A SIMPLE FRAMEWORK FOR INTERNATIONAL MONETARY POLICY ANALYSIS

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ABSTRACT

We study the international monetary policy design problem within an optimizing two-country sticky price model, where each country faces a short run tradeoff between output and inflation. The model is sufficiently tractable to solve analytically. We find that in the Nash equilibrium, the policy problem for each central bank is isomorphic to the one it would face if it were a closed economy. Gains from cooperation arise, however, that stem from the impact of foreign economic activity on the domestic marginal cost of production. While under Nash central banks need only adjust the interest rate in response to domestic inflation, under cooperation they should respond to foreign inflation as well. In either scenario, flexible exchange rates are desirable.

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1   Introduction

The existence of a short run tradeoff between output and inflation is a central obstacle to the smooth management of monetary policy. In the open economy, of course, there are additional complications: Not only must a central bank take account of the exchange rate in this situation, but potentially also the feedback responses of foreign central banks to its policy actions.

In this paper we revisit these classic issues by developing a simple two country model that is useful for international policy analysis. Consistent with a voluminous recent literature, our framework is optimization-based and is sufficiently tractable to admit an analytical solution.[1] In most of this work (particularly the work that is purely analytical), nominal price setting is done on a period by period basis, leading to highly unrealistic dynamics.[2] We use instead the staggered price setting model that has become the workhorse of monetary policy analysis in the closed economy, and augment it by allowing for a short run tradeoff in a way that does not sacrifice tractability. Thus, we are able to investigate qualitatively the implications of international considerations for monetary policy management without having to abstract from the central problem that the tradeoff poses.

Our framework is essentially a two country version of the small open economy model we developed in Clarida, Galí, Gertler (CGG) (2001), which is in turn based on Galí and Monacelli (2001). In this paper we showed that under certain conditions the monetary policy problem is isomorphic to the problem of the closed economy studied in CGG (1999). In this setting, accordingly, the qualitative insights for monetary policy management are very similar to what arises for the closed economy. International considerations, though, may have quantitative implications, as openness does affect the model parameters and thus the coefficients of the optimal feedback policy. In addition, openness gives rise to an important distinction between consumer price index (cpi) inflation and domestic inflation. To the extent there is perfect exchange rate pass through, we find that the central bank should target domestic infla-

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[1]Some examples of this recent literature include: Obstfeld and Rogoff (2000), Corsetti and Pesenti (2001a), Kollmann, McCallum and Nelson (1999), Devereaux and Engle (2000), Lane (2001), and Chari, Kehoe, and McGrattan (2000).

[2]A recent exception is Benigno and Benigno (2001) who have concurrently emphasized some similar themes as in our paper, though the details of the two approaches differ considerably. We introduce a short run tradeoff by allowing for staggered price setting in conjunction with a certain type of labor market friction, as we discuss in the next section.

tion and allow the exchange rate to float, despite the impact of the resulting exchange rate variability on the cpi.

In the two country setting we study here, the monetary policy problem is sensitive to the nature of the strategic interaction between central banks. In the absence of cooperation (the "Nash" case), our earlier "isomorphism" result is preserved. Each country confronts a policy problem that is qualitatively the same as the one a closed economy would face. The two country framework, though, allows us to characterize the equilibrium exchange rate and illustrate concretely how short run tradeoff considerations enhance the desirability of flexible exchange rates.

The strict isomorphism result, however, breaks down when we allow for the possibility of international monetary coordination. There are potentially gains from cooperation within our framework, though they are somewhat different in nature than stressed in the traditional literature, as they are supply side. In particular, the domestic marginal cost of production and the domestic potential output depend on the terms of trade, which in turn depends on foreign economic activity. By coordinating policy to take account of this spillover, central banks can in principle improve welfare. As we show, coordination alters the Nash equilibrium in a simple and straightforward way. Among other things, we show that it is possible to implement the optimal policy under coordination by having each central bank pursue an interest rate feedback rule of the form that was optimal under Nash (a kind of Taylor rule), but augmented to respond to foreign inflation as well as domestic inflation.

In section 2 below we characterize the behavior of households and firms. Section 3 describes the equilibrium. Section 4 describes the policy problem and the solution in the Nash case. In section 5, we consider the case of cooperation. Concluding remarks are in section 6. Finally, the appendix provides explicit derivations of the welfare functions.

2   The Model

The framework is a variant of a dynamic New Keynesian Model applied to the open economy, in the spirit of much recent literature[3]. There are two countries, home and foreign, that differ in size but are otherwise symmetric. The home country (H) has a mass of households $1-\gamma$, and the foreign country (F) has a mass γ. Otherwise preferences and technologies are the same across

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[3]See Lane (2001) for a survey.

countries, though shocks may be imperfectly correlated. Within each country, households consume a domestically produced good and an imported good. Households in both countries also have access to a complete set of Arrow-Debreu securities which can be traded both domestically and internationally.

Domestic production takes place in two stages. First, there is a continuum of intermediate goods firms, each producing a differentiated material input. Final goods producers then combine these inputs into output, which they sell to households. Intermediate goods producers are monopolistic competitors who each produce a differentiated product and set nominal prices on a staggered basis. Final goods producers are perfectly competitive. We assume that the number of final goods firms within each country equals the number of households. Though, we normalize the number of intermediate goods firms at unity in each country.[4]

Only nominal prices are sticky. As is well known, in the absence of other frictions, with pure forward looking price setting there is no short run tradeoff between output and inflation[5]. To introduce a short run tradeoff in a way that is analytically tractable, we assume that households have some market power in the labor market, and then introduce exogenous variation in this market power as a convenient way to generate cost-push pressures on inflation.

We next present the decision problems of households and firms.

2.1 Households

Let C_t be the following index of consumption of home (H) and foreign (F) goods, :

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma} \tag{1}$$

and let P_t be the corresponding consumption price index (that follows from cost minimization):

$$
\begin{aligned}
P_t &= k^{-1} P_{H,t}^{1-\gamma} \ P_{F,t}^{\gamma} \\
&= k^{-1} P_{H,t} \ S_t^{\gamma}
\end{aligned}
\tag{2}
$$

where $S_t = \frac{P_{F,t}}{P_{H,t}}$ is the terms of trade and $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$.

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[4] As will become clear, making the number of intermediate goods firms the same across countries ensures that final goods producers within each country face the same technology.

[5] See, for example, Clarida, Galí and Gertler (1999).

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Let $N_t(h)$ denote the household's h hours of labor, with $W_t(h)$ the corresponding nominal wage. Let D_{t+1} denote the (random) payoff of the portfolio purchased at t, with $Q_{t,t+1}$ the corresponding stochastic discount factor. Finally, let T_t denote lump sum taxes and Γ_t denote lump sum profits accruing from ownership of intermediate goods firms. Then the representative household in the home country maximizes[6]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(N_t(h)) \right] \tag{3}$$

subject to the sequence of budget constraints

$$P_t C_t + E_t \{Q_{t,t+1} D_{t+1}\} = W_t(h) \, N_t(h) + D_t - T_t + \Gamma_t \tag{4}$$

In addition, the household is a monopolistically competitive supplier of labor and faces the following constant elasticity demand function for its services:

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t \tag{5}$$

where N_t is per capita employment and $W_t \equiv \left(\frac{1}{1-\gamma} \int_0^{1-\gamma} W_t(h)^{1-\eta_t} \, dh \right)^{\frac{1}{1-\eta_t}}$ is the relevant aggregate wage index. The elasticity of labor demand, η_t, is the same across workers, but may vary over time. Note that this particular demand curve evolves from a production technology that has labor input a CES aggregate of individual household labor hours, as we describe in the next sub-section.

We specialize the period utility function to be of the form:

$$U(C_t) - V(N_t(h)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi} \tag{6}$$

The first order necessary conditions for consumption allocation and intertemporal optimization are standard:

$$P_{H,t} \, C_{H,t} = (1 - \gamma) \, P_t C_t \tag{7}$$

[6]The assumption of complete markets guarantees that consumption is equated across households.

$$P_{F,t}\ C_{F,t} = \gamma\ P_t C_t \tag{8}$$

$$\beta\ (C_{t+1}/C_t)^{-\sigma}\ (P_t/P_{t+1}) = Q_{t,t+1} \tag{9}$$

Let R_t denote the gross nominal yield on a one-period discount bond. Then by taking the expectation of each side of equation (9) we obtain the following Euler equation:

$$1 = \beta R_t\ E_t\{(C_{t+1}/C_t)^{-\sigma}(P_t/P_{t+1})\} \tag{10}$$

where $R_t^{-1} = E_t\{Q_{t,t+1}\}$ is the price of the discount bond.

The first order condition for labor supply reflects the household's market power

$$\frac{W_t(h)}{P_t} = (1 + \mu_t^w)\ N_t(h)^\phi\ C_t^\sigma \tag{11}$$

where $\mu_t^w = \frac{1}{\eta_t - 1}$ is the optimal wage markup. In contrast to Erceg, Henderson, and Levin (2000), wages are perfectly flexible, implying the absence of any endogenous variation in the wage markup resulting from wage rigidities. On the other hand we allow for exogenous variation in the wage markup arising from shifts in η_t, interpretable as exogenous variation in workers' market power.[7] Note that because wages are flexible, all workers will charge the same wage and have the same level of hours. Thus we can write:

$$W_t(h) = W_t$$

$$N_t(h) = N_t \tag{12}$$

for all $h \in [0, 1 - \gamma]$ and all t.

A symmetric set of first order conditions holds for citizens of the foreign country. In particular, given the international tradability of state-contingent securities, the intertemporal efficiency condition can be written as:

$$\beta\ (C_{t+1}^*/C_t^*)^{-\sigma}\ (P_t^*/P_{t+1}^*)\ (\mathcal{E}_t/\mathcal{E}_{t+1}) = Q_{t,t+1} \tag{13}$$

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[7]To be clear, we assume exogenous variation in the wage markup only for simplicity. In our view this approach provides a convenient way to obtain some of the insights that arise when there is an endogenous markup due to wage rigidity.

The law of one price, which implies $P_t = \mathcal{E}_t \, P_t^*$ for all t, in conjunction with equations (13) and (9), and a suitable normalization of initial conditions, yields:

$$C_t = C_t^* \tag{14}$$

for all t.[8]

2.2 Firms

2.2.1 Final Goods

Each final goods firm in the home country uses a continuum of intermediate goods to produce output, according to the following CES technology:

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \tag{15}$$

where Y_t denotes aggregate output, while $Y_t(f)$ is the input produced by intermediate goods firm f. Both variables are normalized by population size $1 - \gamma$, i.e., they are expressed in per capita terms. Profit maximization, taking the price of the final good $P_{H,t}$ as given, implies the set of demand equations:

$$Y_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} Y_t \tag{16}$$

as well as the domestic price index

$$P_{H,t} = \left(\int_0^1 P_{H,t}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}} \tag{17}$$

2.2.2 Intermediate Goods

Each intermediate goods firm produces output using a technology that is linear in labor input, $N_t(f)$ (also normalized by population size), as follows:

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[8] As in Corsetti and Pesenti (2000), Benigno and Benigno (2000), the complete asset market equilibrium can be achieved in a simple asset market in which only nominal bonds are traded.

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$$Y_t(f) = A_t \, N_t(f) \tag{18}$$

where A_t is an exogenous technology parameter. The labor used by each firm is a CES composite of individual household labor, as follows:

$$N_t(f) = \left(\frac{1}{1-\gamma} \int_0^{1-\gamma} N_t(h)^{\frac{\eta_t-1}{\eta_t}} \, dh \right)^{\frac{\eta_t}{\eta_t-1}} \tag{19}$$

Aggregating across optimizing intermediate goods firms yields the market demand curve for household labor given by equation (5), where the technological parameter η_t is the wage elasticity of hours demand. Because in equilibrium each household charges the same wage and supplies the same number of hours, we can treat the firm's decision problem over total labor demand as just involving the aggregates $N_t(f)$ and W_t. We also assume that each firm receives a subsidy of τ percent of its wage bill.

In addition, intermediate goods firms set prices on a staggered basis as in Calvo (1983), where θ is the probability a firm keeps its price fixed in a given period and $1-\theta$ is the probability it changes it, where probability draws are i.i.d. over time. Firms that do not adjust their price simply adjust output to meet demand (assuming they operate in a region with a non-negative net markup.) In either case, choosing labor to minimize costs conditional on output yields:

$$\begin{aligned} MC_t &= \frac{(1-\tau) \, (W_t/P_{H,t})}{A_t} \\ &= \frac{(1-\tau) \, (W_t/P_t) \, S_t^\gamma}{k \, A_t} \end{aligned} \tag{20}$$

where MC_t denotes the real marginal cost. Observe that, given the constant returns technology and the aggregate nature of shocks, real marginal cost is the same across firms.

Firms that are able to choose their price optimally in period t choose the reset price $P_{H,t}^0$ to maximize the following objective:

$$E_t \sum_{j=0}^{\infty} \theta^j \, Q_{t,t+j} \, Y_t(f) \, \left(P_{H,t}^0 - P_{H,t+j} \, MC_{t+j} \right) \tag{21}$$

subject to the demand curve (16). The solution to this problem implies that firms set their price equal to a discounted stream of expected future nominal marginal cost.

$$E_t \sum_{j=0}^{\infty} \theta^j \ Q_{t,t+j} \ Y_t(f) \ \left(P_{H,t}^0 - (1+\mu^p) \ P_{H,t+j} \ MC_{t+j}\right) = 0 \qquad (22)$$

Note that if a firm was able to freely adjust its price each period, it will choose a constant markup over marginal cost, i.e., $\theta = 0$ implies

$$\frac{P_{H,t}^0}{P_{H,t}} = (1+\mu^p) \ MC_t \qquad (23)$$

Finally, the law of large number implies that the domestic price index evolves according to:

$$P_{H,t} = \left[\theta \ (P_{H,t-1})^{1-\xi} + (1-\theta) \ (P_{H,t}^0)^{1-\xi}\right]^{\frac{1}{1-\xi}} \qquad (24)$$

3   Equilibrium

We begin by characterizing the equilibrium conditional on output. How the model is closed depends on the behavior of prices and monetary policy. We first characterize the flexible price equilibrium, for which an exact solution is available, and then turn to the case of staggered price setting, for which an approximate solution is available.

Goods market clearing in the home and foreign countries implies:

$$(1 - \gamma) \ Y_t = (1 - \gamma) \ C_{H,t} + \gamma \ C_{H,t}^* \qquad (25)$$

$$\gamma \ Y_t^* = (1 - \gamma) \ C_{F,t} + \gamma \ C_{F,t}^* \qquad (26)$$

The demand curves for home and foreign goods by home citizens, equations (7) and (8), respectively, along with the analogues for foreign citizens and the law of one price imply that the cpi-based real exchange rate is unity:[9]

$$\frac{\mathcal{E}_t P_t^*}{P_t} = 1 \qquad (27)$$

It then follows (after also taking into account equations (25) and (26)) that the trade balance is zero within each country

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[9]Note that we assume producer currency pricing and complete-pass through. Devereux and Engle (2000), among others, have emphasized the role of local currency pricing.

$$P_{H,t}Y_t = P_tC_t \tag{28}$$

$$P_{F,t}^*Y_t^* = P_t^*C_t^* \tag{29}$$

In turn, combining equations (2) and (28) implies an aggregate demand schedule that relates domestic per capita output, per capita consumption, and the terms of trade, $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$, as follows:

$$Y_t = k^{-1} \ C_t \ S_t^\gamma \tag{30}$$

with

$$S_t = \frac{Y_t}{Y_t^*} \tag{31}$$

as well as an aggregate demand schedule. Observe that equations (30), (31) and the consumption Euler equation (10) determine domestic output demand, conditional on foreign output and the path of the real interest. In addition, (30) and (31) can be combined to yield an expression for consumption as a function of domestic and foreign output:

$$C_t = k \ (Y_t)^{1-\gamma} \ (Y_t^*)^\gamma \tag{32}$$

On the supply side, notice that $N_t = \int_0^1 N_t(f) \ df = \frac{Y_t}{A_t} \int_0^1 \left(\frac{Y_t(f)}{Y_t}\right) df$, and (16) can be combined to yield the aggregate production function

$$Y_t = \frac{A_t \ N_t}{V_t} \tag{33}$$

where $V_t \equiv \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\xi} df \geq 1$.

Combining the labor supply and demand relations, equations (11) and (20) respectively, and then using the aggregate demand schedule (30) and the aggregate production function (33) to eliminate C_t and N_t yields the following expression for real marginal cost:

$$\begin{aligned} MC_t &= (1-\tau)(1+\mu_t^w) \frac{k^{-1} \ N_t^\phi \ C_t^\sigma \ S_t^\gamma}{A_t} \tag{34} \\ &= (1-\tau)k^{\sigma-1} \ (1+\mu_t^w) \ A_t^{-(1+\phi)} \ Y_t^\kappa \ (Y_t^*)^{\kappa_o}V_t^\phi \tag{35} \end{aligned}$$

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where κ and κ_o are the elasticities of marginal cost with respect to domestic and foreign output, given by

$$
\begin{aligned}
\kappa &\equiv \sigma(1-\gamma)+\gamma+\phi \qquad\qquad (36)\\
&= \sigma+\phi-\kappa_o
\end{aligned}
$$

and

$$
\begin{aligned}
\kappa_o &\equiv \sigma\gamma-\gamma \qquad\qquad\qquad\qquad (37)\\
&= \gamma(\sigma-1)
\end{aligned}
$$

As will become clear, the implications of international considerations for monetary policy within this framework depend critically on how the open economy affects the behavior of marginal cost, as summarized by the behavior of the two key elasticities, κ and κ_o. Note first that the sign of κ_o, the elasticity of marginal cost with respect to foreign output, is ambiguous. There are two effects of a change in Y_t^* on MC_t that work in opposite directions: A rise in Y_t^* causes the terms of trade to appreciate which, holding constant domestic consumption, reduces marginal cost, as equation (35) suggests, and as reflected by the term $-\gamma$ in (37). At the same time, however, holding constant domestic output, the increase in Y_t^* raises domestic consumption due to risk sharing, leading to an increase in marginal cost (since the rise in C_t increases the marginal rate of substitution between consumption and leisure). The latter wealth effect, captured by the term $\sigma\gamma$ in (37), dominates the terms of trade effect if $\sigma > 1$ (implying $\kappa_o > 0$); and vice versa if $\sigma < 1$ (implying $\kappa_o < 0$).

In turn, the impact of the open economy on κ, the elasticity of marginal cost with respect to domestic output, depends inversely on κ_o. An increase in Y_t, for example, causes a depreciation in the terms of trade, raising MC_t. That effect is captured by the term γ in (36). Due to risk sharing, however, consumption increases by proportionately less than the increase in domestic output, which works to dampen the increase in marginal cost (relative to the closed economy), as reflected by the term $\sigma(1-\gamma)$ in (36). Finally, there is a third channel, also found in the closed economy, through which domestic output variations affect marginal cost, and which works through the effect on employment and the disutility of labor, as captured by the term ϕ. In the end, whether openness increases or decreases the elasticity of marginal cost with respect to domestic output (relative to the closed economy) depends on

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the size of σ. Again, the wealth effect dominates the terms of trade effect when $\sigma > 1$,. implying $\kappa_o > 0$. In this instance, the open economy's aggregate marginal cost schedule is flatter than its closed economy counterpart (i.e., since $\kappa_o > 0$, κ is below its value for a closed economy (given by $\sigma + \phi$), holding constant the preference parameters σ and ϕ.

We emphasize that in the knife-edge case of logarithmic utility ($\sigma = 1$), the terms of trade and risk sharing effects cancel. In this instance, there are no direct effects of the open economy on marginal cost: $\kappa_o = 0$ and $\kappa = \sigma + \phi$, exactly as for a closed economy.

To summarize, we have characterized the values of C_t, S_t, MC_t and N_t conditioned on Y_t, V_t (which captures the dispersion of output across firms) and Y_t^*. An analogous set of relations for the foreign country determines C_t^*, $S_t^*(= S_t^{-1})$, MC_t^* and N_t^* conditional on Y_t^*, V_t^* and Y_t . How we close the model depends on the behavior of prices.

3.1 Equilibrium under Flexible Prices

We consider an equilibrium with flexible prices where the wage markup is fixed at its steady state value $1 + \mu^w$. We focus on this case because we would like to define a measure of the natural level of output that has the feature that cyclical fluctuations in this construct do not reflect variations in the degree of efficiency (hence we shut off variation in the wage markup). This approach also makes sense if we think of variations in the wage markup as standing in for wage rigidity.

In addition, we make the distinction between the equilibrium that arises when prices are flexible at home, taking foreign output as given, and the one that arises when prices are flexible across the globe. We refer to the former as the "*domestic* flexible price equilibrium" and the latter as (just) the "flexible price equilibrium". The distinction between these two concepts becomes highly relevant when we compare the Nash versus cooperative equilibria.

3.1.1 The Domestic Flexible Price Equilibrium

Let a variable with an upper bar (e.g. $\overline{X}_t$) denote its value when prices are flexible at home, but foreign output is taken as exogenously given (independently of how it is determined). We proceed to characterize the domestic flexible price equilibrium, as follows:

Under flexible prices, all firms set their price equal to a constant markup over marginal cost, as implied by condition (23). Symmetry, further, implies that all firms choose the same price. Imposing the restriction $\frac{P^o_{H,t}}{P_{H,t}} = 1$ on equation (23) implies that in the flexible price equilibrium, real marginal cost is constant and given by

$$\overline{MC} = \frac{1}{1 + \mu^p} \tag{38}$$

where we use the bar to denote the domestic flexible price equilibrium value of a variable. Symmetry of prices further implies that all firms choose the same level of output, inducing $\overline{V}_t = 1$. Hence, from equation (33),

$$\overline{Y}_t = A_t\, \overline{N}_t \tag{39}$$

Furthermore, using the fact that $\overline{MC}_t = (1 + \mu^p)^{-1}$ and fixing the wage markup at its steady state then permits us to use equation (35) to solve for the natural level of output in the domestic flexible price equilibrium

$$\overline{Y}_t = \left(\frac{k^{1-\sigma}\, A_t^{1+\phi}\, (Y_t^*)^{-\kappa_o}}{(1 - \tau)(1 + \mu^w)(1 + \mu^p)} \right)^{\frac{1}{\kappa}} \tag{40}$$

Note that the impact of foreign output Y_t^* on $\overline{Y}_t$, depends on the sign of κ_o. If $\kappa_o < 0$ (implying that MC_t is decreasing in foreign output Y_t^*), then $\overline{Y}_t$ varies positively with Y_t^*; and vice-versa if $\kappa_o > 0$. With $\kappa_o = 0$, $\overline{Y}_t$ depends only on domestic economic factors.

3.1.2 Flexible Price Equilibrium

We obtained the domestic natural level of output, $\overline{Y}_t$, by taking foreign output as exogenously given. As we discussed earlier, it is also useful to define the natural level of output, $\overline{\overline{Y}}_t$, that arises when prices are flexible worldwide:

$$
\begin{aligned}
\overline{\overline{Y}}_t &= \left(\frac{k^{1-\sigma}\, A_t^{1+\phi}\, (\overline{\overline{Y}}_t^*)^{-\kappa_o}}{(1 - \tau)(1 + \mu^w)(1 + \mu^p)} \right)^{\frac{1}{\kappa}} \\[2mm]
&= \overline{Y}_t \left(\frac{\overline{\overline{Y}}_t^*}{Y_t^*} \right)^{\frac{-\kappa_o}{\kappa}}
\end{aligned}
\tag{41}
$$

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3.2 Equilibrium Dynamics under Sticky Prices

We now express the system with sticky prices as a loglinear approximation about the steady state that determines behavior conditional on a path for the nominal interest rate. We use lower case variables to denote log deviations from the deterministic steady state.

From equation (30), aggregate demand is given by

$$y_t = c_t + \gamma \, s_t \tag{42}$$

where, from the euler equation (10), aggregate consumption evolves according to

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \, (r_t - E_t\{\pi_{t+1}\} - \gamma E_t\{\Delta s_{t+1}\}) \tag{43}$$

where r_t is the nominal rate of interest, π_{t+1} is the rate of domestic inflation from t to $t+1$ and where, from equation (31), the terms of trade is given by

$$s_t = y_t - y_t^* \tag{44}$$

On the supply side, the first order approximation to the aggregate production function (33) implies:

$$y_t = a_t + n_t \tag{45}$$

Further, combining the log-linearized optimal price setting rule (22) with the price index (24) yields

$$\pi_t = \delta \, mc_t + \beta E_t\{\pi_{t+1}\} \tag{46}$$

where $\delta = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Let $\widetilde{y}_t = y_t - \overline{y}_t$ denote the domestic output gap, i.e., the gap between output and the domestic natural level. Then from the loglinearized version of the expression for marginal cost (35) and the production function (45), we obtain[10]

$$mc_t = \kappa \, \widetilde{y}_t + \mu_t^w \tag{47}$$

where from equation (40):

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[10]From the loglinearized version of the expression for marginal cost (35), we obtain $mc_t = \mu_t^w + \kappa y_t + \kappa_o y_t^* - (1 + \phi) \, a_t$. Combining this expression with equation (48) then yields equation (47).

$$\overline{y}_t = \kappa^{-1}[(1 + \phi)\, a_t - \kappa_o\, y_t^*] \tag{48}$$

It is straightforward to collapse the system into an IS and Phillips type equations that determine $\widetilde{y}_t$ and π_t conditional on the path of r_t:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \sigma_o^{-1}\,[r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t] \tag{49}$$

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \lambda\, \widetilde{y}_t + u_t \tag{50}$$

with $\sigma_o = \sigma - \kappa_o$, $\lambda = \delta\kappa$, and $u_t = \delta\widehat{\mu}_t^w$, and where $\overline{rr}_t$ is the domestic natural real interest rate (conditional on foreign output), given by:

$$\overline{rr}_t = \sigma_o\, E_t\{\Delta\overline{y}_{t+1}\} + \kappa_o\, E_t\{\Delta y_{t+1}^*\} \tag{51}$$

An analogous set of equations holds for the foreign country, with $\sigma_o^* = \sigma - \kappa_o^*$, $\kappa_o^* = (1 - \gamma)(\sigma - 1)$, $\kappa^* = \sigma_o^* + \phi$, and $\lambda^* = \delta\kappa^*$. In addition, we assume that the "cost push shock" u_t obeys the following stationary first order process:

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{52}$$

with $0 < \rho < 1$, and where ε_t is white noise.

As discussed in Clarida, Galí and Gertler (2001) and Galí and Monacelli (2001) for the case of a small open economy, the form of the system is isomorphic to that of the closed economy. Open economy effects enter in two ways: first, through the impact of the parameter σ_o which affects both the interest elasticity of domestic demand (equal to σ_o^{-1}) and the slope coefficient on the output gap $\lambda = \delta\kappa$; and, second, via the impact of foreign output on the natural real interest rate and natural output levels. In the special case of log utility (implying $\sigma_o = \sigma$ and $\kappa_o = 0$), the open economy effects disappear and the system becomes identical to that of a closed economy.

Finally, we obtain a simple expression linking the terms of trade to movements in the output gap:

$$\begin{aligned}
s_t &= (\widetilde{y}_t - \widetilde{y}_t^*) + (\overline{y}_t - \overline{y}_t^*) \\
&= (\widetilde{y}_t - \widetilde{y}_t^*) + \overline{s}_t
\end{aligned} \tag{53}$$

where $\overline{s}_t$ is the natural level of the terms of trade.

4 Welfare and Optimal Policy: the Non Co-operative Case

In this section we analyze the problem of a domestic central bank that seeks to maximize the utility of domestic households, while taking as given foreign economic activity. We assume that the fiscal authority chooses a tax rate that makes the natural level of output correspond to the efficient level in a zero inflation steady state. In particular, we assume that the fiscal authority chooses a subsidy rate τ that maximizes the utility of the domestic household in a zero inflation steady state (i.e., in the absence of cost-push shocks), while taking as given foreign variables. As shown in the appendix, the subsidy must satisfy

$$(1 - \tau)(1 + \mu^w)(1 + \mu^p)(1 - \gamma) = 1 \tag{54}$$

and for the foreign country

$$(1 - \tau^*)(1 + \mu^w)(1 + \mu^p)\,\gamma = 1 \tag{55}$$

Note that the subsidy does not simply offset the steady state price and wage markups. Roughly speaking, the fiscal authority must balance the desirability of offsetting the inefficiencies from the steady state price and wage markups, against the need to eliminate the central bank's incentive to generate an unanticipated deflation. As discussed by Corsetti and Pesenti (2000), in the non-cooperative case, a central bank may be tempted to produce a surprise currency appreciation. In turn, Benigno and Benigno (2001) show that in this case, the optimal subsidy allows for a positive steady state markup distortion that creates an incentive for unanticipated inflation that exactly offsets the incentive for a surprise deflation. We refer to the resulting steady state as the Nash steady state.

As we show in the appendix, to derive the central bank's objective function, we take second order approximation of the utility of the representative household around the domestic flexible price equilibrium. After an appropriate normalization, we obtain the following quadratic objective, where the welfare loss from a deviation from the optimum is expressed as a fraction of steady state consumption:

$$\mathbb{W}^H \equiv -(1 - \gamma)\frac{\Lambda}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \alpha\ \widetilde{y}_t^2\right] \tag{56}$$

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with $\Lambda \equiv \frac{\xi}{\delta}$ and $\alpha \equiv \frac{\kappa\delta}{\xi} = \frac{\lambda}{\xi}$. (Recall that ξ is the price elasticity of demand for intermediate goods (see equation (16)).

Given that the home central bank takes foreign output as exogenous, the objective function is of the same form as that for a closed economy. In particular, the central bank minimizes a loss function that is quadratic in the domestic output gap and domestic inflation, with a weight α on the output gap. It is worth noting that the relevant inflation variable is domestic inflation, as opposed to overall cpi inflation.[11]. In particular, the parameter α differs from the closed economy counterpart only to the extent σ_o differs from σ. Otherwise, the objective function (including the underlying parametric specification of α) is identical to the one that would arise in the closed economy.

Given that the IS and AS curves also have the same form as in the closed economy, the overall policy problem is completely isomorphic to the closed economy, as we found in Clarida, Galí and Gertler (2001). Here we focus on the case in which the central bank lacks a commitment technology that would allow it to choose credibly, once and for all, an optimal state-contingent plan.[12] Instead we assume the central bank will choose $\widetilde{y}_t$ and π_t each period to maximize equation (56) subject to the aggregate supply curve given by equation (50), taking expectations of the future as given.

The solution satisfies:

$$\widetilde{y}_t = -\frac{\lambda}{\alpha} \pi_t \tag{57}$$
$$= -\xi \pi_t \tag{58}$$

Substituting this optimality condition into (50) and solving forward yields the following reduced form solutions for the domestic inflation and the domestic output gap in terms of the cost push shock:

$$\pi_t = \psi \ u_t \tag{59}$$

---

[11]See, e.g., Woodford (1999). As in the closed economy, the presence of domestic inflation in the objective function reflects the costs of resource misallocation due to relative price dispersion, where the latter is approximately proportionate to inflation. Note that if there were pricing to market, with sticky prices in the final traded goods sector, then a measure of cpi inflation would instead enter the objective function. The same would be true if trade were in intermediate goods, with final goods prices sticky, as in McCallum and Nelson (1999).

[12]The same assumption applies to both central banks jointly in the cooperative case considered below.

$$\widetilde{y}_t = -\xi\psi\ u_t \qquad (60)$$

where $\psi \equiv [(1 - \beta\rho) + \lambda\xi]^{-1} > 0$ (recall that ρ reflects the serial correlation in the u_t shock.) Similar expressions hold for the foreign economy.

Several points are worth emphasizing. First, the policy response is identical in form to the closed economy case (see Clarida, Gali and Gertler, 1999). There is a lean against the wind response to domestic inflation, as suggested by the optimality condition given by (57). In addition, in the absence of cost push shocks the central bank is able to simultaneously maintain price stability and close the output gap. Otherwise, a cost push shock generates a tradeoff between the output gap and inflation of the same form that applies for the closed economy. Interestingly, openness does not affect the optimality condition that dictates how aggressively the central bank should adjust the output gap in response to deviations of inflation from target.[13] Further, openness affects the reduced form elasticity of π_t and $\widetilde{y}_t$ with respect to the cost push shock only to the extent it affects the slope of the Phillips curve, λ, through its impact on the elasticity of marginal cost with respect to output, κ, as discussed above. Finally, we observe that while π_t and $\widetilde{y}_t$ may be insulated from foreign disturbances, the overall level of domestic output will depend on foreign shocks, since the domestic natural level of output depends on foreign output (via the terms of trade and consumption), as equation (48) makes clear.

We may combine the IS curve (49) with the solutions for $\widetilde{y}_t$ and π_t to obtain an expression for an interest rate rule that implements this policy:[14]

$$r_t = \overline{rr}_t + \vartheta\ E_t\{\pi_{t+1}\} \qquad (61)$$

with

$$\vartheta = 1 + \frac{\xi\sigma_o(1 - \rho)}{\rho} > 1$$

---

[13] This result arises from our explicit derivation of the weight α, which yields a restriction on the ratio λ/α which governs the optimal adjustment of the output gap to inflation. In particular, $\lambda/\alpha = \xi$ which does not depend on openness. The rough intuition for why the optimal policy becomes more aggressive in combating inflation as ξ rises is that the costs of resource misallocation from relative price dispersion depends on the elasticity of demand.

[14] This is one among many possible representations of an interest rate rule that would implement the time consistent policy under the Nash case considered here.

As in the case of the closed economy, the optimal rule may be expressed as the sum of two components: the domestic natural real interest rate, and a term that has the central bank adjust the nominal rate more than one for one with respect to domestic inflation. Openness affects the slope coefficient only to the extent it affects the interest elasticity of domestic spending, given by σ_o. Note also that the terms of trade does not enter the rule directly, but rather does so indirectly via its impact on the domestic equilibrium real rate.

To summarize, we have:

Proposition 1 *In the Nash equilibrium, the policy problem a country faces is isomorphic to the one it would face if it were a closed economy. As in the case of a closed economy, the optimal policy rule under discretion may be expressed as a Taylor rule that is linear in the domestic natural real interest rate and expected domestic inflation. Open economy considerations affect the slope coefficient on domestic inflation in the rule, as well as the behavior of the domestic natural real interest rate.*

Finally, there are some interesting implications for the behavior of the terms of trade and the nominal exchange rate in the Nash equilibrium. To gain some intuition we restrict attention to the symmetric case of $\gamma = \frac{1}{2}$ (implying $\sigma_o = \sigma_o^*$). In this instance

$$
\begin{aligned}
s_t &= (\widetilde{y}_t - \widetilde{y}_t^*) + (\overline{y}_t - \overline{y}_t^*) \\
&= -\xi\psi\,(u_t - u_t^*) + (a_t - a_t^*)
\end{aligned}
\tag{62}
$$

The terms of trade depends not only on the relative productivity differentials, but also on the relative cost push shocks. A positive cost push shock in the home country induces an appreciation in the terms of trade.

Using the definition of the terms of trade, we may write:

$$
\begin{aligned}
e_t &\equiv e_{t-1} + s_t - s_{t-1} + \pi_t - \pi_t^* \\
&= e_{t-1} - \xi\psi\,(\Delta u_t - \Delta u_t^*) + (\Delta a_t - \Delta a_t^*) + \psi\,(u_t - u_t^*) \\
&= e_{t-1} - (\xi - 1)\psi\,(u_t - u_t^*) + \xi\psi\,(u_{t-1} - u_{t-1}^*) + (\Delta a_t - \Delta a_t^*)
\end{aligned}
\tag{63}
$$

Notice that, in general, the nominal exchange rate should respond to relative differences in cost push shocks, in addition to the productivity shocks.

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Given that $\xi > 1$, a country that experiences a relatively high cost-push shock should engineer a short run appreciation of its currency, followed by an eventual permanent depreciation. The initial appreciation is a consequence of the terms of trade appreciation resulting from the contraction in the output gap that is needed to dampen inflationary pressures. The eventual depreciation to a permanently lower plateau results from the permanent effect of the shock on the domestic price level, which has a unit root under the optimal time consistent policy. Notice that it also not optimal to peg the exchange rate in response to relative productivity shocks: the adjustment of the exchange rate makes it possible to fully avoid a change in inflation that would be costly from a welfare standpoint.

It is also interesting to observe that the nominal exchange rate is nonstationary in this instance. This result obtains because each central bank is operating under discretion, which implies that inflation targeting is optimal within this kind of framework (see CGG, 1999). The latter property results in a non stationary price level. Under commitment, price level targeting would be optimal for this type of environment. In that instance, and so long as both central banks optimize under commitment, the nominal exchange rate would be stationary.

5 The Cooperative Equilibrium

Next we consider optimal policy under cooperation. We assume that the two central banks agree to pursue a policy that maximizes a weighted average of the utilities of home and foreign households, with weights determined by the relative size of the two economies. Whereas under Nash, each central bank designs an optimal policy taking the other country's economy as given, in this case there is explicit coordination. As we show further, any gains from policy coordination stem from the effects of foreign economic activity on domestic marginal cost of production.

Since there is complete consumption insurance, the period utility function relevant to the policy maker is the following:

$$U(C_t) - (1 - \gamma) \, V(N_t) - \gamma \, V(N_t^*) \tag{64}$$

We also assume that both economies jointly choose the employment subsidy that maximizes this objective function in the steady state. Under cooperation, each central bank refrains from creating a surprise currency apprecia-

tion that might otherwise be tempting under Nash. As a consequence, as we shown in the appendix, each economy will set the subsidy at a level that exactly offsets the distortions associated with market power in goods and labor markets. More specifically, the common subsidy τ will satisfy the following condition:

$$(1 - \tau)(1 + \mu^w)(1 + \mu^p) = 1 \tag{65}$$

Given that the steady state markups are the same across countries, each fiscal authority will choose the same subsidy. It is important to note that with this subsidy in place, the equilibrium allocation under globally flexible prices and no wage markup shocks is optimal. We refer to the equilibrium as the cooperative steady state.

It is convenient to define the *output gap*, $\widetilde{\widetilde{y}}_t$, as the percent deviation of output from the level that would arise under globally flexible prices and no wage markup shocks, i.e., the level of output that corresponds to the cooperative steady state. We emphasize that $\widetilde{\widetilde{y}}_t$ is distinct from the domestic output gap, $\widetilde{y}_t$, which is the percent deviation of output that would arise when domestic prices are flexible, but taking foreign output as given (along with no wage markup shocks), i.e., the level of output in the Nash steady state. It follows from equation (41) that $\widetilde{\widetilde{y}}_t$ is related to $\widetilde{y}_t$, as follows:

$$\widetilde{\widetilde{y}}_t = \widetilde{y}_t - \frac{\kappa_o}{\kappa} \, \widetilde{\widetilde{y}}_t^* \tag{66}$$

Observe that if foreign output does not affect domestic marginal cost (i.e., $\kappa_o = 0$), then the the distinction between the two gap concepts vanishes.

As we show in the appendix, given the definition of $\widetilde{\widetilde{y}}_t$, a second order approximation to the objective function around the socially optimal (cooperative) steady state takes the form:

$$\mathbb{W}_C \equiv -\frac{1}{2} \Lambda \, E_0 \sum_{t=0}^{\infty} \beta^t \left[(1 - \gamma) \, \left(\pi_t^2 + \alpha \, (\widetilde{\widetilde{y}}_t)^2 \right) + \gamma \, \left(\pi_t^{*2} + \alpha^* (\widetilde{\widetilde{y}}_t^*)^2 \right) - 2 \, \Phi \, \widetilde{\widetilde{y}}_t \widetilde{\widetilde{y}}_t^* \right] \tag{67}$$

with $\Lambda = \frac{\xi}{\delta}$, $\alpha = \frac{\lambda}{\xi}$, $\alpha^* = \frac{\lambda^*}{\xi}$ and,

$$\Phi \equiv \frac{\delta \, (1 - \sigma) \, \gamma (1 - \gamma)}{\xi}$$

where, as in the noncooperative case, losses due to deviations from the steady state are measured in terms of percentages of steady state consumption.

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Combining equations (50) and (66), as well as the analogue equations for the foreign countries, permits us to express the respective aggregate supply curves in terms of the home and foreign output gaps, as follows:

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \lambda\, \widetilde{\widetilde{y}}_t + \lambda_o\, \widetilde{\widetilde{y}}_t^* + u_t \tag{68}$$

$$\pi_t^* = \beta\, E_t\{\pi_{t+1}^*\} + \lambda^*\, \widetilde{\widetilde{y}}_t^* + \lambda_o^*\, \widetilde{\widetilde{y}}_t + u_t^* \tag{69}$$

where $\lambda_o = \delta\kappa_o$ reflects the sensitivity of the home country's domestic inflation rate to the foreign output gap, and $\lambda_o^* = \delta^*\kappa_o^*$ is the analogue for the foreign domestic inflation. Note that the signs of λ_o and λ_o^* depend positively on the respective elasticities of marginal costs with respect to foreign output, κ_o and κ_o^* (see equation (35)). For example, if a rise in foreign output reduces domestic marginal cost ($\kappa_o < 0$), then through this channel, a rise in the foreign output gap will reduce domestic inflation.

In the cooperative case we assume that the central banks jointly maximize the objective given by (67) on a period by period basis, subject to the constraints given by (68) and (69). In addition, expectations of future inflation are taken as given by the policymakers (in other words we solve for the time consistent, jointly optimal policy). Hence, as in the Nash case analyzed in the previous section, we assume that the monetary authorities cannot commit to a state-contingent policy rule that binds them into the future.

The first order conditions are now:

$$\widetilde{\widetilde{y}}_t = -\xi\, \pi_t - \frac{\kappa_o}{\kappa}\left(\widetilde{\widetilde{y}}_t^* + \xi\, \pi_t^*\right) \tag{70}$$

$$\widetilde{\widetilde{y}}_t^* = -\xi\, \pi_t^* - \frac{\kappa_o^*}{\kappa^*}\left(\widetilde{\widetilde{y}}_t + \xi\, \pi_t\right) \tag{71}$$

which can be combined to yield:

$$\widetilde{\widetilde{y}}_t = -\xi\, \pi_t \tag{72}$$

$$\widetilde{\widetilde{y}}_t^* = -\xi\, \pi_t^* \tag{73}$$

Under cooperation, each central bank adjusts the output gap to counter deviations of domestic inflation from target. Indeed, the optimality conditions are identical to the Nash case (equation (57)), but with the output gaps $\widetilde{\widetilde{y}}_t$ and $\widetilde{\widetilde{y}}_t^*$ replacing the domestic output gaps $\widetilde{y}_t$ and $\widetilde{y}_t^*$. This distinction emerges for the following reason: When evaluating the tradeoff between

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output and inflation, the central bank uses as the bliss point for output the natural level under globally flexible prices as opposed to the domestic natural level, since under cooperation it does not take foreign output as given Further, because the aggregate supply curve is unchanged as are the relative weights on domestic inflation in the loss function (α and α^*), the slope coefficient ξ remains the same across the Nash and cooperative cases.

To gain some further perspective on how the solution under cooperation differs from the Nash case, we can use equation (66) to rewrite the optimality conditions in terms of the *domestic* output gaps as follows:

$$\widetilde{y}_t = -\xi \left(\pi_t + \frac{\kappa_o}{\kappa} \pi_t^*\right) \tag{74}$$

$$\widetilde{y}_t^* = -\xi \left(\pi_t^* + \frac{\kappa_o^*}{\kappa^*} \pi_t\right) \tag{75}$$

In contrast to the Nash policy-maker, in setting the domestic output gap, the cooperative policy-maker takes into account foreign inflation as well as home inflation. The weight on foreign inflation, further, depends on the sign and relative strength of the spillover of the foreign output gap on domestic marginal cost measured by the ratio κ_o/κ.

Suppose, for example, that there is a rise in foreign inflation, π_t^*. Under the optimal policy the foreign central bank contracts its output gap, $\widetilde{y}_t^*$, which in turn affects the home country's domestic output gap, $\widetilde{y}_t$, according to equation (66). If the spillover is negative (i.e., the terms of trade effect dominates, implying $\kappa_o < 0$), the contraction in foreign output reduces the home country's domestic natural level of output, implying an increase in $\widetilde{y}_t$. If there are no domestic inflationary pressures (i.e., $\pi_t = 0$), it is optimal under cooperation for the home central bank to accept the increase in $\widetilde{y}_t$ in order to keep the overall output gap, $\widetilde{\widetilde{y}}_t$, fixed at zero. By contrast, under Nash, the central bank instead insulates the domestic output gap, $\widetilde{y}_t$, from foreign inflation.

Under the cooperative equilibrium, accordingly, inflationary pressures generated by cost push shocks can spill over from one country to another. In the symmetric equilibrium, the reduced form expressions for home and foreign inflation are given by

$$\pi_t = \overline{\psi} \cdot \underline{\psi}[\psi^{-1}u_t - \kappa_o\delta\xi u_t^*] \tag{76}$$

$$\pi_t^* = \overline{\psi} \cdot \underline{\psi}[\psi^{-1}u_t^* - \kappa_o\delta\xi u_t]$$

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with $\overline{\psi} = [1 - \beta\rho + \xi(\lambda - \lambda_o)]^{-1}$ and $\underline{\psi} = [1 - \beta\rho + \xi(\lambda + \lambda_o)]^{-1}$, and where, as before, $\psi = [1 - \beta\rho + \xi\lambda]^{-1}$. In the case of negative spillovers ($\kappa_o < 0$), for example, a rise in the foreign cost push shock, u_t^*, leads to an increase in domestic inflation, π_t. This "importing" of inflation is a consequence of permitting a rise in the domestic output gap, $\widetilde{y}_t$, and hence domestic marginal cost as the optimal policy under cooperation dictates in this case. Under Nash, in contrast, inflationary pressures do not spill over across countries.

In general, as long as the marginal cost spillover is present (i.e., $\kappa_o \neq 0$), the equilibrium under cooperation differs from the Nash equilibrium, implying potential gains from policy coordination. Note that in the absence of the spillover (i.e. $\kappa_o = 0$) the the domestic output gap responds to inflation exactly as in the Nash case, as equations (72) and (74) suggest. In turn, domestic inflation in this case is identical to the Nash case, as equation (76) implies (keeping in mind that $\lambda_o = \delta\kappa_o$).

From equation (37), we know that $\kappa_o \neq 0$ iff $\sigma \neq 1$, i.e., if preferences over consumption are not logarithmic. Thus we have:

Proposition 2 *There is a gain to monetary policy cooperation unless $\sigma = 1$.*

We emphasize again that the potential gains come from the spillover of foreign economic activity on domestic marginal cost. This contrasts with Corsetti and Pesenti (2001), who assume $\sigma = 1$, but allow for imperfect exchange rate pass-through. In the special case of their model that features complete pass-through, they also find no gain to monetary policy cooperation.

Finally, we can derive some implications for the behavior of the nominal interest rate and the real exchange rate under cooperation. Combining the optimality condition (74) with the IS curve (49) yields

$$
\begin{aligned}
r_t &= \overline{rr}_t^d + \vartheta \ E_t\{\pi_{t+1}\} + \frac{\kappa_o}{\kappa}(\vartheta - 1)E_t\{\pi_{t+1}^*\} \qquad (77) \\
&= r_t^{nash} + \frac{\kappa_o}{\kappa}(\vartheta - 1) \ E_t\{\pi_{t+1}^*\}
\end{aligned}
$$

Thus, we have:

Proposition 3 *Optimal policy in the cooperative equilibrium can be written as a Taylor rule which is linear in the equilibrium real interest rate, domestic inflation, and foreign inflation.*

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With $\kappa_o < 0$, for example, as we noted earlier, it was optimal for the central bank to permit to domestic output gap to rise in response to an increase in foreign inflation. This suggests that, in response to a rise in foreign inflation, the central bank should increase the interest by less relative to the Nash case. The reverse is true, of course, if $\kappa_o > 0$.

Finally, in the symmetric case, the terms of trade and the nominal exchange rate are given by:

$$
\begin{aligned}
s_t &= (\widetilde{\overline{y}}_t - \widetilde{\overline{y}}_t^*) + (\overline{\overline{y}}_t - \overline{\overline{y}}_t^*) \\
&= -\xi\overline{\psi} \cdot \underline{\psi}(\psi^{-1} + \kappa_o\delta\xi)(u_t - u_t^*) + (a_t - a_t^*)
\end{aligned}
\tag{78}
$$

$$
\begin{aligned}
e_t &\equiv e_{t-1} + s_t - s_{t-1} + \pi_t - \pi_t^* \tag{79} \\
&= e_{t-1} - (\xi - 1)\overline{\psi} \cdot \underline{\psi}(\psi^{-1} + \kappa_o\delta\xi)(u_t - u_t^*) + \xi\overline{\psi} \cdot \underline{\psi}(\psi^{-1} + \kappa_o\delta\xi)(u_{t-1} - u_{t-1}^*) \\
&\quad + (\Delta a_t - \Delta a_t^*) \tag{80}
\end{aligned}
$$

Note that the expressions are the same as in the Nash case, except that the sensitivity of both s_t and e_t to cost push shocks differs in general under cooperation, depending on the sign and magnitude of the spillover parameter, κ_o. Interestingly, the impact of relative productivity shocks on terms of trade and exchange rate variability is the same as under Nash, reflecting the fact that these shocks do not force a tradeoff between output gap and inflation stabilization. In either case, accordingly, under cooperation the nominal exchange rate should be free to vary in response to both relative cost push shocks and relative productivity shocks. Thus we have:

Proposition 4 *Under cooperation, a system of floating exchange rates is optimal. The marginal cost spillover affects the sensitivity of the exchange rate to relative cost push pressures, but it does not affect the sensitivity to relative productivity shocks.*

Finally, we note that it is possible to explicitly calculate the gain from policy coordination by evaluating the loss function (67) under the cooperative equilibrium and Nash equilibrium values of π_t and $\widetilde{\overline{y}}_t$. Since a serious model calibration is beyond the scope of this paper, we save this exercise for subsequent research. We only observe here, that to find a quantitatively important gain from coordination within this framework, at a minimum, there must be a nontrivial spillover effect of foreign output on domestic marginal cost, as measured by the ratio κ_o/κ.

6   Concluding Remarks

A virtue of our framework is that we are able to derive sharp analytical results, while still allowing each central bank to face a short run tradeoff between output and inflation, in contrast to much of the existing literature. Of course, we obtained these results by making some strong assumptions. It would be desirable to consider the implications of relaxing these assumptions, including: allowing for imperfect consumption risk sharing, pricing-to-market, and trade in intermediate inputs. We considering the latter particularly interesting, as it would provide an additional effect of openness on marginal cost, which is the main channel in our model through which there are gains from cooperation.

Finally, while we have considered gains from cooperation, we have not considered the gains from commitment that arise in the closed economy variant of our framework (see, e.g., Clarida, Galí and Gertler, 1999 and Woodford, 1999). It is straightforward to allow for commitment, and we expect that doing so will produce some interesting implications for the behavior of the equilibrium exchange rate.

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Appendix: Policy Objectives and Optimal Subsidies for the Nash and Cooperative Equilibria

We now derive a second order approximation to the central bank's policy objective in the Nash and cooperative cases. Below we make frequent use of the following second order approximation of percent deviations in terms of log deviations:

$$\frac{Y_t - Y}{Y} = y_t + \frac{1}{2}\, y_t^2 + o(\|a\|^3)$$

where $o(\|a\|^n)$ represents terms that are of order higher than n^{th}, in the bound $\|a\|$ on the amplitude of the relevant shocks. We begin with the non-cooperative case and then turn to the cooperative case.

CASE #1: Non-Cooperative Case (Nash equilibrium)

In this case the policymaker seeks to maximize the utility of the domestic representative consumer, taking as given the other country's policy and outcomes. Below we derive a second order approximation of the period utility $U(C_t) - V(N_t)$ about the steady state associated with the optimal non-cooperative choice of a subsidy rate. The latter is chosen to maximize $U(C) - V(N)$ subject to $C = k\,(N)^{1-\gamma}(Y^*)^\gamma$, while taking Y^* parametrically. The associated optimality condition requires $V'(N)N = U'(C)C\,(1-\gamma)$.

Notice that in the equilibrium steady state (with $A = A^* = 1$), we have

$$
\begin{aligned}
(1+\mu^p)^{-1} &= MC = (1+\mu^w)\,(1-\tau)\,\frac{V'(N)}{U'(C)}\,k^{-1}\,S^\gamma \\
&= (1+\mu^w)\,(1-\tau)\,(1-\gamma)
\end{aligned}
$$

where the third equality makes use of the optimality condition derived above, as well as of (31) (evaluated at the steady state). Hence, the optimal choice of τ in the Nash equilibrium satisfies:

$$(1-\tau)(1+\mu^w)(1+\mu^p)(1-\gamma) = 1$$

We start by approximating the utility of consumption $U(C_t)$ about the value of consumption that would prevail under the domestic flexible price equilibrium allocation, *while taking foreign output as given*:

$$U(C_t) \;=\; U(\overline{C}_t) + U'(\overline{C}_t)\overline{C}_t \;\left[\tilde{c}_t + \frac{1}{2}(1-\sigma)\,\tilde{c}_t^2\right] + o(\|a\|^3)$$

$$=\; U(\overline{C}_t) + U'(\overline{C}_t)\overline{C}_t \;\left[(1-\gamma)\,\tilde{y}_t + \frac{1}{2}(1-\sigma)\,(1-\gamma)^2\,\tilde{y}_t^2\right] + o(\|a\|^3)$$

where the second equality follows from (32) in the text, and the fact that foreign output is taken as given in the non-cooperative case considered here.

Furthermore, linearization of $U'(\overline{C}_t)\overline{C}_t$ about the steady state implies:

$$U'(\overline{C}_t)\,\overline{C}_t \;=\; U'(C)C + [U''(C)C + U'(C)]\; C\,\overline{c}_t + o(\|a\|^2)$$

$$=\; U'(C)C + U'(C)C\,(1-\sigma)\,\overline{c}_t + o(\|a\|^2)$$

where $\overline{c}_t \equiv \log \frac{\overline{C}_t}{C}$ denotes the percent deviation of flexible price consumption from its steady state level.

Combining the expressions above we obtain:

$$U(C_t) \;=\; U'(C)C \;\left[(1-\gamma)\,\tilde{y}_t + \frac{1}{2}(1-\sigma)\,(1-\gamma)^2\,\tilde{y}_t^2 + (1-\sigma)(1-\gamma)\,\overline{c}_t\,\tilde{y}_t\right] + t.i.p. + o(\|a\|^3)$$

$$=\; U'(C)C(1-\gamma)\;\left[\tilde{y}_t + \frac{1}{2}(1-\sigma)(1-\gamma)\,\tilde{y}_t^2 + (1+\phi)\,\overline{n}_t\,\tilde{y}_t\right] + t.i.p. + o(\|a\|^3)$$

where the second equality makes use of the equilibrium relationship $(1-\sigma)\,\overline{c}_t = (1+\phi)\,\overline{n}_t$. The latter condition, in turn, can be derived from the first order conditions of the consumer's problem, which, together with optimal price setting under flexible prices imply $\sigma\,\overline{c}_t + \phi\,\overline{n}_t + \gamma\,\overline{s}_t = a_t$, as well as $\overline{y}_t = \overline{c}_t + \gamma\,\overline{s}_t$ (where all lower case variables with an upper bar denote log deviations from steady state)

Similarly, we can approximate the disutility of labor $V(N_t)$ about the domestic flexible price equilibrium as follows:

$$V(N_t) = V(\overline{N}_t) + V'(\overline{N}_t)\overline{N}_t\;\left[\tilde{n}_t + \frac{1}{2}(1+\phi)\,\tilde{n}_t^2\right] + o(\|a\|^3)$$

Furthermore, the term $V'(\overline{N}_t)\overline{N}_t$ can be approximated about the steady state as follows:

$$V'(\overline{N}_t)\overline{N}_t \;=\; V'(N)N + [V''(N)N + V'(N)]\; N\,\overline{n}_t + o(\|a\|^2)$$

$$=\; V'(N)N + V'(N)N\,(1+\phi)\,\overline{n}_t + o(\|a\|^2)$$

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Combining the previous two approximations we obtain

$$V(N_t) = V'(N)N \left[\widetilde{n}_t + \frac{1}{2}(1+\phi) \, \widetilde{n}_t^2 + (1+\phi) \, \overline{n}_t \, \widetilde{n}_t \right] + t.i.p. + o(\|a\|^3)$$

It is easy to show that $N_t = \left(\frac{Y_t}{A_t} \right) \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\xi} di$. Accordingly,

$$\widetilde{n}_t = \widetilde{y}_t + v_t$$

where $v_t \equiv \log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\xi} di$.

Lemma 1: let $\sigma_{p,t}^2 \equiv \int_0^1 (p_{H,t}(i) - p_{H,t})^2 \, di$ the cross-sectional dispersion of prices. Up to a second order approximation, $v_t \simeq \frac{\xi}{2} \, \sigma_{p,t}^2$.
Proof: see Galí and Monacelli (2001).

From the previous Lemma, it follows that $\overline{n}_t \, \widetilde{n}_t = \overline{n}_t \, \widetilde{y}_t + o(\|a\|^3)$. Thus, we can rewrite labor disutility as:

$$V(N_t) = V'(N)N \left[\widetilde{y}_t + \frac{\xi}{2} \, \sigma_{p,t}^2 + \frac{1+\phi}{2} \, \widetilde{y}_t^2 + (1+\phi) \, \overline{n}_t \, \widetilde{y}_t \right] + t.i.p. + o(\|a\|^3)$$

As discussed above, the optimal subsidy under Nash case supports the steady state condition $V'(N)N = U'(C)C(1-\gamma)$. Ignoring terms that depend exclusively on foreign variables (and which are taken as given by the domestic policymaker in the non-coperative case), allows us to rewite the period utility for the home consumer as:

$$U(C_t) - V(N_t) = -\frac{1}{2}U'(C)C \, (1-\gamma) \, \left[\xi \, \sigma_{p,t}^2 + \kappa \, \widetilde{y}_t^2 \right] + t.i.p. + o(\|a\|^3)$$

where $\kappa \equiv \sigma_o + \phi$.
Lemma 2: $\sum_{t=0}^{\infty} \beta^t \, \sigma_{p,t}^2 = \frac{1}{\delta} \sum_{t=0}^{\infty} \beta^t \, \pi_t^2$, where $\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.
Proof: Woodford (2001, NBER WP8071), pp 22-23.

Collecting all the previous results, we can write the second order approximation to the objective function of the home central bank in the non-cooperative case (expressed as a fraction of steady state consumption), as follows:

$$\mathbb{W}_N \equiv -\frac{(1-\gamma)}{2} \; \Lambda \; E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; \left[\pi_t^2 + \alpha \; \tilde{y}_t^2 \right] \right\}$$

with $\Lambda = \frac{\xi}{\delta}$ and $\alpha = \frac{\delta \kappa}{\xi} = \frac{\lambda}{\xi}$.

It is straightforward to obtain an analogous objective function for the foreign central bank.

CASE #2: Cooperative Case

The period utility in the joint objective function to be maximized takes the form:

$$U(C_t) - (1-\gamma)V(N_t) - \gamma V(N_t^*)$$

The choice variables include output, employment, and consumption in both countries, simultaneously. In this joint optimization problem policymakers no longer take the other country's variables as given. This interdependence is taken into account in deriving the second order approximation to the above objective function, which is taken about the steady state associated with the optimal cooperative choice of a subsidy rate. The latter is chosen to maximize the previous objective function to $C = (N)^{1-\gamma}(N^*)^{\gamma}$. The associated optimality condition requires and $V'(N)N = U'(C)C$

Notice that in the equilibrium steady state (with $A = A^* = 1$), we have

$$
\begin{aligned}
(1+\mu^p)^{-1} &= MC = (1+\mu^w)\,(1-\tau)\,\frac{V'(N)}{U'(C)}\,k^{-1}\,S^{\gamma} \\
&= (1+\mu^w)\,(1-\tau)
\end{aligned}
$$

where the third equality makes use of the optimality condition derived above, as well as of (31) (evaluated at the steady state). Hence, the optimal choice of τ in the cooperative equilibrium satisfies:

$$(1-\tau)(1+\mu^w)(1+\mu^p) = 1$$

Recalling that variables with double upper bars denote equilibrium values under globally flexible prices (and double tildes deviations from the latter), a second order approximation of period utility of the home consumer yields:

$$U(C_t) - U(\overline{\overline{C}}_t) \;=\; U'(\overline{\overline{C}}_t)\overline{\overline{C}}_t \left[\widetilde{c}_t + \frac{1}{2}(1-\sigma)\,(\widetilde{c}_t)^2\right] + o(\|a\|^3)$$

$$= \; U'(\overline{\overline{C}}_t)\overline{\overline{C}}_t \left[(1-\gamma)\widetilde{y}_t + \gamma\widetilde{y}_t^* + \frac{1}{2}(1-\sigma)[(1-\gamma)^2\widetilde{y}_t^2 + \gamma^2(\widetilde{y}_t^*)^2 + 2(1-\gamma)\gamma\widetilde{y}_t\widetilde{y}_t^*]\right]\cdot$$

where the second equality follows from (32) in the text.

Furthermore, linearization of $U'(\overline{\overline{C}}_t)\overline{\overline{C}}_t$ about the steady state implies:

$$U'(\overline{\overline{C}}_t)\overline{\overline{C}}_t \;=\; U'(C)C + [U''(C)C + U'(C)]\; C\,\overline{\overline{c}}_t + o(\|a\|^2)$$
$$= \; U'(C)C + U'(C)C\,(1-\sigma)\,\overline{\overline{c}}_t + o(\|a\|^2)$$

where $\overline{\overline{c}}_t \equiv \log \frac{\overline{\overline{C}}_t}{C}$ denotes the percent deviation of consumption from its steady state level, in the world flexible price equilibrium.

Hence,

$$U(C_t) \;=\; U'(C)C\,\{\,(1-\gamma)\,\widetilde{y}_t + \gamma\,\widetilde{y}_t^* + \frac{1}{2}(1-\sigma)\,[(1-\gamma)^2(\widetilde{y}_t)^2 + \gamma^2(\widetilde{y}_t^*)^2 + 2(1-\gamma)\gamma\,\widetilde{y}_t\widetilde{y}_t^*]$$
$$+(1+\phi)\,\overline{\overline{n}}_t\,((1-\gamma)\widetilde{y}_t + \gamma\widetilde{y}_t^*)\} + t.i.p. + o(\|a\|^3)$$

Similarly, we have

$$V(N_t) = V(\overline{\overline{N}}_t) + V'(\overline{\overline{N}}_t)\overline{\overline{N}}_t \left[\widetilde{n}_t + \frac{1}{2}(1+\phi)\,(\widetilde{n}_t)^2\right] + o(\|a\|^3)$$

which, combined with the approximation around the steady state given by

$$V'(\overline{\overline{N}}_t)\overline{\overline{N}}_t \;=\; V'(N)N \;+\; [V''(N)N + V'(N)]\; N\,\overline{\overline{n}}_t + o(\|a\|^2)$$
$$= \; V'(N)N \;+\; V'(N)N\,(1+\phi)\,\overline{\overline{n}}_t + o(\|a\|^2)$$

implies

$$V(N_t) \;=\; V'(N)N \left[\widetilde{n}_t + \frac{1}{2}(1+\phi)\,(\widetilde{n}_t)^2 + (1+\phi)\,\overline{\overline{n}}_t\,\widetilde{n}_t\right] + t.i.p. +\; o(\|a\|^3)$$

$$= \; V'(N)N \left[\widetilde{y}_t + \frac{\xi}{2}\,\sigma_{p,t}^2 + \frac{1}{2}(1+\phi)\,(\widetilde{y}_t)^2 + (1+\phi)\,\overline{\overline{n}}_t\,\widetilde{y}_t\right] + t.i.p. +\; o(\|a\|^3)$$

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A similar expression holds for $V(N_t)$ the foreign country.

Using the second order approximations derived above and imposing the steady state (jointly) optimal condition $V'(N)N = U'(C)C$ (which is assumed to be supported by the appropiate common subsidy), allows us to write the (normalized) period objective as:

$$
\begin{aligned}
\frac{U(C_t) - (1-\gamma)V(N_t) - \gamma V(N_t^*)}{U'(C)C} &= -\frac{1-\gamma}{2} \left[\xi \, \sigma_{p,t}^2 + \kappa \, (\widetilde{y}_t)^2 \right] \\
&\quad -\frac{\gamma}{2} \left[\xi \, \sigma_{p,t}^{*2} + \kappa^* (\widetilde{y}_t^*)^2 \right] \\
&\quad + (1-\sigma) \, \gamma(1-\gamma) \, \widetilde{y}_t \, \widetilde{y}_t^* \\
&\quad + t.i.p. + \ o(\|a\|^3)
\end{aligned}
$$

It follows that we can express the objective function in the cooperative case as:

$$
\mathbb{W}_C \equiv -\frac{1}{2} \Lambda E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma) \left(\pi_t^2 + \alpha \, (\widetilde{y}_t)^2 \right) + \gamma \left(\pi_t^{*2} + \alpha^* (\widetilde{y}_t^*)^2 \right) - 2 \, \Phi \, \widetilde{y}_t \widetilde{y}_t^* \right]
$$

with $\Lambda = \frac{\xi}{\delta}$, $\alpha = \frac{\lambda}{\xi}$, and $\alpha = \frac{\lambda^*}{\xi}$

$$
\Phi \equiv \frac{\delta \, (1-\sigma) \, \gamma(1-\gamma)}{\xi}
$$

where, as in the noncooperative case, losses due to deviations from the steady state are measured in terms of percentages of steady state consumption.