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ABSTRACT

We study the organization of federations - or international unions - which decide together the provision of certain public goods. The benefit of centralization depends on the internalization of the spillovers, that of decentralization on the adaptability to local differences. We individuate as an optimal institutional design a form of fiscal federalism based on decentralization of expenditures and a system of subsidies and transfers between countries. Since this solution can be politically unfeasible, we study institutional compromises between a centralized federation and a decentralized one. "Flexible unions" and federal mandates in which both the state and federal levels are involved in providing public goods are typically superior to complete centralization and politically feasible. Finally, we study the effects of a qualified majority voting rule in a centralized system: we find that it can be a useful device to correct a bias toward "excessive" union level activism.

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1 Introduction

The European countries are engaged in a process of institution building that is unprecedented in their history. After the introduction of a single currency in 1999, a Conference to be convened in 2004 is expected to open the door to 12 new members and introduce a "Constitutional Charter" for the European Union. Although the word "federation" does not belong to the EU institutional lexicon[1], the European Union seems to possess key features commonly attributed to federations; at least if one refers to the classic study by Riker (1964), where a federation is defined as an arrangement in which individual states or countries retain considerable sovereignty, while at the same time important prerogatives are moved to a supranational entity.

Despite its advanced institutional development, however, the European Union still has some way to go to identify clear, systematic and agreed criteria for allocating policymaking prerogatives between national and European authorities. This is indeed a central issue surrounding the current debate on the future European Constitution. The building of European institutions originated, in the early 1950s, from the aim of maintaining security and peace in a continent destroyed by the war[2]. Subsequently, the project developed into a broader one, aiming at establishing a single internal market for goods, services, capital and

[1]The use of this term was ruled out as it proved controversial during the negotiation of the Treaty of Maascricht; see Padoa-Schioppa (2001).

[2]See for example Jean Monnet (1976). According to Riker (1964), security and defence are common motives at the origin of most if not all federations in history.

people. This was eventually completed in the early 1990s. However, in the recent decades, as documented by Alesina, Angeloni and Schuknecht (2001), European level institutions have been entrusted with an increasing range of policy responsibilities.

The question we address in this paper is how alternative institutional arrangements lead to different outcomes concerning the centralisation, or decentralisation, of policy responsibilities in a federation, or international union. For our purpose, we treat the terms "federation" and "union" as interchangeable; in fact, most of our stylised conclusions can equally be applied to the European Union and to explicitly federal structures, such as the US.

To build our argument, we need a model which implies a need for unions to be established in the first place. We believe that a main reason for which countries delegate some power to a supernational institution is the need to coordinate decisions which affect each other. The element that in our view is central to the political economy of all federations is a tension between the heterogeneity between countries and the advantage of taking certain decisions in common. As in Alesina, Angeloni and Etro (2001, AAE from now on), we define a union as a heterogeneous collection of countries that decide together on the provision of a certain common policy, that exerts positive spillovers on all of them. While we focus on heterogeneity in preferences over policy, but heterogeneity in economic fundamentals would have similar effects.[3] It is easier to think of this policy as

[3]See also Alesina and Wacziarg (1999) on this point.

the provision of a public good, like investment in infrastructures, environmental quality or defense. However some of our arguments apply also when we interpret national policies in broader terms (like fiscal, monetary or trade policy) or even as a legal or regulatory framework.

The institutional design of a federation is the fruit of a compromise between two different objectives: the internalization of externalities amongst countries and the adaptability of policies to different national preferences. The benefits from internalization of externalities increase in the degree of interdependence between countries, while the benefits from adaptability of the policy increase with heterogeneity between countries. A union in which every country chooses independently its policy maximizes the adaptability to local differences, but minimizes the internalization of the spillovers, while a union with a uniform policy for each country assures this internalization but does not take into account heterogeneity between countries.

In the present paper we investigate various alternative institutional arrangements which are intermediate between these two extremes. In fact, the need for intermediate institutions is well captured by Oates (1999) in his discussion of federalism, when he writes that: "individual local governments are presumably much closer to the people and geography of their respective jurisdictions; they possess knowledge of both local preferences and cost conditions that a central agency is unlikely to have. And, second, there are typically political pressures (or perhaps even constitutional constraints) that limit the capacity of central

governments to provide higher levels of public services in some jurisdictions than others. These constraints tend to require a certain degree of uniformity in central directives. There are thus important informational and political constraints that are likely to prevent central programs from generating an optimal pattern of local outputs".

Our first investigation concerns a form of redistributive mechanism which is inspired to the way the European Union incentivates investment in national policies with spillovers in the Community. We label this system as fiscal federalism because it leaves complete decisional autonomy to the countries on what to choose, but it induces them to make the "right" choices through a system of taxes and subsidies between countries. This system is able to induce the first best policies, but it may be difficult to implement - because of administrative or political constraints - and it may not be politically feasible.

In the rest of the paper we depart from the rigid structure of the union which was under study in AAE (2001). We define a "flexible" union one which is organized through a two step procedure in which first every country provides some public good and than the federation decides at majority voting on a common level of federal expenditure. This structure turns out to be an improvement over the rigid structure. The adoption of federal mandates in which the union fixes a minimum expenditure for all countries and these can subsequently bottom up is an other example. Finally, we study the effects of a qualified majority voting rule in a centralized system: we find that it can be a useful device to correct a

bias toward "excessive" union level activism.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 discusses three alternative institutional rules for the federation. First we study a federal organization which decentralizes the policy choices at the national level while constraining them with a system of transfers between countries and subsidies l. Then we study two mixed systems in which decisions are partially centralized and partially decentralized: the first one captures the subsidiarity principle, the second one captures the federal mandates. Section 4 focuses on qualified majority voting and its effects on decisions taken by the union. Section 5 concludes. The proofs are in the Appendix.

2 The Model

The basic environment is the same as in AAE (2001), and we describe it below. However, we use it for different purposes: in the previous paper we investigated the equilibrium size of the federation in a simple majority voting institutional setting. Here we hold the size of the federation constant and we investigate alternative and realistic institutions.

Consider a federation of N equally sized countries with the population size normalized at 1. The generic country i has the following objective function:

$$U_i = c_i + \alpha_i H(g_i + \beta \cdot \sum_{j=1, j\neq i}^{N} g_j) \tag{1}$$

where g_i is the per capita and total level of government spending in country i, c_i is private consumption which enters linearly in the utility function for simplicity and $H_g(\cdot) > 0$, $H_{gg}(\cdot) < 0$. The parameter $\alpha_i > 0$ captures how much the representative individual of country i values public expenditure relative to private consumption: we will refer to α_i as the preference of country i and, without loss of generality, we will assume that countries are ordered such that $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{N-1} \leq \alpha_N$. We will assume that these preference parameters are observable but unverifiable information at the federal level. The parameter $\beta \in [0, 1]$ captures the spillover effects from other countries' government spending on the "home" country. When $\beta = 1(0)$ we are in the Samuelson case of a pure public good at the federal (national) level (see Samuelson, 1955).

We assume that income (y) is exogenously given and equal for everyone in every country. Each country has a balanced budget, therefore $g_i = t_i \in (0, y)$ where t_i are the lump sum taxes raised in country i. Using the government budget constraint, we then have

$$U_i = y - g_i + \alpha_i H(g_i + \beta \cdot \sum_{j=1, j\neq i}^{N} g_j) \tag{2}$$

2.1 The utilitarian first best

In principle a federation could internalize the spillovers and accommodate differences in preferences by imposing differentiated policies for each member. Consider in fact the "utilitarian" case, which seems reasonable since all countries

have the same income and the same size. From now on we identify the "utilitarian first best" as simply the "first best." Thus, the optimal choice implies maximizing the following welfare function:

$$W = \sum_{i=1}^{N} \alpha_i H(g_i + \beta \sum_{j \neq i} g_j) + Ny - \sum_{i=1}^{N} g_i \tag{3}$$

The optimality condition for country i is the modified Samuelson rule:

$$\alpha_i H_g(g_i + \beta \sum_{j \neq i} g_j) + \beta \sum_{j \neq i} \alpha_j H_g(g_j + \beta \sum_{k \neq j} g_k) = 1 \tag{4}$$

which implies a system of N equations, with solution $g^*(\alpha_i)$ increasing in α_i. For instance, in the case of isoelastic utility $H(g) = \frac{g^{1-\theta}}{1-\theta}$, solving the system under the assumption of interior solutions for all countries we can obtain:

$$g^*(\alpha_i) = \frac{\alpha_i^{1/\theta}[1+\beta(N-1)]^{1/\theta}}{(1-\beta)} - \frac{\beta \sum_{i=1}^{N} \alpha_j^{1/\theta}[1+\beta(N-1)]^{\frac{1-\theta}{\theta}}}{(1-\beta)}$$

In this case $\partial g^*(\alpha_i)/\partial \beta$ is ambiguous while $\partial g^*(\alpha_i)/\partial \alpha_i > 0$ and $\partial^2 g^*(\alpha_i)/\partial \alpha_i^2 \lessgtr 0$ iff $\theta \gtreqless 1$.

The "first best policy", which implies a different choice for each preference parameter α_i, is not directly achievable because the national preference parameter α_i is not verifiable. We know that a complicated mechanism could implement the first best - since preferences are observable - but we will restrict our attention to simpler, and more realistic, institutional designs which are inspired by real world institutions. To establish a benchmark, we will start with the two simplest arrangements: in the first, there is no federation, and every country independently decides its own public expenditure (extreme decentral-

ization), in the second the federation decides the same level of expenditure for all countries (extreme centralization).

2.2 Extreme decentralization: the Nash equilibrium

Consider a federation in which each country independently chooses its own policy taking as given the policy of all the other countries. In this decentralized equilibrium, assuming interior solutions, the first order conditions for a maximum in country i imply:

$$\alpha_i H_g(g_i + \beta \sum_{j \neq i} g_j) = 1 \tag{5}$$

Let us call the solution of the system of N equations (5) as $g^n(\alpha_i)$ where n stands for Nash equilibrium. This decentralized equilibrium is characterized by complete adaptability of policies to countries' preferences, but it is inefficient because countries do not take into account the effect of their choice abroad: while the national marginal cost of their investment is unitary, the social marginal cost is $1/[1 + \beta(N-1)]$,[4] which is decreasing in the spillover and in the number of countries. Due to the free-riding problem, $g^n(\alpha_i)$ is decreasing in the size of the externality β and increasing in the preference parameter α_i. Moreover, it is lower than $g^*(\alpha_i)$ for each α_i. For instance, under isoelastic utility and assuming

[4] To verify this, notice that for one unit of expenditure in a country, the same country obtains one unit of public good and each of the other $N-1$ countries obtain β units of public good. But if the social cost of $1 + \beta(N-1)$ units of public good is 1, it must be that the social cost of 1 unit of public goods is $\frac{1}{1+\beta(N-1)}$.

interior solutions we have:

$$g^n(\alpha_i) = \frac{\alpha_i^{1/\theta}}{(1-\beta)} - \frac{\beta \sum_{i=1}^{N} \alpha_j^{1/\theta}}{(1-\beta)} < g^*(\alpha_i)$$

and notice that $\partial g^n(\alpha_i)/\partial \beta < 0$, $\partial g^n(\alpha_i)/\partial \alpha_i > 0$ and $\partial^2 g^n(\alpha_i)/\partial \alpha_i^2 \lessgtr 0$ iff $\theta \lessgtr 1$.

2.3 Extreme centralization: the political equilibrium

At the opposite extreme of the options available to the federation is an institutional set up in which every country has to follow the policy rule chosen at the federal level. The main cost of this centralized organization is the lack of adaptability to the differences between countries, the main benefit is that the spillovers between countries can now be internalized at the federal level. This happens either if the federation acts to maximize its welfare in a utilitarian fashion, or if countries vote on the federal policy.

Consider first the case in which the federal policy is chosen to maximize (3) under the constraint $g_j = g$ for each j. We would obtain the first order condition:

$$\frac{\sum_{j=1}^{N} \alpha_j}{N} H_g\{g[1+\beta(N-1)]\} = \frac{1}{1+\beta(N-1)}$$

which equates the average of the marginal utilities from public expenditure to its social marginal cost. Notice that implementation of this policy requires to know the average preference parameter.

2.3.1 Simple majority voting

In this section we will assume a simple majority voting rule; we study qualified majority voting in section 4. Since preference are single-peaked, the median voter theorem applies. If α_m is the median of α_i's, the chosen policy is g_m such that:

$$\alpha_m H_g\{g_m[1+\beta(N-1)]\} = \frac{1}{1+\beta(N-1)} \tag{6}$$

Note from (6) that g_m is increasing in α_m, since:

$$\frac{dg_m}{d\alpha_m} = \frac{g_m}{\alpha_m}\left(\frac{1}{\theta\{g_m[1+\beta(N-1)]\}}\right) > 0$$

where $\theta(g) = -H_{gg}(g)g/H_g(g) > 0$ is the index of relative concavity. The comparative statics with respect to the size of the externality β, and the size of the union N are ambiguous:

$$\frac{dg_m}{d\beta(N-1)} = \frac{g_m}{[1+\beta(N-1)]}\left(\frac{1-\theta\{g_m[1+\beta(N-1)]\}}{\theta\{g_m[1+\beta(N-1)]\}}\right)$$

Hence, the political equilibrium provision of public goods decreases (increases) with the size of the union and of the spillovers if the elasticity of the marginal utility of public goods is higher (lower) than unity, while, under unitary elasticity we have size-independence. Indeed, under isoelastic utility we have:

$$g_m = \alpha_m^{1/\theta}[1+\beta(N-1)]^{\frac{1-\theta}{\theta}}$$

The ambiguous effect of an increase in spillovers on the union policy choice is due to the usual substitution and income effects. But the same argument holds, more surprisingly for an increase in the number of members. Even if a new country is strictly in favor of a small government and it will increase votes in this direction inside the union, its entry could end up determining a bigger size government: this can happen if the concavity of the function $H(\cdot)$ is not too strong. The intuition is that in this case the substitution effect (more public goods because they produce more spillovers) more than compensates the income effect (more consumption because we have already a lot of spillovers). Also the opposite could happen: the increase in size of the union could end up reducing the size of government if the concavity of $H(\cdot)$ is strong enough.[5]

2.3.2 Discussion

The median voter solution, by imposing the same policy to all countries gives up to the flexibility of the decentralized equilibrium, but internalizes the externalities. Given the trade-off between flexibility and internalization of the externalities, this is a "corner" solution, in the sense that it does not allow for any flexibility in tailoring policies to local preferences. Note that in the absence of compensatory schemes, the first best policy characterized above is not necessarily Pareto superior to the median voter solution. This is because the median voter (and some members close to the median) may be better off with the policy

[5] This result holds for any non separable utility function, but the cut-off is lower than unity if public and private goods are complements.

that solves (6) than with the first best.[6] More formally, note that:

$$\alpha_i H_g(g_i^* + \beta \sum_{j \neq i} g_j^*) = \alpha_m H_g\{g_m[1 + \beta(N-1)]\} = \frac{1}{1 + \beta(N-1)} \text{ for any } i$$

Hence, by concavity of $H(\cdot)$,

$$\alpha_i \gtreqless \alpha_m \quad \Leftrightarrow \quad H\left\{g_m[1 + \beta(N-1)]\right\} \gtreqless H\left\{g_i^* + \beta \sum_{j \neq i} g_j^*\right\} \text{ for any } i \qquad (7)$$

Countries with low preference for the public good obtain more utility from public expenditure under the political equilibrium. However, countries with low enough preference for the public good, obtain less consumption under the political equilibrium than in first best (they have to pay higher taxes in the former). Some countries are better off under the political equilibrium than the utilitarian "first best." For instance, the median country obtains the same utility form public expenditure in both cases - see (7), and it prefers the political equilibrium if and only if:

$$g_m < g_m^* \quad \Longleftrightarrow \frac{g_m^* + \beta \sum_{j \neq i} g_j^*}{[1+\beta(N-1)]} < g_m^* \quad \Longleftrightarrow g_m^* > \frac{\sum_{i=1}^N g_i^*}{N}$$

which happens when the median country favors a large expenditure compared to the other members of the federation. In this case, the median country and, in general, some countries around it, would favor the political equilibrium because it forces other countries to spend more on public goods than in the utilitarian first best case.[7] It is also possible that a majority of countries would actually

[6] Remember that the impossibiltiy of transfers contingent on the preferences makes compensative payments unoperative.

[7] More precisely, if the value of α_i of countries which are the closest in preferences to the median voter on either side are not to distant from α_m than the median voter is not the only

prefer the political equilibrium.

The disadvantage of the median voter solution for the union as a whole, is that it forces all the union members to follow the most preferred policy by the median, making those who are far from it much less satisfied than members close to the median, if there are no compensations. In the next section we investigate better institutional arrangements to study when they are efficient and politically feasible.

3 Institutional design of a federation

In this section we will study different institutional rules and compare their properties. In many cases these institutions resemble procedures adopted in some areas from the European Union.

3.1 Tax and subsidy schemes

Fiscal federalism is a system of decentralized decision making on public expenditure financed both through local decentralized taxation and a net of intergovernmental taxes and grants. In his classic book, Oates (1972) states a Decentralization Theorem for which "in the absence of cost-savings from the centralized provision of a good and of interjurisdictional externalities, the level of welfare will always be at least as high (and typically higher) if Pareto-efficient

country which is better off in the political equilibrium.

levels of consumption are provided in each jurisdiction than if any single, uniform level of consumption is maintained across all jurisdictions". Since in our model there are externalities, the theorem does not apply, but we will show that a form of fiscal federalism with redistribution between countries is still optimal in our set up.[8]

In this section we study a simple system of taxes and subsidies which is inspired to the way the European Union incentivates certain regional investment. Indeed, certain kinds of investment, especially those located in poorer regions, aimed at their development or to environmental protection and characterized by spillovers on the Community, is subsidized by the European Union:[9] for each Euro devoted to it, a supplementary fraction is added by the Union.

Suppose that for every unit of income allocated by country i to public expenditure, the union adds s_i units of public expenditure. These subsidies are financed with taxes T_j for $j = 1, ..., N$. By assumption, both s_i and T_i must be independent from the preference parameter of country i, α_i, but they do not need to be the same in all countries. In particular, they could depend on each of the public expenditures, g_1,...,g_i,...,g_N, and we will assume that they

[8]Indeed, Oates (1999) notes that "intergovernmental grants constitute a distinctive and important policy instrument in fiscal federalism that can serve a number of different functions. The literature emphasizes three potential roles for such grants: the *internalization of spillover benefits to other jurisdictions*, fiscal equalization across jurisdictions, and an improved overall tax system" (italics added).

[9]A big part of the European Union budget, about 35% (see Nava, 2000) is devoted to these "Structural Expenditures".

are differentiable in these variables. Hence, we can write the problem of each country i as the choice of g_i to maximize:

$$U_i = y - g_i - T_i(g_1, ..., g_i, ..., g_N) + \alpha_i H \left\{ \begin{array}{c} g_i \left[1 + s_i(g_1, ..., g_i, ..., g_N)\right] + \\ +\beta \sum_{j=1, j\neq i}^{N} g_j \left[1 + s_j(g_1, ..., g_i, ..., g_N)\right] \end{array} \right\} \tag{8}$$

which implies the system of first order conditions:

$$\alpha_i H_g \left(g_i(1 + s_i) + \beta \sum_{j=1, j\neq i}^{N} g_j(1 + s_j) \right) = \frac{1 + \frac{\partial T_i}{\partial g_i}}{\left(1 + s_i + \frac{\partial s_i}{\partial g_i} + \beta \sum_{j=1, j\neq i}^{N} g_j \frac{\partial s_j}{\partial g_i}\right)} \tag{9}$$

where we assumed an interior solution for each country. Despite the complexity of this differential system, we can derive the optimality of a very simple system of taxes and subsidies.

Proposition 1. *The first best allocation of public expenditure can be achieved if the federation complements national public expenditure by a fraction* $s^* = \beta(N-1)$ *for each country and finances these subsidies with taxes* T_i^* *such that* $\sum_{i=1}^{N} T_i^* = \frac{\beta(N-1)}{1+\beta(n-1)} \left(\sum_{i=1}^{N} g_i^*\right)$.

Proof: see the Appendix.

The intuition for this result is quite simple. The additional expenditure in the public good which is provided by the union distorts the incentives to invest in the public good. These incentives are the same for every country as in the first best if the marginal cost of public expenditure equates its social marginal cost. But we know that this is the same for every country and given

by $1/[1+\beta(N-1)]$, hence this equality is trivially satisfied when $\frac{\partial T_i}{\partial g_i} = 0$ while s is as specified in Proposition 1 for all countries, so that $\frac{\partial s_i}{\partial g_j} = 0$ for any j: the subsidy alone can be chosen so as to implement the right allocation of public expenditure. Indeed, in the Appendix we show that:

$$g_i[1+\beta(N-1)] = g_i^* \qquad i = 1, ..., N$$

However, we still need to check that these subsidies can be financed. Indeed, we have:

$$\begin{aligned} \sum_{i=1}^{N} g_i s^* &= \beta(N-1)\sum_{i=1}^{N} g_i = \\ &= \sum_{i=1}^{N} T_i^* = \frac{\beta(N-1)}{1+\beta(n-1)}\left(\sum_{i=1}^{N} g_i^*\right) = \beta(N-1)\sum_{i=1}^{N} g_i \end{aligned}$$

One way to choose taxes so as to satisfy this revenue constraint is closely related to the Clark-Grove-Vickrey mechanism studied in mechanism design under a quasi-linear environment. Since we did not need to constraint $\frac{\partial T_i}{\partial g_j}$ to be zero for $j \neq i$, we can use the lump sum taxes like:

$$T_i^* = \beta \sum_{j \neq i} g_j$$

Clearly, the tax of each country is independent from its own expenditure and the union revenue constraint is satisfied.[10] Another simple way to solve the

[10] Indeed we have:

$$\sum_{i=1}^{N} g_i s^* = \beta(N-1)\sum_{i=1}^{N} g_i = \sum_{i=1}^{N} T_i^* = \sum_{i=1}^{N}\left(\beta \sum_{j \neq i} g_j\right) = \beta(N-1)\sum_{i=1}^{N} g_i$$

problem is to share equally the cost of subsidies by adopting taxes:

$$T_i^* = T^* = \frac{\beta(N-1)}{1+\beta(n-1)} \frac{\left(\sum_{i=1}^{N} g_i^*\right)}{N} \qquad i = 1, ..., N$$

Since preferences are observable, this tax can be directly calcolated and it clearly satisfies the revenue constraint.

In every case, the optimal subsidy is independent from the utility functions and the distribution of preference parameters: we can obtain the first best allocation of public expenditure just by knowing what is the size of the spillovers. The optimal subsidy is the product of this and the number of countries, independently from the details of the model: all of them end up in transfers between countries.

3.2 Discussion

An important feature of the optimal mechanisms we have just studied is that, while they induce the optimal investment in public goods by each country, in general, they do not replicate the exact allocation of consumption of the utilitarian first best. Given the linearity in consumption of our objective function, this is consistent with optimality - the welfare function (3) is still maximized -, but it is clear that departing from this quasi-linear environment we would loose the optimality property of such a mechanism.

In both the particular cases studied in the previous subsection, countries with high public expenditure receive high subsidies but pay less than proportionally for them (indeed, their taxes are independent from their own expenditure):

borrowing the mechanism design jargon, this is the "rent" that we need to leave to the players for whom the right action is more costly. The consequences of this is that countries with high preference for public expenditure end up with higher utility than in the utilitarian optimum: indeed, their expenditure is subsidized by countries with low preferences for public expenditure.[11] In theory, lump sum transfers between countries could reestablish the utilitarian allocation of consumption while preserving the one of the public goods, but unverifiability of the preference parameters prevents this from being feasible.

This is not just a technical problem for our mechanism. Countries with low preference for the public good may end up being worse off than under the decentralized Nash equilibrium, which would make pointless their same adhesion to the union. Also, if income is low enough, some countries may not even be able to pay their transfer, which would directly make unfeasible the all mechanism. Typically, the imposition of individual rationality constraints would make im-

[11] For instance, in the case with $T_i^* = \beta \sum_{j \neq i} g_j$, for country i utility from this mechanism is lower than in the optimal utilitarian case if and only if:

$$g^*(\alpha_i) < g_i + T_i = g_i + \beta \sum_{j \neq i} g_j = \frac{g^*(\alpha_i)}{1 + \beta(N-1)} + \beta \sum_{j \neq i} \frac{g^*(\alpha_j)}{1 + \beta(N-1)}$$

$$\iff \quad g^*(\alpha_i)\beta(N-1) < \beta \sum_{i=1}^{N} g^*(\alpha_j) - \beta g^*(\alpha_i)$$

$$\iff \quad g^*(\alpha_i) < \frac{\sum_{i=1}^{N} g^*(\alpha_j)}{N}$$

that is, if its public expenditure is below the average in the union. It is easy to verify that exactly the same condition emerges when taxes are equal for all countries.

possible the attainment of the first best with such a mechanism even under our quasi-linear specification. But even if this was not the case, the implementation of the redistributive mechanism we have just described, seems quite difficult to sustain from a political point of view, especially in a federation of otherwise identical countries.

Finally we need to remember that despite the first best could be achieved, there may not be a majority in the federation which supports it (remember that compensatory payments would not be feasible since they should be contingent on the preference parameters).

For these reasons we now return to political concerning issues. In the next two sections we study two different institutional arrangements based on majority voting in the union which add flexibility to the political equilibrium previously studied.

3.3 Flexible unions

In section 2.3 we have assumed that the political process in the federation decides a unique level of the public good for all members. A potentially more efficient structure is to allow countries to choose and independently finance public expenditure and to have an additional federal provision of the public good which is decided by the union. This complementary role of the federation depicts a well known feature of the European Community constitution, the *subsidiarity principle*. For instance, as the Article 3b of the Maastricht Treaty says that "*In*

areas which do not fall within its exclusive competence, the Community shall take action, in accordance with the principle of subsidiarity, only if and in so far as the objectives of the proposed action cannot be sufficiently achieved by the Member States and can therefore, by reason of the scale or effects of the proposed action, be better achieved by the Community". In other words, public policies should be assigned to the lowest level of government which is capable to succesfully achieve their objectives (see Inman and Rubenfeld, 1998). In our model we have exactly a situation were each member's policy does not internalize the effects on the other members and so it cannot sufficiently achieve its objectives. Hence, the subsidiarity principle implies that the union should complement countries' expenditure in the public good with an additional expenditure at the union level.

We will refer to a union adopting this principle as to a "flexible union" compared to the rigid one considered in the case of extreme centralization, in which every country had to adopt the same policy. Formally we will describe a flexible union with N countries and median α_m, assuming that the median country chooses an union expenditure g_m^U ,[12] while each country i chooses $g_i^n \in$

[12]We focus on the natural case in which single-peakedness according to the α-ordering holds in the second stage. We cannot exclude the existence of other subgame perfect equilibria in which single peakedness does not hold. However, the nature of the equilibrium would be qualitatively similar.

$(0, y - g_m^U)$. Utility for country i is now:

$$U_i = y - g_i^n - g_m^U + \alpha_i H \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] \qquad (10)$$

The timing assumption in our description of the principle of subsidiarity is that the countries choose their individual public expenditure in a first stage, and the union decides a complementary expenditure in a second stage.[13] In particular, the median country will choose g_m^U to maximize U_m. Given the expectation $E(g_m^U)$, all the countries in the first stage choose $g_i^n = g^n \left(\alpha_i, E(g_m^U) \right)$, $i = 1, 2, ..., N$, to maximize U_i. In equilibrium it must be $E(g_m^U) = g_m^U$. Given this, the sub-game perfect equilibrium is defined by the following system of $N + 1$ equations:

$$\begin{aligned} \alpha_i H_g \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] &\leq \frac{1}{1 + \left[1 + \beta(N-1) \right] \frac{\partial g_m^U}{\partial g_i^n}} \qquad (11) \\ g_i^n &\geq 0 \quad i = 1, 2, ..., N \end{aligned}$$

$$\alpha_m H_g \left[g_m^n + \beta \sum_{j \neq m} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] = \frac{1}{\left[1 + \beta(N-1) \right]} \qquad (12)$$

[13] Our modeling strategy on flexible unions may remind of the theory on budgetary procedures and agenda setting rules in presence of common pool problems *a la* Persson, Roland and Tabellini (1997, 2000). However, in that literature the multi-stage budgeting was separating the problems of the size of the budget and its allocation, while in our model there is not an overall budget to decide. Moreover, there, each player prefered a bigger share of public expenditure, here the opposite holds because of positive externalities between the players. Finally our purpose is not to do comparative politics at a horizontal level - e.g. executive versus legislative powers or presidential versus parliamentary systems -, but to do comparative politics at a vertical level - national versus sovranational powers or state versus federal powers.

Given $\sum_{j\neq m} g_j^n$ as chosen in the first stage, the last equation defines $g_m^U = g_m^U\left(\alpha_m, \sum_{j\neq m} g_j^n\right)$ as a function increasing in the first argument and decreasing in the second one with:

$$\frac{dg_m^U}{dg_i^n} = -\frac{\beta}{[1+\beta(N-1)]}$$

Substituting this in (11) we obtain that for all countries individually providing additional public goods:

$$\alpha_i H_g\left[g_i^n + \beta\sum_{j\neq i} g_j^n + g_m^U\left[1+\beta(N-1)\right]\right] = \frac{1}{1-\beta} \tag{13}$$

Hence, we have shown that $g_m^U\left(\alpha_m, \sum g_j^n\right) \leq g_m^U\left(\alpha_m, 0\right)$ and that $\alpha_i H_g(\cdot) < 1$ for any $i \leq m$. We can summarize our findings as follows:

Proposition 2. *In a flexible union 1) the federal expenditure is lower than the one adopted by a rigid union, 2) the median country and all countries with weaker preferences for the public good do not add any individual expenditure and 3) the countries with strongest preferences for the public good add individual expenditure.*

Proposition 3. *In a rigid union, the adoption of a flexible structure is Pareto-efficient and unanimously supported.*

Proof: see the Appendix.

The outcome under the flexible union is a compromise between the decentralized one and the rigid one. Countries with preferences for a small government

will spend less then in a rigid union, while benefiting from the additional expenditure of countries with opposite preferences: hence they are better off. The median country must be better off because it is always able to replicate the public expenditure of the rigid system (with the opportunity to be better off if some countries provide further expenditure). Hence a strict majority is in favor of the flexible system. The other countries on one side are worse off because the union is providing less public good and on the other side they are better off because to some extent they can individually repair to this. It turns out that the second effect is prevailing under any circumstances. Hence, everybody is better off under the flexible union. Thus, even if in a rigid union a reform in favor of the utilitarian optimum or fiscal federalism is not politically feasible, we have found a Pareto-efficient reform which is always politically feasible. An important feature of this reform is that it does not rely on any information about the preferences of the countries.

The feasibility of flexible unions depends to a large extent on the nature of the policy arena. For certain cases this arrangement seems feasible; think for instance of the provision of public goods such as infrastructures, defense, research, etc. The same applies to aspects of regulation, taxation and labor market policies. In other areas the nature of the policy may make a flexible union impractical; it is hard to adopt "a little bit" of the common currency, although Britain may be trying. The general point is, however, that by adopting "minimum common denominator" approach, the combination of union level and

national level policy choices can achieve Pareto improvements, relative to the "rigid" union.

3.4 Federal Mandates

A related institution is one in which each country could choose and independently finance public expenditure on top of the one decided by the federation. We can think of the situation in which the federation moves first as a description of federal mandates:[14] the choice of the union will be the minimum amount of public good that each country has to provide. In our context we prove in the Appendix that the subgame-perfect equilibrium is characterized by the system:

$$g_i^n = 0 \quad \text{for} \quad i = 1, 2, ..., m, ..., N - \tilde{N}$$

$$\alpha_i H_g \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] = 1 \text{ for } \quad i = N - \tilde{N}, ..., N$$

$$\alpha_m H_g \left[\beta \sum_{j \neq m} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] = \frac{\left[1 + \beta(\tilde{N} - 1) \right]}{\left[1 + \beta(N-1) \right](1-\beta)}$$

so that $\tilde{N}$ countries add individual public expenditure to the amount g_m^U decided as the federal mandate. More precisely we can show:

Proposition 4. *The adoption of a federal mandate implies that 1) the federal mandate is lower than the public expenditure adopted by the rigid union, 2) the*

[14] Cremer and Palfrey (2000) have studied this kind of federal mandates, however in their model there are not externalities between countries. In our model, instead, the federal mandate accomplishes an important role: it limits the free-riding of the decentralized equilibrium internalizing to some extent the externalities produced in public good provision.

median country and all countries with weaker preferences for the public good do not add any individual expenditure and 3) the countries with strongest preference for the public good add individual expenditure.

Proof: see the Appendix.

Even if the adoption of federal mandates in a rigid union may be Pareto-efficient and unanimously supported, as it was true for the flexible structure, it may be the case that some country is made worse off. However we can prove the following encouraging result:

Proposition 5. *In a rigid union there is always a majority in support of federal mandates.*

Proof: see the Appendix.

In subgame perfect equilibrium, the union requires every country to provide a minimum amount of public expenditure, and some countries complement it with further expenditure. Even if there is always a majority in support of federal mandates, in this case we cannot exclude that some countries with stronger preference for the public expenditure than the median are made worse off by their adoption. The reason is that the advantage of the first move for the union as represented by the median country induces an additional free riding (this time at the union level) which may be deleterious for those countries.

4 Qualified majority voting

Define "qualified majority voting" rule (QMV) a situation in which, in order to change a predefined status quo, the union needs a majority Q, where $1 \geq \frac{Q}{N} > \frac{1}{2}$. Unanimity is a special case of QMV in which $Q = N$. Simple majority voting is again a special case of QMV, where Q approaches $N/2$. Hence, the results we derived in previous sections under simple majoriry rule all obtain here as a special case. But the implications of QMV are of particular importance in the context of the European Union. The European Union Treaty stipulates that unanimity in the EU Council is needed to entrust policy-making powers to the EU in a certain number of key areas. In other areas, QMV is sufficient. A livey debate has developed on the possibility of extending the range of areas where QMV applies, correspondingly reducing the scope of unanimity.

Under QMV, no single policy outcome unambiguously emerges from the vote. The outcome is indeterminate within a set of policies each of which cannot be defeated by QMV by any other. The ambiguity is resolved only by the agenda setter, that decides which alternatives are put to vote. We can however characterize this set. The following proposition highlights the policy implications of different super majority rules; a more rigorous treatment is provided in the Appendix.

Proposition 6. *Under a qualified majority voting rule with quorum Q, there exists a "winning set" of public good provisions which cannot be defeated against*

an alternative option. This set collapses to the political equilibrium provision under simple majority voting, g_m, when $Q = 1/2$. Assume that the alternative option is the maximum provision unanimously supported against no provision, g_o: then, the lower bound of the winning set is decreasing in the required majority from g_m to g_o and the upper bound is increasing in the required majority for Q small enough and decreasing for Q big enough.

Proof: see the Appendix.

The intuition behind this proposition is that moving from unanimity to less binding forms of qualified voting tends to make the union more centralized, i.e. the provision of public good by the union "is likely to" increase while the amount of good provided individually (and the number of countries doing so) "is likely to" decrease. As Q declines further towards $N/2$, beyond a certain point the set of policies prevailing under QMV simply shrinks: as the quota approaches simple majority, the dominant policy must converge to that chosen by the median voter. The word "likely" above expresses the presumption that, if both the lower and the upper bounds of the winning set move up, the outcome will also move up (or vice versa).

Moreover, the introduction of QMV would imply that the number of countries individually providing the public good and the total amount of public good individually provided are increasing functions of Q, if and only if the QMV outcome g^U moves in the opposite direction to Q.

4.1 Discussion

The discussion about voting rules and super majority becomes even more heavily loaded if countries have different size. In this case the question is which weight should be given to each country. Obviously the resulting policy decision would depend on the choice of weights.

One possible rule is that the voting share of each country is proportional to the share of the population. Since we are assuming that income per capita is the same everywhere, population weights are identical to GDP weights. If income per capita is different, that would not be the case.

Let's continue to assume that individuals in each country have the same α, so, trivially, α_i represent the median preferences of members of country i. Then, with population weights the median of the federation (α_m) coincide with the median preference of the citizens members of the union. If, instead, the voting scheme favors small countries (like the current EC voting system) than one has a "bias" in favor of the preferred government size of small countries. Alesina and Wacziarg (1998) find that the share of government over GDP is decreasing with the size of the country measured by population or by total GDP.[15] Thus, voting schemes that favor small countries would produce an α for the union government higher than the median α of the population of the union.

Consider now the case in which within each country individuals have different

[15] Note however, that these results are drawn from independent countries, not from countries in a union.

preferences, i.e. there exists within each country a distribution of α such that α_i is the median of the distribution of α in country i. If each country in the union is represented by its median voter, and if the union adopts population weights, then the median of the union defined as the median of country medians, coincides with the median preference of the citizen of the union.

In practice, country representative in federation level institutions may not represent the median of their population, particularly in non-proportional systems. In this case the median of the individual countries' representative in the union do not coincide with the median of the population of the union. Alesina and Grilli (1992) using electoral result from the late eighties, show that the median of EU institutions (like the Council of Ministers) can deviate significantly from the median of European voters.

5 Conclusions

In this paper we study how alternative institutional arrangements lead to different outcomes concerning the centralisation, or decentralisation, of policy responsibilities in a federation.

We show three main results. The first one concerns the possibility of achieving the first best coordination of policies within a union. We have shown how this is possible with a mechanism of transfers and taxes or subsidies. However we also point out many political and administrative limitations of this solution.

The second result is that even in policy arena where externalities are high, a "flexible" union in which both national and super national institutions are involved in choosing policies is superior to a "rigid" one in which the union level institutions have full control of policy decision. We also investigate the properties of alternative rules regulating the relationship between different levels of governments like the adoption of federal mandates.

Finally, we analyse the implications of qualified majority voting (as opposed to unanimity of simple majority rules) in our federal context. We show that moving from unanimity to qualified majority voting tends to lead to greater centralisation. Moreover, adopting qualified (as opposed to simple) majority voting can correct against certain biases towards excessive centralisation that arise in some cases. In AAE (2001), we point out one of such cases, which leads to sub optimally small federations. The idea that once the union is formed, based upon the expectation of a certain level of centralization, a majority of members around the median have an incentive to centralize more. In anticipation of this effect, fewer countries will join to begin with, leading to federations that are too small from the start. We show, in this paper, that qualified majority voting poses limits on the policy involvement of the union. Hence, in the presence of the bias discussed above, super majorities may be welfare improving.

6 Appendix

Proof of Proposition 1.

The proof is composed of three steps. First, we rewrite the system of optimality conditions, and than we show that the proposed mechanism induces an identical equilibrium system. Finally we verify that the proposed mechanism also satisfies budget balance.

If we rewrite (4) as

$$\alpha_i H_g(g_i^* + \beta \sum_{j \neq i} g_j^*) = \frac{1 - \beta \sum_{j=1}^N \alpha_j H_g(g_j^* + \beta \sum_{k \neq j} g_k^*)}{(1-\beta)}$$

and sum over all i's, we obtain:

$$\begin{aligned} \sum_{j=1}^N \alpha_i H_g(g_i^* + \beta \sum_{j \neq i} g_j^*) &= N \left[\frac{1 - \beta \sum_{j=1}^N \alpha_j H_g(g_j^* + \beta \sum_{k \neq j} g_k^*)}{(1-\beta)} \right] = \\ &= \left(1 + \frac{\beta N}{1-\beta}\right)^{-1} \left(\frac{N}{1-\beta}\right) = \frac{N}{1+\beta(N-1)} \end{aligned}$$

Substituting in the previous equation we obtain the equivalent expression:

$$\alpha_i H_g(g_i^* + \beta \sum_{j \neq i} g_j^*) = \frac{1 - \beta \frac{N}{1+\beta(N-1)}}{(1-\beta)} = \frac{1}{1+\beta(N-1)}$$

Now, let us consider the system (9). Under the proposed mechanism, it becomes:

$$\alpha_i H_g \left\{ g_i[1+\beta(N-1)] + \beta \sum_{j \neq i} g_j \left[1+\beta(N-1)\right] \right\} = \frac{1}{1+\beta(N-1)}$$

Since the two systems are identical they must have the same solutions:

$$g_i[1+\beta(N-1)] = g_i^* \quad i = 1, ..., N$$

Finally budget balance is satisfied because:

$$\sum_{i=1}^{N} g_i \cdot s^* = \sum_{i=1}^{N} T_i^* = \frac{\beta(N-1)}{1+\beta(n-1)} \left(\sum_{i=1}^{N} g_i^* \right) = \beta(N-1) \sum_{i=1}^{N} g_i$$

hence the proposed mechanism is feasible and it implements the first best allocation of public expenditure. **Q.E.D.**

Proof of Proposition 3.

Consider the difference between utility from the equilibria under a flexible union and under a rigid union:

$$\begin{aligned} \Phi(\alpha_i) \quad \equiv \quad & \alpha_i \left\{ \begin{array}{c} H\left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] \\ -H_g\{ g_m[1+\beta(N-1)] \} \end{array} \right\} + \\ & + g_m - g_i^n - g_m^U \end{aligned}$$

Comparing equilibrium conditions (6) and (12), it follows that $\beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] = g_m[1+\beta(N-1)]$. Hence, for all countries $i \leq m$ and the other countries for which $g_i^n = 0$, it must be $\Phi(\alpha_i) = g_m - g_m^U > 0$ (by Proposition 2.1) and $\Phi'(\alpha_i) = 0$.

Now, let us consider the countries for which $g_i^n > 0$. In this case we have:

$$\begin{aligned} \Phi'(\alpha_i) \quad = \quad & H \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] - H\{ g_m[1+\beta(N-1)] \} + \\ & + \frac{\partial g_i^n}{\partial \alpha_i} \left[\alpha_i H_g \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] - 1 \right] \\ = \quad & H \left[g_i^n + \beta \sum_{j \neq i} g_j^n + g_m^U \left[1 + \beta(N-1) \right] \right] - H\{ g_m[1+\beta(N-1)] \} + \\ & + \frac{\beta}{1-\beta} \frac{\partial g_i^n}{\partial \alpha_i} \end{aligned}$$

where we used (13) and the fact that:

$$g_i^n+\beta\sum_{j\neq i} g_j^n+g_m^U\left[1+\beta(N-1)\right]>\beta\sum g_j^n+g_m^U\left[1+\beta(N-1)\right]=g_m[1+\beta(N-1)]$$

It follows that $\Phi'(\alpha_i)>0$, and hence, every country is better off under the flexible union. **Q.E.D.**

Proof of Proposition 4.

Let us consider the case of federal mandates, in which the union moves first. The median country chooses $g_m^U\left\{\alpha_m, E\left[\sum_{j\neq m} g_j^n\right]\right\}$, where $E\left[\sum_{j\neq m} g_j^n\right]$ is the expectation of the sum of the individual expenditures of all the countries, to maximize U_m. Then, all the countries choose g_i^n, $i=1,2,...,N$, to maximize U_i. In equilibrium it must be $E\left[\sum_{j\neq m} g_j^n\right]=\sum_{j\neq m} g_j^n$. Given this, the equilibrium is defined by the following (differential) system of $N+1$ equations:[16]

$$\alpha_i H_g\left[g_i^n+\beta\sum_{j\neq i} g_j^n+g_m^U\left[1+\beta(N-1)\right]\right]\leq 1\ ,\ g_i^n\geq 0\qquad i=1,2,...,N$$

$$\alpha_m H_g\left[g_m^n+\beta\sum_{j\neq m} g_j^n+g_m^U\left[1+\beta(N-1)\right]\right]=\frac{1}{[1+\beta(N-1)]+\beta\sum_{j\neq m}\frac{\partial g_j^n}{\partial g_m^U}}$$

Notice that for a given g_m^U, the first N equations define $g_i^n=g^n(\alpha_i,g_m^U)$ as functions increasing in the first argument and decreasing in the second one when positive. Hence $[1+\beta(N-1)]+\beta\sum_{j\neq m}\frac{\partial g_j^n}{\partial g_m^U}\in(1,1+\beta(N-1)]$.[17] This

[16] In this case, it can be verified that single-peakedness always holds in the first stage.

[17] We are implicitly assuming that $g_m^U>0$. otherwise we would be back at the decentralized equilibrium.

implies 1) that $g_m^U\left(\alpha_m, \sum_{j\neq m} g_j^n\right) \leq g_m^U(\alpha_m, 0)$, and that 2) $\alpha_i H_g(\cdot) < 1$ for any $i \leq m$. Finally, 3) positive individual provision is chosen by all countries to the right of the cut-off α_b defined by:[18]

$$\alpha_b H_g\left[\beta\sum_{j>b} g_j^n + g_m^U\left[1+\beta(N-1)\right]\right] = 1$$

Now, by totally differentiating the equilibrium first order condition for the individual contributions, we obtain:

$$\frac{dg_i^n}{dg_m^U} = -[1+\beta(N-1)] - \beta\sum_{j>b, j\neq i}\frac{dg_j^n}{dg_m^U}$$

Subtracting from both sides $\beta\frac{dg_i^n}{dg_m^U}$ and simplifying we have:

$$\frac{dg_i^n}{dg_m^U} = -\frac{[1+\beta(N-1)] + \sum_{j>b}\frac{dg_j^n}{dg_m^U}}{1-\beta}$$

Summing over all $j > b$ we get:

$$\sum_{j>b}\frac{dg_j^n}{dg_m^U} = -\tilde{N}\left\{\frac{[1+\beta(N-1)] + \sum_{j>b}\frac{dg_j^n}{dg_m^U}}{1-\beta}\right\}$$

where $\tilde{N}$ is the number of countries with $\alpha_j > \alpha_b$, that is the number of countries providing individual public expenditure, as previously determined. Solving, we have:

$$\sum_{j>b}\frac{dg_j^n}{dg_m^U} = -\frac{\tilde{N}[1+\beta(N-1)]}{\left[1+\beta(\tilde{N}-1)\right]}$$

Finally, substituting in the equilibrium condition for the federal choice of the median country it follows that:

$$\alpha_m H_g\left[\beta\sum_{j>b} g_j^n + g_m^U\left[1+\beta(N-1)\right]\right] = \frac{\left[1+\beta(\tilde{N}-1)\right]}{[1+\beta(N-1)](1-\beta)}$$

[18] Obviously, if $\alpha_b > \alpha_N$, we are back to the equilibrium with a rigid union.

whose right hand side is strictly greater than $\frac{1}{1+\beta(N-1)}$ as long as $\tilde{N} \geq 1$. Hence, in this case, comparing the last equation with (6) we conclude that:

$$g_m^U \left[1+\beta(N-1)\right] < \beta \sum_{j>b} g_j^n + g_m^U \left[1+\beta(N-1)\right] < g_m[1+\beta(N-1)]$$

that is $g_m^U < g_m$. **Q.E.D**

Proof of Proposition 5.

Let us define the difference between utility from the equilibria under a union adopting the federal mandate and under a rigid union as $\tilde{\Phi}(\alpha_i)$. Consider the median country; by revealed preferences it must be that:

$$\tilde{\Phi}(\alpha_m) = \alpha_m \left[H\left\{\beta \textstyle\sum_{j>b} g_j^n + g_m^U \left[1+\beta(N-1)\right]\right\} - H\{g_m[1+\beta(N-1)]\}\right] + \\ + \left(g_m - g_m^U\right) > 0$$

which clearly implies $\tilde{\Phi}(\alpha_i) \geq 0 \ \forall \ i \leq m$. Hence there is a strict majority in favor of the union**. Q.E.D**

Remark. With federal mandates we do not have necessarily unanimous support for their adoption. Indeed for any $j > b$:

$$\begin{aligned} \tilde{\Phi}'(\alpha_i) &= \left[H\left\{\beta \sum_{j>b} g_j^n + g_m^U \left[1+\beta(N-1)\right]\right\} - H\{g_m[1+\beta(N-1)]\}\right] + \\ &\quad + \frac{\partial g_i^n}{\partial \alpha_i}\left[\alpha_i H_g \left[g_i^n + \beta \sum_{j\neq i} g_j^n + g_m^U \left[1+\beta(N-1)\right]\right] - 1\right] \\ &= \left[H\left\{\beta \sum_{j>b} g_j^n + g_m^U \left[1+\beta(N-1)\right]\right\} - H\{g_m[1+\beta(N-1)]\}\right] \end{aligned}$$

which is negative for α_i close enough α_m.

Proposition 6.

Suppose the provision of g^U is *decided by qualified majority, i.e., a majority* $Q:N$, *where* $1 \geq Q/N > 1/2$, *is required to beat a given default option. Then:*

(i) For each $Q:N$, *there is a corresponding "Dominant Set" of options* D_Q *such that no option in this set can be beaten by* $Q:N$ *by any other.* $D_Q \equiv \{g^U_{N-Q}; ...; g^U_Q\}$, *where* g^U_{N-Q} and g^U_Q *are the options preferred respectively by the* $N-Qth$ *and the* Qth *country (ranked in increasing order of* α*).* g^U_{N-Q} *increases and* g^U_Q *decreases as* Q *decreases between* N *(unanimity) and* $N/2$ *(simple majority).* D_Q *collapses to the median option as* Q *approaches* $N/2$.

(ii) Let g_0 *be the maximum value of* g^U *that is unamimously supported against* $g^U = 0$. *This "consensus option" under weak assumptions is strictly positive* ($g_0 > 0$). *Let this be the default option to be challenged by an alternative* $g^U > g_0$ *by a majority* $Q:N$. *Then, for each* $Q:N$, *a compact "*g_0*-Dominant" set* $Z_Q \equiv \{g_0; ...; x^U_Q\}$ *exisis for which all* $g^U \in Z_Q$ *beat* g_0 *by a majority* $Q:N$. *Moreover,* x^U_Q *is a decreasing function of* Q.

(iii) Let $W_Q = D_Q \cap Z_Q$. *Then,* W_Q *is the set of options that beat* g_0 *and that cannot be beaten by any other by a majority* $Q:N$. *We call this (intersection of the Dominant and the* g_0*-Dominant sets) the "Winning Set". The Winning Set includes all options that will emerge as winners if, starting from unanimity, QMV is applied, one time or repeatedly, with the same quorum* $Q:N$. *The winning option within* W_Q *is indeterminate. Then:*

(a) For Q *above a cutoff* $\overline{Q}$, *both the lower and upper bounds of the Winning*

Set increase as Q decreases.

(b) For Q below the cutoff $\overline{Q}$, the lower (upper) bound of S_Q increases (decreases) as Q decrease. The set W_Q collapses to the median option as Q approaches $N/2$.

(c) $\overline{Q}$ corresponds to the majority quorum for which $\max W_{\overline{Q}} = \max Z_{\overline{Q}}$.

Proof. The properties of QMV have been first analysed by Black (1948), under the assumption that preferences are single-peaked. Proposition 6(i) follows directly from Black's Lemma 5. Intuitively, under single-peaked preferences only options located at the extremes can be beaten, because they can gather sufficient opposition from voters located at the opposite extreme. Options located around the center can never be beaten if the quorum is large enough. Hence there is a compact Dominant Set D_Q, located around the centre, that can never be beaten by QMV by any alternative option. D_Q tends to shrink as the quorum declines from N to $N/2$. For $Q \to N/2$, the median voter theorem applies.

Part 6(ii) follows from Black's (1948) Theorem 13(b). The *"g_0-Dominant"* set is given by $Z_Q \equiv \{g_0; ...; x_Q^U\}$, where x_Q^U is defined as the option that is equivalent to g_0 for the "marginal country", i.e. the country with lowest preference for the public good within the qualified majority[19]. Intuitively, all countries that belong to that majority will support all options in Z_Q against g_0. As Q decreases, the marginal country will be characterised by a stronger preference

[19]Such option must exist by assumption of single-peakedness of individual preferences.

for the public good, hence shifting x_Q^U to the right.

To prove Part 6(iii-a), we first show that W_Q is not an empty set for Q close to (and below) N. We need to show that around (and below) N, $x_Q^U > g_{N-Q}^U$. We know that g_{N-Q}^U is the preferred option by the marginal country, while x_Q^U is, for the same country, the option that is equivalent to g_0. Since $g_{N-Q}^U > g_0$ (otherwise, g_{N-Q}^U would be unanimously supported), $x_Q^U > g_{N-Q}^U$ by assumption of single peakedness, and W_Q is non empty. Hence, for large enough Q, $W_Q = \{g_{N-Q}^U; ...; x_Q^U\}$, and the rest of Proposition 6(iii-a) follows directly from 6(i) and 6(ii).

So show Proposition 6(iii-b) it is sufficient to note that, since g_Q^U decreases with Q, there is a cutoff point $\overline{Q}$ below which $W_Q = D_Q$. This cutoff point is defined by $x_{\overline{Q}}^U = g_{\overline{Q}}^U$, or, equivalently, $\max W_{\overline{Q}} = \max Z_{\overline{Q}}$. **QED**.

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