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HIGHWAY FRANCHISING AND REAL ESTATE VALUES

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### ABSTRACT

It has become increasingly common to allocate highway franchises to the bidder that offers to charge the lowest toll. Often, building a highway increases the value of land held by a small group of developers, an effect that is more pronounced with lower tolls. We study the welfare implications of highway franchises that benefit large developers, focusing on the incentives developers have to internalize the effect of the toll they bid on the value of their land. We study how participation by developers in the auction affects equilibrium tolls and welfare. We find that large developers bid more aggressively than construction companies that own no land. As long as land ownership is sufficiently concentrated, allowing developers in the auction leads to lower tolls and higher welfare. Moreover, collusion among developers is socially desirable. We also analyze the case when the franchise holder can charge lower tolls to those buying her land ('toll discrimination'). Relative to uniform tolls, discrimination decreases welfare when land is highly concentrated, but increases welfare otherwise. Finally, we consider the welfare implications of subsidies and bonuses for proposing new highway projects.

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# 1 Introduction and motivation

The standard way to finance highways leading to new land developments is with general public funds, to be recovered via increased property taxes.<sup>2</sup> One problem with this approach, is that the increase in tax receipts takes place *after* the road is built (and the land developed and sold to new owners), thereby leaving ample space for opportunistic behavior by landowners and land developers. For example, they could vote down a tax increase after the road is built, or, alternatively, lobby for socially undesirable projects that collect far less taxes than suggested by the optimistic forecasts of the proponents.<sup>3</sup> Given this problem, during the last decades franchises have become an increasingly popular way of financing highways,<sup>4</sup> operating under so called build-operate-and-transfer (BOT) contracts.<sup>5</sup> These franchises are usually awarded in competitive auctions to the bidder that offers to charge the lowest toll for a pre-specified period.

In this paper we study the welfare consequences of highway franchises that benefit large land developers. In doing so we address several questions: (a) Should large landowners be allowed to participate in the auction, or do they have an “undue advantage” (i.e., one that leads to a decrease in welfare)? (b) How does an auction compare with a monopoly road owner? (c) Should a franchise holder that owns land be allowed to offer lower tolls to the buyers of his tracts of land? (d) Should collusion among bidders be deterred? (e) What are the welfare consequences of government subsidies for building the road or government bonuses for the proponents of a new project?

We find that large developers bid more aggressively than construction companies that own no land. As long as land ownership is sufficiently concentrated, allowing developers in the auction leads to lower tolls and higher welfare. Moreover, collusion among developers is socially desirable. We also analyze the case when the franchise holder can charge lower tolls to those buying her land (‘toll discrimination’). Relative to uniform tolls, discrimination decreases welfare when land is highly concentrated, but increases welfare otherwise.

A specific example is useful to motivate most of the issues considered. In 1999 the Chilean government decided to franchise the US\$170MM road through the valley of Chicureo, a formerly rural area near Santiago which is slotted for major expansion. The project was proposed by a private group, and would add value to the substantial extensions of land this group owns in the area to be served by the road. According to the Chilean Concessions Law of 1994, anybody can propose a highway project. If approved by the Ministry of Public Works, the project is franchised in an open auction and the proponent of the project receives a bonus in the auction, implying that she may win even if she does not make the best offer. Shortly after the project was approved by the Ministry of Public Works, the original proponents formed an agreement with other large landowners in the valley in order to participate in the auction. Moreover, this consortium is

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<sup>2</sup>The idea is that a substantial part of the highway’s benefits are reflected in higher land prices.

<sup>3</sup>Moreover, even if landholders and land developers do not behave opportunistically, it may still be difficult for the government to determine whether a project is beneficial before undertaking it, since it may be difficult to obtain precise estimates of the future commercial value of the land.

<sup>4</sup>See, for example, Gómez-Ibañez and Meyer [1993], and the collection of papers in Irwin et al. [1997].

<sup>5</sup>Under such a contract, a private firm builds and finances the road and then collects tolls for a long period (usually between 10 and 30 years). When the franchise ends the road is transferred to the state.

considering implementing a scheme by which buyers of their land will not have to pay tolls during the first few years (Magni, 2000). Finally, there were no participants when the road was auctioned in 2000, with the proponents arguing that a more generous subsidy than that offered by the government was required, since toll revenue was insufficient to finance the road.<sup>6,7</sup>

Throughout the paper we consider *socially desirable* roads, that is, roads such that if built, the increase in welfare (reflected in the price of the land) is higher than the cost of building the road when no tolls are charged.<sup>8</sup> Roads that satisfy these conditions should be built. However, there is no guarantee that they will be built, since franchise holders do not necessarily appropriate all of the benefits produced by the road, unless they happen to own all the land that increases its value thanks to the project.

We examine two options for building the road: first, an unplanned or *laissez faire* approach, which serves as a benchmark, in which the owner of a tract of land builds a road and charges a toll for its use. There is no coordination with other landowners and the franchise holder is free to choose the toll at her discretion. The second option is to have the government franchise the project in a competitive auction, to the bidder offering the lowest toll.

Laissez faire is simplest to analyze. Suppose first that toll discrimination is forbidden ('uniform tolls'). If there is competitive demand in the land market, the road owner appropriates all the consumer surplus generated by the road when selling her land plots. Since tolls can be set at any level, the owner of the road will have to balance the increases in toll revenue paid by those who do not own her land, with the fall in the price of her real estate. Hence her optimal toll will vary between the monopoly toll (if she owns no land) and a zero toll (if she owns all of the land). On the other hand, if toll discrimination is possible then, independent of how much land she owns, her best strategy is to set a zero toll on buyers of her plots and a monopoly price on all other users of the road in order to help defray costs. It is not obvious whether toll discrimination should be allowed under laissez faire, except in the obvious case in which the road would not have been built in the absence of discrimination. On the one hand, buyers of the road owner's plots obtain the maximum consumer surplus from the road (which is transferred efficiently via land prices to the road owner), but other users are penalized. We show that under certain conditions, discrimination is bad. However, there are counterexamples in which discrimination is beneficial (even when the road would still have been built under uniform tolls), so it is not obvious whether discrimination should be prohibited under laissez faire.

In the alternative approach, the planner auctions the franchise to all bidders (which may include non-owners of land) on the basis of the lowest toll. A landowner then weighs two effects when deciding how to

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<sup>6</sup>It is also worthwhile noting that there have been many cases of highway franchises associated with land developments in Asia (Guasch, 2000). Note also that the United States awarded land grants to the first transcontinental railways in order to reduce the subsidies required by the developers (Faulkner, 1960).

<sup>7</sup>Another example, in an area far removed from highways, is the creation of a small independent telephone company in a newly developed area in Santiago, whose main object was to raise the price of land at a time when waiting times from the state owned telephone company were very long (Díaz and Soto, 1999). Finally, note that often plots of land are sold with some basic infrastructure in place: roads, water main connections, drainage, electricity supply. In this case there are economies of scale in providing these services (and in their inspection by local authorities) for many plots, and their benefits are extracted through higher prices of land.

<sup>8</sup>In order to simplify the analysis, we assume no congestion.

participate in the auction. If she does not bid aggressively, she avoids paying for the road, but the winning bid may be too high, leading to lower land prices. On the other hand, if she wins (by bidding aggressively), land prices rise and she gets the toll revenue, but has to pay for the road.

With uniform tolls these two effects lead to some surprising results. Assume there is one large landowner. She will bid a fairly low toll that does not generate enough revenues to pay for the road, but will lead to high land prices. If there is another only slightly smaller landowner that participates in the auction, welfare is not higher and may be lower than without his participation. The explanation is simple: since the large landowner will charge a lower toll if she gets the road, it is beneficial for the second owner to have the large owner build the road. However, since the optimal toll set by the second owner is only slightly higher, the large landowner prefers to avoid paying for the road and have the smaller landowner build the road, a solution which is inefficient, since overall tolls are higher. Thus there are two Nash solutions to the bidding game, one of which is worse than the equilibria with no competition. Best of all however, is to allow the two landowners to collude in offering a much lower bid, which raises everyone's welfare.

If discrimination is allowed, or if it cannot be detected, we show that the largest landowner will limit price the second largest landowner in the auction. Thus the lowest toll that the second landowner can credibly bid—the toll that leaves her indifferent between building the road and not doing so—plays a central role in this case. The largest landowner always wins and charges zero tolls to those who buy her real estate. The other landowners participate in order to force her to offer a lower toll.

Discrimination leads to higher welfare than uniform tolls when the difference in land holdings between the two largest landowners is small, since under uniform tolls most of the benefits of the road are received through land appreciation, which requires low tolls for everyone. By contrast, uniform tolls are better when one landowner owns most of the land.

In many cases, governments are led to subsidize BOT projects because they expect welfare to increase due to the externalities associated to the project. As we have argued before (Engel, Fischer and Galetovic, 1997), subsidies eliminate one of the main advantages of infrastructure franchises, namely screening white elephants. In the case of projects where many of its associated externalities are captured by landowners, the fact that toll revenues are not sufficient to finance the road is not a reason for a subsidy. An additional disadvantage of subsidies is that often they amount to a transfer to the franchise holder, without changing the toll or the feasibility of the project. It is only when landowners are small and do not internalize most of the benefits of the road that the subsidy can have a potentially beneficial effect.

This paper is related to the literature on franchise bidding pioneered by Chadwick (1859) and Demsetz (1968), according to which competition for a monopoly infrastructure project will reproduce the competitive outcome (see also Stigler [1968], Posner [1972] and Riordan and Sappington [1987], Spulber [1989, ch. 9], Laffont and Tirole [1993, chs. 7 and 8], Harstad and Crew [1999] and Engel, Fischer and Galetovic [2001 a, b]).<sup>9</sup> Somewhat less related is the limited literature on auctions of objects with externalities. For example, Jehiel, Moldovanu and Stacchetti (1996, 1999) solve a mechanism design problem in which the auctioned object causes an identity-dependent externality on bidders that loose in the auction. We differ from these

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<sup>9</sup>But see Williamson (1976, 1985) for a critique.

papers in that we model the microeconomic origin of the externality—the winning toll affects the welfare of all landowners— which allows us to analyze policy questions.<sup>10</sup>

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the consequences of letting one landowner build the road and giving him free hand to choose tolls (*laissez faire*). Section 4 examines the bidding behavior and outcome of the auction. Section 5 looks at policy implications. Uniform tolls are assumed throughout Sections 3, 4 and 5. We consider toll discrimination in Section 6, characterizing both the outcomes under *laissez faire* and under competitive bidding. In this section we also make welfare comparisons between the cases of uniform and discriminatory tolls. Section 7 concludes. Several appendices with formal proofs follows.

## 2 Basic model

We consider a static model with three types of agents: developers, construction companies and households.<sup>11</sup> There is a land development composed by many identical plots of land and a much larger number (a continuum) of households with identical preferences and monetary income, so that the development under consideration is marginal within the relevant real-estate market and hence does not affect real-estate prices elsewhere.<sup>12</sup> To simplify notation we normalize the total area of land to one. Each household demands only one plot. The highway increases the willingness to pay for each plot of land depending on the number of trips each household makes, viz.

$$V(p) = \int_0^{D(p)} D^{-1}(s) ds,$$

where  $D(p)$  is the demand for trips when the toll is  $p$ , with  $D' < 0$ . Hence, a lower toll increases the number of trips made by the household, thereby raising members' welfare.

The increase in willingness to pay is divided between toll payments,  $pD(p)$ , and payment for the plot, which also depends on  $p$  and is:

$$(1) \quad V(p) - pD(p) = \int_0^{D(p)} [D^{-1}(s) - p] ds.$$

Note that since the land development is small and the number of households is large, in equilibrium all households who buy a plot will obtain the same utility as in any other location in the relevant real estate market. Hence, competition among households pushes up the price of each plot until the increase in value due to the highway is fully capitalized in the land rent.<sup>13</sup>

<sup>10</sup>Also, instead of solving a mechanism design problem, we assume that the road is auctioned to the bidder offering the lowest toll, which is how most of these roads have been auctioned. Possibly, more sophisticated designs have not been used because of the transaction costs involved.

<sup>11</sup>Adding the time dimension changes static values to discounted flows of benefits without significant implications. For this reason, using revenue based auctions, as in Engel, Fischer and Galetovic (2001b), or other auction mechanisms, does not alter the results of this paper.

<sup>12</sup>In terms of standard urban economic theory (see, for example, Fujita [1989]), the development is located at one of a continuum of locations and hence is “small” in terms of the aggregate land market.

<sup>13</sup>More generally, the highway is a neighborhood good (see Fujita [1989, ch. 6.5]).

Developers are indexed by the fraction of plots they own. We often consider two developers,  $\alpha$  and  $\beta$ , with  $\alpha \geq \beta \geq 0$  and  $\alpha + \beta \leq 1$ . For any given toll  $p$ , the developer's objective is to sell her land for as much as possible.

We further assume that developers, and many identical construction companies that own no land, can build the road at cost  $I$ . For simplicity, we ignore maintenance and operation costs. Last, note that when the toll is  $p$ , the increase in welfare is

$$(2) \quad W(p) \equiv V(p) - I.$$

We assume  $W(0) > 0$ , so that it would be efficient to build the road if it could be financed with lump sum transfers.<sup>14</sup>

### 3 Laissez faire

We begin by studying the case of laissez faire, where the agent who builds the road is free to choose the toll. This benchmark will be useful in the next section, when we evaluate auctions for the road. Throughout this and the next section, we assume that the  $\alpha$ -developer must charge the same toll to all users ('uniform tolls').<sup>15</sup> She faces the following tradeoff: on the one hand, she would like to charge the monopoly toll to plot owners that do not buy her land; on the other hand, she would like to charge no toll at all to those that buy land from her, since the revenue she obtains from these individuals (either through tolls or land payment) is larger for lower tolls.

To formally derive this tradeoff, we note that  $\alpha$  chooses the toll that maximizes a weighted average of toll revenues and land sales, viz.

$$(3) \quad \Pi(p; \alpha, I) \equiv pD(p) + \alpha[V(p) - pD(p)] - I = (1 - \alpha)pD(p) + \alpha V(p) - I.$$

The term  $(1 - \alpha)pD(p)$  in (3) corresponds to revenues obtained from those who do not buy the developer's land. Assuming a relatively inelastic demand, as we do, this component of profits increases with  $p$ , as long as  $p$  is below the monopoly toll  $p_m$ . By contrast, the term  $\alpha V(p)$  in (3) is total revenue obtained by the developer, via tolls or via land sales, from those who buy her plots. It is straightforward to see that this component, which equals consumer surplus, is decreasing in  $p$ . The situation is summarized in Figure 1, showing the distribution of consumer surplus between toll income and payment for land plots. The first component is maximized at the monopoly toll, denoted by  $p_m$  in what follows, the second component at  $p = 0$ . Figure 1 also depicts the deadweight loss from charging a positive toll.

We assume that the developer's maximization problem has a solution for all  $\alpha \in [0, 1]$ . Without additional assumptions, a fundamental result on monotonic optimal solutions for supermodular functions then implies that the optimal toll correspondence,  $\operatorname{argmax}_p \Pi(p; \alpha)$ , is decreasing in  $\alpha$ , for  $\alpha \in [0, 1]$ .<sup>16</sup>

<sup>14</sup>Without loss of generality we ignore through-drivers.

<sup>15</sup>Toll discrimination is considered in Section 6.

<sup>16</sup>This follows directly from Theorem 2.8.2 in Topkis (1998). To see this, using the notation in that theorem let  $t \equiv 1 - \alpha$  and

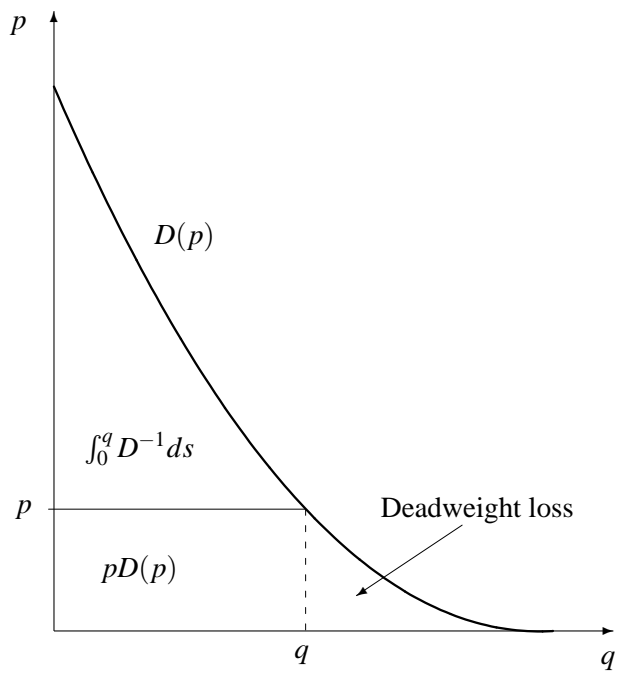


Figure 1: Tolls and welfare



In what follows we simplify the analysis and assume that for each  $\alpha \in [0, 1]$  the solution to the developer's maximization problem is unique (denoted by  $p^*(\alpha)$ ) and satisfies the first order condition:<sup>17</sup>

$$(4) \quad pD'(p) + (1 - \alpha)D(p) = 0,$$

which leads to

$$(5) \quad \varepsilon(p^*(\alpha)) = -(1 - \alpha),$$

where  $\varepsilon(p) \equiv pD'(p)/D(p)$  is the elasticity of the demand for trips at price  $p$ .<sup>18</sup>

It follows from (5) that, as  $\alpha \rightarrow 1$ , the elasticity tends to zero, i.e. to the case of a zero toll. Conversely, as  $\alpha \rightarrow 0$ , the elasticity tends to one, which corresponds to the case of a monopoly. Thus:

**Result 1** *When developer  $\alpha \in [0, 1]$  owns the road, she sets a toll between 0 and the monopoly price. If she owns no land ( $\alpha = 0$ ), she charges the monopoly toll,  $p_m$ . At the other extreme, if she owns all the land ( $\alpha = 1$ ), she sets  $p = 0$ . Furthermore,  $p^*(\alpha)$  is decreasing in  $\alpha$ .*

Result 1 summarizes the central tradeoff faced by the developer between charging high tolls to those who do not buy her land and low tolls to those who do. At one extreme, when  $\alpha = 1$ , there is no tradeoff: the distortions created by charging tolls are borne by the developer. Therefore, since we assume no congestion, she sets  $p = 0$ . By contrast, when  $\alpha = 0$ , the road operator sets the monopoly toll because she does not internalize any of the efficiency losses caused by the distortion.

Total welfare equals

$$(6) \quad W(p^*(\alpha)) = V(p^*(\alpha)) - I,$$

which is clearly increasing in  $\alpha$ . Hence:

**Result 2** *Under laissez faire it is efficient to allocate the right to build the road to the developer who owns the largest amount of land. Moreover, since  $p^*(1) = 0$ , it follows that welfare is maximized when the road is built by a developer who owns all the land.*

From a social perspective tolls are just a distorting transfer. When  $\alpha = 1$ , the owner of the road fully internalizes the social cost of the price distortion, since she replicates the social optimum, acting like a social planner who can charge lump sum taxes.

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define

$$f(x, t) \equiv txD(x) + (1 - t) \int_0^{D(x)} D^{-1}(s) ds - I.$$

We then have that  $\partial^2 f / \partial x \partial t = D(x) > 0$ , showing that the only non-trivial assumption of the theorem is satisfied.

<sup>17</sup>Using supermodularity results, many of the propositions that follow can be shown to hold with considerable more generality.

<sup>18</sup>We show in Appendix A that the solutions of (5) satisfy the second order condition, for all  $\alpha$ , if

$$[D'(p)]^2 > \frac{1}{2} D(p) D''(p)$$

for all  $p$  below the monopoly price. This holds, for example, for linear demand curves.

The last result illustrates a further advantage of concentrated landholding. Consider the case where the road cannot generate enough revenue to finance its cost even when monopoly tolls are charged (i.e.,  $p_m D(p_m) < I$ ). However, profits evaluated at the optimal toll,  $\Pi_\alpha^* \equiv \Pi(p^*(\alpha); \alpha)$ , are increasing in  $\alpha$ .<sup>19</sup> And since, by assumption,  $\Pi_1^* = V(0) - I > 0$ , we have that there exist an  $\bar{\alpha} > 0$  such that for all  $\alpha < \bar{\alpha}$  an  $\alpha$ -landowner does not find it attractive to build the road, even if she is allowed to set the toll she desires.

**Result 3** *Socially desirable roads may not be built when landholding is too dispersed.*

To end this section, note the analogy between the results derived here and the standard double marginalization result of monopoly theory (Spengler, 1950). As in the standard case, vertical integration into the downstream real estate market reduces the incentive to price monopolistically in the upstream road market and simultaneously increases firm's profits. Hence, it is socially desirable.

## 4 Bidding

Behind the trend of using open auctions to franchise roads is Chadwick's (1859) idea, popularized by Demsetz (1968), that competition for the field can substitute for competition in the field. Can a competitive auction improve laissez faire? Auctioning a franchise has two clear advantages. First, it moderates the commitment problem that a developer may face vis-a-vis the road owner, because the franchise contract forces the road owner to charge no more than the toll she bid in the auction.<sup>20</sup> Second, an auction forces potential franchise holders to compete and set a toll at most high enough to defray the road's investment cost.

Yet it is not obvious that competition is useful in all circumstances. For example, it follows from Result 1 that under laissez faire a landowner that owns most of the land will set a toll that does not generate enough income to defray the road's cost. Thus other landowners and construction firms will find it unattractive to participate in the auction and for all practical purposes the outcome of an auction does not differ from laissez faire. Furthermore, a competitive auction could conceivably worsen things compared with laissez faire. Consider, for example, the case where two developers own a similar fraction of plots. If this fraction is large enough, toll revenue may be below road construction costs for both of them, so that both of them prefer that the other one builds the road. Thus unexpected results may follow from the fact that the winning bid affects all developers.

### 4.1 Time line of the game

The timing of actions is as follows:

- Developers  $\alpha$  and  $\beta$  participate in the auction, with  $\alpha \geq \beta > 0$  and  $\alpha + \beta \leq 1$ .<sup>21</sup> There also are at least

<sup>19</sup>Let  $p^*$  denote  $p^*(\alpha)$  and  $q^* \equiv D(p^*)$ . The envelope theorem implies that  $\frac{d\Pi_\alpha^*}{d\alpha} = V(p^*) - p^* q^*$ , which is the household's willingness to pay, net of tolls, and therefore strictly positive (see [1]).

<sup>20</sup>Implicit in this argument is the assumption that a franchise contract is easier to enforce than a private contract.

<sup>21</sup>All results extend trivially to the case of more than two developers.

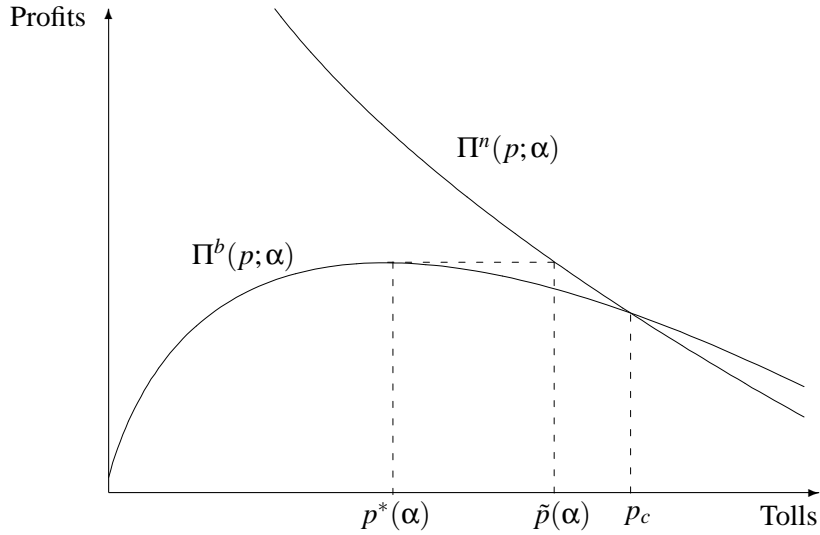


Figure 2: Profits from building and not building. Case of large developer.

two construction firms that participate in the auction. All participants bid a toll in  $[0, \infty]$ , where a bid of  $\infty$  is equivalent to not participating in the auction.

- The road is allocated to the firm which makes the lowest bid, denoted by  $\underline{p}$  in what follows. If bidders tie, the winner is the bidder owning the largest fraction of land (among those offering the lowest toll).
- The winner builds the road and charges at most  $\underline{p}$  for each ride.

Participants in the auction are developers  $\alpha$  and  $\beta$  plus the group of identical building companies who own no land. All participants have construction costs equal to  $I$ .

If there exists a toll  $p_c$  that makes the road self-financing, it is straightforward to see that bidding this toll is the dominant strategy for construction firms.<sup>22</sup> And if no such toll exists—i.e., if  $p_m D(p_m) < I$ —then construction firms will not participate in the auction.

What makes the problem interesting is the behavior of developers. The key strategic interaction in the auction is that developers may prefer to free ride and let someone else build the road. However, not building the road may lead to an unattractively high toll. To appreciate this tradeoff, consider developer  $\alpha$  when making her bid. Conditional on winning the auction she would like to set  $p = p^*(\alpha)$ , which maximizes her revenues by optimally internalizing the effects of tolls on the value of her land. Nevertheless,  $\alpha$  may want to let  $\beta$  win. While tolls would probably be higher than  $p^*(\alpha)$ ,<sup>23</sup>  $\alpha$  would save  $I$  in construction costs.

<sup>22</sup>All that is needed for this result is that there be at least two construction firms.

<sup>23</sup>Recall that, since  $\alpha \geq \beta$ , it follows from Result 1 that  $p^*(\alpha) \leq p^*(\beta)$ .

To analyze this tradeoff, it is useful to compare profits when building and not building the road. If developer  $\alpha$  wins the auction with toll  $\underline{p}$ , her profits are

$$\Pi^b(\underline{p}; \alpha) = \underline{p}D(\underline{p}) + \alpha[V(\underline{p}) - \underline{p}D(\underline{p})] - I,$$

where superscript ‘ $b$ ’ stands for ‘build’. This function is plotted in Figure 2. From the previous section we know that it peaks at  $p^*(\alpha)$ . On the other hand, if some other agent wins the auction and sets toll  $\underline{p}$ , then  $\alpha$  earns

$$(7) \quad \Pi^n(\underline{p}; \alpha) = \alpha[V(\underline{p}) - \underline{p}D(\underline{p})],$$

where superscript ‘ $n$ ’ stands for ‘not build’. This function is also plotted in Figure 2, and is decreasing and convex in the winning toll: the higher the toll, the lower the value of  $\alpha$ ’s plots.

Clearly  $\Pi^n(p; \alpha) = \Pi^b(p; \alpha) - pD(p) + I$ , because building the road enables  $\alpha$  to cash  $pD(p)$  in toll revenue at the cost of investing  $I$ . Thus if both curves intersect, the smallest toll at which they cross, denoted by  $p_c(I)$ , satisfies

$$(8) \quad pD(p) = I.$$

Note that  $p_c$  is independent of  $\alpha$ . Moreover, from Figure 2 it follows that developer  $\alpha$  would rather have someone else build the road for all tolls below  $p_c$ , since in that range  $pD(p) < I$ .

When the road, viewed as a separate project, is not profitable,  $\Pi^n$  remains above  $\Pi^b$  for all (finite) tolls. Letting  $p_c = \infty$ , the results that follow extend (with little effort) to this case, so we do not consider it separately in what follows.

Many of the results in this section hinge on how dispersed land ownership is. For this reason, next we provide a useful definition of ‘‘small developer’’, where ‘‘small’’ is relative to the building cost of the road.

**Definition 1** *Developer  $\alpha$  is small if  $p^*(\alpha) \geq p_c(I)$ . ■*

Figure 3 depicts the case when  $\alpha$  is ‘‘small’’. If allowed to build the road and charge whatever toll she wants, a small developer charges more than  $p_c$ . By contrast, a large developer, who is depicted in Figure 2, prefers to charge less than  $p_c$ .

The existence of (at least) two building companies ensures that the winning bid can never be higher than  $p_c$ . Thus:

**Result 4**  *$p \leq p_c$  in equilibrium.*

In Appendix B we show that this game always has a Nash equilibrium in pure strategies. Recall that under *laissez faire* the toll that results is either  $p^*(\alpha)$  or  $p^*(\beta)$ , depending on which developer builds the road. Thus, if both developers are small, the toll that results under *laissez faire* is above  $p_c$ , and since welfare decreases monotonically with the winning bid:

**Result 5** *When  $\alpha$  (and therefore  $\beta$ ) is small, an auction leads to higher welfare than *laissez faire*.*

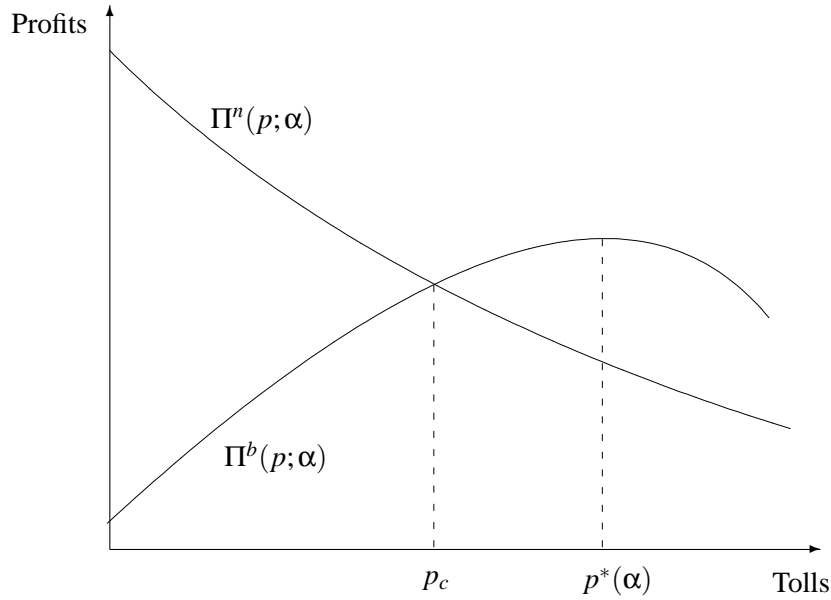


Figure 3: Profits from building and not building. Case of small developer.

Competition for the field is welfare improving when developers are small. Building companies force developers to compete away part of the rents they could obtain under laissez faire from exploiting the road's monopoly power. It is also apparent from Figure 3 that no agent will bid less than  $p_c$  in equilibrium when  $\alpha$  is small: should a developer win with  $p < p_c$ , she could increase her profits by unilaterally deviating and bidding  $p_c$ . By doing so profits increase to  $\Pi^b(p_c, \alpha) = \Pi^n(p_c, \alpha)$  (see Figure 2), independent of whether deviating leads to winning or loosing the auction. Hence:

**Result 6** *When developers are small,  $p = p_c$  in equilibrium. Moreover, it is irrelevant whether developers participate in the auction.*

What happens when developer  $\alpha$  is large? It can be seen from Figure 2 that  $p_c$  can no longer be the equilibrium toll, for if it were it would pay the developer to unilaterally deviate bidding  $p = p^*(\alpha)$ . On the other hand, if developer  $\beta$  is small, she is not willing to bid less than  $p_c$  (as is implied by Figure 3). Hence

**Result 7** *If only one developer is large, then  $\underline{p} = p^*(\alpha) < p_c$  in equilibrium.*

It can easily be shown (see Appendix B) that when  $\alpha$  is sufficiently larger than  $\beta$ , in all Nash equilibria  $\alpha$  wins the auction bidding  $p^*(\alpha)$ . Thus, additional bidders who own little or no land do not force lower tolls than what would obtain under laissez faire. The reason, quite simply, is that a large developer internalizes the effect of higher tolls on land values and this gives him a decisive "advantage" in the auction. For this reason, we have

**Result 8** *Excluding large developers from the auction leads to a higher toll and is welfare decreasing.*

Now take the one large developer case as a benchmark. Can competition between large developers buy an extra reduction in tolls and an increase in welfare? To answer this question, note that when developers  $\alpha$  and  $\beta$  are large,  $p^*(\alpha) \leq p^*(\beta) < p_c$ . Now suppose that  $\alpha$  does not participate in the auction (i.e.,  $\alpha$  “bids”  $p = \infty$ ). Given that strategy,  $\beta$  has no incentive to deviate and (optimally) bids  $p = p^*(\beta)$ . The same holds for  $\alpha$  as long as  $p^*(\beta)$  is sufficiently close to  $p^*(\alpha)$ , where Figure 2 suggests that the precise meaning of “sufficiently close” is that

$$(9) \quad p^*(\beta) < \tilde{p}(\alpha),$$

with  $\tilde{p}(\alpha) < p_c$  defined by:<sup>24</sup>

$$\Pi^n(\tilde{p}(\alpha); \alpha) = \Pi^b(p^*(\alpha); \alpha).$$

The above condition implies that

$$\Pi^n(p^*(\beta); \alpha) > \Pi^n(\tilde{p}(\alpha); \alpha) = \Pi^b(p^*(\alpha); \alpha),$$

where the inequality follows from condition (9) and  $\Pi^n$  being decreasing, and the equality from the definition of  $\tilde{p}$ . Thus condition (9) ensures that  $\alpha$  is better off not bidding, letting  $\beta$  build the road and charge  $p^*(\beta)$ ; the higher toll charged by  $\beta$  is more than compensated by not having to pay  $I$  to build the road. And given that  $\alpha$  bids  $\infty$ , it is optimal for  $\beta$  to bid  $p^*(\beta)$  and build the road. Similarly, one can show that there exists a Nash equilibrium where  $\alpha$  bids  $p^*(\alpha)$  and  $\beta$  does not participate in the auction (see Appendix B). Thus

**Result 9** *Competition among large developers does not increase welfare; it may even bring about higher tolls and lower welfare.*

One can even go beyond Result 9. Not only may competition among large developers be socially harmful, but explicit collusion through joint bidding is clearly welfare improving. To see this, suppose that  $\alpha$  and  $\beta$  costlessly collude and bid to maximize joint profits (this will occur if bargaining is efficient). Then they would bid  $p = p^*(\alpha + \beta) < p^*(\alpha)$ , thereby winning the auction. Hence

**Result 10** *Collusion among large developers brings about lower tolls and unambiguously increases welfare.*

The benefits from collusion are twofold. First, it eliminates the socially inefficient equilibrium where  $\beta$  builds the road. Second, it is profitable for large developers to bid below  $p^*(\alpha)$ . Hence, regulators should not only allow large developers to participate in the auction, but should also encourage them to collude!<sup>25</sup>

Summing up, allowing developers to participate in the auction never hurts social welfare and is socially desirable when at least one developer is large. Also, competition between large developers never increase

<sup>24</sup>That  $\tilde{p}(\alpha) < p_c$  follows from:  $\Pi^n(\tilde{p}(\alpha); \alpha) = \Pi^b(p^*(\alpha); \alpha) > \Pi^b(p_c; \alpha) = \Pi^n(p_c; \alpha)$ , where the first equality follows from the definition of  $\tilde{p}$ ; the following inequality from  $\alpha$  being large and  $\Pi^b$  strictly decreasing for  $p > p^*(\alpha)$ ; and the second equality from the definition of  $p_c$ . Hence, since  $\Pi^n$  is decreasing in  $p$ ,  $\tilde{p}(\alpha) < p_c$ .

<sup>25</sup>This prescription changes when toll discrimination is possible, see Section 6.

welfare and may lower it. For this reason, allowing developers to collude is socially desirable. Far from discouraging joint bidding, auction design should facilitate coordination among bidders who own land and facilitate side payments among them.

## 5 Some policy implications

### 5.1 Subsidies

Governments often subsidize BOT road contracts because of externalities associated with the project.<sup>26</sup> Such subsidies should be scrutinized with particular care, since an unfavorable social evaluation of the road as a stand alone project does not justify a subsidy. Indeed, if benefits obtained by landowners from the appreciation of their real-estate are larger than losses associated with building the road, a government subsidy is not needed to induce landowners to build the road.

A subsidy raises welfare if it leads to the construction of a socially desirable road (i.e., one that satisfies  $W(0) > 0$ , see [2]) or if resulting tolls are lower than without the subsidy.<sup>27</sup> To determine whether the subsidy actually increases overall welfare, its benefits must be weighed against the costs associated with raising the funds to finance it.

Formally, consider a socially desirable road that costs  $I$  to build and a subsidy  $S$ ,  $0 < S < I$ . First we consider the case of laissez faire, where a landowner that owns a fraction  $\alpha$  of the land builds the road and sets the toll she desires. If this landowner would not have built the road without the subsidy,<sup>28</sup> then the subsidy increases welfare generating profits for all landowners, not only the one that builds the road. By contrast, if building the road and selling her land is attractive for the landowner even without the subsidy, introducing the subsidy has no effect on tolls (since  $p^*(\alpha)$  does not depend on  $I$ , see Result 1) and constitutes a pure transfer to the road builder.

Next we consider the case where the franchise holder is determined through a competitive auction. As in Section 4, there are two land developers,  $\alpha$  and  $\beta$ , with  $\alpha > \beta$ , and a fringe of building companies. The cost of building the road is the same for all potential builders and equal to  $I$ . A construction subsidy of  $S$  reduces the toll that leaves a building company indifferent between building and not building the road, from  $p_c(I)$  to  $p_c(I - S)$ .<sup>29</sup>

There are a large number of cases that need to be analyzed, yet all of them fall into two groups. In the first group the subsidy is a pure transfer to landowners; in the second it enables the road to be built or lowers tolls. We present one example within each group.

Consider first the case where, both without and with the subsidy,  $\alpha$  is large and  $\beta$  is small (see Definition 1). Then (see Result 7) the toll, both with and without the subsidy, will be  $p^*(\alpha)$ ; thus the subsidy is a

<sup>26</sup>See Engel, Fischer and Galetovic (1997) for a thorough discussion of government guarantees in infrastructure franchises.

<sup>27</sup>Note that, within the model presented in this paper, the sole beneficiaries of the increases in welfare described above are landowners, since they extract all rents from toll users when selling their plots of land.

<sup>28</sup>That is,  $\Pi(p^*(\alpha); \alpha, I - S) > 0 > \Pi(p^*(\alpha); \alpha, I)$ , with  $\Pi$  defined in (3).

<sup>29</sup>This assumes that such tolls exist. The results that follow do not require this assumption. Also, see (8) for the definition of  $p_c$ .

pure transfer to developer  $\alpha$ .

By contrast, consider the case where  $\alpha$  is small without the subsidy and large with the subsidy, and  $\beta$  is small both with and without the subsidy. Then introducing the subsidy reduces the toll from  $p^*(\alpha)$  to  $p_c(I - S)$ .<sup>30</sup>

## 5.2 Bonus for proposals

As mentioned in the introduction, the Chilean Concessions Law provides a bonus to the bidder that proposes a project that is auctioned. For example, if the bonus is a fraction  $b$  of the toll and the proponent bids  $p$ , then she wins if  $p/(1 + b)$  is lower than the remaining bids, yet she may charge  $p$ .

The reason for the bonus appears to have been an unfortunate analogy with legal monopolies for inventions (patents). According to this reasoning, by providing a compensation for the initial investment in a feasibility study, lobbying for the project and so on, these activities will be encouraged and more projects will be proposed.<sup>31</sup> However, the Chilean legislator seems to have forgotten that assigning a monopoly to an inventor is desirable because of inappropriability problems: social welfare increases with a temporal monopoly on inventions because the profits obtained during the patent period encourage investments in research and development.

The case of the road franchises considered in this paper is quite different, since a developer that owns the road benefits directly from the project. Hence a bonus is not called for, since in some scenarios it has no effect while in others it reduces welfare. Next we present one illustration of each possibility.

Consider first the case where all landowners are small and  $\alpha$  receives the bonus, and assume  $p^*(\alpha)/(1 + b) < p_c$ . A straightforward extension of Result 7 shows that with a bonus  $\alpha$  wins the auction, and users pay  $p^*(\alpha)$ . This toll is above the toll that results without a bonus ( $p_c$ , see Result 6). By contrast, if  $\alpha$  is sufficiently large (or both  $\alpha$  and  $\beta$  are sufficiently large), a bonus for the proponent has no effect on welfare or its distribution.

## 6 Toll discrimination

Regulators often prohibit price discrimination between users by imposing equal access rules. Are such restrictions warranted in the context of this paper? In this section we assume that developer  $\alpha$  can charge different tolls to households that buy her plots and those that do not. We study what happens under *laissez faire* and competitive bidding, and consider policy implications.

### 6.1 Laissez faire

If an  $\alpha$ -developer can set the tolls she desires, discriminating among users, she will sell her plots with a guarantee of zero tolls, while charging the monopoly toll to the rest. Thus, price-discrimination has, in

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<sup>30</sup>As mentioned earlier, the benefit of the toll reduction must be compared with the costs of raising  $S$ .

<sup>31</sup>Proposed projects undergo social evaluation to ensure that the projects that are undertaken raise welfare.



principle, an ambiguous effect on welfare. On the one hand, it creates wealth by eliminating the distortion to those who buy  $\alpha$ 's land. But, on the other hand, it reduces the value of the rest of the land.

In Appendix C we derive the following result:

**Result 11** *If  $\alpha$  is allowed to discriminate, it will charge a zero toll to those who bought her plots and the monopoly toll to the remaining buyers. This unambiguously reduces welfare if  $\alpha$  is sufficiently close to one, but may be welfare increasing otherwise.*

This result assumes that the road is built by  $\alpha$  regardless of whether she is allowed to toll discriminate or not. Since for a given toll, profits for the franchise holder under toll discrimination are always at least as high as with uniform tolls, if  $\alpha$  does not find it attractive to build the road with price discrimination, she will also decline to build it with uniform tolls. By contrast, it may happen that  $\alpha$  wants to build the road only if she is allowed to price discriminate. Under these circumstances, price discrimination is welfare increasing.<sup>32</sup>

## 6.2 Bidding

For any given toll, price discrimination increases the attractiveness of winning the auction and building the road because the developer can eliminate the toll distortion that reduces the value of her plots while at the same time charging monopoly tolls to the remaining users. To analyze the outcome of the auction when the winner can price discriminate, suppose that  $\alpha$  wins the auction bidding  $p$ . As we know, with price discrimination  $\alpha$  will charge a toll equal to 0 to buyers of her plots and  $p$  to the rest of users, thereby making profits equal to

$$(10) \quad \Pi^{bd}(p; \alpha) = (1 - \alpha)pD(p) + \alpha V(0) - I,$$

where superscript ‘*bd*’ denotes “build and discriminate”. This function is plotted in Figure 4.<sup>33</sup> Because  $p$  does not affect the value of  $\alpha$ 's plots,  $\Pi^{bd}(p; \alpha)$  peaks at  $p = p_m$  independently of  $\alpha$ . By contrast, for any given toll, profits made by  $\alpha$  when not building the road are the same as with uniform tolls— $\Pi^n(p; \alpha)$  remains the same as in Section 4. We denote by  $\tilde{p}^d(\alpha)$  the toll (less or equal than  $p_m$ , see Figure 4) that solves

$$(11) \quad \Pi^{bd}(p; \alpha) = \Pi^n(p; \alpha);$$

that is,  $\tilde{p}^d(\alpha)$  leaves  $\alpha$  indifferent between building and not building the road.<sup>34</sup>

Note that for small values of  $\alpha$  the franchise holder may prefer not to build the road, even if she is allowed to set monopoly tolls. That is,  $\Pi^{bd}$  may be below  $\Pi^n$  for all (finite) values of  $p$ . In this case we convene that  $\tilde{p}^d(\alpha) = \infty$ .<sup>35</sup>

<sup>32</sup>The condition for this to be the case is:  $(1 - \alpha)p_m D(p_m) + \alpha V(0) \geq I > (1 - \alpha)p^*(\alpha)D(p^*(\alpha)) + \alpha V(p^*(\alpha))$ .

<sup>33</sup>A straightforward calculation shows that it is increasing and concave in the winning toll.

<sup>34</sup>When the franchise holder builds the road and charges uniform tolls, her optimal toll is  $p^*(\alpha)$ . By contrast no such optimal toll exists when she builds the road and discriminates tolls. This explains the difference in the diagrams depicting the indifference toll  $\tilde{p}(\alpha)$  in the case with uniform tolls and  $\tilde{p}^d(\alpha)$  in the case with toll discrimination. Also note that  $\tilde{p}^d(\alpha)$  is decreasing in  $\alpha$  and strictly positive, see (18) in Appendix D for a proof.

<sup>35</sup>The analogy with the definition of  $p_c$  should be evident. Also note that in Lemma D.2 in Appendix D we show that  $\tilde{p}^d(\alpha)$  is

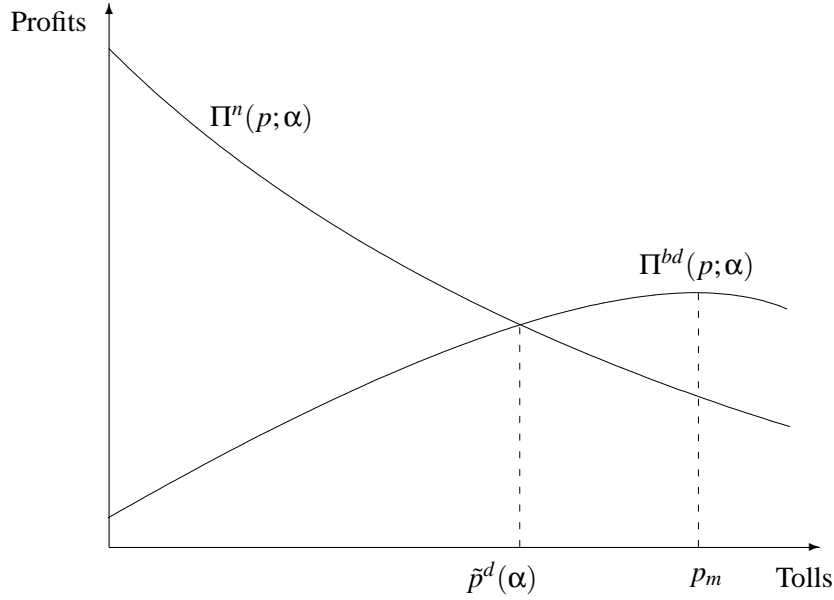


Figure 4: The case of discrimination

**Proposition 6.1** *Assume  $\alpha > \beta$  and that, if allowed to discriminate tolls,  $\alpha$  finds it attractive to build the road. Then, in the case with discrimination, the following possibilities exhaust the Nash equilibria in pure strategies:*

1. *Whenever  $\tilde{p}^d(\beta)$  is finite,  $\alpha$  wins the auction in any pure strategy Nash equilibrium. The set of Nash equilibria is characterized by (i)  $\alpha$ 's winning bid, call it  $p_{\alpha}^{-}$ , belongs to  $[\tilde{p}^d(\alpha), \tilde{p}^d(\beta)]$  and (ii) the lowest bid among the losing bidders is equal to  $p_{\alpha}$ .<sup>36</sup>*
2. *On the other hand, if  $\tilde{p}^d(\alpha)$  is finite and  $\tilde{p}^d(\beta)$  is not, then  $\alpha$  wins the auction in all Nash equilibria with a bid equal to  $p_m$ .*
3. *Finally, if  $\tilde{p}^d(\alpha)$  (and therefore  $\tilde{p}^d(\beta)$ ) are infinite, there are two Nash equilibria in pure strategies. In the first one the winner is  $\alpha$  while in the second one it is  $\beta$ .<sup>37</sup> In both cases the winning toll is  $p_m$ .*

**Proof:** See Appendix D. ■

Thus if developers are sufficiently large,<sup>38</sup> with price discrimination developer  $\alpha$  always wins the auction

finite for all values of  $\alpha$  if and only if  $p_c$  (defined in Section 4.2) is finite. Otherwise,  $\tilde{p}^d(\alpha)$  is finite if and only if  $\alpha \geq \bar{\alpha}$ , with  $\bar{\alpha} = (I - p_m D(p_m)) / (V(0) - V(p_m))$ .

<sup>36</sup>Where  $p_{\alpha}^{-}$  is a shade below  $p_{\alpha}$ , see Appendix D for details.

<sup>37</sup>Existence of the latter requires that  $\Pi^{bd}(p_m; \beta) > 0$ .

<sup>38</sup>Developer  $\alpha$  large in the sense defined in the preceding subsection is sufficient.

and limit–prices the second–highest bid.<sup>39</sup> Since developer  $\alpha$  stands to win most by eliminating the price distortion caused by tolls, she will always bid more aggressively than developer  $\beta$  or any other building company. This explains why  $\alpha$  always wins the auction. Furthermore, contingent on winning the auction, developer  $\alpha$  would like to set the toll as high as possible, which explains why in equilibrium she limit prices the second-highest bid.

That the range of winning tolls is  $[\tilde{p}^d(\alpha), \tilde{p}^d(\beta)]$  follows from noting that it is not in  $\alpha$ 's interest to bid a toll above  $\tilde{p}^d(\beta)$ , since for such a toll  $\beta$  finds it attractive to bid  $\tilde{p}^d(\beta)$  and win the auction. Similarly, the lowest toll that  $\beta$  can credibly bid is  $\tilde{p}^d(\alpha)$ , since if it bids a lower toll  $\alpha$  finds it attractive to let  $\beta$  win.

How do auctions with price discrimination and uniform pricing compare from a welfare perspective? We begin with the following result:

**Result 12** *Whenever  $p^*(\alpha) \geq \tilde{p}(\beta)$ , social welfare with price discrimination is higher than with uniform pricing.*

**Proof:** See Appendix D. ■

The following corollary follows straightforwardly from Result 12:

**Corollary 1** *If developer  $\alpha$  is small (in the sense of Definition 1) and  $\beta > 0$  then welfare is higher under price discrimination than with uniform pricing.*

Consider next the case when  $\alpha$  is close to 1. In that case the uniform price auction is always won by  $\alpha$  bidding  $p^*(\alpha)$  (see Result 7). By contrast, with price discrimination the winning toll is at least  $\tilde{p}^d(\alpha)$ . Moreover,  $p^*(\alpha)$  approaches 0 as  $\alpha$  tends to one. Since  $\tilde{p}^d(1) > 0$ , the following Proposition follows:

**Proposition 6.2** *If developer  $\alpha$  is large ( $\alpha$  close to 1), then uniform pricing leads to higher welfare than toll discrimination.*

**Proof:** See Appendix D. ■

We found that collusion between developers never hurts welfare with uniform pricing. With discrimination, however, collusion eliminates competition between  $\alpha$  and  $\beta$  and rises the upper bound of equilibrium prices to  $p_c$ . Hence, collusion may be welfare decreasing. Yet it may also increase welfare, as happens when  $\alpha + \beta$  is sufficiently close to one, since in this case the coalition of landowners will have incentives to charge a toll close to zero.

## 7 Conclusion

Highways, and more generally infrastructure projects, change the value of land, because their benefits are capitalized into its price. This paper has examined the strategies of large real estate developers and how these strategies affect social welfare. Our results depend on the fraction of the land that is owned by the largest

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<sup>39</sup>Recall that our tie breaking assumption implies that  $\alpha$  can limit price any other's bid by matching it.

landowners, and on the possibility of discrimination between different users of the road. We show that it is always in the interest of the real estate developer to charge a zero toll on buyers of her land, and hence she always prefers to discriminate in tolls. If she is not allowed to discriminate, and this rule is enforceable, we show that welfare is maximized when large landowners are allowed to collude in the bidding process, and that competition may lower welfare. On the other hand, when discrimination is possible, competition among small landowners leads to higher welfare than under uniform tolls.

In the light of this analysis, it is interesting to examine the aftermath of the auction for the road to Chicureo, described in the Introduction. As predicted by our model, the largest landowners formed a group to present a single bid. In the end, however, no one showed up for the auction. The landowners complained that contingent subsidies against losses in the highway project were too small, making the franchise unprofitable. Our analysis suggests that profitability of the highway itself is not a true measure of the overall private value of the project for large developers. There are two possible explanations. Since large landowners internalize most of the social benefits of the highway, building the highway might not have been socially desirable. If this were the case, the fact that there was no participation was welcome. More plausibly however, the large landowners were lobbying for a larger government handout, and were willing to wait given the then low current prices for real estate (due to an economic slowdown), which made waiting costless. A subsidy in this case would be a pure wealth transfer with no allocative effects.

Finally it is worth considering whether the issues considered in this paper are quantitatively relevant. Consider the case in which there are 6000 plots of land, and assume that families make three trips a day in the equilibrium and that the cost of the road is US\$170MM.<sup>40</sup> If there is no toll discrimination, no collusion and there are no other users for the road, small landowners would have to finance the road out of tolls, which implies a toll of US\$2.59. If we assume linear demand, we can calculate the benefits from having a single landowner as compared to dispersed landowners, by measuring the effect of reducing tolls to zero. The increase in welfare depends on the toll at which plot owners stop using the road (i.e., the vertical intercept). For example, it varies between 29 and 91% of the construction cost when the intercept varies between \$7 and \$4.

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<sup>40</sup>These figures come from the Chicureo example. We will use a discount rate of 10%, and no maintenance and operation costs.

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## APPENDICES

### A Results in Section 3

**Proposition A.1** *Assume that*

$$[D'(p)]^2 > \frac{1}{2}D(p)D''(p)$$

*for all  $p$  below the monopoly price. Then any  $p$  satisfying the first order condition (5) also satisfies the corresponding second order condition.*

**Proof:** The second order condition corresponding to  $\max_p \Pi_\alpha(p)$  is:

$$(2 - \alpha)D'(p) + pD''(p) < 0.$$

Substituting the expression for  $p$  that follows from the first order condition (5) in the expression above and rearranging terms shows that the second order condition is equivalent to:

$$(12) \quad \frac{2 - \alpha}{1 - \alpha} [D'(p)]^2 - D(p)D''(p) > 0.$$

If the inequality above holds for all  $p < p_m$  it will also hold for  $p^*(\alpha)$ ,  $\alpha \in [0, 1]$ . The proof concludes by noting that the minimum value of  $(2 - \alpha)/(1 - \alpha)$  over  $\alpha \in [0, 1]$  is 2. ■

### B Results in Section 4.2

We now characterize the Nash equilibria of the auction with uniform tolls. This characterization follows directly from the following lemma, where we derive developer  $\gamma$ 's best-response correspondence:

**Lemma B.1** *Let  $\underline{p}^-$  denote the smallest bid among all bidders, excluding  $\gamma$ . Without loss of generality we may assume  $\underline{p}^- \leq p_c$  (see Result 5). Then, if  $\gamma$  is small her best response correspondence is*

$$\mathcal{P}(\underline{p}^-; \gamma) = \begin{cases} [p_c, \infty] & \text{if } \underline{p}^- = p_c; \\ (\underline{p}^-, \infty] & \text{if } p_c > \underline{p}^-. \end{cases}$$

*And if  $\gamma$  is large it is:*

$$\mathcal{P}(\underline{p}^-; \gamma) = \begin{cases} p^*(\gamma) & \text{if } \underline{p}^- \in [\tilde{p}(\gamma), \infty]; \\ (\underline{p}^-, \infty] & \text{if } \underline{p}^- < \tilde{p}(\gamma), \end{cases}$$

*where  $\tilde{p}(\gamma)$  is defined by  $\Pi^n(\tilde{p}(\gamma); \gamma) = \Pi^b(p^*(\gamma); \gamma)$ .*

**Proof:** Suppose  $p^*(\gamma) \geq p_c$ , i.e.,  $\gamma$  small. Then  $\Pi^b(p, \gamma) \geq \Pi^n(p; \gamma)$  for  $p \geq p_c$ . Hence, if  $\underline{p}^- < p_c$  then  $\Pi^b < \Pi^n$  and  $\gamma$  is better-off not building the road, so that any  $p \in (\underline{p}^-, \infty]$  is a best response. If  $\underline{p}^- = p_c$  then  $\Pi^b(\underline{p}^-, \gamma) = \pi^n(\underline{p}^-; \gamma)$  and  $\gamma$  is indifferent between building and not building the road, so that any  $p \in [\underline{p}^-, \infty]$  is a best response.

Now suppose  $p^*(\gamma) < p_c$ , i.e.,  $\gamma$  is large. If  $\underline{p}^- \in [\tilde{p}(\gamma), p_c]$ , then  $\Pi^n(\underline{p}^-; \gamma) \leq \Pi^b(\tilde{p}(\gamma); \gamma)$  (see Figure 2). Hence, it is optimal for  $\gamma$  to build and charge  $p^*(\gamma)$ , which is a best response. On the other hand, if  $\underline{p}^- < \tilde{p}(\gamma)$ , then  $\Pi^n(\underline{p}^-; \gamma) > \Pi^b(\tilde{p}(\gamma); \gamma)$  and  $\gamma$  is better-off not building the road. Hence, bidding more than  $\underline{p}^-$  is optimal for  $\gamma$  in this case. ■

**Proposition B.1** *Denote by  $\underline{p}$  the lowest (and therefore winning) bid in the auction. Then:*

- (i) *In any Nash equilibrium  $\underline{p} \leq p_c$ .*
- (ii) *If  $\alpha$  (and therefore  $\beta$ ) is small, then any set of bids where two are equal to  $p_c$  and the remainder is larger or equal than  $p_c$  is a Nash equilibrium. Furthermore, this characterizes all Nash equilibria in pure strategies. It follows that the resulting toll is  $p_c$ .*
- (iii) *If  $\alpha$  is large and  $p^*(\alpha) < \tilde{p}(\alpha) \leq p^*(\beta)$ , then any set of bids such that  $\alpha$  bids  $p^*(\alpha)$  and the remaining bidders bid above  $\tilde{p}(\alpha)$  is a Nash equilibrium of the auction. Furthermore, this exhausts all Nash equilibrium in pure strategies.*
- (iv) *If  $p^*(\beta) < \tilde{p}(\alpha)$ , then any set of bids such that (a)  $\alpha$  bids  $p^*(\alpha)$  and the remainder bids above  $\tilde{p}(\alpha)$  or (b)  $\beta$  bids  $p^*(\beta)$  and the remainder bids above  $\tilde{p}(\beta)$  is a Nash equilibrium of the auction. Furthermore, both possibilities exhaust the set of Nash equilibria in pure strategies.*

**Proof:**

- (i) Suppose that in a Nash equilibrium the winning bid,  $\underline{p}$ , is larger than  $p_c$ . Since, by definition,  $p_c D(p_c) = I$ , it follows that if  $\underline{p} > p_m$ , a builder who unilaterally deviates bidding  $p_m$  would win the auction and make a profit. If  $\underline{p} \leq p_m$ , then a builder who unilaterally deviates bidding a shade below  $\underline{p}$  would win the auction and make a profit. It follows that in a Nash equilibrium the winning bid cannot be above  $p_c$ .
- (ii) Clearly in a Nash equilibrium the winning bid,  $\underline{p}$ , cannot be below  $p_c$ , because it would pay to that bidder to unilaterally deviate (see Lemma B.1). Furthermore, Lemma B.1 (which also holds for  $\gamma = 0$ ) shows that bidding in  $[p_c, \infty]$  is a best response to  $\underline{p}^- = p_c$  for any bidder.
- (iii) Strategies induce  $\underline{p} > \tilde{p}(\alpha)$ , and Lemma B.1 implies that  $\mathcal{P}(\underline{p}; \alpha) = p^*(\alpha)$ . Moreover, Lemma B.1 implies that  $\mathcal{P}(p^*(\alpha); \beta) = (p^*(\alpha), \infty]$ , since  $\underline{p} = p^*(\alpha) < p^*(\beta) < \tilde{p}(\beta)$ ; and  $\mathcal{P}(p^*(\alpha); 0) = (p^*(\alpha), \infty]$ , since  $p^*(0) = p_m > p_c$ . Hence any toll above  $\tilde{p}(\alpha)$  is a best response for  $\beta$  and for the building companies.
- (iv) We consider both cases separately:
  - (a) Strategies induce  $\underline{p} > \tilde{p}(\alpha)$ , and Lemma B.1 imply that  $\mathcal{P}(\underline{p}; \alpha) = p^*(\alpha)$ . Consider next developer  $\beta$ . Since  $p^*(\alpha) < p^*(\beta)$ , it follows that  $\Pi^n(p^*(\alpha); \beta) > \Pi^b(p^*(\beta); \beta) > \Pi^b(p^*(\alpha); \beta)$ , which is the highest profit that  $\beta$  can make when building the road. Therefore,  $\mathcal{P}(p^*(\alpha); \beta) = (p^*(\alpha), \infty]$  and any toll above  $\tilde{p}(\alpha)$  is a best response to  $p^*(\alpha)$ . Last,  $\mathcal{P}(p^*(\alpha); 0) = (p^*(\alpha), \infty]$ , since  $p^*(0) = p_m > p_c$  and any toll above  $\tilde{p}(\alpha)$  is a building company's best response.
  - (b) According to strategies,  $\underline{p} > \tilde{p}(\beta)$ , and a Lemma B.1 implies that  $\mathcal{P}(\underline{p}; \beta) = p^*(\beta)$ . Consider next developer  $\alpha$ . Since  $p^*(\beta) < \tilde{p}(\alpha)$ , it follows that  $\Pi^n(p^*(\beta); \alpha) > \Pi^b(p^*(\alpha); \alpha)$ , which is the highest profit that  $\alpha$  can make when building the road. Therefore,  $\mathcal{P}(p^*(\beta); \alpha) = (p^*(\beta), \infty]$ , and any toll above  $\tilde{p}(\beta)$  is a best response to  $p^*(\alpha)$ . Last,  $\mathcal{P}(p^*(\beta); 0) = (p^*(\beta), \infty]$ , since  $p^*(0) = p_m > p_c$  and any toll above  $\tilde{p}(\beta)$  is a building company's best response. ■



## C Results in Section 6.1

In this section we derive three results, which are summarized in the main text under Result 11.

**Proposition C.1** *For  $\alpha$  sufficiently close to one and under laissez faire, uniform tolls lead to higher welfare than toll discrimination.*

**Proof:** Welfare under uniform pricing equals  $W(p^*(\alpha))$  (see equation [6]) while under price discrimination it is

$$\alpha \int_0^{D(0)} D^{-1}(s) ds + (1 - \alpha) \int_0^{D(p_m)} D^{-1}(s) ds - I.$$

Subtracting  $W(p^*(\alpha))$  from this expression yields the difference in welfare between discrimination and uniform pricing for a given  $\alpha$ :

$$(13) \quad \Delta W(\alpha) \equiv \alpha \int_{D(p^*(\alpha))}^{D(0)} D^{-1}(s) ds - (1 - \alpha) \int_{D(p_m)}^{D(p^*(\alpha))} D^{-1}(s) ds.$$

Note that, since uniform tolls and toll discrimination are indistinguishable when  $\alpha = 0$  or  $\alpha = 1$ , we have:

$$(14) \quad \Delta W(0) = \Delta W(1) = 0.$$

Differentiating both sides of (13) with respect to  $\alpha$  yields

$$(15) \quad \begin{aligned} \Delta W'(\alpha) &= \int_{D(p_m)}^{D(0)} D^{-1}(s) ds - p^*(\alpha) D'(p^*(\alpha)) \frac{dp^*(\alpha)}{d\alpha} \\ &= \int_{D(p_m)}^{D(0)} D^{-1}(s) ds + (1 - \alpha) D(p^*(\alpha)) \frac{dp^*(\alpha)}{d\alpha}, \end{aligned}$$

where the second equality follows from (5). Since  $\Delta W(1) = 0$  (see [14]) and  $\Delta W'(1) > 0$  (from [15]), discrimination is welfare decreasing when  $\alpha$  is sufficiently close to one. ■

Next we provide an example where toll discrimination leads to higher welfare than uniform tolls. Thus the condition that  $\alpha$  be close to one in the previous proposition is essential.

**Example 1** *Consider a truncated, isoelastic demand with elasticity  $\varepsilon$  larger than  $-1$ . More precisely, let  $D(p) = 0$  for  $p > p_{\max}$ ,  $D(p) = p^\varepsilon$  for  $p_{\min} < p < p_{\max}$  and  $D(p) = p_{\min}^\varepsilon$  for  $p < p_{\min}$ . A patient but straightforward calculation shows that*

$$\Delta W(\alpha) = \begin{cases} \alpha \int_{D(p_{\max})}^{D(p_{\min})} D^{-1}(s) ds & \text{for } \alpha < 1 + \varepsilon \\ -(1 - \alpha) \int_{D(p_{\max})}^{D(p_{\min})} D^{-1}(s) ds & \text{for } \alpha > 1 + \varepsilon. \end{cases}$$

*It follows that price discrimination is welfare improving for  $\alpha < 1 + \varepsilon$ .* ■

We end this section by showing that by imposing additional conditions on the demand function  $D$  (a convex demand function is sufficient), a similar result to Proposition C.1 can be derived for  $\alpha$  close to zero.

**Proposition C.2** *With the notation and assumptions of Section 3, assume that  $D''(p_m) \geq 0$  and  $[D'(p_m)]^2 > \frac{1}{2}D(p_m)D''(p_m)$ . Then toll discrimination reduces welfare for  $\alpha$  sufficiently close to zero.*

**Proof:** Since  $\Delta W(0) = 0$  (see [14]), it suffices to show that  $\Delta W'(0) < 0$ , which (see [15]) is equivalent to:

$$\int_{D(p_m)}^{D(0)} D^{-1}(s) ds < -q^m \frac{dp^*(\alpha=0)}{d\alpha}.$$

A straightforward calculation shows that the tangent to the inverse demand curve at  $(q_m, p_m)$  intersects the  $q$ -axis at  $q = 2q^m$ . This, combined with the convexity of  $D$  (and therefore concavity of  $D^{-1}$ ) implies that

$$\int_{D(p_m)}^{D(0)} D^{-1}(s) ds < \frac{1}{2} p_m q_m.$$

Thus a sufficient condition for  $\Delta W'(0) < 0$  is

$$(16) \quad \frac{1}{2} p_m q_m < -q^m \frac{dp^*(\alpha=0)}{d\alpha}.$$

From (5) it follows that

$$(17) \quad \frac{dp^*(\alpha)}{d\alpha} = \frac{1}{\varepsilon'(p^*(\alpha))}.$$

Calculating  $\varepsilon'(p)$  from first principles yields:

$$\varepsilon'(p) = \frac{D'(p)}{D(p)} + p \frac{D'(p)}{D(p)} \left[ \frac{D''(p)}{D'(p)} - \frac{D'(p)}{D(p)} \right].$$

Noting that  $p_m D'(p_m)/D(p_m) = -1$ , for  $p = p_m$  the expression above simplifies to:

$$\varepsilon'(p_m) = 2 \frac{D'(p_m)}{D(p_m)} - \frac{D''(p_m)}{D'(p_m)}.$$

Substituting this expression for  $dp^*/d\alpha$  in (17) with  $\alpha = 0$ , and the resulting expression in (16), shows that a sufficient condition for  $\Delta W'(0) < 0$  is

$$\frac{1}{2} p_m q_m < -\frac{q^m}{2 \frac{D'(p_m)}{q^m} - \frac{D''(p_m)}{D'(p_m)}}.$$

A straightforward calculation shows that this condition follows from our assumptions and the fact that  $p_m D'(p_m)/q_m = -1$ . ■

## D Results in Section 6.2

The following result will be used when characterizing the Nash equilibria.

**Lemma D.1**  $0 < \tilde{p}^d(\alpha) \leq \tilde{p}^d(\beta) \leq p_c$ . Furthermore, the second inequality is strict if  $\alpha > \beta$  and  $\beta \geq \bar{\alpha}$ , with  $\bar{\alpha}$  defined in footnote 35. And the third inequality is strict if  $p_c < \infty$ .

**Proof:** In the range  $p < p_m$ ,  $\Pi^{bd}$  is strictly increasing and  $\Pi^n$  strictly decreasing, so that they cross (at most) once. This, together with the fact that  $\Pi^{bd}(0; \alpha) = \alpha V(0) - I < \alpha V(0) = \Pi^n(0; \alpha)$  implies that they cannot cross at zero, so that  $\tilde{p}^d(\alpha) > 0$ .

To show that  $\tilde{p}^d(\alpha) \leq \tilde{p}^d(\beta)$  for  $\alpha \geq \beta$  it suffices to consider the case where  $\beta > \bar{\alpha}$ , with  $\bar{\alpha}$  defined in Lemma D.2. For values of  $\alpha$  in this range, we have that implicitly differentiating (11) leads to

$$(18) \quad \frac{d\tilde{p}^d(\alpha)}{d\alpha} = -\frac{V(0) - V(\tilde{p}^d(\alpha))}{D(\tilde{p}^d(\alpha)) + (1 - \alpha)\tilde{p}^d(\alpha)D'(\tilde{p}^d(\alpha))} < 0$$

since  $D(p) + pD'(p) > 0$  when  $p < p_m$ .

Finally, we show that  $\tilde{p}^d(\alpha) < p_c$  when  $p_c < \infty$ . A straightforward calculation shows that  $\Pi^{bd}(p_c; \alpha) > \Pi^n(p_c; \alpha)$  is equivalent to

$$\alpha[V(0) - V(p_c)] > I - p_c D(p_c),$$

which holds since the right hand side is zero by the definition of  $p_c$ . This, combined with the fact that in the range  $p < p_m$ ,  $\Pi^{bd}$  is strictly increasing and  $\Pi^n$  strictly decreasing, implies that  $\tilde{p}^d(\alpha) < p_c$ . ■

The first inequality in Lemma D.1 says that there is always a low enough, strictly positive toll to make a developer prefer not to build the road. The second inequality shows that larger developers have a stronger preference to build the road.

**Lemma D.2** *If  $p_c < \infty$  then  $\tilde{p}^d(\alpha)$  is finite for all  $\alpha \in [0, 1]$ . And if  $p_c = \infty$  (that is  $p_m D(p_m) < I$ ), then not building the road dominates building the road (and toll discriminating) if and only if  $\alpha \leq \bar{\alpha}$ , with  $\bar{\alpha} \equiv (I - p_m D(p_m)) / (V(0) - V(p_m))$ .*

**Proof:** Not building the road dominates building it and toll discriminating if and only if  $\Pi^{bd}(p_m, \alpha) < \Pi^n(p_m; \alpha)$  which (see (10) and (7)) is equivalent to

$$(19) \quad p_m D(p_m) + \alpha[V(0) - V(p_m)] < I.$$

Hence, if  $p_m D(p_m) \geq I$  we have that (19) does not hold and  $\tilde{p}^d(\alpha)$  is well defined. And if  $\tilde{p}^d(\alpha) = \infty$ , (19) holds for all  $\alpha \leq \bar{\alpha}$ , thereby concluding the proof. ■

Next we characterize bidders' best response function, then we turn to Nash equilibria in pure strategies.

**Lemma D.3** *Let  $\underline{p}^-$  denote the lowest bid among all bidders, excluding  $\alpha$ , where  $\alpha > 0$ . Assume  $\alpha$  finds it attractive to build the road if she is allowed to price discriminate without restriction.<sup>41</sup> Then, if  $\tilde{p}^d(\alpha) < \infty$ ,  $\alpha$ 's best-response correspondence when the franchise holder can discriminate is*

$$\mathcal{P}^d(\underline{p}^-; \alpha) = \begin{cases} p^m & \text{if } \underline{p}^- > p^m; \\ \underline{p}^- & \text{if } \tilde{p}^d(\alpha) < \underline{p}^- \leq p^m; \\ [\underline{p}^-, \infty] & \text{if } \underline{p}^- = \tilde{p}^d(\alpha); \\ (\underline{p}^-, \infty] & \text{if } \underline{p}^- < \tilde{p}^d(\alpha). \end{cases}$$

On the other hand, if  $\tilde{p}^d(\alpha) = \infty$ :

$$\mathcal{P}^d(\underline{p}^-; \alpha) = \begin{cases} p^m & \text{if } \underline{p}^- > p^m; \\ (\underline{p}^-, \infty] & \text{if } \underline{p}^- \leq \tilde{p}^d(\alpha). \end{cases}$$

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<sup>41</sup>That is,  $(1 - \alpha)p_m D(p_m) + \alpha V(0) \geq I$ .

**Proof:** We begin with the case where  $\tilde{p}^d(\alpha)$  is finite.  $\Pi^{bd}(p; \alpha) - \Pi^n(p; \alpha) = pD(p) + \alpha[V(0) - V(p)] - I \stackrel{\geq}{=} 0$  if  $p \stackrel{\leq}{=} \tilde{p}^d(\alpha) < p_c$  (see Lemma D.1). It follows that  $\Pi^{bd}(p; \alpha) - \Pi^n(p; \alpha) > 0$  for  $p > \tilde{p}^d(\alpha)$ . Hence,  $p_m$  is a best response to  $\underline{p}^- > p_m$ , and  $\underline{p}^-$  is a best response to  $\underline{p}^- \in (\tilde{p}^d(\alpha), p_m]$ , since in a tie the largest developer wins.

Next, if  $\underline{p}^- = \tilde{p}^d(\alpha)$ , then  $\Pi^{bd} = \Pi^n$ . Hence,  $\alpha$  is indifferent between building and not building the road and any  $p \in [\underline{p}^-, \infty]$  is a best response.

Last, if  $\underline{p}^- < \tilde{p}^d(\alpha)$ , then  $\Pi^d < \Pi^n$ . Thus,  $\alpha$  is better-off not winning the auction and any  $p$  in  $(\underline{p}^-, \infty]$  is a best response.

The best response function in the case where  $\tilde{p}^d(\alpha) = \infty$  hinges on the fact that  $\alpha$  has no interest in building the road, except if no one else shows interest (i.e.,  $\underline{p}^- = \infty$ ). The remainder of the proof then is straightforward. ■

We can now prove Proposition 6.1, which characterizes equilibria in the auction with price discrimination.

Unfortunately, the best correspondence derived above (Lemma D.3) does not extend trivially to the second largest developer,  $\beta$ . The problem is that when  $\alpha$  bids  $p \in [\tilde{p}^d(\alpha), \tilde{p}^d(\beta)]$ ,  $\beta$ 's best response is to bid a "slightly lower" toll, which does not lead to a well defined correspondence.<sup>42</sup> Discretizing the set of possible tolls solves the problem above, yet makes the notation considerably more cumbersome. For this reason, in the proposition that follows we assume a discrete toll space but avoid referring to it explicitly.

### Proof of Proposition 6.1.

Cases 2 and 3 are straightforward, so we concentrate on case 1.

For all practical purposes we may assume that building companies bid  $p_c$ . Hence we may concentrate on the game between both developers, subject to the constraint above.

First we show that  $p_\alpha \in [\tilde{p}^d(\alpha), \tilde{p}^d(\beta)]$  and  $\underline{p}^- = p_\alpha$  is a Nash equilibrium. Since  $\underline{p}^- = p_\alpha \in [\tilde{p}^d(\alpha), \tilde{p}^d(\beta)]$ ,  $p_\alpha$  is a best response for  $\alpha$ . Also, any toll larger or equal than  $p_\alpha$  is a best response for  $\beta$ . By contrast,  $\underline{p}^- > p_\alpha$  is not Nash, since  $\alpha$  would gain by slightly raising her bid.

Next we show that in a Nash equilibrium the winning bid,  $\underline{p}$ , cannot be strictly larger than  $\tilde{p}^d(\beta)$ . Indeed, in such an equilibrium  $\alpha$  would gain by unilaterally lowering its bid to  $\tilde{p}^d(\beta)$ . Profits under this deviating strategy would increase, since  $\Pi^{bd}$  evaluated at the new toll is larger than  $\Pi^n$  evaluated at that toll, which in turn is larger than  $\Pi^n$  evaluated at  $\alpha$ 's original bid (since this bid is larger than  $\tilde{p}^d(\alpha)$ ).

Finally, note that the winning bid cannot be strictly less than  $\tilde{p}^d(\alpha)$  in equilibrium, since the winner would gain by unilaterally deviating and bidding  $\tilde{p}^d(\alpha)$ . ■

### Proof of Proposition 6.2

Welfare under uniform pricing equals  $W(p^*(\alpha))$  (see equation 6); under price discrimination welfare equals at most

$$\alpha \int_0^{D(0)} D^{-1}(s) ds + (1 - \alpha) \int_0^{D(\tilde{p}^d(\alpha))} D^{-1}(s) ds - I,$$

which occurs when the winning bid equals its lowest feasible value,  $\tilde{p}^d(\alpha)$ . In that case the difference in welfare between discrimination and uniform pricing for a given  $\alpha$  equals

$$\Delta W(\alpha) \equiv \alpha \int_{D(p^*(\alpha))}^{D(0)} D^{-1}(s) ds - (1 - \alpha) \int_{D(\tilde{p}^d(\alpha))}^{D(p^*(\alpha))} D^{-1}(s) ds,$$

<sup>42</sup>Our tie-breaking rule avoids this problem only for the largest developer.

since  $D(\tilde{p}^d(\alpha)) > D(p^*(\alpha))$ , when  $\alpha$  is large. The first term on the right-hand side is the welfare gain from eliminating the distortion that affects the land owned by  $\alpha$  when tolls are uniform. The second term is the loss borne by the rest of the landowners, who now have to pay the toll that won the auction, which is at least  $D(p^*(\alpha))$ .

Now

$$\begin{aligned}\Delta W'(\alpha) &= \int_{D(\tilde{p}^d(\alpha))}^{D(0)} D^{-1}(s)ds - p^*(\alpha)D'(p^*(\alpha))\frac{dp^*(\alpha)}{d\alpha} + (1-\alpha)\tilde{p}^d(\alpha)D'(\tilde{p}^d(\alpha))\frac{d\tilde{p}^d(\alpha)}{d\alpha} \\ &= \int_{D[p^*(\alpha)]}^{D(0)} D^{-1}(s)ds + (1-\alpha)D[p^*(\alpha)]\frac{dp^*(\alpha)}{d\alpha} - (1-\alpha)\tilde{p}^d(\alpha)D'(\tilde{p}^d(\alpha))\frac{d\tilde{p}^d(\alpha)}{d\alpha},\end{aligned}$$

where the second equality follows from (5). Since  $\Delta W(1) = 0$  and, from the derivation above we have  $\Delta W'(1) = \int_{D(\tilde{p}^d(1))}^{D(0)} D^{-1}(s)ds > 0$ , discrimination is welfare decreasing when  $\alpha$  is sufficiently close to one.

■

### Proof of Result 12

Total welfare increases with lower tolls; hence to show that welfare is higher with discrimination it suffices to show that in this case everyone pays lower tolls.

Since  $p^*(\alpha) \geq \tilde{p}^d(\beta)$ , the best possible outcome of a uniform price auction is  $\min\{p^*(\alpha), p_c\}$ . On the other hand, the worst that can happen in an auction with price discrimination is that the winning toll is  $\tilde{p}^d(\beta) \leq p^*(\alpha)$  by assumption. From Lemma D.1 we also have that  $\tilde{p}^d(\alpha) \leq p_c$ . Finally, since some plot owners pay no tolls with discrimination, all are at least as well off as in a uniform price auction and some are better off. ■