NBER WORKING PAPER SERIES

COMPETITION AND INTERVENTION IN SOVEREIGN DEBT MARKETS

Bernhard Paasche
Stanley E. Zin

Working Paper 8679
http://www.nber.org/papers/w8679

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2001

We would like to thank Ravi Bansal, V.V. Chari, Rick Green, Burton Hollifield, Kenneth Kletzer, Uday Rajan, Chris Telmer, and participants of Duke's First Annual Research Workshop on the "Macroeconomics of Global Market Imperfections" in Venice, participants of the Fourth UTDT Workshop in International Economics and Finance in Buenos Aires, and seminar participants at Carnegie Mellon University, for helpful comments. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

Competition and Intervention in Sovereign Debt Markets
Bernhard Paasche and Stanley E. Zin
NBER Working Paper No. 8679
December 2001
JEL No. F33, E44, G15, G12

ABSTRACT

We investigate markets for defaultable sovereign debt in which even though there are many identical lenders and symmetric information (including no hidden actions), perfect competition does not obtain. When a private lender allows a sovereign country to increase its level of indebtedness, that lender implicitly imposes a default externality on others who have lent to that sovereign. That is, in the case where the borrower would be able to pay back the first loan in the absence of a second loan, the borrower may have a strong incentive to take both loans and default on both loans. When a lender has no control over the actions of other lenders, they must anticipate this behavior and devise a lending strategy that is consistent with the strategies not only of the sovereign borrower, but also of other lenders. We develop a model of this strategic lending behavior in the presence of default, and show that even though there are many competing lenders, the perfectly competitive outcome does not necessarily obtain. Moreover, the equilibrium can result in monopoly-like outcomes in prices and quantities. We also study the consequences of intervention in these markets by a seemingly benevolent international financial institution, and find that these interventions, though well-intentioned, can in some cases be welfare reducing for sovereign countries and welfare improving for private lenders.

Bernhard Paasche
GSIA, Carnegie Mellon University
Pittsburgh, PA 15213-3890

Stanley E. Zin
GSIA, Carnegie Mellon University
Pittsburgh, PA 15213-3890
and NBER
zin@bobbyorr.gsia.cmu.edu

1 Introduction

Discussions about the behavior of international credit markets, and markets for sovereign debt in particular, are often predicated on one of two very different assumptions about the structure of the markets. If we assume that markets for sovereign debt are competitive and, if left to their own devices, borrowers and lenders would organize their activities in an efficient manner (as they presumably do in many other markets), any intervention by an institution like the IMF is distortionary and necessarily welfare reducing. On the other hand, if lenders are assumed to be unable to closely monitor the behavior of sovereign borrowers and are unable to enforce international debt contracts, these markets are subject to severe moral hazard problems which can lead to severe market failures. In this case the IMF's role is to enhance private loan markets to reduce fears of market failure and improve the welfare of both borrowers and lenders. Clearly then, the proper role of intervention by an institution like the IMF is strongly influenced by assumptions about the structure of the market for defaultable sovereign debt.

There is a large literature dealing with a variety of aspects of sovereign debt markets (see, for example, Eaton and Fernandez (1995), Eichengreen (1991), Eaton and Gersovitz (1981), Bulow and Rogoff (1989), and Gibson and Sundareson (2001), among many others). The foundation for virtually all of this work, however, is competitive equilibria (with or without frictions). In this paper, we adopt a different perspective on these markets. We maintain the assumptions that there are many competing lenders and there is symmetric information — assumptions that are typically associated with first-best competitive outcomes. However, we show that the competitive outcome need not obtain. The ability of a sovereign to retain some of their income in the case of default induces a "default externality" that lenders must account for when offering defaultable bond contracts to a sovereign borrower. In other words, lenders must behave strategically even though they have no competitive or informational advantage.

Our model is one of strategic interactions between lenders and the sovereign borrower, rather than more abstract bond-market participants. Hence, it is more likely to apply in situations in which a country doesn't have access to liquid international bond markets, but rather must rely on loans from international banks. This is common for small countries. For example, throughout the 1980's and 1990's, Ghana issued no long-term bonds to private creditors, but borrowed significant amounts from commercial banks and other private lenders. On the other hand, of the $87.5 billion of Mexico's long-term public debt outstanding in 1999, only $9.7 billion came from commercial banks and private lenders. Our model may also apply when a sovereign's likelihood of default is very high, hence, it is excluded from further borrowing in international bond markets, *e.g.*, Argentina in 2001. Overall, banks play a significant role in sovereign debt markets. Banks held 97 percent of all emerging markets debt at the end of the 1980s. This number fell to roughly two-thirds by the mid-1990s as the volume of debt grew enormously (see Eichengreen and Mody (1998) and Bernstein and Penicook (1998)), however, it is still the dominant form of credit when one excludes the largest of emerging-market economies *e.g.*, Mexico and Brazil (see Chanda *et al* (2001)). Moreover, two necessary assumptions of our model, first that loan contracts are non-exclusive, and second, that sovereign borrowers can shelter some of their income from creditors in the event of default, seem to apply in many situations involving private institutions lending to sovereign countries.

Empirical work on sovereign loan markets suggest that standard competitive models of sovereign loan markets do not provide a sufficient explanation for much of what we observe. Edwards (1984) shows that spreads on loans to sovereign LDCs in the late 1970's exhibit a lot of variation that cannot be explained with standard economic factors. (Edwards (1986) demonstrates that this applies for sovereign bonds as well as loans.) Others have found that interest-rate spreads on sovereign debt seem puzzling from the perspective of standard competitive models. Eichengreen and Mody (1998) have documented that in the early 1990s, much of the variability in interest rate spreads on emerging market debt is accounted for by a country-specific fixed effect, and that the explanatory power of economic fundamentals is much less than standard theory would suggest. In addition, a recent IMF report (Chadha *et al.* (2001)) finds that emerging-market interest-rates do not have a clear relationship with default episodes. They also find that emerging market debt outperforms other comparably risky investments including corporate debt and even the S&P500 index. Generally, therefore, it is difficult to explain either the high level or the co-movements with standard economic fundamentals for interest rates observed in sovereign debt contracts. Understanding these data, therefore, would seem to require a different approach.

Our analysis of sovereign debt markets applies recent work by Parlour and Rajan (2000) on unsecured personal credit markets. We extend their model to include random productivity shocks which allows default to be an observable outcome, as it is in the international debt markets in which the IMF involves itself. In addition, we introduce an IMF-like institution into the analysis in a number of different ways. We are thereby able to obtain both positive predictions about the role of IMF activity in the industrial organization of these debt markets, and also normative predictions about the welfare consequences of the IMF. Both the Parlour and Rajan (2000) analysis of unsecured personal credit and our analysis of sovereign debt are related to earlier applications of asymmetric information and moral hazard to similar problems. Pauly (1974) has explored equilibria in insurance markets when existing contracts are unobservable, which is analogous to our non-exclusivity requirement. Kletzer (1984) explores the existence of competitive Nash equilibria in sovereign debt markets in a multi-period model with non-observable contracts, however, he does not analyze the non-competitive equilibria implied by the non-exclusive contracts in our model. Bizer and DeMarzo (1992) show that a default externality can arise in a sequential borrowing problem with moral hazard and seniority. They explore the consequences of this for interest rates and indebtedness and compare these outcomes to a model with full commitment.

We show that a range of symmetric, subgame-perfect-Nash equilibria are supportable in the markets we model. Of particular interest is the case where in spite of the large number of lenders with symmetric information, the equilibrium is equivalent (in terms of the equilibrium interest rate and aggregate loan size), to that of a single monopoly lender. We show that the existence of this equilibrium depends critically on the fraction of income/output that the sovereign retains upon default. When this fraction is relatively high, lenders responding strategically to the default externality will be able to extract monopoly rents from borrowers. Counter-intuitively, lenders would be made *ex ante* worse off (and conversely, borrowers made better off) if all of the sovereign's income was lost in the case of default. In other words, removing the default externality would induce competition to the market and eliminate the monopoly rents. We extend these basic results to contracts with state-contingent interest rates and find that comparable symmetric equilibria with monopoly-like behavior can be sustained. We also derived conditions under which these equilibria still exist when debt contracts can be renegotiated *ex* *post*.

Output volatility plays a key role in our model, both in providing an environment conducive to non-competitive outcomes, and also in weakening the link between the terms of loan contracts and competitive fundamentals like default probabilities. As output becomes more volatile, monopoly-like equilibria become more likely, and interest rates can be much larger than would be justified by default probabilities alone.

From this new perspective, the potential roles for intervention by an institution like the IMF that have been traditionally advocated, have very different consequences than in both standard competitive models and market-failure models. We introduce an IMF-like institution into this setting and explore the consequences of different assumptions about its incentives. We show that if the IMF competes with private lenders for loans but, rather than maximizing profits, they lend to maximize the utility of the sovereign, the IMF becomes the "lender of first resort" and drives out all private lenders from a non-competitive equilibrium. If, however, in a monopoly-like equilibrium, the institution can only lend a fraction of what private lenders would provide, the country's total borrowing and total borrowing costs are unaffected, and the institutions surplus is extracted by private lenders. On the other hand, if the IMF-like institution operates as a provider of insurance to sovereign borrowers and lends only in states where the sovereign defaults (*i.e.*, a "lender of last resort"), a country that is already facing monopoly-like pricing will receive an increase in welfare. However, this insurance can have different consequences for different countries. We show that this insurance mechanism can push a country that would otherwise face competitive pricing into a situation where positive-profit equilibria can be sustained.

Section 2 of the paper introduces the theoretical framework for the model. Section 3 derives a variety of results on existence of equilibria and the competitive and welfare implications of various equilibria. Section 4 introduces a benchmark model in which analytical results are much clearer, and Section 5 introduces an IMF-like institution into this benchmark model and studies the positive and normative implications of different incentives for this institution. Section 6 returns to the general specification and explores some numerical examples of the model. In section 7, we show how the monopoly-like equilibria might be affected by changes in the economic environment by allowing state-contingent debt contracts and renegotiation. Section 8 concludes the paper.

2 The market for defaultable sovereign loans

In this section we describe the basic framework for our analysis of competition in sovereign loan markets. There are potentially many identical lenders and one sovereign borrower. All information is symmetric and complete. In other words, there is no adverse selection or moral hazard in this model. There are two time periods. In the initial period, the lenders and the sovereign borrower agree upon a set of debt contracts such that the debt market clears. In the second period, the stochastic output is realized, debts are repaid (or not), and consumption takes place. All agents, both borrowers and lenders, are risk neutral and have rational expectations. Therefore, they each seek to maximize (at least initially) the expected value of second-period consumption. The sovereign country, however, has the ability to abandon its commitment to pay back its loans in the second period, and will do so whenever its *ex post* consumption is larger in default than under the terms of the loan contracts. Lenders can anticipate these strategic defaults and will adjust contract offerings accordingly. This strategic behavior generates complex interactions between borrowers and lenders, and the potential for non-competitive market outcomes in what would appear to be an otherwise simple and transparent setting.

Before exploring these possibilities in detail, we first specify the economic environment. Lenders, denoted $i = 1, 2, \dots, M$, compete by offering a one-period debt contract to the sovereign country. Each contract specifies the loan's principal, d_i, and interest rate, r_i. The sovereign uses these loans to produce output using a production function given by

$$f(x, \tilde{\theta}) = \tilde{\theta} A x^{\beta} \ , \tag{1}$$

where $0 < \beta < 1$, $A > 0$, and $\tilde{\theta}$ is a stochastic productivity shock. The *ex post* value of the shock is public knowledge. For simplicity, we assume that there are two possible realizations for this shock, $\{\theta^L, \theta^H\}$, such that $\theta^H > \theta^L \geq 0$. (We will often assume that $\theta^H = 1$, and $\theta^L = 0$, to simplify the analysis even further.) The probability of the *high* state is denoted as p.

An important assumption of this model is the sovereign's potential to both default *and* maintain positive consumption while in default. To capture this, we assume that in the case of default, the sovereign can shield a fraction γ of output from their creditors, where $0 \leq \gamma \leq 1$. That is, the most that a sovereign can be punished for defaulting is a fraction $(1-\gamma)$ of second-period output. Therefore, when $\gamma = 1$, the sovereign pays nothing back to lenders and consumes all of their output, and when $\gamma = 0$, the sovereign consumes nothing in the case of default. Given this assumption, the sovereign will default on *all* of its creditors whenever *ex post* consumption in default is greater than consumption under the terms of the contracts:

$$\gamma f(\sum_{i=1}^{M} d_i, \theta) > f(\sum_{i=1}^{M} d_i, \theta) - \sum_{i=1}^{M}(1 + r_i)d_i \ , \tag{2}$$

where we assume that the only input to production comes from sovereign debt, and that $\sum_{i=1}^{M}(1 + r_i)d_i$ is the principal and interest owed on all loans. It is obvious from the expression in equation (2) that a lender acting in anticipation of a potential default, must be concerned not only with that lender's own loans, but also the loans of all other lenders.

A second key assumption of this model is that a lender cannot force the sovereign to commit to accepting only one loan contract and, similarly, a lender cannot preclude others from lending to the same sovereign. Therefore, each additional loan contract increases the likelihood of default on all loan contracts. Each loan contract increases the likelihood of default. In other words, if the total amount offered by all lenders is large enough, the sovereign country can choose to accept *all* loans and default on *all* loans, independently of the interest rates charged on these loans. Therefore, by offering a loan contract, each lender creates a negative externality for every other lender, since each loan increases the incentive of the sovereign to default on all loans.

As with most externality problems, there can be multiple equilibria in this economy. We will restrict our attention to symmetric pure-strategy subgame-perfect Nash equilibria in which every lender offers a positive amount and the country accepts all debt contracts. Although these assumptions may rule out some interesting behavior, this approach is relatively tractable analytically, yet yields a sufficiently rich set of equilibria to generate interesting non-competitive outcomes which we can use to frame both positive and normative issues in sovereign debt markets. We make these equilibrium assumptions more specific in the next section.

3 Loan market equilibria

A lender chooses a contract $\{d_i, r_i\}$ to maximize the expected value of profits:

$$q[(1+r_i)d_i] + (1-q)\bar{d} - d_i \ , \tag{3}$$

where q is the probability that they will be repaid by the sovereign borrower, $\bar{d}$ is the amount that the lender receives in the case of default, and the risk-free interest rate is assumed to be zero. (This is without loss of generality, since we could define r_i to be the excess return on sovereign lending.) We also assume for simplicity that the lenders receive nothing from the sovereign in the case of default. Hence, $\bar{d} = 0$, will be maintained from now on. The fraction $(1-\gamma)$ of output is simply a *deadweight loss* to the economy.

Given that there are two states of the world, lenders face two alternatives. They can lend an amount such that the sovereign country decides to default in the low state but pays back in the high state, or they can offer contracts such that the sovereign is better off paying back in both states of the world. Given the default externality described above, the probability of repayment, q, depends on the lender's own actions, d_i and r_i, the set of contracts offered by other lenders, $\{d_{-i}, r_{-i}\}$ (in obvious notation), the stochastic production process with parameters θ, p, A, and β, and also on the default-consumption parameter, γ:

$$q(d_i, r_i, \{d_{-i}, r_{-i}\}, \theta, A, \beta, p, \gamma) \in \{0, p, 1\} \ , \tag{4}$$

The probability of repayment will be equal to 1 if the sovereign country repays the loans in both states of the world, it will equal p if the country pays back only in the high state, and it will equal 0 if the country never repays the loans. Note that if q is equal to 0 the lender's expected profits are maximized at 0, , *i.e.*, the optimal contract is to set d_i equal to 0.

Given the set of contracts offered by all lenders, the sovereign country decides which contracts to accept in the first period, and whether to repay or default in the second period.

Definition: *A symmetric subgame-perfect Nash equilibrium is a set of contracts, $\{d, r\}$, such that*

1. *for each lender i*

$$\{d, r\} = \arg\max_{d_i, r_i} \; q(d_i, r_i, \{d, r\}; \theta, p, A, \beta, \gamma)(1 + r_i)d_i - d_i \; ,$$

2. *the country accepts all contracts offered, and*

3. *$q \in \{0, p, 1\}$ is consistent with the sovereign country's decision to default.*

Finding an equilibrium, therefore, requires further characterization of the repayment probability, q, which is contingent on the country's default decision. The country will evaluate default contingency by contingency, and form an expected value accordingly. For an arbitrary interest rate r and loan d, the country's expected utility equals:

$$\begin{aligned} & p \quad \max\left\{f(d, \theta^H) - (1 + r)d, \; \gamma f(d, \theta^H)\right\} \\ + \quad & (1 - p) \max\left\{f(d, \theta^L) - (1 + r)d, \; \gamma f(d, \theta^L)\right\} \; . \end{aligned} \tag{5}$$

Note that when θ^L is greater than zero, the country might be willing to repay even in the low state.

If the country does not default in either state of the world, debt contracts are completely riskless. If, in addition, markets are competitive, then the equilibrium interest rate must be equal to the risk-free rate, which is assumed to be equal to zero. Expected utility, therefore, is given by

$$f(d, E[\tilde{\theta}]) - d \; .$$

The country's optimal loan size in this no-default case is denoted as $\hat{d}_{ND}$, and is the solution to

$$\arg\max_{d} f(d, E[\tilde{\theta}]) - d \; .$$

The largest loan consistent with repayment in both states, denoted as $\bar{d}^L$, is not necessarily equal to $\hat{d}_{ND}$. Rather, it is determined by the lowest state alone (since if default is not optimal in the low state, it cannot be optimal in the high state), and results in indifference between default and repayment in that state:

$$\gamma f(\bar{d}^L, \theta^L) = f(\bar{d}^L, \theta^L) - (1+r)\bar{d}^L \ .$$

Definition: *A riskless competitive debt contract is defined as* $\{d_{ND}^C, r_{ND}^C\}$*, where* $d_{ND}^C = \min\{\bar{d}^L, \hat{d}_{ND}\}$*, and* $r_{ND}^C = 0$*.*

Note that for γ high enough and a concave production function,[2] the loan amount that maximizes the country's surplus, $\hat{d}^{ND}$, will be larger than the amount that induces repayment in both states of the world, $\bar{d}^L$. For our particular production function we get

$$\bar{d}^L = \left[(1-\gamma)\theta^L A\right]^{\frac{1}{1-\beta}}$$

and

$$\hat{d}_{ND} = \left[E[\tilde{\theta}]A\beta\right]^{\frac{1}{1-\beta}} \ .$$

This leads to our first theorem which states that a competitive equilibrium will not exist for values of γ sufficiently high. We first find the value of γ for which $\hat{d}_{ND} = \bar{d}^L$:

$$\gamma_{ND} = 1 - \beta\left[1 + p\left(\frac{\theta^H}{\theta^L} - 1\right)\right] \ .$$

Theorem 1 *There are no riskless competitive equilibria when* $\gamma > \gamma_{ND}$*.*

(The proof of this theorem is a straightforward extension of the proof of Theorem 3, below.)

The intuition for this result is straightforward. If $\hat{d}_{ND}$ is greater than $\bar{d}^L$, the marginal lender can always offer a contract with a lower principal amount and a higher interest rate instead of the competitive contract. Since accepting this new contract will increase the country's utility, the competitive contract cannot be sustained as an equilibrium.

Equilibria with default

[2] For the competitive contract to make sense in this risky environment, we require that $\beta > \frac{\theta^L}{\theta^H}$, which we assume throughout.

If the country chooses to default in the low state, but repays in the high state, the debt contracts become risky. Since p is the probability of default in this case, the marginal cost of funds to any lender is $1/p$, which would also be the interest rate in a competitive environment. The country's expected utility in this case is given by

$$pf(d, \theta^H) + (1-p)\gamma f(d, \theta^L) - d \ . \tag{6}$$

Analagous to the no-default case, we can define a contract size, $\bar{d}^H$, such that the country is just indifferent to defaulting or repaying in the high state:

$$(1-\gamma) f(\bar{d}^H, \theta^H) = \frac{\bar{d}^H}{p} \ .$$

The loan amount that maximizes the country's surplus, $\hat{d}^D$, is given by

$$\arg\max_d pf(d, \theta^H) + (1-p)\gamma f(d, \theta^L) - d \ .$$

Definition: *A risky competitive debt contract is defined as* $\{d_D^C, r_D^C\}$*, where* $d_D^C = \min\{\bar{d}^H, \hat{d}_D\}$*, and* $r_D^C = 1/p$.

As before, for γ high enough the loan amount that maximizes the country's surplus will be larger than the amount that induces repayment in the high state. For our particular production function we get

$$\bar{d}^H = \left[p(1-\gamma)\theta^H A\right]^{\frac{1}{1-\beta}}$$

and

$$\hat{d}_D = \left[(p\theta^H + \gamma(1-p)\theta^L)A\beta\right]^{\frac{1}{1-\beta}} \ .$$

Hence we have a second theorem that again rules out competitive equilibria when γ is sufficiently high. We first find the value of γ for which $\hat{d}_D = \bar{d}^H$:

$$\gamma_D = \frac{p\theta^H}{p\theta^H + (1-p)\theta^L\beta}(1-\beta) \ .$$

Theorem 2 *There are no risky competitive equilibria when* $\gamma > \gamma_D$.

The intuition behind this result is directly analagous to Theorem 1. The main implication of these two theorems is that in a setting where γ is relatively large, we cannot rely on the predictions of competitive equilibria even when the number of symmetric lenders is extremely large. There are many non-competitive equilibria that one might consider at this point. We will focus our attention primarily on a *monopoly* equilibrium. That is, we explore whether an equilibrium in this sovereign debt market with many lenders can exhibit the same interest rate and loan size as a market dominated by a single monopolist. To develop intuition for this result, we first simplify the stochastic process which, in turn, simplifies the analytical expressions that define the equilibria without affecting the basic qualitative results of the model.

4 A benchmark case

In this section, we simplify the model by assuming $\theta^H = 1, \theta^L = 0$. This guarantees that there is no equilibrium were the country does not default in the low state, since they have no output in that case, $f(d, \theta^L = 0) = 0$. We also write $f(d, \theta^H = 1)$ as simply $f(d)$.

Competitive equilibria

We will begin by defining the competitive contract for this case. Given that investors are risk neutral and that the country will only be able to repay in the high state, in a competitive market the interest rate will be equal to $1/p$. If the country had the ability to choose how much debt to borrow at the competitive market rate it would solve:

$$\max_d p\left(f\left(d\right) - \frac{d}{p}\right),$$

Therefore, the optimal (from the country's perspective) debt level, $\hat{d}$, is such that $f'(\hat{d}) = 1/p$, which is given by

$$\hat{d} = [pA\beta]^{\frac{1}{1-\beta}} \ .$$

However, a commitment by the country to repay lenders in the high state will only be credible if:

$$\gamma f(\sum_{i=1}^{M} d_i) \leq f(\sum_{i=1}^{M} d_i) - \sum_{i=1}^{M}(1 + r_i)d_i \ .$$

Define,$\bar{d}$, as the debt level such that the country is indifferent between repaying and defaulting in the high state, *i.e.*, $\gamma f\left(\bar{d}\right) = f\left(\bar{d}\right) - \frac{\bar{d}}{p}$, which is given by

$$\bar{d} = [pA(1-\gamma)]^{\frac{1}{1-\beta}} \ .$$

Therefore, the value of γ for which $\hat{d} = \bar{d}$ is given by

$$\gamma_{NC} = 1 - \beta \ .$$

Theorem 3 *There are no zero profit equilibria when $\gamma > \gamma_{NC}$.*

Proof: The proof follows the proof of Proposition 3 of Parlour and Rajan (2000). By contradiction, suppose that $\gamma > 1-\beta$ and that $r_i = \frac{1}{p}$. It must be the case that $\sum_i d_i \leq \bar{d}$, otherwise default would be optimal. Furthermore, if this is the case, the country's expected surplus, $p\left(f(d) - \frac{d}{p}\right)$, is maximized at $\bar{d}$.

Consider a lender, i, that offers a positive amount. If there is no other lender offering a positive amount, then the monopolistic contract is optimal. Therefore, there must exist at least another lender, j, with $d_j > 0$. Let $d_{-i} = \sum_{j\neq i} d_j$. It must be the case that $\bar{d} - d_{-i} \geq d_i > 0$. Furthermore, since the surplus is increasing for $d < \bar{d}$, we have that $\gamma f(d_{-i}) < f(d_{-i}) - \frac{d_{-i}}{p}$. Hence, there exists an ε such that $\gamma f(d_{-i}+\varepsilon) < f(d_{-i}+\varepsilon) - \frac{d_{-i}+\varepsilon}{p}$ and $f(d_{-i}) - \frac{d_{-i}}{p} < f(d_{-i}+\varepsilon) - \frac{d_{-i}+\varepsilon}{p}$. Thus, there is a δ such that

$$f(d_{-i}+\varepsilon) - \frac{d_{-i}+\varepsilon}{p} - \delta\varepsilon > \max\left(\gamma f(d_{-i}+\varepsilon), f(d_{-i}) - \frac{d_{-i}}{p}\right).$$

Set $d_i = \varepsilon$ and $r_i = \frac{1}{p} + \delta - 1$. Clearly, ε and δ can be chosen such that lender i makes a profit.

An immediate implication of this theorem is that countries with close to linear production functions, *i.e.*, β close to 1, are likely to face noncompetitive pricing even when γ is close to 0.

If the competitive outcome obtains, then lenders' expected profits are equal to zero and the country's expected utility is given by

$$(pA)^{\frac{1}{1-\beta}}\left((1-\gamma)^{\frac{\beta}{1-\beta}}-(1-\gamma)^{\frac{1}{1-\beta}}\right) .$$

Monopoly equilibria

We can say more about the non-competitive equilibria in this debt market. We now show that for γ sufficiently large, a symmetric monopoly contract can be sustained as an equilibrium. Moreover, this result will hold independently of the number of lenders. Increasing the number of lenders, therefore, does not result in a competitive outcome.

If this market was dominated by a single lender, the monopolist would solve:

$$\begin{array}{ll} \max_{r,d} & p(1+r)d-d \\ s.t. & (1-\gamma)f\left(d\right)\geq(1+r)d\ , \end{array}$$

and the optimal contract would satisfy

$$\begin{aligned} d^M &= [p(1-\gamma)A\beta]^{\frac{1}{1-\beta}}. \\ r^M &= \frac{1}{p\beta}-1\ . \end{aligned}$$

Note that the size of the loan decreases with γ. In fact, if $\gamma=1$, *i.e.*, the country can consume all of its output in the case of default, then $d^M=0$. The interest rate is independent of the value of γ, but it does depend on the value of β. That is, the greater the curvature in the production function, *i.e.*, the smaller the value of β, the larger the monopoly interest rate. This interest rate compounds a compensation for risk, $\frac{1}{p}$, and a monopoly mark-up $\frac{1}{\beta}$.

Lenders who adopt this monopoly-price strategy run the risk of being undercut by a lender who can earn higher expected profits by making a larger loan at a lower interest rate. We will examine these strategies further in section 6 and the Appendix, however, it is worth spending a bit of time developing some intuition for the unprofitability of defection from this symmetric monopoly equilibrium. Imagine first a market dominated by a single monopolist. Another lender considers the possibility of finding a profitable entry strategy. The lender might think of offering an additional loan at a lower rate. Note that the borrower is still credit constrained at the monopoly equilibrium, hence, the country will readily accept this new loan offer. However, note in addition, that the monopolist has extracted all of the possible rents in the good productivity state, so that repayment would be possible only if the new loan was offered below cost. Otherwise, the acceptance of the new loan would push the country to default even in the good state. In either case, this new loan is unprofitable. The lender's only hope is to try to take over the monopolist's position. That is, to offer a large loan at a lower rate that the country will accept instead of the monopolist's contract. However, for γ sufficiently large, and given the non-exclusivity of loan contracts, the country will be better off taking both the monopolist's loan and the entrant's large loan and defaulting on both. Once again, the potential entrant is left without a profitable strategy.

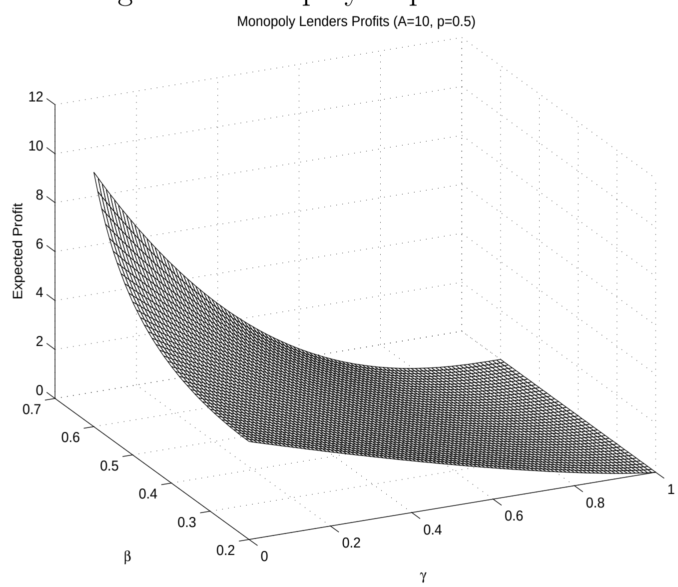
In the case with many lenders, the intuition is very similar. For a lender to offer a bigger loan, without inducing the country to reduce their borrowing from one of the other lenders, simply induces default for the same reason as with the single monopolist. On the other hand, offering a larger loan that is attractive to the borrower and that will displace one of the other lenders is confronted with the same default problem as before. For γ sufficiently large, the country will not substitute the new larger loan for an old one, but will prefer to take all loans offered and default on all loans. Lenders will see that there is no scope for a profitable departure from the monopoly-price equilibrium, and the equilibrium will be sustainable. Note in both cases, however, that the sustainability of this equilibrium depends critically on the value of γ. For small values of γ, rather than defaulting, the country could be better off simply replacing expensive loans with cheaper ones which would result in standard Bertand competition.

In this monopoly default equilibrium, lenders' expected profits are given by

$$(\frac{1}{\beta} - 1)\,[p(1-\gamma)A\beta]^{\frac{1}{1-\beta}} \quad . \tag{7}$$

Figure 1 depicts these monopoly profits for various values of β and γ. Profits are increasing in β and decreasing in γ (recall profits are zero when $\gamma = 1$). Moreover, profits can be extremely large when the country cannot keep much

Figure 1: Monopoly Expected Profits



Note: The figure depicts the expected profits of a monopolist lender.

of their output in the case of default, *i.e.*, γ close to 0, and the production function is close to linear, *i.e.*, β close to 1. Nevertheless, even though profits are potentially large, the monopoly outcome cannot be sustained as an equilibrium for small values of γ.

The main conclusion of this section is that in an environment where sovereign countries are able shelter a large fraction of their income when they default, *i.e.*, γ high, interest rates are higher than the cost of capital, and a country will be able to borrow less than it could in a competitive market.

5 Policies of a benevolent lending institution

In this section we analyze how an international organization such as the IMF could influence equilibrium outcomes in the market for sovereign debt. For simplicity we will continue with the assumption in the benchmark model of the last section: $\theta^L = 0$ and $\theta^H = 1$. We analyze two possible cases. In the first case we consider an IMF-like lender who enters this sovereign debt market and makes loans at competitive interest rates. We ask whether this activity could alter the structure of the market equilibrium and, in particular, whether it would induce competition or promote monopoly-like outcomes. In the second case, we assume that the IMF-like institution acts like an insurance provider for the sovereign country. That is, the sovereign country buys insurance from the institution against the prospect of the low state and potential default. Again we ask whether this activity deters or promotes competition in these markets.

A Lender of "First Resort"

Suppose the international institution is both able and willing to offer a loan as large as the sovereign country needs at the competitive interest rate. In a fairly obvious way, any private lenders not offering competitive loans would be driven out of the market, and all profits would be driven to zero. Therefore, if the IMF-like institution seeks to maximize a sovereign country's welfare by becoming an alternative source of competitively-priced loans, then there is an equilibrium where this institution becomes the "lender of first resort", *i.e.*, the only lender.

Perhaps more realistically we assume that the institution lacks the resources to do this on a large scale. In particular, let the maximum amount that the institution can lend be equal to

$$d_I = \phi d^M,$$

where ϕ is a number between zero and one, and d^M denotes the amount that a monopolistic lender would choose, i.e.,

$$d^M = [p(1-\gamma)A\beta]^{\frac{1}{1-\beta}} .$$

Note that provided that the institution does not lend enough to drive out private lenders, the theorems above will still hold. That is, for γ sufficiently large, there will be no zero-profit equilibria in the market for private loans to sovereign countries. This activity may lead to both increases in the country's utility and decreases in the profits of private lenders, however, it is not sufficient to bring about a competitive equilibrium.

To illustrate the effects that this limited lending at competitive rates can have on a monopolistic equilibrium, we will analyze how the symmetric monopolistic outcome changes with this institutional intervention.

In this case lenders solve:

$$\max_{d_i, r_i} p(1+r_i)d_i - d_i$$

subject to

$$f(\sum_{i=1}^{M} d_i + d_I) - \frac{d_I}{p} - \sum_{i=1}^{M}(1+r_i)d_i \geq \gamma f(\sum_{i=1}^{M} d_i + d_I) .$$

The monopolistic equilibrium in this case will result in private loans in the amount

$$\sum_{i=1}^{M} d_i = (1-\phi)\, d^M ,$$

and private interest rates of

$$r_i = \frac{1-\phi\beta}{(1-\phi)\, p\beta} - 1 .$$

Note that this implies that the cost to the country of servicing the total debt (both private and institutional) is equal to

$$\frac{d_I}{p} + \sum_{i=1}^{M}(1 + r_i)d_i = \frac{d^M}{p\beta} ,$$

which is independent of the value of ϕ. The cost to the country is the same when the IMF-like institution does a lot of low-cost lending or very little. All benefits of these low-interest loans are extracted by the strategic private lenders who implement the monopoly strategy.

An implication of this equilibrium is that as $\phi \to 1$ the interest rate charged by the international lenders goes to infinity and the amount they lend goes to zero. Therefore, if the symmetric monopolistic outcome obtains, limited institutional lending at competitive rates does not change either the total amount lent to the country or the cost to the country of servicing their debt.

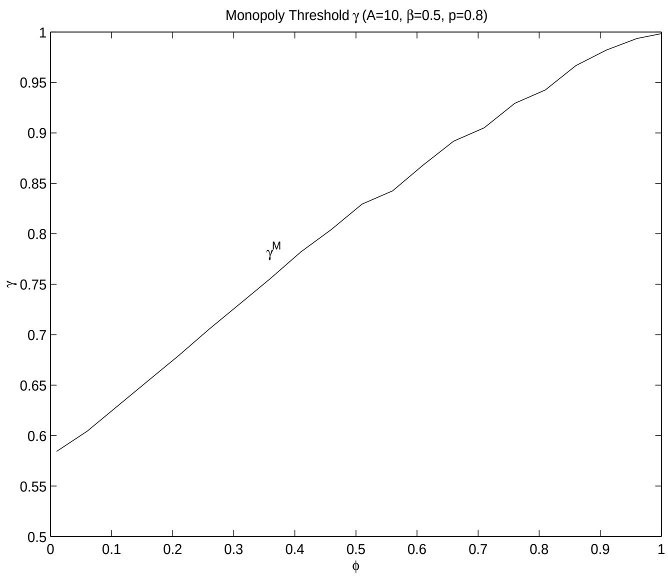
Figure 3 depicts the increase in the value of γ necessary to sustain the monopoly outcome as the size of the IMF loan increases. What this shows is that even when the size of the loan offered by the institution is relatively small, since this loan reduces the scope for monopoly-like equilibria (*i.e.*, such equilibria can only be sustained for larger values of γ), it could ultimately be welfare enhancing for the country.

An Insurance Provider

Suppose the sovereign country can buy insurance against the low state from the IMF-like institution. In particular, assume that if the low state occurs then the institution pays the country's debts to private lenders. In exchange for this insurance, the sovereign country pays a premium, π, when the high state is realized. The insurance is contracted before the private credit market opens. In addition, we assume that the institution pays off the country's creditors in the low state only if the contracts are written such that the country is able to repay both the premium and its debts in the high state. Since there is symmetric information in this economy, ruling out moral hazard in the high state seems reasonable.

The payment of the insurance premium has a consequence on the resulting equilibrium. Recall that whenever $\hat{d} > \bar{d}$ (that is, whenever the amount that

Figure 2: Monopoly Threshold Value of γ *vs.* IMF Loan Size



Note: The figure depicts the threshold value for γ needed to sustain the monopoly outcome as the size of the loan made by the IMF (measured by ϕ) changes.

the country would like to borrow is greater than the amount that they can credibly commit to repay), competitive equilibria cannot exist. In addition, recall that with complete insurance, the competitive interest rate will be driven down to the risk-free interest rate, assumed to be zero. The effect of this lower rate is to increase both the country's demand, *i.e.*, $\hat{d}$ increases, and its ability to credibly repay, *i.e.*, $\bar{d} increases$. This effect on $\bar{d}$, however, is mitigated since the country has to pay the insurance premium in the good state, which lowers the amount of resources available to repay its debts. The net effect is to increase the distance between $\hat{d}$ and $\bar{d}$, which will rule out competitive contracts for even small values of γ. In other words, the benefits of this type of intervention must be weighed against its negative consequences for competition.

To see the potential benefits of intervention in the case of a symmetric monopoly equilibrium, (*i.e.*, γ sufficiently large), consider the objective of a lender operating in the presence of this institutional insurance mechanism. In this case, the typical lender solves:

$$\max_{d_i, r_i} \quad r_i d_i \tag{8}$$

subject to

$$f(\sum_{i=1}^{M} d_i) - \pi - \sum_{i=1}^{M}(1 + r_i)d_i \geq \gamma f(\sum_{i=1}^{M} d_i) \ . \tag{9}$$

The constraint guarantees that in the high state the country pays the insurance premium and the interest rates on the loans. The loan size and interest rate in the monopoly case are given by

$$\sum_{i=1}^{M} d_i = d_M^I = [(1-\gamma)A\beta]^{\frac{1}{1-\beta}} \tag{10}$$

and

$$r_M^I = \frac{1}{\beta} - \frac{\pi}{d_M^I} - 1 \ . \tag{11}$$

When default is insured by the institution, we can obtain lenders' expected profits for the monopoly default equilibrium:

$$\left(\frac{1}{\beta} - 1\right) [(1-\gamma)A\beta]^{\frac{1}{1-\beta}} - \pi \ .$$

Recall that profits in the comparable equilibrium without insurance are given in equation (7). Note that profits in the insured case differ from the uninsured case in two ways. Profits rise as a result of the increase in the total amount lent to the country (this amount is higher since it no longer depends on p). But profits also fall by the amount of the premium paid by the country to the institution. Therefore, even though the structure of the insurance mechanism would lead one to believe that the country is paying for the insurance, in equilibrium, the insurance premium is actually funded through lower profits.

The expected profits of private lenders depend in a fairly obvious way on the insurance pricing. Assuming γ is high, if the insurance is cheap bankers will be better off since the insurance mechanism will allow them to increase the loan amounts and they will still obtain positive profits. On the other hand, if the premium is large, the rents that bankers can extract from the country will drop and they will be worse off.

As a benchmark, consider the situation in which the institution charges the sovereign country an actuarially fair price for this insurance. The premium in this case would be given by

$$\pi = \frac{1-p}{p}(1+r)d_M^I \ ,$$

which implies an equilibrium interest rate of

$$r_M^I = \frac{p}{\beta} - 1 \ .$$

Note that this interest rate will be lower than $\frac{1}{\beta} - 1$, the interest rate in monopoly equilibrium without uncertainty.

Profits in this benchmark case are equal to

$$\left(\frac{p}{\beta} - 1\right) d_M^I \ ,$$

which are smaller than the profits lenders earn without the insurance mechanism. In other words, to make lenders better off under this insurance scheme, the institution would have to subsidize insurance, and charge a lower premium than would be actuarily fair.

The country's expected utility in this monopoly equilibrium with insurance is strictly larger than the comparable uninsured case. Since the monopoly equilibrium will result in equality in the constraint given in equation (9), we know that expected utility in the insured case is given by $p\gamma f(d_M^I)$, compared to the uninsured case, $p\gamma f(d_M)$. Since the country is able to borrow more when their loans are insured, *i.e.*, $d_M^I > d_M$, utility for the country is strictly larger with insurance.

This welfare improvement is somewhat offset by the affect that insurance may have on competition. Since the lending institution is extracting resources from the country in the good state whenever $\pi > 0$, the amount that the country can credibly commit to repay in the good state will be lower. Therefore, the default externality can rule out competitive equilibria for even smaller values of γ. Overall, this insurance mechanism can have very different effects for different countries since the presence of the insurance decreases the value of γ above which the competitive equilibrium cannot be sustained. That is, the insurance mechanism can push a country that faces competitive pricing into an equilibrium in which foreign investors gain positive profits, which can have a substantial negative impact on the country's welfare. On the other hand, if a country is already facing monopoly pricing, then the insurance will increase its welfare. In general, countries with large values of γ will tend to benefit from the insurance, whereas countries with low values of γ may be better off without it.

6 The general model revisited

The simple benchmark model allowed us to study in a relatively clear way, the implications of an intervention in this sovereign debt market by an international financial institution. Given the restrictive assumption made on the stochastic productivity shock, however, the benchmark model does not allow us to study the consequences of different stochastic processes for the equilibrium in this market. We now return to the general specification to explore these cases in more detail, with special emphasis on the monopolistic equilibria that might obtain.

Recall that there are two types of monopolistic equilibria. There is a monopoly-like equilibrium with default in the low state and another where the country never defaults. Given that we do in fact observe defaults in sovereign debt markets, and since the monopoly-like equilibrium without default can only be sustained when the productivity shock has a very small variance, we will restrict our analysis to the equilibria where the country defaults in the low state. Given the need to explore all forms of profitable defections from the symmetric monopoly equilibrium, and the complexity of these possible strategies for the more general stochastic productivity shock, we are unable to find simple closed-form expressions for the restrictions on the structure of the economy that admit monopoly equilibria. We summarize these conditions in the Appendix and show that they correspond to a relatively simple constrained optimization problem. For specific examples of the theoretical economy. These problems are easy to solve numerically. We now analyze these equilibria for a variety of numerical examples.

To study the effects of volatility on equilibrium outcomes, we parameterize the states using the mean, variance and probability:

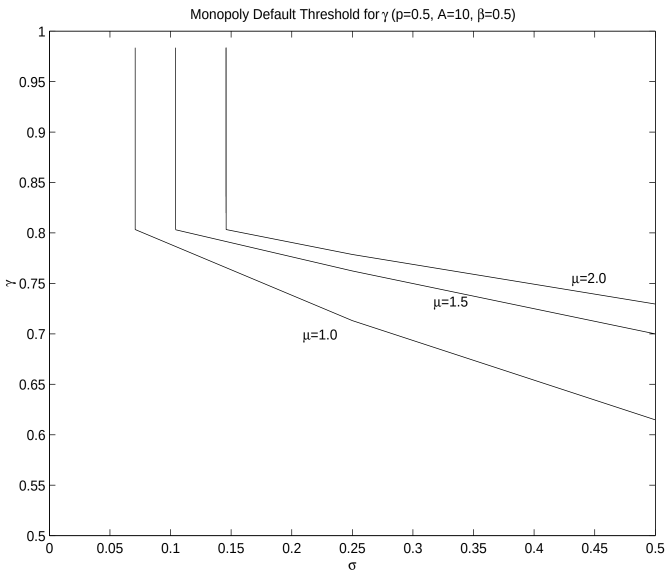
$$\begin{aligned} \theta^H &= \mu + \left(\frac{1-p}{p}\right)^{\frac{1}{2}} \sigma \, , \\ \theta^L &= \mu - \left(\frac{p}{1-p}\right)^{\frac{1}{2}} \sigma \, . \end{aligned}$$

Therefore, $E[\tilde{\theta}] = \mu$ and $Var(\tilde{\theta}) = \sigma^2$. The following graphs show the (γ, σ) pairs for which the symmetric monopolistic outcome obtains in equilibrium. Figures 4, 5, and 6 depict the threshold values of γ that will support the monopoly equilibrium for large values of M.

A number of patterns in the relationship between the productivity-shock parameters and sustainability of a monopoly-price equilibrium emerge from these numerical results. The value of γ that would support a monopoly-price equilibrium decreases as:

1. the mean decreases
2. volatility increases, and

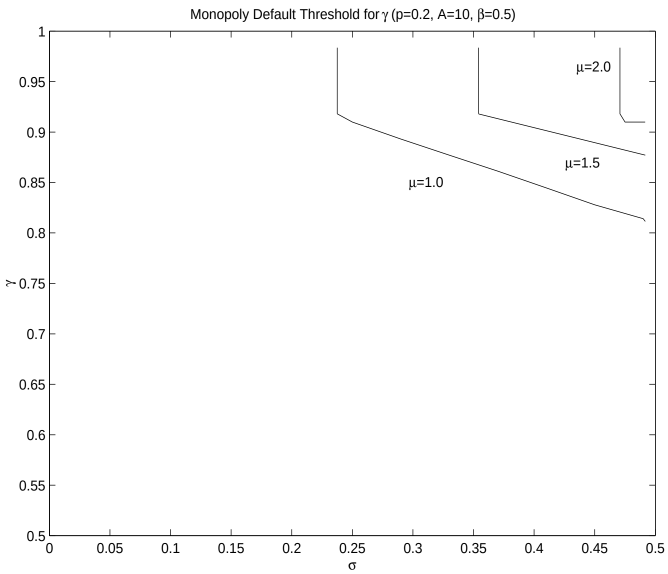
Figure 3: Threshold Values for γ_D (p=0.5)



3. the probability of the high state increases.

The lenders' strategy in case of the monopoly-price equilibrium involves a tradeoff between the cost of default when the low state occurs and the profits earned when the high state occurs. Therefore, to interpret these patterns, it is helpful to think in terms of the debt capacity of the country in the low state and the interest rate charged by lenders who adopt the monopoly-price strategy. In general, when σ is relatively small, the debt burden that the country can repay in both states of the world will also be relatively large, since θ^L is close to θ^H. In a monopoly-price equilibrium a large fraction of this debt burden will be interest rather than principal since the interest rate charged by the typical lender following the monopoly strategy will be very high. To see that monopoly-price equilibria can only be sustained for countries with high values of γ when σ is small (or θ^L is relatively large), consider the incentives of the marginal lender to offer a contract such that the country will repay in the low state. If all other lenders are following the monopoly-price strategy, the marginal lender may have an incentive to

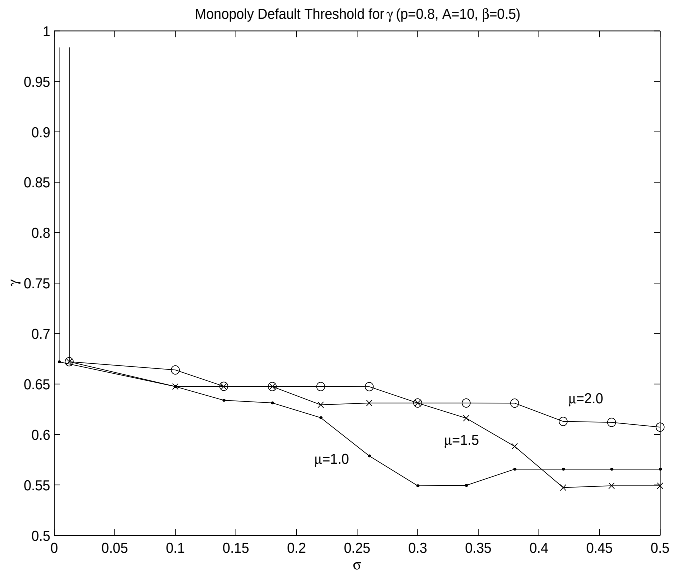
Figure 4: Threshold Values for γ_D (p=0.2)



deviate and offer a larger contract at a lower interest rate. (For example, if this interest rate is sufficiently low and θ^L is sufficiently high that the debt burden is small enough that it can be repaid even in the low state, then the marginal lender may increase profits by deviating from the monopoly strategy.) If this deviation is attempted for a country with a large value of γ, *i.e.*, a country for which there is little penalty associated with default, the country would be willing to accept *but not repay* a contract at a lower interest rate. In that case, a lender would have no incentive to deviate from the monopoly strategy. Therefore, when σ is relatively small, the monopoly-price strategy will only be an equilibrium for countries with relatively high values of γ.

Note that the other patterns displayed in these figures, *i.e.*, mean increase, and high-state probability decrease, result in an increase in θ^L. Their effects on the sustainability of a monopoly equilibrium are all very similar to the decrease in σ described in detail above. As θ^L increases, the monopoly equilibrium can only be sustained for countries with increasingly higher values of γ.

Figure 5: Threshold Values for γ_D (p=0.8)



Another way to see these effects is to look at the condition that allows profitable deviation. Profitable deviation will be possible only if:

$$(1-\gamma)\theta^L f(d^M + d_i) - \frac{1}{p\beta} d^M > d_i \ ,$$

where the d^M is the total amount of loans offered under the monopoly-price contract, and d_i is the size of the defector's loan.[3] Note that the three parameter changes discussed above all act to decrease the left-hand side of this inequality, hence, making deviation impossible.

Instead of analyzing the symmetric monopolistic outcome, one could ask whether a single monopolist could corner the market in such a way that it is optimal for other lenders not to offer any loans. One can show that a single monopolist can indeed corner the market, provided γ and σ are high enough, as shown in Figures 4, 5, and 6. The intuition is basically the same.

[3]This restriction assumes that the country accepts all contracts. The appendix deals with the more general case where the country is allowed to accept only a fraction of the contracts offered.

For σ high enough, offering contracts such that the country repays in both states of the world is not possible because there is just not enough output produced in the low state. Furthermore, for γ high enough, the country will be able to consume a large fraction of the total output in default, and hence any contract offered with an interest rate higher than $\frac{1}{p}$ will induce default. Given that the monopoly contract is being offered, there is no possibility to enter the market and generate a positive profit. Therefore, the other lenders will optimally offer the contract $d = 0$, $r = 0$.[4]

7 Extensions

In this section we examine three important extensions to the basic model presented above. First we consider the case of state-contingent debt contracts. We allow lenders to index the interest rate to the realization of the productivity shock. Given the assumption of full and symmetric information, this seems like a natural contract for lenders to offer. In this context, we also extend the model to allow a more general multiple-state (*i.e.*, more than 2-states) stochastic process. Finally, we allow for renegotiation in the case of default. Working in the context of a relatively static model, our specification of renegotiation is necessarily simple, however, it yields results that are suggestive for the likely consequence of allowing renegotiation in a more dynamic setting.

7.1 Contingent Contracts and Multiple States

In the previous sections we have assumed that lenders offer standard debt contracts. Here we show that if state-contingent contracts are offered instead, the monopoly-like equilibrium can still be sustained for sufficiently large values of γ. Furthermore, we show that in this case the monopoly profits increase relative to non-contingent contracting, since the monopoly rents can be extracted from the country in *all* states by charging different rates in

[4] Numerical examples have also shown that this monopoly approach may also provide a simpler numerical algorithm for finding the symmetric Nash equilibrium, relative to the method detailed in section 6.

the different states of the world. Generalization to a multiple-state stochastic process is straightforward.

Consider the case where there is a single monopolist lender.[5] This lender offers a contract $\{d^M, r^M(\theta_i)\}_{i=1,...K}$. The state θ_i occurs with probability $p(\theta_i)$. Denote the mean of the productivity shock as $\bar{\theta} = \sum_i p(\theta_i)\theta_i$. The risk-neutral monopolist will choose a state-contingent contract to maximize expected profits:

$$\max_{d_i, r(\theta_i)} \sum_i p(\theta_i)(1 + r(\theta_i))d - d \tag{12}$$

subject to the state-by-state repayment constraints

$$(1 - \gamma)f\left(d, \theta_i\right) \geq (1 + r(\theta_i))d \ , \tag{13}$$

for all i.

The monopolist, therefore, will choose a contract such that:

$$\begin{aligned} 1 + r^M(\theta_i) &= \frac{\theta_i}{\bar{\theta}\beta} \ , \text{ and} \\ d^M &= \left[\bar{\theta}(1 - \gamma)A\beta\right]^{\frac{1}{1-\beta}} \ . \end{aligned}$$

The following theorem establishes that this monopoly equilibrium can be sustained provided γ is sufficiently large.

Theorem 4 *There exists a value of γ for which the monopoly contract can be sustained as an equilibrium.*

Proof: Suppose there is a lender offering the monopoly contract and the rest are offering the zero contract. If lender j deviates and offers another contract, say $\{d_j, r_j(\theta_i)\}$, for $i = 1, ...K$, then the country has two options. If it accepts both contracts, then the maximum that the deviating lender can obtain is

$$\pi_j = \sum_i p(\theta_i)\left[(1 - \gamma)\theta_i f\left(d^M + d_j\right) - \left(1 + r^M(\theta_i)\right)d^M\right] - d_j$$

[5] The extension to the symmetric monopoly equilibrium with many lenders is straightforward.

which by concavity of f is less than

$$\sum_i p(\theta_i)\left[(1-\gamma)\theta_i\left(f\left(d^M\right)+f'\left(d^M\right)d_j\right)-\left(1+r^M(\theta_i)\right)d^M\right]-d_j\ .$$

Recall that $\left(1+r^M(\theta_i)\right)d^M=(1-\gamma)\theta_i f'\left(d^M\right)$ *and that* $(1-\gamma)\bar{\theta}f\left(d^M\right)=1$. *Hence we have that*

$$\pi_j<(1-\gamma)\bar{\theta}f'\left(d^M\right)d_j-d_j=0\ .$$

Next, we will show that for γ sufficiently large the country will always prefer to take both contracts. Consider the country's utility. If the country accepts both contracts then its utility equals

$$\sum_i p(\theta_i)\max\left(\gamma\theta_i f\left(d^M+d_j\right),\theta_i f\left(d^M+d_j\right)-r^M d^M-r_j d_j\right)>\gamma\bar{\theta}f\left(d^M+d_j\right)\ .$$

For lender j to make a profit we need $\sum_i p(\theta_i)\left(1+r_j(\theta_i)\right)>1$. *If the country accepts only the contract offered by lender j then its utility equals*

$$\sum_i p(\theta_i)\left[\theta_i f\left(d_j\right)-(1+r_j(\theta_i))d_j\right]<\bar{\theta}f\left(d_j\right)-d_j\ .$$

Now, for γ large and $d_j>0$, we have that

$$\gamma\bar{\theta}f\left(d^M+d_j\right)>\bar{\theta}f\left(d_j\right)-d_j\ .$$

Therefore, when γ is large in the sense of the last equation, there is no profitable deviation possible.

Note that since

$$\left(\frac{1}{\beta}-1\right)\left[p\theta^H(1-\gamma)A\beta\right]^{\frac{1}{1-\beta}}<\left(\frac{1}{\beta}-1\right)\left[\bar{\theta}(1-\gamma)A\beta\right]^{\frac{1}{1-\beta}}\ ,$$

profits are larger with state-contingent contracts than with a non-contingent debt contract. In the monopoly-price equilibrium, the state-contingent contracts allow lenders to extract the maximum amount of profit in each state.

7.2 Renegotiation

Up until now, we have assumed that if the country defaults, lenders loose the entire principal of their loan and the country consumes only a fraction of their output. Therefore, default represents a deadweight loss to the economy. In this section, we would ask how sensitive the results are to this assumption. In particular, we could like to know how an *ex post* renegotiation in which the lender can potentially recapture some of their capital would affect the equilibrium. Clearly this will depend on how renegotiation is modeled. For example, assume that lending is dominated by a single monopolist and another lender, say lender j, offers a contract that pushes the country into default. If lender j is able to extract a large fraction of the output through a renegotiation then it might indeed be profitable for him to enter the market even though it induces the country to default. The logic of this example will also translate to the symmetric monopoly problem

To further illustrate this point, consider a deterministic economy. That is, $\theta^H = \theta^L = 1$. Assume there is a monopolist lender offering a debt contract such that $(1-\gamma)f'\left(d^M\right) = 1$. We've shown that if γ is large, this is an equilibrium. Assume that lender j enters the market and offers a contract such that the country decides to default. Furthermore, assume that lender j is able to extract a fraction $\delta(d_j, d^M)$ of the deadweight loss, $(1-\gamma)f\left(d^M + d_j\right)$, during the renegotiation.[6] Lender j will enter the market if and only if

$$\delta(1-\gamma)f\left(d^M + d_j\right) - d_j > 0 \ .$$

Alternatively, lender j will enter the market provided its marginal revenue is larger than its marginal cost:

$$\frac{\partial\delta(d_j, d^M)}{\partial d_j}(1-\gamma)f\left(d^M + d_j\right) + \delta(1-\gamma)f'\left(d^M + d_j\right) \geq 1 \ .$$

The second term on the right-hand side of this inequality is less than one by concavity of f. The first term is positive, but its magnitude depends on

[6]The fraction δ could conceivably also depend on r_j. We abstract from that case to rule out a strategy in which lender j could offer a contract with $r_j = \infty$ when δ is increasing in r_j, and get all the surplus from the renegotiation.

$\frac{\partial \delta(d_j, d^M)}{\partial d_j}$. Therefore, the entry decision for lender j depends critically on the rate at which the amount recovered during the renegotiation increases with loan size. For example, if δ was a constant that was sufficiently smaller than 1, *i.e.*, there is still a significant dead-weight loss associated with default, the symmetric monopoly equilibrium would be preserved even with the prospect of a renegotiation *ex post*. Note that this logic extends quite naturally to the case of a random productivity shock.

8 Conclusion

We have shown how non-competitive outcomes can be sustained in sovereign credit markets even though there are many lenders and complete and symmetric information. By extending the model of Parlour and Rajan (2000) to include stochastic productivity shocks and equilibrium default, we derive conditions under which perfectly competitive equilibria can be ruled out. Moreover, we show that given a sufficiently large incentive for a sovereign country to default, *i.e.*, the sovereign can consume a relatively large share of their output in the case of default, monopoly-like outcomes for interest rates and loan size can be sustained as equilibria. The existence of monopoly-like equilibria in these types of markets depends critically on the fraction of income that the sovereign can retain in the case of default, and on the distribution of random productivity shocks that governs default probabilities. We, therefore, solve for threshold values of this fraction that change with the production technology and the properties of the stochastic productivity shock. Counter-intuitively, given the possibility of a non-competitive equilibrium, the sovereign country would be better off if it were unable to consume any of its income in the case of default. Lenders, on the other hand, can benefit greatly in equilibrium from a sovereign country's ability to consume in default. In addition, our model has the property that output volatility plays a key role both in providing an environment conducive to non-competitive outcomes, and also to breaking the link between the terms of loan contracts and competitive fundamentals like default probabilities. As output becomes more volatile, monopoly-like equilibria become more likely, and interest rates can be much larger than would be justified by default probabilities alone.

We also explore how a benevolent institution like the IMF can affect equilibrium outcomes in these markets. Depending on the incentives of the institution, the outcomes can be somewhat surprising. We show that if in a monopoly equilibrium the IMF offers loans at competitive interest rates, it drives out all private lenders, thereby becoming the "lender of first resort." If the IMF can only lend a fraction of the loans that private lenders would provide, the country's total borrowing and total borrowing costs are unaffected, and the institution's surplus is extracted by private lenders.

Alternatively, if we model the IMF as an insurance mechanism for sovereign borrowers that lends only in states where the sovereign defaults (*i.e.*, a "lender of last resort"), a country that is already facing monopoly-like pricing will receive an increase in welfare. However, this type of insurance mechanism can push a country that might otherwise face competitive pricing into a situation where positive-profit equilibria, or even monopoly-like equilibria, can be sustained.

Finally, we show how the monopoly-like equilibria might be affected by changes in the economic environment. In particular, we show that most of our results will hold even when lenders are allowed to offer state-contingent debt contracts in an environment with many states, and when we allow for the possibility of renegotiation.

References

Bernstein, Robert J., and John A. Penicook Jr. (1998). "Emerging Market Debt: Practical Portfolio Considerations," in **Emerging Market Capital Flows,** Richard Levich (Ed.), New York: Kluwer Academic Publishers, pp. 335-373.

Bulow, Jeremy, and Kenneth Rogoff (1989). "Sovereign Debt: Is to Forgive to Forget?" **The American Economic Review**, Vol. 79, No. 1, pp. 43-50.

Chanda, Bankim and a Staff Team (2001). "Emerging Market Financing," IMF Quarterly Report on Developments and Prospects, Feb. 13.

Bizer, David S. and Peter M. DeMarzo (1992). "Sequential Banking," **Journal of Political Economy,** Vol. 100, No. 1.

Eaton, Jonathan and Raquel Fernandez (1995). "Sovereign Debt," NBER Working Paper 5131.

Eaton, Jonathan and Mark Gersovitz (1981). "Debt with Potential Repudiation: Theoretical and Empirical Analysis," **Review of Economic Studies,** Vol. 48, pp. 289-309.

Edwards, Sebastian (1984). "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," **The American Economic Review**, Vol. 74, No. 4., pp. 726-734.

Edwards, Sebastian (1986). "The Pricing of Bank Loans and Bonds in International Financial Market," **European Economic Review**, pp. 565-589.

Eichengreen, Barry (1991). "Historical Research on International Lending and Debt," **Journal of Economic Perspectives**, Vol. 5, No. 2, pp. 149-169.

Eichengreen, Barry and Ashoka Mody (1998). "What Explains Changing Spreads on Emerging-Market Debt: Fundamentals or Market Sentiment?" NBER Working Paper 6408.

Gibson, Rajna and Suresh Sundaresan (2000). "A Model of Sovereign Borrowing and Sovereign Yield Spreads," Columbia University Working Paper.

Kletzer, Kenneth (1984). “Asymmetries of Information and LDC Borrowing with Sovereign Risk,” **The Economic Journal**, Vol. 94, No. 374, pp. 287-307.

Parlour, Christine and Uday Rajan (2000). “Competition in Loan Contracts,” *forthcoming in* **The American Economic Review**.

Pauly, Mark V. (1974). “Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection,” **Quarterly Journal of Economics**, Vol. 88, No. 1, pp. 44-62.

Appendix

In this appendix we summarize a set of necessary conditions for the existence of the symmetric monopoly equilibrium when the stochastic productivity shock has a more general distribution than that of the benchmark case of section 4.

A monopolistic lender in the general model would solve:

$$\max_{d,r} \; p(1+r)d - d$$

subject to

$$pf(d,\theta^H)) + (1-p)\gamma f(d,\theta^L) - p(1+r)d \geq \gamma f(d, E[\tilde{\theta}]) \; .$$

This constraint is identical to the one obtained when $\theta^L = 0$. Therefore, as in that case, the equilibrium interest rate is given by

$$r_M^D = \frac{1}{p\beta} - 1 \; ,$$

and the equilibrium loan size is given by

$$d_M^D = [p\theta^H(1-\gamma)A\beta]^{\frac{1}{1-\beta}} \; .$$

Relaxing the assumption that $\theta^L = 0$ complicates things substantially because there are now more deviation possibilities for the marginal lender. For example, the marginal lender could have an incentive to deviate and offer a contract such that the country is better off repaying in both states of the world. This can happen when θ^L and θ^H are very close, *i.e.*, when volatility is low. Nevertheless relaxing the assumption that $\theta^L = 0$ allows us to analyze how increases in volatility affect the equilibria. It is in general not possible to characterize the region where the symmetric monopolistic type outcome obtains analytically. Therefore, we will explore these equilibria numerically.

We now determine conditions for the symmetric monopolistic default contracts to be sustainable as an equilibrium. As in the case of the competitive equilibrium analyzed above, we will start by defining loan amounts that are of particular interest. As before, the threshold loan size for the low state, $\bar{d}^L(r_M^D)$, is defined implicitly by the equation $(1-\gamma)\theta^L f\left(\bar{d}^L(r_M^D)\right) = (1+r_M^D)\bar{d}^L(r_M^D)$. For our particular production function we get

$$\bar{d}^L(r_M^D) = \left[\frac{(1-\gamma)\theta^L A}{1+r_M^D}\right]^{\frac{1}{1-\beta}}.$$

The threshold loan size for the high state, $\bar{d}^H(r_M^D)$, is defined implicitly by the equation $(1-\gamma)\theta^H f\left(\bar{d}^H(r_M^D)\right) = (1+r_M^D)\bar{d}^H(r_M^D)$. In this case, $\bar{d}^H(r_M^D) = d_M^D$. Clearly then, $\bar{d}^H(r_M^D) > \bar{d}^L(r_M^D)$.

The loan size that maximizes the country's welfare in the high state, $\hat{d}^H(r_M^D)$, is defined as $\hat{d}^H(r_M^D) = \arg\max_d pf(d,\theta^H)) + (1-p)\gamma f(d,\theta^L) - p(1+r_M^D)d$. In our case, this is given by

$$\hat{d}^H(r_M^D) = \left[\frac{\bar{\theta}^\gamma A\beta}{p\,(1+r_M^D)}\right]^{\frac{1}{1-\beta}}$$

The loan size that maximizes the country's welfare in the low state, $\hat{d}^L(r_M^D)$, is defined as $\hat{d}^L(r_M^D) = \arg\max_d \bar{\theta} f\,(d) - (1+r_M^D)d$. In our case, this is given by

$$\hat{d}^L(r_M^D) = \left[\frac{\bar{\theta}A\beta}{(1+r_M^D)}\right]^{\frac{1}{1-\beta}}$$

Note that $\hat{d}^H > \hat{d}^L$.

For the symmetric monopolistic contracts to be an equilibrium we need to find conditions such that if all lenders offer the monopolistic contract, the country will accept all contracts. That is, the country's demand for loans at the monopolistic interest rate is at least as big as the total amount offered. This will be the case when the following conditions are satisfied:

1. $\hat{d}^H(r_M^D) > d_M^D$.
2. If $\hat{d}^L \geq d^L(r_M^D)$ then $pf\left(d_M^D,\theta^H\right) + (1-p)\gamma f\left(d_M^D,\theta^L\right) - p(1+r_M^D)d_M^D > f\left(d^L(r_M^D), E[\tilde{\theta}]\right) - (1+r_M^D)d^L(r_M^D)$.

3. If $\hat{d}^L < d^L(r_M^D)$ then $pf\left(d_M^D, \theta^H\right) + (1-p)\gamma f\left(d_M^D, \theta^L\right) - p(1+r_M^D)d_M^D > f\left(\hat{d}^L, E[\tilde{\theta}]\right) - \left(1 + r_M^D\right) \hat{d}^L$.

The first condition indicates that, for the monopolistic outcome to hold, the country has to be rationed. This is the analog to the condition in Theorem 1. The last two conditions guarantee that the country won't be better off accepting only a fraction of the contracts offered and repaying in both states of the world.

Given the contracts offered by all other lenders the marginal lender has three basic options:

1. He can offer the same contract as the others. This needs to be the case for the symmetric equilibrium to obtain.
2. The marginal lender could offer a contract such that the country is better off accepting his contract and a fraction of the contracts offered by the other lenders. There are two possibilities that we need to check in this case, since the fraction accepted plus the marginal loan could be such that the country defaults or not in the low state.
3. Finally, the lender could offer a contract such that the country decides to take only that contract and reject all the other contracts offered. As before, this marginal contract could be such that the country decides to default or not in the low state.

The usual intuition that by offering a lower rate and a larger loan the marginal lender can make a profit doesn't necessarily hold in this model, because the country may have an incentive to take all contracts and default. For the country to be willing to accept only a fraction of the contracts offered at higher rates, the loan with the lower rate has to be relatively large. But, for large values of γ, the country will tend to be better off defaulting than repaying when the loan amounts are large due to the concavity of the production function.

Given that all other lenders are offering the same monopolistic type contract, the marginal lender, i, will seek to maximize its expected profits subject to the best response of the country to the contracts offered. Recall that the country can choose which contracts to accept and whether to default or not. This implies two possible strategies for the country which the marginal lender must consider. The country could default in the low state but repay in the high state, or it could repay in both states. If the marginal lender decides to offer a contract such that the country defaults in the low state but repays in the high state, then the following two conditions need to be satisfied:

$$\begin{aligned} pf\left(d_i + \phi_H D_{-i}, \theta^H\right) &+ (1-p)\gamma f\left(d_i + \phi_H D_{-i}, \theta^L\right) - p(1+r_i)d_i \\ &- p(1+r_M^D)D_{-i}\phi_H \geq \gamma f\left(d_i + D_{-i}, E[\tilde{\theta}]\right) \ , \quad (14) \end{aligned}$$

and

$$\theta^H f\left(d_i + \phi_H D_{-i}\right) - (1+r_i)d_i - (1+r_M^D)D_{-i}\phi_H \geq \gamma\theta^H f\left(d_i + \phi_H D_{-i}\right) \ , \quad (15)$$

where ϕ_H is the fraction of the loans offered by the other lenders that the country chooses to accept.

The first constraint guarantees that the country prefers accepting a fraction ϕ_H of the other loans and repaying in the high state, to taking all loans and defaulting in both states. The second constraint guarantees repayment in the high state. It is straightforward to show that the first constraint implies the second.

The country will choose ϕ_H to maximize its expected utility. Given that the contracts offered are such that the country defaults in the low state and repays in the high state, its expected utility equals:

$$\begin{aligned} \max_{\phi_H} \Big\{ pf\left(d_i + \phi_H D_{-i}, \theta^H\right) &+ (1-p)\gamma f\left(d_i + \phi_H D_{-i}, \theta^L\right) \\ &- p(1+r_i)d_i - p(1+r_M^D)D_{-i}\phi_H \Big\} \ , \quad (16) \end{aligned}$$

subject to $0 \leq \phi_H \leq 1$.

The optimal fraction of loans, ϕ_H, will therefore satisfy:

$$\phi_H = \begin{cases} 1 \text{ if } d_i < \hat{d}^H(r_M^D) - D_{-i} \\ \frac{\hat{d}_H - d_i}{D_{-i}} \text{ if } \hat{d}^H(r_M^D) - D_{-i} \leq d_i < \hat{d}^H(r_M^D) \\ 0 \text{ if } d_i > \hat{d}^H(r_M^D) \end{cases} \quad (17)$$

Given ϕ_H, lender i will solve the following maximization problem:

$$v_{Hi} = \max_{d_i, r_i} \; p(1+r_i)d_i - d_i$$

subject to

$$\begin{aligned} pf\left(d_i + \phi_H D_{-i}, \theta^H\right) &+ (1-p)\gamma f\left(d_i + \phi_H D_{-i}, \theta^L\right) - p(1+r_i)d_i \\ &- p(1+r_M^D)D_{-i}\phi_H \geq \gamma f\left(d_i + D_{-i}, E[\tilde{\theta}]\right) . \end{aligned}$$

Therefore, the marginal lender will have an incentive to deviate if and only if:

$$v_{Hi} > \frac{1}{M}(p(1+r_M^D) - 1)D_M^D.$$

To solve for v_{Hi} we will split the problem into three different maximization problems. We will find the marginal lender's value function for $\phi_H = 1, \phi_H = \frac{\hat{d}_H - d_i}{D_{-i}}$ and $\phi_H = 0$. Denote these value functions as $v_{H1,i}, v_{H2,i}$ and $v_{H3,i}$, respectively. Note that a monopolistic equilibrium requires $v_{H1,i} > v_{H2,i}, v_{H1,i} > v_{H3,i}$.

In an analogous way, we can solve for the case where the country repays in both states, which would yield expected utility for the country denoted by v_{Li}. Solving the optimization problem analogous to equation (16), implies the optimal fraction of loans:

$$\phi_L = \begin{cases} 1 \text{ if } d_i < \hat{d}^L(r_M^D) - D_{-i} \\ \frac{\hat{d}^L(r_M^D) - d_i}{D_{-i}} \text{ if } \hat{d}^L(r_M^D) - D_{-i} \leq d_i < \hat{d}^L(r_M^D) \\ 0 \text{ if } d_i > \hat{d}^L(r_M^D) . \end{cases}$$

As before, to solve for $v_L i$ we will split the problem into three different maximizations, and find the marginal lender's value function for $\phi_L = 1, \phi_L = \frac{\hat{d}^L(r_M^D) - d_I}{D_{-i}}$ and $\phi_L = 0$. For the monopolistic outcome to be an equilibrium we need $v_{H1,i} > v_{L1,i}, v_{H1,i} > v_{L2,i}$ and $v_{H1,i} > v_{L3,i}$.

Note that although solving these optimization problems that define the equilibria is difficult analytically, it is a relatively standard set of constrained optimization problems that can be easily solved numerically.