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A DUAL METHOD OF EMPIRICALLY EVALUATING DYNAMIC COMPETITIVE EQUILIBRIUM MODELS WITH MARKET DISTORTIONS, APPLIED TO THE GREAT DEPRESSION AND WORLD WAR II

Casey B. Mulligan

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ABSTRACT

I prove some theorems for competitive equilibria in the presence of market distortions, and use those theorems to motivate an algorithm for (simply and exactly) computing and empirically evaluating competitive equilibria for dynamic economies. Although a competitive equilibrium models interactions between all sectors, all consumer types, and all time periods, I show how my algorithm permits separate empirical evaluation of these pieces of the model and hence is practical even when very little data is available. I then compute a neoclassical growth model with distortionary taxes that fits aggregate U.S. time series for the period 1929-50 and conclude that, if it is to explain aggregate behavior during the period, government policy must have heavily taxed labor income during the Great Depression and lightly taxed it during the war. In other words, the challenge for the competitive equilibrium approach is not so much why output might change over time, but why the marginal product of labor and the marginal value of leisure diverged so much and why that wedge persisted so long. In this sense, explaining aggregate behavior during the period has been reduced to a public finance question – were actual government policies distorting behavior in the same direction and magnitude as government policies in the model?

Casey B. Mulligan
Department of Economics
University of Chicago
1126 East 59th Street
Chicago, IL 60637
and NBER
c-mulligan@uchicago.edu

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I. Introduction

Explaining aggregate measures of behavior, such as employment, output, consumption, and investment, has for decades been one of the prime interests of macroeconomists, and others. Almost as old is the question of how much aggregate behavior might be explained by private sector impulses (in modern parlance: tastes, technology, market structure, and demographic shocks) rather than public sector impulses such as government regulations, taxes, and subsidies. Somewhat more recent are attempts to quantitatively model private sector behavior as a dynamic competitive equilibrium. Kydland and Prescott (1982) is a pioneering, and rather successful, attempt. This paper reconsiders the interaction between the time series data, construction of competitive models of private behavior, and construction of models of government policy, in order to: (a) further improve and apply the computable dynamic general equilibrium methods that have been used by the many important papers following Kydland and Prescott, and (b) emphasize and exploit some of the economic similarities between computable general equilibrium models and microeconometric models of market supply and demand.

One obstacle in the use of quantitative dynamic general equilibrium models has been their analytic tractability, and the nature of the data required to evaluate them. As they become more realistic, especially as regards to modeling market distortions, quantitative general equilibrium models become more complex to "solve" or "simulate," and this complexity has tempted many taking the competitive equilibrium approach (eg., Braun and McGratten 1993 or Ramey and Shapiro 1998) to ignore, for example, the distortionary effects of taxes, and nearly all studies ignore the distortionary effects of business, labor, and product regulations. A "solution" to a dynamic general equilibrium model also depends on the behavior of time sequences of the exogenous variables into the distant future, so empirically evaluating a model and its "solution" requires measurement of, or guesses about, the nature of those sequences in the future. I argue that these difficulties can be avoided, without the cost of additional approximation and with the benefit of added economic understanding, by changing the computation procedure. I motivate this change by borrowing price theory's concepts of "supply price" and "demand price" and using those concepts to generalize and reinterpret a calculation that has been made by some in the macroeconomic literature on unmeasured time-varying preferences.

Although time-varying preferences are perfectly consistent with the computable general equilibrium methodology, in practice only a very small fraction of quantitative general equilibrium models include them.[1] Two of those studies are Parkin (1988) and Hall (1997) and, although their models do not allow for market distortions, both authors make an interesting calculation. They compare the postwar time series behavior of the consumption-leisure ratio, which they interpret as one of two determinants of a representative consumer's marginal rate of consumption-leisure substitution, to the time series behavior of the average product of labor, which they interpret as the one determinant of the marginal product of labor. When these two series diverge, they suggest, this is evidence of a preference shift.[2] Regardless of whether preferences "should" be modeled as stable or not, the purpose of my paper is to suggest that the consumption-leisure ratio might be interpreted as (proportional to) the supply price of labor, and the average product of labor as (proportional to) the demand price. Hence, the Parkin-Hall calculation might instead be interpreted as generating measures of gaps between supply and demand prices – labor-leisure distortions – and be generalized in order to empirically evaluate *stable* preference competitive equilibrium models from a *public finance perspective*. My evaluation method is computationally economical, easy to adjust for questions of data quality, and economically consistent with a variety of partial and general equilibrium models of market

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[1]Whether a utility function varies over time, or it is a stable function with some unmeasured and time-varying arguments is not relevant for Hall and Parkin's analysis (see Becker 1996 for further discussion of this point). In any case, I use the more common practice of modeling utility as a stable function of measured variables.

Ingram, Kocherlakota, and Savin (1997) have calculations similar Hall and Parkin's, although not identical because Ingram et al have two consumption goods, and interpret deviations between measured consumption and work hours as changes in "home production time" rather than "preferences shifts."

[2]One difference between Hall and Parkin is that Parkin also considers a time-varying labor elasticity in the production function. Hence, the gap between the consumption-leisure ratio and the average product of labor is a measure of the preference shift in Parkin's model only when we correct the average product for any change in the labor elasticity.

distortions.

The computable general equilibrium literature usually, and understandably, constructs competitive equilibrium explanations of aggregate behavior proceeds as follows:

(i) write down a model for government policy (eg., a set of taxes, transfers, and regulations)

(ii) write down a model for private sector behavior, including responses to the modeled government policies

(iii) choose functional forms and numerical parameters for the model of the private sector (eg., rate of time preference, elasticities of substitution in preferences, elasticities of substitution in production)

(iv) choose numerical values for the government policy parameters (a) based on some observations of government policy and (b) so that the model government budget constraint balances in step (v)

(v) compute a competitive equilibrium (eg., time series for employment, consumption, interest rates, etc.)

(vi) compare the equilibrium quantities (and perhaps prices) to observed quantities (and perhaps prices)

Steps (i) - (vi) might be done once, in which case the procedure is called "simulation," and the success of the model might be judged on step (vi)'s metric of the proximity of simulated and observed quantities.[3] This is the approach, for example, of Burnside et al (2000)[4] and, essentially, Mulligan (1998), who conclude that a neoclassical model cannot explain some time series comovements of employment and government expenditure, and Cole and Ohanian (1999) who suggest that fiscal policy cannot explain the Great Depression. Steps (iii) - (vi) might be done many times, perhaps with the objective of choosing numerical values for the private sector parameters in order to maximize step (vi)'s metric of the proximity of simulated and observed

[3]A variety of metrics have been proposed in the literature; Watson (1993) is one study with a detailed proposal, and a review of previous approaches.

[4]One of the methodological contributions of the Burnside et al study is to propose a specific test statistic for evaluating the ability of a fiscal policy model, based on its simulated quantities and prices, to explain observed quantities and prices.

quantities, in which case the procedure is called "estimation." This is the approach, for example, of Hansen and Sargent's (1991, Chapter 7) study of recursive linear competitive equilibrium models. In either procedure, step (v) – computing quantities and prices that maximize utility, maximize profits, and balance the government budget given numerical values for government policy – is not an easy one, especially when government policy is distortionary. Indeed, this step can be so difficult that many taking the competitive equilibrium approach are tempted to ignore market distortions. Even when distortionary taxes are included in the model, it is difficult without the assistance of a computer to understand exactly how changes in private sector parameters or government policies affect equilibrium quantities and prices.

My approach is not to advocate "simulation" versus "estimation," but rather to change steps (iv)-(vi) in order to simplify computation and data requirements by orders of magnitude, and to highlight the public finance dimension of the problem. Here are my proposed steps:

- (i) write down a model for government policy (eg., a set of taxes, transfers, and regulations)
- (ii) write down a model for private sector behavior, including responses to the modeled government policies
- (iii) choose functional forms and numerical parameters for the model of the private sector (eg., rate of time preference, elasticities of substitution in preferences, elasticities of substitution in production)
- (iv)′ use observed quantities to compute marginal rates of substitution and transformation
- (v)′ use the competitive equilibrium conditions, and the results from (iv)′, to compute numerical values for the government policy parameters, and perhaps prices
- (vi)′ compare the equilibrium policies (and perhaps prices) to observed policies (and perhaps prices)

Notice how I have left (i)-(iii) intact, changing only (iv)-(vi). In particular, (iv)′ calculates supply and demand prices, and (v)′ interprets the gap as indicators of market distortions.

I propose to feed observed quantities into the model to infer policies, rather than feeding policies into the model to infer quantities. For this reason, I refer to (i)-(vi) as the "primal" or "policy-quantity" approach and my approach (i)-(vi)′ as the "dual" or "quantity-policy" approach, and highlight their differences in Figure 1.

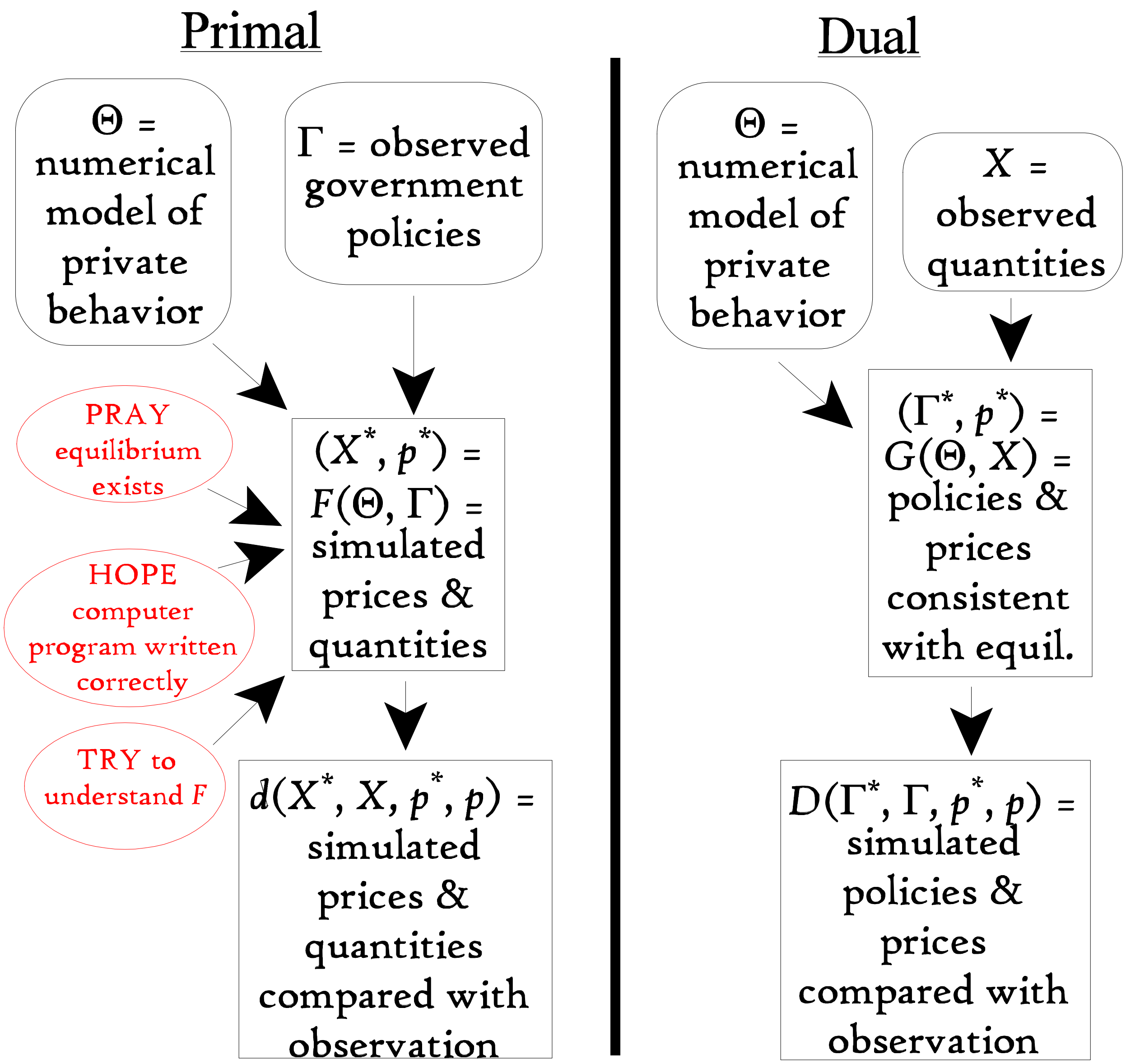

Figure 1 Overview of the Primal and Dual Approaches

We see in the Figure's left panel that the primal approach uses a numerical model of private behavior and observed policies to simulate quantities and prices, and the red ovals emphasize some of the practical difficulties with the approach. As shown in the right panel, the dual approach uses a numerical model of private behavior and observed quantities to simulate policies and prices.

As in the primal approach, the dual approach has both simulation and estimation versions. Steps (i)-(vi)′ could be done once (aka, "simulation") or steps (iii) - (vi)′ might be done many times, perhaps with the objective of choosing numerical values for the private sector parameters in order to maximize step (vi)′'s metric of the proximity of simulated and observed policies (aka, "estimation"). In either procedure, step (v)′ – computing policies that satisfy equilibrium conditions given observed quantities – is a trivial one. Indeed, estimation is much more economical with my dual approach than with the primal approach because performing step (v)′ many times is much easier than performing step (v) many times.

Consider first a very simple static economy with many identical firms producing one unit of the consumption good c for every unit of labor time L it hires. The labor time is supplied by identical consumers who earn w per unit time, pay a price 1 for the consumption good, and have utility function $\ln c + \theta \ln (1\text{-}L)$. Firms also pay a tax at flat rate τ on its payroll wL, and the government uses the revenue to finance lump sum taxes in the amount v to the consumers. Following the usual conventions, let's define a competitive equilibrium to be, for a given tax rate τ, four scalars $\{c,L,w,v\}$ such that: (i) c and L maximize utility $\ln c + \theta \ln (1\text{-}L)$ subject to $c = wL + v$ and given w and v, (ii) the resource constraint $c = L$ binds, (iii) c and L maximize profits $[c - (1+\tau)wc]$ given w and τ, and (iv) the government budget constraint $v = \tau wL$ balances. The primal approach of empirically evaluating this model is picking a positive value of the utility parameter θ, and using a measure of the tax rate τ to simulate equilibrium consumption and labor. In this example, the simulation involves just a simple formula, namely $c^* = L^* = [1+(1+\tau)\theta]^{-1}$,[5] and we can compare the simulated values c^* and L^* with observed values. The dual approach uses a parameter value θ and data on c and L to calculate marginal rates of substitution $(\theta c/(1\text{-}L))$ and transformation (1), to simulate the tax rate τ^* based on the gap (namely, $\tau^* = (1\text{-}L)/(\theta c)\text{-}1$), and to compare the simulated tax rate with the observed tax rate. Both methods would lead to analogous conclusions. For example, if the primal simulation over-predicted labor ("the model says people should work less than they do"), then the dual simulation would under-predict the tax rate ("the model says people are acting like the tax rate is lower than it is"). Both the primal and dual methods simulations involve a single formula in this example, but one important

[5]The formula is just the solution to the three (non-redundant) competitive equilibrium conditions: $c=L$, $w(1+\tau)=1$, and $\theta c=w(1\text{-}L)$.

attribute of the dual method is that, as we complicate the model, the primal simulations get a lot more complicated than do the dual simulations.

Section II exposes the generality and computational simplicity of the dual approach by proving two propositions for a distortionary tax dynamic general equilibrium model with many agents, consumption goods, and investment goods. The first shows that, given a model of private sector behavior and observed quantities, a policy consistent with competitive equilibrium can be computed two first order conditions at a time, and in any order. The second proposition shows that there is one and only one set of government policies that is consistent with a competitive equilibrium. It is well known that analogous results cannot be proven for the primal approach because there can be zero, or multiple, equilibrium responses to a given policy (the "Laffer curve" characterizes some of the well known examples of nonunique or nonexistent competitive equilibria), and the equilibrium quantity in any one sector at any one date depends on policies and technologies in all sectors at all dates. Hence, the dual approach does not present "equilibrium choice" or nonexistence problems, and does not require much accurate data.

Section III then uses my procedure to "explain" the period 1929-50 with a neoclassical growth model. The calculations are so simple that they are reported in a self-contained appendix for easy verification by the interested reader. I show that, in order for the neoclassical growth model to explain aggregate behavior during the period, marginal labor income tax rates must have been quite high during the Depression and quite low during the war. Since it appears that marginal labor income tax rates had a different history, I conclude that the neoclassical growth model cannot explain why there was so little employment during the Depression and so much during the War. Perhaps another defensible conclusion is that marginal labor income tax rates did have a history like that generated by the model, and that the usual measures of marginal tax rates are not capturing all of the distortions introduced by government regulations, taxes, and subsidies during the period. Under this interpretation of my results, explaining the period 1929-50 is reduced to the public finance problem of identifying and quantifying the various government policies driving a wedge between labor supply and labor demand, and showing how actual marginal tax rates had a history like that generated by the model.

II. Competitive Equilibria with Distortionary Taxes

II.A. Setup of the Model

There are a continuum of infinitely lived consumers and firms, each taking prices and policy parameters as given. Consumers are partitioned into h=1,...,H (equally populated) types according to the productivity of their labor, their preferences and their treatment by the government. There are M capital goods, which are used together with labor to produce more capital goods or to produce the N consumption goods. Any firm produces only one of these M+N goods; firms are indexed by their sector j=1,...(M+N) with the first N sectors producing consumption goods, and the rest producing capital goods. Since the economy is assumed to be competitive the ownership of capital does not affect the allocation in the economy. For convenience I assume that all capital is owned by the firm producing it and rented out to the other firms for production purposes, and that there is no uncertainty.[6]

Vectors are denoted by underlined letter: $\underline{x}$, and are column vectors. Matrices are denoted by capped letters: $\hat{x}$. I use $\hat{*}$ to denote multiplication element-by-element. Let $\underline{x}^{-1}$ stand for the vector of reciprocals of $\underline{x}$.

Time is discrete and indexed by $t = 0,\ldots,\infty$. Consumption good prices, gross of taxes, are given by: $\underline{p(t)} = [p_1(t), p_2(t), \ldots p_N(t)]'$

II.A.1 Consumers

The consumption of the N consumption goods by individual h is given by the vector $\underline{c^h(t)} = [c_1^h(t), c_2^h(t), \ldots c_N^h(t)]'$.

Aggregate consumption of the economy is given by $\underline{c(t)} = \sum_h \underline{c^h(t)}$. Total labor supplied by the household is $L^h(t) = \sum_{i=1}^{M+N} L_i^h(t)$ and $L_i^h(t)$ denotes the labor by h to sector i. The vector of ownership shares by h of the N+M firms is given by $\alpha^h = [\alpha_1^h, \alpha_2^h, \ldots, \alpha_{N+M}^h]'$. The interest rate is given by $q(t)$

[6]Mulligan (2001b) studies the application of the dual method to stochastic models.

Preferences are governed by:

$$U^h = \sum_{t=0}^{\infty} e^{\sum_{s=1}^{t} \pi(t)} u^h(\underline{c^h(t)}, L^h(t)) \tag{1}$$

where $\pi(t)$ is the consumer's date (t-1) one period forward rate of time preference.

The budget constraint is given by:

$$\sum_{t=0}^{\infty} \prod_{s=0}^{t} (1+q(s))^{-1}[w^h(t)L^h(t) - \underline{p}(t)'c^h(t) + v^h(t)] + \alpha^h{}'\underline{z} = 0 \tag{2}$$

where $\underline{z}$ is the value at date zero of the firms and $v^h(t)$ denotes the lump sum transfers at date t.

The resource constraint of the individual can also be expressed using a series of constraints as follows:

$$w^h(t)L^h(t) + v^h(t) + a^h(t) = \underline{p}(t)'c^h(t) + [1+q(t+1)]^{-1}a^h(t+1) \qquad t = 0,\ldots,\infty \tag{3}$$

$$a^h(0) = \underline{\alpha}^h{}'\underline{z}$$

where $w^h(t)$ and $a^h(t)$ are scalars denoting household h's date t wage rate and asset holdings.

II.A.2 Firms and Distortionary Taxes

The production functions are given by $f_i(\underline{K_i(t)}, \underline{L_i(t)}, t)$ where $\underline{K_i(t)} = [K_i^{N+1}(t), K_i^{N+2}(t), \ldots, K_i^{M+N}(t)]'$ and $\underline{L_i(t)} = [L_i^1(t), L_i^2(t), \ldots, L_i^H(t)]'$ are the vectors of capital and labor inputs used by firm i at date t. $K^i(t)$ will denote the date t aggregate amount of type i capital.

The rental rate vectors of inputs at date t are given by $\underline{r}(t) = [r^{N+1}(t), r^{N+2}(t), \ldots r^{M+N}(t)]'$ and $\underline{w}(t) = [w^1(t), w^2(t), \ldots w^H(t)]'$

Taxes are levied on labor at the rates $\underline{\tau(t)} = [\tau^1(t), \tau^2(t), \ldots, \tau^H(t)]'$, and on capital inputs at rates $\underline{\gamma(t)} = [\gamma^{N}(t), \gamma^{N+1}(t), \ldots, \gamma^{N+M}(t)]'$. The input prices faced by firms are then

$$\underline{\tilde{r}(t)} = \underline{r(t)} \hat{*} [\underline{1_M} + \underline{\gamma(t)}] \text{ and } \tilde{w}_i(t) = \underline{w(t)} \hat{*} [\underline{1_H} + \underline{\tau(t)}] .$$

The objective in the consumption good sector is:

$$z_i(t) = \max[p_i(t) f_i(\underline{K_i(t)}, \underline{L_i(t)}, t) - \underline{\tilde{r}(t)}' \underline{K_i(t)} - \underline{w_i(t)}' L_i(t)] \tag{4}$$

$$i = 1, \ldots, N \text{ and } t = 0, \ldots, \infty$$

The problem in the production goods sector can be expressed recursively as

$$V(K^i(t)) = \max_{\{K_i(t), L_i(t)\}} \begin{Bmatrix} r^i(t) K^i(t) - \underline{\tilde{r}(t)}' \underline{K_i(t)} - \underline{w_i(t)}' \underline{L_i(t)} \\ +(1+q(t))^{-1} V(K^i(t+1)) \end{Bmatrix} \tag{5}$$

$$\text{s.t. } K^i(t+1) = f_i(\underline{K_i(t)}, \underline{L_i(t)}, t) + (1-\delta_i) K^i(t) \tag{6}$$

$$K^i(0) = K_0^i \quad i = N+1, \ldots N+M$$

The value of the firms at date t is given by $\underline{z(t)} = [z_1(t), z_2(t), \ldots, z_{N+M}(t)]$

In this set up the investment by firms is reversible. The production of capital and consumption goods however is restricted to be non-negative, i.e. $f_i(\underline{K_i(t)}, \underline{L_i(t)}, t) \geq 0 \quad \forall t, i$. An interior solution is assumed to hold, although see Houthakker (1995) or Mulligan (2000, 2001a), for some discrete-choice interpretations of the "interior" conditions.

II.A.3. The Government

The government budget constraint is given by:

$$\underline{g(t)}' \underline{p(t)} = [\underline{\gamma(t)} \hat{*} \underline{K^i(t)}]' \underline{r(t)} + \sum_{i=1}^{N+M} [\underline{\tau_i(t)} \hat{*} \underline{L_i(t)}]' w(t) - \sum_{h=1}^{H} v^h(t) \tag{7}$$

where $g(t) = [g_1(t), \ldots, g_N(t)]'$ is the vector of government consumption. This constraint simply says that government spending (consumption and net transfers) equals the sum of labor and capital income taxes.

II.A.4 Resource Balance Constraints

Markets for consumer goods, capital goods, labor, and assets "clear" at each date. In other words, government and private purchases equal output in each of the N consumption good sectors, capital demanded by firms equal supply (capital type-by-type), labor demanded by firms equal supply (labor type-by-type), and net household asset holdings equal the value of the firm sector. Algebraically, market clearing implies (8)-(11).

$$\underline{g(t)} + \underline{c(t)} = \underline{y_N(t)} = [y_1(t), \ldots, y_N(t)]' \text{ with } y_i(t) = f_i(\underline{K_i(t)}, \underline{L_i(t)}, t) \quad \forall t \tag{8}$$

$$K^i(t) = \Sigma_j K^i_j(t), \quad i = N+1, \ldots N+M, \quad \forall t \tag{9}$$

$$[L^1(t), L^2(t), \ldots, L^H(t)]' = \sum_{i=1}^{N+M} \underline{L_i(t)}, \quad \forall t \tag{10}$$

$$\Sigma_{i=1}^{N+M} z_i(t) = \Sigma_h a^h(t), \quad \forall t \tag{11}$$

II.A.4 Definition of a Competitive Equilibrium

Given a policy sequence $\{\underline{g(t)}, \underline{\gamma(t)}, \{v^h(t)\}_{h=1}^{H}, \{\underline{\tau(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$, initial capital stocks $\underline{K_0}$ and initial ownership shares $\{\underline{\alpha^h}\}_{h=1}^{H}$, a competitive equilibrium is given by quantity sequences $\{\{L^h(t), a^h(t), c^h(t)\}_{h=1}^{H}, \{K^i(t)\}_{i=N+1}^{N+M}, \{\underline{K_i(t)}, \underline{L_i(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$, price sequences $\{\underline{p(t)}, \underline{w(t)}, \underline{r(t)}, \underline{q(t)}\}_{t=0}^{\infty}$ and a sequence of firm values $\{z(t)\}_{t=0}^{\infty}$ such that:

(i) $\{\{L^h(t), a^h(t), \underline{c^h(t)}\}_{h=1}^{H}\}_{t=0}^{\infty}$ maximize (1) subject to (2) (or (3)) for all h

(ii) $\{\{\underline{K_i(t)}, \underline{L_i(t)}\}_{i=1}^{N}\}_{t=0}^{\infty}$ maximize (4) for i=1,...,N and all t

$\{\{\underline{K_i(t)}, \underline{L_i(t)}\}_{i=N+1}^{N+M}\}_{t=0}^{\infty}$ maximize (5) for i=N+1,...,N+M

(iii) $z_i(t) = 0$ for all i=1,...,N and all t

$z_i(t) = \sum_{t=0}^{\infty} \prod_{s=0}^{t} \{1 + q(s)\}^{-1} (1-\delta_i)^s r_i(t) K^i(t)$ for i=N+1,...,N+M and for all t

(iv) (7)-(11) hold at all t

(i) requires that households willingly consume the equilibrium consumption bundle, willingly hold equilibrium assets, and willingly supply equilibrium labor. (ii) requires each type of firm to willingly demand the equilibrium inputs. (iii) is a free-entry condition, and requires that firms are only valued at the value of their assets. (iv) says the government budget constraint must hold and all markets clear.

II.B. Problems with the Primal Approach

Proposition 1a Given a policy sequence $\{\underline{g(t)}, \{v^h(t)\}_{h=1}^{H}, \{\underline{\gamma(t)}, \underline{\tau(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$, initial capital stocks $\underline{K_0}$ and intial ownership shares $\{\underline{\alpha^h}\}_{h=1}^{H}$, a competitive equilibrium may not exist.

Proof (by example) Consider a 1 household, 1 good economy without capital and with a policy having no government consumption, no capital taxes:

$\underline{g(t)} = \underline{\gamma(t)} = 0$ and $v(t) = v^*$, as well as, $\tau(t) = \tau^*$. This implies from the gov't BC that:

$$L(t) = \frac{v^* \tau^*}{w(t)}. \tag{12}$$

Taking v^* and τ^* as given the household problem yields a labor supply function:

$$L(t) = L(w(t); \tau^*, v^*) \tag{13}$$

Unless the relation (12) and (13) intersect in the positive quadrant this economy does not admit an equilibrium.

The existence problem is more severe than suggested by proposition 1a: even if part of the policy sequence is treated as a free parameter, there are situations where no competitive equilibrium exists. This is outlined in proposition 1b:

Proposition 1b Given a policy sequence $\{\underline{g(t)}, \{v^h(t)\}_{h=1}^{H}, \{\underline{\gamma(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$, initial capital stocks $\underline{K_0}$ and intial ownership shares $\{\underline{\alpha^h}\}_{h=1}^{H}$, there might not exist a sequence $\{\underline{\tau(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$ that admits a competitive equilibrium.

Proof (by example) Consider a 1 household, 1 good economy without capital with the policy $g(t)$ = $\gamma(t)$ = 0 and $v(t) = v^*$. Also consider preferences that admit a (monotone, continuous) inverse labor supply function s.t. $w^S(L=0,t)=0$ and a production function s.t. the inverse (monotone, continuous) labor demand function has $w^D(L=0,t) < \infty$. Then in equilibrium the tax revenue is given by the area $\tau(t)w^*(t)^*L^*(t)$. Continuity implies that this area is bounded by the area A between the inverse supply and demand function and is thus finite. Thus for any $v^* > A$, there are no tax rates compatible with a competitive equilibrium.

Here the labor tax rate is taken as a free parameter, and only government spending, transfers and capital taxes are taken as given. Still it is easy to construct an example in which for the given policy sequences there does not exist a equilibrium-compatible labor tax rate. Similar examples can be constructed for cases in which other subsets of the policy sequence are labeled 'free parameters' and givens. Note the proof given for proposition 1b is a simple case of an economy with a continuous, bounded Laffer curve. Any policy allocation requiring revenues greater than the bound on the Laffer curve can not possibly be supported.

A similar idea can be used to prove an analogous idea for the multiple equilibrium case, as in Proposition 2.

Proposition 2 There may be multiple competitive equilibria consistent with a given policy sequence

$\{\underline{g(t)}, \{v^h(t)\}_{h=1}^{H}, \{\underline{\gamma(t)}, \underline{\lambda_i(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$ initial capital stocks $\underline{K_0}$ and initial ownership shares

$\{\underline{\alpha^h}\}_{h=1}^{H}$.

Proof (by example) Consider the same economy as in the proof for proposition 1b. Equations (12) and (13) can have multiple intersections ('backward bending supply curve').

In other words, for a set of required tax revenues there are two or more possible labor tax rates that raise the required revenue in equilibrium.

Proposition 3 If there is any missing data for the policy sequence $\{\underline{g(t)}, \{v^h(t)\}_{h=1}^{H}, \underline{\gamma(t)}, \{\underline{\tau(t)}\}_{i=1}^{N+M}\}_{t=0}^{\infty}$ initial capital stocks $\underline{K_0}$ and initial ownership shares $\{\underline{\alpha^h}\}_{h=1}^{H}$, then competitive equilibria and prices are not computable.

Proof Immediate

Proposition 3 emphasizes how infinite policy sequences are required inputs for the primal approach. In practice, this difficulty is handled by extrapolating future policies from past policies, and often by truncating the horizon. The next section shows how neither of these approximations are required by the dual procedure.

III. The Dual Procedure for Computing and Evaluating the Model

The dual procedure simply uses the first order conditions (i)-(ii) implied by the definition of competitive equilibrium to calculate tax rates.

III.A."Demand" and "Supply" Prices

III.A.1 Consumer Problem

Let $mrs(c_i^h(t), L^h(s)) = \log\left(\frac{\delta u^h(\underline{c}^h(t), L^h(t), t) / \delta c_i^h(t)}{\delta u^h(\underline{c}^h(s), L^h(s), s) / \delta L^h(s)}\right)$. With a known utility function, and known date *t* quantities, *mrs* can readily be calculated. Item (i) of the definition of competitive equilibrium requires that consumers willingly demand the equilibrium quantities. If these quantities are positive, then (i) implies the first order condition equating marginal rates of substitution to the relative after-tax price of goods (note normalization of $p_1(t) = 1 \ \forall t = 0, \ldots, \infty$):

$$mrs(c_i(t), L^h(t)) = \log(p_i(t)) - \log(w^h(t)) \qquad i = 1, \ldots, N, \ \ h = 1 \ldots, H \qquad [C.1]$$

$$mrs(c_1(t), c_1(s)) = \sum_{k=0}^{t} \log(1 + q(k)) - \sum_{k=0}^{s} \log(1 + q(k)) \qquad [C.2]$$

Equations [C.1] are the within-period first order conditions and [C.2] the between-period conditions. These conditions are related to the within- and between- period conditions for firms, as shown below.

III.A.2 Firms

Item (ii) of the definition of competitive equilibrium requires that firms willingly demand the equilibrium quantities. If these quantities are positive, then (ii) implies the first order condition equating marginal products to the net-of-tax input rental rates:

$$\log\left(\frac{\delta f_i(\underline{K_1(t)}, \underline{L_i(t)}, t)}{\delta L_i^h(t)}\right) = \log(w^h(t)) + \log(1 + \tau_i^h(t)) - \log(p_i(t)) \qquad [F.1]$$

i = 1, ...,N, h = 1, ...,H

$$\log\left(\frac{\delta f_i(\underline{K_1(t)}, \underline{L_i(t)}, t)}{\delta L_i^h(t)}\right) = \log(w^h(t)) + \log(1+\tau^h(t)) - \log(\lambda_i(t)) \quad [F.2]$$

i = N+1, ...,N+M, h = 1, ...,H

$$\log\left(\frac{\delta f_i(\underline{K_1(t)}, \underline{L_i(t)}, t)}{\delta K_i^j(t)}\right) = \log(r_j(t)) + \log(1+\gamma^j(t)) - \log(p_i(t)) \quad [F.3]$$

i = 1, ...,N, j = N+1, ...,N+M

$$\log\left(\frac{\delta f_i(\underline{K_1(t)}, \underline{L_i(t)}, t)}{\delta K_i^j(t)}\right) = \log(r_j(t)) + \log(1+\gamma^j(t)) - \log(\lambda_i(t)) \quad [F.4]$$

i = N+1, ...,N+M, j = N+1, ...,N+M

where $\lambda_i(t) = \sum_{s=1}^{\infty} \prod_{m=0}^{s} (1+q(t+m))^{-1} r_i(t+s)(1-\delta_i)^s$ represent the PDV of a unit of capital of type 1.

III.B. "Tax Wedges"

The dual procedure as suggested in Section I allows us to evaluate the model without explicitly calculating the (primal) solution to the maximization problem. Even with limited data we can derive model implications with minimal computational effort. The maximization problems of the consumer and the firms imply a set of FOCs as given above. Given numerical functions for technologies and preferences,[7] and observations of the quantity data, mutual

[7]In other words, the quantity data is sufficient to calculate any marginal product or marginal rate of substitution. For example, with Cobb-Douglas production and preferences, this means that the numerical values of the share parameters and multiplicative productivity

consistency of the FOCs allows the derivation of a set of tax wedges. These tax wedges in turn can be used to deduce the policy sequences consistent with the model. Proposition 4 establishes that in the present set-up for minimal data it is possible to deduce the labor tax rate at date t for households h.

Proposition 4

Given a sample containing no more data than labor supply and consumption by one household h $\{L^h(t), \underline{c^h(t)}\}$ and data on the production inputs for one of the consumption firms $\{\underline{K_i(t)}, \underline{L_i(t)}\}$ at date *t* it is possible to obtain the labor income tax rate $\tau^h(t)$.

Proof: solve C.1 for $\log w^h(t) - \log p_i(t)$ and insert into F.1.

To do this is not even necessary to observe all quantity data at date *t* nor do we need any observations from other time periods.

Proposition 5 shows how using quantity data from 2 adjacent time periods it is possible to deduce the complete set of prices and policies for the first period. These 2 propositions contrast with the result from proposition 3 that the competitive equlibrium in the primal problem is only computable if all policies are observed. Thus it is possible here to evaluate the model without access to the complete set of data and without the computational effort implied by the primal problem.

Proposition 5

Given observations on quantities $\{\{L^h(t), \underline{c^h(t)}\}_{h=1}^{H}, \{K_i(t)\}_{i=N+1}^{N+M}, \{\underline{K_i(t)}, \underline{L_i(t)}\}_{i=1}^{N+M}\}_{t=0}^{T}$

and it is possible to compute the price sequences $\{\underline{p(t)}, \underline{w^h(t)}\}_{t=0}^{T}$, and the policy sequences

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for each date have been chosen. As mentioned above, with some restrictions these parameters could be estimated by iterating on steps (iii)-(vi)′. Or the parameters could be separately estimated other data sets, as is done for "Solow residuals" in much of the macroeconomics literature.

$\{\underline{\gamma(t)}, \underline{\tau(t)}, \underline{v(t)}, \underline{g(t)}\}_{t=0}^{T}$ for t=0, ..., T.

Proof

Step 1. Beginning with i=1 and $p_1(t)$=1, C.1 yields $\underline{w(t)}$ for t=0,...,T. Given $\underline{w(t)}$, $p_i(t)$ for i > 1 can be calculated from C.1's other conditions, t=0,...,T.

Step 2. Use proposition 4 to get $\underline{\tau(t)}$ for t=0,...,T.

Step 3. Use the condition F.2 to get $\underline{\lambda(t)}$ for t=0,...,T.

Step 4. From C.2 for t and t-1 get $q(t)$ for t=0,...,T

Step 5. From the definition of $\lambda_i(t)$:

$$\lambda_i(t-1) = (1+q(t))^{-1}[r_i(t)(1-\delta_i) + (1-\delta_i)\lambda_i(t)]$$

Using $\lambda_i(t), \lambda_i(t-1), q(t)$ (from Step 3 and 4) and δ_i (specified in the set-up) this solves for $r_i(t)$

Step 6. From F.3 obtain $\gamma(t)$

Step 7. Use the RBC (8) and the budget constraint (2) to obtain $\underline{v(t)}$ and $\underline{g(t)}$

Thus proposition 5 shows how to use quantity data and the consistency requirements to identify price and policy sequences consistent with the competitive equilibrium assumption. It is possible to obtain policy data for only a subset of data.

To illustrate proposition 5 consider how it is applied to the canonical example of an economy with one good and leisure, where the good serves both as the consumption and capital good. There exists only one type of labor.

Step 1. Given there is only one good $p(t)$=1 and from C.1 obtain $w(t)$ for all t. This step simply exploits the margin between leisure and consumption for the consumer to deduce the price of leisure faced by the consumer.

Step 2. From F.1 obtain $\tau(t)$. Given the price of leisure faced by the consumer and the marginal product of labor one can obtain the wedge between the wage paid by the firm and the wage received by workers

Step 3. Since the good doubles as a consumption and capital good we immediately have $p(t)=1=\lambda(t)$.

Step 4. C.2 gives the interest factor $q(t)$ (where $q(0)=0$). Here the intertemporal margin is exploited.

Step 5. Here $(1+q(t))=(1-\delta)(1+r(t))$. The return on a capital unit produced equals the nominal interest rate. This gives the rental price for a capital unit net of taxes.

Step 6. From F.3 one obtains $\gamma(t)$. The rental price gross of taxes has to equal the marginal product of capital. This allows deducing the capital tax rate.

Step 7. Finally as before use the RBC and budget constraint to obtain $\underline{v(t)}$ and $\underline{g(t)}$.

IV. Application to the Great Depression and WWII

To see the usefulness of these methods, consider the question "How can aggregate U.S. behavior be explained for the period 1929-50?" A first step in answering this is to pick a model of the economy, say, the neoclassical growth model with distortionary taxes and changing productivity. Second, I use the dual approach to generate the marginal tax rates rendering the observed quantities 1929-50 to be *exactly* a competitive equilibrium of the model. I show how the required marginal labor income tax rates change significantly over time, suggesting that a model without distortionary taxes, or with time-invariant taxes, cannot fit the quantity data. I then look at some of the evidence on taxes and regulation during the period, and suggest that it is implausible for those policies to have generated the large marginal tax rate changes that are required to replicate observed behavior in the model.

IV.A. A Neoclassical growth Model with Labor and Capital Income Taxes as a Special Case

Here we limit our attention to the special case of the model with one type of household ($H=1$), one capital good ($M=1$), and one consumer good ($N=1$) that is perfectly substitutable for investment goods. The model government only consumes, lump sum transfers, taxes labor income, and taxes capital inputs. Given a policy sequence $\{g(t),v(t)\}_0^\infty$, and an initial capital stock K_0, a competitive equilibrium with labor income taxes is simply a constant z and sequences $\{c(t),L(t),K(t+1),w(t),q(t),\gamma(t),\tau(t)\}_0^\infty$ such that:

(i) given z and $\{(1-\tau(t))w(t), q(t+1), v(t)\}_0^\infty$, $\{c(t), L(t)\}_0^\infty$ solve:

$$\sum_{t=0}^{\infty} e^{\sum_{s=0}^{t} \pi(s)} u(c(t), L(t)) \qquad \text{s.t.}$$

$$\sum_{t=0}^{\infty} Q(t)\left[(1-\tau(t))\, w(t)\, L(t) + v(t) - c(t)\right] + z = 0$$

$$\ln Q(t) \equiv -\sum_{s=1}^{t} \ln\left[1 + q(t)\right]$$

where $\pi(t)$ is the consumer's date $(t$-1) one period forward rate of time preference.

(ii) The resource constraint binds at each date t:

$$f(L(t), K(t), t) - \delta K(t) = c(t) + (K(t+1) - K(t)) + g(t)$$

(iii) given $\{w(t), q(t+1), \gamma(t)\}_0^\infty$ and $K(0)$, z and $\{L(t), K(t+1)\}_0^\infty$ solve:

$$z = \max_{\{L(t), K(t+1)\}} \sum_{t=0}^{\infty} Q(t)\left[f(L(t), K(t), t) - (K(t+1) - (1-\delta)\, K(t)) - w(t)\, L(t) - \gamma(t)\, q(t)\, K(t)\right]$$

(iv) $\{\mathrm{g}(t), \mathrm{v}(t), \tau(t), \gamma(t), \mathrm{w}(t), \mathrm{L}(t)\}_0^\infty$ balances the government budget constraint at each date:

$$g(t) + v(t) = \tau(t)\mathrm{w}(t)L(t) + \gamma(t)\mathrm{q}(t)K(t)$$

Given data $(L(t), K(t+1), K(t))$ on quantities for any period t, and numerical utility and production functions, it is straightforward to compute the policy variables $(\tau^*(t), \eta^*(t), v^*(t), g^*(t))$ that are consistent with a competitive equilibrium:

$$\tau^*(t) = 1 + \frac{u_L(c(t),L(t))}{u_c(c(t),L(t))f_L(L(t),K(t),t)} \;,\quad \gamma^*(t) = \frac{f_K(K(t),L(t),t) - \delta}{e^{\pi(t)}\frac{u_c(c(t-1),L(t-1))}{u_c(c(t),L(t))} - 1} - 1$$

$$v^*(t) = c(t) + [K(t+1) - (1-\delta)K(t)] - f(L(t),K(t),t) - \tau^*(t)L(t)f_L(L(t),K(t),t) \tag{14}$$

$$g^*(t) = f(L(t),K(t),t) - c(t) - [K(t+1) - (1-\delta)K(t)]$$

where the term in square brackets is simply gross investment. The simulated tax rates are just the gap between the simulated demand and supply prices.

I use production and utility functions familiar from the real business cycle literature (eg., King, Plosser, and Rebelo 1988):[8]

$$u(c,L) \equiv \ln c + \theta \ln(1-L)$$

$$f(L,K,t) \equiv A(t)L^{\beta}K^{1-\beta}$$

where L is measured as manhours as a ratio of the annual "time endowment" (2500 hours per person) for the population aged 15 and over, and all other quantities are measured per person aged 15+.

Appendix Table 1 reports $\{L(t),c(t)/Y(t)\}$ for t = 1929-50 (where $Y(t)$ is date t output).[9] Four adjustments are made during wartime (1939-48)[10] to reflect the mismeasurement of output and the involuntary nature of wartime military labor supply (not captured in the model above). First, output is measured for the civilian sector only, under the assumption that civilian and

[8] Mulligan (2000) studies two other functional forms as well, finding very similar results for the Great Depression and somewhat different results for WWII and other time periods.

[9] Data sources, and the wartime adjustments below, are explained in Mulligan (2000).

[10] Results are quite insensitive to small changes in the definition of "war years" because these adjustments are trivial when the military is small, or there is a volunteer force.

military personnel produce measured output in proportion to their measured labor income. To be consistent with this adjustment, the second adjustment is to measure labor input as civilian manhours only.

Most wartime soldiers were drafted, so it is questionable whether their consumption and leisure is as voluntary as modeled above. My third adjustment is therefore to calculate consumption as civilian consumption expenditure per civilian aged 15+.[11] This adjustment slightly increases measured wartime consumption.

IV.B. Simulated Policies

Given the numerical utility and production functions, the formulas for the policy variable $\tau^*(t)$ consistent with the model's competitive equilibrium are:

$$\tau^*(t) = 1 - \frac{L(t)}{(1-L(t))}\frac{\theta}{\beta}\frac{c(t)}{f(L(t),K(t),t)} \qquad \textbf{(14)}'$$

Recall that $\tau^*(t)$ is just the gap between the simulated demand and supply prices of labor which, given the assumed Cobb-Douglas functional forms, are proportional to the consumption-leisure and output-labor ratios, respectively.

The last column of Appendix Table 1 calculates $\{\tau^*(t)\}$ for t = 1929-50, using parameters β = 0.615 and θ = 0.7.[12] The dual approach does not have implications for transfers and government consumption that can be tested with national accounts data because the national accounts calculate these to fit the model (at least if we interpret purchases and sales of government debt as lump sum transfers and taxes), so (14)′ neglects the equations simulating transfers and government consumption.

[11]Civilian consumption is measured as the difference between aggregate personal consumption expenditures and one half of military wages (assuming that half of military wages are saved, paid in taxes, or paid to civilian family members).

[12]These are basically those used in the literature, with small differences due to the different time period studied, and my explicit modeling of distortionary taxes.

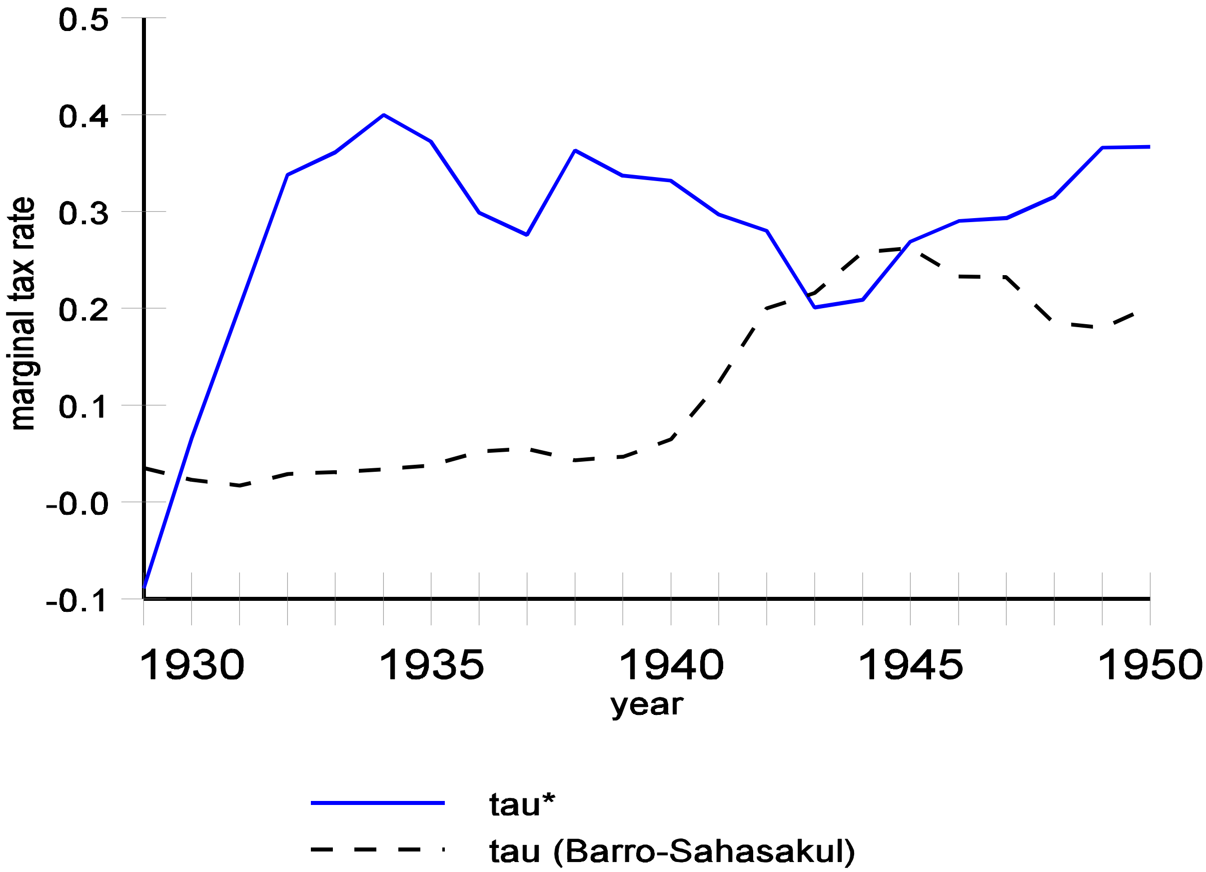

Figure 2 Simulated and Measured Marginal Labor Income Tax Rates Compared, 1929-50

Figure 2 compares the marginal labor income tax rates $\{\tau^*(t)\}$ consistent with the model's competitive equilibrium with the marginal labor income tax rates calculated by Barro and Sahasakul from IRS data (1986). We see that the model predicts Depression tax rates that are much higher, and Wartime tax rates that are lower, than measured directly from government tax records.

It is easy to study the economic and statistical reasons for the fluctuations in the simulated marginal labor income tax rate $\{\tau^*(t)\}$. To understand the statistical reasons, recall from (14)′ that $\tau^*(t)$, up to the ratio θ/β of constants, is one minus the product of the labor-leisure and consumption-output ratios. Figure 3 displays the measured time series for those ratios, and we see how the consumption-output ratio is pretty steady except during the war when it is a bit lower. So most of the variation in $\tau^*(t)$ comes from the labor-leisure ratio which is low in the depression and high in the war, so that simulated marginal tax rates are high during the war and low during the Depression. The basic patterns in the data are hardly controversial – see, for example, Friedman (1957, p. 117f) on low-to-medium frequency constancy of the consumption-output ratio and Lucas and Rapping (1969) labor fluctuations.

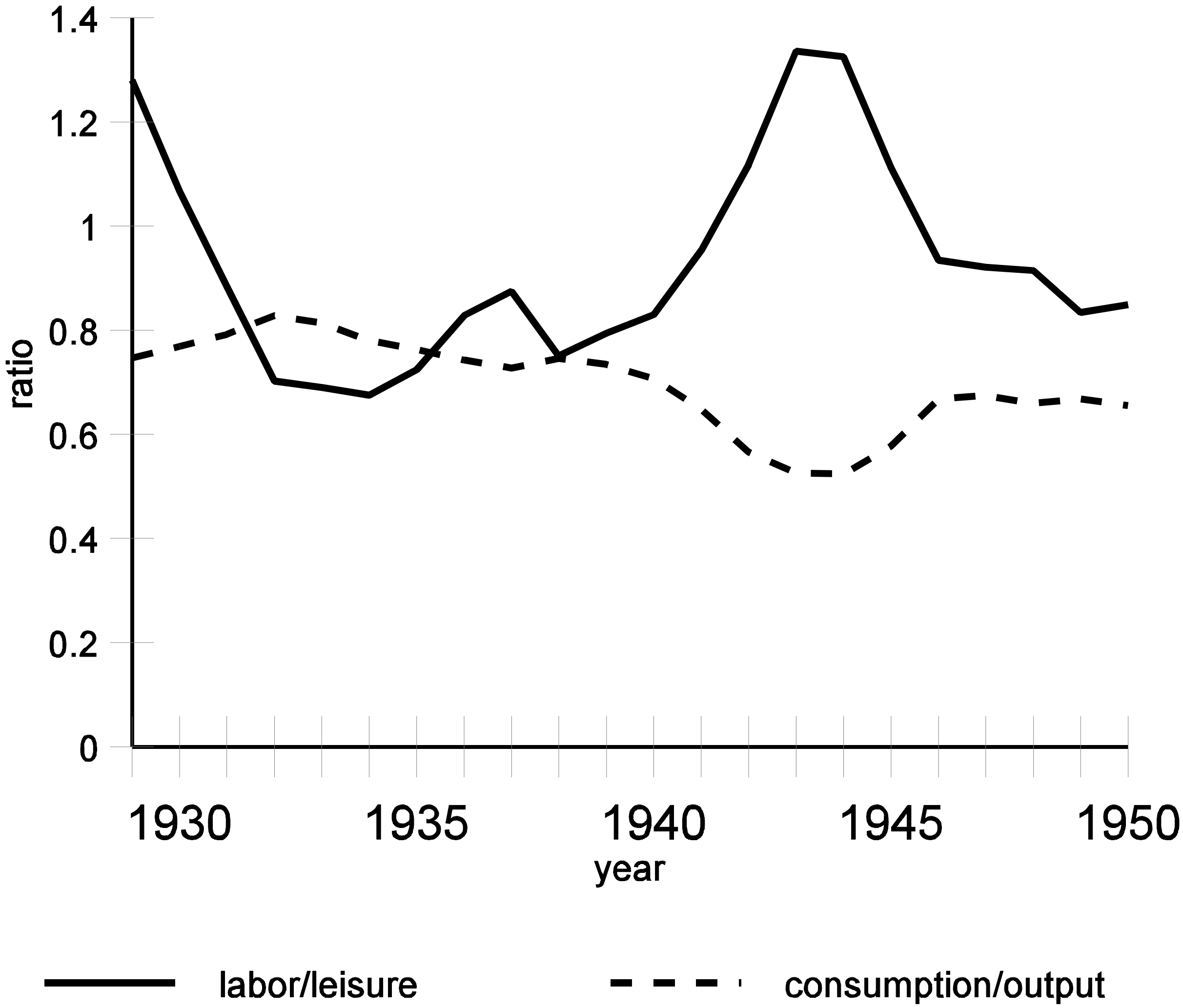

Figure 3 Components of Simulated Labor Tax Rates: By Data Source, 1929-50

I have not removed trends from the data, but we see from Figure 3 that trends are not particularly noticeable in the data I use to simulate marginal tax rates. Perhaps this is one advantage of the dual approach – there is less reason to remove trends of from the basic data (because there is not much trend!) and we might worry less about the sensitivity of results to trend estimation.

Figure 4 displays the economic components of the simulated tax rate, namely the marginal product of labor and the marginal rate of substitution (see equation (14)). The marginal product of labor, computed as 0.615 times the average product of labor, is displayed as a solid line. It follows a pretty steady trend over time, except a bump during the war and no growth 1929-33.

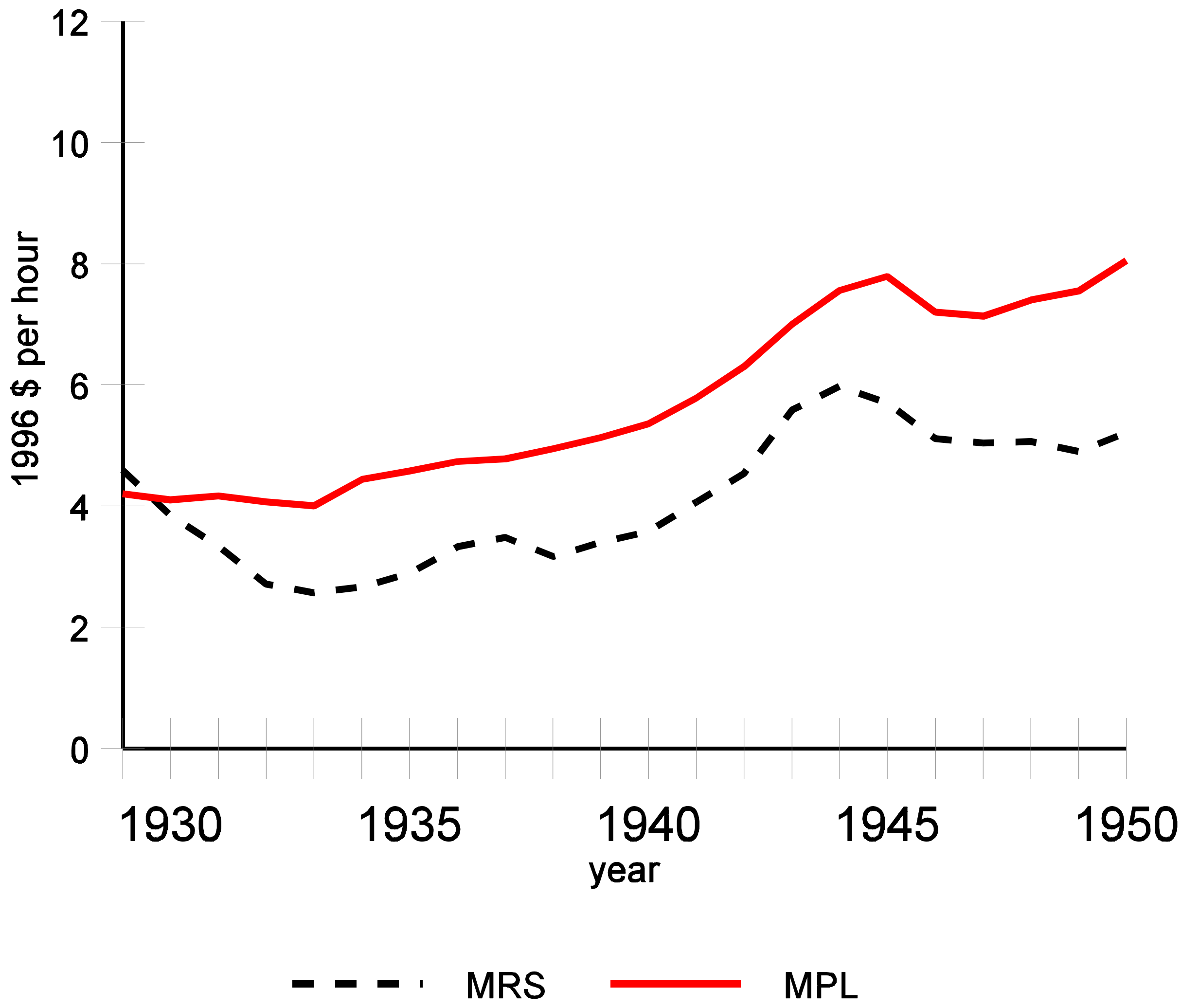

Figure 4 Components of Simulated Labor Tax Rates: By Economic Margin, 1929-50

For the most part, the simulated marginal rate of substitution (*MRS*), or marginal value of leisure time, is less than the marginal product of labor (*MPL*). Perhaps surprising is the dramatic divergence of *MRS* from *MPL* during the 1929-33 period (30 or 40 percentage points!), a wedge which persists until the war. As I discuss in the next subsection, the rapid emergence of this wedge, and its persistence, are crucial for understanding the Great Depression.

IV.C. Understanding the Great Depression

Figure 2 and 4 make an important point – if an aggregative competitive equilibrium model is to explain the Great Depression, at least with Cobb-Douglas production and utility functions, it must explain why *MRS* and *MPL* diverged so dramatically 1929-33 and why the wedge persisted. This point has implications for many theories explored in the literature:

IV.C.1. Productivity Shocks Cannot Explain 1929-33, or 1933-39

Cole and Ohanian (1999, p. 3) suggest that, if it could be argued that productivity shocks ($\{A(t)\}$ in my notation) were large and persistent enough, then a neoclassical growth model could fit the 1930's data pretty well. They reject this explanation because they see no reason why productivity would have been low after 1933, but my analysis rejects it for a very different reason: there is no productivity series $\{A(t)\}$ that can be fed into the neoclassical growth model (without some of the distortions mentioned below) to fit the Depression data because that model equates *MRS* and *MPL* for any realization of the productivity series. In other words, while the productivity parameter A affects the relation between inputs and outputs, it does not affect either the relation between *MRS* and the consumption-leisure ratio or the relation between the average and marginal products of labor. Hence, according to the model, technology shocks should not affect the gap between observed and simulated labor income tax rates, because these simulated rates are calculated from the consumption-leisure and the output-labor ratios.

Similarly, Cole and Ohanian (1999, p. 3) and Prescott (1999, p. 26) suggest that the period 1929-33 is not puzzling for the real business cycle approach, because there are lots of candidates for productivity shocks during that period. Perhaps there are good candidates, but productivity shocks do not cause *MRS* and *MPL* to diverge in the neoclassical growth model – and my Figures 2 and 4 shows that such divergence is what happened 1929-33.[13] In summary, in addition to (or instead of?) the right time series for productivity shocks, the neoclassical growth model needs to be amended to explain why *MRS* and *MPL* diverged and why that wedge persisted.

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[13]To put it another way, an adverse productivity shock decreases the *MRS* and *MPL* *together* in the neoclassical growth model.

IV.C.2. Income and Sales Taxes are not an Important Part of the Labor-Leisure Distortion

Cole and Ohanian (1999, p. 6) suggest that government purchases, or taxes on factor incomes, might help explain some of the Depression economy. However, my analysis suggests that government purchases, and taxes on capital, cannot explain why *MRS* and *MPL* would be different, let along why and how that wedge would persist over time. Of course, taxes on labor income create such a wedge, but Barro and Sahasakul's study suggests that federal taxes on payroll and individual income were trivial, and unchanging, during the period. Indeed, IRS records (IRS, various issues) show that the vast majority of the population did not file individual income tax returns during the 1930's, so that any IRS-induced tax wedge affected very few people (not to mention small for the few affected).[14]

Taxes on consumption expenditure are also expected to drive a wedge between *MRS* and *MPL* (in the absence of other distortions, consumers equate their *MRS* to $MPL/(1+\sigma)$, where σ is the marginal sales tax rate). The federal government did not have a general sales tax, although it does have (and has had) excise taxes on goods such as cigarettes, gasoline, and imports. More general sales taxes have been collected by states and localities. However, the revenues from these taxes are too few, and not changing enough over time, to drive much a of wedge. Furthermore, given the assumed logarithmic functional forms and the fact that my measure of consumption is inclusive of sales taxes, sales taxes do not drive a wedge between *measured MRS* and *MPL*.

IV.C.3. Transfer Programs Have Little or No Effect

Government transfer payments, such as those used by Social Security, welfare, and unemployment systems are also expected to affect the gap between *MRS* and *MPL*. Unfortunately (for the analyst), there are many transfer programs at the federal, state, and local levels that might be expected to drive a wedge, and the incentive effects of even one of those programs are complicated, heterogeneous, and changing over time. Indeed, a entire paper – or literature – might be devoted to the wedge created by *one* entitlement program in *one year*, for one

[14]State and local income taxes are not included in my calculations. However, since these taxes tend to be "flat" (ie, relatively few tax brackets and a relatively broad tax base), and revenues from these taxes were essentially zero during the 1930's (Census Bureau 1975 series Y-658), it seems that these taxes had practically no effect on the wedge between *MRS* and *MPL*.

subset of the population (eg., Feldstein and Samwick 1992 on 1990 Social Security benefit formulas and the working-aged population, Blinder, Gordon and Wise 1980 on 1977 Social Security benefit formulas and the population aged 62-69, or Fraker, Moffitt, and Wolf 1985 on 1981 AFDC). My approach is therefore to calculate an upper bound on the potential aggregate incentive effects to see if transfer programs *might* credibly explain the large tax wedge changes simulated from aggregate behavior.

Figure 5 displays as a solid red line government transfers (including those paid by federal, state, and local governments) as a fraction of labor income for the years 1929-50. Transfers increased slightly in nominal terms during the 1930's while nominal labor income declined, so Figure 5 shows an increase in the transfer-labor income ratio. The transfer-labor income ratio was relatively high between WWII and the Korean War.

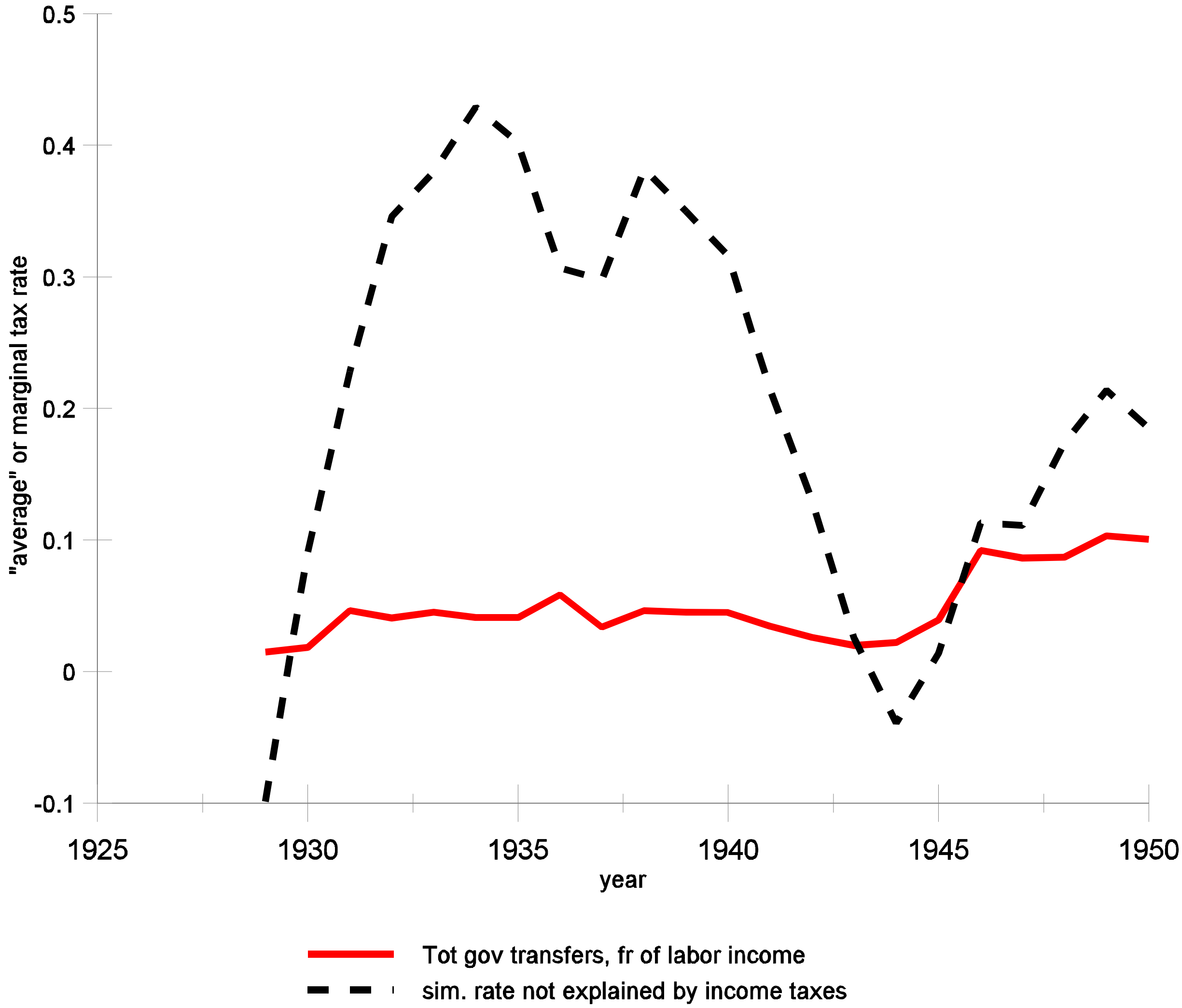

Figure 5 Spending on Transfer Programs vs Simulated Tax Rates, 1929-96

While calculating the average marginal tax rate implicit in the portfolio of federal, state, and local transfer programs is very difficult, the transfer-labor income ratio shown in Figure 5 is probably an upper bound on a more thorough and more accurate calculation of that rate (see Mulligan 2000 for more discussion of this point). Figure 5 displays as a dashed line the simulated tax rate minus the measured income tax rate, which I interpret as that part of the simulated tax wedge that is unexplained by income tax policy. With its solid red line as an upper bound on the composite marginal rate from transfer programs, Figure 5 suggests that, during the 1930's, simulated tax rates increased by an order of magnitude more than did the rates from transfer programs, so that transfer programs cannot be an important part of an explanation of Depression labor markets.[15] To put it quite simply, how could Depression transfer programs simultaneously have large disincentive effects and spend so little money at a time when a lot of people were not employed?

IV.C.4. How Much Can International Trade Explain?

The Great Depression was an important time in the history of international trade, with dramatic increases in tariff *rates* as a result of the Hawley-Smoot Act, other legislation, and other nonlegislation (see, for example, Taussig 1931 or Crucini 1994). Some (eg., Metlzer 1976, Crucini and Kahn 1994) have suggested that international trade was an important influence on aggregate activity during that period. A key question is: would changes in tariffs drive a large wedge between *MRS* and *MPL*, and would that wedge persist for a decade?

Given our assumed functional forms, the answer would be trivial if all consumption were subject to the tariff, and tariffs were levied only on consumption goods, because we could treat tariffs as sales taxes and apply the result above. But Crucini and Kahn's (1994) emphasize that tariffs are levied on intermediate inputs, and therefore have implications for productivity and its measurement. Nevertheless, using Crucini and Kahn's (1994) dynamic general equilibrium trade model, it is easy to show how the tariffs of the 1930's could not drive large wedges between *MRS* and *MPL* as we have measured them. Theirs is a two country model, with a representative agent in each country. That agent consumes three types of goods (home nontraded, home traded, and foreign traded), and supplies his time to each of three sectors (traded consumption, untraded consumption, and traded production materials). Crucini and Kahn do not have labor income taxes, so their model implies an equation of the marginal value of time (in utility) with the marginal net-of-tariff revenue product of labor (in production, in each of the three sectors). Of course, if their model did have labor income taxes, the marginal labor income tax rate would be the wedge between the marginal value of time and the marginal net-of-tariff revenue product

[15]After the 1930's, there are some positive high-frequency correlations between transfer "tax rates" and unexplained simulated rates (see Mulligan 2000), which are consistent with the hypthesis that transfer programs drive a wedge between *MRS* and *MPL*. However, notice that transfer programs tend to grow in size in response to nonemployment, so that fluctuations in the *MRS* might cause fluctuations in measured transfer "tax rates" rather than the other way around.

of labor, computed in much the same way as in the examples above:

$$\tau^*(t) = 1 + \frac{u_L(c(t),L(t))}{u_c(c(t),L(t))f_L(L(t),K(t),t)} \tag{14}$$

There are two differences between (14) and the analogue for the neoclassical one-sector growth model: (1) consumption is a composite good (eg., a CES aggregate of the three consumption goods as in Crucini and Kahn's numerical model), and (2) f_L is the equilibrium marginal revenue product of labor, *net of tariffs*, in either the traded or untraded sectors. But, because Crucini and Kahn (1994, pp. 439, 441) assume production is Cobb-Douglas in labor in both sectors – with the same labor share – my calculations ((14)′, repeated below for convenience) for the neoclassical growth model can be applied to Crucini and Kahn's model with one very minor correction.

$$\tau^*(t) = 1 - \frac{L(t)}{(1-L(t))}\frac{\theta}{\beta}\frac{c(t)}{f(L(t),K(t),t)} \tag{14$'$}$$

The relevant marginal product of labor is net of tariffs, so it is computed as labor's share times *GNP net of tariff revenue* – not total GDP – per unit labor input.[16] However, the sign and magnitude of this correction depends on the sign and magnitude of (net factor income from abroad minus tariff revenue and) share of GDP. According to the Bureau of Economic Analysis (1999), net factor income from abroad was positive in the 1930's, and between 0.4 and 0.8 percent

[16]To derive this from Crucini and Kahn's (1996) equations, first compute aggregate labor income (wL in my notation) by adding the three marginal revenue product of labor equations from their p. 460, weighting by labor income and using the Cobb-Douglas functional forms (with identical labor shares for each sector). Part of this sum is aggregate expenditure on intermediate inputs (see their fifth-to-last equation on p. 460), which in turn is tariff revenue plus the compensation for those selling materials to that constant returns sector (to see this, add the two p. 460 intermediate marginal revenue product equations). Simple subtraction then implies that aggregate labor income (wL in my notation) is labor share times GNP minus tariff revenue. In other words, the marginal revenue product of labor w is GNP minus tariff revenue, times labor share, and divided by aggregate labor input L.

of GDP. Crucini and Kahn (1998, p. 443) suggest that tariff revenues was on the order of 0.7 percent of GDP, so the difference between the two is essentially zero. In other words, the simulated tax wedge is essentially numerically identical for the neoclassical growth and Crucini-Kahn models, even though those models suggest that somewhat different ingredients go into the calculation of the marginal revenue product of labor.[17]

Hence, Crucini and Kahn's (1996, p. 446) explanation of Depression labor supply is grossly inconsistent with Cobb-Douglas functional forms and three basic time series – output, consumption expenditure, and aggregate hours – used in my Figure 3, in the business cycle literature, and by Crucini and Kahn themselves. Their model "explains" Depression labor supply by simulating a counterfactually low average product of labor, rather than driving an important wedge between the marginal value of time and the marginal product of labor. The only hope for a trade model like Crucini and Kahn's to explain such a large wedge is for tariff revenue to be a large share of GDP, and larger than the share of net factor income from abroad.[18] In this sense, Crucini and Kahn's analysis supports, rather than refutes, Lucas's (1994, p. 13) claim that "the effects of [a tariff] policy (in an economy with a five percent foreign trade sector...) would be trivial."[19]

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[17] I have measured c as real consumption expenditures. In principle a price index could be designed based on Crucini and Kahn's utility function so that changes over time in real consumption expenditures would be the same as changes over time in the quantity of the composite consumption good. In practice (ie, using the GDP deflator or the Consumer Price Index), the real consumption expenditures may either over- or under-state composite consumption, because of imperfections in the price index, and a second adjustment of (14)′ may be required. However, we see from (14)′ and our empirical results how, in order to explain the Depression gap between MRS and MPL, any adjustment to real consumption expenditures must (a) *increase* measured real consumption during the Great Depression, and (b) be large enough to drive a 30% wedge.

[18] Or for exported goods to be extremely labor intensive – a possibility that was not explored by Crucini and Kahn and probably not quantitatively interesting.

[19] Perri and Quadrini (2000) suggest that the traded sector was more important in Italy – not only because tariff revenues were substantial, but also because regulations drove a significant wedge between the tradeable and nontradeable *MRS* – so that trade policy had an important effect on output, and perhaps also employment.

IV.C.5. Labor Market Regulations

Prescott (1999, p. 26) suggests that labor market regulations may have hurt employment during the Depression. My Figure 2 can guide future studies of this hypothesis. In particular, were there regulations driving a wedge between MRS and MPL? Did those regulations first appear, or take effect, 1929-33? How big was the wedge – as large as 30 or 40 percent?

On the first point, it should be noted that labor market regulations are varied. Some may have no effect because the regulations require workers and employers to do things that they would already do, or because the regulations are not enforced. Others may lower the marginal product of labor schedule (or raise it?), perhaps by restricting (or helping?) firms from using the most efficient production process. But of particular interest for my study are regulations that drive a wedge between *MRS* and *MPL*. According to the textbook analysis, a binding minimum wage is one example because it puts some people out of work – a movement down the aggregate labor supply schedule – and moves employers up their *MPL* schedule (aka, labor demand curve). Mandatory fringe benefits, if they are valued by employees at less than their cost to employers,[20] also drive such a wedge.

It is hard to identify which regulations drive a wedge between *MRS* and *MPL*, let alone accurately quantify the wedge created by the large and varied portfolio of federal regulation. However, recall from Figure 2 that the changes in implied tax rates to be explained are quite large – on the order of 30 percentage points or more for the entire labor force. Hence, even a rough qualitative analysis of federal labor regulation can reveal whether labor market regulation and its changes over time are a viable explanation. Mulligan (2000) attempts such a qualitative analysis, and his results are summarized here.

First, notice that, according to the Center for the Study of American Business' 1981 *Directory of Federal Regulatory Agencies*, the only federal labor regulations begun in the 1930's and covering more than a few workers were the 1933 National Industrial Recovery Act, the 1935 Wagner Act, and the 1938 Fair Labor Standards Act (FLSA). I consider the effect of unions below, so that leaves the 1938 Fair Labor Standards Act (FLSA) which was at least five years after the large wedge appears in Figure 2.

[20]ie, the mandated benefits exceed the amount workers would demand in the absence of regulation. See, for example, Summers (1989) for some analysis of this point.

Second, labor regulation that was at least as comprehensive of FLSA appeared in the 1960's and 1970's (including in this later regulatory explosion, for example, was the 1970 Occupational Safety and Health Act), but we see nothing like the Great Depression in the 1960's or 1970's and, according to Mulligan (2000), nothing like the 1930's divergence of *MRS* and *MPL*.

IV.C.6. Can Monopoly Unions be Part of the Story?

Textbook monopoly unions, by definition, deliberately drive a wedge between *MRS* and *MPL* in order to raise member incomes.[21] The size of this wedge is *related* to the "relative union wage gap", the percentage gap between a typical union worker and an observably otherwise similar nonunion worker, often measured in the labor economics literature. My approach is to use the estimates from that literature to quantify the potential contribution of monopoly unionism to the gap between *MRS* and *MPL* as measured in the aggregate.

Lewis (1963, 1986) surveys much of a large literature attempting to estimate the union wage gap for various industries. He stresses (1986, pp. 9, 187) that wage gaps vary a lot from industry to industry, and are typically overestimated because union workers are expected to have more unmeasured human capital than nonunion workers (so that measured wage gaps are only part monopoly union power, and part human capital differences). With these caveats in mind, I construct Table 1 below by reproducing and extending Lewis' (1963) Table 50, reporting by time period the relative wage gap for the "typical" unionized worker.

[21]Dunlop (1944) is an early description of the "textbook" model. Other plausible union models have unions raising the payments from employers to employees, but not in a way that distorts the labor-leisure margin (eg., Leontief 1946, and applications by Barro 1977 and MaCurdy and Pencavel 1986). If the latter union model is correct, then we immediately conclude that unions are not contributing to the wedge between *MRS* and *MPL*.

Table 1: Union Relative Wage Gaps by Time Period

	gap estimates	
time period	lower estimate	upper estimate
1923-29	0.15	0.20
1931-33	0.25	
1939-41	0.10	0.20
1945-49	0	0.05
1957-58	0.10	0.15
1967-70	0.12	0.16
1971-79	0.13	0.19

Table lists the difference between the typical union wage and the nonunion wage of observationally similar workers, as a fraction of the nonunion wage.
Source: Lewis (1963, Table 50 and 1986, p. 9)

Notice in particular that the union wage gap is about twice as large during the Great Depression (see also Lewis 1963, pp. 4f).

The measured wage gap need not be exactly the percentage wedge between *MRS* and *MPL* in the union sector. But it is perhaps a reasonable first estimate of that wedge – and would be identical to the wedge in the case that the wedge is zero in the nonunion sector, and the value of time (*MRS*) is the same in both sectors.[22] With this, and Lewis' (1986, p. 9) overestimation caveat, in mind I use the "lower" wage gaps reported in Table 1 as estimates of the *MRS*/*MPL* wedges in the union sector.

My calculations of implied tax wedges are for the entire economy, and not just the union sector. How much can monopoly unionism affect the average tax wedge? Assuming the

[22]The wedge is one minus the ratio of union sector *MPL* to union sector *MRS* which, under these assumptions, is the same as one minus the ratio of union sector wage to union sector *MRS*, which equals one minus the ratio of union sector wage to nonunion sector *MRS*, which is the same as one minus the ratio of union sector wage to nonunion sector wage.

monopoly union wedge is zero for nonunion workers, the size of the monopoly union wedge for the average worker is the product of the union wedge and union density (ie, the fraction of the labor force that is unionized[23]). Using Rees' (1989 Table 1)[24] time series, we see from the dashed line in Figure 6 that union density increased somewhat during the 1930's – reaching 18% – while the largest increases during the century were after the Depression. Union density has declined since the 1950's (see also Freeman and Medoff 1984, Figure 15-1), and perhaps that decline accelerated in the late 1970's and 1980's.

[23]Public sector union members are included. Their contribution to the national union density is small (5% in 1960; Rees 1989, p. 181), but growing steady over the period (Freeman 1986; Rees 1989 p. 181 says that 29% of union members in 1983 were public sector employees). Since 1983, the fraction of union members working in the public sector has grown further, to 44% by 2001 (BLS 2002).

[24]I use Census Bureau (1975, series D-17, 1900 value) to fill in Rees' missing nonagricultural employment for the year 1897, and then Census Bureau (1975) series D-167, 170 and BLS series LFU40000000, LFU11102000000 to convert Rees' ratio to nonagricultural employment to a ratio to the entire labor force.

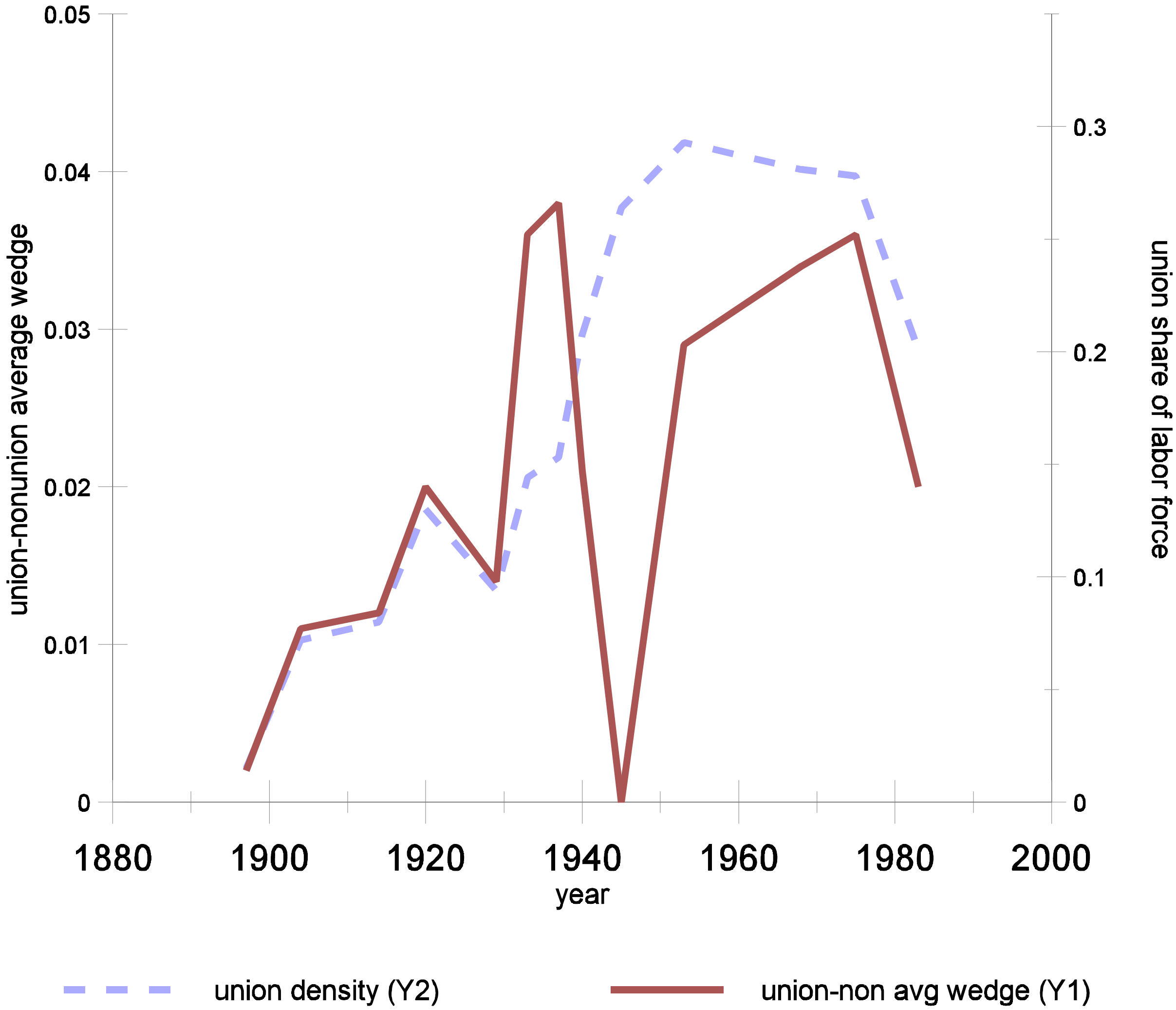

Figure 6 Union Density and Induced Wedges 1897-1983

The solid line in Figure 6 illustrates how changes in union density might affect the time series for the economy's average monopoly union wedge. The solid line assumes a nonunion sector wedge of 0, a union sector wedge of 15% prior to 1923, a union sector wedge equal to the "lower" gap estimates reported in Table 1 for the years 1923-79, and a union sector wedge of 0.10 after 1979. Union membership growth during the Depression, and especially the assumed growth in the union sector wedge, add 2 percentage points to the economy average wedge in the 1930's, and might thereby explain only small part of the Depression's implied tax wedge shown in Figure 2. However, even though it is assumed that the union sector wedge declines dramatically after the Depression, the post-Depression growth in union membership implies that (with the exception of the war) the economy-average wedge is pretty stable until the 1980's. In other words, even if the union wage effect appeared for the first time in the 1930's, monopoly unionism cannot explain a wedge of more than 4%, so most wedge shown in Figure 2 is unexplained.

IV.C.7. What about Monetary Shocks?

Whether monetary shocks can explain what is shown in Figures 2-4 depends on the margins distorted by those shocks. If monetary shocks have their primary effect on credit markets or otherwise distort intertemporal margins (as they do in Lucas 1975 and some other island models), then they cannot explain Figures 2-4. Barro and King (1984) emphasize that changes in intertemporal margins cause consumption and leisure to move together or, in terms of Figure 4, cause the *MRS* and *MPL* to move together.

In Lucas-island models of the confusion of real and nominal magnitudes, *MRS* is still equated to *MPL* (monetary shocks instead create a gap between perceived and actual intertemporal marginal rates of transformation) and thus inconsistent with Figure 4. But perhaps a modified monetary confusion model would predict that *MRS* is equated to *perceived MPL*, which we might expect to be less than the actual *MPL* shown in Figure 4 during those periods when the price level is less than expected. But could the misperception be as large as 30 or 40 percent and could it persist for a decade?

Sticky nominal wages, perhaps as modeled Barro and Grossman (1971) might well drive a wedge between *MRS* and *MPL* in response to monetary shocks. However, the timing and magnitude of such rigidities are difficult to measure *independently* of the average product and consumption series shown in Figure 4. This sets the "rigid wage" hypothesis apart from the public finance distortions (whose magnitude and timing were independently measured using IRS tax rules and return data) and the monopoly union distortions (whose magnitude and timing were independently measured using union density and Lewis's comparisons of union and nonunion sectors). Are there direct measures of wage rigidity for the 1930's? Or are there "flexible wage" sectors that could be compared with "rigid wage" sectors?

According to one special case of the "rigid wage" hypothesis (and one suggested by Lewis, eg., 1963 pp. 5f), wages are rigid only in the union sector, in which case wage rigidity can be measured independently of average productivity by comparing wages in union and nonunion sectors. This is what Lewis does, and his results are transformed into a wedge between *MRS* and *MPL* in the previous section. In other words, rigid wages may only be another interpretation of the calculations I interpreted above as "monopoly union."

Another relation between rigid wages and my "monopoly union" calculations is that any estimate of wage rigidity, however obtained, can be translated into a tax wedge by using Table 1, or using the 2ω rule. For example, if Depression wages should have been 5% lower than they were, Table 1 suggests that this rigidity drove a 25% wedge between *MRS* and *MPL*, while the 2ω rule suggests the wedge was 10%. The calculation can also be done in reverse – what must be the magnitudes derived from a study of wage rigidity in order to "explain" Depression labor supply? Using the 30% implied wedge calculated for the 1930's in Figure 2, we see from Table 2 that Depression wages must be 5-10% too high and from the 2ω rule that Depression wages be 15% too high.

IV.D. Intertemporal Distortions During the Period

The procedure shown in Section III can also be used to simulate corporate profits tax rates for the neoclassical growth model. The formula is (repeated from (14) for the reader's convenience):

$$\gamma^*(t) = \frac{f_K(K(t),L(t),t) - \delta}{e^{\pi(t)}\frac{u_c(c(t-1),L(t-1))}{u_c(c(t),L(t))} - 1} - 1$$

where $\pi(t)$ is the consumer's date $(t$-1) one period forward rate of time preference. With the Cobb-Douglas production and utility functions, the consumer's ratio of marginal utilities is just consumption growth, and the marginal product of capital is proportional to the output-capital ratio. $\gamma^*(t)$ and its two components are graphed in Figure 7.

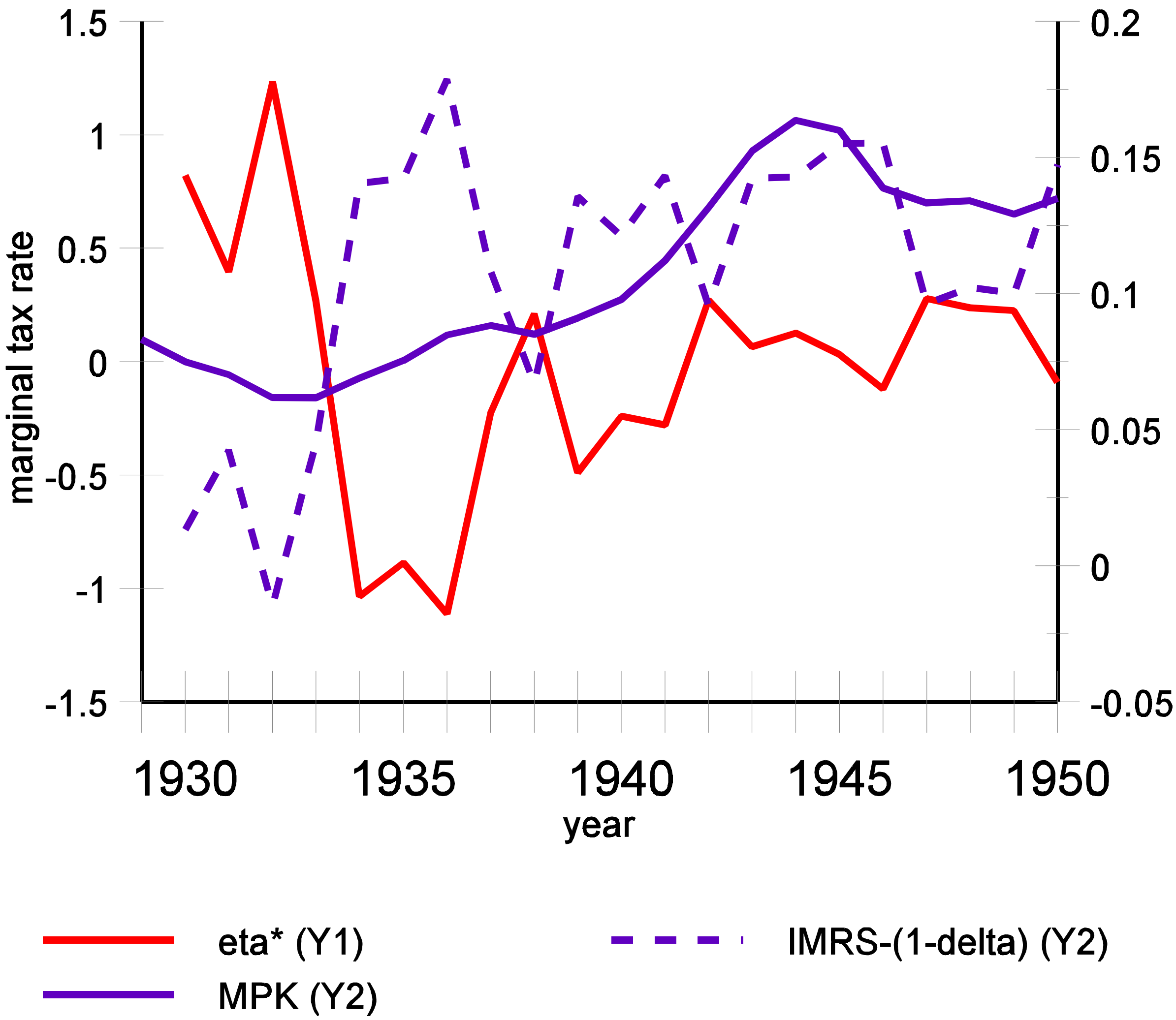

Figure 7 Simulated Marginal Tax Rates and Their Components, 1929-50

Figure 7 shows how the marginal product of capital (MPK, shown as a solid blue line) declined slightly 1929-33, and then increased steadily until the end of the war. Consumers' intertemporal marginal rate of substitution (IMRS, shown as a dashed blue line) followed the same pattern but was less regular from year-to-year. Hence, the average simulated capital income tax rate was zero. Perhaps the IMRS was persistently below MPK early in the Depression, and persistently higher later, so it might be said that the model predicts heavy capital taxation early, and capital subsidies later in the Depression. I explore the intertemporal margin further in Mulligan (2001b), which discusses application of the dual method to stochastic models, and in Mulligan (2001c) which connects Figure 7 with previous studies of intertemporal substitution.

V. Conclusions

Rather than using a numerical model of private behavior and measured government policies to simulate quantities and prices for comparison with measured quantities and prices (the "primal" approach for empirically evaluating competitive equilibrium models), I suggest that it is easier and economically more informative to use a numerical model of private behavior and measured quantities to simulate prices and government policies for comparison with measured prices and policies (the "dual" approach for empirically evaluating competitive equilibrium models). The dual approach, which does nothing more than compute wedges between measured marginal rates of substitution and transformation, is easily applicable to competitive equilibrium models with many (even infinite) time periods, many heterogeneous agents, many sectors, and many government policy instruments.

V.A. Understanding the American Economy 1929-50

I illustrate the method by evaluating the performance of the neoclassical growth model for the 1929-50 American economy. Assuming that the neoclassical growth model is not far off with Cobb Douglas utility and production functions, the data show how the marginal product of labor (*MPL*) diverged from the marginal value of time (*MRS*) by 30-40 percent from 1929-33, and that wedge persisted until WWII, when the wedge between *MPL* and *MRS* was more than 20 percentage points smaller than it was in the early 1930's. This is particularly puzzling in light of federal tax policy during the period, where marginal labor income tax rates were practically zero during the 1930's and at their height during the war.

In principle, the primal and dual methods should lead to the same empirical evaluation. I believe that the computational burden of the primal approach has led to some mistakes, however. For example, Cole and Ohanian (1999, p. 3) and Prescott (1999, p. 26) have suggested that there exists a time sequence of productivity shocks that could reconcile the neoclassical growth model with much of the important economic behaviors during the Great Depression, Crucini and Kahn (1994) have suggested that trade shocks can explain those same behaviors, and Braun and McGratten (1993) have argued that government investment (a productivity shock from the point of view of my calculations for the neoclassical growth model) can explain the amount of work during World War II. But my application of the Dual Method shows how each of these studies ignored the very large discrepancies between model and data in terms of the gap between the consumption-leisure ratio and the after-tax average product of labor. A key question for existing and potential theories of the Great Depression – whether the theory is one of productivity shocks, monetary shocks, factor income taxes, tariffs, labor market regulation, or monopoly unionism – is whether there should be a wedge between *MRS* and *MPL*. If so, when should the wedge first appear? Can the wedge be as large as 30 or 40 percent?

V.B. Lessons for Other Applications

My evaluation of the neoclassical growth model with data from 1929-50 highlights several advantages of the dual approach to the empirical evaluation of dynamic competitive equilibrium models. The first is the simplicity of the calculation: I simulate marginal labor income tax rates merely by multiplying the ratio of labor to leisure by the consumption-output ratio. One byproduct of this simplicity is that there is no need for the approximations often found in the literature – such as discretizing the state space, restricting policy functions to be in the space of low order polynomials, assuming that capital begins the time period in its "steady state," or even assuming that all observations are steady-state observations. Second, the model can be partially evaluated even with very limited data. Although the neoclassical growth model is an infinite period model, with capital and time-specific production possibilities, I made labor distortion calculations based only on 22 years of the labor-leisure and consumption-output ratios – without data on capital or "productivity." Informative calculations could have been made with even fewer years of data. Third, even when infinite data is available, the dual approach partitions complicated models into simpler pieces.[25] In the case of the neoclassical growth model, I show how, assuming Cobb-Douglas utility and production, the Great Depression was a time of departure between the marginal product of labor and the marginal value of time – my argument does not depend on whether there are multiple capital goods, or whether there are adjustment costs to investment, or on other assumptions of the neoclassical growth model. Fourth, the dual method is applicable to noncompetitive equilibrium models, as long as the model has some first order conditions of the form *MRS* = after-tax *MRT*, or that the noncompetitive behavior is

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[25]It does so without approximation, other than those embodied in any numerical model of private behavior.

readily interpreted as a wedge between *MRS* and *MRT*. My discussion of monopoly unionism and neoclassical growth model's labor-leisure first order condition illustrates one such application.

V.C. The Relation Between the Dual Method and GMM

There have been a variety of empirical studies, such as Hansen and Singleton (1983), of "consumption asset pricing models" or the intertemporal first order condition of a representative consumer, and it is worth noting some relationships of my dual method with the methods typically used in the CAPM literature. One feature common to both methods is that one first order condition is studied at a time. Practitioners of both are asking "Is the model's first order condition consistent with observation?" However, my approach differs from that of a typical CAPM study in terms of the reasons why, observation-by-observation, there are gaps between marginal rate of substitution and marginal rate of transformation. CAPM studies use uncertainty to explain the gaps and, together with ergodic and rational expectation assumptions, obtain some predictions for the nature of those gaps. In contrast, I explain the gaps with variation over time in marginal tax rates, an explanation which has predictions for observed indicators of tax and regulatory policy. Not surprisingly, these two approaches can be combined, and Mulligan (2001b) is one attempt at a combination.

The two approaches are also a bit different when it comes to using observations to infer model parameters. CCAPM models have the feature that *MRS* does not equal *MRT* observation-by-observation, but in such a way that the deviations between *MRS* and *MRT* average zero, and are uncorrelated with other variables in the model. These restrictions on the data necessitate nonlinear econometric methods, of which the Generalized Method of Moments is the one most commonly found in the literature. In contrast, my approach has *MRS* equal to after-tax *MRT* for *every observation*. To put it another way, log(*MRS*) is always equal to log(*MRT*) plus the log of the tax factor – a linear relationship which lends itself to study with linear econometric methods. For example, proportional measurement errors in the *MRS*, *MRT*, and/or tax factor in many cases imply that model parameters can be estimated with regression methods. Indeed, matters may be even simpler than this, because the equation of log(*MRS*) to log(*MRT*) plus the log of the tax factor is not terribly different from the partial equilibrium implication "Log demand price equals log supply price plus log tax factor" – so that the large toolbox of econometric techniques that have been developed over the decades for studying linear supply and demand systems are likely to be of use when using the dual method to estimate general equilibrium models with distortions. I offer these observations for contemplation by the interested reader, and leave a detailed analysis and application of dual estimation methods for future research.[26]

V.D. Forecasting vs. Empirical Evaluation

My dual approach uses measured quantities as input and is therefore not directly applicable to forecasting the quantity and price effects of a hypothetical government policy – an important exercise in policy research. However, the dual approach is best for empirically evaluating a competitive equilibrium model, and hence a prerequisite for predicting the effects of a hypothetical government policy – at least for those who only forecast with models shown to have some empirical success.

V.E. Tastes Shifts vs. Market Distortions

One of my departures from Parkin (1988), Hall (1997), and Ingram et al (1997) is to interpret the time series of gaps between a measured marginal rate of substitution and the corresponding marginal rate of transformation as a time series of market distortions rather than as time series of taste or technological parameters. Without direct measures of the causes of market distortions, or of the taste and technology parameters, it is difficult to justify one interpretation over the other.[27] With this in mind, the reader might conclude that the contribution of my paper is only to analyze market distortions in order to show that, although derived from models with time-varying preference parameters, the Hall and Parkin calculations

[26]See Mulligan (2001c) for one application of the dual method together with linear supply-demand econometrics.

[27]Of course, we can use price data to distinguish a taste shock from a technology shock (the former drives a wedge between price and *measured MRS*, while the latter drives a wedge between price and *measured* marginal product). But the price data do not allow us to distinguish a distortion from these two other shocks.

can be generalized in order to empirically evaluate *stable* preference equilibrium models from a public finance perspective. But we do have measures of taxation and other indicators of distortionary public policy, and I believe that many of the "taste shifts" indicated by Hall are in fact changes in distortions. For example, Hall (1997, Figure 8) shows a "shift in preferences" away from leisure during the 1980's which, according to the Dual Method, I interpret as a reduction in labor-leisure distortions (see Mulligan 2000 for details of the 1980's calculations). Is it an accident that these shifts occur at the same time that the federal government cut marginal tax rates, and private sector unions were on a dramatic decline?

I do not want to suggest that all gaps between the consumption-leisure ratio and the average product of labor, or any other measured gap between marginal rates of substitution and transformation, are distortions rather than preference shifts. A suggestion one way or another is unnecessary from a methodological point of view, because in principle we could follow the Dual Method shown in Figure 1, except in the last step compare simulated gaps to empirical measures of tastes, such as the age or ethnic composition of the population. Indeed, I have argued elsewhere (Mulligan 1998) that taste changes – perhaps like those modeled by Hall and Parkin for the postwar years – may have been very important during World War II. Figure 2 shows how, during the war, people acted as if either: (a) there was a labor-leisure taste shift, or (b) there were markedly smaller distortions than in the 1930's or 1950's. I see no indication that taxes, unionism, and other potential causes of distortions were reduced enough in the 1940's, and at least casual empiricism suggests that patriotism and other taste changes were significant.

Figure 2 also shows how people acted in the 1930's as if there were either huge labor market distortions or a big shift in tastes. I and many others are tempted to search for and model causes of labor market distortions, but at this point I must admit that the available measures do not indicate that taxes, unionism, or regulation were unusual enough during the 1930's to serve as significant and plausible explanations of 1930's behavior. Whether the Depression labor market will ultimately be explained by tastes or market distortions is still an open question, and I hope the Dual Method will expedite the formulation of an answer.

VI. Appendix: Data and Calculations for the period 1929-50

Appendix Table 1: 1929-50 data and Labor Distortion Calculations

year	civilian labor	military labor	consumption/ output[†]	simulated marginal labor income tax rate[‡]
t	$L(t)$		$c(t)/Y(t)$	$\tau^*(t) = 1 - \frac{L(t)}{(1-L(t))} \frac{0.7}{0.615} \frac{c(t)}{Y(t)}$
1929	0.560	0.002	0.747	-0.094
1930	0.515	0.002	0.769	0.062
1931	0.468	0.002	0.792	0.199
1932	0.412	0.002	0.828	0.335
1933	0.407	0.002	0.814	0.358
1934	0.402	0.002	0.780	0.397
1935	0.419	0.002	0.763	0.369
1936	0.452	0.002	0.743	0.296
1937	0.466	0.002	0.727	0.272
1938	0.428	0.002	0.746	0.359
1939	0.442	0.002	0.734	0.337
1940	0.452	0.004	0.707	0.332
1941	0.482	0.013	0.648	0.297
1942	0.510	0.034	0.566	0.280
1943	0.525	0.082	0.526	0.201
1944	0.512	0.102	0.524	0.209
1945	0.479	0.091	0.578	0.269
1946	0.472	0.023	0.668	0.290
1947	0.475	0.011	0.674	0.293
1948	0.473	0.010	0.659	0.315
1949	0.450	0.010	0.668	0.351
1950	0.454	0.011	0.655	0.352
Source:	Kendrick	Kendrick	BEA NIPA	

[†] 1939-48 for the civilian sector only

[‡] Leisure time = 1 - civilian and military labor. Labor input = civilian & military labor, except during wartime (1939-48)

VII. References

Aiyagari, S. Rao, Lawrence J. Christiano, Martin Eichenbaum. "The Output, Employment, and Interest Rate Effects of Government Consumption." *Journal of Monetary Economics*. 30(1), October 1992: 73-86.

Barro, Robert J. "Long-Term Contracting, Sticky Prices, and Monetary Policy." *Journal of Monetary Economics*. 3(3), July 1977: 305-16.

Barro, Robert J. and Herschel I. Grossman. "A General Disequilibrium Model of Income and Employment." *American Economic Review*. 61(1), March 1971: 82-93.

Barro, Robert J. and Robert G. King. "Time-separable Preferences and Intertemporal-Substitution Models of Business Cycles." *Quarterly Journal of Economics*. 99(4), November 1984: 817-39.

Baxter, Marianne and Robert G. King. "Fiscal Policy in General Equilibrium." *American Economic Review*. 83(3), June 1993: 315-34.

Becker, Gary S. *Accounting for Tastes.* Cambridge, MA: Harvard University Press, 1996.

Braun, R. Anton and Ellen R. McGrattan. "The Macroeconomics of War and Peace." in Olivier Jean Blanchard and Stanley Fischer, eds. *NBER Macroeconomics Annual 1993*. Cambridge and London: MIT Press, 1993: 197-247.

Burnside, Craig, Martin Eichenbaum, and Jonas D.M. Fisher. "Assessing the Effects of Fiscal Shocks." NBER Working paper #7459, January 2000.

Cole, Harold L. and Lee E. Ohanian. "The Great Depression in the United States from a Neoclassical Perspective." *Federal Reserve Bank of Minneapolis Quarterly Review*. 23(1), Winter 1999: 2-24.

Crucini, Mario J. "Sources of Variation in Real Tariff Rates: The United States, 1900-1940." *American Economic Review*. 84(3), June 1994: 732-43.

Crucini, Mario J. and James Kahn. "Tariffs and Aggregate Economic Activity: Lessons from the Great Depression." *Journal of Monetary Economics*. 38(3), December 1996: 427-67.

Dunlop, John T. *Wage Determination under Trade Unionism*. New York: Macmillan, 1944.

Friedman, Milton. *A Theory of the Consumption Function*. Princeton, NJ: Princeton University Press, 1957.

Hall, Robert E. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics*. 15(1), Part 2 January 1997: S223-50.

Hansen, Lars Peter and Thomas J. Sargent. *Recursive Linear Models of Dynamic Economies*. Manuscript, University of Chicago, January 1991.

Hansen, Lars Peter and Kenneth J. Singleton. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy*. 91(2), April 1983: 249-65.

Houthakker, Hendrik S. "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis." *Review of Economic Studies*. 23, 1955: 27-31.

Ingram, Beth F., Narayana R. Kocherlakota and N. E. Savin. "Using Theory for Measurement: An Analysis of the Cyclical Behavior of Home Production." *Journal of Monetary Economics*. 40(3), December 1997: 435-456.

Kendrick, John W. *Productivity Trends in the United States*. Princeton: Princeton University Press, 1961.

King, Robert G., Charles I. Plosser and Sergio T. Rebelo. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics*. 21(2), March 1988: 195-232.

Kydland, Finn and Edward C. Prescott. "Time to Build and Aggregate Fluctuations." *Econometrica*. 50(6), November 1982: 1345-70.

Leontief, Wassily W. "The Pure Theory of the Guaranteed Annual Wage Contract." *Journal of Political Economy*. 54(1), February 1946: 76-79.

Lucas, Robert E., Jr. "Review of Milton Friedman and Anna J. Schwartz's 'A Monetary History of the United States, 1867-1960.'" *Journal of Monetary Economics*. 34(1), August 1994: 5-16.

Lucas, Robert E., Jr. and Leonard A. Rapping. "Real Wages, Employment, and Inflation." *Journal of Political Economy*. 77(5), September 1969.

Lucas, Robert E., Jr. and Leonard A. Rapping. "Unemployment in the Great Depression: Is There a Full Explanation?" *Journal of Political Economy*. 80(1), January 1972: 186-91.

MaCurdy, Thomas E. and John H. Pencavel. "Testing Between Competing Models of Wage and Employment Determination in Unionized Markets." *Journal of Political Economy*. 94, Part 2, 1986: s3-s39.

Meltzer, Alan. "Monetary and Other Explanations of the Start of the Great Depression." *Journal of Monetary Economics*. 2(4), November 1976: 455-71.

Mulligan, Casey B. "Pecuniary Incentives to Work in the United States during World War II." *Journal of Political Economy*. 106(5), October 1998: 1033-77.

Mulligan, Casey B. "A Century of Labor-Leisure Distortions." Manuscript, University of Chicago, September 2000.

Mulligan, Casey B. "Aggregate Implications of Indivisible Labor." *Advances in Macroeconomics*. 1(1), 2001a: Article 4.

Mulligan, Casey B. "A Dual Method of Empirically Evaluating Dynamic Competitive Equilibrium Models with Distortionary Taxes: Evaluating the Stochastic Margins." Manuscript, University of Chicago, January 2001b.

Mulligan, Casey B. "Capital, Interest, and Aggregate Intertemporal Substitution during the 20th Century." Joint Economic Theory and Econometrics Invited Lecture at the European Meeting of the Econometric Society, August 2001c.

Ohanian, Lee E. "The Macroeconomic Effects of War Finance in the United States: World War II and the Korean War." *American Economic Review*. 87(1), March 1997: 23-40.

Parkin, Michael. "A Method for Determining Whether Parameters in Aggregative Models are Structural." in Karl Brunner and Bennett T. McCallum, eds. *Money, Cycles, and Exchange Rates: Essays in Honor of Allan H. Meltzer*. Carnegie-Rochester Conference Series on Public Policy, 29, Autumn 1988: 215-52.

Perri, Fabrizio and Vincenzo Quadrani. "Understanding the Great Depression: What Can We Learn from the Italian Experience." Working paper, New York University, October 2000.

Prescott, Edward C. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review*. 10, Fall 1986: 9-22.

Prescott, Edward C. "Some Observations on the Great Depression." *Federal Reserve Bank of Minneapolis Quarterly Review*. 23(1), Winter 1999: 25-31.

Ramey, Valerie A., and Matthew D. Shapiro. "Costly Capital Reallocation and the Effects of Government Spending." *Carnegie-Rochester Conference Series on Public Policy*. 48, June 1998: 145-94.

Rees, Albert. *The Economics of Trade Unions*. Chicago: University of Chicago Press, 1989.

Taussig, Frank W. *The Tariff History of the United States*. New York: G.P. Putnam's and Sons, 1931.

United States, Bureau of Economic Analysis. *Survey of Current Business*. November 1999. [data also available at www.bea.doc.gov/bea/dn1.htm]

United States, Bureau of Labor Statistics. *Union Membership*. Table 3. Union affiliation of employed wage and salary workers by occupation and industry. January 2002.

United States Department of the Treasury, Internal Revenue Service. *Statistics of Income*. various issues.

Watson, Mark W. "Measures of Fit for Calibrated Models." *Journal of Political Economy*. 101(6), December 1993: 1011-41.