NBER WORKING PAPER SERIES

THE VALUE OF A STATISTICAL LIFE AND THE COEFFICIENT OF RELATIVE RISK AVERSIONS

Louis Kaplow

Working Paper 9852
http://www.nber.org/papers/w9852

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2003

I am grateful to John Campbell, Steven Shavell, Andrei Shleifer, and Kip Viscusi for comments and the John M. Olin Center for Law, Economics, and Business at Harvard University for financial support. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research

The Value of a Statistical Life and the Coefficient of Relative Risk Aversion
Louis Kaplow
NBER Working Paper No. 9852
July 2003
JEL No. D80, G11, G12, I10, J17

ABSTRACT

Individuals' risk preferences are estimated and employed in a variety of settings, notably including choices in financial, labor, and product markets. Recent work, especially in financial economics, provides estimates of individuals' coefficients of relative risk aversion (CRRA's) in excess of one, and often significantly higher. However, it can be shown that high CRRA's imply equally high values for the income elasticity of the value of a statistical life. Yet estimates of this elasticity, derived from labor and product markets, are in the range of 0.5 to 0.6. Furthermore, it turns out that even a CRRA below one is difficult to reconcile with these elasticity estimates. Thus, there appears to be an important (additional) anomaly involving individuals' risk-taking behavior in different market settings.

Louis Kaplow
Harvard Law School
Hauser 322
Cambridge, MA 02138
and NBER
rroberts@law.harvard.edu

I. Introduction

Individuals' risk preferences have an important influence on a wide variety of behavior, ranging from portfolio selection to occupational choice. Arrow (1971, pp. 97-98), in his seminal work on behavior under uncertainty, suggested on grounds of boundedness of the utility function that individuals' coefficients of relative risk aversion (CRRA's) should be approximately 1. For a constant-relative-risk-aversion utility function, this implies that utility would depend on the log of income, so that when income doubled, marginal utility would be halved. Subsequently, Maitel (1973) suggested that a range of evidence favored an estimate of CRRA of approximately 1.5, and other earlier work suggested similarly low values for CRRA.[1]

More recently, a growing body of empirical work, especially in financial economics, provides estimates of the value of individuals's CRRA's that exceed earlier suggestions. Most of this work indicates a CRRA of 2 or more, and some, including that attempting to reconcile the equity risk premium with rational behavior, indicates that individuals' CRRA's may be above 10.[2] Similarly high estimates are also reported in studies of risk-taking behavior in other markets.[3]

Separate bodies of empirical work study individuals' risk-taking behavior in labor, product, and housing markets, examining compensating wage differentials or individuals' willingness to pay for safer products or homes in areas with lower environmental hazards. An important strand of this literature measures the value of a statistical life (VSL) based on individuals' tradeoffs between wages or prices on one hand and job or product safety on the other. Researchers have long recognized that VSL will depend positively on income. Viscusi and Aldy (2003) survey the large body of pertinent literature and extend it through their own meta-analysis, finding that the income elasticity of VSL is in the range 0.5 to 0.6, with the upper bound of the 95 percent confidence interval falling below 1.0.

It is worth considering whether estimates of individuals' behavior in risky situations are consistent across contexts. A priori, one would suppose that the same individuals, with the same utility functions, should display similar risk-taking behavior in different settings. What does not seem to have been recognized, however, is that CRRA's, estimated by financial economists and others, have particular implications for the income elasticity of VSL, measured in studies of

---

[1]For example, the work surveyed in Mehra and Prescott (1985) in their presentation of the equity premium puzzle finds CRRA's in the range from near 0 to 2.

[2]See, for example, Blake (1996), Brav, Constantinides, and Geczy (2002), Campbell (1996, 2003), Kocherlakota (1996), Mankiw and Zeldes (1991), and Palsson (1996).

[3]See, for example, Barsky, Juster, Kimball, and Shapiro (1997) (survey on willingness to gamble on lifetime income) and Mankiw (1985) (consumption spending). Some studies report lower CRRA's. See, for example, Choi and Menezes (1992) (discussing wide range of estimates in earlier literature, some below one; article examines willingness to accept gambles and suggests the implausibility of CRRA's below one and the probably that they are much greater than one) and Halek and Eisenhauer (2001) (life insurance purchases; median estimate of 0.888 and mean estimate of 3.735).

labor and product markets (and conversely).

The connection between CRRA and the income elasticity of VSL is fundamental and can be understood intuitively as follows. VSL depends (in significant part) on the marginal utility cost of expenditures to protect one's life. It follows from this that the income elasticity of VSL will depend on how the marginal utility cost of such expenditures varies with income, which is to say on the rate at which the marginal utility of income falls as income rises. An individual's CRRA is a measure of that rate.

To illustrate this relationship between CRRA and the income elasticity of VSL, consider again the case in which the CRRA is constant and equal to 1. When utility thus equals the log of income, marginal utility equals the inverse of income. Therefore, if an individual's income doubles, marginal utility and hence the utility cost of a precautionary expenditure that reduces the risk of death falls to half the prior level, so individuals with twice the income would on this account be willing to pay twice as much for a given safety measure. This factor taken alone suggests an income elasticity of VSL equal to 1 when CRRA is equal to 1; likewise, when CRRA equals 2, 10, or some higher value, the income elasticity of VSL should on this account equal 2, 10, or that higher value, respectively.

There is, however, another effect of income on VSL: the value of preserving one's life is higher when income is higher, because utility is accordingly higher. This suggests that the income elasticity of VSL should tend to exceed CRRA. It turns out that this second effect is rather small when CRRA is large (a high CRRA implies that the marginal utility of income falls rapidly, so total utility rises little as income increases), but this effect is larger when CRRA is low. Indeed, it turns out that very low values of CRRA (well below 1) do not necessarily (or even probably) imply that the income elasticity of VSL is below 1, so estimates of this elasticity in the empirical literature, which are about half the lowest value apparently obtainable from utility-maximization, seem difficult to reconcile with rational behavior.

More important for present purposes, the low estimates of this income elasticity – tightly clustered in the range of 0.5 to 0.6 – which in theory are an upper bound on CRRA, seem quite inconsistent with the high estimates of CRRA – at least 2 and perhaps 10 or more – obtained in the financial economics and other literatures. Thus, there appears to be an important anomaly involving individuals' risk-taking behavior in different market settings, one that exists even if one accepts the lower estimates of CRRA that researchers find most plausible. Of course, it is hardly novel to suggest that individuals' behavior under uncertainty may not be fully rational and, in particular, may exhibit inconsistencies in different contexts. Nevertheless, the new anomaly identified here seems significant, and its exploration may contribute to the range of efforts that seek to improve our understanding of risk-taking behavior.

The foregoing heuristic analysis of the relationship between the income elasticity of VSL and CRRA is formalized in a model in section II.[4] Section III presents extensions and discusses

---

[4]The model (in particular the extension that allows for a bequest motive) can readily be interpreted as applying as well to risks of injury rather than death. Though there is less empirical

how the analytical results relate to the pertinent empirical literatures. Section IV concludes.

II. Analysis

Individuals are assumed to choose expenditures x that reduce the probability $p(x)$ of death, $p'(x) < 0$ and $p''(x) > 0$, to maximize their expected utility

$$(1)\quad U(x) = (1 - p(x))u(y - x),$$

where y is exogenous income and u is a concave function of consumption, $c = y - x$. (The possibility of a bequest motive is considered in section III.) An individual's first-order condition regarding the choice of x is[5]

$$(2)\quad p' = -\frac{(1-p)u'}{u},$$

where $u' = du/dc$.

Expression (2) can readily be transformed into an expression for VSL.

$$(3)\quad VSL = -\frac{1}{p'} = \frac{u}{(1-p)u'}.$$

The numerator on the right side is simply the utility benefit of saving one's life, and the denominator is the marginal utility cost (in units of utility per dollar), so the ratio gives the value of a statistical life (in dollars).

Clearly, because u depends on exogenous income y, so does VSL. Let η denote the elasticity of VSL with respect to y, that is,

$$(4)\quad \eta = \frac{dVSL}{dy}\frac{y}{VSL}.$$

To analyze this elasticity, we first find

$$(5)\quad \frac{dVSL}{dy} = \frac{(1-p)u'^2(1-x_y) - u[(1-p)u''(1-x_y) - u'p'x_y]}{(1-p)^2u'^2},$$

---

work on the subject, estimates of the income elasticity of the value of statistical occupational injuries reported in Viscusi and Evans (1990) are in the range 0.6 to 1.0, similar to the values for the income elasticity of VSL, thereby reinforcing the anomaly discussed in the text.

[5]The second-order condition holds globally, justifying later substitutions using this first-order condition.

where $x_y = dx/dy$ indicates how individuals adjust their investment in reducing the probability of death as their income rises. Combining expressions (3)-(5),

$$(6)\quad \eta = y\frac{(1-p)u'^2(1-x_y) - u[(1-p)u''(1-x_y) - u'p'x_y]}{(1-p)uu'}$$

$$= \frac{yu'}{u}(1-x_y) - \frac{yu''}{u'}(1-x_y) + \frac{yp'x_y}{1-p}.$$

In the third term on the right side of (6), one can substitute for pℕ using the first-order condition (2), simplify, and combine with the first term to yield

$$(7)\quad \eta = \frac{yu'}{u}(1-2x_y) - \frac{yu''}{u'}(1-x_y).$$

Examining expression (7), the first term (ignoring the $1\text{-}2x_y$ for the moment) looks much like the elasticity of utility with respect to income, 0_{uy}. More precisely, however, the utility and marginal utility in (7) correspond to u rather than U. And the second term (ignoring the $1\text{-}x_y$ for the moment) looks like the CRRA, though again the utilities in (7) involve u rather than U. This suggests that it would be illuminating – and better relate to empirical estimates – to restate (7) in terms of 0_{uy} and CRRA.

To begin, using expression (1), we can evaluate the derivatives of U with respect to y, which are as follows:

$$(8)\quad \frac{dU}{dy} = (1-p)u',$$

and

$$(9)\quad \frac{d^2U}{dy^2} = (1-p)[u''(1-x_y) + \frac{u'^2}{u}x_y].$$

In (8), x_y does not appear on account of the envelope theorem – i.e., the x_y terms drop out when we substitutes for pℕ using the first-order condition for x, expression (2). In (9), the first-order condition (2) is used to substitute for pℕ.

Using (8) and (9), we can express 0_{uy} and CRRA – with each defined in terms of the derivatives of U with respect to y – as follows (after appropriate substitutions and simplifications):

$$(10)\ \eta_{uy} = \frac{dU}{dy}\frac{y}{U} = \frac{yu'}{u},$$

and

$$(11)\ CRRA = -\frac{y\frac{d^2U}{dy_2}}{\frac{dU}{dy}} = -\frac{yu''}{u'}(1-x_y) - \frac{yu'}{u}x_y = -\frac{yu''}{u'}(1-x_y) - \eta_{uy}x_y.$$

We can now use expressions (10) and (11) to write expression (7) as follows:

$$(12)\ \eta = \eta_{uy}(1-x_y) + CRRA.$$

The two terms on the right side of expression (12) correspond to the two effects identified in the introduction. First, as income rises, the utility value of saving one's life rises at a rate indicated by η_{uy}, the elasticity of utility with respect to income. This effect is weighted by 1-x_y because some of one's increased income is spent on safety and this expenditure does not contribute to utility directly. Second, as income rises, the cost, measured in utility, of safety expenditures falls at a rate indicated by the CRRA (which is (the negative of) the elasticity of the marginal utility of income with respect to income).

To assist in interpreting expression (12) quantitatively, we will make a crude approximation, namely, by assuming that x_y will be close zero. (This approximation is discussed further in section III.) This allows us to write the income elasticity of VSL as

$$(13)\ \eta \approx \eta_{uy} + CRRA.$$

For concreteness, it is also useful to consider constant relative risk aversion utility functions. To begin, consider the case in which CRRA = 1, that is, $u = \ln c$, where, recall, c (consumption) is $y - x$. For this case, we have

$$(14)\ \eta \approx \frac{y}{c}\frac{1}{\ln c} + 1.$$

The first term is strictly positive. As y and thus c become large, the term approaches zero.[6] For a reasonable range of income levels, the first term – ignoring the difference between y and c, which only makes it somewhat larger – is close to 0.1, so $\eta \approx 1.1$. (For example, for c = 10,000, $1/(\ln c) \approx 0.11$, and for c = 100,000, $1/(\ln c) \approx 0.09$.) Thus, in this simple case, our

---

[6]This assumes, plausibly (see section III) that individuals do not spend virtually all increases in income on x.

approximation of the elasticity of VSL with respect to income is substantially in excess – about double – the typical magnitudes obtained in the literature of 0.5 to 0.6.

To generalize, consider other constant CRRA utility functions of the form $u = (c^{1-\alpha}-1)/(1-\alpha)$, for $\alpha \neq 1$. Now we have

$$(15)\ \eta \approx \frac{y}{c}\frac{c^{1-\alpha}}{c^{1-\alpha}-1}(1-\alpha)+\alpha.$$

For low levels of CRRA, specifically, for $\alpha < 1$, the first components of the first term in (15) approach y/c (from above) as y and thus c become large. Unless y is significantly greater than c (i.e., x is a high fraction of income, which we implicitly assumed is not the case in making the approximation in (13); see section III), our approximation for η approaches 1 (from above). Hence, the result – that η is (perhaps slightly) greater than 1.0 – is not appreciably different from that in the case in which $\alpha = 1$. (To reinforce the intuition about why low values of CRRA do not imply that $\eta < 1$, consider the case of risk neutrality, in which $\alpha = 0$. The second term in (15) equals zero, but the first term slightly exceeds y/c, which in turn exceeds one: as income rises, the marginal utility cost of expenditures to increase safety does not fall at all, but the utility value of the life one might save is rising roughly proportionately in this case.)

Finally – and of particular interest given the high empirical estimates of CRRA – consider the case in which $\alpha > 1$. For nontrivial values of c, $c^{1-\alpha}-1 < 0$, and $1-\alpha < 0$; hence, the first term is positive. Further analysis reveals that, as y and thus c become large, the first term approaches zero (from above).[7] As a consequence, our approximation for η is (slightly) greater than the value of α. Thus, if one accepts moderate estimates for CRRA, say 2, or much larger estimates, the inconsistency with the estimated value of the income elasticity of VSL (0.5 - 0.6) is striking.

III. Extensions and Discussion

A. *Bequest Motive*

The analysis in section II assumed that individuals' obtain no utility if they die. To incorporate a bequest motive, one can revise expression (1) for expected utility as follows:

---

[7]It is helpful to multiply the numerator and denominator of the first term in (11) by $c^{\alpha-1}$ and to rearrange terms to yield

$$\frac{y}{c}\frac{1}{c^{\alpha-1}-1}(\alpha-1).$$

For $\alpha > 1$, as y and thus c become large, $c^{\alpha-1}-1$ approaches infinity, so the first term in (15) approaches zero from above (subject to the caveat in note 6).

$$(1')\ U(x) = (1 - p(x))u_L(y - x) + p(x)u_D(y - x),$$

where u_L and u_D are concave state-contingent functions of consumption in the states in which the individual lives and dies, respectively.[8] As shown in the Appendix, if one undertakes analysis parallel to that in section II for this more general case, one obtains

$$(12')\ \eta = \frac{y(u_L' - u_D')}{u_L - u_D}(1 - x_y) + CRRA.$$

The only difference between expressions (12') and (12) is that, in the first term, in place of η_{uy} in expression (12) – which from (11) equaled yu'/u – we have in expression (12'), in place of u' and u, the difference in these values between the two states.

To interpret expression (12'), it is useful to consider briefly the case in which, for some $\lambda \in [0,1)$, $u_D = \lambda u_L$. That is, utility in state D, if positive, is taken to be proportional to (and lower than) utility in state L. It is straightforward to show that, under this assumption, expression (12') reduces precisely to expression (12), restoring our original result.[9] The intuition for this conclusion is that η depends only on how pertinent terms reflecting differences between the two states change as income rises; as long as an individual's utilities in the two states are in fixed proportions, the manner in which the relevant differences change with income is the same regardless of the particular proportion (including the case in which the proportion is zero, that is, the original case in which there is no bequest motive).

There is, however, no a priori basis for this particular assumption of fixed proportions, and empirical evidence on bequest motives has not been able to pin down even the functional form, much less the pertinent magnitudes, regarding the utility of bequests.[10] It does seem plausible, however, to conjecture that the utility value of consumption in the death state, at least for some individuals, is not subject to as rapid diminishing returns as is utility in the life state. Consider, for example, cases in which the bequest motive is altruistic and involves children, of which there are many, or charities. The first term on the right side of expression (12') indicates that this alternative assumption implies a lower value of η. But as long as we suppose that the marginal utility of income in the life state remains higher than in the death state, the first term will remain positive, in which case the value of CRRA continues to place a lower bound on η. Thus, it is conceivable that bequest motives could help to explain why estimates of η are slightly lower than suggested in section II, but unless CRRA is substantially less than 1, it still seems difficult to explain estimates of η in the range 0.5 to 0.6.

---

[8]This model is similar to that in Viscusi (1994).

[9]Let $u_L = u$. Now, in the first term on the right side of (12'), $1-\lambda$ will appear in each of the numerator and denominator, which cancels.

[10]See, for example, Davies (1996), Masson and Pestieau (1997), and Stark (1995).

B. *Assumption that x_y . 0*

In moving from expression (12) to the approximation in expression (13), it was assumed that x_y roughly equals zero when in fact it is positive – which is intuitive (expenditures on reducing the risk of death should rise with income) and would be straightforward to demonstrate. Thus, the discussion in the latter part of section II overstates the value of 0. However, for values of CRRA exceeding one, the entire first term was ignored (on the ground that it approaches zero, from above, in any event[11]), so this approximation is of little consequence in such cases. (Likewise, for CRRA equal to one, this term was small, pushing the value of 0 from one to a slightly higher level.)

Furthermore, when CRRA is significantly below one, it also seems difficult to generate a value of 0 that is significantly less than one – and thus in the range of empirical estimates – even if the value of x_y is large. For example, suppose that " = 0.2 and, moreover, that $x_y = 0.5$ (i.e., that half of all increases in income are spent reducing the risk of death). In expression (15), the first term, with 1-", which now equals 0.8, will be weighted by $1\text{-}x_y$, equal to 0.5 in this case, to produce a value of only 0.4. Furthermore, the immediately preceding component of the first term, as noted previously, will approach 1 for high values of y and thus of c. However, when x_y is large, the first component of the first term, y/c, is no longer close to 1, as assumed above. Here, if one assumes for example that $x_y = 0.5$ at all levels of y, y/c will equal 2. As a result, the first term will be approximately 0.8 as before – the effects of y/c and of $1\text{-}x_y$ precisely cancelling – so the total value of 0, adding in the second term which here is 0.2, is again approximately 1.0.[12] For 0 to fall significantly below 1.0 on account of x_y being significantly greater than zero (and CRRA being very low), therefore, it would also be necessary for x_y to be low for lower values of y but to become large in the relevant range. If, in our example with " = 0.2 and $x_y = 0.5$, one also made the extreme assumption that y/c was nevertheless close to 1.0, then for high incomes the value of 0 would approach 0.60, which is in the range of the empirical estimates.

Of course, the empirical literature on CRRA's does not support such low estimates of ". Moreover, empirical evidence most closely related to x_y, that concerning how health expenditures rise with income, does not suggest high values of x_y. Estimates reported in Viscusi (1994) indicate values on the order of 0.1 or perhaps smaller. In sum, the approximation made in moving from expression (12) to expression (13) does not contribute significantly to the main result herein.

C. *Empirical estimates of 0*

The foregoing analysis indicates that, as a matter of rational behavior, we would expect 0 to be at least 1 and, if empirical estimates of CRRA are to be believed, possibly far greater. This may be juxtaposed against the empirical evidence bearing on 0. Notably, Viscusi and Aldy

---

[11] If x_y > 1, this term would instead approach zero from below.

[12] Indeed, whenever x_y is constant, these two components will precisely cancel, for the first, y/c, simply equals $y/(y\text{-}x)$, and the second, $1\text{-}x_y$, then is $1\text{-}(x/y)$, which equals $(y\text{-}x)/y$.

(2003, p. 44) state: "Based on the approximately 50 wage-risk studies from 10 countries, we can conclude from these results that the income elasticity for the value of a statistical life is less than 1.0. Across a number of specifications with our data, our point estimates of the income elasticity range between about 0.5 and 0.6. Note that in none of our specifications did the income elasticity's 95 percent confidence interval upper bound exceed 1.0."[13]

Despite the apparent confidence in these estimates as an econometric matter, perhaps there are aspects of the underlying behavior that produce a substantial downward bias. One possibility might be that individuals are imperfectly informed, greatly underestimating the pertinent risks, or that they behave myopically. There is, however, a difficulty with such an explanation: it implies that estimates of VSL itself are biased downward but carries no immediate implication for estimates of the elasticity of VSL with respect to income. Moreover, if one had to make a conjecture, it seems plausible that such biases would become less severe as income rises. That is, individuals who are better informed, better processors of information, and in particular less myopic and thus more inclined to invest in their own human capital, would tend to have higher rather than lower incomes. If so, as income rises, the degree of downward bias in estimated VSL would fall, which would produce empirical estimates of the income elasticity of VSL that were biased *upward* relative to values that reflect rational, fully informed decisionmaking.

Other possibilities might also be explored. For example, life insurance – by making the state involving death more attractive (or, put more gently, less unattractive), will produce moral hazard with regard to market decisions, such as occupational choices and product purchases (if insurance premiums do not fully reflect such choices). This too would cause estimates of VSL drawn from observed behavior to be biased downward. In this instance, it might be more plausible to imagine that the effect rises with income, if the relative extent (not absolute amount) of insurance coverage rises with income.[14] Then, empirical estimates of the income elasticity of VSL would be biased downward. Even if this is true, it is difficult to imagine a sufficient downward bias to reconcile estimates of 0 with the high measured values of CRRA.[15]

Perhaps additional explanations could be adduced. Of course, biases in measuring VSL and its elasticity with respect to income are important in their own right. However, the attempt

[13]More precisely: "For the OLS specifications, the income elasticity varies from 0.49 to 0.60. The 95 percent confidence intervals never range below 0.2 and never exceed 0.95. For the robust regression specifications, the income elasticity varies from 0.46 to 0.48. The 95 percent confidence intervals never fall below 0.15 and never exceed 0.78."

[14]Indirect evidence appears in Bernheim, Carman, Gokhale, and Kotlikoff (2001), which indicates that the extent of financial vulnerability from the death of a spouse is lower (in part due to life insurance coverage) for individuals in higher income groups.

[15]Considering that estimates of VSL are in the range of $4 to $7 million (Viscusi and Aldy 2003) whereas individuals at the income level of those in such studies typically have life insurance of no more than a few hundred thousand dollars, the moral hazard effect is almost certainly small; hence, the extent to which it might change with income could not plausibly be very significant.

to reconcile such estimates with evidence on high values for CRRA – and with a model of rational behavior that strongly suggests that regardless of CRRA, 0 should be at least equal to 1 – provides further motivation for exploring the issue.

D. *Empirical estimates of CRRA*

As noted in the introduction, empirical estimates of CRRA have been derived in the financial economics literature – notably in attempts to explain the equity premium puzzle – and in various other literatures. In most more recent work, estimates of CRRA exceed 2 and a number of them exceed 10. Some literature has suggested various adjustments to individuals' utility functions and has introduced other features in attempts to explain equity premiums with what many view as more plausible estimates of CRRA, namely, single-digit estimates, often closer to 2 rather than substantially higher values.[16] If, indeed, CRRA is close to 2, the extent of the discrepancy with estimates of the income elasticity of VSL is greatly reduced, but hardly eliminated. And if those income elasticity estimates are not substantially off, perhaps work in financial economics and other fields should attempt to reconcile their empirical evidence with a value of CRRA closer to 1.

Another possibility is that CRRA is not constant at different income levels and the measures of CRRA in the different literatures involve individuals who, on average, are at different levels of income. Specifically, estimates of the income elasticity of VSL tend to be derived from data on ordinary workers who probably have lower incomes than individuals who would be marginal investors. However, there is no strong evidence that CRRA rises with income or wealth; it may even be decreasing, which would add to the difficulty of reconciling these disparate results.[17]

IV. Conclusion

This paper is motivated by the view that theoretical and empirical work on individuals' risk-taking behavior ought to be reconciled across contexts. It turns out that individuals' CRRA's – typically estimated and employed in the study of behavior in financial and some other markets – have direct implications for the income elasticity of VSL (and conversely) – typically examined in the study of behavior in labor and product markets. In the model analyzed here, it is shown that the income elasticity of VSL should, roughly, be as high as the CRRA (and plausibly higher, at least equal to 1, if the CRRA is below 1). Yet estimates of the income elasticity of VSL across a range of studies seem to cluster tightly in the range 0.5 to 0.6 whereas estimates of

---

[16]See, for example, Benartzi and Thaler (1995), Brav, Constantinides, and Geczy (2002), Campbell (2003), Constantinides (1990), Epstein and Zin (1990), McGrattan and Prescott (2003), and Shrikhande (1997). For a skeptical view of the success of most such attempts, see Kocherlakota (1996).

[17]Palsson (1996) finds no significant correlation between income or wealth and CRRA and also surveys the mixed evidence in prior literature which tends, perhaps, to favor a view that CRRA does not vary greatly with income. By contrast, Blake (1996) finds that CRRA's are much higher for poorer investors than for rich investors.

CRRA tend to be at least 2, with some estimates exceeding 10. This discrepancy suggests the existence of a previously unrecognized (additional) anomaly concerning individuals' risk-taking behavior in different market settings.

This paper raises more questions than it answers. It may be that individuals in fact behave quite inconsistently in different contexts, in which case the causes of this divergence should be explored. Alternatively, the estimates in one of these literatures may be off substantially. Finally, the present model may not adequately capture some important factor that might explain the gap. In any case, there seems to be value in researchers paying greater attention to the models and empirical results concerning individuals' risk-taking behavior across different settings.

APPENDIX: EXTENSION FOR BEQUEST MOTIVE

As indicated by expression (1'), expected utility for this case is

$$(A1)\ U(x) = (1 - p(x))u_L(y - x) + p(x)u_D(y - x).$$

The first-order condition for *x* now is

$$(A2)\ p' = -\frac{(1-p)u_L' + pu_D'}{u_L - u_D},$$

so we have

$$(A3)\ VSL = -\frac{1}{p'} = \frac{u_L - u_D}{(1-p)u_L' + pu_D'}.$$

Analogous to expression (3), here the numerator is the difference between utility when alive and when dead (the net value of life in utils), and the denominator is the average marginal utility of income across the two states (the expected cost in utils of expenditures on life).

As before, to derive η, we begin with *dVSL*/*dy*, which in this case is

$$(A4)\ \frac{[(1-p)u_L' + pu_D'](u_L' - u_D')(1-x_y) - (u_L - u_D)\Big([(1-p)u_L'' + pu_D''](1-x_y) - (u_L' - u_D')p'x_y\Big)}{[(1-p)u_L' + pu_D']^2}.$$

Combining (A3), (A4), and the definition of η in (4), we have

$$(A5)\ \eta = \frac{[(1-p)u_L' + pu_D'](u_L' - u_D')(1-x_y) - (u_L - u_D)\Big([(1-p)u_L'' + pu_D''](1-x_y) - (u_L' - u_D')p'x_y\Big)}{(u_L - u_D)[(1-p)u_L' + pu_D']}$$

$$= y\frac{u_L' - u_D'}{u_L - u_D}(1-x_y) - y\frac{(1-p)u_L'' + pu_D''}{(1-p)u_L' + pu_D'}(1-x_y) + y\frac{u_L' - u_D'}{(1-p)u_L' + pu_D'}p'x_y.$$

To derive an expression for CRRA, we differentiate (A1), as follows:

$$(A6)\ \frac{dU}{dy} = (1-p)u_L' + pu_D',$$

and

$$(A7)\ \frac{d^2U}{dy^2} = [(1-p)u_L'' + pu_D''](1-x_y) - p'x_y(u_L' - u_D').$$

Thus, using the definition of CRRA in (11), we can write

$$(A8)\ CRRA = -y\frac{(1-p)u_L'' + pu_D''}{(1-p)u_L' + pu_D'}(1-x_y) + y\frac{u_L' - u_D'}{(1-p)u_L' + pu_D'}p'x_y.$$

Finally, substituting (A8) into the right side of (A5), we obtain

$$(A9)\ \eta = \frac{y(u_L' - u_D')}{u_L - u_D}(1-x_y) + CRRA,$$

which is expression (12') in the text.

REFERENCES

Arrow, Kenneth J. 1971. *Essays in the Theory of Risk-Bearing*. Chicago: Markham Publishing Co.

Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro. 1997. "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study." *Quarterly Journal of Economics* 112(2): 537-579.

Benartzi, Shlomo and Richard H. Thaler. 1995. "Myopic Loss Aversion and the Equity Premium Puzzle." *Quarterly Journal of Economics* 110(1): 73-92.

Bernheim, B. Douglas, Katherine Grace Carman, Jagadeesh Gokhale, and Laurence J. Kotlikoff. 2001. "The Mismatch between Life Insurance Holdings and Financial Vulnerabilities: Evidence from the Survey of Consumer Finances." NBER Working Paper 8544.

Blake, David. 1996. "Efficiency, Risk Aversion and Portfolio Insurance: An Analysis of Financial Asset Portfolios Held by Investors in the United Kingdom." *Economic Journal* 106: 1175-1192.

Brav, Alon, George M. Constantinides, and Christopher C. Geczy. 2002. "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence." *Journal of Political Economy* 110(4): 793-824.

Campbell, John Y. 1996. "Understanding Risk and Return." *Journal of Political Economy* 104(2): 298-345.

Campbell, John Y. 2003 (forthcoming). "Consumption-Based Asset Pricing." In *Handbook of the Economics of Finance*, vol. 1B, edited by G.M. Constantinides, M. Harris, and R.M. Stulz. Amsterdam: Elsevier.

Choi, E. Kwan and Carmen F. Menezes. 1992. "Is Relative Risk Aversion Greater than One?" *International Review of Economics and Finance* 1(1): 43-54.

Constantinides, George M. 1990. "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy* 98(3): 519-543.

Davies, James B. 1996. "Explaining Intergenerational Transfers." In *Household and Family Economics*, edited by Paul L. Menchik, 47-82. Boston: Kluwer Academic Publishers.

Epstein, Larry G. And Stanley E. Zin. 1990. "'First-Order' Risk Aversion and the Equity Premium Puzzle." *Journal of Monetary Economics* 26(3): 387-407.

Halek, Martin and Joseph G. Eisenhauer. 2001. "Demography of Risk Aversion." *Journal of Risk and Insurance* 68(1): 1-24.

Kocherlakota, Narayana R. 1996. "The Equity Premium: It's Still a Puzzle." *Journal of Economic Literature* 34(1): 42-71.

Maitel, Shlomo. 1973. "Public Goods and Income Distribution: Some Further Results." *Econometrica* 41(3): 561-568.

Mankiw, N. Gregory. 1985. "Consumer Durables and the Real Interest Rate." *Review of*

Economics and Statistics 67(3): 353-362.

Mankiw, N. Gregory and Stephen P. Zeldes. 1991. "The Consumption of Stockholders and Nonstockholders." *Journal of Financial Economics* 29: 97-112.

Masson, André, and Pierre Pestieau. 1997. "Bequest Motives and Models of Inheritance: A Survey of the Literature." In *Is Inheritance Legitimate? Ethical and Economic Aspects of Wealth Transfers*, edited by Guido Erreygers and Toon Vandevelde, 54-88. Berlin: Springer-Verlag.

McGrattan, Ellen R. And Edward C. Prescott. 2003. "Average Debt and Equity Returns: Puzzling?" *American Economic Review Papers and Proceedings* 93(2): 392-397.

Mehra, Rajnish and Edward C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15(2): 145-161.

Palsson, Anne-Marie. 1996. "Does the Degree of Relative Risk Aversion Vary with Household Characteristics?" *Journal of Economic Psychology* 17: 771-787.

Shrikhande, Milind M. 1997. "Non-addictive Habit Formation and the Equity Premium Puzzle." *European Financial Management* 3(3): 293-319.

Stark, Oded. 1995. *Altruism and Beyond: An Economic Analysis of Transfers and Exchanges within Families and Groups*. Cambridge: Cambridge University Press.

Viscusi, W. Kip. 1994. "Mortality Effects of Regulatory Costs and Policy Evaluation Criteria." *Rand Journal of Economics* 25(1): 94-109.

Viscusi, W. Kip and Joseph E. Aldy. 2003. "The Value of a Statistical Life: A Critical Review of Market Estimates throughout the World." NBER Working Paper 9487.

Viscusi, W. Kip and William N. Evans. 1990. "Utility Functions that Depend on Health Status: Estimates and Economic Implications." *American Economic Review* 80(3): 353-374.