

NBER WORKING PAPER SERIES

ANNEXATION OR CONQUEST? THE ECONOMICS OF EMPIRE BUILDING

Herschel I. Grossman  
Juan Mendoza

Working Paper 8109  
<http://www.nber.org/papers/w8109>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
February 2001

The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

© 2001 by Herschel I. Grossman and Juan Mendoza. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Annexation or Conquest? The Economics of Empire Building  
Herschel I. Grossman and Juan Mendoza  
NBER Working Paper No. 8109  
February 2001  
JEL No. D74, F02, N40

**ABSTRACT**

This paper develops an economic theory of empire building. This theory addresses the choice among three strategies that empire builders historically have used. We call these strategies Uncoerced Annexation, Coerced Annexation, and Attempted Conquest. The theory shows how the choice among these strategies depends on such factors as the economic gains from imperial expansion, the relative effectiveness of imperial armies, the costs of projecting imperial military power, and liquidity constraints on financing imperial armies. This theory also addresses the scope of imperial ambitions. The paper uses examples from the history of the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.

Herschel I. Grossman  
Brown University  
Russell Sage Foundation  
and NBER

Juan Mendoza  
State University of New York at Buffalo

This paper develops an economic theory of empire building. This theory addresses both the choice among strategies that can be used to expand an empire as well as the scope of imperial ambitions. The paper uses examples from the history of the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.

To focus the analysis, we consider the following story: The Romans are thinking about expanding their empire by incorporating a country now ruled by Barbarians. Historical accounts suggest that the Romans, like other empire builders, can choose among three different strategies.

In an **Uncoerced Annexation** the Romans compensate the Barbarians sufficiently to induce the Barbarians to agree to the annexation of their country by Rome. This strategy requires that incorporating the country into the Roman empire would yield either economic or military advantages and that the Romans share these gains with the Barbarians. The attraction of this strategy to the Romans is that it avoids the expense of sending Legions either to threaten or to attempt to conquer the Barbarians. (Although Uncoerced Annexation implies subjugation, the formal arrangements of an Uncoerced Annexation can range from direct Roman rule to an alliance with Rome under Roman leadership.)

In a **Coerced Annexation** the Romans induce the Barbarians to agree to the annexation of their country under the threat that the Romans will attack and try to conquer the country. This strategy requires the Romans to send Legions of sufficient strength to the borders of the Barbarian country that the Barbarians decide to capitulate rather than to defy the Romans. The attraction of this strategy to the Romans is that it avoids having to share the gains from annexation with the Barbarians.

In an **Attempted Conquest** the Romans attack the Barbarian country. This strategy is cheaper than the strategy of Coerced Annexation. It requires sending Legions to the Barbarian country, but fewer Legions than would be necessary to induce the Barbarians to capitulate. But, this strategy is risky in that an Attempted Conquest can fail.

Our theory assumes that the Romans will employ one of these strategies only if its expected value to the Romans is positive and is at least as large as the expected value to the Romans of any other strategy. If none of these strategies have a positive expected value, then the Romans will not attempt to incorporate the Barbarian country into their empire.<sup>1</sup>

### 1. Uncoerced Annexation

Let  $\omega$  denote the present value of the expected stream of rents that the Barbarians receive as rulers of the country, and let  $\Omega$  denote the expected gross present value to the Romans of annexing the country, where  $\Omega = k\omega$ ,  $k \geq 0$ . A value of  $k$  larger than one implies that annexing the country would yield expected benefits to the Romans that are larger than the expected losses to the Barbarians. Thus, if  $k$  is larger than one, then the Romans can profitably compensate the Barbarians for agreeing to annexation by paying the Barbarians an amount whose present value is not smaller than  $\omega$ .<sup>2</sup>

The factor  $k$  could be larger than one for a variety of reasons. One set of possibilities is directly economic. For example, Roman rule could bring more efficient public administration. It is also possible that as a part of the Roman empire or as an ally of Rome the country would have expanded trading opportunities.

Another set of possibilities is military and, hence, indirectly economic. For example, by annexing the Barbarian country the Romans could gain either geopolitical advantages or access to resources that would make it easier to incorporate other desirable countries into their empire.

---

<sup>1</sup>A related paper by Ronald Findlay (1996) develops an economic analysis of the size of empires. Findlay's model abstracts from the distinction between annexation and conquest, which is the main concern of the present paper. Also, Findlay treats territorial expansion as a continuous choice variable, rather than modeling the incorporation of discrete countries together with their populations into an empire, as in the present paper.

<sup>2</sup>This compensation can take the form of a lump sum paid at the time of annexation or a promise of future payments. This timing is likely to depend in part of the credibility of Roman and Barbarian commitments. The present theory abstracts from the issue of credibility.

Let  $R_A$  denote the expected value to the Romans of a strategy of Uncoerced Annexation. Allowing for the possibility that  $k$  is larger than one, we have

$$(1) \quad R_A = \max\{R_a, 0\}, \quad \text{where } R_a \leq (k-1)\omega.$$

In equation (1),  $R_a$  equals the difference between the expected gross present value to the Romans of annexing the country and the present value of the compensation that the Romans pay to the former Barbarian rulers.

## 2. Coerced Annexation or Attempted Conquest

Let  $Q$  denote the probability that, if the Romans send Legions to the Barbarian country, then they will successfully incorporate the Barbarian country into the Roman empire. Let  $g$  denote the fighting strength of the Legions that the Romans send to the country, and let  $h$  denote the amount of resources that the Barbarians allocate to defending their country from a Roman threat. Assume that

$$(2) \quad Q = \begin{cases} \frac{1}{1 + h/\theta g} & \text{for } g > 0, \text{ where } \theta > 0, \\ 0 & \text{for } g = 0. \end{cases}$$

Equation (2) says that  $Q$  is positive if and only if  $g$  is positive. Also, if  $g$  is positive, then  $Q$  is an increasing function of  $g$  and a decreasing function of  $h$ . If  $g$  is positive and if  $h$  equals zero, then  $Q$  equals one. The parameter  $\theta$  quantifies the effectiveness of the Roman Legions against the Barbarian defenses.<sup>3</sup>

To analyze Roman strategy, we must analyze first how the Barbarians respond to a Roman threat. Let  $B_C$  denote the expected value to the Barbarians of resisting a Roman attempt to conquer their country. If the Romans send Legions to their country, then the

---

<sup>3</sup>Equation (2) abstracts from the choice of military tactics, such frontal assault or siege warfare. In this respect equation (2) is a generic black box, like a standard black-box production function.

Barbarians choose  $h$  to maximize  $B_C$ , where

$$(3) \quad B_C = (1 - Q) \omega - h.$$

Assume that in choosing  $h$  the Barbarians take  $g$  as given. This assumption implies that a Roman decision to allocate resources to sending Legions to the country is irreversible. In effect, this model assumes that the Romans are a Stackelberg leader.

From equations (2) and (3) the solution to the Barbarians' choice problem is

$$(4) \quad h = \max\{\sqrt{\theta g \omega} - \theta g, 0\}.$$

Equation (4) implies that the Barbarians choose a positive value of  $h$  if and only if  $g$  is positive but smaller than  $\omega/\theta$ . Thus, a strategy of Coerced Annexation requires the Romans to allocate sufficient resources to sending Legions to the Barbarian country such that  $g$  is at least as large as  $\omega/\theta$ , which is the minimum positive value of  $g$  that would induce the Barbarians to capitulate by choosing  $h$  equal to zero. Alternatively, if the Romans use a strategy of Attempted Conquest, then they choose a positive, but smaller, value of  $g$ , and the Barbarians resist the Romans by choosing a positive value of  $h$ .

Let  $R_C$  denote the expected value to the Romans of sending Legions to the Barbarian country. Assume for now that the cost of sending Legions of fighting strength  $g$  is  $g$ . Thus, if the Romans send Legions to the country, then they choose  $g$  to maximize  $R_C$ , where

$$(5) \quad R_C = Q \Omega - g.$$

In choosing  $g$  the Romans take into account both the direct effect of  $g$  on  $Q$ , as given by equation (2), as well as the indirect effect on  $g$  on  $Q$  through the effect of  $g$  on  $h$ , as given by equation (4).

From equations (2), (4), and (5), the solution to the Romans' choice problem is

$$(6) \quad g = \min\{k^2 \theta \omega / 4, \omega / \theta\}.$$

Because  $k^2\theta\omega/4$  is smaller than  $\omega/\theta$  only if the product of  $\theta$  and  $k$  is smaller than two, equation (6) implies that, if the Romans send Legions to the country, then they choose  $g$  large enough to induce the Barbarians to capitulate if and only if the product of  $\theta$  and  $k$  is at least as large as two. If  $\theta k$  is smaller than two, then the Romans choose a smaller value of  $g$  the smaller is  $k$ ,  $\theta$ , or  $\omega$ .

Substituting equations (4) and (6) into equation (2), we find that, if the Romans send Legions to the country, then in equilibrium the probability that they will successfully incorporate the country into the Roman empire is

$$(7) \quad Q = \min\{\theta k/2, 1\}.$$

Equation (7) implies that, if and only if the product of  $\theta$  and  $k$  is as large as two, in which case the Barbarians capitulate, then  $Q$  equals one. But, if  $\theta k$  is smaller than two, in which case the Barbarians resist, then  $Q$  is smaller the smaller is  $\theta k$ . Equation (7) also implies that, although  $Q$  depends on  $k$ , which is the ratio of  $\Omega$  to  $\omega$ ,  $Q$  does not depend on the absolute value of  $\omega$ , because in equilibrium both  $g$  and  $h$  are proportionate to  $\omega$ .

Substituting equations (6) and (7) into equation (5), the expected value to the Romans of sending Legions to the country is

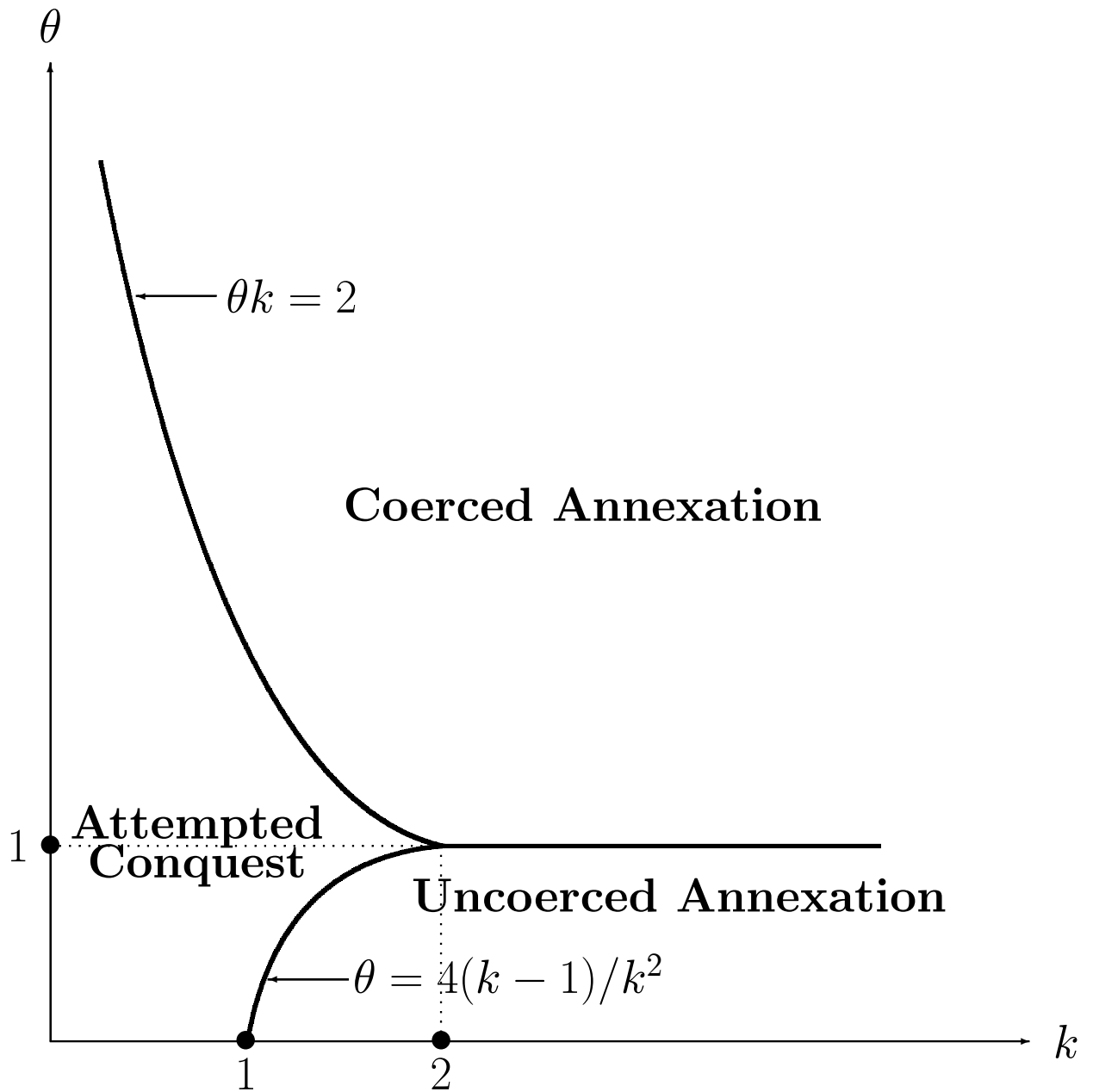
$$(8) \quad R_C = \begin{cases} \theta k^2\omega/4 & \text{for } \theta k < 2 \\ (\theta k - 1)\omega/\theta & \text{for } \theta k \geq 2. \end{cases}$$

Equation (8) says that  $R_C$  is larger the larger are  $\theta$ ,  $k$  and  $\omega$ .

### 3. Roman Strategy

From equations (1) and (8) we can infer the conditions under which the Romans will use each of the three possible strategies. Figure 1 illustrates these results.

- **Uncoerced Annexation:** The Romans compensate the Barbarians sufficiently to induce the Barbarians to agree to the annexation of their country by the Roman empire



**Figure 1: Annexation or Conquest?**

$k$ : gain from imperial expansion

$\theta$ : relative effectiveness of imperial armies



only if  $0 < R_A \geq R_C$ . Assuming, for simplicity, that  $R_a$  equals  $(k - 1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(9) \quad \begin{array}{l} \text{either } 1 < k < 2 \quad \text{and} \quad \theta \leq 4(k - 1)/k^2 \\ \text{or} \quad k \geq 2 \quad \text{and} \quad \theta \leq 1. \end{array}$$

[See Appendix A for the derivation of condition (9).] As we see in Figure 1, condition (9) obtains only if  $k$  is larger than one and  $\theta$  is smaller than one. Thus, the Romans use a strategy of Uncoerced Annexation only if annexing the country would yield large economic or military benefits to the Romans and only if the Barbarian defenses are more effective, sestertium for sestertium, than the Roman Legions.<sup>4</sup>

- **Coerced Annexation:** The Romans send Legions of sufficient fighting strength to induce the Barbarians to capitulate only if  $\theta k \geq 2$  and  $0 < R_C \geq R_A$ . Assuming again that  $R_a$  equals  $(k - 1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(10) \quad \theta \geq \max\{1, 2/k\}$$

As we see in Figure 1, condition (10) obtains if  $k$  is larger than two and  $\theta$  is larger than one. If  $k$  is not larger than two,  $\theta$  must be larger the smaller is  $k$ . Thus, if their Legions are more effective, sestertium for sestertium, than the Barbarian defenses and annexing the country would not be seriously disadvantageous, either economically or militarily, then the Romans use a strategy of Coerced Annexation.

---

<sup>4</sup>The assumption that  $R_a$  equals  $(k - 1)\omega$  implies that the former Barbarian rulers of the country receive an amount whose present value is  $\omega$  and that the Romans get all of the net gain from incorporating the country into the Roman Empire. If the Romans do not get all of the net gain, then, to satisfy the condition  $0 < R_A \geq R_C$ , either  $k$  must be larger or  $\theta$  must be smaller. An interesting extension would be to model the bargaining game between the Romans and the Barbarians.

- **Attempted Conquest:** The Romans attack and try to conquer the country only if  $\theta k \leq 2$  and  $0 < R_C \geq R_A$ . Assuming again that  $R_a$  equals  $(k - 1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(11) \quad \max\{4(k - 1)/k^2, 0\} < \theta \leq 2/k.$$

As we see in Figure 1, condition (11) obtains only if neither  $\theta$  nor  $k$  are too large. Thus, the Romans attack and try to conquer a country only if their Legions are not too effective, sestertium for sestertium, than the Barbarian defenses and annexing the country would not yield large economic or military advantages. To understand this result, observe that equation (6) implies that, with a small values of  $\theta$  and  $k$ , although the Romans attack and try to conquer the country, they do not allocate as large an amount of resources to this effort as an alternative strategy of Coerced Annexation would require.

Conditions (9), (10), and (11) show how the Romans' choice of strategy depends on the parameters,  $\theta$  and  $k$ . These conditions also reveal that the Romans' choice of strategy does not depend on the absolute value of  $\omega$ , again because in equilibrium both  $g$  and  $h$  are proportionate to  $\omega$ .

#### 4. The Scope of Imperial Ambitions

In the preceding section, either condition (9), or condition (10), or condition (11) was satisfied. Thus, this analysis implies that the Romans want to incorporate every Barbarian country into the Roman empire. This implication is counter factual.

In order to bound the scope of imperial ambitions, assume that the cost to the Romans of sending Legions of fighting strength  $g$  to the Barbarian country is not  $g$ , but  $c$ , where

$$(12) \quad c = \begin{cases} 0 & \text{for } g = 0 \\ g + f & \text{for } g > 0. \end{cases}$$

The parameter  $f$  in equation (12) represents a fixed cost of sending Legions. Equation (12) says that, if  $g$  is positive, then  $c$  equals the sum of  $g$  and  $f$ .

The preceding analysis implicitly assumed that  $f$  equals zero. Because the value of  $f$  does not affect the marginal cost of sending Legions to the Barbarian country, equations (6) and (7) obtain whether or not  $f$  is positive. But, if  $f$  is positive, then equation (8) generalizes to

$$(13) \quad R_C = \max\{R_f, 0\},$$

$$\text{where } R_f = \begin{cases} \theta k^2 \omega / 4 - f & \text{for } \theta k < 2 \\ (\theta k - 1) \omega / \theta - f & \text{for } \theta k \geq 2. \end{cases}$$

Equation (13) implies that  $R_C$  is not positive if and only if

$$(14) \quad \begin{array}{l} \text{either } \theta k < 2 \text{ and } f/\omega \geq \theta k^2/4 \\ \text{or } \theta k \geq 2 \text{ and } f/\omega \geq (\theta k - 1)/\theta. \end{array}$$

Condition (14) is satisfied for a combination of a sufficiently large value of the ratio,  $f/\omega$ , and sufficiently small values of the parameters,  $k$  and  $\theta$ . Thus, condition (14) implies that, if the fixed cost of sending Legions is large relative to the wealth of the Barbarian country and the Barbarian defenses are sufficiently effective against Roman Legions, then the Romans will expect neither Coerced Annexation nor Attempted Conquest to be profitable.

Recall that equation (1) implies that  $R_A$  is not positive if  $k$  is not larger than one. Thus, if both  $k$  is not larger than one and condition (14) obtains, then neither  $R_A$  nor  $R_C$  is positive. Countries for which both  $k$  is not larger than one and condition (14) obtains are beyond the scope of Roman imperial ambitions. The theory implies that the Romans are not interested in incorporating such countries into the Roman empire either through Uncoerced Annexation, Coerced Annexation, or Attempted Conquest.

Even if  $R_A$  and  $R_C$  are positive, a positive value of  $f$  makes  $R_C$  smaller and, hence, would imply that  $R_A$  is larger than  $R_C$  for a larger range of values of  $k$  and  $\theta$ . In other

words, a fixed cost of sending Legions makes Coerced Annexation and Attempted Conquest less profitable and makes Uncoerced Annexation relatively more attractive. [See Appendix B for a derivation of the conditions for  $0 < R_A \geq R_C$  with  $f > 0$ .]

## 5. Liquidity Constraints

The preceding discussion has assumed implicitly that the Romans can mobilize sufficient resources to send to the Barbarian country Legions with a fighting strength that equals the value of  $g$  given by equation (6). This assumption abstracts from the possibility that the Romans face a binding liquidity constraint on their ability to mobilize resources.

To relax this abstraction, let  $\bar{g}$  denote the maximum amount of resources that the Romans can mobilize to send Legions to the Barbarian country. If condition (10) obtains, but  $\bar{g}$  is smaller than  $\omega/\theta$ , which is the value of  $g$  that a strategy of Coerced Annexation requires, then  $\bar{g}$  is a binding liquidity constraint. This constraint would prevent the Romans from using a strategy of Coerced Annexation that they otherwise would prefer.<sup>5</sup>

If a binding liquidity constraint precludes a strategy of Coerced Annexation, then the Romans instead would use either a strategy of Attempted Conquest or a strategy of Uncoerced Annexation. The Romans would use a strategy of Attempted Conquest, instead of a strategy of Coerced Annexation, only if  $\bar{g}$  is large enough, and  $k$  is small enough, that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is equal to or larger than  $R_A$ . Alternatively, the Romans would use a strategy of Uncoerced Annexation, instead of a strategy of Coerced Annexation, only if  $\bar{g}$  is small enough, and  $k$  is large enough, that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is equal to or smaller than  $R_A$ .<sup>6</sup>

---

<sup>5</sup>A binding liquidity constraint also could cause the Romans to use a strategy of Uncoerced Annexation instead of a strategy of Attempted Conquest that they otherwise would prefer. This situation could occur only if condition (11) obtains, with  $k$  larger than one, but  $\bar{g}$  is much smaller than  $k^2\omega\theta/4$ , which is the value of  $g$  that maximizes the expected value of a strategy of Attempted Conquest.

<sup>6</sup>Substituting equations (2) and (4) into equation (5), and replacing  $g$  with  $\bar{g}$ , we find that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is  $k\sqrt{\theta\bar{g}\omega} - \bar{g}$ . From equation (1),  $R_A$  equals  $\max\{R_a, 0\}$ , where

## 6. The Theory Applied to Roman History

From the fifth century BCE until the early part of the third century BCE, the incipient Roman empire was often engaged in bloody conflicts with its neighbors, both Latin and Etruscan. The fact that the Romans won most of these wars suggests that in confronting the armies of the Latin League and Etruria the Roman Legions enjoyed a large value of  $\theta$ . Also, it seems likely that the value of  $k$  associated with incorporating the Latin and Etruscan states into the Roman empire was at least equal to one. Given that  $\theta$  was large and that  $k$  was not too small, the fact that early on the Romans did not use a strategy of Coerced Annexation suggests that the incipient Roman empire faced binding liquidity constraints on its ability to finance its Legions. This implication is also consistent with the fact that in those days the Legions were composed mainly of conscripts and other part-time soldiers, whose mobilization was ad hoc.

During the third century BCE the Romans finally incorporated most of the states in the Latin League and Etruria into the Roman empire, and during the second century BCE the Romans added most of the Greek states to their empire. The strength of the Roman Legions was critical for imperial expansion. But, as many of the previously independent states of Italy and Greece capitulated without the Roman Legions attacking, it appears that in the third and second centuries BCE, in contrast to earlier times, the Romans used mainly the strategy of Coerced Annexation.<sup>7</sup> As Edward Luttwak (1976, page 2) tells us,

Forces visibly ready to fight but held back from battle could serve...to control lands and peoples by intimidation —ideally to the point where... effective domination could be achieved without any use of force at all... indeed [the Romans]

---

$R_a$  is equal to or smaller than  $(k - 1)\omega$ .

<sup>7</sup>The Epirians, whom Pyrrhus led to Pyrrhic victory, and the Macedonians were exceptions. The Romans eventually conquered both of these states, but it seems that in confronting the Epirians and the Macedonians the Romans did not have a large enough value of  $\theta$  to use the strategy of Coerced Annexation.

conquered the entire Hellenistic world with few battles and much coercive diplomacy.

This change in the Romans' predominant strategy suggests that by the third century BCE the Romans no longer faced binding liquidity constraints. This implication is also consistent with the fact that by the third century BCE the Legions comprised a standing army of professional soldiers, whom the Roman state fed, clothed, and equipped.<sup>8</sup>

It is noteworthy that at about the same time that the Roman empire was consolidating its control of Italy and expanding into Greece the Romans and the Carthaginians were alternately attempting to conquer parts of each other's empires. Historical accounts of the Punic Wars suggest that neither Rome nor Carthage enjoyed a large enough value of  $\theta$  to warrant a strategy of Coerced Annexation with respect to the other. Also, it seems unlikely that incorporation of any part of either one of the empires into the other one would have involved a large enough value of  $k$  to warrant a strategy of Uncoerced Annexation. Hence, as the theory implies, both Rome and Carthage chose the strategy of Attempted Conquest.

In later years, as the Roman empire expanded into central and northern Europe, Asia Minor, and the Middle East, the Romans' predominant strategy changed again. Now they used mainly the strategy of Uncoerced Annexation. Historians refer to the small countries that the Romans peacefully incorporated into their empire as "client states". Examples include Judea, the countries of Germanic peoples like the Chauci, Cherusi, Quadi, and Marcomanni, and various countries in Asia Minor. Our theory also suggests an explanation for this change in strategy.

---

<sup>8</sup>Luttwak proposes the alternative hypothesis that the increased use of the strategy of Coerced Annexation resulted from a learning process. He writes (page 2),

Having learned in the earlier republican period how to defeat neighbors in battle by sheer tactical strength, having later mastered the strategic complexities of large-scale warfare in fighting the Carthaginians, the Romans finally learned that the most desirable use of military power was not military at all, but political...

In attempting to push out the northern and eastern frontiers of the empire the Romans encountered both relatively small values of  $\theta$  and relatively large values of  $k$ . On these frontiers the Roman Legions were not highly effective against Barbarian defenses, as evidenced by the mixed success of the Legions in their campaigns against the Germanic tribes, the Dacians, and Parthians. But, annexing the client states yielded large military advantages to the Romans. Luttwak (pages 19-20, 26, 27) describes these advantages,

Since clients would take care to prevent attacks against provincial territory, their obedience lessened the need to provide local security at the periphery of the empire against low-intensity threats...Against high-intensity threats, such as invasions on a provincial or even a regional scale, client states and client tribes could contribute both their own interposed forces and their capacity to absorb the threat — in other words, they could provide geographic depth...Another obvious contribution of client states and client tribes to Roman security was the supply of local forces to augment Roman armies on campaign. Naturally, these troops would fall into the Roman category of *auxilia*, i.e., cavalry and light infantry, rather than legionary forces of heavy infantry...Auxiliary troops contributed by clients had played an important role in the campaigns of the republic, not least because they could provide military specialties missing from the regular Roman arsenal, such as archers, and especially mounted archers...The complementarity between *auxilia* and legionary forces was an important feature of the Roman military establishment; moreover, the forces maintained by the client states were substantial.

Also, many of these client states had relatively primitive economies, and incorporating these countries into the Roman empire resulted in large economic gains. With a small value of  $\theta$  and a large value of  $k$ , our theory implies that the Romans would chose a strategy of Uncoerced Annexation.

How did the Romans compensate the rulers of their client states to induce them to agree to being annexed into the Roman empire? Luttwak (pages 32, 37) tells us,

Loyal and efficient client rulers were rewarded by personal honors, ordinarily receiving Roman citizenship (which Augustus's highly restrictive citizenship policy made an important privilege)...More tangible rewards were also given, primarily territorial. That model client, Polemo I, king of Pontus, received Lesser Armenia from Anthony, and when Augustus detached that territory from Pontus, Polemo received instead the important (but, as it turned out, ungovernable) Bosphoran state. Similarly, when Herod - a very efficient client ruler indeed - was still in Augustus's good graces, he was granted in 24-23 B.C. part of the plateau country of Ituraea (Golan-Hauran), at the expense of another client, Zenodorus, who had failed to control the nomadic raiding of his subjects...The major active instrument of client management among the primitive peoples of continental Europe was a systematic policy of subsidization...By channeling money and favors through chosen client chiefs, the Romans helped the latter gain power over their subjects, while the Romans gained power over them...Speaking of the once formidable Marcomanni and Quadi, Tacitus describes both as ruled by client rulers maintained in power — and controlled — by a combination of occasional armed assistance and financial support.

Luttwak (page 37) also suggests that to enforce deals with client states after they were annexed the Romans supplemented Uncoerced Annexation with elements of Coerced Annexation.

The control mechanism was complex. It was necessary to manipulate the tribes through their chiefs, while controlling the chiefs by means of personal threats and personal inducements; always there was the latent threat of force against the tribe as a whole.



Dacia, a relatively rich country, was an exception to the use of a strategy of Uncoerced Annexation. But, the case of Dacia also accords with our theory. According to Peter Wilcox and G. A. Embleton (1987, page 25),

Dacian culture at this time was far in advance of that of their fellow European barbarians. It was, in all recognizable aspects, an embryo civilization...Trade was well organized and encouraged; silver and gold work, pottery, iron implements and weapons, of extremely high quality, were produced for home consumption and export to the sophisticated Roman world in the south.

Because Dacia already was economically advanced, the economic gains associated with adding Dacia to the Roman empire apparently were not large enough to allow the Romans profitably to compensate the Dacians for agreeing to annexation. Instead, just as our theory implies, with neither  $\theta$  nor  $k$  being large the Romans attempted to conquer Dacia, and eventually were successful. Note that in the case of Dacia  $k$  was not large because Dacia was already a rich country.<sup>9</sup>

Our analysis of the scope of imperial ambitions also seems to be consistent with Roman history. Luttwak (page 96) attributes the Roman decision not to incorporate Scotland into the Roman empire to the Scotland's being "inherently difficult to settle, urbanize, and Romanize". In addition, because of its location, annexation of Scotland was not attractive for geopolitical reasons. In the terms of our theory, Scotland apparently differed from the client states of central and northern Europe, Asia Minor, and the Middle East in that, in addition to  $\theta$  being too small to make a strategy of either Coerced Annexation or Attempted Conquest attractive,  $k$  was too small to warrant a strategy of Uncoerced Annexation. Note that in the case of Scotland  $k$  was small because Scotland was hopelessly poor.

---

<sup>9</sup>Another exception was Judea, where a strategy of Uncoerced Annexation, supplemented by elements of Coerced Annexation, worked well until the Jewish revolt (66-70CE) forced the Romans to shift to a strategy of Attempted Conquest, which, as in Dacia, was eventually successful.

## 7. The Theory Applied to the History of The Mongol and Ottoman Empires

This analysis does not only offer insights into the building of the Roman empire. In his discussion of the expansion of the Mongol empire, E.L. Jones (1998, pages 108-115) suggests that the Mongols used mainly a strategy of Attempted Conquest, which was usually successful. The account provided by Jones also suggests that for the Mongols, with their superior technology of warfare and their effective terror tactics,  $\theta$  was much larger than one. For example, according to Desmond Martin (1971) and Michel Hoang (1990), between 1211 and 1215 an army of little more than 110,000 Mongols defeated an army of 500,000 Chinese and conquered the Chin Empire of Northern China. According to Martin and Leo de Hartog (1989) in 1223 a Mongol army of 20,000 decisively defeated a Russian army of 80,000 in the Battle of the River Khalka. As Martin writes (pages 11-12),

Whenever it is possible to obtain reliable information on the strength of Mongol forces, one finds that often they were heavily outnumbered by their enemies. We shall see that in 1211 Chingis Khan marched against the Chin with little more than 110,000 men — decidedly less than a quarter of the forces of his opponent. During 1219 he mobilized perhaps 150,000 effectives for the war against the Khwarazm Shah. On that occasion, while the army opposed to him was neither quite so large — approximately 400,000 — or as well organized as that of the Chin, he had to march west nearly one thousand miles from his last home base before reaching the enemy's border. . . . In the troops of Chingis Khan numerical inferiority, both on the battlefield and on campaign, was common.

Also, there seems to be no reason to presume that, even if it was not larger than one, the value of  $k$  associated with expansion of the Mongol empire typically was much smaller than one. Thus, abstracting from liquidity constraints, it would seem puzzling that the Mongols did not mainly use a strategy of Coerced Annexation. But, if for the Mongols  $\bar{g}$  was a binding constraint, then this puzzle is solved.

Indeed there are other good reasons to think that the Mongols were subject to liquidity constraints. Most importantly, the Mongols started with a small population relative to their imperial ambitions. According to Martin and Hoang, the Mongol population at the beginning of the 13th century was about one million people, whereas by 1260 the Mongol empire had a population of about 100 million people. In addition, because the Mongols were originally nomads, they probably did not begin their imperial expansion with large amounts of accumulated capital or other resources. Also, at the beginning their empire building it is likely that the difficulty of collecting taxes from a nomadic population constrained their ability to mobilize resources.

Turning to the Ottoman empire, our theory suggests an explanation for the clear difference in the strategies that the Ottomans used in Anatolia and in the Balkans. The Ottomans seemed to have used a strategy of Uncoerced Annexation to incorporate Anatolia into their empire. Metin Kunt (1995, page 10) writes, “Ottoman historical tradition describes the incorporation of Muslim lands as a peaceful process, in some cases as voluntary submission.” In contrast, to incorporate the Balkans into their empire the Ottomans used a combination of Coerced Annexation, especially in annexing Bosnia in 1453 and Herzegovina in 1454, and Attempted Conquest in other countries, notably Serbia.

Kunt suggests that the Ottoman armies were more effective in the Balkans than in Anatolia because the Black Death affected the Balkans much more than Anatolia. Kunt writes (page 11),

Consideration should also be given to the effects of the mid-fourteenth century Black Death on the demography of Anatolia and the Balkan peninsula. In later Ottoman sources we find no mention of the plague which devastated so much of the Mediterranean and Europe, though event-book ‘calendars’ have scattered references for various years. Constantinople and other Byzantine territories suffered, as did part of Islamic West Asia such as Syria and Egypt, but not, it seems,

the *gazi* Türkmén emirates. It may be that the Turkic and Mongolian population of Anatolia and parts of Iran were relatively immune to the plague strain, which seems to have had its origin in the great Eurasian steppe, carried by Genoese ships from Crimea through Constantinople to the Mediterranean and beyond to northern Europe. If it is true the plague had less of an impact on the Oğuz of Anatolia than on the Greeks and other peoples of the Balkans, this would have been another factor in the swift success of Murad Bey's troops in the Balkans.

As our theory suggests, enjoying a larger  $\theta$  in the Balkans, the Ottomans put aside the strategy of Uncoerced Annexation that they used in Anatolia.

## 8. The Theory Applied to Nazi Germany

Turning to modern times, our theory also can help us to understand the strategies that Nazi Germany used to build its empire. We focus on German territorial ambitions in central and eastern Europe.<sup>10</sup> On our reading of history, the Germans used a strategy of Coerced Annexation in taking over Austria and then the Sudetenland and the rest of Czechoslovakia, a successful strategy of Attempted Conquest of Poland, a strategy of Uncoerced Annexation in inducing Hungary and Romania to join the Axis alliance, and finally an unsuccessful strategy of Attempted Conquest of the Soviet Union. Our analysis implies that these differences in Nazi strategy resulted either from differences in  $\theta$  or in  $k$  or from liquidity constraints.

Prior to the German annexation of Austria in March 1938, the existence of a substantial pro-Nazi faction within Austria undermined the ability of the Austrian leaders to resist German aggression without the support of Austria's ostensible foreign friends. As William

---

<sup>10</sup>Whether the Nazis had ultimate territorial ambitions elsewhere is not clear. In September 1939 Great Britain and France, being unwilling to acquiesce in further German aggression in the East, declared war on Germany. A.J.P. Taylor (1961, page 70) tells us, "Against all expectations, Hitler found himself at war with the Western Powers before he had conquered the East. Nevertheless, Eastern expansion was the primary purpose of his policy, if not the only one." But, other historians suggest that the Nazis intended all along to incorporate western Europe into their empire. See, for example, Gerhard Weinberg (1994, page 107).

Shirer (1960, page 323) reminds us,

Throughout 1937, the Austrian Nazis, financed and egged on by Berlin, had stepped up their campaign of terror. Bombings took place nearly every day in some part of the country, and in the mountain provinces massive and often violent Nazi demonstrations weakened the governments position.

But, Great Britain, France, and Italy failed to support Austrian independence and to oppose the *Anschluss*. Shirer (1960, page 327), drawing on Schuschnigg's book, *Austrian Requiem*, and on Schuschnigg's Nuremberg affidavit provides the following paraphrasing of Hitler's boasting to then Austrian chancellor Kurt von Schuschnigg when they met at Berchtesgaden, Hitler's mountain retreat, in February 1938.

Don't think for one moment that anybody on earth is going to thwart my decisions. Italy? I see eye to eye with Mussolini. . . England? England will not move one finger for Austria. . . And France? France could have stopped Germany in the Rhineland and then we would have had to retreat. But now it is too late for France.

Shirer (page 330) also tells us vividly how Schuschnigg was then forced into signing an agreement that effectively ended Austria's independence "under the terrible threat of armed attack". British and French timidity, together with the existence of a substantial pro-Nazi faction within Austria, meant that in confronting Austria the Germans enjoyed a large value of  $\theta$ . Hence, as our theory implies, the Germans used a strategy of Coerced Annexation.

At Munich in September 1938 Great Britain and France, in their efforts to appease Hitler and avoid war, effectively handed the Sudetenland to Germany. Manfred Messerschmidt (1990, page 657) summarizes the British and French reluctance to support the Czechs in the months leading to the Munich pact.

As the British saw, Hitler was in a position to create *faits accomplis* that could only be reversed by a long war. At the beginning of such a war, British aid must

be basically confined to economic pressure. Hence, the British government placed more hopes in an Anglo-French policy of urging Prague to be more conciliatory to its German minority. In other words, London basically thought that an initial German success could not be avoided. At the beginning of May [of 1938] Daladier indicated to Bullit, the American ambassador, what conclusion the French drew from this: they would not be able to fight a war in defence of Czechoslovakia. Their attitude was also affected by fear of German air superiority.

According to Shirer (page 417), reporting what a British official told two Czech representatives at Munich,

If you do not accept, he admonished them, as he prepared to go, you will have to settle your affairs with the Germans absolutely alone. Perhaps the French may tell you this more gently, but you can believe me that they share our views. They are disinterested.

Again, because of British and French timidity, the Germans enjoyed a high value of  $\theta$ , and again, as our analysis implies, the Germans used a strategy of Coerced Annexation.

Following the annexation of the Sudetenland, and after again threatening the Czechs with military action, the Germans occupied the rest of Czechoslovakia in March 1939. Once again, it was clear that Great Britain and France would not intervene in defense of Czechoslovakia. Shirer (page 450) writes, “Neither Great Britain nor France made the slightest move to save it, though at Munich they had solemnly guaranteed Czechoslovakia against aggression.” In his discussion of the British and French “guarantee” of Czech independence, Messerschmidt (page 674) writes,

Chamberlain felt committed to it on moral grounds, but he did not envisage the commitment leading to a confrontation with Germany. The French view was much the same. Bonnet and his political friends wanted an accord with Germany.

Halifax considered that they must avoid getting into a position in which Great Britain and France might be called on to act against Germany and Italy.

Furthermore, after the Munich pact, the rest of Czechoslovakia laid almost defenseless to the German army. Shirer (pages 421-422) writes

The final settlement of November 20, 1938, forced Czechoslovakia to cede to Germany 11,000 square miles of territory in which dwelt 2,800,000 Sudeten Germans and 800,000 Czechs. Within this area lay all the vast Czech fortifications which hitherto had formed the most formidable defense line in Europe, with the possible exception of the Maginot Line in France.

When the Germans confronted Czechoslovakia in March 1939, again they had a large value of  $\theta$ , and again they used a strategy of Coerced Annexation.

Now we come to Poland. Several factors suggest that, in confronting Poland, the Germans did not have a value of  $\theta$  as large as they enjoyed in confronting Austria or Czechoslovakia. First, the probability of an Allied intervention was higher than in the case of Austria or Czechoslovakia. After the *Anschluss* and the annexation of Czechoslovakia, public opinion in Great Britain and France was opposed to any further concessions to Germany. The British and French governments acted accordingly. Shirer (page 454) reports that on March 31st, 1939 Chamberlain said before the Commons:

In the event of any action which clearly threatened Polish independence and which the Polish Government accordingly considered it vital to resist with their national forces, His Majesty's Government would feel themselves bound at once to lend the Polish Government all support in their power. They have given the Polish Government an assurance to this effect. I may add that the French Government have authorized me to make it plain that they stand in the same position in this matter.

Second, the Poles were firmer than the Austrians or the Czechs in resisting German threats. As the Germans pressed claims on the port of Danzig and the Polish corridor, Shirer (page 464) writes,

Warsaw was not so easily intimidated as Vienna or Prague. The next day, March 28, Beck sent for the German ambassador and told him, in answer to Ribbentrop's declaration that a Polish coup against Danzig would signify a *casus belli*, that he in turn was forced to state that any attempt by Germany or the Nazi Danzig Senate to alter the status of the Free City would be regarded by Poland as a *casus belli*.

With  $\theta$  in the case of Poland not being large enough to warrant a strategy of Coerced Annexation, and with  $k$  apparently not being large enough to warrant a strategy of Uncoerced Annexation, the Germans, as our theory predicts, used a strategy of Attempted Conquest.

In contrast to Poland, both Hungary and Romania provide examples of Uncoerced Annexation. The distinguishing feature of Hungary and Romania seems to be that  $k$  was large because annexation, which took the form of Hungary and Romania joining the Axis alliance under German leadership, offered large military advantages to the Germans. These advantages were both geopolitical and material. The Germans needed manpower in order to confront the Soviet Union, and the military forces of both Hungary and Romania would fight, more or less effectively, along side of the Wehrmacht. In addition, the Germans coveted the Romanian oil fields.<sup>11</sup>

To induce the Hungarians and Romanians to join the Axis alliance, the Germans used policies similar to those that the Romans had used in annexing their client states. As a result of the First World War Hungary had lost territory and population, whereas Romania

---

<sup>11</sup>From a military standpoint there was little difference between direct German rule, as in the case of Austria, and joining the Axis alliance under German leadership. Also, when it suited other Nazi objectives, especially pursuit of their genocidal policies, the Germans imposed direct rule on Hungary and Romania.



had gained territory and population. In the interwar period Hungarian policy focused on recovering its lost territory and population, whereas Romania was concerned with maintaining its gains. The Germans offered the Hungarians, as well as the Romanians, a share of the spoils of war. Jürgen Förster (1998, pages 412, 416, and 417) tells us,

After the outbreak of the European war Hungarian policy, while maintaining its independence and avoiding an open rupture with the Western powers, was aimed at achieving its revisionist objectives through close alignment with the Axis...[Later] with her Third Army (approximately 146,000 men) Hungary participated in the military and political smashing of Yugoslavia...Hungary reacquired Bácska, the Mur territory, and the Baranya triangle...In consequence, Hungary had nearly doubled her territory, compared with her 1920 frontiers, both in area and in population...

The Germans also offered the Romanians security against the territorial claims of the Soviet Union, Hungary, and Bulgaria. In describing Romanian policy after the Soviet annexation of Bessarabia, Hertza, and northern Bukowina in 1940, Förster writes (page 393)

As no help was to be expected from Great Britain, [King] Carol was feverishly trying to gain German support against the Hungarian and Bulgarian revisionist claims...He saw his country's only salvation in a very close alignment with Germany, with which he was willing to co-operate in all fields...On 1 July 1940 Romania renounced the now worthless Anglo-French guarantee of her frontiers and on 11 July left the League of Nations. Carol, moreover, formed a government of respected pro-German ministers...

Then, on June 22nd, 1941, Germany attacked the Soviet Union, launching, with the active support of their Hungarian and Romanian allies, "Operation Barbarossa". In confronting the Soviet Union the Germans surely enjoyed a high value of  $\theta$ . They had amply

demonstrated their military might in easily conquering Poland, France, and the Low Countries. Undoubtedly, the Wehrmacht was qualitatively superior to the Soviet army in terms of both weapons and leadership.

But, the Soviet Union had the ability to mobilize massive armed forces. Moreover, the Nazis were not yet in a position to demand large sacrifices for the war effort. Rolf-Dieter Müller (1998, p. 187-188) writes

Once the euphoria of the early summer of 1940 had evaporated, it became obvious that Germany, while dominating large parts of the European continent, had not in fact achieved any abundance of material assets as a result. The performance of the German war economy was still considerably below the pre-war level... Germany was unable to draw on any additional manpower reserves. Repeated demands from military quarters that greater use should be made of female labour were rejected by the political leadership. Any more intensive exploitation of the manpower potential was prevented not only by ideological and administrative obstacles, but also by a lack of enthusiasm for the war among broad circles of the population.

The high command of the German army calculated that “while the enemy would have approximately 155 divisions, German strength would be about the same”. (Shirer, page 822).

Thus, even if  $\theta$  was large enough to warrant a strategy of Coerced Annexation, liquidity constraints apparently prevented the Germans from using this strategy. Accordingly, as our theory suggests, the Nazi Empire chose a strategy of Attempted Conquest of the Soviet Union. From an ex-ante perspective this choice presumably was sound, albeit risky. Unfortunately for the Germans, a combination of bad luck and unexpected Russian heroism caused Operation Barbarosa to fail.

## 9. Summary

This paper has developed an economic theory of empire building. This theory addresses the choice among three strategies that empire builders historically have used: Uncoerced Annexation, Coerced Annexation, and Attempted Conquest. Annexation, whether uncoerced or coerced, is peaceful, whereas Attempted Conquest involves the application of force. Our theory views these strategies as merely different ways to achieve the goal of building a profitable empire in the most profitable way. In the famous words of the 19th century military strategist Carl von Clausewitz (1976, page 87), “War is merely the continuation of policy by other means.”

Our theory implies that the key factors in choosing among these strategies are the economic gains from imperial expansion, the relative effectiveness of imperial armies, the costs of projecting imperial military power, and liquidity constraints on financing imperial armies. The theory also addressed the scope of imperial ambitions. The paper used historical examples from the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.

We conjecture that the theory also would help us to understand the strategies used by other empire builders as well as the scope of their imperial ambitions. Applying the theory to the building of overseas empires by European powers from the 16th through the 19th centuries would seem to be an especially interesting extension, but we leave this exercise for another paper.

## References

- Clausewitz, Carl von (1976) *On War*. Edited and translated by Michael Howard and Peter Paret. Princeton University Press, Princeton.
- Deist, Wilhelm (1990) "The Rearmament of The Individual Services, 1933-1939" in Deist, Wilhelm, Messerschmidt, Manfred, Volkmann, Hans-Erich, and Wette, Wolfram *Germany and the Second World War. Volume I. The Build-up of German Aggression*, pages 405-504. Translated by P.S. Falla, Dean S. McMurry, and Ewald Osers. Clarendon Press, Oxford.
- Findlay, Ronald (1996) "Towards a Model of Territorial Expansion and the Limits of Empire" in Garfinkel, Michelle and Skaperdas, Stergios, (eds.) *The Political Economy of Conflict and Appropriation*, pages 41-56, Cambridge University Press, New York.
- Förster, Jürgen (1998) "Germany's Acquisition of Allies in South-East Europe" in Boog, Horst, Förster, Jürgen, Hoffmann, Joachim, Klink, Ernst, Müller, Rolf-Dieter, and Ueberschär, Gerd R. *Germany and the Second World War. Volume IV. The Attack on the Soviet Union*, pages 386-428. Translated by Dean S. McMurry, Ewald Osers, and Louise Willmot. Clarendon Press, Oxford.
- Hartog, Leo de (1989) *Genghis Khan. Conqueror of the World*. I.B. Tauris & Co. Ltd, London.
- Hoang, Michel (1990) *Genghis Khan*. Translated by Ingrid Cranfield. Saqi Books, London.
- Jones, E. L. (1988) *Growth Recurring. Economic Change in World History*, Clarendon Press, Oxford.
- Kunt, Metin (1995) "State and Sultan up to the Age of Suleyman: Frontier Principality to World Empire" in Kunt, Metin and Woodhead, Christine (eds.) *Suleymen the Magnificent and His Age. The Ottoman Empire in the Early Modern World*, pages 3-29. Longman, London.
- Luttwak, Edward (1976) *The Grand Strategy of the Roman Empire from the First Century*

- A.D. to the Third*, Johns Hopkins University Press, Baltimore.
- Martin, H. Desmond (1971) *The Rise of Chingis Khan and His Conquest of North China*. Octagon Books, New York.
- Messerschmidt, Manfred (1990) "The Road to War, 1936-1938" in Deist, Wilhelm, Messerschmidt, Manfred, Volkmann, Hans-Erich and Wette, Wolfram *Germany and the Second World War. Volume I. The Build-up of German Aggression*, pages 615-679. Translated by P.S. Falla, Dean S. McMurry, and Ewald Osers. Clarendon Press, Oxford.
- Müller, Rolf-Dieter (1998) "From Economic Alliance to a War of Colonial Exploitation" in Boog, Horst, Förster, Jürgen, Hoffmann, Joachim, Klink, Ernst, Müller, Rolf-Dieter and Ueberschär, Gerd R. *Germany and the Second World War. Volume IV. The Attack on the Soviet Union*, pages 118-224. Translated by Dean S. McMurry, Ewald Osers, and Louise Willmot. Clarendon Press, Oxford.
- Shirer, William L (1960) *The Rise and Fall of the Third Reich. A History of Nazi Germany*. Simon and Schuster, New York.
- Taylor, A. J. P. (1961) *The Origins of the Second World War*. Atheneum, New York.
- Weinberg, Gerhard L. (1994) *A World at Arms: A Global History of World War II*. Cambridge University Press, Cambridge.
- Wilcox, Peter and Embleton, G. A. (1987) *Rome's Enemies: Germans and Dacians*, Osprey Men-at-Arms Series 129, London.

## Appendix A: Derivation of Condition (9)

$R_A > 0$  obtains only if  $k > 1$ .

Assuming that  $R_a$  equals  $(k-1)\omega$ ,  $R_A \geq R_C$  obtains if and only if

$$\begin{aligned} &\text{either (A) } \theta k < 2 \text{ and } (k-1)\omega \geq \theta k^2 \omega / 4 \\ &\text{or (B) } \theta k \geq 2 \text{ and } (k-1)\omega \geq (\theta k - 1)\omega / \theta. \end{aligned}$$

Observe that  $(k-1)\omega \geq \theta k^2 \omega / 4$  is equivalent to  $\theta k \leq 4(k-1)/k$ .

Also observe that we have  $4(k-1)/k \gtrless 2$  as  $k \gtrless 2$ .

Thus, condition (A) obtains if and only if

$$\begin{aligned} &\text{either (A.1) } k < 2 \text{ and } \theta \leq 4(k-1)/k^2 \\ &\text{or (A.2) } 2 \leq k < 2/\theta. \end{aligned}$$

Next, observe that  $(k-1)\omega \geq (\theta k - 1)\omega / \theta$  is equivalent to  $\theta \leq 1$ .

Thus, condition (B) obtains if and only if

$$(B.1) \quad 1 \geq \theta \geq 2/k.$$

Furthermore, either condition (A.2) or condition (B.1) obtains if and only if

$$(C) \quad k \geq 2 \text{ and } \theta \leq 1.$$

Thus,  $R_A \geq R_C$  obtains if and only if either condition (A.1) or condition (C) obtains.

**Appendix B: Derivation of Conditions for  $0 < R_A \geq R_C$  with  $f > 0$**

$R_A > 0$  obtains only if  $k > 1$ .

Recall that  $R_C \leq 0$  obtains if condition (14) is satisfied.

Thus, assuming that condition (14) is satisfied,  $0 < R_A \geq R_C$  obtains if and only if  $k > 1$ .

Assuming that condition (14) is not satisfied and that  $R_a$  equals  $(k - 1)\omega$ ,  $R_A \geq R_C$  obtains if and only if

$$\begin{aligned} &\text{either (A) } \theta k < 2 \text{ and } (k - 1)\omega \geq \theta k^2 \omega / 4 - f \\ &\text{or (B) } \theta k \geq 2 \text{ and } (k - 1)\omega \geq (\theta k - 1)\omega / \theta - f. \end{aligned}$$

Observe that  $(k - 1)\omega \geq \theta k^2 \omega / 4 - f$  is equivalent to  $\theta k \leq 4(k - 1 + f/\omega)/k$ .

Also observe that we have  $4(k - 1 + f/\omega)/k \gtrless 2$  as  $k \gtrless 2(1 - f/\omega)$ .

Thus, condition (A) obtains if and only if

$$\begin{aligned} &\text{either (A.1) } k < 2(1 - f/\omega) \text{ and } \theta \leq 4(k - 1 + f/\omega)/k^2 \\ &\text{or (A.2) } 2(1 - f/\omega) \leq k < 2/\theta. \end{aligned}$$

Next, observe that  $(k - 1)\omega \geq (\theta k - 1)\omega / \theta - f$  is equivalent to  $\theta \leq 1/(1 - f/\omega)$ .

Thus, condition (B) obtains if and only if

$$(B.1) \quad 1/(1 - f/\omega) \geq \theta \geq 2/k.$$

Furthermore, either condition (A.2) or condition (B.1) obtains if and only if

$$(C) \quad k \geq 2(1 - f/\omega) \text{ and } \theta \leq 1/(1 - f/\omega).$$

Thus, assuming that condition (14) is not satisfied,  $R_A \geq R_C$  obtains if and only if  $k > 1$  and either condition (A.1) or condition (C) obtains. Consequently, assuming that condition (14) is not satisfied, the conditions for  $0 < R_A \geq R_C$  are satisfied if and only if  $k > 1$  and either condition (A.1) or condition (C) obtains.