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ABSTRACT

A popular view about social security, dating back to its early days of inception, is that it is a means for young, unemployed workers to "purchase" jobs from older, employed workers. The question we ask is: Can social security, by encouraging retirement and hence creating job vacancies for the young, improve the allocation of workers to jobs in the labor market? Using a standard model of labor market search, we establish that the equilibrium with no policy-induced retirement can be efficient. Even under worst-case parameterizations of our model, we find that public retirement programs pay the elderly substantially more than labor market search theory implies that their jobs are worth. An important effect, ignored by the popular view, is that the creation of a vacant job by a retirement reduces the value of other vacant jobs.

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The objective of this paper is to formalize a popular argument about the economic rationale for Social Security and public pension programs in other countries. We construct a model in which there can be significant frictions slowing the process of matching workers with firms, and social security is a labor market policy designed to remove old workers from the labor market. In this sense, it can be said that social security is a means for young, unemployed workers to "purchase" jobs from older, employed workers. President Franklin D. Roosevelt, in particular, in one of his "fireside chats" suggested that this would be an important goal for social security:

> "The program for social security now pending before the Congress is a necessary part of the future unemployment policy of the Government...It proposes, by means of old-age pensions, to help those who have reached the age of retirement to give up their jobs and thus give to the younger generation greater opportunities for work and to give all a feeling of security as they look toward their old age..." (Roosevelt, p. 134-135.)

This position on Social Security continues to receive some support, at least enough to receive serious consideration in recent social security debates. In recent discussion about the retirement earnings test in Congress, many agreed with the following perspective:

> "Social Security, when it was created in 1935, sought to achieve two goals–moving older workers out of the workforce to make way for younger workers, and to partially replace lost income due to retirement."
> (Testimony of Honorable John J. Rhodes III in *Social Security Retirement Test*)

Can discouraging work among the elderly be understood as a way of creating job vacancies for the young, and improving the allocation of workers to jobs in the labor market? This paper explores a model of labor market search in order to answer this question. We re-interpret the Hosios (1990) environment to include retirement decisions and show how the answer depends on the model's parameter values, some of which imply that the social value of retirement exceeds the private value while other parameter values imply that the private value exceeds the social value. We then consider the parameter values implying the largest wedge between private and social retirement values and show that, even in this extreme case, optimal retirement policy does much less to encourage retirement than do the policies used by governments around the world. To put it simply, labor market search theory provides at best only a partial justification for publicly induced retirement; public retirement programs pay the elderly substantially more than labor market search theory implies that their jobs are worth. In this regard, our results might be interpreted as either a critique of public retirement policy or as a puzzle to be explained by positive theories of the public sector.

Section I begins with an overview of retirement-inducing policies used around the world. Section II presents the Hosios (1990) model of search, simplified, amended and reinterpreted to allow for a retirement decision. Section III considers an extreme parameterization of the model in order to show that, even if it is the case that the social value of retirement exceeds the private value, an optimal retirement program does much less to induce retirement than do observed programs. Section IV concludes.

I. An Overview of Retirement-Inducing Policies Around the World

There is a growing literature comparing public pension systems and their retirement incentives across countries and over time. We report some of the main results from that literature. The purpose of our report is not to conduct a detailed statistical analysis, but merely to highlight the empirical regularities relevant to a theoretical study of publicly induced retirement. The most conspicuous, and theoretically most relevant, regularity is that implicit earnings tax rates are highest for the elderly.

I.A Public Policies Encourage Retirement

As of 1995, over 100 countries had public pension programs.[2] Among the 88 of those countries reporting to the U.S. Social Security Administration sufficient detail of their public pension benefit formulas, 75% pay pension benefits in such a way as to discourage work by its elderly citizens. The most typical means by which benefit formulas induced retirement is remarkably

[2]Data in this paragraph are reported and described in more detail by Mulligan and Sala-i-Martin (1999a,b) and Sala-i-Martin (1996).

transparent: retirement is a necessary condition for receiving public pension benefits, and no credit is given to those who decide to retire later and collect benefits for fewer years. Other countries had more complicated benefit formulas extending some less-than-actuarially fair credits to those who delay retirement, or allowing employed elderly to collect partial benefits, or both (the case, until this year, for U.S. Social Security for elderly aged 65-69). But the more complicated formulas have much the same effect as the simple one: elderly labor income is implicitly taxed.

At least in higher income countries, the rates of implicit taxation are enormous. Although an exact calculation of marginal tax rates is complicated due to nonlinearities and other details of benefit formulas, the reason for the high rates is simple: the elderly must retire to obtain full benefits and full benefits are typically a very large fraction of the earnings enjoyed if one does not retire. Gruber and Wise (1999, Table 1, based on even more detailed computations of their coauthors) attempt to quantify the rates of implicit taxation for 11 countries. According to their calculations for the early 1990's, the "typical" implicit tax rate for "someone of retirement age" ranges from roughly 20% for Japan, U.S., and Canada, to more than 80% for Belgium and the Netherlands.[3]

Another way to appreciate the quantitative significance of the implicit taxation of elderly labor income by public pension programs is to notice the prevalence of 100%(!) marginal tax rates. Mulligan (1998) discusses in some detail a number of examples, including U.S. Social Security benefit formulas between 1939 and 1971, under which retirees lost *all* of their Social Security benefit if their earnings exceeded a rather low earnings limit by even one dollar. Other American examples of 100% marginal tax rates can be found prior to the Social Security Act in U.S. state administered Old Age Assistance programs, which typically implicitly taxed earnings at a 100 percent rate (Joint Committee 1966, pp. 26-27). Spain has one of several international examples, where their elderly

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[3]Gruber and Wise point out that, in any one country, marginal implicit rates vary with earnings, age, calendar year, and other variables. For a person of age t in the early 1990's, where t is between the early retirement age (age 60 in 9 of the 11 countries they study) and 69, they compute for a worker of median earnings the present value of public pension benefits foregone by delaying retirement one year, and express it as a fraction of earnings (after income and payroll taxes) for that year, a fraction τ_t which can be interpreted as an implicit tax rate. They sum τ_t between the early retirement age and $t = 69$, and I divide their sum by the number of years in the sum (10 years are in the sum for 9 of the 11 countries they study) to arrive at the "typical" implicit tax rate for "someone of retirement age" reported in the text.

are not allowed to collect a government pension if they earn any labor income at all (Boldrin et al 1997 p. 16, SSA 1997 p. 330) and those benefits are typically close to or more than what the pensioner would have earned after taxes if working (Boldrin et al 1997).

Perhaps these implicit taxes are not distortionary, because they are not enforced or because other government regulations prohibit people from changing their behavior in response to them? There are two reasons to be skeptical of such a claim. First, Gruber and Wise (1999) show that retirement *behavior* is highly correlated across countries and across age groups with the measured incentives. Second, the stated purpose of the implicit tax provision is often to encourage retirement (Sala-i-Martin 1996; Gruber and Wise 1999, p. 31).

Pensions are not the only public programs encouraging retirement. "Disability insurance" and "unemployment insurance" programs "essentially provide early retirement benefits before the official social security early retirement age" (Gruber and Wise 1999, p. 9) in many countries. Tax-favoring company pensions, mandatory defined benefit company pensions, and public health insurance are some other government policies that may substantially induce retirement.

I.B Marginal (Implicit + Explicit) Tax Rates are Highest for the Old

Perhaps it is unsurprising that public policies discourage work, since governments need to raise revenue, or may want to assist the poor. But another feature of public pension programs, and government policy in general, is that elderly work is discouraged more than young work. Hence, while payroll tax rates are paid by young and old workers and can be large in many countries – more than 10% in the U.S. and nearly 50% in Egypt, Italy, and the Netherlands – public pension benefit formulas in many countries substantially reduce the incentive to work beyond its reduction due to payroll and income taxation.

Income taxes, payroll taxes, and public pension benefits are not the only public policies discouraging work. Minimum wages, unemployment compensation, welfare payments, workweek restrictions, and other policies have the effect of discouraging work, and a full analysis of public policy and work incentives would include detailed calculations of the effects of these programs. However, two observations strongly suggest that, taken together, the various public policies tax elderly labor income at much higher marginal rates. First of all, a number of these programs – such as unemployment and welfare – affect work incentives for both elderly and young people. Often unemployment and welfare payments are most generous for the elderly, and implicitly tax elderly labor earnings at higher rates. Indeed, the unemployment insurance programs in Belgium, Finland, and other countries are hard to distinguish from public pension programs in terms of their intergenerational incidence and their age profile of marginal tax rates.[4] Second, it seems that, because of public pension programs, the prevalence of 100% and near-100% marginal tax rates is much higher among the elderly than among the young (as a consequence of tax and other policies) and, as a result, work is so much more prevalent among the young.

I.C Pensions Designed this Way Have Existed for Many Decades

For decades, Social Security benefit formulas have implicitly taxed labor income of the elderly. To prove this, Mulligan (2000) constructs a data set for the years 1958, 1975, and 1995 based on SSA reports (SSA, various issues). It was somewhat more common internationally in 1958 and 1975 for benefit formulas to induce retirement with the simpler formula making retirement a necessary condition for receiving public pension benefits (eg., the U.S. did so in 1958, but not in 1995). Delayed retirement credits and gradual phaseout of benefits with earnings were more common in 1995, so it might be said that retirement was induced more dramatically in 1958 and 1975. However, the size of the benefit foregone by the elderly worker has grown over time relative to what a retiree would have earned, so in this sense benefit formulas induce retirement more in recent years. More research is required to determine exactly how the incentive to retire has changed over the years in various countries, but it is clear that public pension benefits have for decades provided an important incentive to retire.

II. The Basic Model

II.A. Tastes and Technology

[4]For international examples, see SSA (1995) and Gruber and Wise (1999). Leimer (1998, pp. 16-17) reports results for the American DI program.

With these observations of public policies in mind, we ask whether publicly-induced retirement can alleviate labor market frictions. We consider a simple one-period extension of the standard Mortensen (1982) - Pissarides (2000) model. As in the standard search model of the labor market, there are two groups of agents: workers and firms. Workers are heterogeneous in exactly two dimensions. First, some workers are matched with an employer and some are unmatched. We refer to workers that begin the period matched with employers as "old" workers and the initially unemployed as "young" workers, and their population shares as λ and $(1-\lambda)$, respectively. [5] Second, workers also differ in their nonpecuniary costs of working.[6] To be specific, we let γ_i represent the cost of working for worker *i*. One may interpret this cost as the opportunity cost of working in terms of lost leisure time. These costs of working may vary across cohorts. The cost of working for old workers is described by the cumulative distribution function, $F_1(\gamma)$, while the cost for young workers is described by $F_0(\gamma)$. Both distributions have a lower support of 0. One interesting case has old workers having a higher value of leisure time than young workers, so that $F_0(\gamma) \leq F_1(\gamma)$. We let the productivity of each match be given by p. When worker *i* is matched with a firm, and work occurs, the total surplus created is $p - \gamma_i$.

Worker-firm matches are made in one of two ways. First, matches are part of the initial conditions for so-called "old" workers. Second, a young worker can be matched with a firm, or a unmatched firm with a young worker, by "search." Job search costs s for each worker and, for

[5]In Shimer (2001), all workers are infinitely-lived, but in each period a new generation of workers are born."Young" workers are those who were born in the recent past. Since young workers have had less time to participate in the labor market, they are more likely to be unemployed than "older" workers.

[6]Technically, this is a departure from the standard Mortensen-Pissarides model which provides a straightforward calculation of a "retirement rate." In the standard Mortensen-Pissarides model, workers are heterogeneous only in their employment status. All workers obtain the same utility from working and all the unemployed have the same value of leisure. Our model also adds heterogeneity within and across the initially employed and unemployed groups according to the way they value work (or leisure time). The across-group difference provides a rationale for retirement – the utility cost of work for the initially unemployed ("young" workers) is less than the utility cost of continued work for the initially employed group ("old" workers). The within-group differences provide a rationale for retirement of some (but not all) of the old, and thereby permiting calculation of a retirement rate in the interval (0,1).

simplicity, worker search costs nothing for firms. Jobs searchers and firms with vacancies are brought together according to a matching technology M. The matching technology, $M(U,V)$, denotes the aggregate number of matches as a function of the aggregate number of searchers U and aggregate number of vacancies V. The matching technology is stochastic and undiscriminating – namely, all searchers enjoy the same ex ante probability of a match $m = M/U$ and all employers posting a vacancy enjoy the same ex- ante probability of a match M/V. Since the matching technology exhibits constant returns to scale, we let $m = m(\theta) = M(1,V/U)$ where $\theta = V/U$ is the number of vacancies per worker and therefore may be viewed as the degree of labor market tightness.

Initial worker-firm matches can be dissolved, in which case there is no surplus associated with the initial worker. We interpret this situation as "retirement," with the retiree consuming leisure and his former employer participating in the aforementioned matching process by posting his vacancy. There are two possibilities in which young workers may consume leisure. The first, which might be called "unemployment," occurs when a worker searches for a job but does not find one. The second, which might be called "out of the labor force" occurs when a worker does not search at all.

II.B. Efficient Allocations

An efficient allocation is the aggregate surplus-maximizing list of retirees, job searchers, and firms posting vacancies, given the economy's matching technology and the costs of searching. An efficient allocation involves: (a) all unmatched employers posting their vacancies, (b) retirement for the initially matched workers with high nonpecuniary costs of work (relative to the others initially matched), and (c) job search among the initially unmatched workers with low nonpecuniary costs of work (relative to the others initially unmatched).

Let γ_1^* and γ_0^* represent the planner's choices for the critical values of the costs of working for the initially matched and unmatched respectively. Let $\Phi_1^* = F_1(\gamma_1^*)$ and $\Phi_0^* = F_1(\gamma_0^*)$ represent the planner's choices for the fractions of the initially matched and unmatched who retire and do not search, respectively. The social surplus W in the economy is, as functions of the number of old workers Φ_1 and young searchers Φ_0, given by:

$$W = \lambda\left[\int_0^{F_1^{-1}(\Phi_1)} p\, dF_1(\gamma) - \int_0^{F_1^{-1}(\Phi_1)} \gamma\, dF_1(\gamma)\right] +$$

$$M((1-\lambda)\Phi_0, \lambda(1-\Phi_1))\left[p - \frac{\int_0^{F_0^{-1}(\Phi_0)} \gamma\, dF_0(\gamma)}{\int_0^{F_0^{-1}(\Phi_0)} dF_0(\gamma)}\right] - (1-\lambda)\int_0^{F_0^{-1}(\Phi_0)} s\, dF_0(\gamma) \tag{1}$$

The first term is the aggregate surplus of the old, calculated by adding p - γ for each old person who works (i.e., who has $\gamma < F_1^{-1}(\Phi_1)$). The second term is the average surplus of successful young searchers, whose quantity are M and whose average surplus is in square brackets. The final term is the aggregate surplus of the unsuccessful young searchers, which is necessarily negative because they end up paying the search cost without producing anything.

From the above description of aggregate surplus, we can derive conditions for the efficient amount of retirement and job search. Efficient retirement is described by:

$$p = F_1^{-1}(\Phi_1^*) + \frac{m'}{\Phi_0^*}\left[\Phi_0^* p - \int_0^{F_0^{-1}(\Phi_0^*)} \gamma\, dF_0(\gamma)\right] \tag{2}$$

The left-hand side represents the marginal social surplus of less retirement while the right-hand side is the social marginal cost of less retirement. Of particular importance for our analysis is the last term, which is the effect of retirement by the marginal worker on the surplus of inframarginal workers because, as we show below, it will be ignored by a person choosing retirement solely to maximize the joint surplus of he and his employer.

The efficient amount of labor market participation by the young is given by:

$$[p - F_0^{-1}(\Phi_0^*)]\, m \;=\; [p - E_0\gamma]\, \frac{\lambda(1 - \Phi_1^*)}{(1 - \lambda)\,\Phi_0^*}\, m' + s$$

$$E_0\gamma \;\equiv\; \frac{\int_0^{F_0^{-1}(\Phi_0^*)} \gamma\, dF_0(\gamma)}{\Phi_0^*} \tag{3}$$

The efficient choice of job search recognizes that the marginal searcher has probability *m* of finding a job and hence probability *m* of enhancing surplus by *p* minus γ_0^*. But there are two social costs of search: the search cost *s* and the marginal searcher's effect on the surplus of others. This second cost is the product of the inframarginal searchers' potential surplus (p-$E_0\gamma$) and the effect $\theta m'$ of additional search on job finding by the inframarginal searchers.

II.C. Equilibrium Retirement, with Public Policy

In this section, we seek to determine the equilibrium allocations of workers to jobs and the unemployment rate, and model some simple public policies that might affect these decisions. We suppose that the government can observe whether a worker is matched and producing, and levies a tax T_1 on (or, if $T_1 < 0$, pays a subsidy to) each old person, regardless of his retirement status, and a tax T_0 on each young person. The government pays B_1 to (or, if $B_1 < 0$, taxes) old nonworkers (aka, retirees) and B_0 to young nonworkers.

We follow those in the literature and suppose that the postfisc surplus derived from work is split between workers and firms with shares β and (1- β), respectively. Each initially matched old worker is assumed to decide jointly with his employer whether or not to retire. Each young agent, initially unemployed, is assumed to decide on his own whether or not to search, based only on his expected costs and benefits. In other words, the search decision is *not* made jointly by the young, old, and the young's ultimate employer, because the essence of the search friction is that searchers and employers do not know who are their ultimate match partners. As we shall show, distortionary public policy can in principle help coordinate some of these decisions.

Given matching probabilities, the cost of searching, distributions of working costs, government policy (B_1, T_1, B_0, T_0), and a sharing parameter β, an equilibrium allocation is a list of retirees, job searchers, and firms posting vacancies, so that (i) the government budget balances, (ii) a job searcher's match probability is $M(U,V)/U$ and an employee searcher's match probability is $M(U,V)/V$, where U and V are the population measure of job searchers and employee searchers, respectively, (iii) a young person cannot improve his expected surplus by changing his decision to search or not, and (iv) an old person and his employer cannot improve their joint expected surplus by changing the old person's retirement status.

An equilibrium can be characterized algebraically as a pair of scalars Φ_1^* and Φ_0^* satisfying:

(i) $$\lambda T_1 + (1-\lambda) T_0 = \lambda(1-\Phi_1^*) B_1 + [1-\lambda-M(U,V)] B_0$$

(ii) $$U = (1-\lambda)\Phi_0^*,\ V = \lambda(1-\Phi_1^*)$$

(iii) $$p - F_1^{-1}(\Phi_1^*) = B_1 + \frac{M(U,V)}{V}(1-\beta)(p - E_0\gamma)$$

(iv) $$B_0 = m\beta[p - F_0^{-1}(\Phi_0^*)] + (1-m)B_0 - s$$

where $E_0\gamma$ is the average cost of work among those young people searching for a job.

(i) is the government budget constraint, where taxes are collected from all persons and benefits are paid to retirees and young nonworkers. (ii) accounts for U and V, the number of job searchers and of job vacancies. (iii) defines the marginal retiree who, together with his original employer, is indifferent between remaining employed and retiring (in which case benefit B_1 is collected and a vacancy is created). Notice that T_0 and T_1 do not enter (iii) because those particular taxes are collected (subsidies paid) regardless of labor market status, while B_1 is viewed as benefit of retirement from the point of view of a potential retiree and his employer because it is a outside source of income that, unlike T_1, is paid only conditional on retirement. Potential retiree and employer are maximizing joint surplus, so β enters (iii) only to the extent that it determines the employer's surplus after retirement. (iv) defines the marginal job searcher, who is indifferent between the employment benefit and the expected proceeds from search.

This algebraic characterization (i)-(iv) of an equilibrium permits easy proofs of our two main propositions.

Proposition 1 For any β, λ, $s \geq 0$, any distribution functions $F_0(\cdot)$ and $F_1(\cdot)$, and any homogeneous function $M(\cdot)$ defined on $[0,1]^2$, there exists a government policy (T_0, T_1, B_0, B_1) consistent with an efficient equilibrium.

Proof We prove Proposition 1 by constructing a government policy that makes the efficient Φ_1^* and Φ_0^* consistent with equilibrium. First, pick $B_1 = [m' - (1-\beta)m\frac{U}{V}](p - E_0\gamma)$, calculating U, V, and $E_0\gamma$ from an efficient allocation. Plugging this into equilibrium condition (iii), we have that the equilibrium retirement margin is efficient. Second, pick $B_0 = \beta\frac{\theta m'}{m}(p - E_0\gamma) - (1-\beta)\frac{s}{m}$. Plugging this into equilibrium condition (iv), we have that the equilibrium search margin is efficient. Third, pick any (T_0, T_1) satisfying the government budget constraint for the (B_0, B_1) calculated above.

Proposition 2 If the elasticity of $m(\theta)$ is constant and equal to 1- β, then a government policy consistent with an efficient equilibrium has $B_1 = 0$, and $B_0 \geq 0$.

Proof Use Proposition 1's formula for efficient B_1 to show that it is zero when the elasticity of $m(\theta)$ is constant and equal to 1- β. Use Proposition 1's formula for efficient B_0, and the equilibrium condition (iv), to show it is > (=) zero as the work cost of the marginal searcher is > (=) the work cost of the average searcher.

Proposition 1 says that efficient allocations are consistent with equilibrium with the right government policy. Proposition 2 explores a special case explored by Hosios (1990), where the sharing parameter β is related to the elasticity of the matching function. Hosios finds that the supply of job vacancies is efficient without taxes or transfers; we find that the supply of vacancies (aka, amount of retirement) is efficient without any government distortion of the retirement margin. Efficient search is induced by subsidizing young nonworkers, except in the special case (considered by Hosios) where the average and marginal searcher have the same surplus from work.

Propositions 1 and 2 are important for understanding the quantitative relationships between search and retirement, so we explore them further. Notice that, from the social point of view, there are three distortions of the equilibrium retirement decision. The first is the retirement subsidy (or tax, if $B_1 < 0$) seen on the right-hand-side of (iii). Second, the potential retiree and his employer put some value on creating a vacancy according to the *average* match rate M/V. When M is homogeneous, and the number of matches are at least somewhat elastic to the number unemployed, the average match rate exceeds the *marginal* match rate $\partial M/\partial V$ that is relevant from the social point of view. This second distortion tends to cause employers (in agreement with their potential retirees) to excessively encourage retirement. Third, equilibrium retirement decisions do not consider the creation of surplus for the unemployed group, which is the product of the marginal match rate and the average surplus $\beta(p - E_0\gamma)$ for those matched. By itself, this third distortion means that an equilibrium has too little retirement. The second distortion can overwhelm the third, as is the case when β is small and/or the gap between marginal and average match rates is large, or vice versa. The optimal retirement subsidy is positive when the third distortion dominates, and negative when the second dominates.

In order to isolate and quantitatively evaluate the third distortion, consider the limiting case of a linear matching function $M(U,V) = V$. Search is very efficient in this limiting case, because all vacancies costlessly find a match with probability one,[7] although inefficient in an important sense which we demonstrate below. Now equilibrium condition (iii) becomes (iii)′:

$$\text{(iii)}' \quad p - F_1^{-1}(\Phi_1^*) = B_1 + (1 - \beta)(p - E_0\gamma)$$

(iii)′ allows us to compute the efficient retirement subsidy (namely, that for which equilibrium and planned retirement coincide) in this case: $B_1 = \beta(p - E_0\gamma)$. In words, the efficient retirement subsidy is that surplus received by the employee who takes over the job of the retiree because that future employee is not at the table when employer and potential retiree make the retirement decision.

[7]For the sake of simplicity, we do not consider the case $V > U$, in which case the more reasonable linear matching function would be $M(U,V) = U$.

Or, using the optimal retirement condition for the planner facing a linear matching function, we see that the optimal retirement subsidy equates the disutility of work of the marginal retiree ($F_1^{-1}(\Phi_1^*)$) with the average disutility of work among the unemployed ($E_0\gamma$).

Although these formulas overstate the size of the optimal retirement subsidy that would be optimal with other matching functions (see the second distortion above), we explore some of its quantitative implications and use the results to clarify the more realistic calculation in Section III. Namely, is the retirement subsidy optimal under linear matching of similar magnitude to observed retirement subsidies? A precise answer requires precise estimates of the sharing parameter β and the average disutility of work among the unemployed ($E_0\gamma$), but notice how $B_1 = \beta(p - E_0\gamma)$ implies that an average *young* worker *with a job* who suffered a reduction in his after-tax earnings[8] in the amount B_1 would still have a nonnegative surplus from working. Since it is likely that, in reality, 50-80% reductions in earnings would eliminate the surplus – and then some – from working for a great many young workers (e.g., practically all women workers, if it is indeed the case that their labor supply is wage elastic), it seems the retirement subsidies of 50% and above seen in several European countries are excessive. Perhaps implicit elderly earnings tax rates on the order of 25-50% might be optimal when the matching function is linear, since we might expect (and often observe) a lot of young people to continue working even when wages are reduced by 25-50%.

III. Limits on the Optimal Retirement Incentive

As long as the matching function is nonlinear, the formula $B_1 = \beta(p - E_0\gamma)$ overstates the optimal retirement subsidy because it ignores the negative effect of a retirement on the matching success of other owners of vacant jobs. The optimal retirement subsidy for nonlinear matching functions is shown above in the proof of Proposition 1, which we rewrite below:

[8]Remember that a worker's surplus is less than his earnings; earnings are computed in our model by subtracting the employer surplus from worker productivity (ie, p - $(1-\beta)(p-E_0\gamma)$).

$$B_1 = (\beta - \alpha)\frac{M}{V}(p - E_0\gamma)$$
$$\alpha \equiv \left(1 - \frac{\partial M}{\partial V}\frac{V}{M}\right) \tag{4}$$

The first two terms show exactly how the optimal retirement subsidy is less in the nonlinear case. First, subtracted from β in parentheses is the elasticity of M with respect to U, which is proportional to the gap between the average and marginal match rates (M/V and $\partial M/\partial V$). This first reduction can be substantial, since α has been estimated in the range of 0.4 to 0.7.[9] In other words, the worker's share β has to exceed 0.4 if the optimal retirement subsidy were even to be positive! With β = 0.5, and the lower estimate of α = 0.4, the (β-α) alone is 80% smaller than β ((0.5-0.4)/0.5 = 0.2). In other words, even if M/V were arbitrarily close to one, and $E_0\gamma$ were zero (so that $\beta(p - E_0\gamma)$ were as large as employee earnings), the optimal implicit tax on elderly earnings is only 20%. Most of the countries studied by Gruber and Wise have implicit taxes in excess of this amount.

The second term is M/V, which of course is less than one with the nonlinear matching function, and reflects the fact that some retirements will create vacancies that go unmatched. To quantify this term, we need to further parameterize our model, and carefully distinguish stocks from flows. In particular, "additional" matches in our single period model are more realistically interpreted as matches that occur more rapidly in the presence of an additional vacancy than they would otherwise. In other words, all vacancies find a worker with very high probability if they wait long enough. $M/V < 1$ is a way to capture, in our static model, the fact that a new vacancy can expect to go unused for some period of time. If we let r denote the interest rate, t time, and δ the instantaneous hazard at which a vacancy finds a match, a continuous-time expression for M/V is:

[9]Blanchard and Diamond (1989) find α to be 0.4 for the U.S. and Layard, Nickell, and Jackman (1991) find it to be 0.7 for the U.K.

$$\frac{\int_0^\infty \left[\delta e^{-\delta t} \left(\int_t^\infty e^{-rs} ds \right) \right] dt}{\int_0^\infty e^{-rt} dt} = \frac{\delta}{r+\delta}$$

For a vacancy with productive capacity one, the numerator is the expected present value of production, accounting for the fact that a match may take some time to occur. The denominator is the present value of production if the match were instantaneous. Since the expected duration of a vacancy ($1/\delta$) is measured in months,[10] δ is much larger than r and, for quantitative purposes, we can treat M/V as one.

Table 1 presents calculations of the optimal retirement subsidy using the formula (4), and various values for the parameters α, β, and $E_0\gamma$. In order to facilitate comparisons with quantitative studies of taxes and labor supply, we express the optimal subsidy, and work disutility, as fractions of worker earnings. Worker earnings are computed in our model by subtracting the employer surplus from worker productivity (ie, earnings are p - $(1-\beta)(p-E_0\gamma)$).

[10]Ours and G. Ridder's (1992) calibration implies an average vacancy duration of about 45 days, while Blanchard and Diamond's (1989) implies one month.

Table 1: The Optimal Tax Rate on Elderly Earnings ($B_1/[p - (1-\beta)(p-E_0\gamma)]$, assuming $M/V \approx 1$)					
		young work disutility as a fraction of earnings, $E_0\gamma/[p - (1-\beta)(p-E_0\gamma)]$			
alpha	beta	0	0.2	0.4	0.6
0.4	0.5	0.20	0.16	0.12	0.08
0.5	0.5	0	0	0	0
0.6	0.5	-0.20	-0.16	-0.12	-0.08
0.7	0.5	-0.40	-0.32	-0.24	-0.16
0.5	0.6	0.17	0.13	0.10	0.07
0.5	0.7	0.29	0.23	0.17	0.11
0.5	0.8	0.38	0.30	0.22	0.15
0.5	0.9	0.44	0.36	0.27	0.18
0.5	1.0	0.50	0.40	0.30	0.20

The upper left cell reports the calculation discussed above: the optimal implicit tax rate is 20% when $\alpha = 0.4$, $\beta = 0.5$, and $E_0\gamma = 0$. The top half of the table shows how the optimal rate falls with α and $E_0\gamma$. The bottom half fixes $\alpha = 0.5$, and varies the employee's share β. Optimal tax rates are small unless we use β well in excess of 0.5. Even $\beta = 1$ cannot justify elderly implicit tax rates in excess of 50%, as observed in a number of European countries.

IV. Conclusions

Many countries around the world use public policy, especially social security programs, to induce their elder citizens to retire. An important motive behind such policy is to create job vacancies that can be occupied by the young. In this paper, we seek to evaluate the strength of the foundations underlying this rhetoric. To that end, we produce a relatively-standard search model with young and old workers and firms. That model allows for the possibility of significant frictions in the process of matching workers with firms. We ask whether policy-induced retirement can be part of an efficient labor market search and matching equilibrium. In other words, is there really any reason for governments to intervene in elderly labor markets on behalf of the young and the unemployed? Even when the model exhibits significant frictions, and inefficient labor market allocations without government intervention, it may be the case that there is *too much* retirement, in which case the optimal policy discourages retirement rather than encourage it. Other versions of the model imply that efficient equilibria are supported with no government subsidies (social security) to the elderly. Still other cases imply that an efficient equilibrium can be supported with a positive subsidy to the old, but that its size is much smaller than what real-world governments routinely provide. This is primarily because the planner takes into account the negative effect on aggregate matching possibilities of an additional vacancy, an effect the empirical literature suggests to be strong. The social security rhetoric, on the other hand, ignores this effect, over-emphasizing the beneficial effects of an additional vacancy on those searching for jobs. In short, many societies excessively induce retirement by the elderly, at least from the standpoint of efficiency as understood in standard search models.

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