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ABSTRACT

This study demonstrates how constrained efficient allocations can arise endogenously as equilibria in an economy with a limited ability to enforce contracts and with private agents behaving competitively, taking a set of taxes as given. The taxes in this economy limit risk-sharing and arise in an equilibrium of a dynamic game between governments of sovereign nations. The equilibrium allocations depend on governments choosing to tax both the repayment of international debt and the income from capital investment in their countries.

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Applied general equilibrium models have proven useful for analyzing a variety of issues, ranging from international business cycles to asset pricing. However, many of these models assume the existence of complete asset markets, which in turn implies complete risk-sharing among agents in the economy. Complete risk-sharing has implications that are often far from the data (Backus, Kehoe, and Kydland [4]). Researchers have thus developed models in which risk-sharing is limited for some reason. One approach is to exogenously restrict the set of tradable assets (Baxter and Crucini [5]); another is to introduce a friction in the environment that endogenously limits the amount of achievable risk-sharing (Kehoe and Perri [13]).

Recently, we have examined a model in which limited risk-sharing arises endogenously from the limited ability to enforce credit arrangements between sovereign nations (Kehoe and Perri [13]). This type of friction goes a long way toward reconciling theory and data. The limited ability to enforce international credit arrangements manifests itself in enforcement constraints which require that in each period and state, allocations can be enforced only if their value is greater than it would be if the country were excluded from all further intertemporal and interstate trade. This friction captures in a simple way the difficulties of enforcing contracts between sovereign nations that involve large transfers of resources backed only by promises to repay.

Our recent work focuses on planning problems with enforcement constraints, or *constrained efficient allocations*, but does not analyze in detail how these allocations can be *decentralized.* Here we do that detailed analysis. We show that constrained efficient allocations arise as equilibria of a dynamic game between governments, with private agents acting competitively. In this game, private agents solve standard competitive equilibrium problems, while the government of each country can choose to prevent its agents from repaying their outstanding international debts by taxing such repayments, and if there is capital, the government can also tax capital income. We show that the allocations that solve the constrained planning problem can be supported as equilibria of this game if and only if they satisfy the enforcement constraints.

The main contribution of this work is to show how limited international risk-sharing can endogenously arise in the equilibrium of an appropriately defined game with competitive private agents. As such, this work builds on both the literature on international debt—such as the studies of Eaton and Gersovitz [9], Kletzer and Wright [14], and Manuelli [17] and those surveyed by Eaton and Fernandez [8]—and the literature on debt-constrained asset markets, particularly the studies of Alvarez and Jermann [2], Attanasio and Ríos-Rull [3], Kehoe and Levine [11, 12], Kocherlakota [15], and Ligon, Thomas, and Worrall [16].

In those studies, the equilibrium is modeled in one of two ways. In the international debt literature, private competitive agents are not explicitly modeled; instead, a game is set up between two large agents, often thought of as the governments of the countries. In the debt-constrained asset market literature, private agents are explicitly modeled as competitive, but the constraints that private consumers face are not explicitly chosen by any agent as part of the equilibrium. For example, in the work of Kehoe and Levine [11], the enforcement constraints are built directly into the commodity space. Alvarez and Jermann [2] go the farthest and show how appropriately set constraints on debt can decentralize the constrained efficient allocations as a competitive equilibrium. Even in that work, however, the debt constraints are not chosen by any agent. Alvarez and Jermann [2] show, rather, that if the debt constraints are appropriately set, then the allocations of interest can be decentralized. Jeske [10] and Wright [19] also analyze competitive equilibria with limited enforcement, but they focus on the case in which the decision to repudiate the debt is made by private agents and not by governments, so the strategic element of default decision is not explicitly modeled.

Our work goes beyond the literatures on international debt and debt-constrained asset markets. Here the debt taxes, which are the mechanism through which international risk-sharing is limited, are derived endogenously as equilibria of a dynamic game between governments.

We begin with a pure exchange economy with two countries and a large number of identical consumers in each. We set up a planning problem with enforcement constraints and show how the resulting constrained efficient allocations can be characterized by a transition law for the ratio of marginal utilities of consumers across countries together with a resource constraint. We show that the constrained efficient allocations can be decentralized as either a competitive equilibrium with appropriately set debt constraints, as in the work of Alvarez and Jermann [2], or as a competitive equilibrium with debt taxes. In both notions of competitive equilibrium, the frictions faced by private agents, the debt constraints or the debt taxes, while appropriately set, are exogenous.

We then define a dynamic game in which the governments of the countries optimally choose the debt taxes as part of the equilibria, while private agents act competitively, taking the debt taxes as exogenous. We show that any constrained efficient allocation can be supported as an equilibrium of this dynamic game. In this sense, our economy is a standard competitive environment in which limited international risk-sharing arises endogenously from the limited enforcement of international contracts and the strategic interactions between governments.

We then add capital to the model, so that the economy is a standard two-country growth model with enforcement constraints. We show that the constrained efficient allocations cannot be decentralized with only the type of debt constraints used by Alvarez and Jermann [2]. This is because in the planning problem with enforcement constraints, the Euler equation for capital accumulation is necessarily distorted away from the first-best, but with debt constraints alone, there is no such distortion. If we add a constraint limiting the amount of capital that can be saved, as suggested by Seppälä [18], the constrained efficient allocations can be decentralized as competitive equilibria. However, we find this decentralization in which consumers are limited in the amount they can borrow as debt and the amount they can save in the form of capital not intuitively appealing.

Finally, we show that if the economy includes capital income taxes as well as debt taxes, then the constrained efficient allocations can be decentralized as competitive equilibria. It is then easy to show that any constrained efficient allocation can be supported as the equilibrium of a dynamic game in which governments choose both types of taxes.

1. CONSTRAINED EFFICIENT ALLOCATIONS

Consider the following deterministic pure exchange economy, which is a special case of the stochastic pure exchange economy studied by Alvarez and Jermann [2] and the stochastic production economy studied by Kehoe and Perri [13]. We will show here that constrained efficient allocations in this economy can be decentralized either by limits on borrowing or by taxes on debt payments to agents outside a country.

1.1. The World Economy

Our theoretical world economy consists of two countries, $i = 1, 2$, each represented by a large number of identical, infinitely lived consumers and a time-varying deterministic endowment of a single homogeneous consumption good. The endowment of country i in time period t is y_{it} while consumers in country i have utility, or *preferences,* of the form $\sum_{t=0}^{\infty} \beta^t U(c_{it})$,where c_{it} denotes consumption of the endowment good by consumers in country i in t and β denotes the discount factor. The *resource constraints* are given by

$$c_{1t} + c_{2t} = y_{1t} + y_{2t}. \tag{1}$$

We assume that for country $i = 1, 2$, all endowments $y_{it} \in [\underline{y}, \bar{y}]$ for some finite, strictly positive constants $\bar{y}$ and $\bar{y}$.

This economy has, besides the resource constraints, *enforcement constraints* which require that at every point in time, each country prefers the allocation it receives over the allocation it could get if it were in *autarky,* or self-sufficient, from then on. These enforcement constraints are of the form

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it} = \sum_{s=t}^{\infty} \beta^{s-t} U(y_{is}), \tag{2}$$

where V_{it} denotes the value of autarky for country i from period t on, which is given by the value of utility in which consumers simply consume their endowment for t on.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities:

$$\max \left[\lambda_1 \sum_{t=0}^{\infty} \beta^t U(c_{1t}) + \lambda_2 \sum_{t=0}^{\infty} \beta^t U(c_{2t}) \right] \tag{3}$$

subject to the resource constraints (1) and the enforcement constraints (2) for country $i = 1, 2$ and all periods t, where λ_1 and λ_2 are nonnegative initial weights on the two countries' utilities.

An allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative planning weights λ_1 and λ_2. We characterize these allocations as follows. Let $\beta^t \mu_{it}$ denote the multipliers on the enforcement constraints. Let $M_{it} = M_{it-1} + \mu_{it}$ and $M_{i,-1} = \lambda_i$. Notice that M_{it} is the initial planning weight on country i, λ_i, plus the sum of the multipliers on country i's enforcement constraint from period 0 through period t. By grouping terms, we can write the planning problem as

$$\max \sum_{t=0}^{\infty} \sum_{i} \beta^t \big[M_{it-1} U(c_{it}) + \mu_{it}\big(U(c_{it}) - V_{it}\big)\big]$$

subject to the resource constraints (1). The first-order conditions are summarized by

$$\frac{U'(c_{1t})}{U'(c_{2t})} = \frac{M_{2t}}{M_{1t}}.$$

For notational simplicity, we use the normalized weight $z_t = M_{2t}/M_{1t}$ and the normalized multiplier $v_{it} = \mu_{it}/M_{it}$. Then the transition law for the z along with the first-order conditions can be written as

$$z_t = \left(\frac{1 - v_{1t}}{1 - v_{2t}}\right) z_{t-1} \tag{4}$$

$$z_t = \frac{U'(c_{1t})}{U'(c_{2t})}, \tag{5}$$

where $z_{-1} = \lambda_2/\lambda_1$. Thus, constrained efficient allocations are characterized by (4) and (5) along with the resource constraints and enforcement constraints for some sequence of relative weights z and multipliers v_i. Notice that, if in equilibrium some enforcement constraint is binding, then the first-order condition for relative consumption (5) is distorted away from those conditions for the unconstrained efficient allocations in which $U'(c_{1t})/U'(c_{2t}) = \lambda_2/\lambda_1$, and the allocation will display less than perfect risk-sharing.

We can get some intuition for how the binding pattern of the enforcement constraints is related to the allocations as follows. Combine (4) and (5) to give

$$\frac{U'(c_{1t})}{U'(c_{2t})} = \left(\frac{1 - v_{1t}}{1 - v_{2t}}\right) \frac{U'(c_{1t-1})}{U'(c_{2t-1})}, \tag{6}$$

and realize that there are three possible binding patterns for the enforcement constraints: either country 1's constraint binds and country 2's constraint is slack ($v_{1t} > 0, v_{2t} = 0$), country 2's constraint binds and country 1's constraint is slack ($v_{2t} > 0, v_{1t} = 0$), or both countries' constraints are slack ($v_{1t} = v_{2t} = 0$). If country 2's constraint binds in period t, then

$$\frac{U'(c_{1t})}{U'(c_{2t})} > \frac{U'(c_{1t-1})}{U'(c_{2t-1})}, \tag{7}$$

so that the ratio of country 1's marginal utility to country 2's marginal utility increases relative to this ratio in period $t-1$, with the reverse when country 1's constraint binds. If neither constraint binds, then the ratio of marginal utilities stays the same. Of course, (7) also implies that if country 2's constraint binds, then the consumers in country 1 have the higher intertemporal marginal rate of substitution from period $t-1$ to t in that

$$\frac{U'(c_{1t})}{U'(c_{1t-1})} > \frac{U'(c_{2t})}{U'(c_{2t-1})}, \tag{8}$$

with the reverse when country 1's constraint binds.

This simple manipulation of (7) into (8) gives the intuition for the following lemma established by Alvarez and Jermann [2]:

Lemma 1. If $\{c_{1t}, c_{2t}\}$ is a constrained efficient allocation with

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) > \sum_{s=t}^{\infty} \beta^{s-t} U(y_{js}), \tag{9}$$

then

$$\frac{U'(c_{jt+1})}{U'(c_{jt})} = \max_i \frac{U'(c_{it+1})}{U'(c_{it})}. \tag{10}$$

In words, unconstrained consumers have the highest marginal rate of substitution. Alvarez and Jermann [2] prove this using a simple variational argument, but for our purposes, the algebra of

(7) and (8) makes the lemma obvious. We use this lemma when we construct asset prices for the decentralization with debt constraints. In that decentralization, the asset prices are determined by the marginal rate of substitution for the unconstrained consumers, which, as this lemma shows, is whatever marginal rate of substitution is the highest among the consumers.

We will be most interested in allocations for which the present value of the allocation, at the appropriately defined prices, is finite for each consumer. Letting $q_{0,t} = q_{0,1}q_{1,2} \ldots q_{t-1,t}$ with

$$q_{t,t+1} = \max_i \frac{\beta U'(c_{it+1})}{U'(c_{it})},$$

we say that an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ has *high implied interest rates* if for $i = 1, 2,$

$$\sum_{t=0}^{\infty} q_{0,t}(y_{1t} + y_{2t}) < \infty. \tag{11}$$

Here $q_{t,t+1}$ is the marginal rate of substitution for whichever country's representative consumer is unconstrained between periods t and $t+1$. Typically, in some periods one country's consumer will be unconstrained while in other periods the other country's consumer will be unconstrained. Thus, the product of these marginal rates $q_{0,t}$ does not represent any single consumer's marginal rate of substitution between periods 0 and t, but rather is a mixture of both representative consumers' marginal rates.

1.2. Decentralization With Debt Constraints

Here we consider how to decentralize the constrained efficient allocations with debt constraints along the lines of Alvarez and Jermann [2]. We show that any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with appropriately chosen debt constraints and initial assets.

In this economy, the price of a claim to one unit of the consumption good in period $t+1$ in period t units is denoted by $q_{t,t+1}$ and the amount of such asset claims purchased by consumer

i in period t is denoted by a_{it+1}. In this decentralization, the consumers in country i choose $\{c_{it}, a_{it+1}\}$ to solve

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{it}) \tag{12}$$

subject to

$$c_{it} + q_{t,t+1} a_{it+1} = y_{it} + a_{it}, \tag{13}$$

$$a_{it+1} \geq B_{it+1},$$

with a_{i0} given, where $B_{it+1} \leq 0$ specifies the lowest amount of assets that a consumer in country i in period t is permitted to have. Thus, $\{B_{it+1}\}$ is a sequence of exogenous, time-varying, country-specific debt constraints.

A *competitive equilibrium with debt constraints* $\{B_{1t+1}, B_{2t+1}\}$ together with initial assets a_{10} and a_{20} is a set of allocations $\{c_{1t}, c_{2t}\}$, asset holdings $\{a_{1t+1}, a_{2t+1}\}$, and asset prices $\{q_{t,t+1}\}$ for which $\{c_{it}, a_{it+1}\}$ solves (12) for each i, and markets clear, so that $a_{1t+1} + a_{2t+1} = 0$ and (1) holds.

Let $\beta^t \theta_t$ denote the multiplier on the debt constraints. Then the first-order conditions are summarized by

$$q_{t,t+1} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\theta_{it}}{U'(c_{it})} \tag{14}$$

and the transversality conditions

$$\lim_{t \to \infty} \beta^t U'(c_{it})(a_{it} - B_{it}) = 0. \tag{15}$$

Hence, the allocations and prices that constitute a competitive equilibrium are summarized by the resource constraints (1), the budget constraints (13), the first-order conditions (14) with $\theta_{it} \geq 0$, and the transversality conditions (15).

Given a constrained efficient allocation $\{c_{1t}, c_{2t}\}$ with normalized multipliers $\{v_{1t}, v_{2t}\}$, we construct the asset prices, asset holdings, and debt constraints that decentralize this allocation as follows. Let

$$q_{t,t+1} = \max_i \beta \frac{U'(c_{it+1})}{U'(c_{it})} \tag{16}$$

be the asset price, and given this price and the allocations, use (14) to define the multipliers θ_{it}. It is immediate that these multipliers have the right properties. If consumer i has the higher marginal rate of substitution, so that $q_{t,t+1} = \beta U'(c_{it+1})/U'(c_{it})$, then $\theta_{it} = 0$. If consumer i has the lower marginal rate of substitution, $U'(c_{it+1})/U'(c_{it}) < U'(c_{jt+1})/U'(c_{jt})$, then (14) and (16) imply that $\theta_{it} > 0$.

Using the transversality condition, we can iterate on the consumer budget constraints to get an expression for the assets as

$$a_{it} = \sum_{s=t}^{\infty} q_{t,s}(c_{is} - y_{is}), \tag{17}$$

where $q_{t,s} = q_{t,t+1}q_{t+1,t+2}\dots q_{s-1,s}$. We set initial assets $a_{i0} = \sum_{t=0}^{\infty} q_{0,t}(c_{it} - y_{it})$.

We set the debt constraints as follows. If a debt constraint binds for consumer i in t, so that $v_{it+1} > 0$, then we set the debt constraint $B_{it+1} = a_{it+1}$, so that the constrained consumer can borrow no more than the consumer's actual borrowing.

If a debt constraint is slack for consumer i in t, so that $v_{it} = 0$, then there are many ways to set the borrowing limit, all of which will be slack. The loosest is to set the limit equal to the present discounted value of future endowments, so that $B_{it+1} = -\sum_{s=t+1}^{\infty} q_{t+1,s}y_{is}$. Alvarez and Jermann [2] choose to set it according to the following counterfactual thought experiment. If at the constructed prices an unconstrained consumer happens to borrow exactly up to the limit in period t and then acts optimally from then on, this consumer will be indifferent between the proposed allocations and autarky. More formally, let $J_{it}(a_{it})$ denote the maximized value in the consumer's problem (12) for some arbitrary level of initial assets, where we have suppressed the dependence of this value on the current and future prices and debt constraints $\{q_{s,s+1}, B_{is+1}\}_{s=t}^{\infty}$. Then define debt constraints to be *not too tight* if the sequence $\{B_{it+1}\}$ satisfies

$$J_{it}(B_{it}) = V_{it}. \tag{18}$$

Notice that (18) not only defines the debt constraints for the unconstrained consumer as we have discussed, but applied to the constrained consumer, it also automatically implies that $B_{it+1} = a_{it+1}$.

To make our argument complete, we need to show that any constrained efficient allocation that satisfies the high implied interest rate condition (11) also satisfies the transversality condition (15). To see this, note that with debt constraints that satisfy $B_{it+1} = -\sum_{s=t+1}^{\infty} q_{t+1,s} y_{is}$ for the unconstrained consumer and $B_{it+1} = a_{it+1}$ for the constrained consumer, from (17) it follows that $a_{it} - B_{it}$ is equal to $\sum_{s=t}^{\infty} q_{t,s} c_{is}$ for the unconstrained consumer and equal to 0 for the constrained consumer. In either case, since c_{is} is nonnegative and satisfies the resource constraint, $a_{it} - B_{it} \leq \sum_{s=t}^{\infty} q_{t,s}(y_{1s} + y_{2s})$, and hence,

$$\begin{aligned} \lim_{t\to\infty} \beta^t U'(c_{it})(a_{it} - B_{it}) &\leq U'(c_{i0}) \lim_{t\to\infty} \beta^t \frac{U'(c_{it})}{U'(c_{i0})} \sum_{s=t}^{\infty} q_{t,s}(y_{1s} + y_{2s}) \\ &\leq U'(c_{i0}) \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,s}(y_{1s} + y_{2s}) = 0, \end{aligned} \tag{19}$$

where the second inequality in (19) follows since, by construction, $q_{0,t} \geq \beta^t U'(c_{it})/U'(c_{i0})$, and the second equality in (19) follows from the high implied interest rate condition (11).

From the construction, it is immediate that the constrained efficient allocations $\{c_{1t}, c_{2t}\}$ together with the constructed asset positions $\{a_{1t}, a_{2t}\}$, debt prices $\{q_{t,t+1}\}$, and debt constraints

$\{B_{1t+1}, B_{2t+1}\}$ form a competitive equilibrium with debt constraints. We have thus established the following:

Proposition 1. Any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with debt constraints.

1.3. Decentralization With Debt Taxes

Now we discuss how to decentralize the constrained efficient allocations as a competitive equilibrium with debt taxes. We show that if these debt taxes are appropriately chosen, then the constrained efficient allocations can be decentralized. (In the next section, we will allow the governments to purposefully choose these taxes.)

In this economy, the government of each country can tax payments made to consumers in the other country and then rebate the proceeds in a lump-sum fashion to its own consumers. Except for these government policies, private markets function perfectly.

We begin by setting up a competitive equilibrium with debt taxes. Consider the consumer problem and the government budget constraint for some arbitrarily given sequence of government policies and prices. Throughout we will focus on country 1; the notation for country 2 is analogous. It is convenient to define separate variables for saving and for borrowing. We let $s_{1t+1} \geq 0$ denote the savings, or assets, of a consumer in country 1, $b_{1t+1} \geq 0$ denote that consumer's borrowings, or liabilities, and $\tau_{1t} \in [0, 1]$ denote the tax rate levied by the government of country 1 on payments from country 1 consumers to country 2 consumers.

The problem for a consumer in country 1 is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) = y_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t} + T_{1t}; \tag{20}$$

the nonnegativity constraints $s_{it+1}, b_{it+1} \geq 0$; and bounds on debt $b_{1t+1} \leq \bar{b}$, where $\bar{b}$ is a large positive constant. Here $p_{t,t+1}$ is the price of a consumption good in $t+1$ in period t units, τ_{2t} is country 2's tax rate on payments s_{1t} that country 2 consumers make to country 1 consumers, and T_{1t} is lump-sum transfers from the government of country 1 to its own consumers. The initial assets s_{i0} and liabilities b_{i0} are given.

The government of country 1 chooses a tax rate on payments to country 2 consumers τ_{1t} and rebates the revenues to its own consumers in a lump-sum fashion, so that the government budget constraint in country 1 is $T_{1t} = \tau_{1t} b_{1t}$.

A *competitive equilibrium with debt taxes* $\{\tau_{1t}, \tau_{2t}\}_{t=0}^{\infty}$ together with initial assets and liabilities $\{s_{i0}, b_{i0}\}_{i=1,2}$ consists of an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$, assets $\{s_{1t+1}, s_{2t+1}\}_{t=0}^{\infty}$, liabilities $\{b_{1t+1}, b_{2t+1}\}_{t=0}^{\infty}$, and prices $\{p_{t,t+1}\}_{t=0}^{\infty}$ such that $\{c_{it}, s_{it+1}, b_{it+1}\}$ solves the consumer problem for each i, and markets clear, so that $s_{1t+1} = b_{2t+1}$ and $b_{1t+1} = s_{2t+1}$ and the resource constraint (1) holds.

To understand the budget constraints of the consumer and the government, suppose that in period $t-1$ a consumer in country 1 lends $p_{t-1,t}s_{1t}$ in exchange for a promise to receive, in period t, s_{1t} minus the taxes $\tau_{2t}s_{1t}$ levied by country 2 on repayments to country 1. Consumers in country 2 repay a total of $s_{1t} = b_{2t}$, with $(1 - \tau_{2t})s_{1t}$ going to country 1 consumers and $\tau_{2t}s_{1t} = \tau_{2t}b_{2t}$ going to the government of country 2. The government of country 2 then redistributes its tax revenue in a lump-sum fashion to its own consumers.

For brevity, from now on we let U'_{it} denote $U_{ct}(c_{it})$. With this notation, the first-order conditions for the consumer's problem are

$$p_{t,t+1}U'_{1t} \geq \beta U'_{1t+1}(1 - \tau_{2t+1}), \tag{21}$$

with equality if $s_{1t+1} > 0$, so that country 1 is lending to country 2; and

$$p_{t,t+1}U'_{1t} \leq \beta U'_{1t+1}, \tag{22}$$

with equality if $b_{1t+1} > 0$, so that country 2 is lending to country 1. Here and throughout we assume that the debt constraint $b_{1t+1} \leq \bar{b}$ does not bind. The transversality condition is

$$\lim_{t\to\infty} \beta^t p_{t,t+1}U'_{1t}(s_{it+1} - b_{it+1} + \bar{b}) = 0. \tag{23}$$

We now show the following analog of Proposition 1.

Proposition 2. Any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with debt taxes.

Proof. We decentralize a constrained efficient allocation with high implied interest rates as follows. We set the prices

$$p_{t,t+1} = \beta \min_i \frac{U'_{it+1}}{U'_{it}}. \tag{24}$$

We set the taxes as follows. If at the given allocations $U'_{1t+1}/U'_{1t} \geq U'_{2t+1}/U'_{2t}$, then we set $\tau_{1t+1} = 0$ and

$$1 - \tau_{2t+1} = \frac{U'_{2t+1}/U'_{2t}}{U'_{1t+1}/U'_{1t}}. \tag{25}$$

If $U'_{1t+1}/U'_{1t} < U'_{2t+1}/U'_{2t}$, then we set $\tau_{2t+1} = 0$ and

$$1 - \tau_{1t+1} = \frac{U'_{1t+1}/U'_{1t}}{U'_{2t+1}/U'_{2t}}. \tag{26}$$

Notice, for later, that the constructed tax rates lie between 0 and 1 and that

$$(1-\tau_{1t+1})(1-\tau_{2t+1}) = \frac{\min_i U'_{it+1}/U'_{it}}{\max_i U'_{it+1}/U'_{it}}. \tag{27}$$

For assets and liabilities, we set

$$s_{it+1} - b_{it+1} = \frac{\max_i U'_{it+1}/U'_{it}}{\min_i U'_{it+1}/U'_{it}} \sum_{s=t+1}^{\infty} q_{t+1,s}(y_{is} - c_{is}) \tag{28}$$

with $q_{t,s} = q_{t,t+1}q_{t+1,t+2}\dots q_{s-1,s}$ and $q_{t,t+1} = \beta \max_i U'_{it+1}/U'_{it}$. If the right side of (28) is nonnegative, we set $b_{it+1} = 0$; if the right side of (28) is negative, we set $s_{it+1} = 0$.

We can see that the constructed prices, taxes, and assets and liabilities are a competitive equilibrium with taxes as follows. To check the constructed prices, notice that in equilibrium in any period t, either country 1 is lending to country 2, so that $s_{1t+1} = b_{2t+1} > 0$, $s_{2t+1} = b_{1t+1} = 0$, and

$$p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}}(1-\tau_{2t+1}) = \beta \frac{U'_{2t+1}}{U'_{2t}}, \tag{29}$$

or country 2 is lending to country 1, so that $s_{2t+1} = b_{1t+1} > 0$, $s_{1t+1} = b_{2t+1} = 0$, and

$$p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}} = \beta \frac{U'_{2t+1}}{U'_{2t}}(1-\tau_{1t+1}), \tag{30}$$

or neither is lending, so that $s_{1t+1} = b_{2t+1} = s_{2t+1} = b_{1t+1} = 0$ and thus

$$\beta \max_i \frac{U'_{it+1}}{U'_{it}} \leq p_{t,t+1} \leq \beta \min_i \frac{U'_{it+1}}{U'_{it}}. \tag{31}$$

When either (29) or (30) hold, it is clear that the price $p_{t,t+1}$ satisfies (24), while if (31) holds, the price $p_{t,t+1}$ can take on a range of values, one of which is given by (24). By inspection, we know that the constructed taxes satisfy (29)–(31).

To check the constructed assets and liabilities, substitute the budget constraint of the government $T_{1t} = \tau_{2t}b_{1t}$ into that of the consumer to obtain

$$c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) = (1-\tau_{1t})s_{1t} - (1-\tau_{2t})b_{1t} + y_{1t}, \tag{32}$$

where if $s_{1t} > 0$ and $b_{1t} = 0$, then $(1-\tau_{1t})(1-\tau_{2t}) = (1-\tau_{1t})$; and if $s_{1t} = 0$ and $b_{1t} \geq 0$, then $(1-\tau_{1t})(1-\tau_{2t}) = (1-\tau_{2t})$. Hence, in general, we can write (32) as

$$c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) = (1-\tau_{1t})(1-\tau_{2t})(s_{1t} - b_{1t}) + y_{1t}. \tag{33}$$

Using the transversality condition, we can iterate on (33) to obtain

$$s_{1t+1} - b_{1t+1} = \frac{1}{(1-\tau_{1t+1})(1-\tau_{2t+1})} \sum_{s=t+1}^{\infty} \rho_{t+1,s}(y_{1s} - c_{1s}), \tag{34}$$

where $\rho_{t,s} = \rho_{t,t}\rho_{t,t+1}\ldots\rho_{s-1,s}$ and $\rho_{t,t} = 1$ and

$$\rho_{t,t+1} = \frac{p_{t,t+1}}{(1-\tau_{1t})(1-\tau_{2t})}.$$

Using (24) and (27), we can see that $\rho_{t,t+1} = q_{t,t+1}$ and, hence, that $\rho_{t,s} = q_{t,s}$. This relation used along with (27) in (34) lets us reduce (34) to (28).

The final step is to show that at the constructed allocations, if the high implied interest rate condition (11) holds, then the transversality condition (23) holds. Notice first that

$$\bar{b} \lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t} = 0. \tag{35}$$

To see this, note that since $p_{t,t+1}$ satisfies (24), $p_{t,t+1}U'_{1t} \leq U'_{1t+1}$, while the high implied interest rate condition implies that

$$0 = \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,t}(y_{1s} + y_{2s}) \geq \underline{y} \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,t} \geq \underline{y} \lim_{t\to\infty} q_{0,t} \geq \frac{\underline{y}}{U'(c_{i0})} \lim_{t\to\infty} \beta^t U'_{1t} \geq 0,$$

so $\lim_{t\to\infty} \beta^t U'_{1t} = 0$. Since $p_{t,t+1}$ satisfies (24), $p_{t,t+1}U'_{1t} \leq U'_{1t+1}$, and hence, (35) holds. Thus, we need only show that $\lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t}(s_{1t+1} - b_{1t+1}) = 0$:

$$\lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t}(s_{1t+1} - b_{1t+1})$$

$$= \lim_{t\to\infty} \beta^t U'_{1t} \max_i \left(\frac{U'_{it+1}}{U'_{it}} \right) \sum_{s=t+1}^{\infty} q_{t+1,s}(y_{is} - c_{is}) \tag{36}$$

$$= \frac{U'_{10}}{\beta} \lim_{t\to\infty} \beta^t \frac{U'_{1t}}{U'_{10}} \sum_{s=t+1}^{\infty} q_{t,s}(y_{is} - c_{is}) \tag{37}$$

$$\leq \frac{U'_{10}}{\beta} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{is} - c_{is}) \tag{38}$$

$$\leq \frac{U'_{10}}{\beta} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{1s} + y_{2s}) = 0, \tag{39}$$

where (36) follows from (24) and (28), (37) from the definition of $q_{t,t+1}$, (38) from the definition of $q_{0,s}$, and the inequality in (39) from the resource constraint, while the equality follows from (11). ■

1.4. Differences

Although we have shown that either debt constraints or debt taxes can be used to decentralize an allocation, there is an important difference in how interest rates and prices are defined in the two decentralizations. In the debt constraints economy, the interest rate $(1/q_{t,t+1})$ is given by the marginal rate of substitution of the agent whose enforcement constraint is not binding, while in the economy with debt taxes, the interest rate $(1/p_{t,t+1})$ is given by the marginal rate of substitution of the agent whose enforcement constraint is binding. So, in general, the decentralization with debt taxes will produce higher interest rates than the decentralization with debt constraints. At an intuitive level, we know that in the decentralization with debt constraints, interest rates are low, and the debt constraint is needed to prevent the agent with the binding enforcement constraint from borrowing "too much." Conversely, in the decentralization with debt taxes, interest rate are high, and taxes are needed to prevent the agent whose enforcement constraint is not binding from saving "too much."

One implication of these different decentralizations is how interest rates respond to enforcement frictions. If we start from a frictionless economy and add enforcement problems, then we see that the interest rate moves differently in the two decentralizations. Relative to the frictionless rate, the interest rate falls in the debt constraint decentralization and rises in the debt tax decentralization. We find this feature of the debt tax decentralization somewhat appealing.

2. ENDOGENIZING THE DEBT TAXES

In our decentralizations, we have used the constrained efficient allocations to construct the appropriate debt constraints or debt taxes that decentralize the given allocations, but we have not offered a story about where these constraints or taxes come from. Here we provide a story for how the constructed debt taxes may come out of an equilibrium of a dynamic game with both government behavior and consumer behavior endogenous.

2.1. The Dynamic Game

We set up this dynamic game as follows. In each period, the governments and the consumers can vary their decisions, depending on the history of government policies up to the time the decision is made. We let $\pi_t = (\tau_{1t}, \tau_{2t})$ denote the two governments' policies in period t. At the beginning of period t, the government of each country chooses a current policy as a function of the history of past government policies $h_{t-1} = (\pi_0, ..., \pi_{t-1})$ together with a contingency plan for setting future policies for all possible future histories. Let $\sigma_{it}(h_{t-1})$ denote the period t tax on debt repayments chosen by the government of country i when faced with history h_{t-1}. After the government sets the current policies, consumers make their decisions. Faced with the history $h_t = (h_{t-1}, \pi_t)$, consumers in country i choose their period t consumption, assets, and liabilities, denoted $f_{it}(h_t) = \big(c_{it}(h_t),\ s_{it}(h_{it}), b_{it}(h_t)\big)$. The prices are a function of the government policy history and are denoted $p_{t,t+1}(h_t)$. Let $\sigma = (\sigma_1, \sigma_2)$, and let σ_i denote the infinite sequence of functions (σ_{it}). Use similar notation for the other variables.

For some given initial assets and liabilities, a *sustainable equilibrium* is a triple (σ, f, p) such that three conditions are satisfied:

(i) For $i = 1, 2$, for every history of government policies h_t, the consumer allocations $f_{is}(h_s)$ for $s = t, ...,$ solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is})$$

subject to

$$c_{1s} + p_{s,s+1}(h_s)(s_{1s+1} - b_{1s+1})$$

$$= y_{1s} + [1 - \tau_{2s}(h_{s-1})s_{1s}(h_{s-1})] - b_{1s}(h_{s-1}) + T_{1s}(h_{s-1}),$$

where the future histories' policies and prices are induced from h_t, σ, and p in the obvious way. That is, $h_{t+1} = \big(h_t, \sigma_{t+1}(h_t)\big)$, $h_{t+2} = \big(h_t, \sigma_{t+1}(h_t), \sigma_{t+2}(h_t, \sigma_{t+1}(h_t))\big)$, and given these induced future histories, the policies and prices are given by $\sigma_s(h_{s-1})$ and $p_s(h_s)$.

(ii) For every history h_t, markets clear and the government budget constraint holds for $s = t, ...,$ so that $c_{1s}(h_s) + c_{2s}(h_s) = y_{1s} + y_{2s}$, as well as $s_{1s}(h_s) = b_{2s}(h_s)$, $s_{2s}(h_s) = b_{1s}(h_s)$, and $T_{1s}(h_{s-1}) \equiv \tau_{1s}(h_{s-1})b_{1s}(h_{s-1})$, where the future histories h_s are induced from σ in the obvious way.

(iii) For every history h_{t-1}, country 1's government policies from t on, σ_{1s} for all $s \geq t$, solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U\big(c_{1s}(h'_{s-1})\big),$$

where $h'_t = \big(h_{t-1}, (\sigma'_{1t}(h_{t-1}), \sigma'_{2t}(h_{t-1}))\big)$ and $h'_{t+1} = \big(h_t, (\sigma'_{1t+1}(h_t), \sigma'_{2t+1}(h_t))\big)$ and so on. A similar condition holds for the government of country 2.

Notice that in this definition of a sustainable equilibrium we require that both the governments and the consumers act optimally for every history of policies—even for histories not induced by the governments' policy plans. This requirement is analogous to the requirement of perfection in a game. In this definition, the consumers act competitively in that they take current policies and prices and the evolution of future histories as unaffected by their actions. The governments are not competitive. The government of country 1, for example, takes the allocation rules f_1 and f_2, the price function p, and the policy plan of the government of country 2, σ_2, as given. But the government of country 1 realizes that it can affect outcomes both directly, by having its consumers face a different tax on payments to the other country's consumers, and indirectly, by affecting the evolution of the future history and thus affecting the policies chosen by the other government, the allocations chosen by the consumers, and the prices.

2.2. Outcomes of a Sustainable Equilibrium

Recall that a sustainable equilibrium (σ, f, p) is a sequence of functions that specify policies, allocations, and prices for all possible government policy histories. Thus, when we start from the null history in period 0, a sustainable equilibrium induces a particular sequence of policies, allocations, and prices that we denote by (π, x, p). We call this the *outcome* induced by the sustainable equilibrium. In what follows, we adapt the work of Chari and Kehoe [6, 7], which builds on the work of Abreu [1], to characterize this outcome.

We first construct a sustainable equilibrium that we call the *autarky equilibrium.* We then characterize the allocations that can be induced by reverting to this autarky equilibrium after deviations. We define the autarky policy plans σ^a, allocation rules f^a, and price rules p^a starting from some given initial assets and liabilities as follows. The policy plan $\sigma^a_{it}(h_{t-1}) = 1$, for all i and t. Given any history h_t, the autarky allocations $\left(c^a_{it}(h_t), s^a_{it+1}(h_t), b^a_{it+1}(h_t)\right)$ are given by $c^a_{it}(h_t) = y_{it}$, while the autarky prices of debt and the quantities of assets and liabilities are identically zero, so $p^a_{t,t+1}(h_t) = s^a_{it+1}(h_t) = b^a_{it+1}(h_t) = 0$. The utility of autarky for consumer i in period t is V_{it}.

We now characterize the outcomes that can be sustained by a set of plans called the *revert-to-autarky* plans, which are defined as follows. For an arbitrary sequence of policies, allocations, and prices (π, x, p), these plans specify continuation with the candidate sequences (π, x, p) as long as the specified policies have been chosen in the past; otherwise, the plans specify the revert-to-autarky plans (σ^a, f^a, p^a). We then have

Proposition 3. An arbitrary triple of sequences (π, x, p) can be sustained by the revert-to-autarky plans if and only if the sequence is a competitive equilibrium with debt taxes and if, for $i = 1, 2$ for every t, the following inequality holds:

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}. \tag{40}$$

Proof. Suppose, first, that the sequences of policies, allocations, and prices (π, x, p) can be sustained by the revert-to-autarky plans; that is, suppose the associated revert-to-autarky plans (σ, f, p) constitute a sustainable equilibrium. From the definition of a sustainable equilibrium, consumer optimality requires that x maximize consumer welfare in period 0. This requirement together with market-clearing ensures that this sequence is a competitive equilibrium in period 0.

Next, we claim that inequality (40) holds for all i and t. Note that a feasible policy for the government of i in t is to choose the autarky policies for all $s \geq t$ by taxing repayments to consumers in the other country at rate 1. This policy will lead to a continuation utility of V_{it}^a, and hence, optimality of government policy ensures that (40) holds.

Now suppose that some arbitrary triple of sequences (π, x, p) satisfies the proposition's conditions. We show that the associated revert-to-autarky plans constitute a sustainable equilibrium. Consider, first, histories for which there have been no deviations from π before t. Since (π, x, p) is a competitive equilibrium in period 0, x is optimal for consumers in period 0 given π and p, and thus, the continuation of x is optimal for consumers when they are faced with the continuation of π and p. In terms of government optimality, consider the situation of the government of country 1. If it deviates in period t, then the consumers in both countries and the government of country 2 will revert to the autarky policy plans and the autarky allocation rules from period t on. Under these allocation rules, country 2 consumers will never lend to country 1 consumers, regardless of the policies chosen by the government of country 1. Thus, the best the government of country 1 can obtain is the value of autarky from then on given by the right side of (40). Given the assumed inequality, then, sticking to the specified plan is optimal.

Consider, next, histories with a deviation from π before t. Clearly, the autarky plans from then on are sustainable. From a consumer's point of view, since no debt will be repaid, lending is not optimal. The price of debt is zero since the value to a potential lender in the other country of a promise to pay one unit tomorrow, net of taxes equal to one unit, is worthless. Thus, the consumer is indifferent among all amounts to borrow or lend because all have value 0 and all pay 0. From a government's point of view, given that the other government never allows its consumers to repay their debts outside the country, regardless of the first government's actions, it is optimal to prevent its own consumers from repaying their debts outside the country. ■

Combining Propositions 2 and 3, we immediately obtain the following proposition:

Proposition 4. Any constrained efficient allocation is the outcome of a sustainable equilibrium.

3. ADDING CAPITAL

We now explore how our results change when we move from a pure exchange economy to a growth model with capital. We first show in a constrained efficient allocation that if the enforcement constraints bind, then the Euler equation for capital is distorted. This result implies that a competitive equilibrium with debt constraints alone cannot decentralize such an allocation. But if in addition to the debt constraints, we allow for constraints that limit the amount of capital that can be accumulated, *capital constraints*, then the constrained efficient allocations can be decentralized. However, we argue that such capital constraints are not intuitively appealing and turn to our preferred decentralization: a combination of debt taxes and capital income taxes. This combination can decentralize the constrained efficient allocations. Finally, we sketch out how these taxes may endogenously arise in a dynamic game similar to that just described.

3.1. A Growth Model

We modify our pure exchange economy in several ways. The preferences are the same as before. The resource constraints are now

$$c_{1t} + c_{2t} + k_{1t+1} + k_{2t+1} = A_{1t}f(k_{1t}) + A_{2t}f(k_{2t}) + (1-\delta)(k_{1t} + k_{2t}) \tag{41}$$

with k_{i0} given, where k_{it+1} is the capital stock chosen in period t for use in production in period $t+1$; $f(k)$ is a standard production function that is increasing, concave, and continuously differentiable and satisfies the standard Inada conditions; A_{it} is country-specific deterministic exogenous shocks; and δ is the depreciation rate of capital. The enforcement constraints are now

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}(k_{it}), \tag{42}$$

where

$$V_{it}(k_{it}) = \max \sum_{t=0}^{\infty} \beta^{t} U(c_{it}) \tag{43}$$

subject to

$$c_{it} + k_{it+1} = A_{it} f(k_{it}) + (1-\delta) k_{it}. \tag{44}$$

Notice that the problem with (financial) autarky reduces to that of a planning problem of a closed-economy growth model. Notice also that the value of utility under autarky in period t depends on the amount of capital located in country i in that period, k_{it}. The derivatives of this value, $V'(k_{it})$, will be the root problem behind why the equilibrium with debt constraints alone cannot decentralize the constrained efficient allocations.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities:

$$\max \left[\lambda_1 \sum_{t=0}^{\infty} \beta^{t} U(c_{1t}) + \lambda_2 \sum_{t=0}^{\infty} \beta^{t} U(c_{2t}) \right] \tag{45}$$

subject to the resource constraints (41) and the enforcement constraints (42) for country $i = 1, 2$ and all periods t, where λ_1 and λ_2 are nonnegative initial weights on the two countries' utilities.

An allocation $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative weights λ_1 and λ_2. Let $\beta^t \mu_{it}$ denote the multiplier on the enforcement constraints. Let $M_{it} = M_{it-1} + \mu_{it}$ and $M_{i,-1} = \lambda_i$. Then, by grouping terms, we can write the planning problem as

$$\max \sum_{t=0}^{\infty} \sum_i \beta^t \big[M_{it-1} U(c_{it}) + \mu_{it}\big(U(c_{it}) - V_{it}(k_{it})\big)\big]$$

subject to the resource constraints (41). The first-order conditions are summarized by

$$\frac{U'(c_{1t})}{U'(c_{2t})} = \frac{M_{2t}}{M_{1t}}$$

$$U(c_{it}) = \beta \left\{ \frac{M_{it+1}}{M_{it}} U'(c_{it+1})[f'(k_{it+1}) + 1 - \delta] - \frac{\mu_{it+1}}{M_{it}} V'(k_{it+1}) \right\}.$$

Rewriting these using $z_t = M_{2t}/M_{1t}$ and $v_{it} = \mu_{it}/M_{it}$ gives that the transition law for the z along with the first-order conditions can be written as (4), (5), and

$$U(c_{it}) = \beta \left\{ \frac{U'(c_{it+1})}{1 - v_{it+1}} [f'(k_{it+1}) + 1 - \delta] - \frac{v_{it+1}}{1 - v_{it+1}} V'(k_{it+1}) \right\}, \tag{46}$$

where $z_{-1} = \lambda_2/\lambda_1$. Equation (46) is the Euler equation for capital accumulation in the economy with enforcement constraints. This equation is distorted away from the familiar Euler equation of the standard growth model that would arise in the absence of such constraints,

$$U(c_{it}) = \beta U'(c_{it+1})[f'(k_{it+1}) + 1 - \delta]. \tag{47}$$

Notice that if v_{it+1} were equal to zero, then (46) would reduce to (47).

3.2. Decentralization With Debt Constraints

We now show that a competitive equilibrium with debt constraints alone cannot decentralize the constrained efficient allocations, except for the trivial case in which the enforcement constraints never bind. In general, to decentralize such allocations, we need constraints limiting the amount of capital that can be saved on capital as well as constraints limiting the amount of borrowing with debt.

Consider an economy with two countries $i = 1, 2$, each of which has a representative consumer. Each consumer owns a production unit. Only that consumer can work with that unit, but the consumer can borrow and lend from anyone else, subject to some time-varying debt constraints. The representative consumer in country i solves the problem

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{it}) \tag{48}$$

subject to

$$c_{it} + q_{t,t+1} a_{it+1} + k_{it+1} = f(k_{it}) + (1-\delta)k_{it} + a_{it} \tag{49}$$

$$a_{it+1} \geq B_{it+1}, \tag{50}$$

where B_{it+1} is an exogenous, time-varying, agent-specific debt constraint. (We can imagine that $k_{it} \geq 0$, but when f satisfies the Inada conditions, this will never bind, so we ignore it.) Letting $\beta^t \theta_{it}$ denote the multiplier on the debt constraint, we can summarize the first-order conditions by

$$q_{t,t+1} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\theta_{it}}{U'(c_{it})}$$

$$U'(c_{it}) = \beta U'(c_{it+1})[f'(k_{it+1}) + (1-\delta)] \tag{51}$$

with $\theta_t \geq 0$.

Proposition 5. If the enforcement constraint ever binds in the constrained efficient allocation, then that allocation cannot be decentralized as an equilibrium with debt constraints.

Proof. By construction, the normalized multiplier $v_{it+1} \in [0, 1]$. Suppose, by way of contradiction, that the enforcement constraint binds in some period t, so that v_{it+1} is strictly positive. Since U' and f' are positive and V' is negative, the right side of (46),

$$\beta \left\{ \frac{U'(c_{it+1})}{1 - v_{it+1}} [f'(k_{it+1}) + 1 - \delta] - \frac{v_{it+1}}{1 - v_{it+1}} V'(k_{it+1}) \right\},$$

is strictly larger than the right side of (51). Thus if (46) holds at the constrained efficient allocation, then the Euler equation in the decentralized equilibrium (51) cannot also hold. ■

Now imagine that besides the debt constraint, we also add a capital constraint, a time-varying constraint on the amount of capital that can be saved of the form

$$k_{it+1} \leq D_{it+1}, \tag{52}$$

where $\{D_{it+1}\}$ is a sequence of constants. It should be fairly obvious that if we choose these constants appropriately, then we can decentralize the constrained efficient allocations.

To see this, imagine adding (52) to the consumer's problem (48). If we let $\beta^t \lambda_{it}$ denote the multiplier on (52), then the first-order condition for capital (51) is changed to

$$U'(c_{it}) = \beta U'(c_{it+1})[f'(k_{it+1}) + (1 - \delta)] - \lambda_{it}. \tag{53}$$

With the appropriate choice of the capital constraint, the multiplier λ_{it} can be set so that (53) coincides with (46). (For an alternative approach that arrives at the same conclusion, see Seppälä [19].)

3.3. Decentralization With Taxes

Consider now decentralizing the constrained efficient outcome as a competitive equilibrium with taxes on capital income as well as on debt. With these two taxes, we can mimic the distorted first-order conditions that define the constrained efficient outcome.

The problem for a representative consumer in country 1 who faces both types of taxes is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) + k_{it+1} = w_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t} + R_{it}k_{it} + T_{1t} \tag{54}$$

and the nonnegativity constraints $s_{it+1}, b_{it+1} \geq 0$, and s_{i0}, b_{i0} and k_{i0} given. Here $R_{it} = 1 + (1 - \eta_{it})(r_{it} - \delta)$ is the gross return on capital after taxes and depreciation, r_{it} is the before-tax return on capital, and η_{it} is the tax on capital income net of depreciation $(r_{it} - \delta)$. In this decentralization, there are firms whose behavior we can summarize by conditions for rental rates and wage rates:

$$r_{it} = f'(k_{it}) \text{ and } w_{it} = f(k_{it}) - k_{it}f'(k_{it}). \tag{55}$$

In this economy, a *competitive equilibrium with debt and capital income taxes* $\{\tau_{1t}, \tau_{2t}, \eta_{1t}, \eta_{2t}\}_{t=0}^{\infty}$ together with initial assets, liabilities, and capital stocks $\{s_{i0}, b_{i0}, k_{i0}\}_{i=1,2}$ consists of allocations $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$, assets $\{s_{1t+1}, s_{2t+1}\}_{t=0}^{\infty}$, liabilities $\{b_{1t+1}, b_{2t+1}\}_{t=0}^{\infty}$, and prices $\{p_{t,t+1}, r_{it}, w_{it}\}_{t=0}^{\infty}$ such that $\{c_{it}, s_{it+1}, b_{it+1}, k_{it+1}\}$ solves the consumer problem for each i and markets clear, so that $s_{1t+1} = b_{2t+1}$ and $b_{1t+1} = s_{2t+1}$ and the resource constraint (41) holds.

The construction of the debt taxes, assets, liabilities, and prices is nearly identical to that for the pure exchange economy. The rental rates r and wage rates w are given by (55) while the tax on capital income η is backed out from the Euler equation

$$U'(c_{it}) = \beta U'(c_{it+1})[1 + (1 - \eta_{it+1})(f_{kit} - \delta)].$$

The following proposition is then immediate:

Proposition 6. Any allocation that satisfies the resource constraint and has high implied interest rates can be decentralized as a competitive equilibrium with debt and capital income taxes.

It is straightforward to show that any constrained efficient outcome is the outcome of a suitably defined sustainable equilibrium.

4. CONCLUSION

We have proposed a new decentralization of constrained efficient allocations in which the forces that give rise to the limited risk-sharing are more explicitly modeled than in the existing literature. The decentralization is intuitively appealing when applied to international risk-sharing problems for economies with capital and a limited ability to enforce contracts. It may be possible to similarly model the forces that limit risk-sharing in other decentralizations, for example, an equilibrium in which the debt constraints studied by Alvarez and Jermann [2] are explicitly chosen by financial intermediaries in an appropriately defined dynamic game.

Here we have focused on a deterministic economy in order to economize on notation, but all our results immediately generalize to a stochastic economy, provided that debt constraints, capital constraints, and taxes can be state-contingent.

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