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ABSTRACT

This paper provides a baseline general-equilibrium model of optimal monetary policy among interdependent economies, with monopolistic firms that set prices one period in advance. Strict adherence to inward-looking policy objectives such as the stabilization of domestic output cannot be optimal when firms' markups are exposed to currency fluctuations. Such policies induce excessive volatility in exchange rates and foreign sales revenue, leading exporters to set higher prices in response to higher profit risk. In general, optimal rules trade off a larger domestic output gap against lower import prices. Monetary rules in a world Nash equilibrium lead to smaller exchange rate volatility relative to both inward-looking rules and discretionary policies, even when the latter do not suffer from any inflationary (or deflationary) bias. Gains from international monetary cooperation are related in a non-monotonic way to the degree of exchange rate pass-through.

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1 Introduction

What are the basic principles underlying the design of optimal monetary policies among open and interdependent economies? From the recent policy and academic debate, it is far from obvious that monetary policy should have *any* 'international' dimension at all. In fact, there is little or no consensus on such questions as of whether domestic monetary policy should react to exchange rate fluctuations and overseas macroeconomic developments; what role is played by external factors in the choice between different rules, and between rules and discretion in policy-making; what (if anything) can be achieved through international cooperation and 'global compacts' with trading partners.

This paper presents a baseline general-equilibrium framework to assess these issues and provide some tentative answers. The paper shares the unifying research agenda of most recent contributions on optimal monetary policy, and adopts a common methodology based upon choice-theoretic stochastic models with nominal rigidities, imperfect competition in production, and forward-looking price-setting.[1] Our main results can be summarized as follows.

First, inward-looking monetary policies that do not react to world cyclical conditions cannot be optimal for an open economy in which firms' profits are exposed to exchange rate fluctuations. Recent literature has stressed that, in a closed economy with nominal rigidities, it is desirable to pursue a policy that closes the output gap completely.[2] In fact, if one can disregard real rigidities and 'cost-push' inflation, such policy can replicate the allocation that would prevail if prices where fully flexible. In an open economy, however, a policy attempting to close the domestic output gap could induce excessive volatility in the exchange rate, affecting foreign exporters' profits. Risk-averse foreign producers would then hedge against the risk of losses in their

[1]The literature on monetary rules in open economy has boomed in recent years. A far from complete list of theoretical contributions includes Ball [1999], Benigno and Benigno [2000], Benigno, Benigno and Ghironi [2000], Carlstrom and Fuerst [1999], Clarida, Gali and Gertler [2001], Devereux and Engle [2000], Gali and Monacelli [1999], Ghironi and Rebucci [2000], Leitmo and Söderström [2000], McCallum and Nelson [1999], Monacelli [1999], Obstfeld and Rogoff [2000b], Parrado and Velasco [2000], Sutherland [2000], Svensson [2000], and Walsh [1999].

[2]See for instance Gali [2000], Goodfriend and King [2000] and Woodford [2000].

export market by charging higher prices in domestic currency, thus reducing the purchasing power of domestic consumers.

Only when markups are independent of exchange rate movements the optimal domestic policy does not react to policy shocks and cyclical conditions in the world economy. This may be the case, for instance, when goods are produced with domestic inputs only and are priced in the producers' currency, or when exporters are fully insured against currency volatility. Otherwise, relative to an inward-looking policy conduct, the domestic policy maker can improve welfare by trading off, at the margin, a larger domestic output gap against lower import prices. To act in the best interest of domestic consumers and reduce the overall magnitude of distortions affecting national welfare, policy makers must stabilize world producers' profits in the domestic market.

Second, commitment is superior to discretion even when discretionary policies do not suffer from any inflationary (or deflationary) bias. This is because, given pre-set prices by firms, a discretionary policy maker has an incentive, ex post, to 'over-stabilize' the domestic economy and use monetary policy as to tilt terms of trade in favor of domestic agents. But any systematic attempt to follow this conduct is doomed to lower domestic welfare on average, as foreign exporters react by pre-setting higher prices in the domestic market.

Third, relative to the case of suboptimal inward-looking policies, implementing optimal monetary policies reduces exchange rate volatility. With optimal policies in place, the lower the degree of exchange rate pass-through and profits exposure, the lower the equilibrium volatility in the foreign exchange market, both in nominal and real terms. There is, however, a case for exchange rate flexibility. Fixed exchange rates can be supported by optimal monetary policies only when all shocks are correlated worldwide or when local prices are fully inelastic to exchange rate fluctuations. Otherwise, the relative price adjustment associated with the implementation of the optimal policy requires exchange rate flexibility.

Fourth, the magnitude of gains from cooperation are related in a nonlinear way to the degree of pass-through. In our model there are no welfare gains from entering international binding agreements either when there are no deviations from the law of one price or when local prices are completely insulated from exchange rate fluctuations. That is, under the extreme assumptions of complete and zero pass-through, monetary policies are strategically independent and there are no policy spillovers in equilibrium. Yet, gains from cooperation materialize in economies with intermediate levels of pass-through.

Building upon our previous contribution (Corsetti and Pesenti [2001]), we setup a model that can be solved in closed form, without resorting to (loglinear) approximations. Different from our previous work, as well as from many other contributions in the 'new open-economy' macro literature, all our welfare results are derived without specifying a particular distribution of the stochastic disturbances underlying the economy.

Also, our results are derived for intermediate degrees of exchange rate pass-through. Our analysis thus complements and generalizes recent models that explore the implications of two polar cases of nominal rigidities. In the first case (Producer Currency Pricing, or PCP) firms set prices in the currency of the country where they are located. The domestic price of imports then moves one to one with the exchange rate, pass-through is 100% and profits are completely insulated from exchange rate movements. In the second case (Local Currency Pricing, or LCP) firms set the price of their goods in the currency of the market where they sell their products. The domestic price of imports does not change with the exchange rate and pass-through is zero, corresponding to a high degree of profits' exposure to exchange rate movements.[3]

Both PCP and LCP assumptions are extreme in light of the empirical evidence. Pass-through to export and import prices is generally below 1 but seldom 0, and a large body of empirical studies have documented that exchange rate pass-through is rather low at consumer price level but it is far from nul at producer price level.[4] Our modelling strategy takes imperfect

[3]For instance, Benigno and Benigno [2000], Corsetti and Pesenti [2001], and Obstfeld and Rogoff [1995, 2000a,b] focus on the PCP case. Bacchetta and Van Wincoop [2000], Chari, Kehoe and McGrattan [2000], and Duarte and Stockman [2001] focus on the LCP case. Corsetti, Pesenti, Roubini and Tille [2000], Devereux and Engel [2000], and Tille [2001] compare PCP and LCP allocations. Devereux, Engel and Tille [1999] consider the interdependence between a country with zero pass-through and a country with complete pass-through. A few exceptions include Betts and Devereux [2000a], who study the effect of different degrees of pass-through on exchange rate dynamics in a non-stochastic environment, and Monacelli [1999], who simulates numerically the effects of pass-through in a calibration exercise.

[4]See the discussion in Goldberg and Knetter [1997] and Obstfeld and Rogoff [2000a]. Campa and Goldberg [2000] provide updated estimates of exchange rate pass-through

pass-through as a proxy for transaction and distribution costs affecting firms profitability in export markets,[5] as to analyze the implications of deviating from the extreme assumptions of zero or full pass-through on the international transmission mechanism and the design of optimal monetary policy in open economy.[6]

The paper is organized as follows. Section 2 introduces the model. Section 3 summarizes the mechanism of international transmission of real and monetary shocks implied by the model. Section 4 analyzes optimal policy. We first establish a general principle of policy design and discuss it by deriving three equivalent representations of the policy problem. Second, we characterize the optimal monetary rules and discuss their efficiency under different degrees of pass-through. Third, we revisit the 'rules-vs.-discretion' debate for an open economy. In Section 5 we assess the case for cooperation and discuss the choice of exchange rate regimes. Section 6 concludes.

2 The model

2.1 Preferences and technology

We model a world economy with two countries, each specialized in one *type* of goods. Countries and types of goods are denoted H (Home) and F (Foreign). Within a country, the national type of goods is produced in a number of varieties or *brands* defined over a continuum of unit mass. Brands are indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. All brands are traded worldwide. Each country is populated by households defined over a continuum of unit mass. Households are indexed by $j \in [0, 1]$ in the Home country and $j^* \in [0, 1]$ in the Foreign country.

Households consume all brands of both types of goods. $C_t(h, j)$ is consumption of Home brand h by Home agent j at time t; $C_t(f, j)$ is consumption of Foreign brand f by Home agent j at time t. Similarly we define consumption of Foreign agent j^* as $C^*_t(h, j^*)$ and $C^*_t(f, j^*)$. Each Home brand is an

across countries.

[5] Obstfeld and Rogoff [2001] emphasize the role of international trade costs in open economy macroeconomic analysis.

[6] A recent discussion of the policy implications of firms' pass-through is provided by Taylor [2000].

imperfect substitute to all other Home brands, with constant elasticity of substitution $\theta > 1$. Similarly the elasticity of substitution among Foreign brands is $\theta^* > 1$. For each agent j in the Home country, the consumption indexes of Home and Foreign goods are defined as:

$$C_{\mathrm{H},t}(j) \equiv \left[\int_0^1 C_t(h,j)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \quad C_{\mathrm{F},t}(j) \equiv \left[\int_0^1 C_t(f,j)^{\frac{\theta^*-1}{\theta^*}} df\right]^{\frac{\theta^*}{\theta^*-1}} \tag{1}$$

Similarly we define the consumption indexes of agent j^* in the Foreign country, $C^*_{\mathrm{H},t}(j^*)$ and $C^*_{\mathrm{F},t}(j^*)$.

Consistent with the idea that each country specializes in the production of a single type of good, the elasticity of substitution among goods produced in one country should not be lower than the elasticity of substitution across goods produced in different countries. Specifically, while both θ and θ^* are greater than one, we assume that the elasticity of substitution between Home and Foreign types is one. Under this assumption the consumption baskets of individuals j and j^* can be written as a geometric average of the Home and Foreign consumption indexes:

$$C_t(j) \equiv C_{\mathrm{H},t}(j)^{\gamma} C_{\mathrm{F},t}(j)^{1-\gamma}, \quad C^*_t(j^*) \equiv C^*_{\mathrm{H},t}(j^*)^{\gamma} C^*_{\mathrm{F},t}(j^*)^{1-\gamma} \quad 0 < \gamma < 1 \tag{2}$$

where the weights γ and $1-\gamma$ are identical across countries.

We denote $p_t(h)$ and $p_t(f)$ the prices of brands h and f in the Home market (thus expressed in the Home currency), and $p^*_t(h)$ and $p^*_t(f)$ the prices of brands h and f in the Foreign market. The utility-based price of a consumption bundle of domestically produced goods, denoted $P_{\mathrm{H},t}$, is derived as:[7]

$$P_{\mathrm{H},t} = \left[\int_0^1 p_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}. \tag{3}$$

Similarly we derive the other price indexes $P^*_{\mathrm{H},t}$, $P_{\mathrm{F},t}$ and $P^*_{\mathrm{F},t}$, as well as the utility-based CPIs:

$$P_t = \frac{P^{\gamma}_{\mathrm{H},t} P^{1-\gamma}_{\mathrm{F},t}}{\gamma_W}, \quad P^*_t = \frac{\left(P^*_{\mathrm{H},t}\right)^{\gamma} \left(P^*_{\mathrm{F},t}\right)^{1-\gamma}}{\gamma_W} \tag{4}$$

[7]The utility based price index P_{H} is defined as the minimum expenditure required to buy one unit of the composite good C_{H}, given the prices of the brands. See Appendix.

where $\gamma_W \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}$.

Each brand h is produced and sold in both countries by a single Home household (that is, a 'yeoman farmer' as in Obstfeld and Rogoff [1996]), using labor ℓ as the only input in production.[8] Technology is linear in household's h labor, so that the resource constraint for brand h is:

$$\frac{\ell_t(h)}{\alpha_t} \geq \int_0^1 C_t(h,j)dj + \int_0^1 C_t^*(h,j^*)dj^* \tag{5}$$

where α_t is a country-specific productivity shock. Similarly the resource constraint for brand f is:

$$\frac{\ell_t^*(f)}{\alpha_t^*} \geq \int_0^1 C_t(f,j)dj + \int_0^1 C_t^*(f,j^*)dj^* \tag{6}$$

where $\ell^*(f)$ is labor of Foreign household f and α_t^* is a productivity shock in the Foreign country.

Home agent j's lifetime expected utility $\mathcal{U}$ is defined as:

$$\mathcal{U}_t(j) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U \left[C_\tau(j), \frac{M_\tau(j)}{P_\tau}, \ell_\tau(j), \Omega_\tau \right] \tag{7}$$

where $\beta < 1$ is the discount rate and the instantaneous utility U is a positive function of the consumption index $C(j)$ and real balances $M(j)/P$ and a negative function of labor effort $\ell(j)$. Utility can be state-dependent, as indexed by the vector of random variables Ω.

To keep algebraic complexities at a bare minimum, without sacrificing the substance of the argument, our baseline model adopts the following additively-separable parameterization:

$$U\left[C_t(j), \frac{M_t(j)}{P_t}, \ell_t(j)\right] = \ln C_t(j) + \chi_t \ln \frac{M_t(j)}{P_t} - \kappa \ell_t(j) \tag{8}$$

In our framework money demand shocks can be introduced in the form of a stochastically varying elasticity of real balances, so that $\Omega_t = \{\chi_t\}$.[9] In what

[8] Obstfeld [2000] considers an extension of the model to encompass trade in intermediate inputs.

[9] Obstfeld and Rogoff [2000a] include κ_t in the set Ω_t, intepreting the Home elasticity of labor disutility as a productivity shock. Their parameterization is isomorphic to our specification with the Home productivity shock indexed by α_t.

follows, we will refer to $\mathcal{W}_t$ as the non-monetary component of $\mathcal{U}_t$, or $\mathcal{W}_t \equiv \mathcal{U}_t - E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \chi_\tau \ln M_\tau(j)/P_\tau$.

Foreign agents' preferences are similarly defined as:

$$\mathcal{U}_t^*(j^*) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\ln C_\tau^*(j^*) + \chi_\tau^* \ln \frac{M_\tau(j^*)}{P_t^*} - \kappa^* \ell_\tau^*(j^*) \right] \tag{9}$$

where the discount rate β is the same as in the Home country. Preferences and velocity shocks can be asymmetric across countries. For instance, in the baseline model χ_t^* and κ^* in the Foreign country need not coincide with χ_t and κ in the Home country.

2.2 Consumer optimization

Given the specification of the consumption baskets, agent j's demand for brand h is a function of the relative price of h and total consumption of Home goods:

$$C_t(h,j) = \left(\frac{p_t(h)}{P_{\mathrm{H},t}} \right)^{-\theta} C_{\mathrm{H},t}(j) \tag{10}$$

Similar expressions can be derived for the other brands. Accounting for the demand functions above, we can rewrite the resource constraint for agents h in the Home country as:

$$\frac{\ell(h)}{\alpha_t} \geq p_t(h)^{-\theta} P_{\mathrm{H},t}^{\theta} C_{\mathrm{H},t} + p_t^*(h)^{-\theta} \left(P_{\mathrm{H},t}^* \right)^{\theta} C_{\mathrm{H},t}^* \tag{11}$$

Home agents hold Home currency M, and two international bonds, B_{H} and B_{F}, respectively denominated in Home and Foreign currency. They receive a revenue $R(j)$ from the sale of the good they produce — since each agent is the sole producer of a specific brand, we associate individual j with brand h. They pay non-distortionary (lump-sum) net taxes T, denominated in Home currency. Using the above specified consumption and price indexes, the individual flow budget constraint[10] for agent j in the Home country is:

$$M_t(j) + B_{\mathrm{H},t+1}(j) + \mathcal{E}_t B_{\mathrm{F},t+1}(j) \leq M_{t-1}(j) + (1+i_t)B_{\mathrm{H},t}(j) + \\ +(1+i_t^*)\mathcal{E}_t B_{\mathrm{F},t}(j) + R_t(j) - T_t(j) - P_{\mathrm{H},t}C_{\mathrm{H},t}(j) - P_{\mathrm{F},t}C_{\mathrm{F},t}(j) \quad (12)$$

In the expression above the nominal yields i_t and i_t^* are paid at the beginning of period t and are known at time $t-1$; $\mathcal{E}$ is the nominal exchange rate, expressed as Home currency per unit of Foreign currency. The sales revenue of agent j, expressed in Home currency, is:

$$\begin{aligned} R_t(j) &\equiv p_t(h)\int_0^1 C_t(h,j)dj + \mathcal{E}_t p_t^*(h)\int_0^1 C_t(h,j^*)dj^* \\ &= p_t(h)^{1-\theta}P_{\mathrm{H},t}^{\theta}C_{\mathrm{H},t} + \mathcal{E}_t p_t^*(h)^{1-\theta}\left(P_{\mathrm{H},t}^*\right)^{\theta}C_{\mathrm{H},t}^* \end{aligned} \quad (13)$$

Similar expressions hold for the Foreign country, after associating household j^* with brand f.

Taking prices as given, Home agent j maximizes (7) subject to (12), accounting for (11) and (13).[11] A similar optimization problem is solved by Foreign agent j^*. In Section 3 we will focus on symmetric equilibria in which agents are equal within countries (though not necessarily symmetric across countries), dropping the indexes j and j^* and interpreting all variables in per-capita (or aggregate) terms.

2.3 Producer optimization and price setting

Our baseline model allows for nominal rigidities, but abstracts from the complexities associated with the dynamics of price adjustment.[12] To enhance analytical tractability, it is assumed that agents set the nominal price of their product one period in advance, and stand ready to meet (domestic and

[10] We adopt the notation of Obstfeld and Rogoff [1996, ch.10]. Specifically, our timing convention has $M_t(j)$ as agent j's nominal balances accumulated during perod t and carried over into period $t+1$, while $B_{\mathrm{H},t}(j)$ and $B_{\mathrm{F},t}(j)$ denote agent j's bonds accumulated during period $t-1$ and carried over into period t.

[11] The optimization problem is fully characterized in the Appendix.

[12] The implications of Calvo-style adjustment of prices are explored in many closed or small open-economy models mentioned in the references. Benigno and Benigno [2000] consider the extension to a general equilibrium framework under full exchange rate pass-through.

foreign) demand at given prices for one period. The number of producers is large enough so that firms ignore the impact of their pricing decision on the aggregate price indexes.

In terms of our notation, Home firms selling in the Home market choose $p_t(h)$ at time $t-1$. As shown in the appendix, in a symmetric environment where all prices $p_t(h)$ are equal and $p_t(h) = P_{\mathrm{H},t}$, Home firms optimally set domestic prices such that:

$$p_t(h) = P_{\mathrm{H},t} = \frac{1}{\Phi} E_{t-1} \left[\alpha_t P_t C_t \right] \tag{14}$$

where we define $\Phi \equiv (\theta - 1) / \theta\kappa$. Interpreting (14), domestic firms fix prices equal to expected nominal marginal cost (that is, $E_{t-1} [\kappa \alpha_t P_t C_t]$), augmented by the equilibrium markup $\theta / (\theta - 1)$.

To account for the possibility of deviations from the law of one price and the impact of exchange rate fluctuations on firms' markups, it is (realistically) assumed that pass-through to export prices is less than perfect. The simplest way to model imperfect pass-through is to consider the following pricing function:

$$p_t^*(h) \equiv \tilde{p}_t(h) \mathcal{E}_t^{-\eta^*} \qquad 0 \leq \eta^* \leq 1 \tag{15}$$

The variable $\tilde{p}_t(h)$ is the predetermined component of the Foreign currency price of good h that cannot be adjusted to variations of the exchange rate during period t. Home firms choose $\tilde{p}_t(h)$ one period in advance at time $t-1$, while the actual $p_t^*(h)$ depends on the realization of the exchange rate at time t. The parameter η^* indexes the degree of pass-through in the Foreign markets. For instance, if $\eta^* = 1$, pass-through in the Foreign country is complete — as in the PCP case. If $\eta^* = 0$, we have $p_t^*(h) = \tilde{p}_t(h)$ which coincides with the price chosen by the Home producer in the LCP case.

In a symmetric environment, optimal pricing also yields:

$$p_t^*(h) = P_{\mathrm{H},t}^* = \tilde{p}_t(h) \mathcal{E}_t^{-\eta^*} = \frac{1}{\Phi} \frac{1}{\mathcal{E}_t^{\eta^*}} \frac{E_{t-1} \left[\alpha_t P_t^* C_t^* \mathcal{E}_t^{\eta^*} \right]}{E_{t-1} \left[\frac{\mathcal{E}_t P_t^* C_t^*}{P_t C_t} \right]} \tag{16}$$

Interpreting (16), domestic firms fix $\tilde{p}_t(h)$ so that at the margin the expected disutility from an increase in labor effort is equal to the expected utility from consumption financed by additional sales revenue.

Analogous expressions can be derived for the pricing by Foreign firms in the Foreign and the Home market. Define the function:

$$p_t(f) = \tilde{p}_t^*(f)\mathcal{E}_t^{\eta}, \qquad 0 \leq \eta \leq 1. \tag{17}$$

As before, $\eta = 1$ corresponds to PCP, $\eta = 0$ to LCP. The degree of pass-through in the Home country, η, need not be equal to that in the Foreign country, η^*. The optimal pricing strategy is such that:

$$P_{\mathrm{F},t}^* = \frac{1}{\Phi^*} E_{t-1}\left[\alpha_t^* P_t^* C_t^*\right], \quad P_{\mathrm{F},t} = \frac{\mathcal{E}_t^{\eta}}{\Phi^*} \frac{E_{t-1}\left[\alpha_t^* P_t C_t \dfrac{1}{\mathcal{E}_t^{\eta}}\right]}{E_{t-1}\left[\dfrac{P_t C_t}{\mathcal{E}_t P_t^* C_t^*}\right]} \tag{18}$$

where $\Phi^* \equiv (\theta^* - 1)/\theta^* \kappa^*$.

In what follows, it will be useful to compare equilibrium prices with and without nominal rigidities. We denote variables under a flex-price equilibrium with a $flex$ superscript. If prices were fully flexible, firms would use their monopoly power charging a fixed percentage over marginal costs. For instance, in the Home market we would have:

$$P_{\mathrm{H},t}^{flex} = \left(1 + \frac{1}{\theta - 1}\right) \kappa \alpha_t P_t C_t \tag{19}$$

In a fix-price equilibrium instead, for given $P_{\mathrm{H},t}$, the profit margin at time t is not constant, endogenously depending on how macroeconomic and policy shocks affect the marginal cost $\kappa \alpha_t P_t C_t$.

Let Θ denote the ratio between the price and marginal cost of a good, that is the profit margin or markup. Clearly, firms are willing to supply goods at given prices as long as their ex-post percentage markup is non-negative:

$$\Theta_{\mathrm{H},t} \geq 1 \quad \Leftrightarrow \quad P_{\mathrm{H},t} \geq \kappa \alpha_t P_t C_t \tag{20}$$

Otherwise, agents would be better off by not accommodating shocks to demand. In what follows, we restrict the set of shocks so that the 'participation constraint' (20) and its analogs are never violated.

2.4 Government budget constraint and asset markets

The government budget constraint in the Home country is:

$$\int_0^1 \left[M_t(j) - M_{t-1}(j)\right] dj + \int_0^1 T_t(j)dj = 0 \tag{21}$$

where $M_t(j)$ are the money demand functions as determined above.[13] Only Home residents hold Home money. The government affects the stock of Home liquidity by controlling the short-term rate i_{t+1}. Similar considerations hold for the Foreign country, where the government controls the interest rate i^*_{t+1}.

The model is closed by posing that international bonds are in zero net-supply:

$$\int_0^1 B_{\mathrm{H},t}(j)dj + \int_0^1 B^*_{\mathrm{H},t}(j^*)dj^* = 0, \quad \int_0^1 B_{\mathrm{F},t}(j)dj + \int_0^1 B^*_{\mathrm{F},t}(j^*)dj^* = 0 \tag{22}$$

so that, in a symmetric environment, $B_{\mathrm{H},t} = -B^*_{\mathrm{H},t}$ and $B_{\mathrm{F},t} = -B^*_{\mathrm{F},t}$.

3 The international transmission of real and monetary shocks

3.1 The solution of the model

We solve the model under the assumption that, at some initial time t_0, net non-monetary wealth is zero in each country, that is, $B_{\mathrm{H},t_0} = B_{\mathrm{F},t_0} = 0$. Then, we can study an equilibrium in which non-monetary wealth is zero at *any* subsequent point in time: Home imports from Foreign are always equal in value to Foreign imports from Home. As trade and the current account are invariably balanced, countries consume precisely their sales revenue:

$$R_t - P_t C_t = 0, \quad R^*_t - P^*_t C^*_t = 0 \tag{23}$$

[13]The model could easily be extended to encompass government spending and public debt. Note that fiscal shocks, modeled as random fluctuations in government spending, are isomorphic to productivity shocks in our framework under the assumptions that each national government spends exclusively on domestically produced goods and utility is additively separable in private and public consumption. For a detailed treatment of fiscal interdependence and the interaction between monetary and fiscal policies see Corsetti and Pesenti [2001].

This stems from the combination of three hypotheses: (i) Cobb-Douglas consumption indexes; (ii) logarithmic consumption preferences; (iii) zero initial non-monetary wealth. As shown in Corsetti and Pesenti [2001], under PCP the result holds also when assumption (ii) is relaxed.

Before we derive the general solution of the model, it is analytically helpful to introduce the variables μ_t and μ_t^*, defined as the reciprocal of the marginal utility of domestic consumers' nominal wealth.[14] Note that, from the Euler equation μ_t is related to the Home interest rate according to the following expression:

$$\frac{1}{\mu_t} = \beta(1 + i_{t+1})E_t\left(\frac{1}{\mu_{t+1}}\right) \tag{24}$$

The above expression shows that, given the time path of μ (and $1/\mu$), there is a corresponding sequence of Home nominal interest rates: Home monetary easing at time t is associated with both a higher μ_t and a lower i_{t+1}. A similar expression holds for the Foreign country.

Table 1 presents the general solution of the model whereas all endogenous variables (25) through (35) are expressed in closed form as functions of real and monetary shocks (α_t, α_t^*, χ_t and χ_t^*)[15] and the Home and Foreign monetary stances μ_t and μ_t^*.

Interpreting Table 1: since the equilibrium current account is always balanced, the nominal exchange rate $\mathcal{E}_t$ in (25) is proportional to $P_tC_t/P_t^*C_t^*$, that is, a function of the relative monetary stance. Domestic prices of domestic goods are predetermined according to (26) and (29), while import prices vary with the exchange rate, depending on the degree of exchange rate pass-through according to (27) and (28). Given interest rates, money is determined residually according to (30) and (31). Equilibrium consumption is determined in (32) and (33), where Φ_W is defined as $\Phi_W \equiv \Phi^\gamma\left(\Phi^*\right)^{1-\gamma}$. Finally employment levels are determined according to (34) and (35). Note that it is always the case that:

$$E_{t-1}\ell_t = \Phi, \qquad E_{t-1}\ell_t^* = \Phi^*. \tag{36}$$

[14]That is, μ_t is the reciprocal of the Lagrangean multiplier, denoted by λ_t in the Appendix. With logarithmic utility, μ_t is also equal to nominal spending P_tC_t.

[15]Recall that the analysis is restricted to shocks whose size does not lower the ex-post markup below zero.

Table 1: The solution of the fix-price model

$$\mathcal{E}_t = \frac{1-\gamma}{\gamma}\frac{\mu_t}{\mu_t^*} \tag{25}$$

$$P_{\mathrm{H},t} = \frac{1}{\Phi}E_{t-1}\left(\alpha_t\mu_t\right) \tag{26}$$

$$P_{\mathrm{F},t} = \frac{1}{\Phi^*}\left(\frac{\mu_t}{\mu_t^*}\right)^{\eta}E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right]\frac{1-\gamma}{\gamma} \tag{27}$$

$$P_{\mathrm{H},t}^* = \frac{1}{\Phi}\left(\frac{\mu_t^*}{\mu_t}\right)^{\eta^*}E_{t-1}\left[\alpha_t\mu_t^{\eta^*}\left(\mu_t^*\right)^{1-\eta^*}\right]\frac{\gamma}{1-\gamma} \tag{28}$$

$$P_{\mathrm{F},t}^* = \frac{1}{\Phi^*}E_{t-1}\left(\alpha_t^*\mu_t^*\right) \tag{29}$$

$$M_t = \frac{\chi_t\mu_t}{1-\beta\mu_t E_t\left(\mu_{t+1}^{-1}\right)} \tag{30}$$

$$M_t^* = \frac{\chi_t^*\mu_t^*}{1-\beta\mu_t^* E_t\left(\mu_{t+1}^{*-1}\right)} \tag{31}$$

$$C_t = \frac{\gamma\Phi_W\mu_t^{1-\eta(1-\gamma)}\left(\mu_t^*\right)^{\eta(1-\gamma)}}{\left[E_{t-1}\left(\alpha_t\mu_t\right)\right]^{\gamma}\left[E_{t-1}\left(\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right)\right]^{1-\gamma}} \tag{32}$$

$$C_t^* = \frac{\left(1-\gamma\right)\Phi_W\left(\mu_t^*\right)^{1-\eta^*\gamma}\mu_t^{\eta^*\gamma}}{\left[E_{t-1}\left(\alpha_t^*\mu_t^*\right)\right]^{1-\gamma}\left[E_{t-1}\left(\alpha_t\mu_t^{\eta^*}\left(\mu_t^*\right)^{1-\eta^*}\right)\right]^{\gamma}} \tag{33}$$

$$\ell_t = \Phi\left[\gamma\frac{\alpha_t\mu_t}{E_{t-1}\left(\alpha_t\mu_t\right)} + \left(1-\gamma\right)\frac{\alpha_t\mu_t^{\eta^*}\left(\mu_t^*\right)^{1-\eta^*}}{E_{t-1}\left[\alpha_t\mu_t^{\eta^*}\left(\mu_t^*\right)^{1-\eta^*}\right]}\right] \tag{34}$$

$$\ell_t^* = \Phi^*\left[\left(1-\gamma\right)\frac{\alpha_t^*\mu_t^*}{E_{t-1}\left(\alpha_t^*\mu_t^*\right)} + \gamma\frac{\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}}{E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right]}\right] \tag{35}$$

Under rational expectations, for any given distribution of the shocks and policy rule, agents optimally predetermine prices so as to stabilize expected employment at the constant level Φ or Φ^*, that we will soon show to be the flex-price equilibrium employment.

3.2 The transmission mechanism

The essence of the international transmission mechanism in our model is captured by three features of the equilibrium allocation in Table 1.

First, for given monetary stances μ and μ^*, productivity shocks affect current *employment* but neither *output* nor *consumption*. The negative response of employment to productivity shocks is a common finding in the so-called New Keynesian or New Neoclassical synthesis model.[16] Intuitively, nominal aggregate demand is controlled by the government through monetary policy, while the CPI is a function of predetermined prices and the exchange rate, in itself a function of relative monetary stances. In the presence of price rigidities, for given μ and μ^*, an unanticipated shock to the relative supply of the Home goods cannot be matched by a fall of their relative price: all prices (26), (28), (27), and (29), are in fact invariant to the realizations of the productivity shocks. As a consequence, with a given world demand nailing down the level of output in each country, an increase in productivity simply lowers employment (34) without affecting the supply of goods available for consumption.

Second, current consumption and employment in each country are non-decreasing functions of monetary policy shocks abroad. The transmission of monetary shocks works through both world aggregate demand — which always increases after a policy shock eases the global monetary stance — and movements of the terms of trade. A high degree of pass-through ($\eta^* = \eta \simeq 1$) implies that a depreciation of the Home currency is associated with a worsening of the Home terms of trade. The relative price of Home goods falls in each market, so that more units of Home labor are required to buy one unit of Home consumption. Correspondingly, the Foreign terms of trade improve.[17] A low degree of pass-through ($\eta^* = \eta \simeq 0$) , instead, leads to the

[16] Gali [1999] provides empirical evidence in support of such a view of the business cycle effects of supply shocks.

[17] On the 'beggar-thyself' vs. 'beggar-thy-neighbor' effects of exchange rate depreciations see Corsetti and Pesenti [2001] and Tille [2000].

opposite result. Local prices are invariant to the exchange rate, but for each unit of goods sold abroad, Home firms' revenue in domestic currency rises in proportion to the rate of depreciation. This additional revenue finances higher consumption, thus leading to higher production and imports. Foreign exporters now have to supply more goods while suffering a fall in the domestic-currency value of their sales revenue.

Third, observe that the level of the Home good price, (27), depends on $E_{t-1}(\alpha_t \mu_t)$ and is therefore an increasing function of the covariance between α and μ. This implies that the systematic tightening of monetary policy (lower μ) in response to shocks affecting Home producers' marginal costs (higher α) is associated, in equilibrium, with lower domestic prices P_{H} (and higher expected utility). Similar considerations hold for the predetermined component of the Foreign good price in the domestic market, (26): as long as $\eta > 1$, P_{F} depends on the covariance between Home monetary policy and Foreign productivity shock. Of course, in the absence of 'fundamental' shocks α and α^*, monetary volatility would only have a negative effect on Home expected utility, by increasing the volatility of consumption.[18]

3.3 The flex-price equilibrium benchmark

Before delving into policy analysis, it is useful to characterize the equilibrium allocation we would obtain if prices were flexible. In fact, we will use such allocation as a welfare benchmark in the rest of the paper. With flexible prices, the pricing equations (26) through (29) hold in any state of nature, not in expectation, so that the degree of pass-through is irrelevant, the law of one price is valid and purchasing power parity holds. It is easy to verify that the level of employment is constant and equal to:

$$\ell_t^{flex} = \alpha_t \frac{P_t^{flex}}{P_{\mathrm{H},t}^{flex}} C_t^{flex} = \Phi \quad \ell_t^{*flex} = \alpha_t^* \frac{P_t^{*flex}}{P_{\mathrm{F},t}^{*flex}} C_t^{*flex} = \Phi^*. \tag{37}$$

As opposed to the fixed-price case, shocks to productivity affect *output*, not *employment*. Note that the flex-price level of employment is determined by the magnitude of monopolistic distortions — the competitive level of Home employment corresponds in fact to $1/\kappa > \Phi$. Consumption at Home

[18] See the considerations in Obstfeld and Rogoff [1998].

and abroad is a function of global shocks and global monopolistic distortions Φ_W:

$$C_t^{flex} = \gamma \frac{\Phi_W}{\alpha_t^{\gamma} \left(\alpha_t^*\right)^{1-\gamma}}, \quad C_t^{*flex} = \left(1-\gamma\right) \frac{\Phi_W}{\alpha_t^{\gamma} \left(\alpha_t^*\right)^{1-\gamma}}. \tag{38}$$

Observe that, first, consumption and employment are higher in an economy where global monopolistic distortions Φ_W are low. Second, a productivity shock anywhere in the world economy raises global consumption. To see why, consider a positive productivity innovation in the Home country. Employment being constant in equilibrium, such shock raises Home output and consumption. However, it also lowers the Home goods price, raising the real income of Foreign consumers. The terms of trade between domestic and foreign goods are in fact a function of relative output supply:

$$\left(\frac{P_{\mathrm{F},t}^* \mathcal{E}_t}{P_{\mathrm{H},t}}\right)^{flex} = \frac{1-\gamma}{\gamma} \frac{\alpha_t^*}{\alpha_t} \frac{\Phi}{\Phi^*} \tag{39}$$

Movements of the terms of trade ensure that the benefits (costs) from a positive (negative) productivity shock spread around the world. By the same token, large monopolistic distortions anywhere in the world economy raise the level of consumption prices worldwide, thus reducing global consumption.

4 A framework for the analysis of monetary policy in open economy

4.1 The goals of monetary policy

In this section we set and discuss the policy problem faced by a benevolent policy maker whose welfare objective is the maximization of Home agents' expected utility. In our analysis, we focus on the non-monetary components of utility — i.e. we assume that χ is arbitrarily small — adopting $E_{t-1}\mathcal{W}_t$ as a measure of national welfare.

4.1.1 The utility-based policy loss function...

Using the flex-price solution for consumption (38) with the corresponding expression (32), we can calculate the gap between expected utility with flexible prices, indexed $\mathcal{W}^{flex}$, and expected utility with predetermined prices:

$$E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t = E_{t-1}\left[\gamma \ln\left(\frac{E_{t-1}\left(\alpha_t\mu_t\right)}{\alpha_t\mu_t}\right) + \right.$$
$$\left. + (1-\gamma)\ln\left(\frac{E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^\eta \mu_t^{1-\eta}\right]}{\alpha_t^*\left(\mu_t^*\right)^\eta \mu_t^{1-\eta}}\right)\right] \geq 0 \tag{40}$$

By Jensen's inequality, the gap $E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t$ cannot be negative: expected utility with price rigidities is *never* above expected utility with flexible prices. It follows that, at best, what monetary policy rules can do is to bridge the gap between the two: a policy rule that minimizes (40) is also a policy that maximizes the expected utility of the Home representative consumer.

Below we discuss three equivalent representations of the policy loss function (40), each contributing to our understanding of the policy problem.

4.1.2 ... can be expressed as a function of expected markups...

First, we equate the policy loss function to an index of markups charged by producers selling in the Home market. The markup charged by Home firms on domestic sales and the markup charged by Foreign firms exporting to the Home market are, respectively:

$$\Theta_{\mathrm{H},t} = \frac{P_{\mathrm{H},t}}{\kappa\alpha_t P_t C_t} = \frac{\theta}{\theta-1}\frac{E_{t-1}\left(\alpha_t\mu_t\right)}{\alpha_t\mu_t} \tag{41}$$

$$\Theta_{\mathrm{F},t} = \frac{P_{\mathrm{F},t}}{\mathcal{E}_t\kappa^*\alpha_t^* P_t^* C_t^*} = \frac{\theta^*}{\theta^*-1}\frac{E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^\eta \mu_t^{1-\eta}\right]}{\alpha_t^*\left(\mu_t^*\right)^\eta \mu_t^{1-\eta}} \tag{42}$$

Using these definitions, it is easy to show that the policy loss function (40) is equal to the expected value of the (log) average markup in the Home markets, minus a constant depending on θ and θ^*:

$$E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t = E_{t-1}\left[\gamma \ln\Theta_{\mathrm{H},t} + \right.$$
$$\left. +(1-\gamma)\ln\Theta_{\mathrm{F},t} - \gamma\ln\frac{\theta}{\theta-1} - (1-\gamma)\ln\frac{\theta^*}{\theta^*-1}\right] \tag{43}$$

The logic of this result is as follows. Producers optimally set their monopoly markups and prices given their expectations of future monetary policy. The higher these prices, the lower the representative consumer's welfare. Under the constraint of rational expectations, domestic policy makers cannot systematically push markups below their flex-price equilibrium levels; they can, however, design policy rules aimed at minimizing expected markups when firms set their prices. This goal is achieved by stabilizing producers' income from the domestic market.

Note that, in the case $\eta = 1$, the markup of foreign producers in the Home country $\Theta_{\mathrm{F},t}$ does not depend on domestic monetary policy. In this case, Home monetary authorities are only concerned with domestic producers. Conversely, if $\eta = 0$, $\Theta_{\mathrm{F},t}$ does not depend on Foreign monetary policy — ruling out an important channel of policy spillovers.

4.1.3 ... or a function of the consumer price index under flexible and fixed prices...

Second, we translate the above index of markups in terms of consumer prices, with and without nominal rigidities. Expressions for prices of domestic and foreign goods in the Home country in a flex-price equilibrium can be easily derived from (26) and (27), without the expectation operator. The relation between markups and prices can thus be written as:

$$\Theta_{\mathrm{H},t} = \frac{\theta}{\theta - 1} \frac{P_{\mathrm{H},t}}{P_{\mathrm{H},t}^{flex}}, \qquad \Theta_{\mathrm{F},t} = \frac{\theta^*}{\theta^* - 1} \frac{P_{\mathrm{F},t}}{P_{\mathrm{F},t}^{flex}} \tag{44}$$

Substituting, the policy loss function becomes

$$E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t = E_{t-1} \ln \frac{P_t}{P_t^{flex}} \geq 0 \tag{45}$$

The policy loss function is equal to the expected ratio between the fixed-price CPI and the flex-price CPI. For any given monetary stance, the consumer price index under sticky prices cannot be expected to be lower than the CPI under flexible prices. We therefore conclude that optimal monetary policy rules minimize the expected deviation of the (log) CPI from a target given by the (log) equilibrium CPI under flexible prices.[19]

[19] For a recent analysis of the case for price stability in closed and small open economy see Goodfriend and King [2000].

4.1.4 ... or a function of output gaps and deviations from the law of one price

Third, we express (40) in terms of output gaps, in the Home and Foreign economy, and deviations from the law of one price in each market. With a linear production function, the output gap can be measured as the distance between actual and equilibrium employment levels. Using the benchmark of the flex-price allocation, we express the output gap in terms of the ratio ℓ_t/Φ.[20]

Observe that output gaps for the Home and the Foreign countries can be easily derived from (34) and (35). Rearranging these expressions and using the definition of markups, we obtain:

$$\frac{\ell_t}{\Phi} = \frac{\theta}{\theta - 1}\frac{1}{\Theta_{\mathrm{H},t}}\left[\gamma + (1-\gamma)\frac{P_{\mathrm{H},t}}{\mathcal{E}_t P_{\mathrm{H}^*,t}}\right] \tag{46}$$

Substituting into (43) we can see that the policy loss function is equal to the expectation of a weighted average of the output gaps, domestically and abroad, and a weighted average of the deviations from the law of one price in each country:

$$\begin{aligned} E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t = E_{t-1}\Bigg\{&\gamma \ln\frac{\ell_t}{\Phi} + (1-\gamma)\ln\frac{\ell_t^*}{\Phi^*} + \\ +\gamma\ln\left[\gamma + (1-\gamma)\frac{P_{\mathrm{H},t}}{\mathcal{E}_t P_{\mathrm{H}^*,t}}\right] &+ (1-\gamma)\ln\left[\gamma + (1-\gamma)\frac{P_{\mathrm{F},t}}{\mathcal{E}_t P_{\mathrm{F}^*,t}}\right]\Bigg\} \end{aligned} \tag{47}$$

If the law of one price holds, i.e. if η is equal to 1, the two terms in square brackets disappear, and the foreign output gap does not respond to domestic monetary policy. As we will see below, in this case welfare maximization coincides with closing the domestic output gap and the optimal policy is fully inward-looking. In general, however, the policy loss function for an interdependent economy includes more than the domestic output gap — it also depends on the exchange rate and international cyclical conditions.

[20] A different measure of output gap is $\ell/(1/\kappa)$, where the flex-price equilibrium allocation is derived ruling out monopolistic distortion.

4.2 Stabilization and interdependence in the Nash equilibrium

We are now ready to derive the optimal monetary stances in a world Nash equilibrium, whereas national policy makers are able to commit to preannounced rules. The policy problem faced by the Home monetary authority is to maximize the (non-monetary) indirect utility of the Home representative consumer:

$$\max_{\{\mu_{t+\tau}\}_{\tau=0}^{\infty}} E_{t-1}\mathcal{W}_t = E_{t-1}\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(\ln C_\tau - \kappa\Phi\right) \tag{48}$$

taking $\{\mu_\tau^*, \alpha_\tau, \alpha_\tau^*\}_{\tau=t}^{\infty}$ as given. Note that this is equivalent to minimizing the policy loss $E_{t-1}\mathcal{W}_t^{flex} - E_{t-1}\mathcal{W}_t$ in (40). The Foreign authority faces a similar problem. Table 2 presents the closed-form reaction functions, respectively (49) for Home and (50) for Foreign, whose solution is the global Nash equilibrium. The reaction functions are expressed in three ways: as explicit functions of shocks and monetary policies; as implicit functions of output gaps and deviations from the law of one price; and as implicit functions of the ex-post profit margins.

With an optimal policy in place, the Home monetary stance is eased (μ_t increases) in response to a positive domestic productivity shock (low α_t). Monetary policy is pro-cyclical, in the sense that it accommodates domestic productivity improvements. Yet, as seen above, in the absence of a policy reaction a productivity shock would lower employment below Φ — that is, raise Home potential output above the current level. In light of this observation, monetary policy leans against the wind and moves to close the employment and output gaps.

The optimal response of Home interest rates to a Home productivity shock, however, is not necessarily aimed at closing the output gap completely: unless $\eta = 1$, ℓ will not be equal to Φ at an optimum. To gain intuition, consider two symmetric productivity shocks in the Home country, leading Home monetary authorities to tighten (if the shock is negative) or to relax (if the shock is positive) their policy stance. With a negative shock, the appreciation of the Home currency reduces the revenue of Foreign firms selling goods at Home, as $P_{\mathrm{F}}/\mathcal{E}$ falls by a percentage $1-\eta$ with the movement in the exchange rate. Even though the Foreign firms revenue increases with a

Table 2: The Nash equilibrium under commitment

$$
\begin{aligned}
1-\eta(1-\gamma) &= \frac{\gamma\alpha_t\mu_t}{E_{t-1}\left(\alpha_t\mu_t\right)} + \frac{(1-\gamma)\,(1-\eta)\,\alpha_t^*\,(\mu_t^*)^{\eta}\,\mu_t^{1-\eta}}{E_{t-1}\left(\alpha_t^*\,(\mu_t^*)^{\eta}\,\mu_t^{1-\eta}\right)} \\
&= \frac{\gamma\dfrac{\ell_t}{\Phi}}{\gamma+(1-\gamma)\,\dfrac{P_{\mathrm{H},t}}{\mathcal{E}_t P^*_{\mathrm{H},t}}} + \frac{(1-\gamma)\,(1-\eta)\,\dfrac{\ell_t^*}{\Phi^*}}{\gamma+(1-\gamma)\,\dfrac{P_{\mathrm{F},t}}{\mathcal{E}_t P^*_{\mathrm{F},t}}} \\
&= \frac{\gamma\theta}{\theta-1}\frac{1}{\Theta_{\mathrm{H},t}} + \frac{(1-\gamma)\,(1-\eta)\,\theta^*}{\theta^*-1}\frac{1}{\Theta_{\mathrm{F},t}}
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
1-\eta^*\gamma &= \frac{(1-\gamma)\,\alpha_t^*\mu_t^*}{E_{t-1}\left(\alpha_t^*\mu_t^*\right)} + \frac{\gamma\,(1-\eta^*)\,\alpha_t\mu_t^{\eta^*}\,(\mu_t^*)^{1-\eta^*}}{E_{t-1}\left(\alpha_t\mu_t^{\eta^*}\,(\mu_t^*)^{1-\eta^*}\right)} \\
&= \frac{(1-\gamma)\,\dfrac{\ell_t^*}{\Phi^*}}{\gamma+(1-\gamma)\,\dfrac{\mathcal{E}_t P^*_{\mathrm{F},t}}{P^*_{\mathrm{F},t}}} + \frac{\gamma\,(1-\eta^*)\,\dfrac{\ell_t}{\Phi}}{\gamma+(1-\gamma)\,\dfrac{\mathcal{E}_t P^*_{\mathrm{H},t}}{P_{\mathrm{H},t}}} \\
&= \frac{(1-\gamma)\,\theta^*}{\theta^*-1}\frac{1}{\Theta^*_{\mathrm{F},t}} + \frac{\gamma\,(1-\eta^*)\,\theta}{\theta-1}\frac{1}{\Theta^*_{\mathrm{H},t}}
\end{aligned} \tag{50}
$$

positive shock, the welfare costs to Foreign agents from a Home monetary contraction are larger than the gains from higher income in the event of a Home monetary expansion.

Ex ante, Foreign agents will hedge against the expected loss of sales revenue in the Home market by pre-setting a higher price on their exports, lowering Home consumption on average. This is why the Home monetary stance required to close the domestic output gap is not optimal. Relative to such stance, domestic policy makers can improve utility by adopting a policy that equates, at the margin, the benefit from keeping domestic output close to its potential level with the loss from lower consumption, as prescribed by equation (49). The corollary of these considerations is that, in general, optimal monetary rules in a world Nash equilibrium imply smaller exchange rate volatility *vis-à-vis* inward-looking rules focused on domestic developments regardless of global cyclical conditions.

As long as η is below one, the Home monetary stance tightens when productivity worsens abroad and loosens otherwise. Rising costs abroad (an increase in α_t^*) lower the markup of Foreign goods sold at Home. If Home policy makers were not expected to intervene and stabilize the markup by hiking rates and appreciating the exchange rate, Foreign firms would charge higher prices onto Home consumers. Only when $\eta = 1$ Foreign firms realize that any attempt by the Home authorities to stabilize the markup is bound to fail as both P_{F} and the exchange rate fall in the same proportion.

Note that, in general, domestic monetary authorities react to Foreign policy changes: a contraction abroad is matched by an expansion at Home so that monetary stances are strategic substitutes. The only cases in which Home monetary policy does not react to Foreign policy are either when $\eta = 0$ or when $\eta = 1$. In the case $\eta = 1$ Foreign interest rate volatility increases the fluctuations of Foreign marginal costs, thus raising the markup on Foreign exports. Yet, there is nothing the Home authorities can do to reduce the volatility of Foreign firms' profits, since $P_{\mathrm{F}}/\mathcal{E}$ does not respond to Home interest rates. In the case $\eta = 0$, instead, movements in the exchange rate and Foreign marginal costs exactly offset each other and P_{F} does not respond to the current exchange rate. Once again, there is nothing for the Home authorities to stabilize and Home interest rates do not respond to Foreign policies.

Besides these two extreme cases of PCP and LCP, however, there is always a rationale for Home policies to react negatively to Foreign policies: a Foreign expansion lowers the markup on Foreign exports, requiring a Home monetary response to sustain the Foreign firms' profits by appreciating the exchange rate. Thus, in general, there will be strategic interdependence among policy makers.[21]

4.3 Pass-through and Pareto-efficiency

The utility gap in expression (40) does not depend on monopolistic distortions, whose magnitude is indexed by Φ_W. As discussed by Obstfeld and Rogoff [2000a], suppose that governments in both countries set non-monetary instruments (taxes and subsidies) so as to offset the distortions due to monopolistic production. If a global monetary policy exists such that the gap (40) is bridged, such policy would implement the first-best allocation. Then, starting from the first-best allocation, removing taxes and subsidies will lower the *level* of both $E_{t-1}\mathcal{W}_t^{flex}$ and $E_{t-1}\mathcal{W}_t$, but will not affect the *difference* between the two. The global monetary policy in place would still be optimal, and the corresponding allocation would be constrained-Pareto efficient (that is, efficient for the world economy subject to a given level of monopolistic distortion).

Can monetary rules close the world utility gap completely? In the presence of asymmetric shocks the answer is negative in all but one important case, that is PCP. With complete pass-through, the Nash equilibrium policy is:

$$\mu_t = \frac{\Phi P_{\mathrm{H},t}}{\alpha_t} \quad \mu_t^* = \frac{\Phi^* P_{\mathrm{F},t}^*}{\alpha_t^*} \tag{51}$$

Monetary authorities optimally stabilize employment at its flex-price level Φ and Φ^*.[22] Domestic and global consumption optimally co-moves with productivity shocks according to (38). Thus, the Nash optimal monetary policy supports the same allocation as with flexible prices, that is constrained Pareto-efficient.

[21] Betts and Devereux [2000b] examine the problem of international monetary coordination, in environments of differing pass-through, with discretion in policy making.

[22] Note that, because of our utility parameterization, domestic monetary policy only responds to domestic shock. This will not be true, for instance, with a power utility of consumption. Yet, even with power utility the Nash allocation under PCP coincides with the flex-price allocation.

This result provides an extreme version of the case for flexible exchange rates made by Friedman [1953]: even without price flexibility, monetary authorities can engineer the right adjustment in relative prices through exchange rate movements. In our model with PCP, expenditure switching effects makes exchange rate and price movements perfect substitutes.

The Nash equilibrium will however *not* coincide with a flex-price equilibrium when the pass-though is less than perfect in either market. Consider for instance the case of LCP.[23] Suppose monetary authorities set interest rates so as to achieve the flex-price level of consumption in both countries:

$$\mu_t = \frac{\gamma \Phi_W P_t}{\alpha_{W,t}},\ \mu_t^* = \frac{(1-\gamma)\,\Phi_W P_t^*}{\alpha_{W,t}} \tag{52}$$

Note that both monetary policies would react to the same global index of productivity shocks. Then, the employment levels would be quite different from their flex-price levels, varying over time as functions of relative productivity shocks:

$$\ell_t \propto \left(\frac{\alpha_t}{\alpha_t^*}\right)^{1-\gamma}, \quad \ell_t^* \propto \left(\frac{\alpha_t}{\alpha_t^*}\right)^{-\gamma} \tag{53}$$

Only if shocks were global ($\alpha_t = \alpha_t^*$) would the level of employment be constant. In fact, with global shocks the terms of trade externality disappears, relative prices need not adjust, and the Nash equilibrium trivially supports the flex-price allocation.

4.4 Commitment vs. discretion in open economy

So far we have assumed that policy-makers worldwide are able to commit to optimal policies. This section contrasts the policy problem under commitment with the policy problem under discretion. First, we discuss inflationary bias in open economy, identifying the determinants of its magnitude and sign. Second, we show that the optimal policy is not time consistent, even in the absence of an inflationary or deflationary bias.

Under discretion, in each period t the Home policy maker solves

$$\max_{\{\mu_{t+\tau}\}_{\tau=0}^{\infty}} \mathcal{W}_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\ln C_\tau - \kappa \ell_\tau\right) \tag{54}$$

[23] See the discussion in Devereux and Engel [2000].

taking $\{\mu_\tau^*, \alpha_\tau, \alpha_\tau^*\}_{\tau=t}^{\infty}$ as given. The first order condition is:

$$\frac{1-\eta(1-\gamma)}{\kappa\Phi} = \frac{\gamma\alpha_t\mu_t}{E_{t-1}\left(\alpha_t\mu_t\right)} + \frac{(1-\gamma)\,\eta^*\alpha_t\left(\mu_t^*\right)^{1-\eta^*}\mu_t^{\eta^*}}{E_{t-1}\left(\alpha_t\left(\mu_t^*\right)^{1-\eta^*}\mu_t^{\eta^*}\right)} \tag{55}$$

A similar expression holds in the Foreign country.

4.4.1 Inflationary vs. deflationary bias

In a closed economy, the presence of monopolistic distortions are clearly associated with an inflationary bias in policy making under discretion. This need not be true in an open economy. As analyzed in Corsetti and Pesenti [2001], the bias in policy making may be *deflationary* rather than inflationary if the economy is sufficiently small so as to suffer from the consumption and welfare consequences of adverse terms of trade movements. Taking expectations of both sides of (55), it is clear that a discretionary policy will be subject to either inflationary or deflationary bias in the Home country if:

$$1-\eta(1-\gamma) \gtrless \left[\gamma + (1-\gamma)\,\eta^*\right]\kappa\Phi. \tag{56}$$

Under discretion, there is an inflationary bias in Home monetary policy if the economy is sufficiently 'large', in the sense that a large share of consumption falls on Home products and γ is sufficiently close to 1. As a depreciation of the exchange rate affects the price of a relatively small share of consumption goods, policy makers are less concerned with adverse import price movements than with the distortions associated with monopoly power in production.

The reverse is true when γ is sufficiently small and η and η^* sufficiently large. A monetary expansion, while raising output and employment, also increases the price of a substantial proportion of consumption goods. The terms of trade movement becomes a dominant concern in discretionary policy making, leading to a deflationary bias. Reducing the degree of pass-through lowers the concern about adverse terms of trade movements associated with a monetary expansion. For given γ, Φ, and η^*, a lower pass-through in the domestic market (i.e. a lower η) is associated with a stronger inflationary bias.[24]

[24]Recent literature provides evidence of a negative relationship between openness and

4.4.2 Time consistency issues independent of inflationary bias

Assume now that condition (56) holds as an equality, or (alternatively) that the government is able to use an appropriate fiscal instrument to offset the impact of monopolistic distortions,[25] so that there is no inflationary/deflationary bias in policy making. One may expect that the first order conditions of the policy problems under discretion would then coincide with the first order conditions under commitment. Comparing (55) and (49), it is apparent that this is true only in the PCP case ($\eta = \eta^* = 1$), when firms profits are not exposed at all to exchange rate variability. In all other cases, discretionary policies do not coincide with policies under commitment. The reason is the following.

Under discretion, for given firms prices, the Home policy maker responds to domestic productivity shocks and foreign monetary shocks depending on the degree of pass-through *in the Foreign market*, η^*, that is relevant for Home producers/exporters. Ex post, the optimal policy reaction to a shock moves exchange rates as to tilt the external terms of trade in favor of domestic producers. While optimal ex post, however, this rule ignores the effects of domestic monetary policy on the markup of foreign producers. The variability of this markup will be reflected in the *level* of domestic prices of Foreign goods, that are set one period in advance.

The ability to commit to a rule allows policy makers to incorporate these effects in policy design. Under commitment, in fact, the Home policy makers respond to both Home and Foreign shocks depending on the degree of pass-through *at Home*, η, that is relevant for Home consumers. In other words, policy makers do not attempt to manipulate terms of trade systematically (as under discretion). On the contrary, they contain exchange rate and terms of trade movements as to reduce their effects on the income of Foreign producers, giving up to some extent the stabilization of Home producers' income. This result of over-stabilization under discretion complements and extends a similar result discussed in the context of closed-economy models

inflation in a large cross-section of countries (Romer [1993], Campillo and Miron [1997], Lane [1997], and Cavallari [2001]). In light of this contribution their empirical findings can be interpreted as evidence that the incentive to expand monetary policy is lower in economies that are more open, and therefore more likely to suffer from the price effects of a depreciating currency. We would expect the degree of pass-through to be a relevant variable in these empirical models.

[25]See the discussion in Benigno and Benigno [2000] for the PCP case.

with staggered price adjustments by firms and inflation dynamics.[26] Our analysis shows the applicability of the same principle to an open economy, even in the absence of persistent inflation dynamics.

5 Implications for international cooperation and exchange rate volatility

5.1 The case for cooperation

Are there welfare gains from cooperating in the design and implementation of optimal monetary rules? How are these gains influenced by the degree of pass-through? To address this issue, consider a binding cooperative agreement between the two countries such that the policy makers jointly maximize the following objective function:

$$\max_{\left\{\mu_{t+\tau},\mu^*_{t+\tau}\right\}_{\tau=0}^{\infty}} E_{t-1}\left[\xi \mathcal{W}_t + (1-\xi)\,\mathcal{W}^*_t\right] \quad 0 \leq \xi \leq 1 \tag{57}$$

where ξ is a constant parameter indexing the bargaining power of the Home country. A country's bargaining power is not necessarily related to its economic size, so that ξ need not be equal to γ.

The optimal cooperative policy is the solution to the system below, expressed in terms of profit margins for notational simplicity:

$$\begin{aligned} \xi\left[1-\eta(1-\gamma)\right] + (1-\xi)\,\eta^* &= \frac{\xi\gamma\theta}{\theta-1}\frac{1}{\Theta_{\mathrm{H},t}} + \\ &+ \frac{\xi\,(1-\gamma)\,(1-\eta)\,\theta^*}{\theta^*-1}\frac{1}{\Theta_{\mathrm{F},t}} + \frac{(1-\xi)\,\gamma\eta^*\theta}{\theta-1}\frac{1}{\Theta^*_{\mathrm{H},t}} \end{aligned} \tag{58}$$

$$\begin{aligned} (1-\xi)\,(1-\eta^*\gamma) + \xi\eta\,(1-\gamma) &= \frac{(1-\xi)\,(1-\gamma)\,\theta^*}{\theta^*-1}\frac{1}{\Theta^*_{\mathrm{F},t}} + \\ &+ \frac{(1-\xi)\,\gamma\,(1-\eta^*)\,\theta}{\theta-1}\frac{1}{\Theta^*_{\mathrm{H},t}} + \frac{\xi\,(1-\gamma)\,\eta\theta^*}{\theta^*-1}\frac{1}{\Theta_{\mathrm{F},t}} \end{aligned} \tag{59}$$

[26] See for instance Gali [2000].

To verify whether there are gains from cooperation, we can compare the above system (58)-(59) with the Nash equilibrium (49)-(50) in Table 2. It is straightforward to verify that the two systems are identical — and therefore there are no welfare gains from cooperation — in three special cases. The first — and well known — case is when all shocks are global, i.e. $\alpha_t = \alpha_t^*$ for any t. The optimal Nash policies can be written as in (51) with $\alpha_t = \alpha_t^*$, implying a constant exchange rate. Consumption and employment coincide with the flex-price allocation.

The other two cases are PCP — as discussed in Obstfeld and Rogoff [2000b] — and (perhaps surprisingly) LCP. Under the extreme assumptions of either complete or zero pass-through, the allocation is the same as whether or not monetary authorities cooperate, regardless of the value of ξ. We have established above that these are precisely the case in which monetary policies are strategically independent of each other. This new result establishes that, in either case, there are no policy spillovers in equilibrium.

In the case of PCP, we have shown that the Nash equilibrium coincides with the flex-price allocation. This implies that the distortions induced by nominal rigidities and the monopoly power of a country on its terms of trade have no welfare consequences: the utility level is the same in each country, both ex-ante and ex-post.

The absence of gains from cooperation in the case of LCP is less straightforward. While there is no strategic interaction among policy makers, one may expect potentially large spillovers from monetary policy: recall that an expansion at Home increases Foreign employment without affecting Foreign demand, thus depreciating the Foreign terms of trade between labor and consumption. Key to our result is that, whether or not monetary authorities cooperate, the optimal monetary policy in each country cannot be inward-looking, and must instead respond *symmetrically* to shocks anywhere in the world economy. The optimal monetary policy can in fact be written as:

$$\mu_t = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t)} + (1-\gamma)\frac{\alpha_t^*}{E_{t-1}(\alpha_t^* \mu_t)}\right]^{-1} \tag{60}$$

$$\mu_t^* = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t^*)} + (1-\gamma)\frac{\alpha_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)}\right]^{-1}, \tag{61}$$

expressions that are satisfied by setting $\mu = \mu^*$. For any given shock, consumption increases by the same percentage everywhere in the world economy.

Even if ex-post labor moves asymmetrically (so that ex-post welfare is not identical in the two countries, as is the case under PCP), ex ante the expected disutility from labor is constant. Thus, expected utility at Home is identical to expected utility in the Foreign country up to a constant that does not depend on monetary policy: the Nash and the cooperative allocation coincide.

Besides such extreme and unrealistic cases, a country can in general do better than simply 'keeping its own house in order' by engaging in binding international agreements. Whether these gains are sizable for different degrees of pass-through, however, remains an open issue left to further research.

5.2 Volatility and exchange rate regimes

In our model the exchange rate remains constant during the period t as long as $\mu_t = \mu_t^*$. It follows that optimal rules can be consistent with a fixed exchange rate target provided that the implied monetary stances are perfectly correlated across countries. Under what conditions will this be the case?

The Nash equilibrium conditions (49) and (50) are solved by $\mu_t = \mu_t^*$ only in two cases: either when $\alpha_t = \alpha_t^*$, or when $\eta = \eta^* = 0$. In other words, fixed exchange rates can be supported by optimal monetary policies only when productivity shocks are global, or when there is no pass-through worldwide. Note that in these two cases a Nash equilibrium will coincide with a cooperative equilibrium.[27] In general, however, the implementation of optimal policy is not consistent with a fixed exchange rate regime.

To what extent will the exchange rate fluctuate as domestic policy makers follow optimal rules? Our analysis suggests that exchange rate volatility will be higher in a world economy close to purchasing power parity, and lower in a world economy where deviations from the law of one price are large. In fact, if the exposure of firms' revenue to exchange rate fluctuations is limited, inward-looking policy makers assign high priority to stabilizing domestic output and prices, with 'benign neglect' of exchange rate movements. Otherwise, policy makers 'think global', taking into account the repercussions of exchange rate volatility on import prices; hence, the monetary stances in the

[27]Benigno, Benigno and Ghironi [2000] consider the feedback rules leading to exchange rate determinacy in a fixed exchange rate regime.

world economy come to mimic each other, reducing currency volatility.

Note that in our analysis we have ignored money velocity shocks χ_t and χ_t^*. In fact, when utility from real holdings is additively separable, money velocity shocks do not play any role in the determination of consumption and employment. As (30) and (31) show, for given interest rate policies the amount of money held by domestic residents adjusts one-to-one to velocity shocks. In other words, money balances are determined *residually* in the model as functions of exogenous shocks and policy variables. An obvious corollary of this analysis is that, when velocity shocks are the *only* source of asymmetric disturbance hitting the two economies, optimal monetary policies imply that interest rates and exchange rate all remain constant over time.

6 Conclusion

The key message of our contribution is that strict adherence to inward-looking goals in policy making — such as the stabilization of domestic output — cannot be optimal in interdependent economies in which firms' revenues are exposed to currency fluctuations. Intuitively, producers set higher prices in response to higher profits volatility. Unless markups are insulated from exchange rate movements, inward-looking policies make the profits of foreign exporters suboptimally volatile, thus inducing higher average import prices. At an optimum, the welfare costs from higher consumer prices must equate the benefits from bringing domestic output towards its potential, flex-price level.

In our study, the degree of pass-through and exchange rate exposure in domestic and foreign markets emerges as a key element determining the properties of optimal monetary rules, as well as the welfare features of equilibrium allocations. Ceteris paribus, economies with higher degrees of pass-through should be characterized by a higher level of exchange rate volatility and by monetary stances more closely focused on internal conditions.

A corollary of this principle is that gains from cooperation materialize only for intermediate levels of pass-through. Interdependent economies gain little from cooperating in the design of policy rules either when the pass-through is very high (so that inward-looking policies are fully optimal) or when it is very low (so that, even without international agreement, domestic policies respond symmetrically to worldwide cyclical developments).

References

[1] Bacchetta, Philippe, and Eric Van Wincoop [2000]. "Does Exchange-Rate Stability Increase Trade and Welfare?" *American Economic Review* 90 (5), December, 1093-1109.

[2] Ball, Lawrence [1999]. "Policy Rules for Open Economies," in John B. Taylor (ed.), *Monetary Policy Rules*, Chicago, IL: University of Chicago Press.

[3] Benigno, Gianluca, and Pierpaolo Benigno [2000]. "Price Stability as a Nash Equilibrium in Monetary Open-Economy Models," mimeo, New York University.

[4] Benigno, Gianluca, Pierpaolo Benigno and Fabio Ghironi [2000]. "Interest Rates Rules for Fixed Exchange Rate Regimes," mimeo, New York University.

[5] Betts, Caroline, and Michael B. Devereux [2000a]. "Exchange Rate Dynamics in a Model of Pricing-to-Market." *Journal of International Economics* 50 (1), February, 215-244.

[6] Betts, Caroline, and Michael B. Devereux [2000b]. "International Monetary Policy Coordination and Competitive Depreciation: A Reevaluation." *Journal of Money, Credit and Banking* 32 (4), November.

[7] Campa, Jose, and Linda Goldberg [2000]. "Exchange Rate Pass-Through into Import Prices: A Macro or Micro Phenomenon?," mimeo, December.

[8] Campillo, Marta, and Jeffrey Miron [1997]. "Why Does Inflation Differ Across Countries?," in Christina Romer and David Romer (eds.), *Reducing Inflation: Motivation and Strategy,* Chicago, IL: University of Chicago Press.

[9] Carlstrom, Charles, and Timothy S. Fuerst [1999]. "Optimal monetary policy in a small, open economy: a general-equilibrium analysis." Federal Reserve Bank of Cleveland Working Paper No. 9911.

[10] Cavallari, Lilia [2001]. "Inflation and Openness with Non-Atomistic Wage-Setters." *Scottish Journal of Political Economy*, forthcoming.

[11] Chari, V.V., Patrick Kehoe, and Ellen McGrattan [2000]. "Can Sticky Prices Generate Volatile and Persistent Real Exchange Rates?" Federal Reserve Bank of Minneapolis Staff Report No.277.

[12] Clarida, Richard, Jordi Gali and Mark Gertler [2001]. "Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach," mimeo, New York University.

[13] Corsetti, Giancarlo, and Paolo Pesenti [2001]. "Welfare and Macroeconomic Interdependence." *Quarterly Journal of Economics*, forthcoming.

[14] Corsetti, Giancarlo, Paolo Pesenti, Nouriel Roubini and Cédric Tille [200]. "Competitive Devaluations: Toward a Welfare-Based Approach." *Journal of International Economics* 51 (1), June, 217-241.

[15] Devereux, Michael B., and Charles Engel [2000]. "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility." National Bureau of Economic Research Working Paper No. 7665.

[16] Devereux, Michael B., Charles Engel and Cédric Tille [1999]. "Exchange Rate Pass-Through and the Welfare Effects of the Euro." National Bureau of Economic Research Working Paper No. 7382.

[17] Duarte, Margarida, and Alan C. Stockman [2001]. "Rational Speculation and Exchange Rates," mimeo, March.

[18] Friedman, Milton [1953]. "The Case for Flexible Exchange Rates," in *Essays in Positive Economics*, Chicago, IL: University of Chicago Press, 157-203.

[19] Gali, Jordi [1999]. "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review* 89, 249-271.

[20] Gali, Jordi [2000]. "New Perspectives on Monetary Policy, Inflation and the Business Cycle," mimeo, December.

[21] Gali, Jordi, and Tommaso Monacelli [1999]. "Optimal Monetary Policy and Exchange Rate Variability in a Small Open Economy," mimeo, Boston College.

[22] Ghironi, Fabio and Alessandro Rebucci [2000]. "Monetary Rules for Emerging Market Economies," mimeo.

[23] Goldberg, Pinelopi, and Michael M. Knetter [1997]. "Goods Prices and Exchange Rates: What Have We Learned?" *Journal of Economic Literature* 35, 1243-1272.

[24] Goodfriend, Marvin, and Robert G. King [2000]. "The Case for Price Stability," mimeo, October.

[25] Lane, Philip [1997] "Inflation in Open Economies." *Journal of International Economics* 42, 327-347.

[26] Leitmo, Kai and Ulf Söderström [2000] "Simple Monetary Policy Rules and Exchange Rate Uncertainty," mimeo, Norges Bank and Sveriges Riksbank.

[27] McCallum, Bennett and Edward Nelson [1999]. "Nominal Income Targeting in an Open Economy Optimizing Model." *Journal of Monetary Economics* 43 (3), 553-578.

[28] Monacelli, Tommaso [1999]. "Open Economy Policy Rules under Imperfect Pass-Through." Boston College working Paper.

[29] Obstfeld, Maurice [2000]. "International Macroeconomics: Beyond the Mundell-Fleming Model," mimeo, December.

[30] Obstfeld, Maurice, and Kenneth Rogoff [1995]. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103, 624-660.

[31] Obstfeld, Maurice, and Kenneth Rogoff [1996]. *Foundations of International Macroeconomics.* Cambridge, MA: MIT Press.

[32] Obstfeld, Maurice, and Kenneth Rogoff [1998]. "Risk and Exchange Rates." National Bureau of Economic Research Working Paper No. 6694.

[33] Obstfeld, Maurice, and Kenneth Rogoff [2000a]. "New Directions for Stochastic Open Economy Models." *Journal of International Economics* 50 (1), February, pp.117-153.

[34] Obstfeld, Maurice, and Kenneth Rogoff [2000b]. "Do we Really Need a New International Monetary Compact?" National Bureau of Economic Research Working Paper No. 7864, August.

[35] Obstfeld, Maurice, and Kenneth Rogoff [2001]. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" in Ben Bernanke and Kenneth Rogoff (eds.), *NBER Macroeconomics Annual 2000*, Cambridge, MA: MIT Press, forthcoming.

[36] Parrado, Eric, and Andrés Velasco [2000]. "Optimal Monetary Policy in a Small Open Economy," mimeo, New York University.

[37] Romer, David [1993]. "Openness and Inflation: Theory and Evidence." *Quarterly Journal of Economics* 108, pp.870-903.

[38] Sutherland, Alan [2000]. "Inflation Targeting in a Small Open Economy," mimeo, University of St. Andrews.

[39] Svensson, Lars E.O. [2000]. "Open-Economy Inflation Targeting." *Journal of International Economics* 50 (1), 155-183.

[40] Taylor, John B. [2000]. "Low Inflation, Pass-Through, and the Pricing Power of Firms." *European Economic Review* 44 (7), June, 1389-1408.

[41] Tille, Cédric [2001]. "The Role of Consumption Substitutability in the International Transmission of Shocks." *Journal of International Economics*, forthcoming.

[42] Walsh, Carl E. [1999]. "Monetary Policy Trade-Offs in the Open Economy," mimeo, UC Santa Cruz.

[43] Woodford, Michael [2000]. *Interest and Prices*, mimeo, Princeton University.

Appendix

Intratemporal allocation. For given consumption indexes, utility-based price indexes are derived as follows. P_{H} is the price of a consumption bundle of domestically produced goods that solves:

$$\min_{C(h,j)} \int_0^1 p(h)C(h,j)dh \tag{A.1}$$

subject to $C_{\mathrm{H}}(j) = 1$. This constraint can be rewritten as:

$$[\theta/(\theta-1)] \ln \left[\int_0^1 C(h,j)^{\frac{\theta-1}{\theta}} dh \right] = 0. \tag{A.2}$$

The solution is equation (3) in the main text.

Consider now the problem of allocating a given level of nominal expenditure $\bar{Z}$ among domestically produced goods:

$$\max_{C(h,j)} C_{\mathrm{H}}(j) \;\; s.t. \int_0^1 p(h)C(h,j)dh = \bar{Z} \tag{A.3}$$

Clearly, across any pair of goods h and h', it must be true that:

$$\frac{C(h',j)}{C(h,j)} = \left(\frac{p(h')}{p(h)} \right)^{-\theta} \tag{A.4}$$

or, rearranging:

$$C(h,j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} = C(h',j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta} \tag{A.5}$$

Integrating both sides of the previous expression we obtain:

$$\left(\int_0^1 C(h,j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = \left(\int_0^1 C(h',j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} \tag{A.6}$$

so that:

$$C(h,j) \left(\int_0^1 p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = p(h)^{-\theta} \left(\int_0^1 C(h',j)^{\frac{\theta-1}{\theta}} dh' \right)^{\frac{\theta}{\theta-1}} \tag{A.7}$$

which can be rewritten as expression (10) in the main text. Note that:

$$\int_0^1 p(h)C(h,j)dz = P_{\mathrm{H}} C_{\mathrm{H}}(j). \tag{A.8}$$

Intertemporal allocation. Consider the optimal allocation by Home agent j. The maximization problem can be written in terms of the following Lagrangian:

$$\mathcal{L}_t(j=h) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ \ln C_\tau(j) + \chi_\tau \ln M_\tau(j)/P_\tau - \kappa \ell_\tau(j)$$
$$+\lambda_\tau(j) [-B_{\mathrm{H},\tau+1}(j) + (1+i_\tau) B_{\mathrm{H},\tau}(j) - \mathcal{E}_\tau B_{\mathrm{F},\tau+1}(j) + (1+i^*_\tau)\mathcal{E}_\tau B_{\mathrm{F},\tau}(j)$$
$$-M_\tau(j) + M_{\tau-1}(j) + R_\tau(j) - T_\tau(j) - P_{\mathrm{H},\tau} C_{\mathrm{H},\tau}(j) - P_{\mathrm{F},\tau} C_{\mathrm{F},\tau}(j)]\} \quad \text{(A.9)}$$

where $R_\tau(j)$ and $\ell_\tau(j)$ are defined in (13) and (11). The first order conditions with respect to $C_{\mathrm{H},t}(j)$, $C_{\mathrm{F},t}(j)$, $B_{\mathrm{H},t+1}(j)$, $B_{\mathrm{F},t+1}$ and $M_t(j)$, are, respectively:

$$\frac{\gamma}{C_{\mathrm{H},t}(j)} = \lambda_t(j) P_{\mathrm{H},t} \quad \text{(A.10)}$$

$$\frac{1-\gamma}{C_{\mathrm{F},t}(j)} = \lambda_t(j) P_{\mathrm{F},t} \quad \text{(A.11)}$$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j)(1+i_{t+1}) \quad \text{(A.12)}$$

$$\mathcal{E}_t \lambda_t(j) = \beta E_t \mathcal{E}_{t+1} \lambda_{t+1}(j)\left(1+i^*_{t+1}\right) \quad \text{(A.13)}$$

$$\frac{\chi}{M_t(j)} = E_t[\lambda_t(j) - \beta \lambda_{t+1}(j)] \quad \text{(A.14)}$$

First, we solve for the multiplier $\lambda_t(j)$. Take a geometric average of (A.10) and (A.11) with weights γ and $1-\gamma$, respectively:

$$\gamma^\gamma (1-\gamma)^{1-\gamma} = \lambda_t(j) P^\gamma_{\mathrm{H},t} P^{1-\gamma}_{\mathrm{F},t} C^\gamma_{\mathrm{H},t}(j) C^{1-\gamma}_{\mathrm{F},t}(j) \quad \text{(A.15)}$$

This yields:

$$\lambda_t(j) = \frac{1}{P_t C_t(j)} \quad \text{(A.16)}$$

so that, at the optimum, the individual demand for Home and Foreign consumption goods are a share of total consumption expenditure:

$$P_t C_t(j) = \frac{1}{\gamma} P_{\mathrm{H},t} C_{\mathrm{H},t}(j) = \frac{1}{1-\gamma} P_{\mathrm{F},t} C_{\mathrm{F},t}(j). \tag{A.17}$$

Using these expressions, it is easy to verify that

$$P_t C_t(j) = P_{\mathrm{H},t} C_{\mathrm{H},t}(j) + P_{\mathrm{F},t} C_{\mathrm{F},t}(j). \tag{A.18}$$

Combining (A.10), (A.11) and (A.12) the intertemporal allocation of consumption is determined according to the Euler equation:

$$\frac{1}{C_t(j)} = \beta \left(1 + i_{t+1}\right) E_t \left(\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}(j)} \right) \tag{A.19}$$

Finally, condition (A.14) can be written as the money demand function:

$$\frac{M_t(j)}{P_t} = \chi_t \frac{1 + i_{t+1}}{i_{t+1}} C_t(j) \tag{A.20}$$

Define now the variable $Q_{t,t+1}(j)$ as:

$$Q_{t,t+1}(j) \equiv \beta \frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)} \tag{A.21}$$

The previous expression can be interpreted as agent j's stochastic discount rate. Comparing (A.21) with (A.12) and (A.13) we obtain:

$$E_t Q_{t,t+1}(j) = \frac{1}{1 + i_{t+1}}, \qquad E_t \left[Q_{t,t+1}(j) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] = \frac{1}{1 + i^*_{t+1}} \tag{A.22}$$

from which we obtain the risk-adjusted uncovered interest parity linking domestic and foreign nominal interest rates:

$$\frac{1 + i_{t+1}}{1 + i^*_{t+1}} = E_t \left(\frac{\mathcal{E}_{t+1}}{P_{t+1} C_{t+1}(j)} \right) \left[E_t \left(\frac{\mathcal{E}_t}{P_{t+1} C_{t+1}(j)} \right) \right]^{-1} \tag{A.23}$$

Note that in the absence of uncertainty the previous condition collapses to the familiar expression $1 + i_{t+1} = \left(1 + i^*_{t+1}\right) \mathcal{E}_{t+1} / \mathcal{E}_t$.

Using (A.22) we can write:

$$M_t(j) + B_{\mathrm{H},t+1}(j) = \frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left\{ Q_{t,t+1}(j) \left[M_t(j) + (1 + i_{t+1}) \, B_{\mathrm{H},t+1}(j) \right] \right\} \tag{A.24}$$

and:

$$\mathcal{E}_t B_{\mathrm{F},t+1}(j) = E_t \left\{ Q_{t,t+1}(j)(1 + i^*_{t+1}) \mathcal{E}_{t+1} B_{\mathrm{F},t+1}(j) \right\} \tag{A.25}$$

It follows that the flow budget constraint (12) can also be written as:

$$\frac{i_{t+1} M_t(j)}{1 + i_{t+1}} + E_t \left\{ Q_{t,t+1}(j) W_{t+1}(j) \right\} \leq W_t(j) + R_t(j) - T_t(j) - P_t C_t(j) \tag{A.26}$$

where W_{t+1} is wealth at the beginning of period $t+1$, defined as:

$$W_{t+1}(j) \equiv M_t(j) + (1 + i_{t+1}) \, B_{\mathrm{H},t+1}(j) + (1 + i^*_{t+1}) \mathcal{E}_{t+1} B_{\mathrm{F},t+1}(j). \tag{A.27}$$

Optimization implies that households exhaust their intertemporal budget constraint: the flow budget constraint hold as equality and the transversality condition:

$$\lim_{N \to \infty} E_t \left[Q_{t,N}(j) W_N(j) \right] = 0 \tag{A.28}$$

is satisfied, where $Q_{t,N} \equiv \prod_{s=t+1}^{N} Q_{s-1,s}$. If an interior solution exists (as is the case given our parameterization), the resource constraint (11) holds as equality as well.

Foreign constraints and optimization conditions. Similar results characterize the optimization problem of Foreign agent j^*. For convenience, we report here the key equations. The resource constraint for agent f is:

$$\frac{\ell^*(f)}{\alpha^*_t} \geq p_t(f)^{-\theta^*} P^{\theta^*}_{\mathrm{F},t} C_{\mathrm{F},t} + p^*_t(f)^{-\theta^*} \left(P^*_{\mathrm{F},t} \right)^{\theta^*} C^*_{\mathrm{F},t} \tag{A.29}$$

The flow budget constraint is:

$$\frac{B^*_{\mathrm{H},t+1}(j^*)}{\mathcal{E}_t} + B^*_{\mathrm{F},t+1}(j^*) + M^*_t(j^*) \leq (1+i_t)\frac{B^*_{\mathrm{H},t}(j^*)}{\mathcal{E}_t} + (1+i^*_t)B^*_{\mathrm{F},t}(j^*) + \\ + M^*_{t-1}(j^*) + R^*_t(j^*) - T^*_t(j^*) - P^*_{\mathrm{H},t}C^*_{\mathrm{H},t}(j^*) - P^*_{\mathrm{F},t}C^*_{\mathrm{F},t}(j^*) \tag{A.30}$$

and, associating individual j^* with brand f, sales revenue is:

$$R^*(j^*) = \frac{1}{\mathcal{E}_t}\left(p_t(f)\right)^{1-\theta^*} P^{\theta^*}_{\mathrm{F},t}C_{\mathrm{F},t} + p^*_t(f)^{1-\theta^*}\left(P^*_{\mathrm{F},t}\right)^{\theta^*} C^*_{\mathrm{F},t}. \tag{A.31}$$

First order conditions yield:

$$P^*_t C^*_t(j^*) = \frac{1}{\gamma}P^*_{\mathrm{H},t}C^*_{\mathrm{H},t}(j^*) = \frac{1}{1-\gamma}P^*_{\mathrm{F},t}C^*_{\mathrm{F},t}(j^*) \tag{A.32}$$

$$\frac{1}{C^*_t(j^*)} = \beta\left(1+i^*_{t+1}\right)E_t\left(\frac{P^*_t}{P^*_{t+1}}\frac{1}{C^*_{t+1}(j^*)}\right), \tag{A.33}$$

$$\frac{M^*_t(j^*)}{P^*_t} = \chi^*_t\frac{1+i^*_{t+1}}{i^*_{t+1}}C^*_t(j^*), \tag{A.34}$$

$$Q^*_{t,t+1}(j^*) = \beta\frac{P^*_t C^*_t(j^*)}{P^*_{t+1}C_{t+1}(j^*)} = \beta\frac{\lambda^*_{t+1}(j^*)}{\lambda^*_t(j^*)}, \tag{A.35}$$

and:

$$\frac{1+i_{t+1}}{1+i^*_{t+1}} = E_t\left(\frac{1}{P^*_{t+1}C^*_{t+1}(j^*)}\right)\frac{1}{\mathcal{E}_t E_t\left(\frac{1}{\mathcal{E}_{t+1}P^*_{t+1}C^*_{t+1}(j^*)}\right)}. \tag{A.36}$$

Optimal price setting. Optimal pricing in the Home country can be derived as follows. Maximizing the Lagrangian with respect to $p_t(h)$ yields:

$$E_{t-1}\left[-\kappa\theta p_t(h)^{-\theta-1}P^{\theta}_{\mathrm{H},t}\alpha_t C_{\mathrm{H},t} + (\theta-1)\,p_t(h)^{-\theta}P^{\theta}_{\mathrm{H},t}\lambda_t(h)C_{\mathrm{H},t}\right] = 0 \tag{A.37}$$

or, in a symmetric environment:

$$P_{\mathrm{H},t} = \frac{\theta\kappa}{\theta-1}\frac{E_{t-1}\left[\alpha_t C_{\mathrm{H},t}\right]}{E_{t-1}\left[\dfrac{C_{\mathrm{H},t}}{P_t C_t}\right]} \tag{A.38}$$

and, using (A.17), we can rewrite the previous expression as (14) in the main text.

Maximizing the Lagrangian with respect to $\tilde{p}_t(h)$ yields:

$$\begin{aligned} 0 = E_{t-1} \Big[& -\kappa\theta p_t^*(h)^{-\theta}\tilde{p}_t(h)^{-1}\left(P_{\mathrm{H},t}^*\right)^{\theta}\alpha_t C_{\mathrm{H},t}^* \\ & + (\theta-1)\, p_t^*(h)^{1-\theta}\mathcal{E}_t\tilde{p}_t(h)^{-1}\left(P_{\mathrm{H},t}^*\right)^{\theta}\lambda_t(h) C_{\mathrm{H},t}^*\Big] \end{aligned} \tag{A.39}$$

that is:

$$1 = \frac{\theta\kappa}{\theta-1}\frac{E_{t-1}\left[p_t^*(h)^{-\theta}\left(P_{\mathrm{H},t}^*\right)^{\theta}\alpha_t C_{\mathrm{H},t}^*\right]}{E_{t-1}\left[p_t^*(h)^{1-\theta}\left(P_{\mathrm{H},t}^*\right)^{\theta}\dfrac{\mathcal{E}_t C_{\mathrm{H},t}^*}{P_t C_t(h)}\right]} \tag{A.40}$$

In a symmetric environment with $p_t^*(h) = P_{\mathrm{H},t}^*$, we can rewrite the above as:

$$1 = \frac{\theta\kappa}{\theta-1}\frac{E_{t-1}\left[\alpha_t C_{\mathrm{H},t}^*\right]}{E_{t-1}\left[P_{\mathrm{H},t}^*\dfrac{\mathcal{E}_t C_{\mathrm{H},t}^*}{P_t C_t}\right]} \tag{A.41}$$

and, using (A.32), we obtain:

$$1 = \frac{\theta\kappa}{\theta-1}\frac{E_{t-1}\left[\alpha_t\dfrac{P_t^* C_t^*}{P_{\mathrm{H},t}^*}\right]}{E_{t-1}\left[\dfrac{\mathcal{E}_t P_t^* C_t^*}{P_t C_t}\right]} \tag{A.42}$$

or:

$$\tilde{p}_t\left(h\right) = \frac{\theta\kappa}{\theta-1}\frac{E_{t-1}\left[\alpha_t P_t^* C_t^* \mathcal{E}_t^{\eta^*}\right]}{E_{t-1}\left[\dfrac{\mathcal{E}_t P_t^* C_t^*}{P_t C_t}\right]} \tag{A.43}$$

from which we obtain (16) in the main text.

Equilibrium. Given initial wealth levels W_{t_0} and $W^*_{t_0}$ (symmetric among country residents), and the processes for α_t, α^*_t, χ_t, χ^*_t, T_t, T^*_t, i_{t+1} and i^*_{t+1} for all $t \geq t_0$, the symmetric fix-price equilibrium is a set of processes for $C_{\mathrm{H},t}$, $C_{\mathrm{F},t}$, C_t, $Q_{t,t+\tau}$, $B_{\mathrm{H},t+1}$, $B_{\mathrm{F},t+1}$, M_t, ℓ_t, R_t, W_{t+1}, $P_{\mathrm{H},t}$, $P_{\mathrm{F},t}$, P_t, $C^*_{\mathrm{H},t}$, $C^*_{\mathrm{F},t}$, $Q^*_{t,t+\tau}$, C^*_t, $B^*_{\mathrm{H},t+1}$, $B^*_{\mathrm{F},t+1}$, M^*_t, ℓ^*_t, R^*_t, W^*_{t+1}, $P^*_{\mathrm{H},t}$, $P^*_{\mathrm{F},t}$, P^*_t, and $\mathcal{E}_t$ such that, for all $t \geq t_0$ and $\tau > t_0$, (i) the Home government budget constraint (21) and its Foreign analog are satisfied; (ii) Home consumer optimization conditions (A.17), (2), (A.21), (A.19), (A.23), (A.20), (11), (13), (A.27), and their Foreign analogs hold as equalities; (iii) Home firm optimization conditions (14), (16), (4), and their Foreign analogs hold; (iv) the Home transversality condition (A.28) and its Foreign analog hold; (v) the markets for the international bonds clear, that is conditions (22) hold.

A flex-price equilibrium is similarly defined, after imposing that for all $t \geq t_0$ Home conditions (14), (16), and their Foreign analogs hold in any state of nature at time t rather than in expectation at time $t-1$.

Equilibrium balanced current account. Aggregating the individual budget constraints and using the government budget constraint we obtain an expression for the Home current account:

$$E_t \{Q_{t,t+1} A_{t+1}\} = A_t + R_t - P_t C_t \tag{A.44}$$

where A is defined as wealth net of money balances, or

$$A_{t+1} \equiv W_{t+1} - M_t, \tag{A.45}$$

and, by the definition of sales revenue:

$$R_t = P_{\mathrm{H},t} C_{\mathrm{H},t} + \mathcal{E}_t P^*_{\mathrm{H},t} C^*_{\mathrm{H},t} = \gamma \left(P_t C_t + \mathcal{E}_t P^*_t C^*_t\right). \tag{A.46}$$

Assume now that, at time t_0, $B_{\mathrm{H},t_0} = B_{\mathrm{F},t_0} = 0$ so that initial non-monetary wealth $A_{t_0} = 0$. It can then be easily showed that, for all $t \geq t_0$, the equilibrium conditions above are solved by the allocation:

$$R_t - P_t C_t = -\mathcal{E}_t \left(R^*_t - P^*_t C^*_t\right) = 0, \quad A_t = -\mathcal{E}_t A^*_t = 0 \quad t \geq t_0. \tag{A.47}$$

The utility gap. To derive (40) and to show its sign consider the following steps:

$$
\begin{aligned}
& E_{t-1}U_t^{flex} - E_{t-1}U_t = E_{t-1}\left(\ln C_t^{flex} - \ln C_t\right) + \kappa\Phi - \kappa\Phi \\
= \; & E_{t-1}\ln\frac{C_t^{flex}}{C_t} = E_{t-1}\ln\frac{\left\{E_{t-1}\left[\alpha_t\mu_t\right]\right\}^{\gamma}\left\{E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right]\right\}^{1-\gamma}}{\alpha_t^{\gamma}\left(\alpha_t^*\right)^{1-\gamma}\mu_t^{1-\eta(1-\gamma)}\left(\mu_t^*\right)^{\eta(1-\gamma)}} \\
= \; & E_{t-1}\left[\gamma\ln\frac{E_{t-1}\left[\alpha_t\mu_t\right]}{\alpha_t\mu_t} + (1-\gamma)\ln\frac{E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right]}{\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}}\right] \\
= \; & E_{t-1}\left[\gamma\left(\ln E_{t-1}\left[\alpha_t\mu_t\right] - \ln\alpha_t\mu_t\right) + (1-\gamma)\left(\ln E_{t-1}\left[\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right] - \ln\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right)\right] \\
\geq \; & E_{t-1}\left[\gamma\left(E_{t-1}\left[\ln\alpha_t\mu_t\right] - \ln\alpha_t\mu_t\right) + (1-\gamma)\left(E_{t-1}\left[\ln\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right] - \ln\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right)\right] \\
= \; & \gamma\left(E_{t-1}\ln\alpha_t\mu_t - E_{t-1}\ln\alpha_t\mu_t\right) + (1-\gamma)\left(E_{t-1}\ln\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta} - E_{t-1}\ln\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right) \\
= \; & 0. \qquad\qquad (A.48)
\end{aligned}
$$

Nominal equilibrium. The optimal monetary rules derived in the main text do not provide a nominal anchor to pin down nominal expectations. For instance, recalling the solution for $P_{\mathrm{H},t}$, we can write the Home country reaction function (49) in order to emphasize the relation between μ_t and $P_{\mathrm{H},t}$:

$$
\alpha_t\mu_t = \left[1-\eta(1-\gamma) - \frac{(1-\gamma)(1-\eta)\,\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}}{E_{t-1}\left(\alpha_t^*\left(\mu_t^*\right)^{\eta}\mu_t^{1-\eta}\right)}\right]\frac{\Phi P_{\mathrm{H},t}}{\gamma} \qquad (A.49)
$$

Now, using (26) we obtain $\Phi P_{\mathrm{H},t} = E_{t-1}\alpha_t\mu_t$ but taking the expectation of (A.49) we obtain $E_{t-1}\alpha_t\mu_t = \Phi P_{\mathrm{H},t}$ for any arbitrary choice of $P_{\mathrm{H},t}$. Similar considerations hold for (50) and $P_{\mathrm{F},t}^*$.

To address this issue, following Woodford [2000], take μ_t and μ_t^* defined by the system (49) and (50) and define two rules $\hat{\mu}_t$ and $\hat{\mu}_t^*$ such that:

$$
\hat{\mu}_t = \mu_t\left(\frac{P_{\mathrm{H},t}}{\bar{P}_{\mathrm{H}}}\right)^{\delta} \quad \hat{\mu}_t^* = \mu_t^*\left(\frac{P_{\mathrm{F},t}^*}{\bar{P}_{\mathrm{F}}^*}\right)^{\delta^*} \quad \delta,\delta^* < 0 \qquad (A.50)
$$

where δ and $\delta^*$are two negative constants, arbitrarily small, and $\bar{P}_{\mathrm{H}}$ and $\bar{P}_{\mathrm{F}}^*$ are the two nominal targets for the Home and Foreign monetary authorities, respectively. Under the rules above, we obtain:

$$\Phi P_{\mathrm{H},t} = E_{t-1}\alpha_t\hat{\mu}_t = E_{t-1}\alpha_t\mu_t\left(\frac{P_{\mathrm{H},t}}{\bar{P}_{\mathrm{H}}}\right)^{\delta} = \gamma\frac{\Phi\left(P_{\mathrm{H},t}\right)^{\delta+1}}{\gamma\left(\bar{P}_{\mathrm{H}}\right)^{\delta}} \tag{A.51}$$

solved by $P_{\mathrm{H},t} = \bar{P}_{\mathrm{H}}$. This implies that, in equilibrium, $\hat{\mu}_t$ is identical to μ_t once we replace $P_{\mathrm{H},t}$ with $\bar{P}_{\mathrm{H}}$. Similar considerations allow to pin down $P^*_{\mathrm{F},t} = \bar{P}^*_{\mathrm{F}}$. The government sets a nominal anchor and credibly threatens to tighten monetary policy if the price of domestic goods deviates from a given target. Such threat, however, is never implemented in equilibrium.

Equilibrium import prices: some special cases. Given the targets $\bar{P}_{\mathrm{H}}$ and $\bar{P}^*_{\mathrm{F}}$, we can determine the import prices in both countries according to the rules (49) and (50). For instance, under PCP the rules (51) imply:

$$P_{\mathrm{F},t} = \bar{P}_{\mathrm{H}}\frac{\Phi}{\Phi^*}\frac{1-\gamma}{\gamma}\frac{\alpha^*_t}{\alpha_t}. \tag{A.52}$$

Given $\bar{P}_{\mathrm{H}}$, the domestic price of Foreign goods will optimally increase with a positive productivity shock in the Home country.

In the LCP case instead the rule (60) holds, and

$$P_{\mathrm{F},t} = \frac{1}{\Phi^*}\frac{1-\gamma}{\gamma}E_{t-1}\left(\alpha^*_t\mu_t\right) \tag{A.53}$$

where $E_{t-1}\left(\alpha^*_t\mu_t\right)$ is the solution to:

$$\frac{1}{\bar{P}_{\mathrm{H}}\Phi} = E_{t-1}\left[\frac{\alpha^*_t}{\gamma\alpha_t E_{t-1}\left(\alpha^*_t\mu_t\right) + (1-\gamma)\,\alpha^*_t\bar{P}_{\mathrm{H}}\Phi}\right]. \tag{A.54}$$

If $\eta = 1$ and $\eta^* = 0$, we have:

$$P_{\mathrm{F},t} = \bar{P}_{\mathrm{H}}\frac{\Phi}{\Phi^*}\frac{1-\gamma}{\gamma}\left[(1-\gamma)\frac{\alpha^*_t}{\alpha_t} + \gamma\frac{\Phi^*\bar{P}^*_{\mathrm{F}}}{E_{t-1}\left(\alpha_t\mu^*_t\right)}\right] \tag{A.55}$$

where $E_{t-1}\left(\alpha_t\mu^*_t\right)$ is the solution to:

$$\frac{1}{\bar{P}^*_{\mathrm{F}}\Phi^*} = E_{t-1}\left[\frac{\alpha_t}{(1-\gamma)\,\alpha^*_t E_{t-1}\left(\alpha_t\mu^*_t\right) + \gamma\alpha_t\bar{P}^*_{\mathrm{F}}\Phi^*}\right]. \tag{A.56}$$