8 | 3.40 |
| S4B1 | 8.4 | 24.0 | 8.4 | 24.5 | 0.37 |
| S4B2 | 9.2 | 25.6 | 9.4 | 26.2 | 0.69 |
| S4B3 | 11.1 | 25.9 | 11.4 | 26.9 | 1.01 |
| S4B4 | 12.1 | 27.0 | 12.4 | 27.8 | 1.49 |
| S4B5 | 13.6 | 34.5 | 14.3 | 36.6 | 3.35 |
| S5B1 | 7.6 | 21.6 | 6.9 | 21.1 | 0.33 |
| S5B2 | 7.2 | 19.5 | 8.4 | 20.3 | 0.68 |
| S5B3 | 8.8 | 22.1 | 9.5 | 23.7 | 1.00 |
| S5B4 | 9.5 | 25.4 | 10.6 | 27.3 | 1.50 |
| S5B5 | 11.0 | 33.7 | 11.5 | 34.4 | 1.59 |

Table 11: Annual Portfolio Returns 1927-2002. Time-series mean and standard deviation of gross portfolio returns. All returns are in excess of a 1 month T-bill return. The first two columns are value-weighted portfolios, the next two for equally-weighted portfolios and the last column denotes the value weighted portfolio average book-market ratio. All data are from Kenneth French.

| mymo | $\varpi$ | $\vartheta$ | $\chi$ | LHR | ADF |
|---|---|---|---|---|---|
| 1889-2002 | -1.5164 | -.0066 | 1.8010 | 21.07* | -3.46** |
| 1925-2002 | -1.2064 | -.0164 | 2.3546 | 35.04*** | -5.38*** |
| 1945-2002 | -1.2987 | -.0176 | 2.6511 | 30.77*** | -3.06 ** |
| 1889-2002 | -1 | -.0102 | 1.6974 | | -3.08** |
| 1925-2002 | -1 | -.0148 | 2.0624 | | -4.16*** |
| 1945-2002 | -1 | -.0233 | 2.8302 | | -2.89* |
| myrw | $\varpi$ | $\vartheta$ | $\chi$ | LHR | ADF |
| 1889-2002 | -1.8255 | .0084 | -.3659 | 15.16 | -3.46** |
| 1925-2002 | -.5480 | -.0120 | .1895 | 34.00*** | -4.01*** |
| 1945-2002 | -.4108 | -.0147 | .3311 | 25.00* | -3.32** |
| 1889-2002 | -1 | .0011 | -.4434 | | -2.29 |
| 1925-2002 | -1 | -.0023 | -.1720 | | -3.42** |
| 1945-2002 | -1 | -.0083 | .3784 | | -3.51** |
| myfa | $\varpi$ | $\vartheta$ | $\chi$ | LHR | ADF |
| 1925-2001 | -1.0137 | -.0004 | -.2257 | 52.01*** | -4.70*** |
| 1945-2001 | -1.0055 | -.0011 | -.1624 | 28.45** | -3.41** |
| 1925-2001 | -1 | -.0005 | -.2254 | | -4.65*** |
| 1945-2001 | -1 | -.0026 | -.0365 | | -2.88* |

Table 12: Cointegration Relationship. The second through fourth columns show coefficient estimates for the cointegration relationship. The cointegration relationship is estimated for 1889-2002, 1925-2002, and 1945-2002 for $hv = mo$ and $hv = rw$, and for 1925-2001 and 1945-2001 for $hv = fa$. Coefficient estimates for $D_k$ are not reported. We set $K = 8$ in the VECM. The fifth column shows the likelihood ratio statistic of the Johansen cointegration test. It assumes a constant and a trend in the cointegration relationship. The last column shows the value of the ADF test statistic (with K=8 lags) of the null hypothesis of a unit root in the resulting $my$ series. For both test, significance at the 10% level is denoted by a *, significance at the 5% level by **, and at the 1% level by ***. The second part of each block gives the parameter estimates of an OLS regression of $\log hv - \log ylt$ on a constant and a trend.

| Dependent var: | $\Delta \log(hv_t^{mo})$ | $\Delta \log(y_t)$ | $\Delta \log(hv_t^{rw})$ | $\Delta \log(y_t)$ | $\Delta \log(hv_t^{fa})$ | $\Delta \log(y_t)$ |
|---|---|---|---|---|---|---|
| $\Delta \log(hv_{t-1})$ | .75 | -.15 | .14 | -.39 | .33 | .21 |
| (s.e.) | (.09) | (.13) | (.10) | (.13) | (.12) | (.19) |
| $\Delta \log(hv_{t-2})$ | -.20 | .07 | -.17 | -.11 | -.29 | -.50 |
| (s.e.) | (.09) | (.13) | (.11) | (.13) | (.12) | (.19) |
| $\Delta \log(y_{t-1})$ | -.09 | .24 | -.06 | .34 | .04 | .73 |
| (s.e.) | (.07) | (.10) | (.09) | (.11) | (.06) | (.11) |
| $\Delta \log(y_{t-2})$ | -.11 | -.05 | -.02 | -.01 | -.05 | -.18 |
| (s.e.) | (.07) | (.10) | (.09) | (.11) | (.07) | (.12) |
| $my_{t-1}$ | **-.038** | **.023** | **-.037** | **.042** | **-.072** | **.110** |
| (s.e.) | (.015) | (.021) | (.027) | (.033) | (.032) | (.052) |
| $R^2$ | 49.3 | 7.6 | 5.8 | 15.3 | 22.8 | 48.4 |

Table 13: Estimates from Bivariate VAR. For each of the three measures of the housing collateral ratio, a bivariate VAR is estimated. The dependent variables are the current growth rate in housing wealth and in labor income plus transfers. The dependent variables are the first two lags of these variables and the one-period lagged housing collateral ratio (mymo, myrw, and myfa respectively). A constant is included in the system, but its coefficients are not reported. The sample period is 1889-2002 for the first two VAR's and 1925-2001 for the last VAR.

| | 1926-2002 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $mymo$ | .16 | .28 | .39 | .48 | .60 | .73* | .89** | 1.05*** | 1.20*** | 1.31*** |
| | $R^2$ | .02 | .03 | .04 | .05 | .07 | .10 | .16 | .20 | .23 | .24 |
| 2 | $\log d - \log p$ | .14*** | .25*** | .33** | .36* | .42 | .51* | .66** | .83*** | 1.03*** | 1.20*** |
| | $R^2$ | .07 | .09 | .09 | .08 | .08 | .10 | .17 | .23 | .31 | .35 |
| 3 | $cay$ | -.18 | -.36 | -.63 | -.87 | -.98 | -.95 | -.89 | -.89 | -.84 | -.88 |
| | $R^2$ | .01 | .01 | .03 | .05 | .05 | .05 | .04 | .04 | .03 | .03 |
| 4 | $lc$ | .22 | .41 | .57 | .66 | .73 | .72 | .73 | .74 | .74 | .81 |
| | $R^2$ | .01 | .02 | .03 | .04 | .04 | .04 | .04 | .04 | .03 | .03 |
| 5 | $mymo$ | .09 | .17 | .25 | .35 | .46 | .57* | .69** | .80*** | .90*** | .97*** |
| | $\log d - \log p$ | .13** | .22** | .30* | .30 | .34 | .40 | .53* | .67** | .85*** | 1.00*** |
| | $R^2$ | .07 | .10 | .11 | .10 | .12 | .16 | .25 | .34 | .43 | .47 |
| 6 | $mymo$ | .14 | .23 | .30 | .44 | .65 | 1.00 | 1.42** | 1.83*** | 2.22*** | 2.43*** |
| | $lc$ | .04 | .10 | .19 | .09 | -.12 | -.59 | -1.12 | -1.63 | -2.14* | -2.35* |
| | $R^2$ | .02 | .03 | .04 | .05 | .07 | .11 | .19 | .27 | .34 | .35 |
| 7 | $mymo$ | -.08 | -.14 | -.21 | -.08 | .11 | .45 | .80 | 1.09** | 1.33*** | 1.36*** |
| | $\log d - \log p$ | .15** | .27** | .38** | .39* | .42 | .43 | .50* | .59** | .73*** | .89*** |
| | $lc$ | .32 | .61 | .89 | .82 | .65 | .21 | -.21 | -.54 | -.81 | -.74 |
| | $R^2$ | .08 | .11 | .14 | .12 | .13 | .17 | .26 | .35 | .44 | .48 |

| | 1945-2002 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | $mymo$ | -.01 | .08 | .23 | .41** | .62*** | .81*** | 1.00*** | 1.22*** | 1.49*** | 1.71*** |
| | $R^2$ | .00 | .00 | .03 | .08 | .13 | .18 | .23 | .29 | .37 | .43 |
| 9 | $\log d - \log p$ | .14* | .24** | .32** | .41** | .54** | .64** | .79*** | .93*** | 1.12*** | 1.27*** |
| | $R^2$ | .11 | .14 | .17 | .20 | .22 | .23 | .28 | .33 | .39 | .44 |
| 10 | $cay$ | 6.52*** | 12.27*** | 14.27*** | 14.61*** | 17.26*** | 21.34*** | 21.26*** | 20.06*** | 23.66 | 26.66 |
| | $R^2$ | .27 | .49 | .54 | .42 | .40 | .49 | .44 | .35 | .39 | .41 |
| 11 | $lc$ | -.42 | -.75 | -1.18 | -1.59 | -1.76 | -1.91 | -2.06 | -2.46 | -1.93 | -1.61 |
| | $R^2$ | .01 | .02 | .04 | .06 | .06 | .05 | .05 | .06 | .03 | .02 |
| 12 | $mymo$ | -.08 | -.07 | -.01 | .11 | .24 | .39* | .52** | .67** | .86*** | 1.01*** |
| | $\log d - \log p$ | .15** | .24** | .32** | .39 | .47* | .51* | .59** | .66** | .76*** | .83*** |
| | $R^2$ | .12 | .14 | .17 | .21 | .24 | .26 | .33 | .41 | .50 | .57 |
| 13 | $mymo$ | .00 | .07 | .21 | .42** | .63*** | .85*** | 1.07*** | 1.33*** | 1.59*** | 1.80*** |
| | $lc$ | -.42 | -.81 | -1.36 | -1.97* | -2.39* | -2.83** | -3.33** | -4.17*** | -4.11*** | -4.16*** |
| | $R^2$ | .01 | .03 | .07 | .15 | .20 | .26 | .34 | .44 | .50 | .54 |
| 14 | $mymo$ | -.06 | -.04 | .04 | .19 | .34 | .52* | .70** | .92*** | 1.12*** | 1.28*** |
| | $\log d - \log p$ | .19** | .31*** | .40*** | .47*** | .53** | .54** | .58*** | .62*** | .67*** | .72*** |
| | $lc$ | -1.01** | -1.60** | -2.12** | -2.57** | -2.81** | -3.00** | -3.28** | -3.93*** | -3.61** | -3.43*** |
| | $R^2$ | .18 | .23 | .29 | .36 | .37 | .38 | .45 | .55 | .60 | .64 |

Table 14: Long-Horizon Predictability Regressions.

The regressor in the first row is the rescaled housing collateral ratio $mymo$, estimated for the period 1925-2002. In row 2, it is the log dividend yield. In row 3, $cay$ is the consumption-wealth ratio $cay = \log c - .259 \log fw - .682 \log y + 0.035$, estimated with data for 1930-2002. It stays always positive. In row 4, the regressor is the labor income plus transfers to total consumption expenditures ratio. Row 5 includes both the housing collateral ratio and the log dividend yield as regressors. Row 6 includes both the housing collateral ratio and the labor income to consumption ratio as regressors. Row 7 includes the housing collateral ratio, the log dividend yield, and the labor income to consumption ratio as regressors. The regressor in the row 8 is the housing collateral ratio $mymo$, estimated for the post-war period. In row 10, we use the cay ratio constructed by Lettau and Ludvigson for the period 1948-2001: cay = c - 0.292 fw - 0.597 y. The predictability results in this row are for 1948-2001. The first row of each panel reports least squares estimates for $b_1$. Newey-West HAC standard errors are used to denote significance at the 1 percent (***), 5 percent (**) and 10 percent level (*). The second row reports the $R^2$ for this regression.

| | $\lambda_0$ | $\lambda_c$ | $\lambda_\rho$ | $\lambda_{my}$ | $\lambda_{my,c}$ | $\lambda_{my,\rho}$ | $\lambda_{my,c_{-1}}$ | $\lambda_{my,\rho_{-1}}$ | $\lambda_{my,c_{-2}}$ | $\lambda_{my,\rho_{-2}}$ | $\lambda_{my,c_{-3}}$ | $\lambda_{my,\rho_{-3}}$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Separable Prefs. myrw | 3.28 (2.31) [3.55] | 2.51 (1.00) [1.60] | | -.03 (.03) [.06] | 1.52 (.47) [.75] | | | | | | | | **90.3** 89.0 |
| 2 Separable Prefs. myrw | 3.47 (2.20) [3.37] | 2.39 (1.12) [1.77] | | -.02 (.03) [.05] | 1.48 (.51) [.80] | | .13 (.81) [1.25] | | | | | | **90.7** 88.9 |
| 3 Separable Prefs. myrw | 3.48 (2.26) [3.45] | 2.38 (.98) [1.55] | | -.02 (.04) [.06] | 1.48 (.46) [.72] | | .13 (.81) [1.25] | | -.65 (.98) [1.51] | | | | **90.7** 88.3 |
| 4 Separable Prefs. myrw | 3.42 (2.26) [3.45] | 1.44 (1.10) [1.72] | | -.04 (.04) [.06] | .79 (.52) [.81] | | -.17 (.88) [1.36] | | -.62 (.97) [1.49] | | -1.42 (.60) [.94] | | **94.9** 93.2 |
| 5 Non-Sep. Prefs. myrw | 1.80 (2.69) [4.63] | 3.11 (.84) [1.51] | .25 (.23) [.41] | -.03 (.03) [.06] | 1.82 (.43) [.76] | .06 (.10) [.17] | | | | | | | **90.8** 88.5 |
| 6 Non-Sep. Prefs. myrw | 1.94 (2.56) [4.36] | 2.91 (.82) [1.46] | .20 (.21) [.39] | -.02 (.05) [.08] | 1.74 (.43) [.76] | .04 (.09) [.15] | .13 (.45) [.78] | -.04 (.17) [.29] | | | | | **91.1** 87.7 |
| 7 Non-Sep. Prefs. myrw | 1.94 (2.69) [4.60] | 2.83 (.89) [1.58] | .13 (.19) [.36] | -.01 (.05) [.08] | 1.75 (.48) [.83] | .04 (.08) [.15] | .17 (.44) [.78] | -.03 (.15) [.26] | -.66 (.53) [.92] | .05 (.18) [.31] | | | **91.4** 86.5 |
| 8 Non-Sep. Prefs. myrw | 2.57 (2.66) [5.05] | 1.46 (.80) [1.57] | -.05 (.21) [.42] | -.01 (.05) [.09] | .88 (.40) [.79] | -.05 (.08) [.16] | -.75 (.64) [1.22] | -.20 (.18) [.35] | -.81 (.55) [1.05] | -.01 (.19) [.37] | -1.33 (.60) [1.15] | .03 (.20) [.38] | **95.9** 92.8 |

Table 15: Cross-Sectional Results with Lagged Aggregate Pricing Factors. The asset pricing factors are $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ in row 1, row 2-4 contain in addition the one through four period lagged interaction terms $(my^{max} - my_t)\,\Delta\log(c_{t+1-k})$, $k = 1 - 4$. Row 5 contains the factors $\Delta\log(c_{t+1})$, $\Delta\log(\alpha_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ and $(my^{max} - my_t)\,\Delta\log(\alpha_{t+1})$. Rows 6-8 contain in addition the one through four period lagged interaction terms $(my^{max} - my_t)\,\Delta\log(c_{t+1-k})$, $k = 1 - 4$ and $(my^{max} - my_t)\,\Delta\log(\alpha_{t+1-k})$, $k = 1 - 4$. The housing collateral variable is $myrw$ in all rows. The estimation is done using the Fama-MacBeth procedure. The same returns are used in each row: 1930-2002, i.e. in row 4, consumption growth up to 1926 is used. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the $R^2$ and the adjusted $R^2$ just below it.

| | $\lambda_0$ | $\lambda_c$ | $\lambda_\rho$ | $\lambda_{my}$ | $\lambda_{my.c}$ | $\lambda_{my.\rho}$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| 1 | 9.30 | 2.43 | | | | | **49.1** |
| CCAPM | (2.69) | (.84) | | | | | 47.0 |
| | [4.64] | [1.47] | | | | | |
| 2 | 8.73 | 2.46 | .16 | | | | **49.4** |
| HCAPM | (4.09) | (.91) | (.17) | | | | 45.0 |
| | [7.15] | [1.60] | [.30] | | | | |
| 3 | 7.12 | 1.66 | | -.11 | -.03 | | **70.0** |
| Separable Prefs. | (2.19) | (.66) | | (.09) | (.31) | | 65.4 |
| mymo | [4.43] | [1.36] | | [.18] | [.54] | | |
| 4 | 9.61 | 1.15 | | .02 | -.04 | | **69.9** |
| Separable Prefs. | (3.23) | (.67) | | (.06) | (.13) | | 65.8 |
| myrw | [6.50] | [1.37] | | [.12] | [.27] | | |
| 5 | 4.30 | 1.28 | | -.06 | -.15 | | **83.8** |
| Separable Prefs. | (2.62) | (.68) | | (.05) | (.13) | | 81.6 |
| myfa | [5.37] | [1.42] | | [.10] | [.27] | | |
| 6 | 9.06 | 1.22 | .18 | -.08 | -.17 | -.04 | **78.4** |
| Non-Sep. Prefs. | (2.79) | (.61) | (.12) | (.07) | (.37) | (.04) | 73.1 |
| mymo | [6.24] | [1.39] | [.28] | [.16] | [.83] | [.09] | |
| 7 | 10.44 | 1.06 | .18 | .00 | -.12 | -.01 | **70.7** |
| Non-Sep. Prefs. | (3.72) | (.58) | (.15) | (.05) | (.11) | (.01) | 63.4 |
| myrw | [8.05] | [1.29] | [.33] | [.10] | [.24] | [.03] | |
| 8 | 7.38 | .53 | .07 | -.05 | -.27 | -.01 | **85.8** |
| Non-Sep. Prefs. | (2.98) | (.67) | (.10) | (.03) | (.17) | (.01) | 82.2 |
| myfa | [5.66] | [1.29] | [.20] | [.06] | [.33] | [.03] | |

Table 16: Post-war Cross-Sectional Results. The sample period is 1945-2002. The asset pricing factors are $\Delta \log(c_{t+1})$ in row 1, $\Delta \log(c_{t+1})$ and $\Delta \log(\alpha_{t+1})$ in row 2, $\Delta \log(c_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t) \Delta \log(c_{t+1})$ in rows 3-5 and $\Delta \log(c_{t+1})$, $\Delta \log(\alpha_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t) \Delta \log(c_{t+1})$ and $(my^{max} - my_t) \Delta \log(\alpha_{t+1})$ in rows 6-8. The housing collateral variable is $mymo$ in rows 3 and 6, $myrw$ in row 4 and 7 and $myfa$ in row 5 and 8. $my$ is estimated with data from 1945-2002. The variables used are $\Delta c_2$, $\Delta \alpha_2$ described in the data appendix. The estimation is done using the Fama-MacBeth procedure. The set of test assets is $T1$. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the $R^2$ and the adjusted $R^2$ just below it.

| Portfolio | All States | Good States | Bad States |
|-----------|------------|-------------|------------|
| RVW | 1.56 | 0.89 | 2.34 |
| S1B1 | 1.48 | 1.27 | 1.76 |
| S1B2 | 2.28 | .47 | 4.34 |
| S1B3 | 3.20 | 1.70 | 4.95 |
| S1B4 | 3.39 | -.70 | 8.00 |
| S1B5 | 3.96 | .85 | 7.50 |
| S2B1 | 1.94 | .98 | 3.04 |
| S2B2 | 2.23 | 1.25 | 3.37 |
| S2B3 | 2.42 | .33 | 4.80 |
| S2B4 | 2.68 | .49 | 5.17 |
| S2B5 | 2.89 | .58 | 5.51 |
| S3B1 | 1.80 | .33 | 3.46 |
| S3B2 | 2.27 | .61 | 4.17 |
| S3B3 | 2.52 | 1.16 | 4.08 |
| S3B4 | 2.53 | .91 | 4.39 |
| S3B5 | 2.99 | .53 | 5.79 |
| S4B1 | 1.05 | .37 | 1.84 |
| S4B2 | 1.58 | .04 | 3.33 |
| S4B3 | 1.71 | .65 | 2.93 |
| S4B4 | 2.34 | .04 | 4.93 |
| S4B5 | 1.62 | -1.10 | 4.67 |
| S5B1 | 1.54 | 1.20 | 1.95 |
| S5B2 | 1.21 | .76 | 1.74 |
| S5B3 | 1.57 | .77 | 2.50 |
| S5B4 | 1.69 | .32 | 3.25 |
| S5B5 | 1.57 | -1.66 | 5.18 |

Table 17: Consumption Betas. Consumption betas are computed as $\beta_t = \beta_c + \beta_{c.my}(my^{max} - my_t)$. Good states are states where $(my^{max} - myrw_t)$ is below below zero and bad states are times where $myrw$ is one standard deviation above zero (11 observations each). The third and fourth column report the average consumption betas in good states and bad states respectively. The sample is 1926-2002. Lettau and Ludvigson (2001b) do the same exercise for their scaling variable, the consumption-wealth ratio.

| Model | $\tilde{\lambda}_0$ | $\tilde{\lambda}_1$ | $\tilde{\lambda}_2$ | $\tilde{\lambda}_3$ | Size | $R^2$ |
|---|---|---|---|---|---|---|
| 1 | 2.64 | 6.31 | | | -1.28 | 44.2 |
| Static CAPM | (4.11) | (4.72) | | | (.35) | 39.2 |
| Sharpe-Lintner | [5.27] | [6.49] | | | [.48] | |
| 2 | 5.53 | 3.80 | 4.12 | | -1.32 | 54.5 |
| Human Capital-CAPM | (4.08) | (4.67) | (1.29) | | (.35) | 48.0 |
| Jagannathan-Wang | [5.93] | [7.18] | [1.99] | | [.55] | |
| 3 | 12.93 | −4.00 | .05 | −5.26 | -1.46 | 64.7 |
| $lc$-conditional CAPM | (5.67) | (5.96) | (0.03) | (6.09) | (.43) | 57.6 |
| Santos-Veronesi | [10.09] | [10.87] | [0.05] | [11.11] | [.78] | |
| 4 | 6.20 | .86 | | | -1.40 | 44.2 |
| Static CCAPM | (2.33) | (.93) | | | (.36) | 39.2 |
| Breeden-Lucas | [3.14] | [1.33] | | | [.52] | |
| 5 | 4.57 | .64 | −.62 | | -1.03 | 57.5 |
| HCAPM | (2.26) | (.91) | (.27) | | (.32) | 51.4 |
| Non-Separabililty | [2.91] | [1.25] | [.36] | | [.45] | |
| 6 | 3.60 | 2.34 | −.04 | 1.43 | .07 | 87.8 |
| Collateral-CAPM | (2.21) | (.90) | (.04) | (.43) | (.45) | 85.4 |
| this paper | [3.38] | [1.45] | [.07] | [.68] | [.71] | |
| Model | $\tilde{\lambda}_0$ | $\tilde{\lambda}_1$ | $\tilde{\lambda}_2$ | $\tilde{\lambda}_3$ | Value | $R^2$ |
| 1 | 4.30 | 4.20 | | | .24 | 34.8 |
| Static CAPM | (4.28) | (4.87) | | | (.09) | 28.9 |
| Sharpe-Lintner | [4.62] | [5.76] | | | [.12] | |
| 2 | 7.80 | .99 | 3.43 | | .23 | 43.4 |
| Human Capital-CAPM | (4.37) | (4.93) | (1.25) | | (.08) | 35.3 |
| Jagannathan-Wang | [5.30] | [6.44] | [1.65] | | [.12] | |
| 3 | 14.91 | −6.92 | .05 | −8.04 | .22 | 68.8 |
| $lc$-conditional CAPM | (6.08) | (6.45) | (0.02) | (6.62) | (.08) | 62.5 |
| Santos-Veronesi | [9.38] | [10.23] | [0.04] | [10.49] | [.14] | |
| 4 | 8.05 | 2.10 | | | -.06 | 17.7 |
| Static CCAPM | (2.46) | (.83) | | | (.05) | 10.2 |
| Breeden-Lucas | [2.82] | [1.05] | | | [.09] | |
| 5 | 7.20 | 1.37 | −.70 | | -.10 | 43.1 |
| HCAPM | (2.42) | (.79) | (.28) | | (.05) | 35.0 |
| Non-Separabililty | [3.13] | [1.10] | [.39] | | [.10] | |
| 6 | 3.53 | 2.71 | −.01 | 1.76 | -.03 | 78.0 |
| Collateral-CAPM | (2.33) | (.77) | (.04) | (.42) | (.06) | 73.6 |
| this paper | [3.95] | [1.37] | [.06] | [.73] | [.13] | |

Table 18: Residual Size and Value Effects, 1926-2002. Row 1: factor is $R_{t+1}^{vw,e}$. Row 2: factors are $R_{t+1}^{vw,e}$ and $R_{t+1}^{hc,e}$. Row 3 factors: $R_{t+1}^{vw,e}$, $lc_t$ and $lc_t R_{t+1}^{vw,e}$. Row 4: $\Delta\log(c_{t+1})$. Row 5: $\Delta\log(c_{t+1})$, $A_t\Delta\log(\rho_{t+1})$. Row 6 is the collateral model: $\Delta\log(c_{t+1})$, , $myrw^{max} - myrw_t$, and $(myrw^{max} - myrw_t)\Delta\log(c_{t+1})$. Size is log of portfolio's market capitalization. Value is log of portfolio's book-to-market ratio.

|       |         | B1    | B2    | B3    | B4    | B5    | B6    | B7    | B8    | B9    | B10   |
|-------|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $mymo$ | $b_0$  | -4.60 | -4.47 | -4.26 | -4.46 | -3.75 | -3.80 | -4.05 | -3.27 | -3.50 | -3.44 |
|       | $b_1$   | -1.93 | -.81  | -1.35 | -1.62 | -.76  | -.53  | -1.12 | -.10  | 1.01  | 1.79  |
|       | $R^2$   | .27   | .07   | .22   | .21   | .10   | .18   | .49   | .01   | .30   | .34   |
|       | $s.e.$  | .47   | .41   | .36   | .46   | .33   | .16   | .17   | .18   | .22   | .36   |
| $myrw$ | $b_0$  | -4.69 | -4.51 | -4.32 | -4.53 | -3.78 | -3.83 | -4.10 | -3.27 | -3.45 | -3.36 |
|       | $b_1$   | -3.08 | -1.72 | -2.30 | -2.52 | -2.05 | -1.09 | -1.08 | .21   | 1.69  | 2.57  |
|       | $R^2$   | .57   | .29   | .56   | .43   | .62   | .65   | .39   | .03   | .72   | .60   |
|       | $s.e.$  | .36   | .36   | .27   | .39   | .21   | .11   | .18   | .18   | .14   | .28   |
| $myfa$ | $b_0$  | -4.69 | -4.51 | -4.32 | -4.53 | -3.78 | -3.83 | -4.10 | -3.28 | -3.46 | -3.36 |
|       | $b_1$   | -3.53 | -3.15 | -3.4  | -3.52 | -3.03 | -1.38 | -.91  | 1.59  | 2.28  | 2.39  |
|       | $R^2$   | .13   | .17   | .22   | .15   | .24   | .19   | .05   | .24   | .23   | .09   |
|       | $s.e.$  | .51   | .39   | .37   | .48   | .30   | .16   | .23   | .16   | .23   | .42   |

Table 19: Dividend Share and the Housing Collateral Ratio. The results are for the regression $log(d_t) - log(y_t) = b_0 + b_1 my_t + \epsilon_t$, where $d_t$ is the nominal dividend on each of 10 decile value portfolios and $y_t$ is the nominal aggregate labor income plus transfers. The regressor in the first (second and third) row is the annual housing collateral ratio $mymo$ ($myrw$ and $myfa$), estimated for the period 1889-2002. The first row of each panel reports least squares estimates for the intercept $b_0$, the second row for the slope coefficient $b_1$. The third row reports the OLS $R^2$ for this regression. The last row gives the standard error of the regression (i.e. of $\epsilon_t$). The dividend data are annual for 1952 to 1999, constructed from the monthly dividend yield provided by Bansal, Dittmar and Lundblad (2002).

Figure 10: Response of the One-Year Excess Return to Impulse in Collateral Ratio $myfa$. Sample is 1925-2002, The standard errors are computed from 50,000 Monte Carlo simulations. The bottom panel displays the response of the market excess return.



55