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THE COMOVEMENTS BETWEEN REAL ACTIVITY AND PRICES IN THE G7

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ABSTRACT

In this paper, we study the short-run and long-run comovement between prices and real activity in the G7 countries during the postwar period using VAR forecast errors and frequency domain filters. We find that there are several patterns of the correlation coefficients that are the same in all countries. In particular, the correlation at the "long-run" horizon is virtually always negative and the correlation at the "short-run" horizon is typically substantially higher. Although there is evidence of positive "short-run" correlations for some countries it is not very robust to the choice of the price and output variables. In addition, we propose a more efficient method to calculate the covariances of VAR forecast errors and—in contrast to claims made in the literature—we show that band-pass filters isolate the desired set of frequencies not only when the series are stationary but also when they are first or second-order integrated processes.

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The Comovements between Real Activity and Prices in the G7

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1. Introduction

Descriptions of the cyclical behavior of the price level play an important role in macroeconomics. Clearly, there are many ways to describe the properties of economic variables. Typically, a concise set of moments is used to estimate and/or test a model after a filter has been used to render the data stationary. The focus on only a limited set of statistics is often motivated either by the idea that the model is not intended to be an accurate description of the data in all dimensions, or by the fact that more efficient econometric techniques, like maximum likelihood, cannot be used because of technical difficulties.[1] A crucial question, therefore, is what moments to use. Compounding the difficulty in answering this question is the fact that the moments of interest (standard deviation and correlation coefficients) typically require the data to be stationary.

This paper examines the correlation between prices and output for the G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom, and the United States) during the postwar period. We use both the correlation of VAR forecast errors as proposed in Den Haan (2000) and the correlations of prices and output after a frequency domain filter has been used to isolate the frequencies of interest as proposed by Baxter and King (1994). Den Haan (2000) shows that the VAR procedure can be used for stationary as well as integrated stochastic variables. In this paper we similarly show that

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[1] Geweke (1999), for example, shows that the likelihood function of a dynamic economic model is typically zero, either because obtaining numerical solutions of the model requires discrete support of the stochastic driving process which happens in the data only with probability zero or requires the number of unobservable state variables to be small which leads to relationships between variables that are rejected with probability one in the data. The last problem can be avoided by following the standard practice in regression analysis to add a stochastic error term to each policy function, which would, of course, be ludicrous in a dynamic stochastic model.

the frequency domain filters can be used for stationary and I(1) and I(2) stochastic variables. By considering different forecast horizons and frequency domain filters that isolate different frequency bands, we capture important dynamic information about the comovement of prices and output. By considering these two alternative sets of dynamic statistics we offer a more complete description of the comovement between prices and output than other empirical studies in the literature, and are capable of drawing stronger conclusions about the kind of theoretical models that are consistent and inconsistent with the observed comovements.[2] In particular, we find that, virtually always, the "long-run" correlations between prices and output are significantly negative and that the "short-run" correlations are substantially higher. Although there is evidence of positive "short-run" correlations for some countries, it is not very robust to the choice of the price and output variables.

Backus and Kehoe (1992), Cooley and Ohanian (1991), and Fiorito and Kolintzas (1994) show that the correlation between HP-filtered prices and output is negative for several countries during the postwar period.[3] Initially these negative correlation coefficients were believed to support models in which supply shocks play a dominant role. Chadha and Prasad (1993), Judd and Trehan (1995), and Ball and Mankiw (1994), however, showed that these negative correlation coefficients do not provide much identifying information because sticky-price models with only demand shocks can easily generate a negative correlation between prices and output when the HP-filter is used to filter the data. Den Haan (2000) shows, however, that the negative correlation between VAR forecast errors that we find in this paper cannot be generated by these type of models under sensible assumptions. This paper, therefore, provide support for the claim made by Kydland and Prescott (1990) that *"any theory in which procyclical prices figure crucially in accounting for postwar business cycle fluctuations is doomed to failure"*.

In this paper, we also offer some methodological contributions. First, we propose a new procedure to implement the method proposed in Den Haan (2000). We show that

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[2] Backus and Kehoe (1992) analyze the empirical comovement between annual prices and output for ten OECD countries for both the postwar and several prewar periods, Cooley and Ohanian (1991) provides a description of the comovement of US output and prices for both the postwar and several prewar periods, and Fiorito and Kollintzas (1994) investigates the behavior of output and prices for the G7 during the postwar period.

[3] Similarly, Pakko (2000) shows using a postwar sample that the cospectrum of US GDP and its deflator is negative at those frequencies corresponding roughly to the HP filter.

imposing the restrictions implied by the estimated VAR in calculating the correlation coefficients of VAR forecast errors results in substantial efficiency gains. Second, in contrast to claims made in the literature[4], this paper shows that band-pass and high-pass frequency domain filters succeed in eliminating that part of the series associated with frequencies outside of the specified band for stationary as well as integrated processes.

The paper is organized as follows. The following section describes the methodology to calculate the correlation coefficients of the VAR forecast errors discussed in Den Haan (2000) and the new procedure used to implement the method. Section 3 discusses how frequency domain filters can be used to provide a concise set of statistics to describe the comovement of stationary as well as I(1) and I(2) variables. Section 4 discusses the empirical findings and the last section concludes.

2. Measuring correlations at different forecast horizons

In Section 2.1, we review the procedure proposed in Den Haan (2000) to measure the comovement between economic variables. In Section 2.2, we discuss the relationship between this procedure and the impulse response functions from structural VARs.

2.1 Using forecast errors to calculate correlation coefficients

Consider an N-vector of random variables, X_t. The vector X_t is allowed to contain any combination of stationary processes and processes that are integrated of arbitrary order. If one wants to describe the comovement between prices, P_t, and output, Y_t, then X_t has to include at least P_t and Y_t. Consider the following VAR:

$$X_t = a + bt + ct^2 + \sum_{l=1}^{L} A_l X_{t-l} + v_t \qquad (2.1)$$

where A_l is an $N \times N$ matrix of regression coefficients; a, b and c are N-vectors of constants; v_t is an N-vector of innovations; and the total number of lags included is equal to L. The elements of v_t are assumed to be serially uncorrelated but they can be correlated with one another. We denote the K-period ahead forecast and the K-period ahead forecast error of the variable Y_t by $\mathrm{E}_t Y_{t+K}$ and $Y^{ue}_{t+K,t}$, respectively. We do the same

[4] See Cogley and Nason (1995) and Harvey and Jaeger (1993).

for P_t. We denote the covariance between the random variables $P^{ue}_{t+K,t}$ and $Y^{ue}_{t+K,t}$ by $COV(K)$ and the correlation coefficient between these two variables by $COR(K)$.

If the series are stationary, then the correlation coefficient of the forecast errors will converge to the unconditional correlation coefficient of the two series as K goes to infinity. Den Haan (2000) shows that if some of the time series are not stationary then $COV(K)$ and $COR(K)$ can still be estimated consistently for a fixed K. It is important to note that no assumption on the order of integration of the elements of X_t has to be made. For example, it is possible that X_t contains stationary as well as integrated processes. However, an important assumption for the derivation of the consistency results is that Equation (2.1) is correctly specified. In particular, the lag order must be large enough to guarantee that ν_t is serially uncorrelated and not integrated. That is, if X_t contains I(1) stochastic processes, then the lag order has to be at least equal to 1. Likewise, if X_t contains I(2) stochastic processes, then the lag order has to be at least equal to 2. When X_t includes integrated processes one might prefer to estimate a VAR in first differences or an error-correction system. When the restrictions that lead to these systems are correct, then imposing the restrictions may lead to more efficient forecasts in a finite sample. If they are not correct then the system is misspecified and the estimator might be biased.[5]

There are two ways to construct estimates of the covariance terms. Den Haan (2000) constructs time series for the forecast errors using the difference between the realizations and their forecasts and calculates the covariance of the created time series. A disadvantage of using the actual forecast errors is that one looses several observations, which shortens the sample size especially for longer forecast horizons. The second way to construct estimates is to use the covariance that is implied by the VAR coefficients and the variance-covariance matrix of ν_t. In the appendix we show that there are substantial efficiency gains by using the second method.

The correlation coefficients are calculated as follows using the second method. The VAR given in Equation (2.1) for a sample of T observations can be written as

(2.2) $$X_T = \sum_{l=1}^{L} X_{T-l} A_l^{'} + v_T \ ,$$

[5] See Hamilton (1994, page 516) for a discussion.

where for simplicity we have set the constant and the trend terms equal to zero. This system can be written as the following first-order VAR system

(2.3) $$Z_T = Z_{T-1}F' + u_T,$$

where Z_T is a $(T \times LN)$ matrix equal to $[\, X_T \;\; X_{T\text{-}1} \cdots \;\; X_{T\text{-}L+1}]$, u_T is a $(T \times LN)$ matrix equal to $[\, \nu_T \;\; 0_{T,N} \cdots \;\; 0_{T,N}]$, and

$$F' = \begin{bmatrix} A_1' & I_N & 0_N & \cdots & 0_N \\ A_2' & 0_N & I_N & \cdots & 0_N \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_L' & 0_N & 0_N & \ldots & I_N \end{bmatrix},$$

where I_N is an $(N \times N)$ identity matrix and 0_N is an $(N \times N)$ zero matrix. Let COV(K) now denote the $(LN \times LN)$ variance-covariance matrix of the K-period ahead forecast errors. It is then relatively straightforward to show that

(2.4) $$COV(K) = E[Z_{T+K} - Z_T F'^K]'[Z_{T+K} - Z_T F'^K]/T = \sum_{j=0}^{K-1} F^j \Omega F'^j,$$

where $F^0 = I_{NL}$ and $\Omega = \mathrm{E}\,(u_T' u_T)/T$.

2.2 The relationship with impulse response functions

There is an alternative way to use the VAR to construct measures of comovements at different forecast horizons that clarifies the relationship between our procedure and impulse response functions. We can write the K-period ahead forecast error, $Y^{ue}_{t+K,t}$, as follows:

(2.5) $$Y^{ue}_{t+K,t} = \left(Y_{t+K} - \mathrm{E}_{t+K-1}Y_{t+K}\right) + \left(\mathrm{E}_{t+K-1}Y_{t+K} - \mathrm{E}_{t+K-2}Y_{t+K}\right) + \cdots + \left(\mathrm{E}_{t+1}Y_{t+K} - \mathrm{E}_t Y_{t+K}\right)$$

In this equation, the K-period ahead forecast error is written as the sum of the updates in the forecast of Y_{t+K}, starting at period t+1. The first term on the right hand side is just the one-period ahead forecast error realized at period $t+K$. The second term is the update of the two-period ahead forecast and the other terms are defined similarly. We denote the covariance between $\left(\mathrm{E}_{t+K-k+1}Y_{t+K} - \mathrm{E}_{t+K-k}Y_{t+K}\right)$ and $\left(\mathrm{E}_{t+K-k+1}P_{t+K} - \mathrm{E}_{t+K-k}P_{t+K}\right)$ by $COV^{\Delta}(k)$. Since the terms on the right hand side of Equation (2.5) are serially uncorrelated, there is a simple relationship between $COV(K)$, defined in Section 2.1, and $COV^{\Delta}(k)$. That is, (2.6) $$COV(K) = \sum_{k=1}^{K} COV^{\Delta}(k) .$$

When $K= k =1$, then the two covariances are identical. The "$COV^{\Delta}(k)$" covariances, therefore, contain the same information as the "$COV(K)$" covariances. Calculating standard errors for the $COV^{\Delta}(k)$ statistics may seem easier since the updates of the K-period ahead forecasts are serially uncorrelated and the K-period ahead forecast errors are not. However, in both cases calculating standard errors is a complicated exercise because the forecasts are obtained from an estimated VAR and the standard errors of the covariance statistics should incorporate the sampling uncertainty due to the estimation of the VAR. Den Haan (2000) shows that the sign of the $COV(K)$ terms has more identifying information than the sign of the $COV^{\Delta}(k)$ terms.[6] Therefore, we will focus on the $COV(K)$ terms not the $COV^{\Delta}(k)$ terms.

The "$COV^{\Delta}(k)$" covariances are helpful in clarifying the relationship between the proposed statistics and impulse response functions. Suppose that $v_t = B\varepsilon_t$, where B is an $N{\times}M$ matrix of coefficients and ε_t is an M-vector of (independent) fundamental shocks. Without loss of generality assume that each element of ε_t has unit variance. Let $Y_k^{imp,m}$ be the effect on output in response to a one standard deviation shock in the m^{th} element of ε_t after k periods. Thus, $Y_k^{imp,m}$ is the impulse response of Y_t after k periods. We define $P_k^{imp,m}$ in the same manner. Then, $COV^{\Delta}(k)$ is equal to the sum of the products of the k-step impulse responses across all fundamental shocks. That is,

(2.7) $$COV^{\Delta}(k) = \sum_{m=1}^{M} Y_k^{imp,m} P_k^{imp,m} .$$

When there is only one fundamental shock, i.e. $M = 1$, then $COV^{\Delta}(k)$ is equal to the product of the impulse response functions. For the special situation, where Y_k^{imp} and P_k^{imp} always have the opposite sign, the $COV^{\Delta}(k)$ will be negative for every value of k. To understand Equation (2.7) for the case when $M > 1$, note that shocks that have a

[6] In particular, $COV^{\Delta}(k)$ could be negative for some k in models with only demand shocks as long as the effect of a demand shock on output and prices has the opposite sign at some point while $COV(K)$ can only be negative when the *accumulated* effect of a demand shock on output and prices has the opposite sign.

bigger quantitative impact on output and prices obviously obtain more weight in the calculation of $COV^{\Delta}(k)$.

A set of impulse response functions provides complete information about the comovements of output and prices after any type of shock. Estimating impulse response functions, however, requires making identifying assumptions. The results often depend on the particular type of identifying assumptions, and the assumptions are often ad hoc. The advantage of the procedure proposed in this paper is that it does not require making these types of ad hoc assumptions. The disadvantage of this procedure is that it does not identify all the different impulse response functions.

3. Measuring correlations at different frequencies

This section describes how to use spectral analysis to decompose series by frequency and to measure the correlations of two series at different frequencies.[7] The literature on frequency domain analysis typically assumes that the series of interest are stationary. A short description of the relevant techniques is given in Section 3.1. In contrast to some claims made in the literature, we show in Section 3.2 that the procedures used in this paper can be easily extended to the case where the series are first or second-order integrated processes, or the case where the series contain a deterministic linear or quadratic time trend.

3.1 Frequency-domain filters for stationary processes

From the Wold-theorem, we know that any covariance stationary series has a time-domain representation.[8] Similarly, any covariance stationary series has a frequency-domain representation. Informally, this implies that the variable x_t can be represented as a weighted sum of periodic functions of the form $\cos(\omega t)$ and $\sin(\omega t)$, where ω denotes a particular frequency. The frequency domain representation is given by

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[7] Diebold, Ohanian, and Berkowitz (1998) propose to compare the cross spectrum of the data with that of a model. The procedure developed in this section is closer to the commonly used method of filtering the data with the Hodrick-Prescott filter described in Hodrick and Prescott (1997) but like Diebold, Ohanian, and Berkowitz (1998) we consider a more complete description of the data. Recall that the commonly used version of the Hodrick-Prescott filter is an approximate high-pass filter that eliminates cycles with a periodicity of more than 32 quarters.

[8] See Hamilton (1994) for regularity conditions.

(3.1) $$x_t = \mu + \int_0^{\pi} \alpha(\omega)\cos(\omega t)d\omega + \int_0^{\pi} \delta(\omega)\sin(\omega t)d\omega$$

Here, $\alpha(\cdot)$ and $\delta(\cdot)$ are random processes. The spectrum of a series x_t is given by

(3.2) $$S_x(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_j e^{-i\omega j}, \ -\pi \le \omega \le \pi,$$

where γ_j is the j^{th} autocovariance and $i^2 = -1$. The spectrum is useful in determining which frequencies are important for the behavior of a stochastic variable. If the spectrum has a peak at frequency $\omega = \pi/3$, then the cycle with periodicity equal to 6 (= 2π / (π/3)) periods is quantitatively important for the behavior of this stochastic variable.

Consider the following examples. If x_t is white noise, then the spectrum is flat. A flat spectrum indicates that all cycles are equally important for the behavior of the variable x_t. Intuitively, this makes sense because the existence of cycles implies forecastibility, and white noise is, by definition, unforcastable. As a second example, suppose that x_t is an AR(1) with coefficient ρ, where $0 < \rho < 1$. The spectrum for this random variable has a peak at $\omega = 0$ and is monotonically decreasing with $|\omega|$. Since the periodicity of a cycle with zero frequency is "infinite", this stochastic process does not have an observable cycle. Finally, suppose that the stochastic variable x_t has a unit root, then the spectrum would be infinite at frequency zero.

Baxter and King (1994) show how to construct filters that isolate specific frequency bands, while removing stochastic and deterministic trends. Suppose one wants to isolate that part of a stochastic variable x_t that is associated with frequencies between ω_1 and ω_2, with $0 < \omega_1 < \omega_2 \le \pi$. If $\omega_2 = \pi$, then the filter is called a high-pass filter since all frequencies higher than ω_1 are included. If $\omega_2 < \pi$, then the filter is called a band-pass filter. The filters are two-sided symmetric linear filters and can be expressed as follows.

(3.3) $$x_t^F = B(L)x_t$$

where x_t^F is the filtered series, L is the lag operator, and

(3.4) $$B(L) = \sum_{h=-\infty}^{\infty} b_h L^h, \qquad \text{with } b_h = b_{-h}.$$

Let the Wold-representation for x_t be given by

(3.5) $$x_t = C(L)\varepsilon_t.$$

Then,

(3.6) $$x_t^F = B(L)C(L)\varepsilon_t.$$

A useful result in spectral analysis is that the spectrum of x_t^F is given by

(3.7) $$S_{x^F}(\omega) = |B(e^{-i\omega})|^2 S_x(\omega),$$

where $|B(e^{-i\omega})|$ is the gain of the filter $B(L)$. The spectrum of the filtered series x_t^F has to be equal to S_x if $|\omega| \in [\omega_1, \omega_2]$ and equal to zero if ω is outside this set. Therefore, the gain of the filter has to be equal to one if $|\omega| \in [\omega_1, \omega_2]$ and equal to zero otherwise. Using the converse of the Riesz-Fischer theorem, one can find the time-series representation, i.e. $B(L)$, that corresponds to these conditions for the gain of the filter. The formulas are as follows

(3.8) $$b_0 = \frac{\omega_2 - \omega_1}{\pi} \textit{ and } b_h = \frac{\sin(\omega_2 h) - \sin(\omega_1 h)}{\pi h}. \quad h = \pm 1, \ldots$$

The ideal filter is an infinite moving average and cannot be applied in practice. In practice one has to truncate $B(L)$. This gives an approximate filter $A^H(L)$, where

(3.9) $$A^H(L) = \sum_{h=-H}^{H} a_h^H L^h,$$

and H is the truncation parameter.[9] Note that a higher value of H means a more accurate band-pass filter, but also the loss of more data points. The ideal filter $B(L)$ has the property that $B(1) = 0$. To ensure the same property for the feasible filter $A^H(L)$, we adjust the coefficients of $A^H(L)$ in such a way that they add up to zero. Let θ be equal to

(3.10) $$\theta = \frac{-\sum_{h=-H}^{H} b_h}{2H+1}.$$

As in Baxter and King (1994), we adjust the coefficients as follows

9 Other approaches in the frequency domain also have to deal with the endpoints. The HP filter deals with the endpoints by using a different filter for each observation in the sample. See Christiano and Den Haan (1996) for a discussion. Engle (1974) points out that band spectrum regressions assume that the last observation is at the same point in the cycle as the first observation.

(3.11) $$a_h^H = b_h + \theta$$

Note that the distortion introduced by this adjustment approaches zero as H goes to infinity.

3.2 Frequency-domain filters for non-stationary processes[10]

In this section, we analyze the properties of frequency-domain filters for more general stochastic processes. In particular, we show that the properties derived in Section 3.1 remain valid when the input series are integrated stochastic processes or when the input series have a linear or quadratic time trend. To prove this, we have to define the spectrum of a non-stationary random process. Although most of the literature on spectral analysis focuses on stationary processes, there are some exceptions. In fact, Hannan (1970) and Priestley (1988) consider much more general non-stationary processes than the ones considered in this section.

First, we will consider first-order integrated processes. When the series x_t is integrated, then the covariances used to define the spectrum in Equation (3.2) are not well-defined. Therefore, we will define the spectrum of an integrated process as the limit of the spectrum of a stationary stochastic process. The motivation for this definition is the following. According to the Beveridge-Nelson decomposition, one can, under mild regularity conditions, write an I(1) process as the sum of a random walk, initial conditions, and a stationary process.[11] Thus,

(3.12) $$x_t = x_{t-1} + e_t \, ,$$

where e_t is a stationary process. Consider the following "AR(1)-type" process:[12]

(3.13) $$x_t = \rho x_{t-1} + e_t \text{ or } x_t = [1/(1\text{-}\rho L)]\, S_e(\omega).$$

Note that, as long as $|\rho| < 1$, the process defined in (3.13) is stationary and has a well-defined spectrum $S_\rho(\omega) = |\, 1/(1\text{-}\rho\, e^{-i\omega})\,|^2\, S_e(\omega)$. Equation (3.12) can be written as:

(3.14) $$x_t = \lim_{\rho \to 1} \rho x_{t-1} + e_t.$$

Equation (3.14) motivates the following definition of the spectrum of an I(1) process:

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[10] This section has benefited a lot from discussions with Clive Granger.
[11] See, for example, Hamilton (1994).
[12] This process is not necessarily an AR(1), since e_t could be serially correlated.

(3.15) $$S_x(\omega) \equiv \lim_{\rho\to 1} S_\rho(\omega) = \lim_{\rho\to 1}\left|\frac{1}{1-\rho e^{-i\omega}}\right|^2 S_e(\omega).$$

Note that $S_x(\omega)$ is finite for all frequencies except possibly zero. Similarly, we can define the spectrum of an I(2) stochastic process as

(3.16) $$S_x(\omega) = \lim_{\rho\to 1}\left|\frac{1}{1-2\rho e^{-i\omega}+\rho^2 e^{-2i\omega}}\right|^2 S_e(\omega)$$

Since $A^H(L)$ is a symmetric filter with $A^H(1) = 0$ we can write $A^H(L)$ as

(3.17) $$A^H(L) = (1-L)\,\overline{A}^H(L) \text{ with } \overline{A}^H(1) = 0.$$

Consequently, $A^H(L)$ has the property that it can make first-order integrated processes stationary. Let $x_t^{F,H} = A^H(L)\,x_t$. We want to show that even when the law of motion for x_t is given by Equation (3.12) and x_t is, thus, a first-order integrated process, the frequency domain filter still correctly eliminates that part of the series associated with frequencies outside the specified band. That is, we want to show for $0 < \omega_1 < \omega_2 \le \pi$ that[13]

(3.18) $$\begin{aligned} S_{x^F}(\omega) &\equiv \lim_{H\to\infty} S_{x^{F,H}}(\omega) = S_x(\omega) && \text{if } \omega_1 \le \omega \le \omega_2 \text{ and} \\ S_{x^F}(\omega) &\equiv \lim_{H\to\infty} S_{x^{F,H}}(\omega) = 0 && \text{if } 0 \le \omega < \omega_1 \text{ and } \omega_2 < \omega \le \pi. \end{aligned}$$

Note that

(3.19) $$x_t^{F,H} = A^H(L)x_t = A^H(L)\frac{e_t}{(1-L)}.$$

The spectrum of $x_t^{F,H}$ is given by

(3.20) $$S_{x^{F,H}}(\omega) = \left|\frac{A^H(e^{-i\omega})}{1-e^{-i\omega}}\right|^2 S_e(\omega) = \lim_{\rho\to 1}\left|\frac{A^H(e^{-i\omega})}{1-\rho e^{-i\omega}}\right|^2 S_e(\omega) = \left|A^H(e^{-i\omega})\right|^2 S_x(\omega)$$

for $0 < \omega \le \pi$. Since $S_x(\omega)$ is well-defined for all values of ω bigger than zero, Equation (3.20) directly implies the desired result described in (3.18) for $0 < \omega \le \pi$. It remains to be shown that S_{x^F} is equal to zero when ω is equal to zero. Because of (3.17) we have that $S_{x^{F,H}}(0)$ is equal to zero for all H, which implies that

[13] Note that calculating the spectrum of x^F involves taking two limits, namely the limit as $\rho \to 1$ and the limit as $H \to \infty$. In practice one would use a finite-order filter on integrated series and let the order of the filter, H, increase as the sample size increases. Therefore, we first let ρ go to one and then let H go to infinity.

(3.21) $$S_{x^F}(0) \equiv \lim_{H\to\infty} S_{x^{F,H}}(0) = \lim_{H\to\infty} 0 = 0 .$$

Harvey and Jaeger (1993) and Cogley and Nason (1995) argue that the properties of frequency-domain filters depend on the order of integration of the input series. This clearly contradicts the analysis above. These papers reach a different conclusion because they always focus on the stationary part of the series, although the filter is always applied to the level. Consider, for example, the process

(3.22) $$x_t = \rho x_{t-1} + e_t ,$$

where e_t is an arbitrary stationary process. When $|\rho| < 1$, these papers compare the filtered series $B(L)\, x_t$ with the stationary part of the series, i.e. x_t. But, when $\rho = 1$, they compare $B(L)\, x_t$ with the stationary part of the series, i.e. $(1-L)x_t$. Thus, when $|\rho| < 1$, they analyze the properties of the filter $B(L)$, and when $\rho = 1$, they analyze the properties of the filter $B(L)/(1-L)$. Therefore, there is a discontinuity in the focus of their analysis when ρ equals 1. We prefer the analysis above that uses the definition of the spectrum for integrated processes. Note that if a researcher is interested in the first-difference of an integrated process instead of the level he can, of course, apply the filter to Δx_t as opposed to x_t.

Now we turn our attention to second-order integrated processes. Since the filter $A^H(L)$ is a symmetric filter it can be written as

(3.23) $$A^H(L) = (1-L)\,(1-L^{-1})\,\overline{\overline{A}}^H(L) = -L^{-1}\,(1-L)^2\,\overline{\overline{A}}^H(L) \text{ with } \overline{\overline{A}}^H(1) < \infty .$$

Now Equation (3.20) would be equal to

(3.24) $$\begin{aligned} S_{x^{F,H}}(\omega) &= \left| \frac{A^H(e^{-i\omega})}{1-2e^{-i\omega}+e^{-2i\omega}} \right|^2 S_e(\omega) \\ &= \lim_{\rho\to 1} \left| \frac{A^H(e^{-i\omega})}{1-2\rho e^{-i\omega}+\rho e^{-2i\omega}} \right|^2 S_e(\omega) = \left| A^H(e^{-i\omega}) \right|^2 S_x(\omega) \end{aligned}$$

Again, it immediately follows that the spectrum of x^F is equal to the squared gain of the filter $B(L)$ times the spectrum of x for all $\omega > 0$. As a consequence, the spectrum of x^F is equal to the spectrum of x for the included range of frequencies and equal to zero for the excluded range of frequencies. However, since it is not necessarily true that $\overline{\overline{A}}^H(1) = 0$

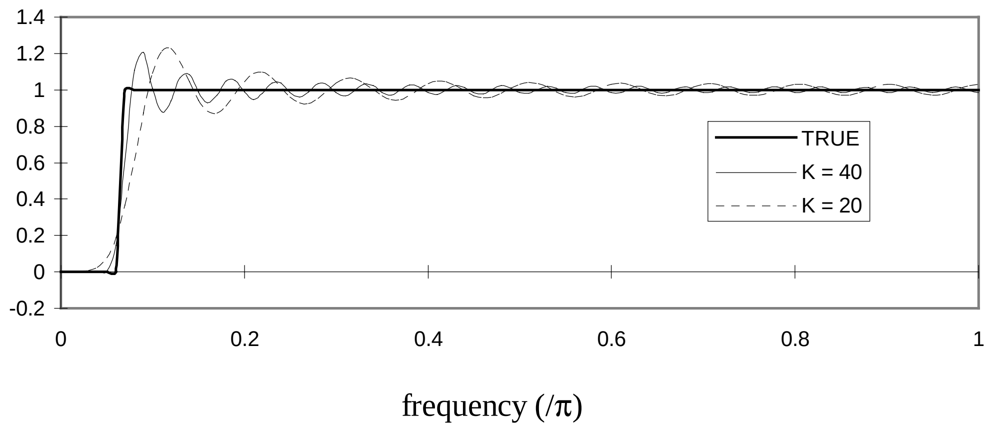
for second-order processes, it is no longer guaranteed that the spectrum of $x^{F,H}$ is equal to zero at $\omega = 0$. Hence, the result for second-order integrated processes is slightly less general then the result for first-order integrated processes.

For finite-order filters, however, there might be another reason why it matters whether the input series is integrated or not. Consider the approximation of the filter that eliminates all frequencies below ω_1. The squared gain of the approximate filter is not exactly equal to zero for frequencies less than ω_1, and not exactly equal to 1 for frequencies bigger than ω_1. Since the spectrum of the filtered series is equal to the squared gain times the spectrum of the input series, the approximation error may be bigger for processes for which the value of the spectrum goes to infinity as the frequency goes to zero—that is, for integrated processes.

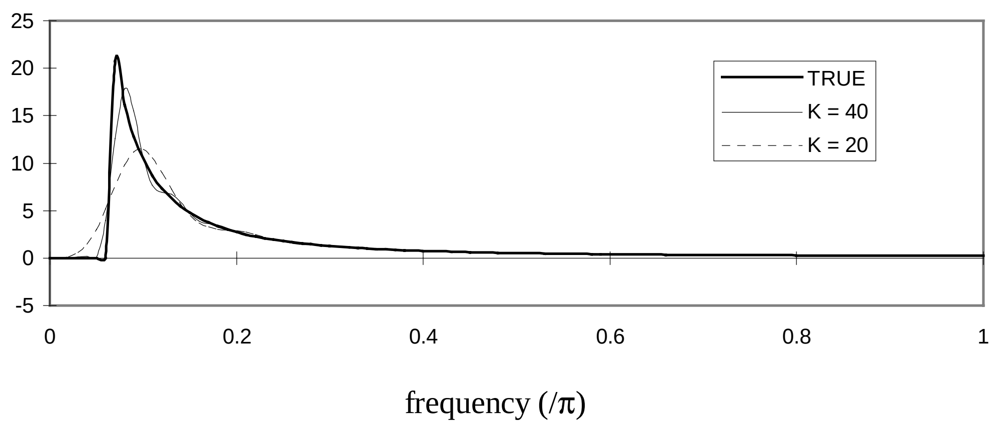
To address this question, we calculate the spectrum of three stochastic processes that are filtered using high-pass filters. Figures 3.1 and 3.2 document the results for the high-pass filters that eliminate all cycles associated with periods bigger than 32 periods and 10 periods, respectively. In addition to examining the ideal infinite-order high-pass filter, we also consider two approximate filters with truncation parameters equal to 20 and 40. Panel A in Figures 3.1 and 3.2 presents the results for the case where the process is a white noise process. Panel B reports the results for the second process, an AR(1) with a coefficient equal to 0.95. Finally, Panel C reports the results for the third process, an integrated AR(1) with a coefficient equal to 0.4. The variance of the white noise process in Panel A is chosen in such a way that the spectrum of the filtered series presented in Panel A is equal to the squared gain of the filter used in Panels A, B, and C.

FIGURE 3.1: SPECTRA OF FILTERED PROCESSES ($\omega_1 = \pi/16, \omega_2 = \pi$)

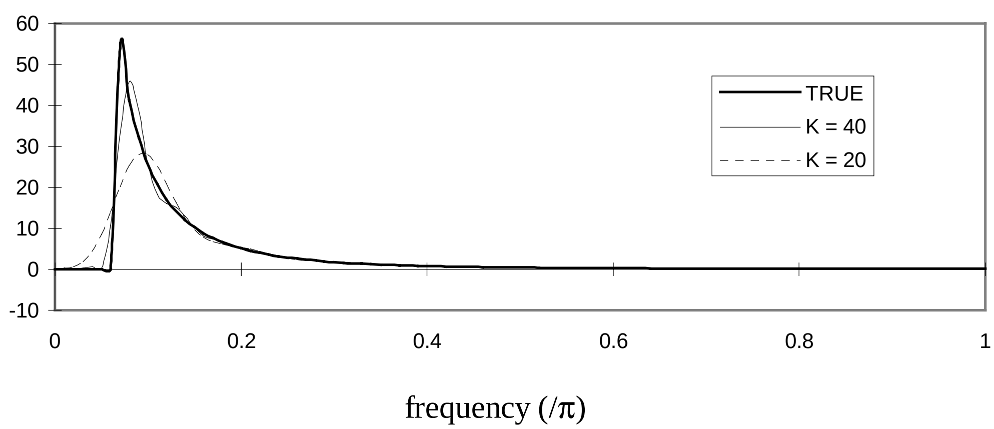
A: White Noise (Squared Gain)



B: AR(1) with Coefficient equal to 0.95



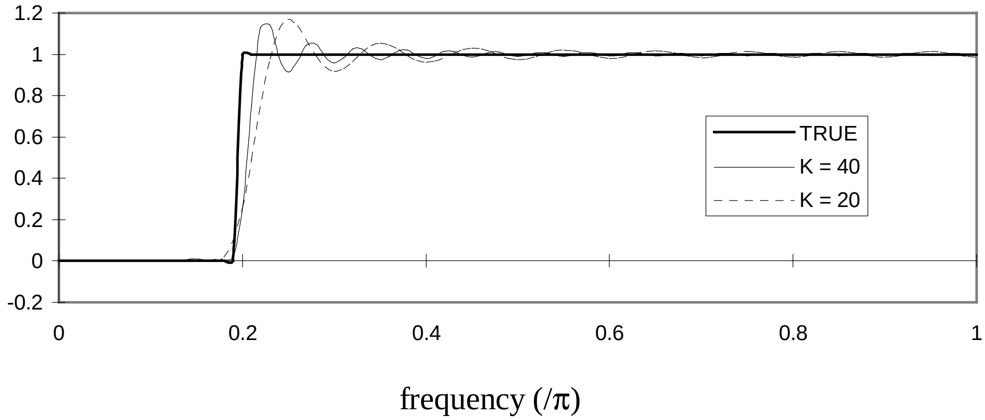
C: Integrated AR(1) with Coefficient equal to 0.4



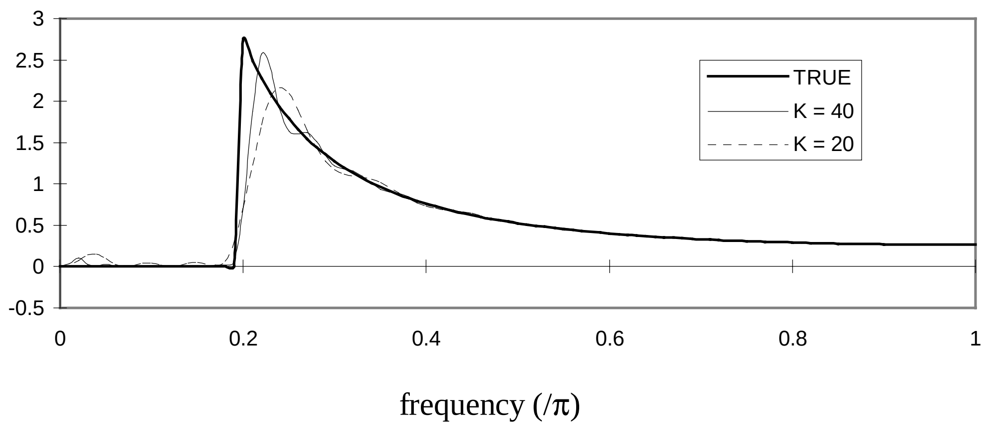
Note: The variance of the white noise process in Panel A is chosen in such a way that Panel A also represents the squared gain of the filter used in these three panels.

FIGURE 3.2: SPECTRA OF FILTERED PROCESSES ($\omega_1 = \pi/5$, $\omega_2 = \pi$)

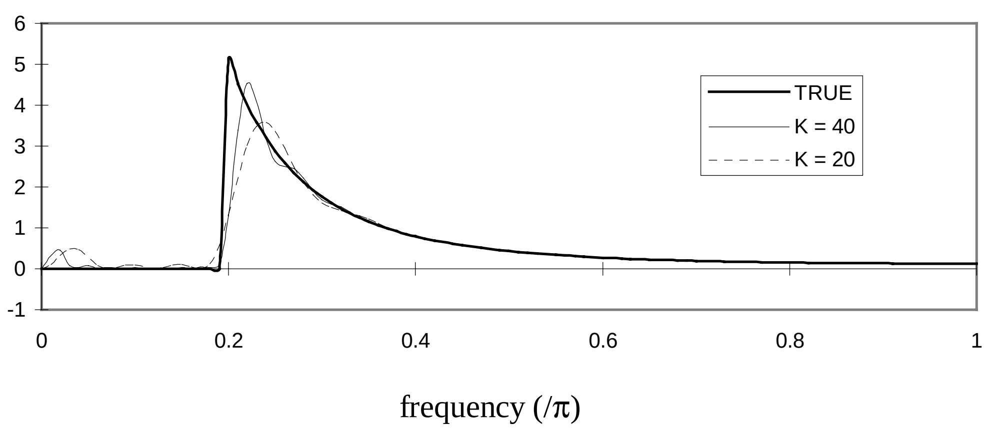
A: White Noise (Squared Gain)



B: AR(1) with Coefficient equal to 0.95



C: Integrated AR(1) with Coefficient equal to 0.4



Note: The variance of the white noise process in Panel A is chosen in such a way that Panel A also represents the squared gain of the filter used in these three panels.

As documented in the graph, for all stochastic processes the approximation is better for the frequencies that are less than ω_1 than for the frequencies just above ω_1. Also, the approximation errors are bigger for the two serially correlated processes than for the white noise process. This suggests that the truncation parameter that one uses should depend on the persistence of the underlying process; i.e., a higher truncation parameter is needed for more persistent processes. The graph also shows that the study of approximation errors does not reveal a fundamental difference between the persistent stationary process and the integrated process studied here. For example, for K equal to 40 and ω_1 equal to $\pi/5$, the peak of the approximated spectrum is 14% and 12% less than the peak of the true spectrum for the stationary persistent process and the integrated process, respectively.

Now suppose that the series x_t has a linear time trend. That is, x_t can be written as

$$x_t = b\,t + y_t, \tag{3.25}$$

where y_t is a stationary or integrated process. When one applies the filter $A^H(L)$ and uses the results shown in (3.17), then

$$x_t^{F,H} = A^H(L)x_t = \overline{A}^H(1)b + A^H(L)y_t \tag{3.26}$$

and

$$\begin{aligned} &S_{x^F}(\omega) = S_y(\omega) \text{ if } \omega_1 \leq \omega \leq \omega_2, \text{and} \\ &S_{x^F}(\omega) = 0 \text{ if } \omega < \omega_1, \omega > \omega_2. \end{aligned} \tag{3.27}$$

Thus, the filter removes a linear trend, and (3.18) holds for the non-deterministic part of the series. When one uses a finite-order filter to approximate $B(L)$, then one can use the results in (3.23), and the filter will also take out a quadratic time trend.

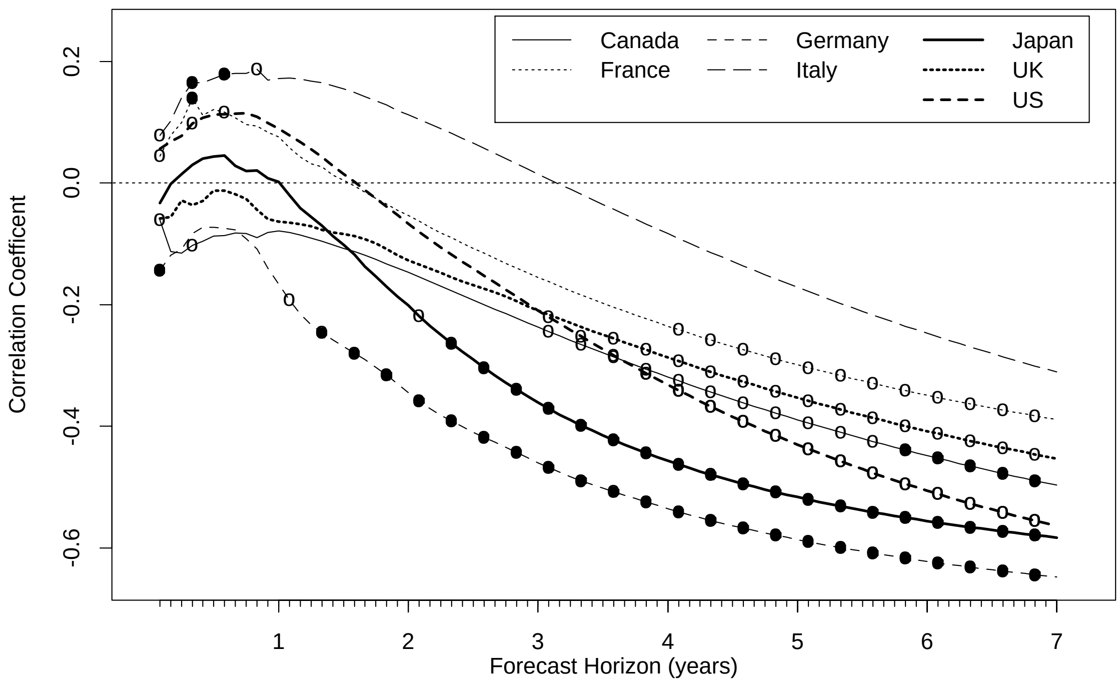
4. Empirical results

In this section, we discuss the empirical comovements between prices and output during the postwar period for the G7 countries. We use both monthly data for industrial production and the CPI index, and quarterly data for GDP and its deflator. Details about data sources and sample periods are given in Appendix A. In Section 4.1, we discuss the results for the VAR forecast errors and in Section 4.2 we discuss the results for the frequency domain filters.

4.1 Comovement of prices and output using VAR forecast errors

In this section, we discuss the comovement of prices and output using the VAR forecast errors. But instead of using the actual realized VAR forecast errors as in Den Haan (2000), we use the correlation coefficients implied by the estimated VAR coefficients and the estimated covariance matrix of the VAR residuals. In Appendix B, we document the efficiency gains of using this alternative procedure. First, we will discuss the results when the monthly CPI index and industrial production is used and then we will discuss the results when quarterly GDP and its deflator are used.

Figure 4.1: The correlation coefficients for CPI and industrial production forecast errors



Note: This figure plots the correlation coefficients of the *k*-period ahead price and output forecast errors of a monthly VAR when a unit root is imposed in the estimation. The open circles indicate that the estimate is significant at the 10% level and the filled-in circles indicate that the estimate is significant at the 5% level. The sample period is from 1957 (from 1958 for Germany) to 1999 (to 1998 for Italy). See Appendix A for details.

Figure 4.1 plots the results for the comovement of the price level and output for the period starting in 1957 and ending in 1999[14] when a unit root is imposed in the

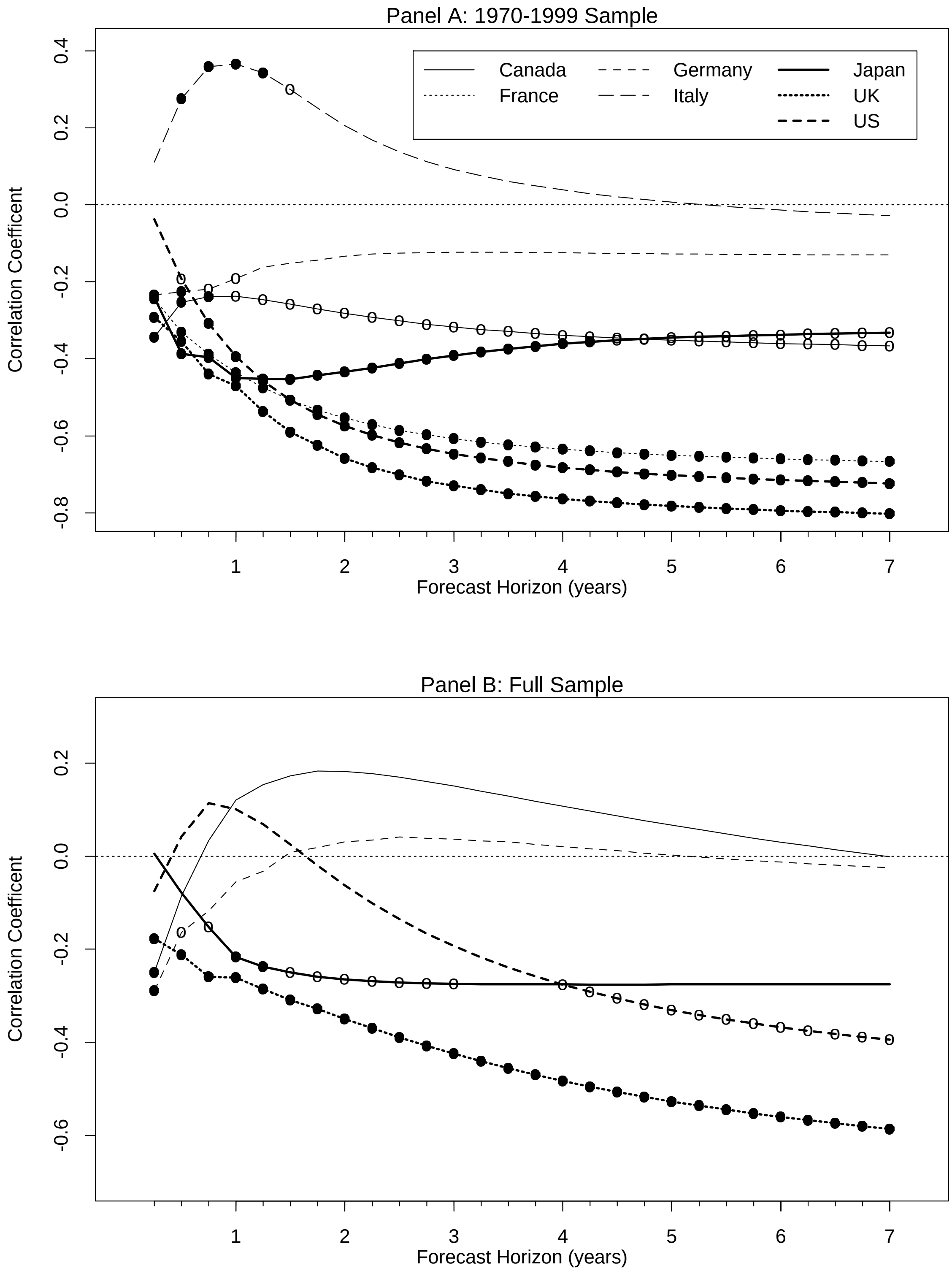
[14] The time series for Germany start in January 1958. The time series for Italy end in December 1998. See Appendix A for details.

estimation of the VAR.[15] For all G7 countries the correlation coefficients at long-term forecast horizons are negative and significantly so at either the 5% or 10% level for all countries except Italy. Moreover, for all countries the short-term correlation coefficients are substantially higher. For France, Italy, and the US there are short-term positive correlation coefficients at either the 5% or 10% significance level. Furthermore, these results are robust to relaxing the unit root restriction in the estimation of the VAR. The main difference between the specifications imposing a unit root and those not imposing a unit root is that, when no unit-root is imposed in the estimation of the VAR, all countries (including Italy) display significant negative long-term correlation coefficients at the 5% significance level and the (negative) short-run correlation coefficients for Canada are no longer significant.

Panel A of Figure 4.2 plots the comovement of quarterly GDP and its deflator for the longest sample period for which we have data for all seven countries, that is, from the first quarter of 1970 to the last quarter of 1999. Again a unit root is imposed in the estimation of the VAR. There are some similarities and some differences with the CPI and industrial production data. For all countries the long-term correlation coefficients are negative. Furthermore, for all countries except Italy and Germany, the long-term correlation coefficients are significant at either the 5% or the 10% level. Also, the results are somewhat less robust to not imposing a unit root. For example, without the unit-root restriction the Italian correlation coefficients become negative for forecast horizons longer than 2 years, but the German correlation coefficients are no longer negative for forecast horizons between 2 and 6 years, although both effects are insignificant. As demonstrated in Panel B of Figure 4.2, the results change for some countries when we consider the full sample. For Japan, the UK, and the US the long-term correlation coefficients remain significantly negative, but the US now shows evidence of positive short-run correlation coefficients. Both Canada and Germany now show (insignificant) positive correlation coefficients.[16]

[15] The figures presented in this paper are all based on VARs estimated in first differences. See Appendix A for a description of the unit-root tests.
[16] For France and Italy the longest available sample period is from 1970 to 1999.

Figure 4.2: The correlation coefficients for GDP and the GDP deflator's forecast errors



Note: These figures plot the correlation coefficients of the *k*-period ahead price and output forecast error of a quarterly VAR estimated over the indicated sample period when a unit root is imposed in the estimation. The open circles indicate that the estimate is significant at the 10% level and the filled-in circles indicate that the estimate is significant at the 5% level. The full sample period is from 1957:1 to 2000:2 for Canada and the U.K., from 1960:1 to 2000:2 for Germany, from 1947:1 to 2000:2 for the U.S., and from 1955:2 to 1999:4 for Japan. See Appendix A for details.

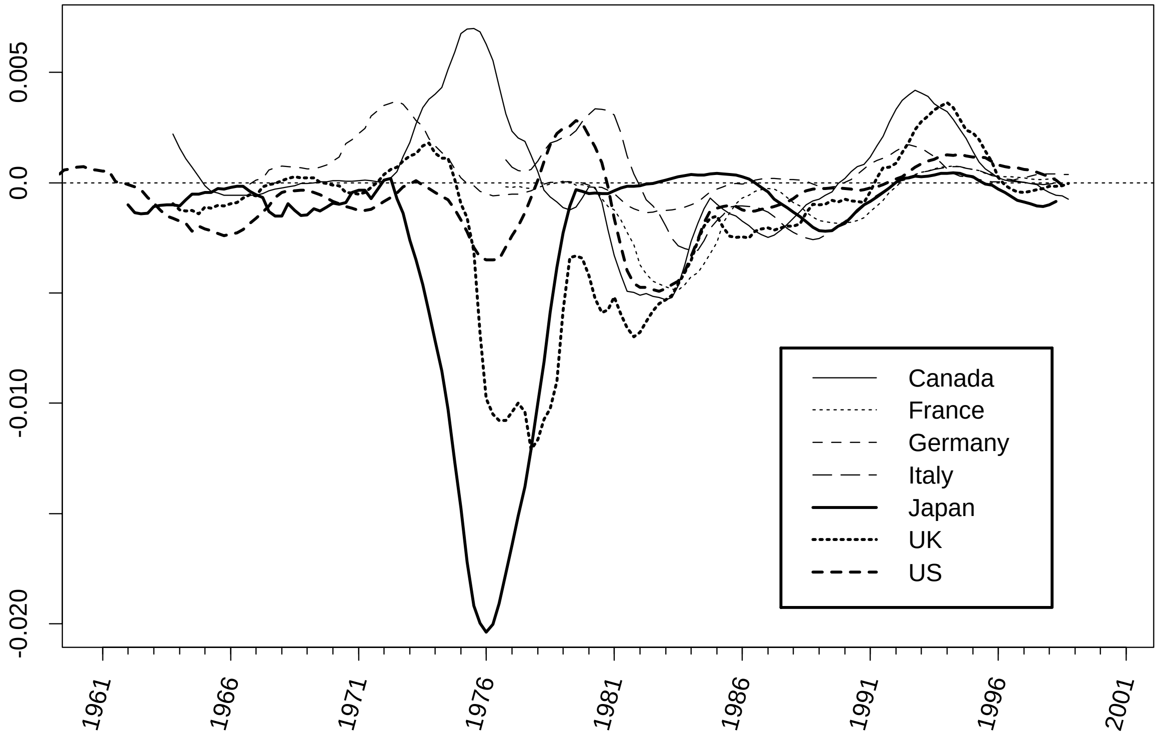
To understand better what time periods are mainly responsible for the observed correlation pattern, we plot in Figure 4.3 a two-sided six-quarter moving average of the cross product of the quarterly price and output forecast errors for the four-year ahead forecast period.[17] For each country we use forecast errors from the VAR estimated with the longest possible sample. Several interesting observations can be made. First, it is clear from the graph that there are factors hitting the different countries at the same time and in the same way, but that the correlation across countries is far from perfect. Second, it is clear that the seventies play a major role in contributing to the magnitude of several of the negative correlation coefficients, although a quantitatively important negative long-run comovement is also present in the early eighties for Canada, France, Italy, the UK, and the US. In fact, for this set of countries and this forecast horizon, the moving average is not often above zero.[18] Third, in the seventies there was no negative comovement in Canada, Germany, and Italy. Canada experienced strong economic growth in the early seventies and experienced a reduction in economic growth that was mild compared to what was observed in the other countries. Although Germany did show a considerable reduction in real growth rates, inflation rates did not rise in the seventies. Italy experienced a strong increase in real growth rates and inflation rates in the early seventies. When the oil crises lead Italy into a recession, inflation rates in fact first decreased. Inflation rates in Italy only started to increase again, when economic growth started to recover. Finally, in the middle of the eighties there was a long-run positive comovement in Canada, France, Italy, and the UK and to some extent Germany and the US. Although the sample is too short to say anything definitive, the last two findings suggest that the correlation between prices and output could very well be time-varying and depend on the particular circumstances and economic institutions of the particular country, and the time period.[19]

[17] At other forecast horizons and for the monthly data the graphs displayed more noise and less interesting patterns. Davis and Kanago (2000) look at the cross products of one-quarter ahead forecast errors for Canada, the UK, and the US.

[18] Results not shown here document the positive correlation coefficients at short-term forecast horizons are due to positive cross products in the fifties and sixties.

[19] An interesting paper that estimates the conditional covariance of the one-quarter ahead forecast errors of GDP and its deflator is Cover and Hueng (2000).

Figure 4.3: Time-varying comovement of the CPI and industrial production



Note: This figure plots a two-sided 6th-order moving average of the cross product of the 4-year ahead forecast errors of the price level and output. A unit root was imposed in the estimation of the (monthly) VAR. The sample period is from 1957:1 to 2000:2 for Canada and the U.K., from 1960 to 2000:2 for Germany, from 1947:1 to 2000:2 for the U.S., and from 1955:2 to 1999:4 for Japan, from 1970:1 to 2000:1 for France, and from 1970:1 to 1999:4 for Italy. See Appendix A for details.

4.2 Comovement of prices and output using frequency-domain filters

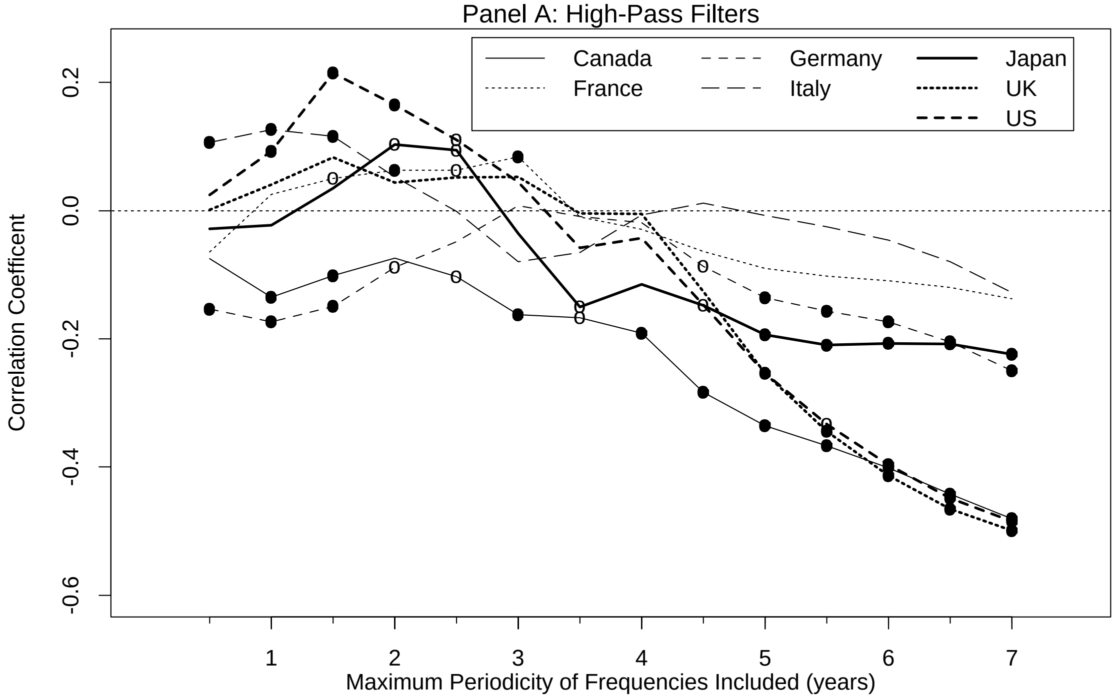
In this section, we analyze the comovement of prices and output using frequency domain filters. Den Haan (2000) argues that the *sign* of the correlation coefficient of filtered prices and output may not have as much identifying power[20] but, of course, that does not mean that the actual numerical values of these correlation coefficients provide any less information. In Figure 4.4, we plot the correlation coefficients of (monthly) CPI and industrial production when frequency domain filters have been used to render the data stationary. Panel A reports the results for the high-pass filters where the filter isolates that part of the series associated with cycles that have a periodicity less than the indicated periodicity. Panel B reports the results for the band-pass filters where the filter

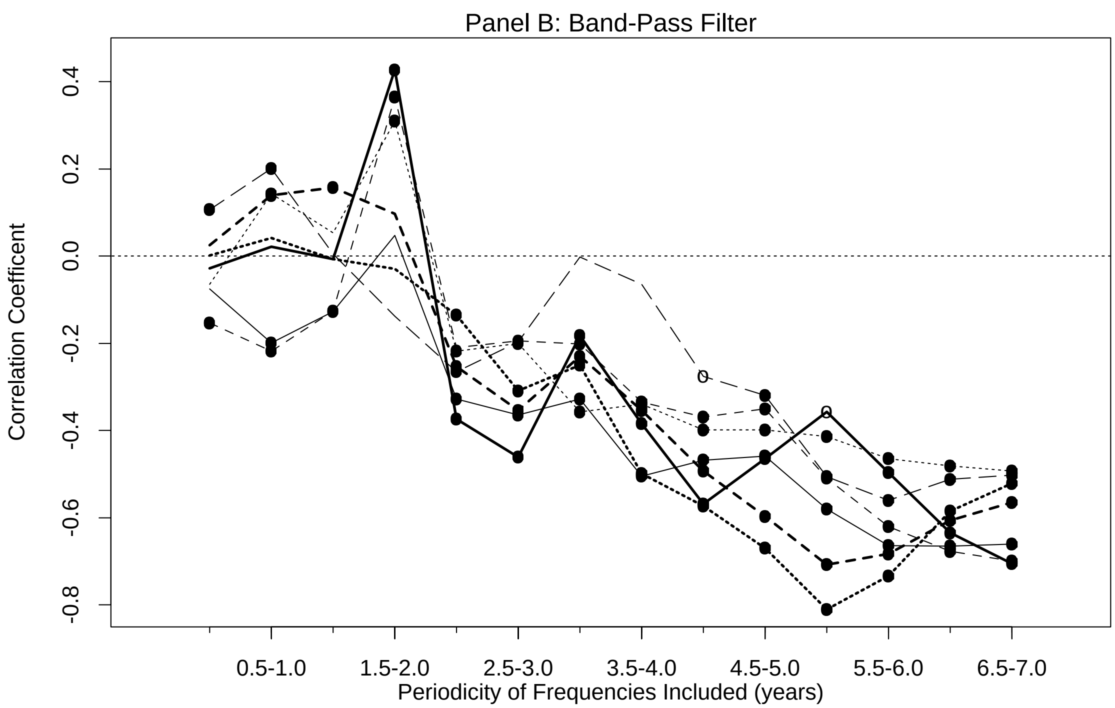
[20] Because negative correlation coefficients for filtered price and output series are consistent with models that only have demand shocks and models that have both demand and supply shocks. In contrast, negative correlation coefficients of VAR forecast errors cannot be generated by models with only demand shocks unless unreasonable assumptions are made.

isolates that part of the series associated with cycles that have the indicated periodicities. The truncation parameter H was set equal to 60, which means that five years of data are discarded at both sides of the sample. For smaller values of H the results are sensitive to changes in H. The following observations can be made. First, consistent with the results for the VAR forecast errors we find for all countries that the long-run correlation coefficients are negative and they are significant at the 5% significance level for five of the seven countries. As shown in Panel B, when band-pass filters are used the correlation coefficients are significantly negative for all countries when the periodicity of the included frequencies exceeds four and one-half years and typically earlier. Second, for all countries the short-term correlations are substantially higher than the long-term correlation coefficients. Using high-pass filters, positive correlation coefficients are observed for France, Italy, Japan, the UK, and the US and significant positive correlation coefficients at the 10% level are observed for France, Italy and the US.

In Figure 4.5, we plot the correlation coefficients of quarterly GDP and its deflator when a high-pass filter has been used to filter the data both for the sample from 1970 to 1999 (Panel A) and for the full sample (Panel B). The truncation parameter is set equal to 20 so that again 5 years of data are discarded at both ends of the sample. The main conclusion to be drawn from Figure 4.5 is that there is substantial evidence of negative correlation coefficients. Also, just as we found with the VAR forecast errors, there is evidence of positive comovement in Italy for the 1970-99 sample period.

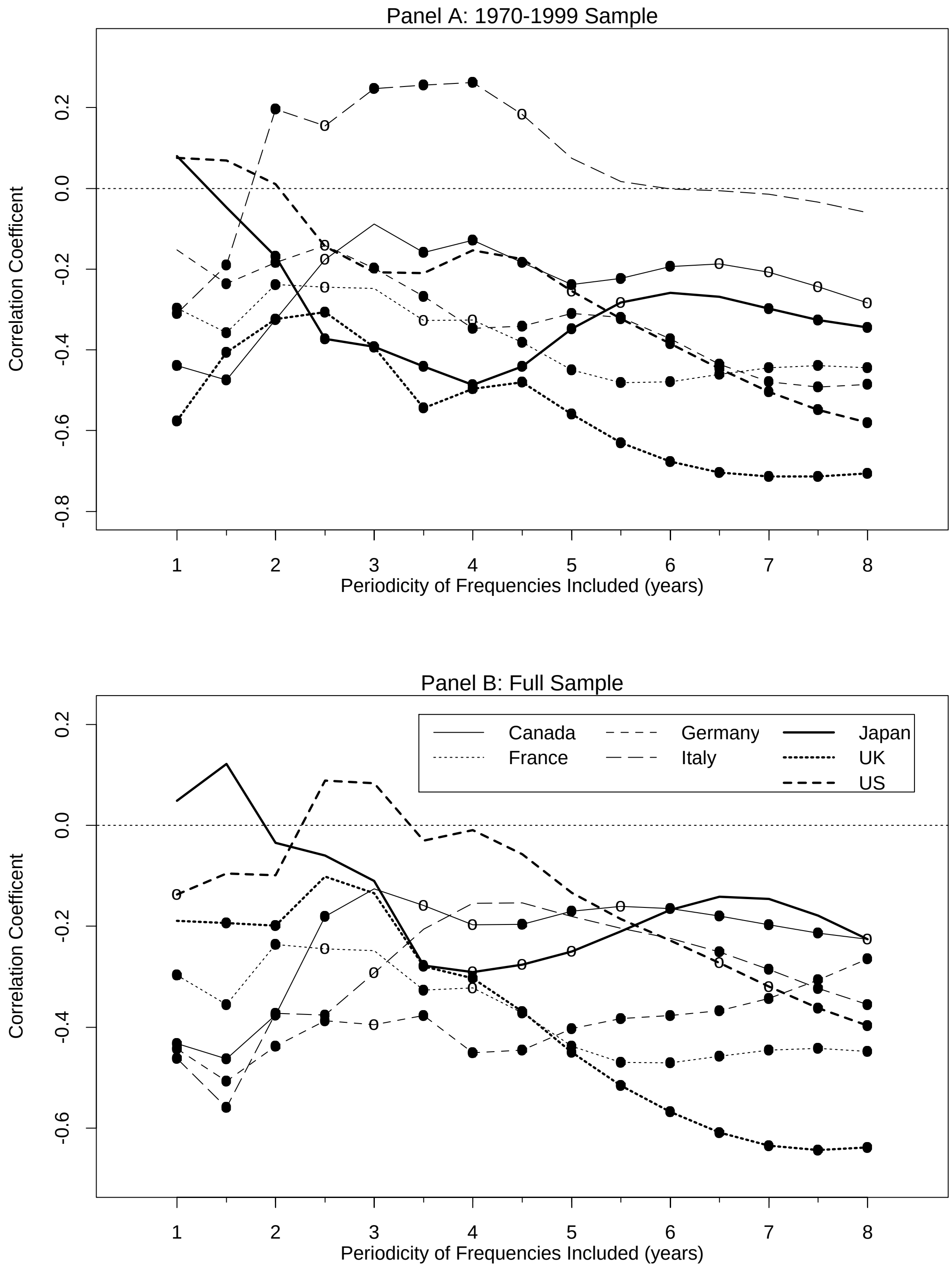
Figure 4.4: The correlation coefficients for filtered CPI and industrial production





Note: These figures plot the correlation coefficients of filtered monthly price and output series using the indicated filter. The high-pass filter isolates that part of the series associated with cycles with a periodicity that is less than the indicated periodicity. The band-pass filter isolates that part of the series associated with cycles with the indicated periodicity. The open circles indicate that the estimate is significant at the 10% level and the filled-in circles indicate that the estimate is significant at the 5% level. Standard errors are calculated using the VARHAC procedure proposed in Den Haan and Levin (1996). The sample period is from 1957 (from 1958 for Germany) to 1999 (to 1998 for Italy).

Figure 4.5: The correlation coefficients for filtered GDP and the GDP deflator



Note: These figures plot for the indicated sample period the correlation coefficients of filtered quarterly price and output series, where an (approximate) high-pass filter is used to isolate that part of the series associated with cycles with a periodicity that is less than the indicated periodicity. The open circles indicate that the estimate is significant at the 10% level and the filled-in circles indicate that the estimate is significant at the 5% level. Standard errors are calculated using the VARHAC procedure proposed in Den Haan and Levin (1996). The full sample period is from 1957:1 to 2000:2 for Canada and the U.K., from 1960:1 to 2000:2 for Germany, from 1947:1 to 2000:2 for the U.S., and from 1955:2 to 1999:4 for Japan. See Appendix A for details.

5. Concluding comments

The results in this paper clearly provide more evidence for negative correlation coefficients than for positive correlation coefficients. For some countries there is some evidence of a positive short-run correlation coefficient during some sample periods and for some price and output measures. The paper also shows that the observed negative correlation coefficients are not just due to the price and output movements during the oil crisis of the seventies when prices soared and output dropped in many countries. Moreover, during that same period there did not seem to be a negative comovement of prices and output in Canada, Germany, and Italy. This suggests that the correlation between prices and output may very well be time varying and depend on, for example, the particular monetary policies being followed. Support for the assertion that the comovement is time varying can also be found in the observation that in the middle of the eighties there was a long-run positive comovement in Canada, France, Italy, and the UK, and to some extent, in Germany and the US, even though this comovement is typically negative.

Appendix A: Data sources and time-series properties

In this appendix, we describe the data sources and sample periods necessary to duplicate all the results in this paper. The actual period for which correlation coefficients are calculated is shorter, since the estimation of the VAR and the use of frequency filters reduces the length of the sample period. We also provide the results of unit-root and cointegration tests performed on the data and the VAR specifications used in the calculations.

A.1 Data sources and sample periods

Monthly Data:[21]

- Series:
  - Industrial Production Index (###66…IZF)
  - Consumer Price Index (###64…ZF)
- Data is from the International Monetary Fund's *International Financial Statistics* CD-ROM published in June 2000
- Sample period for which observations are available for both series:[22]
  - Canada (156): 1957:1-1999:11
  - France (132): 1957:1-1999:11
  - Germany (134): 1958:1-1999:10
  - Italy (136): 1957:1-1998:12
  - Japan (158): 1957:1-1999:12
  - United Kingdom (112): 1957:1-1999:10
  - United States (111): 1957:1-1999:12

Quarterly Data:

- Series:[23]
  - Real Gross Domestic Product, seasonally adjusted (###99B.R.F)[24]
  - Nominal Gross Domestic Product, seasonally adjusted (###99B.CZF)

[21] The IFS time series code is given in parentheses after the variable name. The "###" symbol represents the three digit country code. For more information on the code descriptions, see the documentation file in the PRINT_ME directory of the IFS CD-ROM.

[22] The country code is given in parentheses following the country name. This represents the "###" in the series code.

[23] Real and nominal Gross Domestic Product is in billions of units of the country's national currency, except for Italy, which is in trillions of units of Italy's national currency.

[24] Various versions of the series are spliced together to create the complete time series. The series version is represented by the next to last character in the series code. Canada, France, Germany, and Italy use versions Y and Z. Additionally, France uses version X. The United States and the United Kingdom use only version Z. The most current observations were updated directly from the IFS monthly publication, including those for Japan.

- GDP Deflator—computed from the real and nominal GDP series (Nom. GDP/Real GDP)

- Data for Japan is from the Bank of Japan. For all other countries, the nominal GDP series is from the International Monetary Fund's *International Financial Statistics* CD-ROM published in June 2000 and the real GDP series were obtained by request directly from the IMF.

- Sample period for which observations are available for all series:
  - Canada: 1957:1-2000:2
  - France: 1970:1-2000:1
  - Germany: 1960:1-2000:2[25]
  - Italy: 1970:1-1999:4
  - Japan: 1955:2-1999:4
  - United Kingdom: 1957:1-2000:2
  - United States: 1947:1-2000:2

[25] A trend break in real and nominal GDP in Germany 1990:4/1991:1 was adjusted by multiplying the observations before the break by AVE(t)*ANNUAL(t-1)/(AVE(t-1)*ANNUAL(t)), where AVE(t) is the year t average of the quarterly series with a trend break, and ANNUAL(t), is the year t observation of the available annual series without a trend break.

A.2 Unit-root tests

A.4.2.A: Unit-root Test for Monthly Data

Country	Full Sample		1970-1999 Sample	
	CPI	IP	CPI	IP
Canada	-0.826	-1.663	0.4934	-2.703
France	0.5102	-1.805	0.5235	-3.076
Germany	-0.021	-2.828	-1.490	-2.805
Italy	-1.202	-1.907	0.5326	-2.811
Japan	1.3453	-0.977	-2.052	-1.960
United Kingdom	-0.615	-2.520	-0.247	-2.845
United States	-1.361	-2.383	0.0339	-2.802

A.4.2.B: Unit-root Test for Quarterly Data

Country	Full Sample		1970-1999 Sample	
	GDP Deflator	GDP	GDP Deflator	GDP
Canada	-1.851	-1.399	-1.203	-3.264*
France	N/A	N/A	-0.471	-3.488**
Germany	-0.276	-2.110	-0.671	-3.119
Italy	N/A	N/A	-0.187	-1.797
Japan	0.5631	-0.407	-2.383	-0.616
United Kingdom	-1.533	-2.965	-1.116	-2.509
United States	-1.906	-2.527	-1.046	-3.486**

Note: These tables report the results of the augmented Dickey-Fuller test for a unit root using the estimated OLS autoregressive coefficient. Four lags have been included in the regression along with a constant and linear trend. When the null of a unit root is rejected at the 10% (5%, 1%) level, it is indicated with a * (**,***). See Hamilton (1994) for a description of these tests.

A.3 VAR specifications

Table A.4.1: Specification for Monthly Data (Unit Root Imposed)

Country	Full Sample		1970-1999 Sample	
	# of Lags	Linear Trend Included?	# of Lags	Linear Trend Included?
Canada	12	No	12	Yes
France	12	Yes	6	Yes
Germany	12	Yes	2	Yes
Italy	12	Yes	6	Yes
Japan	12	Yes	12	Yes
United Kingdom	12	No	12	Yes
United States	12	No	12	Yes

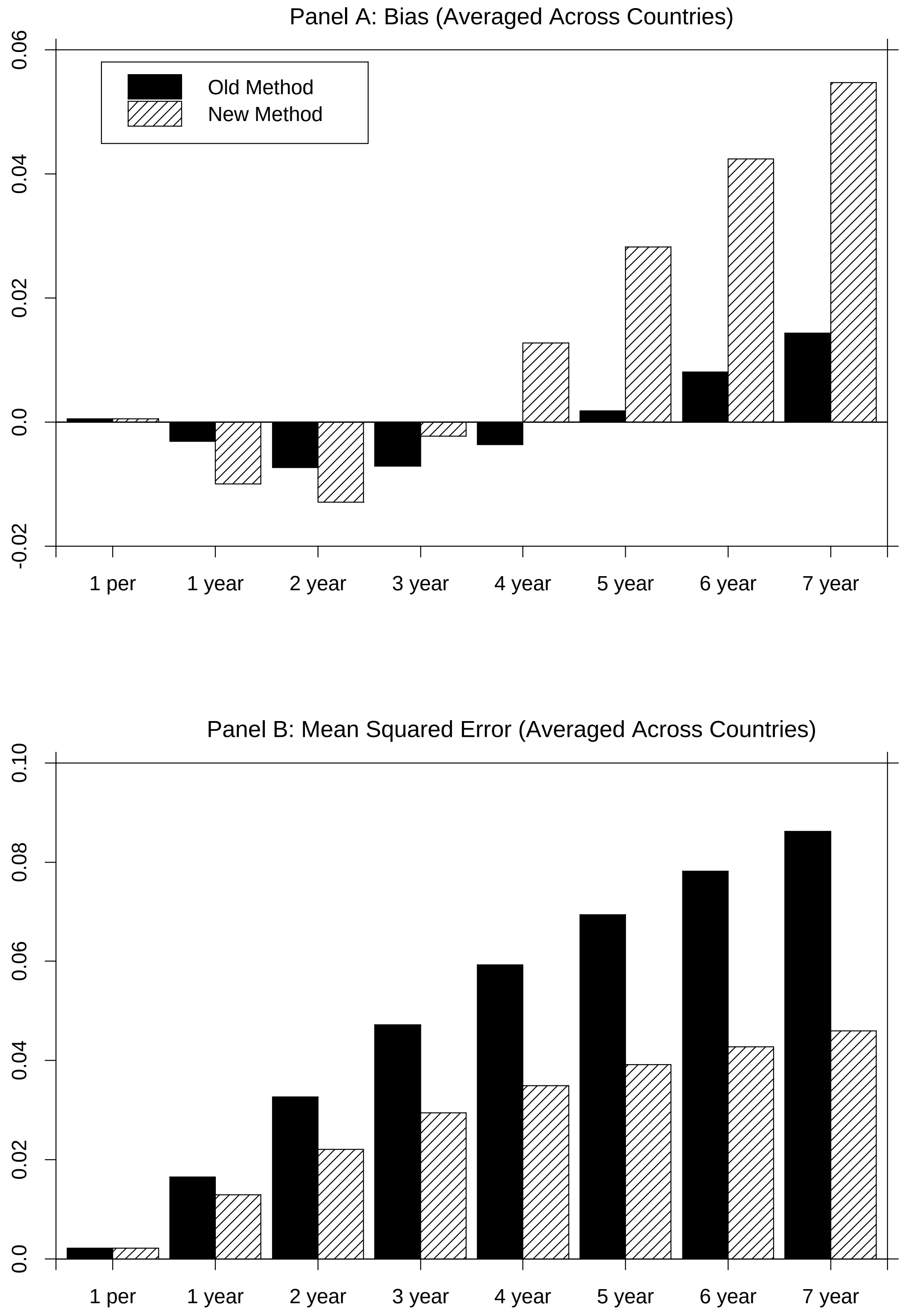
A.4.2: Specification for Quarterly Data (Unit Root Imposed)

Country	Full Sample		1970-1999 Sample	
	# of Lags	Linear Trend Included?	# of Lags	Linear Trend Included?
Canada	4	Yes	2	Yes
France	N/A	N/A	2	Yes
Germany	4	Yes	4	Yes
Italy	N/A	N/A	3	Yes
Japan	4	Yes	4	Yes
United Kingdom	3	No	4	Yes
United States	3	No	1	Yes

Appendix B: Efficiency gains of imposing VAR restrictions

In this appendix, we summarize the efficiency gains that can be made by imposing the VAR restrictions. Den Haan (2000) calculates the correlation coefficient of VAR forecast errors by constructing a time series of forecast errors and calculating the sample correlation coefficients. But, as was shown in Section 2.1, the estimated VAR directly implies a particular correlation coefficient. Using a VAR as the true data generating process, we performed the following Monte Carlo analysis. For each Monte Carlo replication we used the generated data to calculate the correlation coefficient with both procedures. After all 10,000 Monte Carlo replications are completed we calculate the bias and the Mean Squared Error. Such a Monte Carlo experiment was performed using virtually all estimated VARs as the true DGP. Since the results were very similar we present here just a summary of the results. In particular, Figure B.1 shows the bias and the (square root of the) mean squared error averaged across the seven countries for the VAR estimated with the monthly CPI and industrial production data over the full sample. The results in panel B confirm our conjecture that imposing the VAR restrictions leads to substantial efficiency gains especially in estimating the correlation coefficients for the long-term forecast horizons. This despite the fact that imposing the constraints typically increases the bias as documented in panel A. Given that the VAR coefficients are biased estimates and the correlation coefficients are non-trivial functions of these coefficients, it isn't easy to understand the source of the bias. It seems plausible to us that the method that imposes the VAR restrictions has a larger bias because it makes more use of the estimated VAR coefficients in calculating the correlation coefficients.

Figure B.1: Efficiency gains of imposing VAR restrictions



Note: The "old method" referred to in the legend calculates the correlation coefficients using the realized forecast errors as in Den Haan (2000). The "new method" referred to in the legend calculates the correlation coefficient by imposing the VAR restrictions and using the estimated VAR coefficients.

REFERENCES.

Backus, D.K., and P.J. Kehoe, 1992, International evidence on the historical properties of business cycles, *American Economic Review* 82, pp. 864-885.

Ball, L., and N.G. Mankiw, 1994, A sticky-price manifesto, *Carnegie-Rochester Conference Series on Public Policy* 41, pp. 127-151.

Baxter, M., and R.G. King, 1994, Measuring business cycles: Approximate band-pass filters for economic time series, manuscript University of Virginia.

Bernanke, B., and I. Mihov, 1998, Measuring monetary policy, *Quarterly Journal of Economics* 113, pp. 869-902.

Chadha, B., and E. Prasad, 1993, Interpreting the cyclical behavior of prices, *IMF Staff Papers* 40, pp. 266-298.

Christiano, L.J., and W.J. Den Haan, 1996, Small sample properties of GMM for business cycle analysis, *Journal of Business and Economic Statistics* 14, pp. 309-327.

Cogley, T., and J.M. Nason, 1995, *Journal of Economic Dynamics and Control* 19, pp. 253-278.

Cooley, T.F., and L.E. Ohanian, 1991, The cyclical behavior of prices, *Journal of Monetary Economics* 28, pp. 25-60.

Cover, J.P. and C.J. Hueng, 2000, The correlation between shocks to output and the price level: Evidence from a GARCH model, manuscript, University of Alabama.

Davis, G.K. and B.E. Kanago, 2000, The contemporaneous correlation between price shocks and output shocks, manuscript, Miami University.

Den Haan, W.J., and A.T. Levin, 1996, Inferences from parametric and non-parametric spectral density estimation procedures, National Bureau of Economic Research technical working paper #195.

Den Haan, W.J., 2000, The comovement between output and prices, *Journal of Monetary Economics* 46, pp. 3-30.

Diebold, F.X., L.E. Ohanian, and J. Berkowitz, 1998, Dynamic equilibrium economies: a framework for comparing models and data, *Review of Economic Studies* 65, pp. 433-451.

Engle, R.F., 1974, Band spectrum regression, *International Economic Review* 15, 1-11.

Fiorito, R., and T. Kolintzas, 1994, Stylized facts of business cycles in the G7 from a real business cycles perspective, *European Economic Review* 38, pp. 253-269.

Geweke, J.F., 1999, Computational experiments and reality, manuscript, University of Iowa.

Hamilton, J.D., 1994, *Time Series Analysis*, Princeton University Press.

Hannan, E.J., 1970, *Multiple time series*, Princeton University Press.

Harvey, A.C., and A. Jaeger, 1993, Detrending, stylized facts and the business cycle, *Journal of Applied Econometrics* 8, pp. 231-247.

Hodrick, R.J., and E.C Prescott, 1997, Postwar US business cycles: An empirical investigation, *Journal of Money, Credit and Banking* 29, pp. 1-16.

Judd, J.P., and B. Trehan, 1995, The cyclical behavior of prices: Interpreting the evidence, *Journal of Money, Credit, and Banking* 27, pp. 789-797.

Kydland, F.E., and E.C. Prescott, 1990, Business cycles: real facts and a monetary myth, *Federal Reserve Bank of Minneapolis Quarterly Review*, pp. 3-18.

Pakko, M.R., 2000, The cyclical relationship between output and prices: An Analysis in the frequency domain, *Journal of Money, Credit and Banking* 32, pp. 381-399.

Priestley, M.B., 1988, *Non-linear and non-stationary time series analysis*, Academic Press.