NBER WORKING PAPER SERIES

RATIONAL SPECULATION AND EXCHANGE RATES

Margarida Duarte
Alan C. Stockman

Working Paper 8362
http://www.nber.org/papers/w8362

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2001

Rational Speculation and Exchange Rates
Margarida Duarte and Alan C. Stockman
NBER Working Paper No. 8362
July 2001
JEL No. F3, F4

ABSTRACT

In this paper we develop a general equilibrium model of exchange rates where expectations of future variables directly affect the current exchange rate through an "asset-market" term. This term, which results from the assumptions of incomplete asset markets and segmented product markets, does not appear in most models of exchange rates and it allows for changes in expectations about variables at t+1 to affect the date-t exchange rates without requiring changes in other contemporaneous variables. Therefore, the model has the potential to deliver changes in exchange rates, resulting from rational speculation, without much change in consumption allocations or goods' prices, making it consistent with the common view that exchange rates behave like asset prices.

To implement the idea that exchange rates respond to expectations about future economic conditions, we introduce a regime variable governing the covariance structure of shocks to productivity and money growth in each country. Changes in the information variable are intended to generate changes in home and foreign agents' perceptions of the relative risks of holding the nominal asset. The model is roughly consistent with the common view that exchange rates behave like asset prices. However, it does not generate a sufficient degree of rational speculation to explain either observed variation of risk premia in foreign exchange markets or observed variation in exchange rates.

Margarida Duarte
Federal Reserve Bank of Richmond

Alan C. Stockman
University of Rochester and NBER
alan@stockman.net

Rational Speculation and Exchange Rates*

Margarida Duarte
Federal Reserve Bank of Richmond

Alan C. Stockman
University of Rochester

March 2001

1 Introduction

The purpose of this paper is to propose a new approach to explaining exchange rates and to implement this approach within a specific model. A new approach is needed because old approaches have failed:

(1) Purchasing power parity performs miserably, so it cannot play any key role in a satisfactory explanation of exchange rates.

(2) The pioneering work of Richard Meese and Ken Rogoff two decades ago has held up remarkably well: with minor caveats, a simple random-walk model of exchange rates forecasts as well or better than alternative statistical models or statistical implementations of existing economic models.

(3) Exchange rates fail to follow the strong cyclical patterns predicted by most models in which the *same* shocks (whether monetary or real) drive both business cycles and exchange rates. Standard exchange-rate models based on sticky prices and monetary shocks can generate exchange-rate movements but typically predict a strong relationship between exchange rates and cross-country ratios of GDPs. A monetary shock simultaneously raises domestic real GDP (by more than it raises foreign GDP) and creates (tem-

* This is a preliminary version. The views expressed in this paper are those of the authors and not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

porary) depreciation of home currency. Consequently, these models almost generically predict a strong positive correlation between depreciations and (relative) business-cycle booms.[1] However, the data show almost *no* relationship of any kind between exchange rates and ratios of business cycles.[2] Chari, Kehoe, and McGrattan (2000), for example, develop a sophisticated, quantitative (calibrated) model of exchange rates based on monetary shocks operating through sticky prices. While their model succeeds well in matching many features of the data, it predicts a strongly counterfactual relation between exchange rates and international ratios of (detrended) GDP. DGE models based on technology shocks have the same kind of problem. A model that tries to generate both business cycles and exchange-rate changes with technology shocks almost invariably implies counterfactual correlations between exchange rates and business cycles.[3]

(4) As Flood and Rose have elegantly documented, the exchange rate appears to have "a life of its own," disconnected from other macroeconomic variables. Like stock prices or other asset prices, exchange rates show little relation to current or past macroeconomic variables or international-trade variables. The new approach we suggest in this paper is intended to address this puzzling fact.

(5) Practitioners in foreign-exchange markets typically believe that large exchange-rate swings are not justified by fundamentals. Most attribute those swings to speculation. Speculation in foreign exchange markets involves sales or purchases of interest-bearing assets denominated in some currency (with opposite transactions in assets denominated in another currency). When speculators sell yen-denominated assets to buy Euro-denominated assets, interest rates rise on yen-denominated assets and fall on Euro-denominated assets. These interest-rate changes can be interpreted as representing changes in risk premia. (Although the nominal return on a short-term treasury bill

[1] Some models, such as Obstfeld and Rogoff, 1995, do not have that implication for GDP, but have corresonding implications involving consumption.

[2] See Stockman (1998). Leonard and Stockman (forthcoming 2001) documents the joint statistical behavior of exchange rates and GDP-ratios in a nonparametric framework.

[3] These are far from the only serious criticisms of standard exchange-rate models. For example, identification of monetary shocks, like identification of technology shocks, poses a key problem for models of exchange rates as well as models of business cycles. Sticky-price models based on monetary shocks must confront the persistence puzzle: the problem that half-lives of exchange rates (even when statistically detrended in ways that reduce their half-lives!) are far longer than half-lives of business cycles (while the models imply that they should be about the same).

may be riskless, its real return is not.)

Risk premia on forward foreign-exchange markets are highly variable. Unlike exchange rates, evidence suggests that they have predictable components and are strongly correlated with expected changes in exchange rates.[4] Perhaps participants on foreign exchange markets have rational expectations and the compensation that they require for bearing risk corresponds to standard finance models (e.g. Hodrick, 1987). Or, perhaps, foreign exchange markets are dominated by noise traders or other irrational speculators (e.g. Krugman, 1989, or Krugman and Miller, 1993) whose actions implicitly reflect changes in risk premia that have little to do with those models. Regardless, the variation in implied risk premia is substantial.

We propose a new approach that focuses on effects of speculation - perhaps rationally reflecting new information or perhaps "irrationally" exuberant or fearful - and the resulting changes in risk premia by combining (a) the old idea that exchange rates are determined in asset markets with (b) macroeconomic models that incorporate international segmentation in product markets as in Dumas (1992), Sercu, Uppal, and van Hulle (1995), Ohanian and Stockman (1997), Sercu and Uppal (2000), and Obstfeld and Rogoff (2000). Loosely, this segmentation eliminates the "marginal rate of substitution equals relative price" conditions that would otherwise bind real exchange rates to contemporaneous *product market* conditions. Consequently, it allows asset markets to determine the (expected) *growth rate* of the exchange rate (through a forward-looking stochastic difference equation). Asset markets alone, however, do not tie down the *level* of the exchange rate path; that depends on other features of the model, including the possibility of future product-market arbitrage (which puts endpoint restrictions on

[4]Backus, Gregory, and Telmer (1993) estimated that standard deviations of the predictable components of the excess return from currency speculation, interpretable as risk premia average about 0.7% to 0.8% per month, and we have updates of their work Updating their results through 2000 produces similar (and slightly higher) standard deviations estimates of variability in risk premia. Similarly updated estimates show that the forward-premium puzzle is alive and well, implying (by Fama's argument) a strong negative correlation between the risk premium and the expected change in the exchange rate (expected depreciation), implying that currencies perceived to be riskier than others are more likely to be expected to appreciate. This strong relation between the risk premium and expected depreciation contrasts markedly the absence of relationships between exchange rates and other macro variables that Flood and Rose document, and the predictability of the risk premium contrasts markedly with the lack of much predictability of exchange rates by the variables that standard models suggest.

the forward-looking difference equation) and wealth effects of exchange rate changes (operating through household wealth constraints) that affect the marginal utilities that play the role of "parameters" in the difference equation. While traditional macroeconomic forcing variables, such as monetary shocks and productivity shocks, can (and must) play roles in the model, this framework naturally focuses attention on changes in expectations.

The idea that speculation plays a key role in exchange-rate volatility is, of course, not new. Krugman (1989) and Krugman and Miller (1993), for example, have stressed the role speculation in exchange-rate behavior.[5] Krugman and Miller, for example, argue that "the real-world case for exchange-rate stabilization has always rested on fears of excessive speculation, not on the microeconomic concerns of the optimal-currency-area approach." Assuming an exchange-rate equation of the form,

$$s = m + v + \gamma[E(ds)/dt - \beta s'],$$

where s and m denote the exchange rate and the money supply, v is a shift factor, and β is a term that reflects the foreign-exchange risk premium, they discuss a "decision by... investors to shift from domestic- to foreign- currency assets." An exogenous change in the risk premium, in their discussion, is associated with the (non-rational) behavior of "stop-loss" traders. A sudden decision by these traders to sell a currency – represented by a fall in the risk premium β , causes an abrupt depreciation. Although Krugman and Miller characterize speculation as "irrational," their argument and the equations they use to explain it apply equally to "rational" speculation that reflects new information relevant to the size of the risk premium. Missing from their analysis, however, is a full general-equilibrium model to justify their exchange-rate equation and to explain how the model avoids either (a) constraints on the exchange rate imposed by product-market equilibrium (such as equalities between marginal rates of substitution and relative prices involving the exchange rate), or (b) effects of exchange-rate changes on other macroeconomic variables (so that the extra exchange-rate volatility under a floating-rate system, relative to a fixed-rate system, does not translate into systematically higher volatility in macroeconomic and trade variables). That

[5] Of course, the idea that exchange rates reflect expectations about future fundamentals is commonplace in exchange-rate models, and goes back at least to the more sophisticated monetary models of exchange rates in the 1970s, such as Robert J. Hodrick, (1978?) and to Rudiger Dornbusch's famous overshooting model.

is, the model must be consistent with both sides of what Obstfeld and Rogoff (2000) term the "exchange-rate disconnect puzzle." The approach we outline and implement in this paper involves the same solution that Obstfeld and Rogoff suggest: arbitrage costs that segment product markets.[6]

Our motivation for focusing on speculation derives from both casual observation evidence and statistical evidence. Within two years after the Euro was introduced in January 1999 with high expectations and great fanfare, it had lost 30 percent of its value against the U.S. dollar. The Euro fell from \$1.18 in January 1999 to less than \$0.83 in October 2000, before rising again to \$0.95 three months later, in January 2001. Other seemingly inexplicable changes in exchange rates are almost daily occurrences. From 1991 to 1995, the Japanese yen rose in value from 130 yen per U.S. dollar to less than 90 in 1995, then fell to more than 140 yen per dollar in 1998 before rising to 101 in December 1999 and then falling to almost 120 by March, 2001. Is there a reasonable explanation of such episodes that does *not* rely upon speculation?

Standard models of exchange rates, however, *preclude* changes in speculators' perceptions of risk from playing any major role in explaining exchange rates. Models derived from individual optimization typically involve optimization conditions that equate the real exchange rate (defined as the relative price of foreign to domestic goods) with a marginal rate of substitution. In fact, that condition played the central role in the equilibrium approach to exchange rates proposed in Stockman (1980) and developed in Lucas (1982), and Svensson (1985).[7] Similar conditions have appeared more

[6] Krugman and Miller (1993) raise and attempt to answer the objection that "in exchange markets,... exchange rates are ultimately tied down by macroeconomic factors." They point out that "even in Mundell-Fleming models, currency depreciation raises money demand via a rise in net exports that produces an economic expansion. Does this not prevent the exchange rate from moving drastically because of selling by a relatively small group of investors? Our answer would be that in practice, the combination of sticky prices and lags in the responsive trade flows to relative prices means that any macroeconomic anchor to the exchange rate is on a very long chain. If the rate moves to a basically crazy level, this will eventually become apparent, but 'eventually' may mean several years." Their answer, however, basically introduces lags (in price adjustment and other behaviors) into the analysis, but does not necessarily save it from the Flood-Rose critique: that macroeconomic fundamentals (even with a lag) are vastly less volatile than exchange rates. The model we propose in this paper divorces (partially) the exchange rate from macroeconomic variables not only through sticky prices and pricing-to-market, but also through product market segmentation that replaces the "lags" that Krugman and Miller postulate, while leaving open the possibility or surviving the Flood-Rose critique.

[7] Also see Stockman (1987).

recently in other models. For example, the Obstfeld and Rogoff (1995) model has producers equate the marginal rate of substitution in production between foreign and domestic inputs with the relative price of those inputs, which involves *only* the exchange rate and predetermined nominal input prices. In a complete-markets model that implements the first-best allocation, that kind of equation (evaluated at the first-best allocation) completely determines the exchange rate, precluding *any* effects of speculation. In models with incomplete markets, that equation – while not fully determining the exchange rate – nevertheless ties it closely to other macroeconomic variables and severely reduces the scope for speculative effects. In the Obstfeld and Rogoff model, for example, that equation generates a tight relationship between exchange rates and cross-country ratios of consumption.

Substantial evidence has made it clear that these equations are grossly at variance with the data. Any good theoretical explanation of exchange rates must find a way to *avoid* these conditions. Two strands of recent literature can help accomplish this goal: (1) pricing to market, which removes the exchange rate from its role in the relative price that appears in the optimization condition (Betts and Devereux, 1996); and (2) product-market segmentation along the lines of Dumas (1992), Sercu, Uppal, and van Hulle (1995), Ohanian and Stockman (1997), Sercu and Uppal (2000), and Obstfeld and Rogoff (2000). Breaking the strong link between product markets and exchange rates (in one or both of these ways) opens the possibility for speculative activities on asset markets to play a key role in exchange-rate determination. While models based on shocks to macroeconomic fundamentals must essentially *ignore* the Flood-Rose critique (or exchange-rate disconnect puzzle), a focus on information shocks and speculation raises the possibility of *explaining* that disconnect.

This paper implements these ideas by examining a stochastic two-country general-equilibrium model with monopolistic competition and sticky prices, along the lines of Svensson and Wijnbergen (1989), Obstfeld and Rogoff (1995), Betts and Devereux (1996), and Chari, Kehoe, and McGrattan (2000), and by adding an information variable that generates "rational" speculation. We address the question of whether rational speculation when introduced into such a model, can generate changes in risk premia and exchange rates that match the data.

2 The Model

The world economy consists of two countries, denominated home and foreign, each specialized in the production of a composite traded good. We assume that the two markets for final products are segmented. For simplicity, we consider an extreme version of the incomplete-markets model in Ohanian and Stockman (1997), which combined the iceberg-cost model of segmentation (Sercu and Uppal, 2000) with shifts in risk premia. The extreme case we analyze here involves *complete* segmentation of markets for final products, as in Betts and Devereux (1996), and Chari, Kehoe, and McGrattan (2000).[8] This segmentation allows expectations about period $t+1$ variables to affect exchange rates at period t directly, *aside* from any interest-rate effects on money-demand . In our model, new information that affects the risk premium can induce a change in the period-t exchange rate *without*, in principle, creating large changes in period-t macroeconomic variables.

Both countries are subject to shocks to productivity levels and to growth rates of money supplies. The processes for these exogenous shocks, described below, depend on the realization of an information, or regime, variable. In what follows, we describe the home country's economy. The foreign country's economy has an identical structure, with all foreign variables denoted by an $*$.

2.1 Households

Preferences. The lifetime expected utility of the home representative household is

$$U_t = E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, l_t, \frac{M_t}{P_t} \right) \right]$$

where E_0 denotes the mathematical expectation conditional on information available in period $t = 0$, $\beta \in (0, 1)$ is the discount rate, and u is the momentary utility function, assumed to be concave and twice continuously differentiable. The household's instantaneous utility depends positively on c_t, an index of consumption to be defined below,and real money balances, $\frac{M_t}{P_t}$, where M_t is nominal balances held at the beginning of period t and P_t is a

[8] In ongoing work, we are examining a related model with iceberg costs of arbitraging product markets across countries.

consumption price index for period t. Instantaneous utility depends negatively on labor effort, l_t.

Consumption and Price Indexes. There is a continuum of domestic goods indexed by $i \in [0,1]$ and a continuum of foreign goods indexed by $j \in [0,1]$, which are imperfect substitutes in consumption. Let the composite goods $c_{h,t}$ and $c_{f,t}$ be defined as

$$c_{h,t} = \left(\int_0^1 c_{h,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \text{ and } c_{f,t} = \left(\int_0^1 c_{f,t}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \ \theta > 0,$$

where $c_{h,t}(i)$ and $c_{f,t}(j)$ denote date t domestic consumption of the home and the foreign goods of types i and j, respectively and θ denotes the elasticity of substitution between any two goods produced in the same location.

The consumption index c_t is defined as

$$c_t = \left[\omega^{\frac{1}{\gamma}} c_{h,t}^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} c_{f,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \ \gamma > 0 \text{ and } \omega \in (0,1), \tag{1}$$

where the parameter γ represents the elasticity of substitution between the two composite goods, $c_{h,t}$ and $c_{f,t}$, and the weight ω determines the household's bias for the domestic composite good.

Let $P_{h,t}(i)$ and $P_{f,t}(j)$ be the home-currency prices of the home and foreign goods of types i and j, respectively. Given these prices, the consumption-based money price index P_t is defined by

$$P_t = \left[\omega P_{h,t}^{1-\gamma} + (1-\omega) P_{f,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \tag{2}$$

where the price indexes $P_{h,t}$ and $P_{f,t}$ for each composite good are given by[9]

$$P_{h,t} = \left(\int_0^1 P_{h,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \text{ and } P_{f,t} = \left(\int_0^1 P_{f,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

[9] The price indexes P, P_h, and P_f are defined as the minimum expenditure necessary to buy one unit of composite goods c, c_h, and c_f, respectively, taking as given the prices for individual goods $P_h(i)$ and $P_f(j)$, i, $j \in [0,1]$. For example, P is the value of

$$\min_{c_h, c_f} P_h c_h + P_f c_f$$

subject to $c = \left[\omega^{\frac{1}{\gamma}} c_h^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} c_f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} = 1$, for given P_h and P_f.

Taking prices for all individual goods as given, each period the consumer allocates optimally a given level of total consumption among the differentiated goods. This allocation problem yields the demand functions[10]

$$c_{h,t}(i) = \omega \left(\frac{P_{h,t}}{P_{h,t}(i)} \right)^{\theta} \left(\frac{P_t}{P_{h,t}} \right)^{\gamma} c_t \tag{3}$$

and

$$c_{f,t}(j) = (1-\omega) \left(\frac{P_{f,t}}{P_{f,t}(j)} \right)^{\theta} \left(\frac{P_t}{P_{f,t}} \right)^{\gamma} c_t. \tag{4}$$

Household's Budget Constraint. Home and foreign households can trade nominally riskless discount bonds denominated in home and foreign currencies. Let Q_t denote the time t price (in home currency units) of one discount bond paying with certainty one unit of home currency at $t+1$, and let B_{t+1} denote the number of these bonds held by the home household between time t and $t+1$. Similarly, let Q_t^* denote the time t price (in foreign currency units) of one discount bond paying with certainty one unit of foreign currency at $t+1$ and let D_{t+1} denote the number of these bonds held by the home household between time t and $t+1$. To rule out equilibria which admit unbounded borrowing, or Ponzi schemes, we impose exogenous upper bounds, a_t and a_t^*, on the number of one-period bonds that a household can issue.[11]

[10] Formally, the problem

$$\max_{c_h, c_f} c = \left[\omega^{\frac{1}{\gamma}} c_h^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} c_f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

subject to $P_h c_h + P_f c_f = Z$, yields demand functions for c_h and c_f that depend on c and the price ratios $\frac{P}{P_h}$ and $\frac{P}{P_f}$, respectively. The problem

$$\max_{c_h(i)} c_h = \left(\int_0^1 c_h(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

subject to $\int c_h(i) P_h(i) di = Z$, and the analogous problem for foreign goods, yields demand functions for each differentiated good as a function of consumption of the composite good and the ratio of composite and individual good prices.

[11] The borrowing constraint is time dependent, reflecting the fact that the model is nonstationary. We assume the borrowing constraint to be constant in the stationary version of the model.

The household's intertemporal budget constraint, in units of home currency, is

$$P_t c_t + M_t + Q_t B_{t+1} + e_t Q_t^* D_{t+1} \leq P_t w_t l_t + M_{t-1} + B_t + e_t D_t + \Pi_t + P_t T_t, \quad (5)$$

where T_t denotes real transfers paid from the domestic government (which can be negative in the case of taxes), Π_t represents profits of domestic firms (which we assume to be owned by the domestic household) and $P_t w_t l_t$ represents nominal labor earnings.

Summarizing, the household's optimization problem is described by

$$\max_{c_t, l_t, B_{t+1}, M_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, l_t, \frac{M_t}{P_t} \right) \right] \quad (6)$$

subject to

$$P_t c_t + M_t + Q_t B_{t+1} + e_t Q_t^* D_{t+1} \leq P_t w_t l_t + M_{t-1} + B_t + e_t D_t + \Pi_t + P_t T_t, \ \forall t \geq 0$$

$$B_{t+1} \geq -a_t, \ \forall t \geq 0$$

$$D_{t+1} \geq -a_t^*, \ \forall t \geq 0$$

$$B_0, \ D_0 \text{ and } M_{-1} \ given.$$

2.2 Firms and Market Structure

The production function for each intermediate good i is given by $z_t F(l_t(i))$, where $l_t(i)$ represents labor input, z_t is an aggregate (country-specific) productivity shock and F is a production function displaying decreasing returns to scale. Because all goods are imperfect substitutes in consumption, each individual firm has some market power determined by the parameter θ.

We assume that, due to high costs of arbitrage to consumers, each individual monopolist can price-discriminate across countries. Furthermore, we assume pricing-to-market: that firms set prices (separately) in the currencies of each set of buyers.[12] Finally, we assume that prices must be set one period in advance and cannot be revised until the following period. Thus, the home monopolist sets $P_{h,t}(i)$ and $P_{h,t}^*(i)$ optimally at the end of period $t-1$, and these prices cannot be changed during period t.

[12] See Devereux (1997) for a discussion of evidence on pricing-to-market.

The price setting problem of monopolist i is then to maximize expected profits conditional on $t-1$ information, by choosing $P_{h,t}(i)$ and $P^*_{h,t}(i)$. That is, firm i solves

$$\max_{P_{h,t}(i),P^*_{h,t}(i)} E_{t-1}\left[\rho_t \Pi_t(i)\right] \tag{7}$$

subject to $z_t F(l_t(i)) = c_{h,t}(i) + c^*_{h,t}(i)$ and the downward sloping demand functions for $c_{h,t}(i)$ and $c^*_{h,t}(i)$. The term ρ_t denotes the pricing kernel used to value date t profits, which are random as of $t-1$.[13]

Date t profits of monopolist i (in home currency units), $\Pi_t(i)$, are given by:

$$\Pi_t(i) = P_{h,t}(i)\, c_{h,t}(i) + e_t P^*_{h,t}(i)\, c^*_{h,t}(i) - P_t w_t l_t(i),$$

where w_t denotes real wages in units of consumption good c. Note that $P_{h,t}(i)$ and $P^*_{h,t}(i)$ are denominated in units of home and foreign currency, respectively. The country's nominal exchange rate in period t, e_t, converts the revenues from sales in the foreign country into home currency.

2.3 Government

The government issues the local currency, has no expenditures and runs a balanced budget every period. Therefore, nominal transfers are given by

$$P_t T_t = M_t - M_{t-1}.$$

The domestic money stock evolves according to

$$M^s_t = (1+g_t)\, M^s_{t-1},$$

where g_t is a random variable to be described later.

3 Bond Markets, FX Markets, and Risk Premia

Note that bond markets do *not* determine exchange rates through an interest-parity condition. Indeed, while covered interest parity is an arbitrage condition, uncovered interest parity typically would not hold in our model. Instead, the exchange rate equation follows from the optimality conditions of

[13] Because all firms are owned by the representative consumer, it follows that in equilibrium ρ_t equals $\beta \frac{u_{1,t}}{u_{1,t-1}} \frac{P_{t-1}}{P_t}$, the intertemporal marginal rate of substitution in consumption between periods $t-1$ and t.

households, with different intertemporal consumption profiles, for holding the *same* nominal bonds.

To see these connections, rewrite the first-order conditions for bonds and the exchange rate equation as

$$Q_t \lambda_t \geq \beta E_t \left[\lambda_{t+1}\right] \tag{8}$$

and

$$Q_t \frac{\lambda_t^*}{e_t} \geq \beta E_t \left[\frac{\lambda_{t+1}^*}{e_{t+1}}\right], \tag{9}$$

where λ_t represents the nominal marginal utility of consumption of the home household, $\frac{u_c(t)}{P_t}$, and λ_t^* is the analogous term for the foreign household. Similar conditions for foreign bond holdings are

$$e_t Q_t^* \lambda_t \geq \beta E_t \left[e_{t+1} \lambda_{t+1}\right] \tag{10}$$

and

$$Q_t^* \lambda_t^* \geq \beta E_t \left[\lambda_{t+1}^*\right]. \tag{11}$$

Note that equation (8) or equation (9) holds with equality when the household is not borrowing-constrained. Because there is no upper bound on the quantity of home bonds that can be purchased, and this asset has zero net supply worldwide, it follows that at all times at least one of the two households is not borrowing constrained. Therefore, at least one of the above equations always holds with equality, allowing us to always determine the bond price, Q_t. A similar argument applies to Q_t^* and equations (10) and (11).

Using (10) and (11), and assuming interior solutions, we obtain the following expression for the nominal exchange rate

$$e_t = \frac{\lambda_t^*}{\lambda_t} \frac{E_t \left[e_{t+1} \lambda_{t+1}\right]}{E_t \left[\lambda_{t+1}^*\right]}. \tag{12}$$

The first-order conditions above also imply $\frac{E_t[e_{t+1}\lambda_{t+1}]}{E_t\left[\lambda_{t+1}^*\right]} = \frac{E_t[\lambda_{t+1}]}{E_t\left[\frac{\lambda_{t+1}^*}{e_{t+1}}\right]}$.

Note that the nominal exchange rate depends explicitly on expectations about future variables. Moreover, this dependence reflects more than the usual effects of expectations operating through nominal interest rates on the demand for money (as in "monetary models" of exchange rates). This feature of the model results from the combination of asset-market incompleteness and product market segmentation in the model. If the model were to include a complete set of state-contingent nominal assets (as in Sercu and Uppal, 2000), then the nominal exchange rate would be determined every period by the optimal risk-sharing condition $e_t = \frac{P_t}{u_{1,t}} \frac{u^*_{1,t}}{P^*_t}$, which equalizes nominal marginal utilities of consumption across countries every period. If product markets were not segmented, then the equilibrium nominal exchange rate would then satisfy the law of one price (for the final consumption good) each period (and would be fully determined by the equilibrium in product markets). The combination of asset market incompleteness and product market segmentation implies that the nominal exchange rate is determined (partly) by international trade in nominal bonds, and therefore depends on expectations of future variables.

The forward exchange rate, f_t, must satisfy covered interest parity (which is a no-arbitrage condition):

$$\frac{f_t}{e_t} = \frac{Q^*_t}{Q_t}.$$

Equations (8) and (10) imply that the forward price of foreign exchange is

$$f_t = \frac{E_t\left[e_{t+1}\lambda_{t+1}\right]}{E_t\left[\lambda_{t+1}\right]}.$$

Defining the risk premium on foreign assets from the perspective of home households as $rp_t = f_t - E\left[e_{t+1}\right]$, it follows that

$$rp_t = \frac{cov\left(e_{t+1}, \lambda_{t+1}\right)}{E_t\left[\lambda_{t+1}\right]},$$

and (12) can be rewritten as

$$e_t = \frac{\lambda^*_t}{\lambda_t} \frac{E_t\left[\lambda_{t+1}\right]}{E_t\left[\lambda^*_{t+1}\right]} \left(rp_t + E_t\left[e_{t+1}\right]\right).$$

This expression shows that the exchange rate depends on the home household's perception of the relative risk of holding the two nominal assets, rp_t. It also shows that changes in the risk premium cause changes in the exchange rate. Moreover, new information that leads households to revise their perceptions of risk can affect the exchange rate *without*, in principle, affecting (much) current macroeconomic variables. That observation is the source for optimism that an approach to exchange rates based on speculation can avoid what Obstfeld and Rogoff (2000) term the exchange-rate-disconnect puzzle.

The measured size of variation in the risk premium, while large, is smaller than variation in exchange rates themselves. Estimates of standard deviations of U.S. dollar risk premia from Backus, Gregory, and Telmer (1993) range from 0.36 percent per month against the Canadian dollar to 0.93 percent per month against the British pound, with a mean standard deviation of 0.70 percent per month and a median of 0.78. These high standard deviations generate large expected returns to speculators. The simple foreign-exchange investment strategy discussed by Backus, Gregory, and Telmer, based solely upon the sign of the forward premium, $(f_t - e_t)/e_t$, yields Sharpe ratios (ratios of mean returns to standard deviations of returns) ranging from 0.17 to 0.29, significantly higher than Sharpe ratios of around 0.14 for investments in the stock market. Using a sample of data on forward and spot exchange rates for France, Germany, Japan, Switzerland, and the UK, against the U.S. dollar, from June 1973 through December 2000, we calculate that standard deviations of predictable excess returns - using only lagged forward premia for predictions - range from 0.63 percent per month against the Japanese yen to 0.90 percent per month against the Swiss franc, with a mean of 0.76 percent per month and a median of 0.80. The first-order stochastic difference equation for the exchange rate generated by the model has the potential to amplify this variation in risk premia, depending on how the other terms in that equation covary with the risk premium. Apparently, some amplification of this variability will be essential for a successful explanation of exchange-rate variability based on changes in speculators' perceptions of risk.

3.1 Generating Changes in Risk Premia

To implement the idea that exchange rates respond to changes in asset market conditions (that is, to changes in rp) we introduce an information variable, Ω_t, that affects rp_t, that is, the household's perception of the relative risk of holding the two assets. For simplicity, we assume that this information variable takes two possible values and therefore induces two possible levels of the risk premium.

One might be tempted to model changes in the risk premia via changes in the cross-country covariance of monetary shocks. However, changes in that covariance *cannot* generate changes in the foreign-exchange risk premium. To see why, suppose we define the two regimes such that $cov\,(m_t, m_t^*) > 0$ if

$\Omega_t = 1$ and $cov\,(m_t, m_t^*) < 0$ if $\Omega_t = 2$.

Consider a change in Ω_t from 1 to 2. Because this raises the variability of the exchange rate, we expect both $cov\,(\lambda_{t+1}, e_{t+1})$ and $cov\left(\lambda_{t+1}^*, \frac{1}{e_{t+1}}\right)$ to rise. Foreign assets then become relatively riskier to home households (because they tend to give a higher return when marginal utility of consumption is lower) while home assets also become more risky to the foreign household.[14] This increase in the foreign exchange risk premium translates directly into a reduction of the right hand side of equations (9) and (10)[15]: in the new regime, next period's benefit from holding the other country's bond is lower than in the previous regime.

However, equations (11) and (10) - home and foreign households' first-order conditions for foreign bonds - show that the expected benefit of holding the foreign bond falls *only* for the home household. That household would be willing to hold the foreign bond only if the exchange rate *falls*. Similarly, equations (9) and (8) - the first-order conditions for home bonds - imply that the expected benefit of holding the home bond falls for foreign households (and not domestic households). They would be willing to hold the home bond in the new equilibrium only if the exchange rate *rises*.

Consequently, if the initial equilibrium in regime 1 is characterized by interior solutions for bond holdings, then regime 2 cannot have such an interior solution, as equations (8) to (11) cannot all hold with equality. This reflects the fact that a change in $cov\,(m_t, m_t^*)$ makes the home bond relatively *less* risky to home households but relatively *more* risky to foreign households..

We conclude, therefore, that a regime shift in our model should affect *all* households' perception of the relative risk in the same direction. In other words, a switch in regime must imply opposite effects in $cov\,(\lambda_{t+1}, e_{t+1})$ and $cov\left(\lambda_{t+1}^*, \frac{1}{e_{t+1}}\right)$. We therefore model regime shifts as changes in the covariance between monetary shocks and productivity shocks within a country (holding variances and cross-country covariances fixed). While the model outlined above includes completely-predetermined nominal prices, shifts in

[14] A positive money shock in the home country raises home consumption, reducing home marginal utility, as it causes *foreign*-currency *appreciation.* As a result, foreign-currency bonds become riskier to home households: they pay off well in these states of the world - when home consumption is already high – but pay off poorly in the opposite states (following negative home monetary shocks) in which home consumption is low.

[15] Assuming that this change in regime does not affect the variables' first moments, but it only affects second moments.

the within-country covariance of monetary and productivity shocks can change risk premia only if prices are *not completely* predetermined. Consequently, we modify the model so that prices are only partially preset. This corresponds to a model in which a subset of firms can re-adjust nominal prices after realizations of current shocks. Details appear in Appendix 1.

Each period, a realization of the regime variable Ω_t determines the covariance between shocks to productivity and the money supply within each country. The variable Ω_t is perfectly observable by all households at the beginning of period t. Because it is persistent, its realization affects the probability distribution of period's $t+1$ variables, which in turn affect the current exchange rate, e_t. Ideally, we would like to model changes in the risk premium as reflecting information relevant to predicting a *future* regime shift.[16] One could model this with an information variable that signals future shifts in regime. However, for computational simplicity, the results reported below abstract from a separate information variable; instead, the regime variable conveys information about the future because it is persistent; consequently, changes in regime affect the risk premia set in asset markets.[17]

Below, we measure the effects on current nominal and real exchange rates of a change in the information variable. In the numerical implementation, we restrict agents to trade only the home bond, by setting $D_{t+1} = 0$, $\forall t$. The particular relation between the information variable and the covariance structure of productivity and money shocks that we study in this paper is described in Section 5.

We focus on the symmetric and stationary equilibrium. That is, we focus on the equilibrium in which all firms located in the same country make the same choices and the endogenous variables are stationary functions of the current state of the world (to be defined below). To make the economy stationary, we deflate all nominal variables by the level of the relevant money supply[18] and restrict attention to Markov stochastic processes for all exogenous shocks, z, z^*, g, g^*, and Ω.

The aggregate state of the world when the pricing decisions are made (before the realization of current shocks) is fully characterized by the realization

[16] In ongoing work, we are attempting to separate the information and regime variables.

[17] To the extent that a regime shift affects current allocations and other macroeconomic variables, this modeling strategy reduces the ability of the model to replicate the facts of the "exchange-rate disconnect puzzle."

[18] The nominal exchange rate is deflated by the ratio of foreign to home money supplies. Let nominal variables in the stationary model be denoted with a hat, $\widehat{x}$.

of the shocks in the previous period, $\lambda_{-1} \equiv \left(z_{-1}, z^*_{-1}, g_{-1}, g^*_{-1}, \Omega_{-1}\right)$ and by the distribution of wealth between the two countries, $\widehat{B}$. Let $s \equiv \left(\lambda_{-1}, \widehat{B}\right)$ denote the aggregate state for the monopolists. Consumers make their choices after the realization of current period shocks. Consequently, the relevant aggregate state of the world for their decisions also includes these shocks; we denote this state by $s^c = (s, \lambda)$.

A stationary and symmetric equilibrium for this economy is defined as a collection of:

- optimal decision rules for the home and foreign consumers,[19] $l(s^c)$, $B'(s^c)$, $M(s^c)$, $c_h(s^c)$, $c_f(s^c)$ and similarly for the foreign consumer;
- optimal pricing rules for home and foreign firms, $P_h(s)$, $P^*_h(s)$[20] and similarly for the foreign firm;
- equilibrium wage rates $w(s^c)$, $w^*(s^c)$ and
- equilibrium bond price $Q(s^c)$ and nominal exchange rate $e(s^c)$

that satisfy the following conditions:

i) consumers' decision rules solve the consumers' problem,

ii) firms' pricing rules solve the firms' problem and

iii) market clearing conditions for bond and money markets hold.[21]

In this simplified version of the model, the Euler equations for the home bond imply that

$$e_t = \frac{\lambda^*_t}{\lambda_t} \frac{E_t\left[\lambda_{t+1}\right]}{E_t\left[\frac{\lambda^*_{t+1}}{e_{t+1}}\right]}. \tag{13}$$

Let $e_{1,t}$ denote the term $\frac{\lambda^*_t}{\lambda_t}$ in e_t and let $e_{2,t}$ denote the expectation term $\frac{E_t[\lambda_{t+1}]}{E_t\left[\frac{\lambda^*_{t+1}}{e_{t+1}}\right]}$. Note also that in this version of the model, f_t cannot be determined by covered interest parity. See Appendix 2 for a description of how the risk premium is computed.

[19] For notational simplicity, the hats will be omitted hereafter.

[20] In the symmetric equilibrium all firms located in the same country make the same pricing decisions. We therefore drop the firm index.

[21] Equilibrium in labor and goods markets was already imposed in the firm's problem.

4 Exchange Rate Level

The first-order stochastic difference equation (13) determines the relation between the current level of the exchange rate and expectations of the future exchange rate (interacting with other variables). The remainder of the model interacts with this equation to determine the *levels* of current and future exchange rates. A key factor in the remainder of the model involves the relative wealth of the two countries at date t. (As noted above, with complete markets, the ratio of expectations would drop out of equation (13), preventing speculation in asset markets from influencing the exchange rate.)

The simplest way to understand this interaction between equation (13) and the remainder of the model is to simplify the model by assuming, for the moment, a very simple set of behavioral responses. In particular, consider a two-period nonstochastic model, with $t = 1, 2$, for given initial conditions $B_1 = -B_1^* \neq 0$, M_0, and M_0^*, and terminal conditions $B_3 = B_3^* = 0$.

The home representative household's intertemporal budget constraint can be rewritten as

$$B_1 + \varphi_1 + Q\varphi_2 = 0$$

where

$$\varphi_t \equiv P_t w_t l_t + M_{t-1} + \Pi_t + P_t T_t - M_t - P_t c_t,$$

and the intertemporal budget constraint for the foreign representative household can be rewritten as

$$B_1^* + e_1 \varphi_1^* + e_2 Q \varphi_2^* = 0$$

where

$$\varphi_t^* \equiv P_t^* w_t^* l_t^* + M_{t-1}^* + \Pi_t^* + P_t^* T_t^* - M_t^* - P_t^* c_t^*.$$

Assume that φ_t, φ_t^*, and Q are fixed, and approximate the exchange-rate equation (13) by

$$e_1 = \Theta e_2, \tag{14}$$

for some parameter Θ. Although this simplified set of equations is nonstochastic, we can loosely identify a change in the parameter Θ with a change in the real risk of holding nominal bonds. Because the real return on a nominal bond is proportional to the inverse of the inflation rate, an increase in Θ corresponds to a rise in the risk premium. That is, it corresponds to an increase in the risk of holding home-currency-denominated bonds (which, in the complete model, is due to a rise in the covariance between the marginal utility of consumption and the inverse of the inflation rate). (An increase in the parameter Θ can also be identified with a *fall* a risk of holding *foreign* nominal bonds, since the model is symmetric in the two countries.) Such an increase in home-currency risk (or fall in foreign-currency risk) requires an offsetting increase in the rate of home-currency appreciation (or fall in the rate of depreciation), i.e., an increase in Θ.

Solving for the exchange rate from the foreign intertemporal budget constraint and equation (14), and using the domestic budget constraint, we obtain

$$e_2 = \frac{-(\varphi_1 + Q\varphi_2)}{\Theta\varphi_1^* + Q\varphi_2^*}$$

and

$$e_1 = \frac{-\Theta(\varphi_1 + Q\varphi_2)}{\Theta\varphi_1^* + Q\varphi_2^*}.$$

A rise in Θ affects the exchange rate in each period, raising it at $t = 1$ and reducing it at $t = 2$. If the home country is a net international creditor at the beginning of the first period (implying that both the numerators and the denominators in the exchange-rate expressions are positive), the extent to which an increase in Θ reduces the *future* exchange rate is proportional to the share of initial debt that the foreign country repays in the first period,

$$\frac{\Theta}{e_2}\frac{de_2}{d\Theta} = \frac{\Theta\varphi_1^*}{\Theta\varphi_1^* + Q\varphi_2^*},$$

so the change in the *current* exchange rate depends inversely on that share.

Extending the argument above to $N \geq 2$ periods, it is easy to see that the longer the household's horizon and the smaller the share of initial debt repaid in the first period, the smaller the change in the future exchange rate and the larger the change in the current exchange rate. Similarly, the longer-lasting the change in risk, the larger the changes in future exchange rates and the smaller the effect on the current exchange rate.

This simplified set of equations illustrates the interaction between the effects of the exchange-rate difference equation (13) and the rest of the model, for which the exchange rate matters because it affects the value of nominal debt in the households' budget constraints (in particular, it affects the value of domestic-currency debt in the foreign household's budget constraint, and foreign-currency debt in the home household's constraint). The example also shows incomplete markets are necessary to generate a unique equilibrium exchange rate (as mentioned earlier), because allocations would not depend on individual households' budget constraints in a complete-market model; instead, complete contingent securities would provide, on a state-by-state basis, the resources to finance optimal allocations. Consequently, the exchange rate would appear in the model *only* in the difference equation (13). While the simplified set of equations can help illustrate these points, it relies on a very loose approximation and ignores households' optimal behavioral responses. The next section returns to the more general model.

5 Calibration

We study the properties of this economy by approximating numerically the stationary and symmetric equilibrium of the model. In this section we specify the functional forms and the parameter values used in solving the model. Our calibration assumes that the world economy is symmetric, implying that both countries share the same specific functional forms and parameter values. Moreover, we assume that each time period corresponds to one quarter.

Preferences. The momentary utility function is given by:

$$u\left(c, l, \frac{M}{P}\right) = \frac{1}{1-\sigma}\left[\left(ac^{\eta} + (1-a)\left(\frac{M}{P}\right)^{\eta}\right)^{\frac{\zeta}{\eta}}(1-l)^{1-\zeta}\right]^{1-\sigma},$$

where $\sigma > 0$, $\eta > 0$, $\zeta \in (0,1)$, and $a \in (0,1)$.

TABLE 1
Preference Parameters

β	σ	η	ζ	a	γ	ω
0.99	2	−1.56	0.32	0.73	1.5	0.85

The preference parameter values used are described in Table 1. See Duarte (2000) for a discussion of these values.

Production function. We assume that each firm operates a decreasing returns to scale production function $F(l) = zl^{\alpha}$. We set α equal to 2/3.

Price setting. We used different values for the parameter λ in equation (17), which determines the degree to which firms can adjust their preset prices, after uncertainty is resolved, to the prices that would occur in the flexible price equilibrium. We will specify the value used when presenting the results in the next section.

Exogenous shocks. The information variable, Ω, is assumed to determine the covariance between shocks to productivity and money growth in both countries. This variable can take two values, Ω^1 and Ω^2, and evolves according to the symmetric Markov process with transition probabilities: $\pi_{ii} = \pi$ and $\pi_{ij} = 1 - \pi$, $i, j = 1, 2$. In our benchmark calibration, we set $\pi = 0.9$.

The vector of exogenous shocks to productivity and money growth rates, $\mathbf{s}_t = (z_t, z_t^*, g_t, g_t^*)$, follows the autoregressive process

$$\mathbf{s}_t = A\mathbf{s}_{t-1} + \boldsymbol{\varepsilon}_t \tag{15}$$

where A is a (4×4) matrix of coefficients and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_t|\Omega_t)$. Note that the variance-covariance matrix $\Sigma_t|\Omega_t$ depends on the realization of the regime variable in period t, Ω_t. In particular,

$$\Sigma_t|\Omega_t = \begin{bmatrix} \sigma_z^2 & 0 & \sigma_{zg,t} & 0 \\ 0 & \sigma_{z^*}^2 & 0 & \sigma_{z^*g^*,t} \\ \sigma_{zg,t} & 0 & \sigma_g^2 & 0 \\ 0 & \sigma_{z^*g^*,t} & 0 & \sigma_{g^*}^2 \end{bmatrix}$$

where

$$\sigma_{zg,t} = \begin{cases} \sigma_1 & \text{if } \Omega_t = \Omega^1 \\ \sigma_2 & \text{if } \Omega_t = \Omega^2 \end{cases}$$

and

$$\sigma_{z^*g^*,t} = \begin{cases} \sigma_2 & \text{if } \Omega_t = \Omega^1 \\ \sigma_1 & \text{if } \Omega_t = \Omega^2 \end{cases}$$

with $\sigma_1 < \sigma_2$.

In all the exercises in this paper we set

$$A = \begin{bmatrix} 0.9825 & 0.0155 & 0 & 0 \\ 0.0155 & 0.9825 & 0 & 0 \\ 0 & 0 & 0.81 & 0 \\ 0 & 0 & 0 & 0.81 \end{bmatrix}$$

and $\sigma_z = \sigma_{z^*} = 0.00675$, and $\sigma_g = \sigma_{g^*} = 0.0114$. These values are obtained from estimating separately a bivariate autoregressive process for (z_t, z_t^*) and univariate autoregressive processes for g_t and g_t^*. The bivariate autoregressive process for (z_t, z_t^*) was estimated using estimated Solow residuals for the US and Canada, while the univariate autoregressive processes for g_t and g_t^* where estimated using US data for M1. See Duarte(2000) for a detailed description of these regressions.

To completely specify the process in (15) we also need to assign values to σ_1 and σ_2. We chose these two values such that the correlation between the innovations to productivity and money growth shocks is -0.9 and $+0.9$, respectively. With this choice for σ_1 and σ_2 we try to magnify the importance of switches in regime in our results.

In the next section we investigate whether the information variable introduced in the model can help explain movements in exchange rates. We approximate numerically the stationary and symmetric equilibrium described above by iterating on the mapping defined by the system of first-order conditions of the problem. The algorithm requires that all first-order conditions hold exactly on a discrete number of gridpoints defined over the state space. We put 2 gridpoints on each exogenous shock, z, z^*, g, and g^*. This was accomplished by approximating with a discrete Markov chain the estimated continuous autoregressive process in (15) under each possible realization of the information variable using Tauchen and Hussey's (1991) method. The space for bond holdings was replaced by 11 gridpoints centered around 0, with the two extreme gridpoints defined by the two household's borrowing constraints. The grid is not evenly spaced but, instead, is finer closer to the extreme gridpoints.

6 Results

Impulse Response Functions to Money and Productivity Shocks. We start this section by looking at the impulse response functions for both shocks to the growth rate of money and productivity. In these figures, the price adjustment parameter λ was set equal to 0.5.

Figures 1a through 1c depict the impulse response functions to a shock to the growth rate of money in the home country. In period $t = 2$ the rate of money growth rises by 1.975%, which corresponds roughly to 1.75 standard deviations of σ_g.

On impact, prices in the home country increase approximately 3% and the nominal and real exchange rates depreciate approximately 7% and 3.7%, respectively. Because in this period the home consumer needs to hold more real money balances, home consumption rises approximately 3.8%. Accordingly, also home output and labor rise, roughly 3.4% and 5.2%, respectively.[22]

This period, the shock to home money supply is transmitted to the foreign country only through the increased home demand for foreign goods. Therefore also foreign consumption, labor, and output increase slightly in period $t = 2$.

Finally, in period $t = 2$ the home household also increases its average bond holdings, in order to intertemporally substitute its temporary increase in wealth. The long-run change in bond holdings is very small, implying that this monetary shock does not generate relevant permanent wealth effects.[23]

One period after the shock, prices denominated in the home currency adjust fully to their new long-run level, while the home money supply rises gradually to its new long-run level. Because all firms are allowed to reset prices in period $t = 3$ and the path for the home money supply is known, all real effects of the money shock die out after one period. Therefore, all real variables (including the real exchange rate) return to their original level in period $t = 3$.

We now turn to the productivity shock. Figures 2a through 2c depict the impulse response functions to a 1.17% increase in home productivity (which also corresponds to approximately 1.75 standard deviations of σ_z).

On impact, all prices adjust partially to the increased home productivity. Therefore, this period all prices decrease, and due to the higher productivity of home firms, the relative price of home goods falls in both countries. Because both consumers have a bias for the local good, the consumer price index decreases more in the home country than in the foreign country. In response to the higher labor productivity, home firms demand less labor, and in equilibrium, the home household works less but consumes more in period $t = 2$. The home productivity shock affects the foreign country only by low-

[22] If prices could not adjust on impact ($\lambda = 0$), then in the period of the shock the behavior of the real exchange rate would mimic the nominal exchange rate, by depreciating approximately 7%. The adjustment of all other real variables would be bigger as well. Home consumption, output, and labor would increase, on impact, 6.5%, 8.7%, and 5.7%, respectively.

[23] See Chari, Kehoe, and McGrattan (2000) for an explanation for these small permanent wealth effects.

ering home demand for its goods, and we observe a very small reduction in foreign labor and output. On impact, the real exchange rates depreciates reflecting the adjustment in the price levels, while the nominal exchange rate appreciates slightly.[24]

The following period, firms adjust their prices to the new productivity level. All prices decrease further and the relative price of home goods falls further in both countries. Thus total consumption increases in both countries and both households substitute consumption of home goods for foreign goods. Consequently, home output and labor rise, while foreign output and labor decrease.

In response to the higher labor productivity, the home household also accumulates bond holdings, although this effect is small. Therefore, as the productivity shock dies out, variables return gradually to approximately their original levels. The new long-run level of the nominal exchange rate is slightly lower than the initial one, consistent with the home household's higher bond holdings.

Impulse Response Functions to a Change in Regime. We now examine the effects of regime shocks, which alter the covariance between productivity and monetary shocks. These regime shocks are intended to represent episodes of "speculation" by generating changes in risk premia that speculators demand on foreign exchange markets. Those changes in risk premia are then intended to operate through asset markets to generate first-order effects on the exchange rate while creating much smaller effects on other macroeconomic variables such as production, employment, investment, and consumption.

The model implies that changes in the risk premium require changes in the cross-country difference of covariances between the real returns on nominal bonds and the marginal utility of consumption. Regime changes can affect this covariance because monetary and productivity shocks generate different patterns of responses in consumption, the price level, and the exchange rate. However, the results discussed above show that the model generates extremely small responses of macroeconomic aggregates to productivity shocks (compared to the responses to monetary shocks). We conclude that a model

[24] If, on impact, all goods' prices are fixed ($\lambda = 0$), then in response to the higher home labor productivity, the home agent works and consumes less in period $t = 2$. The fall in consumption reflects the substitutability of leisure and consumption in the utility function.

with this structure probably requires some source of shocks other than productivity to generate the sizes of the risk premia observed in the data and sufficient to generate large fluctuations in exchange rates.

In interpreting the figures, note that the *only* source of persistence in the model is the regime. The model has abstracted from *all* other sources of persistence, such as longer-lived, staggered price setting, or capital. Consequently, all responses in the figures after the impact response reflect regime persistence.

Figure 3 depicts the impulse response functions to a change in the regime, from $\Omega_t = \Omega^1$ in period 1 to $\Omega_t = \Omega^2$ in period 2.[25] That is, at $t = 1$ the covariance between domestic monetary and productivity shocks is negative, and the covariance between foreign monetary and productivity shocks is positive, while at $t = 2$ the covariance between domestic monetary and productivity shocks becomes positive, and the covariance between foreign shocks becomes negative. To raise the response of the risk premium to the shocks, the Figure presents results after arbitrarily raising the standard deviation of productivity shocks by a factor of 4.4.

While the regime shock generates a larger percentage change in the nominal exchange rate than in other variables, as expected, the magnitudes of all the responses are extremely small. The exchange rate rises by only 0.17% on impact, and the risk premium changes from -0.00041 to -0.00034.[26] Moreover, a larger fraction of the change in the exchange rate occurs through contemporaneous changes in allocations and prices, represented by the term e_1 in (13), than through expectations of future variables working through asset markets and represented by the term e_2.[27] The impact effect on the exchange rate reflects two factors: (1) the (permanent) difference across regimes in risk premia, and (2) the (temporary) effects of (partially-) preset prices.

In period 3 (one period after the shock is realized), the chance that Ω_t remains at Ω^2 is .9. Over time, that probability falls toward .5, and the economy moves toward an unconditional steady state. This is the sole source of persistence in the figures. At period 3, the real exchange rate jumps

[25] These figures depict the average response of the system to a change in the information variable in 500 simulations of 10000 periods each.

[26] Engel (1999) also reports the inability of standard models to generate sufficient variation in the foreign-exchange risk premium.

[27] The steady-state level of the risk premium is negative due to the asymmetry in the model: with trade *only* in home-currency bonds, foreign households, but not home households, bear exchange-rate risk in holding those bonds.

downward, reflecting the higher level of optimal price setting by domestic firms in the new regime, and the lower optimal level of prices chosen by foreign firms.[28] While the dynamics of the real exchange rate from $t = 3$ onward are strongly at variance with the data, they also reflect the absence of any other sources of persistence in the model.

To study the possibility that the small response of the risk premium and the exchange rate reflect standard asset-pricing puzzles that have appeared in the equity-premium literature and elsewhere, we raise the coefficient of relative risk aversion from $\sigma = 2$ to $\sigma = 20$. Figure 4 depicts the impulse response functions to a change in the regime, from $\Omega_t = \Omega^1$ in period 1 to $\Omega_t = \Omega^2$ in period 2, when $\sigma = 20$. The response of the risk premium remains extremely small, rising from -0.003 to -0.0027 in response to the change in regime, while the domestic currency depreciates by 0.40% on impact. The increase in risk aversion substantially raises the asset-market component of exchange-rate variation. With $\sigma = 20$, the asset-market term e_2 rises by 0.56% on impact, while the product-market term e_1 rises by only 0.15%. Consequently, the asset-market term accounts for more than 3/4 of the impact effect on the exchange rate.

Figure 4 shows that the largest fraction of the change in the asset-market term e_2 occurs through a fall in its denominator, which involves the future exchange rate as well as the future foreign price level and future marginal utility. Persistence in the regime generates persistence in the exchange rate.

As Figure 4 shows, the two components of e_1, the ratio of marginal utilities and the ratio of price levels, each respond to the regime shift, but their responses tend to cancel. Domestic marginal utility rises by 0.57% on impact, while foreign marginal utility falls by a corresponding amount. The percentage changes in home and foreign consumption are only about 1/20 as large as the changes in marginal utility, as $\sigma = 20$. Consequently, the change in the exchange rate is *not* accompanied by correspondingly large changes in consumption. Instead, it is accompanied by changes in expectations about the future. Because domestic and foreign price levels also respond to the regime shift, the real exchange rate rises by only 0.10% on impact. As mentioned above, the abrupt reversal in the real exchange rate at $t = 3$ that appears in Figure 4 is an artifact of our simplifying assumption of one-period price setting, combined with an absence of other features in the model to

[28] Devereaux and Engel (1998),and Bacchetta and van Wincoop (2000), analyze the effects of risk-aversion by firms in optimal price-setting decisions.

generate persistence, such as capital.

Our results, at least with high risk aversion and counterfactually-large technology shocks, illustrate the possibility of large asset-market "speculation" effects on exchange rates. However, our model has not been able to generate sufficiently large changes in risk premia to match the data (or, as a result, such large Sharpe ratios as Backus, Gregory, and Telmer find in the data). Consequently, the model does not imply sufficient exchange-rate variability to match the data.

Clearly, a full theory of speculative effects on exchange rates requires that economists revise models, or develop new models, that can explain the substantial risk premia observed on equities markets and foreign exchange markets, as well as translating how those changes in speculators' perceptions of risk affect exchange rates and other macroeconomic variables. Our model, like previous models (e.g. Engel, 1999), has been unable to generate a sufficient variation in foreign-exchange risk premia to match the evidence. For example, Backus, Gregory, and Telmer develop a theoretical model of the risk premium (the expected return from currency speculation) and, even when choosing parameters to maximize its standard deviation, can generate at most a standard deviation that is about half the standard deviation in the data. Short of pursuing the grand model of asset pricing that has thus far eluded economists, one strategy for subsequent research is a two-tiered approach. On the one hand, continue to pursue a better theory of asset pricing. On the other hand, develop models of how changes in speculators' perceptions of risk (even if not fully explained by the models) affect exchange rates and other macroeconomic variables. Our results are suggestive of the possibility that speculative asset-market effects play a large role in exchange-rate variation.

7 Conclusion

This paper has argued that speculation, with implied changes in risk premia, is likely to play a key role in explaining the behavior of exchange-rates. With sufficient international segmentation in product markets, we have argued that exchange rates follow a forward-looking, first-order stochastic difference equation that includes terms involving risk premia. Consequently, the current exchange rate can be affected by changes in that risk premium. This opens the possibility for speculative effects on exchange rates, generating exchange-rate movements that are *not* strongly correlated with movements in current macroeconomic variables (aside from the risk premium itself).

We have implemented this idea in a standard two-country monopolistic-competition model with sticky prices and pricing to market, with markets for final products that are completely segmented internationally, and a model of regime shifts – affecting the covariances of shocks – intended to create "rational" speculation in the sense of altering equilibrium risk premia.

The resulting model fails to generate a sufficient degree of rational speculation to explain either observed variation of risk premia in foreign exchange markets or observed variation in exchange rates. Future research might increase that variation somewhat by introducing an information variable that signals future changes in regime.[29] It might also adopt model variations from the equity-premium literature to (try to) raise the implied variation in risk premia. A third area for research would be to examine the degree of "irrational" speculation necessary to generate sufficient exchange-rate variability from the difference-equation for the exchange rate. One could ask whether models like the one in this paper can generate sufficient exchange-rate variability *conditional* on variation in the risk premium. Further work will also be required to determine whether the model can generate exchange-rate variation with "a life of its own," moving nearly independently of standard macroeconomic aggregates. Another area for future research involves exploiting information in the term structure of the risk premium to infer the expected persistence of risk-premium changes and the implied magnitude of exchange rate changes.

[29] Our modeling choice represented a compromise between two extremes. On the one hand, the regime must show persistence to generate speculation in our model. (If changes in regime were serially uncorrelated, then the asset-market term in our exchange rate equation, as a ratio of expectations, would be a constant, independent of the current state.) On the other hand, high persistence in regimes reduces the magnitude of changes in exchange rates. One interesting area for future research would be to introduce an information variable, conveying information about future regime changes before they occur. That information variable would introduce rational speculation into the model without requiring the high degree of serial correlation in the regime that we assume. Consequently, the model may be able to generate larger changes in exchange rates for any given change in risk premia.

References

[1] Backus, David, Allan Gregory, and Chris Telmer (1993), "Accounting for Forward Rates in Markets for Foreign Currency," *The Journal of Finance* **XLVIII** (5), 1887-1908.

[2] Baillie, Richard and Tim Bollerslev (1990), "Intra-Day and Inter-Market Volatility in Foreign Exchange Rates," *Review of Economic Studies* **58 (3)**, 565-585.

[3] Betts, Catherine and Michael B. Devereux (1996), "The Exchange Rate in a Model of Pricing to Market," *European Economic Review* **40**, 1007-1021.

[4] Chari, V. V., Patrick Kehoe and Ellen McGrattan (2000), "Can Sticky Prices Models Generate Volatile and Persistent Real Exchange Rates?," Federal Reserve Bank of Minneapolis Research Department Staff Report 223.

[5] Cox, John, Jonathan Ingersoll, and Stephen Ross (1981), "The Relation Between Forward Prices and Futures Prices," *Journal of Financial Economics* **9**, 321-346.

[6] Devereux, Michael (1997), "Real Exchange Rates and Macroeconomics: Evidence and Theory," *Canadian Journal of Economics* **30 (4a)**, 773-808.

[7] Dornbusch, Rudiger (1976), "Expectations and Exchange Rate Dynamics," *Journal of Political Economy* **91**, 141-153.

[8] Dumas, Bernard (1992), "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World," *The Review of Financial Studies* **5 (2)**, 153-180.

[9] Duarte, Margarida (2000), "Why Don't Macroeconomic Quantities Respond to Exchange Rate Variability? Comparing Fixed and Floating Exchange Rate Systems," unpublished paper.

[10] Engel, Charles (1996), "The Forward Discount Anomaly and the Risk Premium: a Survey of Recent Evidence," *Journal of Empirical Finance*, 123-191.

[11] Engel, Charles (1999), "On the Foreign Exchange Risk Premium in Sticky-Price General Equilibrium Models," in *International Finance and Financial Crises: Essays in Honor of Robert P. Flood*, Peter Isard, Assaf Razin and Andrew Rose eds., IMF and Kluwer, 71-85.

[12] Fama, Eugene (1984), "Forward and Spot Exchange Rates," *Journal of Monetary Economics* **14**, 319-338.

[13] Flood, Robert and Andrew Rose (1995), "Fixing Exchange Rates: A Virtual Quest for Fundamentals," *Journal of Monetary Economics* **36**, 3-38.

[14] Frankel, Jeffrey and Kenneth Froot (1990), "Exchange Rate Forecasting Techniques, Survey Data, and Implications for the Foreign Exchange Rate Market," NBER Working Paper no. 3470.

[15] Gallant, Ron, Peter Rossi, and George Tauchen (1992), "Stock Prices and Volume," *The Review of Financial Studies* **5 (2)**, 199-242.

[16] Hodrick, Robert (1987), *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*, Harwood Academic Publishers.

[17] Judd, Kenneth (1998), *Numerical Methods in Economics*, MIT Press.

[18] Karpoff, Jonathan (1987), "The Relation Between Price Changes and Trading Volume: a Survey," *Journal of Financial and Quantitative Analysis* **22 (1)**, 109-126.

[19] Krugman, Paul (1989), "The J-Curve, the Fire Sale, and the Hard Landing," *American Economic Review Papers and Proceedings* **79 (2)**, 31-35.

[20] Krugman, Paul (1989), *Exchange-Rate Instability*, MIT Press.

[21] Krugman, Paul, and Marcus Miller (1993), "Why Have a Target Zone?," Carnegie-Rochester Conference Series 38, June, 279-314.

[22] Lucas, Robert (1982), "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics* **10**, 335-360.

[23] Obstfeld, Maurice and Kenneth Rogoff (1995), "Exchange Rate Dynamics Redux,", *Journal of Political Economy* **103**, 624-660.

[24] Obstfeld, Maurice and Kenneth Rogoff (1996), *Foundations of International Macroeconomics*, MIT Press.

[25] Obstfeld, Maurice and Kenneth Rogoff (2000), "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," NBER Working Paper no. 7777.

[26] Ohanian, Lee and Alan Stockman (1997), "Arbitrage Costs and Exchange Rates," unpublished paper.

[27] Sercu, Piet and Raman Uppal (2000), *Exchange Rate Volatility, Trade, and Capital Flows under Alternative Exchange Rate Regimes*, Cambridge University Press.

[28] Sercu, Piet, Raman Uppal and Cynthia van Hulle (1995), "The Exchange Rate in the Presence of Transaction Costs: Implications for Tests of Purchasing Power Parity," *Journal of Finance* **50 (4)**, 1309-1319.

[29] Stockman, Alan (1980), "A Theory of Exchange Rate Determination," *Journal of Political Economy* **88**, 673-698.

[30] Stockman, Alan (1987), "The Equilibrium Approach to Exchange Rates," *Federal Reserve Bank of Richmond Economic Review* **73 (2)**, 12-30.

[31] Svensson, Lars (1985), "Currency Prices, Terms of Trade, and Interest Rates: A General Equilibrium Asset Pricing Cash-in-Advance Approach," *Journal of International Economics* **18**, 17-41.

[32] Svensson, Lars and Sweder van Wijenbergen (1989), "Excess Capacity, Monopolistic Competition, and International Transmission of Monetary Disturbances," *Economic Journal* **99**, 785-805.

[33] Tauchen, George and Robert Hussey (1991), "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica* **59 (2)**, 371-396.

8 Appendix 1: The Model with Partially Adjustable Prices

Using the algebra of expected values, the term e_2 can be rewritten as

$$e_{2,t} = \frac{E\left[u_{c,t+1}\right] E\left[\frac{1}{P_{t+1}}\right] + cov\left(u_{c,t+1}, \frac{1}{P_{t+1}}\right)}{E\left[u^*_{c,t+1}\right] E\left[\frac{1}{e_{t+1}P^*_{t+1}}\right] + cov\left(u^*_{c,t+1}, \frac{1}{e_{t+1}P^*_{t+1}}\right)}. \tag{16}$$

A change in the information variable, as long as this variable is persistent, will affect the covariance terms in the above expression by affecting the covariance between future shocks to money growth and productivity in both countries. In our framework, however, firms set prices one period in advance and, on impact, prices do not respond to shocks. Consequently, the covariance terms in equation (16) are zero in our model. However, if prices where to adjust to current shocks, future money and productivity shocks would affect both future price levels and marginal utilities of consumption. This would allow a change in regime to affect the term e_2 by affecting the covariance terms in equation (16).

In this appendix we describe the price setting problem of firms, in order to allow them to adjust partially their prices to current shocks. As before, firms set prices for period t one period in advance at $t-1$, before observing the shocks. However, we assume that after uncertainty is resolved, all firms can adjust partially their prices to the ones that would occur in the flexible price equilibrium. That is, for home firm i, the price effectively charged to home consumers in period t, $P^e_{h,t}(i)$, is a linear combination of the price preset in advance, $P_{h,t}(i)$, and the price that would occur in the flexible price equilibrium, $\widetilde{P}_{h,t}(i)$; that is

$$P^e_{h,t}(i) = \lambda P_{h,t}(i) + (1-\lambda)\widetilde{P}_{h,t}(i) \tag{17}$$

and similarly for all other three prices.[30]

The price level in the home country in period t is now given by

$$P_t = \left[\omega\left(P^e_{h,t}\right)^{1-\gamma} + (1-\omega)\left(P^e_{f,t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}.$$

[30] This price setting structure should be equivalent to assume that after the realization of uncertainty a fraction λ of firms can reset prices while the remaining fraction $(1-\lambda)$ does not adjust its preset prices.

When firms choose their prices at $t-1$, they know that the effective price at t is a given linear combination of the price they set at $t-1$ and the price that would occur in the flexible price equilibrium, which they take has given. So, the price setting problem of home firms in period $t-1$ becomes

$$\max_{P_{h,t}(i),P^*_{h,t}(i)} E_{t-1}[\rho_t(\left(\lambda P_{h,t}(i)+(1-\lambda)\widetilde{P}_{h,t}(i)\right)c_{h,t}(i)+$$
$$+e_t\left(\lambda P^*_{h,t}(i)+(1-\lambda)\widetilde{P}^*_{h,t}(i)\right)c^*_{h,t}(i)-P_t w_t l_t(i))]$$

subject to the resource constraint $z_t F(l_t(i)) = c_{h,t}(i)+c^*_{h,t}(i)$ and the downward sloping demand functions for $c_{h,t}(i)$ and $c^*_{h,t}(i)$. This problem's first-order conditions with respect to $P_{h,t}(i)$ and $P^*_{h,t}(i)$ are:

$$E_{t-1}\left[\rho_t\left((1-\theta)c_{h,t}(i)+\frac{\theta}{\alpha}\frac{P_t w_t l_t(i)^{1-\alpha}c_{h,t}(i)}{\lambda P_{h,t}(i)+(1-\lambda)\widetilde{P}_{h,t}(i)}\right)\right]=0$$

and

$$E_{t-1}\left[\rho_t\left((1-\theta)e_t c^*_{h,t}(i)+\frac{\theta}{\alpha}\frac{P_t w_t l_t(i)^{1-\alpha}c^*_{h,t}(i)}{\lambda P^*_{h,t}(i)+(1-\lambda)\widetilde{P}^*_{h,t}(i)}\right)\right]=0.$$

9 Appendix 2: The Risk Premium

In this section we describe how the risk premium is computed in our model. Note that model households are not allowed to trade a risk free bond denominated in the foreign currency. Therefore the forward exchange rate, f_t, cannot be derived from the condition of covered interest parity. This simply reflects the model's asymmetry: households can invest in risk-free bonds denominated in the home currency but not in risk-free bonds denominated in the foreign currency.

An alternative derivation relates the forward exchange rate to the future spot rate (see Cox *et al* (1981)) and will enable us to calculate the risk premium for the home household. Consider the following investment strategy in period t:

1. to invest $\frac{f_t}{1+i_{t+1}}$ dollars in the risk-free asset and;
2. to buy 1 one-period forward contract at price f_t.

Investment 1 delivers f_t with certainty in period $t+1$, while the payoff at time $t+1$ of investment 2 is $(e_{t+1} - f_t)$. Therefore the net payoff at $t+1$ of this investment strategy is e_{t+1}. Since this investment strategy involves an initial investment of $\frac{f_t}{1+i_{t+1}}$ with expected return equal to e_{t+1}, the following arbitrage condition always needs to hold in equilibrium:

$$\frac{f_t}{1+i_{t+1}} \frac{u_{1,t}}{P_t} = \beta E\left[e_{t+1} \frac{u_{1,t+1}}{P_{t+1}}\right]. \tag{18}$$

This condition imposes that in equilibrium the forward exchange rate is such that the home household's demand for forward contracts is zero.

Using the equilibrium condition for the nominal interest rate (in an interior solution), we obtain:

$$f_t = \frac{E\left[e_{t+1}\beta \frac{u_{1,t+1}}{P_{t+1}} \frac{P_t}{u_{1,t}}\right]}{E\left[\beta \frac{u_{1,t+1}}{P_{t+1}} \frac{P_t}{u_{1,t}}\right]} = \frac{E\left[e_{t+1}\lambda_{t+1}\right]}{E\left[\lambda_{t+1}\right]}, \tag{19}$$

where λ_{t+1} represents the nominal marginal rate of substitution, $\frac{u_{1,t+1}}{P_{t+1}}$. However, if the home household is borrowing constrained then the previous expression does not hold with equality. Instead,

$$f_t < \frac{E\left[e_{t+1}\lambda_{t+1}\right]}{E\left[\lambda_{t+1}\right]}$$

Defining the risk premium as $rp_t \equiv f_t - E\left[e_{t+1}\right]$, we obtain that

$$rp_t = \frac{Cov\left(e_{t+1}, \lambda_{t+1}\right) + E\left[\lambda_{t+1}\left(f_t - e_{t+1}\right)\right]}{E\left[\lambda_{t+1}\right]}.$$

Note that from (19), the second term in the right hand side of this expression is always zero in an interior solution.

Figure 1a - Monetary Shock: Money Supply, Exchange Rates, and Bond Holdings

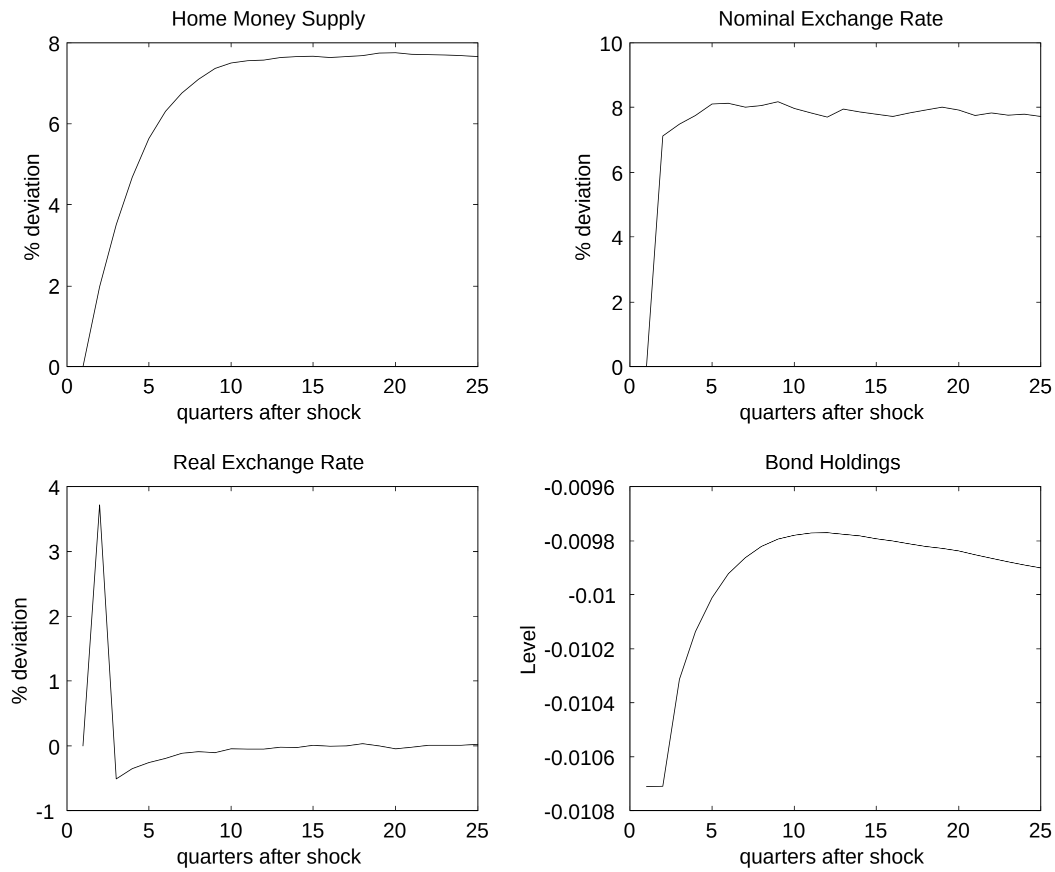

Figure 1b - Monetary Shock: Consumptions, Outputs, Labor Supplies, and Price Levels

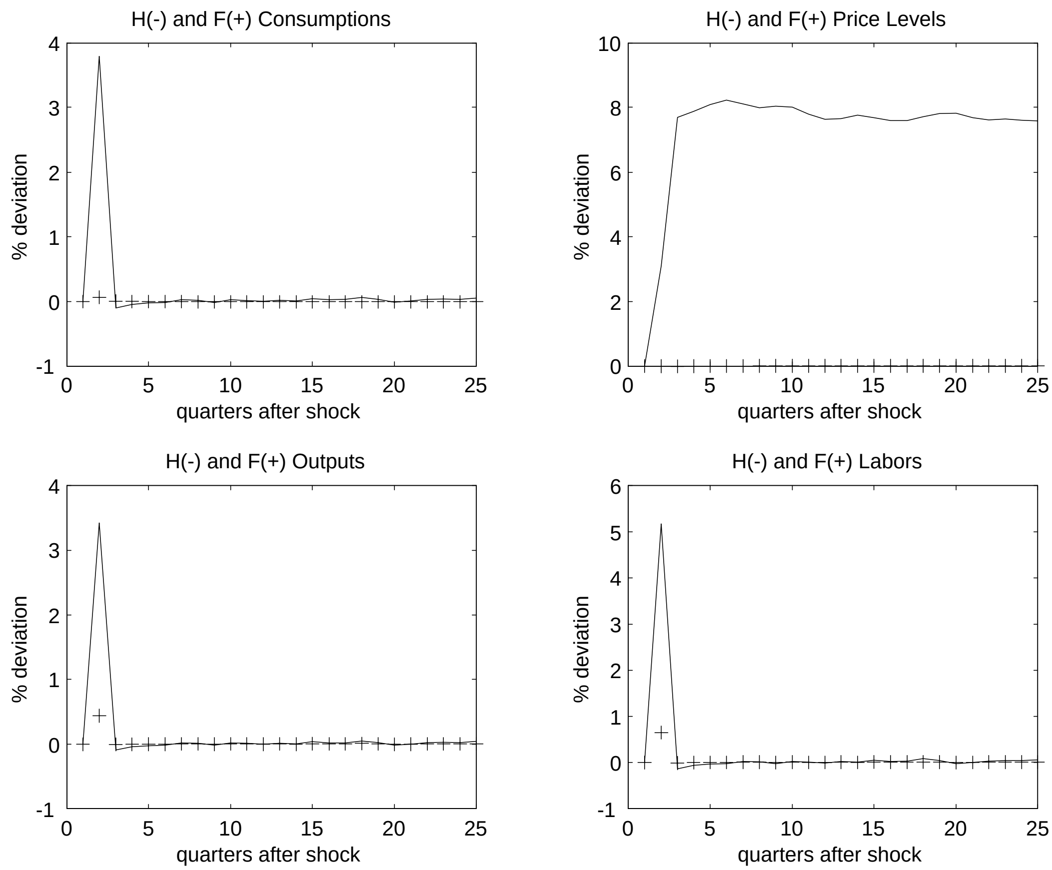

Figure 1c - Monetary Shock: Intermediate Goods

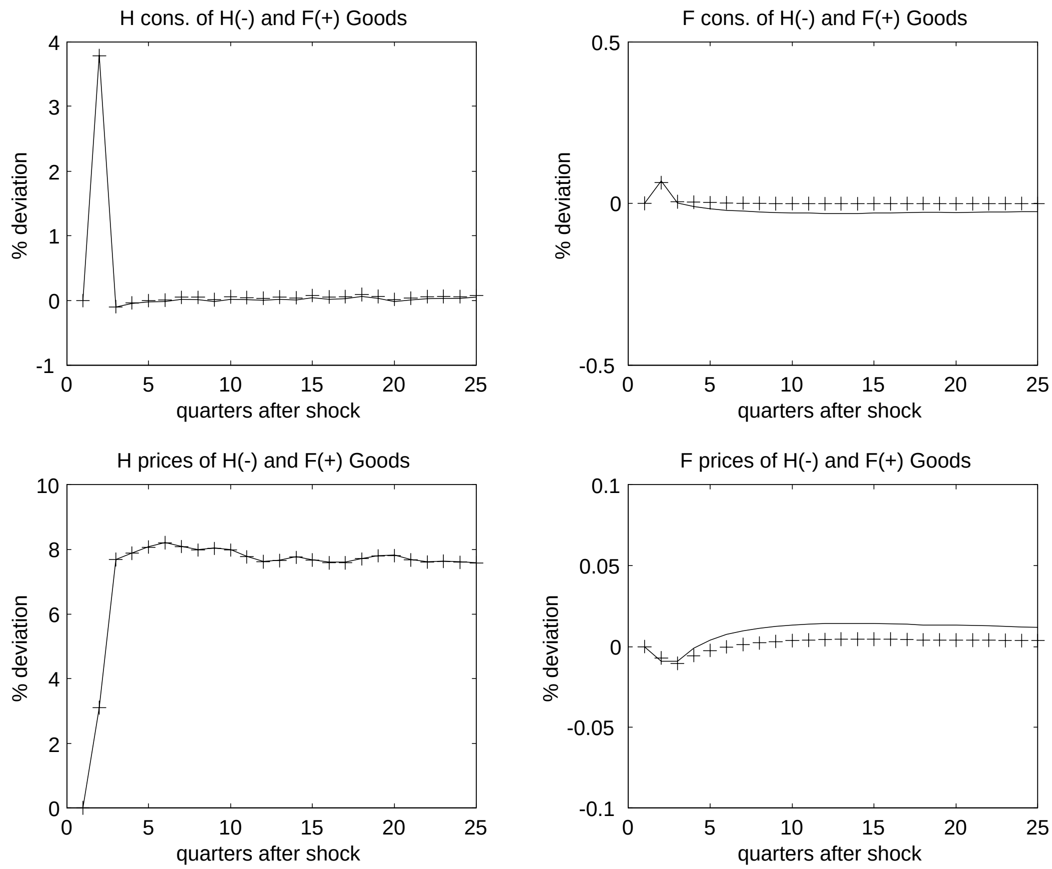

Figure 2a - Productivity Shock: Productivity Level, Exchange Rates, and Bond Holdings

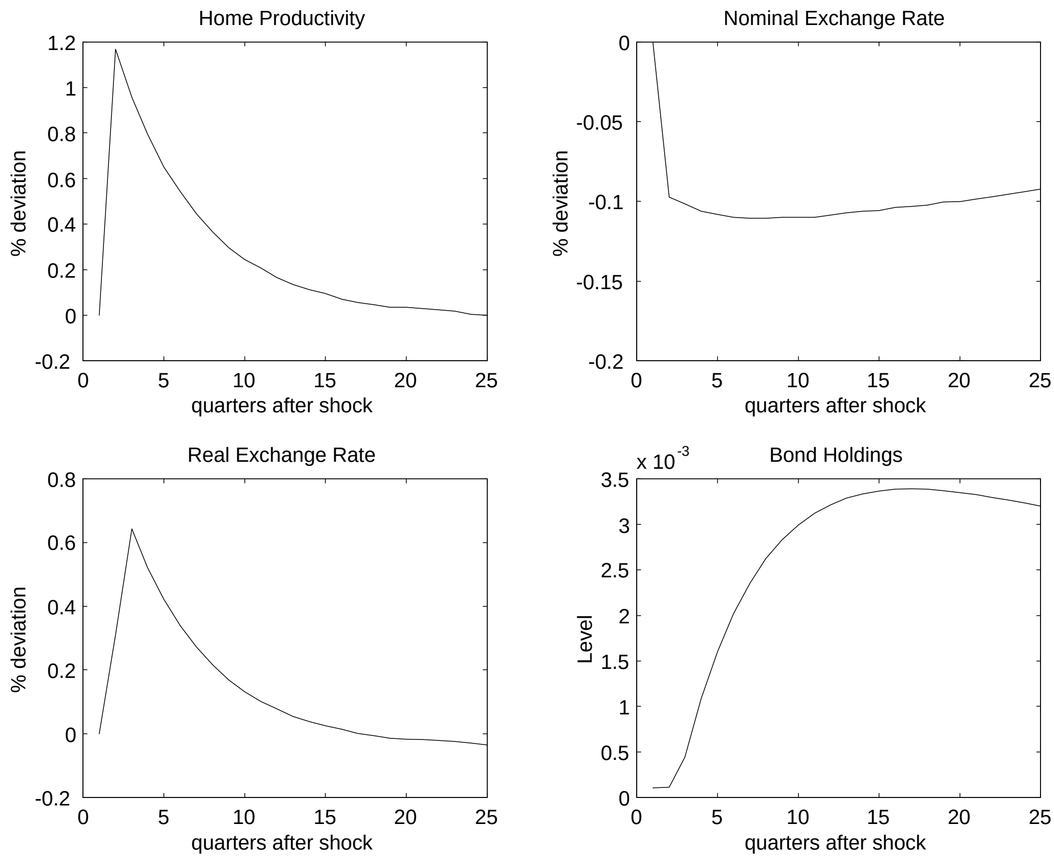

Figure 2b - Productivity Shock: Consumptions, Outputs, Labor Supplies, and Price Levels

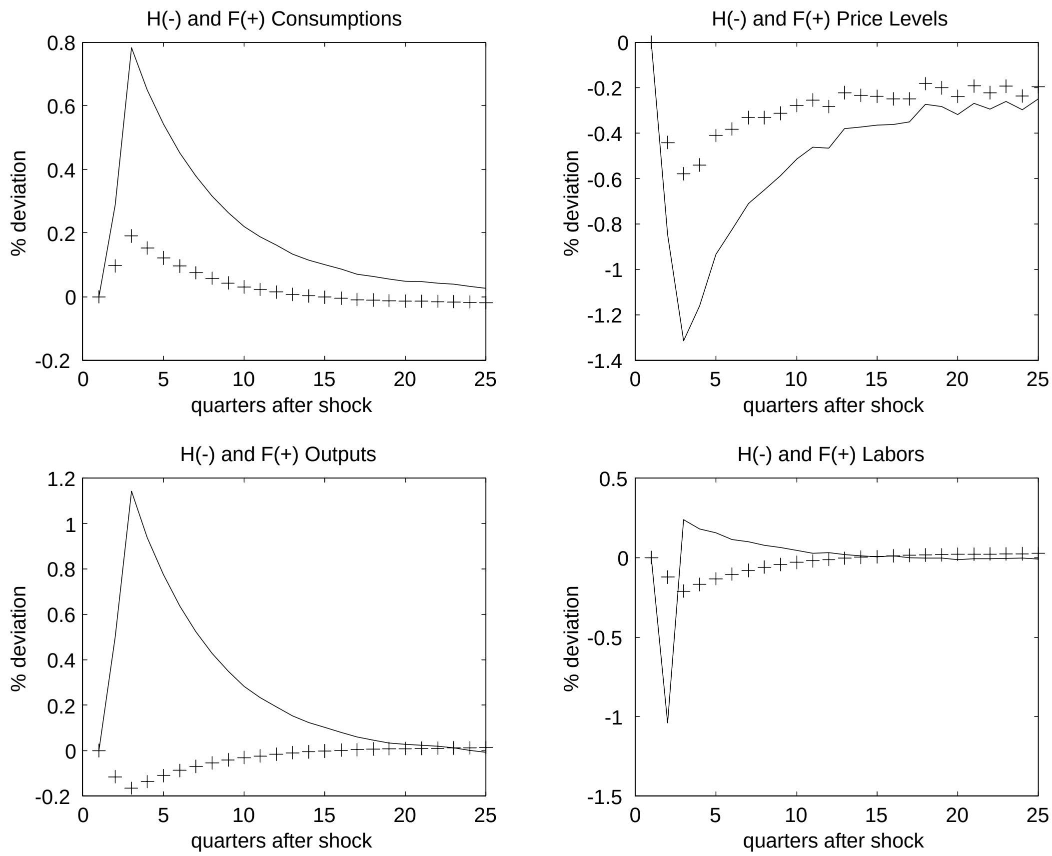

Figure 2c - Productivity Shock: Intermediate Goods

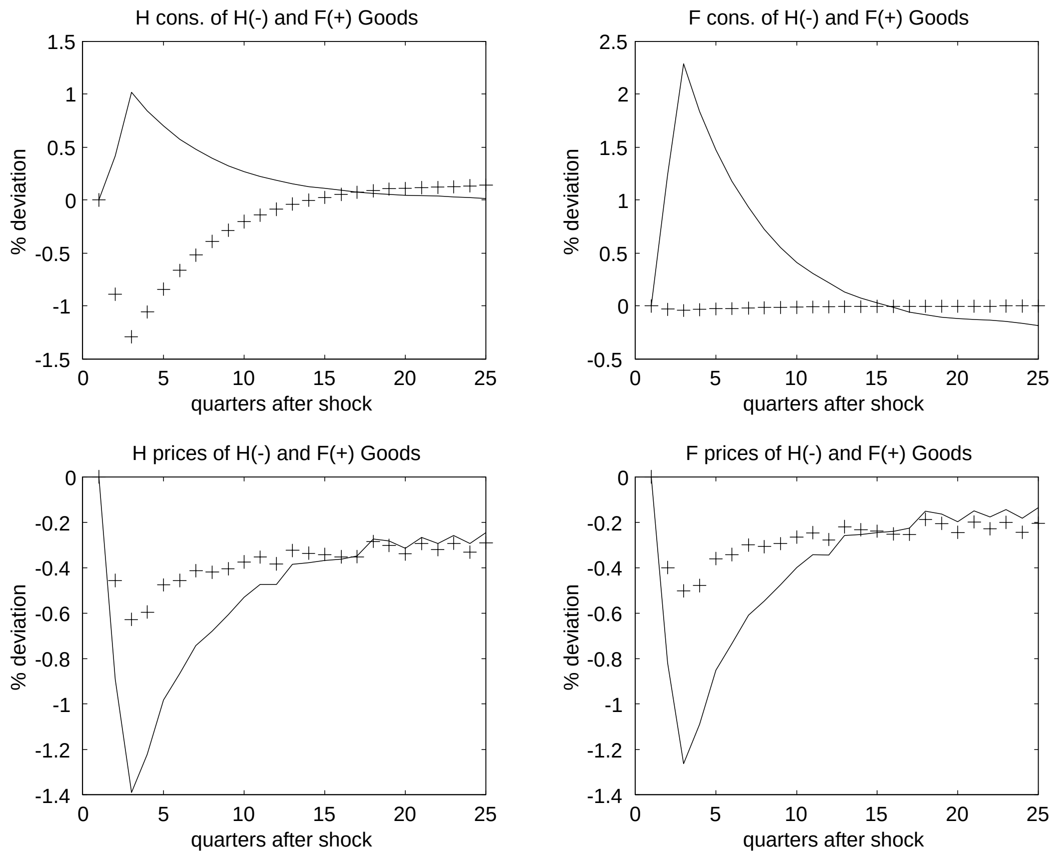

Figure 3 - Change in Regime ($\sigma = 2$)

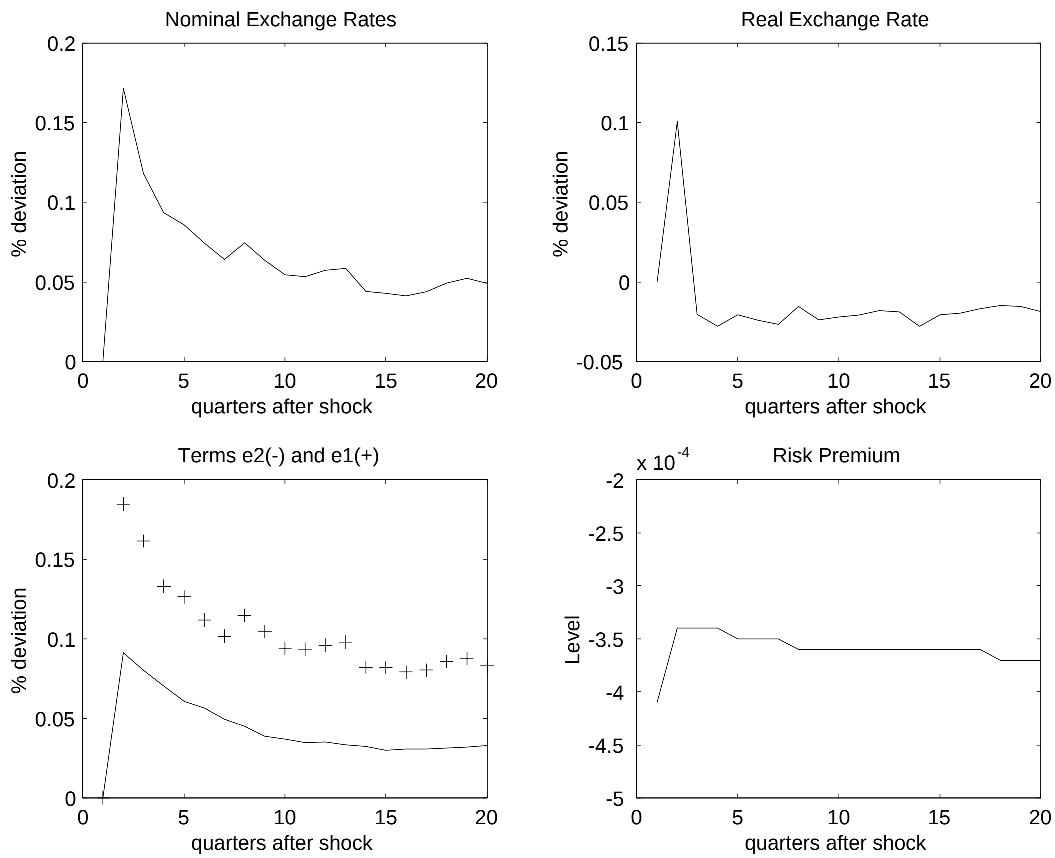

Figure 4 - Change in Regime ($\sigma = 20$)