NBER WORKING PAPER SERIES

NEIGHBORHOOD SCHOOLS, CHOICE, AND THE DISTRIBUTION OF EDUCATIONAL BENEFITS

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Working Paper 7850
http://www.nber.org/papers/w7850

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2000

This paper was originally titled "Public School Choice and Finance Policies, Neighborhood Formation, and the Distribution of Educational Benefits." We thank David Figlio, Jon Hamilton, Eric Hanushek, Steve Slutsky, David E. Wildasin and seminar participants at the University of British Columbia, the University of Montreal, Stanford University, the 1995 NBER School Finance Workshop, the 1996 American Association Meetings, and the 1997 joint PURC-IDEI-CIRANO Conference for comments. This research is supported by the NSF and MacArthur Foundation with our gratitude. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

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NBER Working Paper No. 7850
August 2000
JEL No. H70, H73

ABSTRACT

School districts in the U.S. typically have multiple schools, centralized finance, and student assignment determined by neighborhood of residence. In many states, centralization is extending beyond the district level as states assume an increasing role in the finance of education. At the same time, movement toward increased public school choice, particularly in large urban districts, is growing rapidly. Models that focus on community-level differences in tax and expenditure policy as the driving force in determination of residential choice, school peer groups, and political outcomes are inadequate for analysis of multi-school districts and, hence, for understanding changing education policies. This paper develops a model of neighborhood formation and tax-expenditure policies in neighborhood school systems with centralized finance. Stratification across neighborhoods and their schools is likely to arise in equilibrium. Consequences of intra-district choice with and without frictions are characterized, including effects on the allocation of students across schools, tax and expenditure levels, student achievement, and household welfare.

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1. Introduction.

Public education in the U.S. is increasingly characterized by centralized finance but with school determined by neighborhood of residence. Within jurisdictions households frequently may choose among neighborhoods to reside and send their children to school, but with limited opportunity to choose alternative tax-expenditure schooling packages. In 1998, nine states had school districts that were coterminous with counties or very nearly so, and the ratio of school districts to counties was 2 or below in four more states.[1] Since counties are relatively large geographic entities, county districts are typically multi-school districts. Moreover, these thirteen states do not include a number of states that have the lowest variation among districts in expenditure per student due to relatively centralized finance systems. Examples are Delaware, Colorado, and California, whose ratios of 95 percentile to 5 percentile district expenditures in 1994 were respectively, 1.26, 1.31, and 1.34.[2] While uneven and of limited "success," a central theme in educational finance reform over the last several decades has been expenditure equalization.[3] For example, the same district expenditure ratios in 1972 for the three states just mentioned were respectively 1.81, 1.61, and 1.95.[4] Most students live in districts with multiple schools: In 1993-94, 91 (66) percent of elementary- (high-)school students lived in districts with multiple elementary (high) schools (Common Core of Data 94). Analysis of the provision of education using a Tiebout model with community-level finance of education is inappropriate in such a setting. One purpose of this paper is to examine neighborhood formation and schooling provision when no choice among political jurisdictions is practical, but choice among neighborhoods within the jurisdiction can be exercised. Here any neighborhood sorting of households and variation in school quality that results is driven by peer-group effects.

As school finance is under reform, a school choice movement is gaining momentum.[5] Choice policies and proposals include inter- and intra-district open enrollment, formation of magnet and charter schools, and vouchers for private schools. While it is anyone's guess where the choice movement will go, it seems likely that increased school choice will play a role in the future. Another purpose of this paper is to investigate the effects of public-school choice. We examine the consequences of elimination of territorial (neighborhood) restrictions on school attendance in our model, with and without friction in the exercise of school choice.[6]

We develop a theoretical and complementary computational model to study equilibria in the above policy regimes, with attention to the distribution of educational benefits. School quality is presumed to depend both on per student educational expenditure and on the make up of the student body, the latter measured by mean student ability. Households differ continuously by income and student ability, with normal demand for educational quality. The economy is made up of an exogenous number of neighborhoods, characterized by their housing supplies. Households choose in which neighborhood to reside, where to send their child to school as constrained by policy, and vote over the economy's tax-expenditure package. With neighborhood schooling, students must attend school in their neighborhood of residence. With choice, students may attend school in another neighborhood, which may or may not entail a private (transportation) cost. We also compare these equilibria to that of the traditional Tiebout environment, where neighborhoods provide schooling only to their residents who then also collectively determine their neighborhood's tax-expenditure package.

Key results are quite intuitive. Neighborhood schooling is enough to lead to stratified equilibrium and a school-quality hierarchy if ability and income are positively correlated or if demand for school quality rises with student ability. Thus stratification typically arises even though there is no variation in expenditure per student. Introducing frictionless school choice in such a setting eliminates schooling differences while increasing per student expenditure due to an income effect from reduced housing prices in rich communities. We calculate aggregate welfare losses from frictionless choice. When intra-district choice programs do not induce movement across districts, residents of poor neighborhoods typically experience a welfare gain though this is dependent to some degree on how housing ownership is distributed. Choice with friction can lead the poorest to stay behind in weakened schools. De-centralization of finance in Tiebout equilibrium can be expected to increase further the dispersion of school qualities relative to the neighborhood equilibrium as expenditure rises in richer neighborhoods and declines in poorer neighborhoods. We calculate higher aggregate welfare in Tiebout equilibrium relative to the neighborhood equilibrium, but, again, with uneven distribution.

Our analysis is in the tradition of multicommunity models of local public good provision begun by Tiebout (1956).[7] A number of papers study provision of public education in such a framework including Inman (1978), de Bartolome (1990,1997), Glomm and Ravikumar (1992), Benabou (1993, 1996), Silva and Sonstelie (1995), Durlauf (1996), Fernandez and Rogerson (1996, 1997a, 1997b, 1998), and Nechyba (1999,2000). Central to our model are peer effects which play a role only in de Bartolome (1990), Benabou (1993,1996), Durlauf (1996), and Nechyba (1999,2000) among the latter list. Further narrowing the overlap, only Benabou (1993,1996) has an analogue to neighborhood provision of schooling within a jurisdiction.[8] Similar to one of "our" findings, he shows that complementarities in individual and group characteristics can lead to community stratification without expenditure differences. The models and emphasis differ substantially. Our model has types that differ continuously on two dimensions (income and ability), while his model has two types (arising endogenously in the 1993 paper). We focus strictly on education: Our model is specified to permit quantitative analysis, while his is very general to accommodate other interpretations. We examine school choice while he does not. Other related research is noted at various points below.

2. The Model.

Each household has one child of ability b who will attend public school. Household income is denoted y. The population of households is normalized to one and characterized by joint probability density function f(b,y), with f(b,y) continuous and strictly positive on its support $S \equiv [b_m,b_x] \times [y_m,y_x] \subset R^2_+$. Whether income and ability are correlated in the population is important. To simplify, we will assume either $E[b|y]$ is strictly increasing in y or independent of y implying either positive or zero correlation.

Household utility depends on numeraire and housing consumption, and the educational achievement of the child denoted a. Every household consumes exactly one unit of housing at price denoted p, the simple housing market discussed further below. The student's educational achievement depends on the quality, q, of the school attended, and on the student's ability, denoted b: a = a(q,b). School quality depends on per student educational expenditure in the school, X, and on the mean ability of the school's peer group, denoted θ. The latter peer group effect in education is central to our model and discussed further below. Educational expenditure is financed by a proportional income tax, t. Thus utility is given by $U = U[y(1-t)-p, a(q(X,\theta),b)]$, with all functions increasing, continuous, and differentiable. Demand for educational quality is assumed normal:

$$\frac{U_q}{U_y} \text{ increases with } y. \tag{A-1}$$

In much of the analysis we employ a Cobb-Douglas utility-achievement specification:[9]

$$U = [y(1-t)-p]X^{\alpha}\theta^{\gamma}b^{\beta}; \quad q = X^{\alpha}\theta^{\gamma}; \quad a = qb^{\beta}; \quad \alpha,\gamma,\beta > 0. \tag{1}$$

The economy consists of $N \in \{2,3,....\}$ neighborhoods, with exogenously defined boundaries and characterized by their housing supplies. Each neighborhood has a backward-L housing supply, horizontal at magnitude c until neighborhood land capacity is reached. Interpret c to be the construction cost of a unit of housing, each housing unit requiring one lot of land. For now, assume the economy land capacity is exactly enough to house the population. We offer a more appealing interpretation below, but it can introduce additional equilibria that we wish to avoid initially. Within neighborhood schooling is of homogeneous quality under every policy configuration we study.

Households make a residence choice, choose a school for their child, and participate in a vote over tax rates. The nature of these choices depends on a binary policy parameter that we vary exogenously. The "choice" of school may or may not be restricted by a neighborhood residence requirement. A residence requirement implies the neighborhood school's peer group corresponds to that of the neighborhood's residents.[10] Of course, the absence of such a requirement permits real school choice, and this is what we mean below by a school-choice or open-enrollment policy. We will consider the effects of a school choice policy with and without any frictions (costs of exercising choice).

We focus on the case where school finance is centralized. A single income tax rate is determined by majority preference in the entire economy under centralized finance. There is constant returns to scale in schooling and per student expenditure in the economy is invariant in this case. This conforms to cases with district-level finance and large enough districts that nonschool factors determine district choice or to some cases with statewide finance.[11] For purposes of comparison, we will also examine the more traditional Tiebout public-finance problem with multiple political jurisdictions by assuming neighborhoods correspond to jurisdictions.

Equilibria are determined under the following timing and behavioral assumptions, summarized in Figure 1. In the first stage, (atomistic) households make neighborhood residence choices as price takers and the housing markets clear. Households then choose a school, although a residence requirement renders this a degenerate choice. Voting over taxes takes place last, voters taking the (now committed) residences and schools as given.[12] The model has a single period; no real time elapses between the stages. Households correctly anticipate the ensuing properties of equilibrium when making choices, of course.

RESIDENCE CHOICES & HOUSING MARKETS CLEAR → SCHOOLS CHOSEN → VOTING OVER PROPORTIONAL INCOME TAX

Figure 1

Some elements of the model warrant further discussion. School quality is determined in part by a peer-group externality, which influences neighborhood formation and school choice. Ability-based peer effects in the classroom are confirmed by numerous studies, but this is not without controversy.[13] This aspect of the model can be given a more generic interpretation. Any household variable that positively impacts both the child's and the child's school's performance conforms to the model.[14] Parental input in the education process that entails both helping the child and the child's school is an example. Parental input in the school might come in the form of direct participation in education (e.g., classroom volunteer work) and/or in monitoring and disciplining teachers and administrators.[15] Parental educational attainment may be a good proxy for the latter. Correlation with income will be important as noted above and is thus relevant in the interpretation of b. An extended model might introduce additional inputs into the educational process. We will continue to refer to b as student ability.

That housing prices serve as screens to accessing neighborhoods is not controversial (see Black, 1999 and Barrow, 1999) . We felt it important to consider explicitly housing markets but have avoided complication by adopting such a simple specification. We examine income taxation rather than property taxation so that tax liabilities rise continuously with income. The model can be varied to examine property taxation, with results that are qualitatively the same.

3. School Policy and Equilibrium.

A. Equilibrium with a Residence Requirement.

This model applies whenever the political jurisdiction encompasses multiple neighborhoods, each providing schooling for its own residents, if household entry into and exit from the jurisdiction can reasonably be ignored. This provides a basis for analyzing cases with multiple districts consisting of multiple neighborhoods as we discuss in the concluding section. Toward developing the properties of equilibrium, note first that here the school-choice stage is trivial, committed in the initial residential-choice stage. Providing conditions for and describing a voting equilibrium is not problematic except for one minor issue. We must guarantee equilibrium permits everyone to purchase a house and consider this issue in the determination of equilibrium. Below we address this issue and develop equilibrium properties for the case of Cobb-Douglas preferences (equation (1)). For now, take as given the existence of a unique voting equilibrium in the third stage, with everyone able to afford a house. By definition, centralized finance implies a single tax rate and $X_1 = X_2 = ... = X_N$, where here and henceforth, subscripts indicate the neighborhood.

We focus now on the residential choices. Although the voting equilibrium depends on the residential allocation, it is not influenced by individual (atomistic) household choice, implying households treat the anticipated voting outcome as parametric in the residential-choice stage. Our primary concern is with the existence and nature of equilibria having differentiated neighborhoods, school peer groups, and school qualities. An issue relevant to the equilibrium allocation of households about which there is little evidence is how student ability affects the demand for school quality. We restrict consideration to two possibilities:

$$\frac{U_q}{U_y} \text{ is invariant to ability.} \tag{A2-1}$$

$$\frac{U_q}{U_y} \text{ increases with ability.} \tag{A2-2}$$

In the latter case the demand for school quality is *normal in student ability*. Case (A2-1) is neutral on this issue and is a property of the Cobb-Douglas specification.

The first proposition describes some necessary properties of any equilibrium with differentiated schools/neighborhoods. These will also be used below to derive existence conditions.

Proposition 1: For $K \leq N$, suppose $q_1 < q_2 < ... < q_K$. (If $K < N$, then some neighborhoods have schools of the same quality.) Then:

a) Housing prices ascend: $p_1 < p_2 < ... < p_K$.

b) The allocation exhibits *income stratification*: If household with income y_1 chooses neighborhood having school quality q_i and household with child of the same ability but income y_2 necessarily chooses neighborhood having higher school quality q_j (j > i), then $y_2 > y_1$.

c) If (A2-2), the allocation exhibits *ability stratification* analogously defined.[16] Under (A2-1), household residential choice is invariant to the student's ability.

d) The allocation exhibits boundary indifference and strict preference within boundaries: Type space (the (b,y) plane) is partitioned into neighborhoods by (measure zero) boundaries along which the corresponding households are indifferent to adjacent neighborhoods and for which interior households have strict preference for their neighborhood over differentiated neighborhoods.

Proof: Since everyone pays the same tax rate, part (a) follows simply. Housing price must be lower if school quality is lower to attract any residents. Given part (a), the converse of part (b) contradicts (A1). Part (c) is proved analogously. Part (d) is implied by continuity of U(·). ■

Figure 2 illustrates a potential equilibrium allocation for a case with K = N = 3 and assuming (A2-1). Stratification by income arises but not by ability, households with incomes y_1 (y_2) are indifferent to residing in neighborhoods 1 or 2 (2 or 3), and all other households strictly prefer their neighborhood of residence. For preferences instead satisfying (A2-2), the *boundary loci* in type space separating differentiated neighborhoods are downward sloping, exhibiting stratification by both ability and income.

The next proposition establishes conditions for existence of equilibrium with differentiated neighborhoods and schools (continuing to take as given existence and uniqueness of voting equilibrium).

<u>Proposition 2</u>: a) Constancy of $E[b|y]$ and preferences satisfying (A2-1) are inconsistent with

existence of equilibrium having differentiated schools.

b) Equilibrium with differentiated schools exists if preferences satisfy (A2-1) and $E[b|y]$ is increasing in y.

c) Equilibrium with differentiated schools sometimes exists if preferences satisfy (A2-2) and either $E[b|y]$ is increasing in y or constant.

Proof: Under (A2-1), Proposition 1 shows an equilibrium with differentiated schools must exhibit stratification by income but not by ability, e.g. as in Figure 2. But constancy of $E[b|y]$ then implies equal θ's in all schools, hence schools of equivalent quality, a contradiction. We show part (b) below by construction. We have worked out examples demonstrating part (c) (available on request). ■

School qualities can vary only due to variation in peer groups since expenditures are equalized across neighborhoods. Access to neighborhoods with better peer groups is rationed by higher housing prices. This rationing must be consistent with differentiated peer groups for such an equilibrium. If willingness to pay for school quality depends only on income (i.e. under (A2-1)), then income and ability must be positively correlated to sustain a differentiated equilibrium. If, however, willingness to pay for school quality also increases with the child's ability, then positive correlation between ability and income is unnecessary. In differentiated equilibrium in the latter cases, the (b,y) plane is partitioned into neighborhoods by downward sloping boundary loci, with relatively high-income and low-ability households mixing with relatively low-income and high-ability households. Such partitions will tend to produce a hierarchy of peer groups, e.g., will always if b and y are independently distributed. In trying to demonstrate existence generally we encounter a variation of the lemon's problem. Given a partition and quality hierarchy, lowering housing price premiums in better neighborhoods to clear housing markets can lower quality differentials, require lower yet housing price premiums that lower further quality differentials, etc. We have, however, consistently found differentiated equilibria in simulations of specific cases.

In all that follows (without constant repetition), we adopt assumption (A2-1) and thus also assume $E[b|y]$ is increasing in y. The next proposition and proof describes the multiplicity of *maximally stratified equilibria* that arise and completes the proof of part (b) of Proposition 2.

Proposition 3: Divide the N neighborhoods into k (≤ N) sets, each set consisting of all neighborhoods having the same housing capacity. Let m_i, i = 1,2,...,k, equal the number of neighborhoods in set i. The maximum number of neighborhoods/schools having different peer groups in an equilibrium equals N. There are $\# \equiv \frac{N!}{m_1! \cdot m_2! \cdots m_k!}$ distinct such equilibria.

Proof: Obviously N is the maximum number of different school qualities that is feasible in an equilibrium. # is the number of distinct ways neighborhoods can be ordered by their housing capacities. We now show each distinct order is consistent with an equilibrium having N different school qualities by construction. Refer to Figure 2 for an example. (Note that Figure 2 has 3 neighborhoods but not necessarily with distinct housing capacities (populations sizes), the latter depending on f(b,y).)

Take any order of neighborhoods and number them 1, 2,..., N. Let $F_y(y)$ denote the marginal c.d.f. of y in the population. Set $y_0 \equiv y_m$ and find y_i, i = 1, 2, ... , N-1, such that $F_y(y_i) - F_y(y_{i-1})$ equals the land capacity of neighborhood i. Recalling that we normalized the population to 1 and also set the aggregate housing capacity equal to 1, it is clear that the ordering of neighborhoods results in a unique vector $(y_1, y_2, \ldots, y_{N-1})$. These delineate the equilibrium partition. Set $y_N \equiv y_x$, and let

$$\theta_i \equiv \frac{\int_{y_{i-1}}^{y_i}\int_{b_m}^{b_x} b f(b,y)\,db\,dy}{\int_{y_{i-1}}^{y_i}\int_{b_m}^{b_x} f(b,y)\,db\,dy} = \frac{\int_{y_{i-1}}^{y_i} E[b|y]\cdot[\int_{b_m}^{b_x} f(b,y)\,db]\,dy}{\int_{y_{i-1}}^{y_i}\int_{b_m}^{b_x} f(b,y)\,db\,dy} \tag{2}$$

denote the implied peer quality measures. Since $E[b|y]$ is increasing, $\theta_1 < \theta_2 < ... < \theta_N$. Then, since X_i is constant across neighborhoods, the q_i's also ascend. Let p_i denote the housing price in neighborhood i, and set $p_1 = c$. Find p_i, $i = 2,3, ..., N$, recursively from

$$U[y_{i-1}(1-t)-p_i,a(q_i,b)] = U[y_{i-1}(1-t)-p_{i-1},a(q_{i-1},b)], \tag{3}$$

noting that (A2-1) implies unique solutions independent of b.[17] Since $q_i > q_{i-1}$, $p_i > p_{i-1}$. By (A1) the assigned residential choices are utility maximizing, and the housing markets clear by construction. We have then described an equilibrium consistent with the given ordering of neighborhoods. A distinct equilibrium can be so constructed from each of the # distinct orderings. ■

Other equilibria also exist. We proceed by restricting the model so as to eliminate the multiplicity of equilibria and simplify the exposition. Except when indicated, we assume that school administrators choose neighborhood boundaries so that schools are of equal size, thus eliminating the multiplicity just identified by Proposition 3 (i.e., # = 1).

Two types of multiplicity remain. Any subset of neighborhoods can be combined into one, with housing capacity equal to their aggregate capacity, and alternative equilibria found as in the proof of Proposition 3. Each neighborhood in the set making up the combination would have a homogeneous population, the same housing price, and the same peer group and school quality. In Figure 2 for example, another equilibrium has neighborhoods 1 and 2 with homogeneous populations consisting of those with incomes below y_2. Neighborhood 3 would have residential population as in the alternative maximally stratified equilibrium, although the housing price would differ in general. These alternative equilibria include the allocation with no neighborhood differentiation. Such equilibria will be unstable under reasonable adjustment assumptions, and also are simply less interesting.[18] Hence, we ignore them in what follows.

A final type of multiplicity concerns a degree of freedom in the determination of housing prices. One neighborhood's price needs to be anchored to determine uniquely the remaining prices. To resolve this, we set housing price in the lowest priced neighborhood equal to c. One justification for this is a variation in the housing supply specification. Suppose that one neighborhood has elastic supply of housing at the construction cost c, while the others continue to have (the same) capacity constraints. Any equilibrium we find is the unique maximally differentiated equilibrium under the latter specification of the housing market. Limited housing capacity, hence access, to the more desirable neighborhoods is necessary for our results, but not limited aggregate capacity. We retain the original model in the presentation because it is easier to work with analytically, but the alternative interpretation is always applicable.

The exposition is also facilitated by setting N = 2 in what follows. Hence, the neighborhood housing capacities are ½, and $y_1 = y_{med}$ in equilibrium, y_{med} denoting the median income.

We now analyze voting equilibrium. To obtain precise results, we restrict consideration henceforth to the Cobb-Douglas utility specification (1).[19] We know that the equilibrium partition of households has $y_1 = y_{med}$ as the boundary locus, and we number the poorer neighborhood 1 and the wealthier neighborhood 2. Using (1) and setting $p_1 = c$, find p_2 from (3):

$$p_2 = c + [y_{med}(1-t)-c][1-(\theta_1/\theta_2)^\gamma]. \tag{4}$$

Since the partition implies $\theta_1 < \theta_2$ (and assuming the median income household can afford a house), inspection of (4) confirms that $p_2 > p_1$. We also see that housing prices are independent of per student expenditure.

To solve for voting equilibrium, we follow the same methodology employed in Epple and Romer (1991). Take as given the household's residence and consider the preference mapping in the (X,t) plane. An indifference curve is defined: $U(X,t;y,p,b,\theta)$ = constant. Having in mind an independent income policy that guarantees housing for all, we wish to assure that everyone can afford housing in equilibrium. For the moment, we simply assume t values above $(y_m\text{-}c)/y_m$ cannot be voted on (by law), and provide a more detailed policy below. Using (1), it is straightforward to confirm:

Lemma 1: a) Indifference curves in the (X,t) plane are upward sloping and concave, with lower (southeasterly) indifference curves corresponding to higher utility.
b) The indifference curve mapping is independent of b and θ, depending only on y/p and α.
c) Looking across households, the slope of indifference curves through any point (X,t) increases with y/p. Hence, any pair of indifference curves cross at most once.

Lemma 1 implies households with higher y/p have a stronger preference for (X,t) in the following sense. If a household is indifferent to choices (X_2,t_2) and (X_1,t_1) where $X_2 > X_1$ and $t_2 > t_1$, then all households with higher (lower) y/p strictly prefer point (X_2,t_2) (point (X_1,t_1)) over the alternative. The latter can be verified using Lemma 1 by drawing indifference curves in the (X,t) plane. It also follows that, whether or not the feasible choice set of (X,t) values voters face is well behaved (e.g., convex):

Lemma 2: A most preferred choice of a voter with median preference (i.e., median y/p) from the feasible choice set is a majority voting equilibrium. Only a most preferred choice of a voter with median preference is a voting equilibrium if the density of the preference parameter y/p is positive in the vicinity of the median.

Proof: The argument follows the graphic technique of Epple and Romer (1991), and is presented here for the reader's convenience. Refer to Figure 3 where U_{med} is an indifference curve of a voter with median preference and suppose point (X^*,t^*) is a most preferred choice of this voter in the feasible choice set (not shown). We argue first that no feasible points in the (X,t) plane are majority preferred to (X^*,t^*), establishing it is an equilibrium point. The indifference curve U_{med} and point (X^*,t^*) partitions the (X,t) plane into four regions.[20] No points below U_{med} are feasible choices since this would contradict the median voter's preference for (X^*,t^*). Point (X^*,t^*) is preferred unanimously over all points in the rectangle with lower right-hand corner at (X^*,t^*). Region I (see Figure 3) is made up of points above and including U_{med} and with $X > X^*$, e.g., point A. Since those with below median preferences have flatter indifference curves through point (X^*,t^*), e.g., $U_{<med}$ in Figure 3, they prefer (X^*,t^*) to all points in Region I. Since the median voter prefers (X^*,t^*) or is indifferent (i.e. if the alternative point in on U_{med}), at least a weak majority prefers (X^*,t^*). By an analogous argument, (X^*,t^*) is not defeated by any points in Region II. We have established that any most preferred point of a voter with median preference is a majority voting equilibrium.

Any most preferred point of the median voter is preferred by a strict majority over any other feasible point assuming a positive density of types in the vicinity of the median. A positive measure of households with y/p in the vicinity of y/p of the median voter will share the latter's strict preference, as will either all those with lower or higher y/p (or both). Hence, only most preferred points of a voter with median preference are voting equilibria. ■

Lemma 2 points toward two potential cases of multiple equilibria. One has a gap in the density of the preference parameter at the 50th percentile and two median preference voters with distinct preferences. The other has multiple most preferred points of a unique median preference voter. The former will be ruled out by (reasonable) parameter restriction, and the latter will not arise in our problem.[21]

In each of the two neighborhoods, there is an income and corresponding (measure zero) set of households having median preference. We rule out the case having multiple equilibria with the reasonable assumption:[22]

$$y_x \geq \frac{p_2}{c} y_m. \tag{A3}$$

Let y_{pi}, i =1,2, denote the income of the "pivotal" (median preference) voter residing in neighborhood i. By Lemma 1 and (A3), $y_{p2} = (p_2/c)y_{p1}$. Also using Lemma 1, y_{p1} satisfies:

$$F_y(y_{p1}) + [F_y(\frac{p_2}{c} y_{p1}) - .5] = .5. \tag{5}$$

We have:

Proposition 4: The solution to:

$$\begin{aligned} &MAX_{t,X}\ (y_{p1}(1-t) - c)X^{\alpha} \\ &\text{s.t. } X = t\bar{y}\ ;\ t \leq \frac{y_m - c}{y_m}; \end{aligned} \tag{6}$$

is the unique majority voting equilibrium, where $\bar{y}$ denotes the mean income in the population.

Proof: Lemma 2 implies that the solution to (6) is a majority voting equilibrium. Uniqueness follows because (6) has a unique solution and, by Lemma 1 and (A3), preferences of the other median preference households are identical. ■

We assume an interior solution to (6)[23]:

$$t^* = \frac{\alpha}{1+\alpha}(1 - \frac{c}{y_{p1}}) \quad \text{and} \quad X^* = \frac{\alpha}{1+\alpha}(1 - \frac{c}{y_{p1}})\bar{y}. \tag{7}$$

Having in mind a policy that precludes equilibrium taxation such that the poorest household cannot afford housing, we have thus far assumed such tax rates are not in voters' consideration set. A specific policy that yields the same outcome and places no restrictions on the tax rate dictates no additional tax liability once a household is driven down to subsistence: Household with income y pays a maximum of y-c in taxes. The effect on preferences is that a household with income y obtains minimum (zero) utility at all points (X,t) such that $t \geq (y-c)/y$. The feasible set of (X,t) pairs has expenditure below $t\bar{y}$ for $t > (y_m-c)/y_m$. It is easy to confirm that Proposition 4 continues to apply under our assumption that problem (6) has an interior solution (see the previous footnote).

Summarizing to this point, for Cobb-Douglas preferences, $E[b|y]$ increasing in y, and two neighborhoods of equal size, (stable) equilibrium splits households by income at the median into the two neighborhoods, with the wealthy neighborhood having higher θ (equation (2)). With $p_1 = c$, the remaining equilibrium variables are described by (4), (5), and (7). In addition to the pivotal voters in neighborhood 1, those with income $p_2 y_{p1}/c$, who reside in neighborhood 2, also have median voting preferences.

The appendix derives the following comparative static results:

$$\frac{dp_2}{d(\theta_2/\theta_1)} > 0 \quad \text{and} \quad \frac{dX}{d(\theta_2/\theta_1)} < 0. \tag{8-1}$$

$$\frac{dp_2}{d\gamma} > 0 \quad \text{and} \quad \frac{dX}{d\gamma} < 0. \tag{8-2}$$

$$\frac{dp_2}{d\alpha} < 0 \quad \text{and} \quad \frac{dX}{d\alpha} > 0. \tag{8-3}$$

Not surprisingly, increases in θ_2/θ_1 increase the housing price in the wealthier neighborhood. The negative income effect of this explains the associated decline in per-student expenditure. All households in neighborhood 2 prefer lower tax rates and expenditure leading to a lower equilibrium choice. A change in the distribution of types that increases θ_2/θ_1 and preserves the population mean ability (e.g., a mean preserving spread that increases θ_2) then has a double whammy on a given type residing in neighborhood 1. Neighborhood 1 school quality declines both due to lower expenditure and a lower-ability peer group. We should add, however, that some poorer residents of neighborhood 1 will prefer the lower tax. Welfare analysis is complex, and taken up computationally in Section 4.

Results in (8-2) are similarly explained. It is clearer to understand (8-3) in the reverse order. Everyones' preference for expenditure rises with α. Lower disposable incomes then reduce the demand to live in neighborhood 2.

B. Equilibrium With School Choice.

The analysis of school choice with no frictions is simple. Households select schools without constraint in the second stage of Figure 1. We assume schools face no capacity constraints and must admit all comers.[24] School finance continues to entail an allocation of funds to schools so as to equalize expenditure per student. Those that send their child to school in the "other neighborhood" bear no transportation or other transactions costs (introduced below).[25] Lacking evidence on productivity effects of intra-district choice, we hold fixed the schooling production function $q(\cdot)$.

The immediate implication is that the exercise of school choice must lead to equal school qualities in equilibrium, and, since $X_1 = X_2$, $\theta_1 = \theta_2$. Further, indifference to residence is implied, so that $p_2 = p_1$ (= c). Regarding voting equilibrium, Lemmas 1 and 2 continue to apply. Since housing prices do not vary, only one type of household has median preference, those with median income (who can live in either neighborhood). Assuming again an interior solution, the solution to problem (6) substituting y_{med} for y_{p1} is the unique voting equilibrium.[26] We have established:

Proposition 5: Equilibrium values with frictionless choice satisfy:

$$p_1 = p_2 = c; \; \theta_1 = \theta_2 = \bar{\theta}; \text{ and } E_1 = E_2 = \frac{\alpha}{1+\alpha}\left(1 - \frac{c}{y_{med}}\right)\bar{y}; \tag{9}$$

where $\bar{\theta}$ denotes the mean ability in the population. Households with median income are pivotal in the voting equilibrium. The residential allocation is indeterminate. Any allocation assigning ½ the population to each neighborhood is an equilibrium. Any set of school choices resulting in $\theta_1 = \theta_2$ is an equilibrium set.

The multiplicity of equilibrium allocations that arise are inconsequential in the strong sense that utilities do not vary across them. Comparing equilibria, introduction of (frictionless) school choice leads higher θ_1 and lower θ_2. Using (7) and (9) and that $y_{med} > y_{p1}$, the tax rate and expenditure are higher under choice. This is explained by the income effect on voting of a lower p_2. The strongest implication is that households with income below the median attend better schools unambiguously. Further normative analysis is in Section 4.

Inter-Neighborhood Transportation Costs. We introduce friction in the exercise of school choice in a simple way. We assume that it costs any household T to send their child to school in the other neighborhood. Hence, for example, intra-neighborhood transportation is costless (or provided) but households bear a private cost of T to transport their children between neighborhoods as across a "Hoxby (forthcoming) river."

Transportation costs are prohibitive and equilibrium is as though there is no choice if T exceeds the housing price differential that arises without choice. Letting p_2^* denote neighborhood 2's housing price in the equilibrium without choice, we then assume:

$$T < p_2^* - c. \tag{A4}$$

We first describe intuitively an "interior equilibrium," one where some but not all households exercise choice. More formal analysis follows the discussion. Figure 4 depicts an interior equilibrium allocation. A threshold income below the median, y_I, divides households according to the neighborhood where their children attend school. Those with income below y_I live in neighborhood 1 and their children attend school there. Those with higher income send their children to the better school in neighborhood 2, but are indifferent to their neighborhood of residence. Any mass equal to ½ drawn from the latter group must live in neighborhood 2 for housing-market clearance. Their residential indifference is supported by a housing price differential equal to the transportation cost:

$$p_2 - c = T. \tag{10}$$

It is equivalent to live in neighborhood 1 and pay the transportation cost, or avoid it but pay the higher housing price in neighborhood 2.

Let $\theta_i^*(y_I)$, i =1,2, denote the implied mean ability in neighborhood i. Since $E[b|y]$ is increasing, $\theta_2^* > \theta_1^*$ for all y_I, although housing-market clearance requires $y_I < y_{med}$. Indifference to transporting one's child from neighborhood 1 to 2 for schooling identifies y_I:

$$T = (y_I(1-t) - c)[1 - R(y_I)^\gamma]; \tag{11}$$

where $R(y_I) \equiv \theta_1^*/\theta_2^*$.

Voting equilibrium is determined analogously to the previous models. Lemmas 1 and 2 continue to apply with T+c replacing the housing price for those who live in neighborhood 1 and transport their child to school in neighborhood 2. All those with $y > y_I$ pay "effective housing price" equal to p_2. There is always a median preference voter with $y > y_I$, whose income we denote y_{p2} (and may or may not be one having $y < y_I$). By ordering households according to their income divided by effective housing price, it is straightforward to identify three exhaustive cases that identify income y_{p2} of a pivotal voter:

$$\text{For } y^* \text{ defined in } F_y(y^*) - F_y(y_I) = .5; \tag{12-0}$$

$$y_{p2} = y_{med} \text{ if } \frac{y_{med}}{p_2} \geq \frac{y_I}{c}; \tag{12-1}$$

$$y_{p2} = y^* \text{ if } \frac{y_{med}}{p_2} < \frac{y_I}{c} \text{ and } \frac{y^*}{p_2} \leq \frac{y_m}{c}; \tag{12-2}$$

or

$$y_{p2} \text{ satisfies } F_y(y_{p2}) - F_y(y_I) + F_y(\frac{c}{p_2}y_{p2}) = .5$$
$$\text{if } \frac{y_{med}}{p_2} < \frac{y_I}{c} \text{ and } \frac{y^*}{p_2} > \frac{y_m}{c}. \tag{12-3}$$

Replacing y_{p1} with y_{p2} in problem (6), and assuming again an interior solution:[27]

$$t = \frac{\alpha}{1+\alpha}(1 - \frac{p_2}{y_{p2}}); \text{ and } X = t\bar{y}. \tag{13}$$

Proposition 6 contains a more formal statement of the results:

Proposition 6: Assume (A4) is satisfied and any voting equilibrium permits everyone to afford housing. A solution (p_2, y_I, t, y_{p2}, X) with $y_I < y_{med}$ to (10) - (13) and $\theta_i = \theta_i^*(y_I)$, i=1,2, is an interior equilibrium with allocation depicted in Figure 4 (and with any mass equal to ½ of households having $y > y_I$ living in neighborhood 2).

Proof: Confirming that a solution to the system of equations (and inequalities in (12)) is an interior equilibrium is straightforward. Using (10) and Lemma 2, the voting equilibrium described by (12) and (13) is easily confirmed. The housing markets clear by construction. Given a preference for schooling in neighborhood 2, the indifference of households with $y > y_I$ to their residence is immediate from (10). The preference of those with $y > y_I$ for schooling in neighborhood 2 follows from (11) and that the utility difference, $[y(1-t)-p_2]\theta_2^{\gamma}X^{\alpha}b^{\beta} - [y(1-t)-c]\theta_1^{\gamma}X^{\alpha}b^{\beta}$, is increasing in y (and independent of b). This also implies a preference of households having $y < y_I$ to live in neighborhood 1 and send their children to school there. ■

Existence and uniqueness of an interior equilibrium is not guaranteed. One complication is that the $R(y_I)$ function (see (11)) need not be well behaved. However, we find unique interior equilibrium for a range of T in our computational model of Section 4. Another possibility is a corner equilibrium having $p_2 = T+c$, but where everyone attends school in neighborhood 2. Here T is low enough that choice induces everyone to get the same schooling.

Relative to the no-choice equilibrium, the exercise of choice by those with below median income is on average associated with a negative peer-group externality to both those who stay behind and attend school in neighborhood 1, and those with above-median income. Both θ_1 and θ_2 decline since $E[b|y]$ is an increasing function of y. The decline in neighborhood 1, born by the poorest segment of the population, supports the concerns of some critics of choice. Because the voting equilibrium is also affected, we defer further discussion of normative issues to Section 4.

C. Multiple Jurisdictions: Tiebout Equilibrium.

It is of interest to compare the single-jurisdiction equilibria above to the more de-centralized provision regime where education finance is localized. Here the two neighborhoods are assumed to correspond to two political jurisdictions for the determination of the tax rate and per student expenditure. Households' children attend school in their neighborhood of residence. Otherwise, we maintain the properties of the model including Cobb-Douglas preferences (hence (A2-1)), the same housing capacities of ½ in each now jurisdiction, and $E[b|y]$ increasing in y. As in the single-jurisdiction model without choice, the school-choice stage is degenerate. This version of our model is more akin to a city-suburb environment with small school districts.

We focus again on the existence and properties of equilibrium in which households are not generally indifferent to their choice of residence (as opposed to a homogeneous neighborhood equilibrium). Conditions for existence are presented below. First, we have:

Proposition 7: Any equilibrium without residential indifference of all households exhibits:
a) stratification by income (independent of ability); and
b) boundary indifference and strict preference within boundaries.

Proof: Using (1), calculate the sign of the utility difference (Δ) from residing in neighborhood 2 versus 1: $\text{sgn}\,\Delta = \text{sgn}\,\{[(1-t_2)q_2 - (1-t_1)q_1]y + (p_1q_1 - p_2q_2)\}$. Housing market clearance implies that one-half the population lives in each neighborhood. Linearity of Δ in y then implies either income stratification (that is independent of ability) or that Δ vanishes for every income. The latter case is that having everyone indifferent. Given Δ does not vanish for some y, boundary indifference for $y = y_{med}$ and strict preference otherwise are then implied by the linearity. ■

Residential choices and peer groups are then the same as in the single-jurisdiction model without choice. Each neighborhood/jurisdiction chooses its own tax rate, however. Households within a neighborhood face the same housing price and Lemmas 1 and 2 apply. The households with median preferences in neighborhoods 1 and 2 have first-quartile income (y_{q1}) and third-quartile income (y_{q3}), respectively.[28] The voter's problems are analogous to (6) with the obvious substitutions. Assuming interior solutions:[29]

$$t_1 = \frac{\alpha}{1+\alpha}(1 - \frac{p_1}{y_{q1}}); \; X_1 = t_1\bar{y}_1; \tag{14}$$

$$t_2 = \frac{\alpha}{1+\alpha}(1 - \frac{p_2}{y_{q3}}); \text{ and } X_2 = t_2\bar{y}_2; \tag{15}$$

where $\bar{y}_i$, i = 1,2, denotes mean income in neighborhood i. We have purposely not set $p_1 = c$ since it is possible, though unlikely, that the housing price is lower in the wealthier neighborhood. This is discussed below. Boundary indifference implies prices satisfy:

$$[y_{med}(1-t_2) - p_2]q_2 = [y_{med}(1-t_1) - p_1]q_1. \tag{16}$$

We can now show:

<u>Proposition 8</u>: A sufficient condition for existence of equilibrium without residential indifference is:

$$\begin{gathered}\{y_{med} - c[1+\alpha(1-\frac{y_{med}}{y_{q3}})]\}(1-\frac{c}{y_{q3}})^{\alpha} > \\ \{y_{med} - c[1+\alpha(1-\frac{y_{med}}{y_{q1}})]\}(1-\frac{c}{y_{q1}})^{\alpha}\Omega; \text{ where } \Omega \equiv (\frac{\theta_1}{\theta_2})^{\gamma}(\frac{\bar{y}_1}{\bar{y}_2})^{\alpha} < 1,\end{gathered} \tag{17}$$

and θ_i and $\bar{y}_i$ are calculated assuming income stratification with $y_1 = y_{med}$ (recall (2)). Given (17) is satisfied and setting $p_1 = c$, equilibrium (without residential indifference) is unique. Two of many sufficient conditions for satisfaction of (17) are: (i) c sufficiently low; or (ii) $\alpha < y_{med}/y_{q3}$.

Proof: To show existence, we must show (14) - (16) has a solution (t_1,t_2,p_1,p_2) with $p_i \geq c$, i =1,2, and consistent with the residential preferences of Proposition 7. Clearance of housing markets will be implied. We show (17) implies an equilibrium exists with $p_1 = c$. Set $p_1 = c$, substitute for p_1 in (14), and substitute (14) and (15) into (16):

$$H(p_2) = \{y_{med} - c[1 + \alpha(1 - \frac{y_{med}}{y_{q1}})]\}(1 - \frac{c}{y_{q1}})^{\alpha}\Omega;$$
$$\text{where } H(p_2) \equiv \{y_{med} - p_2[1 + \alpha(1 - \frac{y_{med}}{y_{q3}})]\}(1 - \frac{p_2}{y_{q3}})^{\alpha}. \quad (18)$$

Using $y_{q3} > y_{med}$, observe that $H'(p_2) < 0$ for p_2 such that $H(p_2) > 0$, and $H(p_2) \downarrow 0$ as p_2 rises. Note that the left-hand side of the inequality in condition (17) is H(c). The right-hand side of the inequality in (17) is positive since it has the same sign as the utility of median income households when $p_1 = c$. It follows that, given (17), a unique p_2 satisfying (18) exists. Note, too, that this $p_2 > c$. Hence, one can find a solution to (14) - (16) with $p_i \geq c$, i =1,2.

To show existence, it remains to be confirmed that the residential choices associated with the presumed allocation are actually optimal. This requires Δ (defined in the proof of Proposition 7) is increasing in y, or:

$$(1 - t_2)q_2 > (1 - t_1)q_1. \quad (19)$$

Rewrite the latter and substitute from (16):

$$\frac{(1 - t_2)}{(1 - t_1)} > \frac{q_1}{q_2} = \frac{y_{med}(1 - t_2) - p_2}{y_{med}(1 - t_1) - p_1} \rightarrow$$

$$p_2(1 - t_1) > p_1(1 - t_2). \quad (19\text{-}1)$$

Substitute from (14) and (15) and again rewrite the condition:

$$p_2(1 + \alpha\frac{p_1}{y_{q1}}) > p_1(1 + \alpha\frac{p_2}{y_{q3}}). \quad (19\text{-}2)$$

Since $y_{q3} > y_{q1}$, $p_2 \geq p_1$ is sufficient for satisfaction of (19-2). We have shown p_2 exceeds $p_1 = c$, completing the proof of existence.

We have already shown uniqueness given $p_1 = c$. Sufficiency of (i) for (17) uses $\Omega < 1$. The left-hand side of the inequality in (17) converges to y_{med} as $c \downarrow 0$ while the right-hand side converges to Ωy_{med}. To show (ii), let g(y) denote the expression on the left-hand side of the inequality in (17), where $y = y_{q3}$. With this notation, the inequality (17) is $g(y_{q3}) > \Omega g(y_{q1})$. It follows that the inequality is satisfied if g(y) is an increasing function over $y \in [y_{q1}, y_{q3}]$. After straightforward manipulation one obtains:

$$g'(y) = \frac{\alpha c y}{y^2(y-c)}\left(1-\frac{c}{y}\right)^{\alpha}\left[c\left(\frac{y_{med}}{y}-\alpha\right)+\alpha c\frac{y_{med}}{y}\right].$$

Condition (ii) is sufficient for $g' > 0$ in this range, proving the result. ■

We emphasize that we have only provided sufficient conditions for satisfaction of (17). Since (17) involves 9 parameters (counting θ_1/θ_2 as one), we make no attempt to partition fully the parameter space. Note also that as Ω declines, it is "more likely" (17) will be satisfied and existence guaranteed. Hence, high correlation of (y,b), implying relatively low θ_1/θ_2, favors existence. Moreover, (17) is not necessary for existence of equilibrium. Absent satisfaction of (17), $p_1 > p_2$ in an equilibrium. It appears that this is possible (but we have not worked out any such examples).[30] Such an equilibrium would have a much higher tax rate in neighborhood 2 than 1, reflecting a relatively high y_{q3}, and such that a lower housing price in neighborhood 2 is necessary to keep the median-income household indifferent to residence.

Assuming (17) holds, hence $p_2 > p_1 = c$, it is not clear which neighborhood has higher taxes. Keep in mind it is the ratio of income to the housing price that measures the preference for tax rates (and that preference is independent of $\bar{y}_i$)[31]. However, schools in neighborhood 2 must be of higher quality whether or not $p_2 > p_1$ in equilibrium.

Proposition 9: $q_2 > q_1$ in equilibrium without residential indifference.

Proof: If $p_2 \leq p_1$, then $t_2 > t_1$ and $X_2 > X_1$, both by (14) and (15). The better peer group in neighborhood 2 then implies the result. For the case of $p_1 < p_2$, suppose to the contrary that $q_2 \leq q_1$. From (16) then, $[y_{med}(1-t_2)-p_2] \geq [y_{med}(1-t_1)-p_1]$, implying $p_2-p_1 \leq (t_1-t_2)y_{med}$. Substitute from (14) and (15) for the t's in the latter yielding:

$$p_2 - p_1 \leq \frac{\alpha}{1+\alpha}(p_2\frac{y_{med}}{yq3} - p_1\frac{y_{med}}{y_{q1}}). \tag{20}$$

Since $\alpha/(1+\alpha) < 1$ and $y_{q3} > y_{med} > y_{q1}$, (20) contradicts $p_2 > p_1$. ■

Relative to the single-jurisdiction, multi-neighborhood environment without choice, one would expect here lower per student expenditure in the poor neighborhood and the opposite in the wealthy neighborhood due to the changes in the tax base. This would imply a wider dispersion in school qualities with multiple jurisdictions. The theoretical analysis is obscured by changes across the equilibria in the identities of the pivotal voters and housing prices. This issue is explored computationally in Section 4.

For our Cobb-Douglas specification with fixed housing capacities, residential choices and thus peer groups are exactly the same without school choice whether or not neighborhoods are also political jurisdictions. This illustrates a central result of this paper: *Peer-group effects alone can lead to income-stratified equilibrium, like in a Tiebout equilibrium with local public finance*.[32] If we depart from a Cobb-Douglas specification and/or assume upward sloping housing supplies, then the residential allocation will vary somewhat across the two regimes. But this is only to the extent that educational expenditures is important to school qualities. In the Cobb-Douglas case with upward-sloping housing supplies for example, the allocations converge as $\alpha \to 0$. If educational expenditure has small effects at the margin as most evidence indicates (see Hanushek , 1986), then policies that more evenly distribute educational funds will *not* much reduce stratification so long as school choice is absent.[33]

Inter-jurisdictional school choice is also worthy of study.[34] The analysis of inter-jurisdictional school choice depends on how the choice policy implements school finance when choice is exercised. Here we briefly summarize some results, as space constraints prevent a complete presentation. In an earlier version of this paper (Epple and Romano, 1995), we analyzed frictionless inter-jurisdictional choice assuming that those who cross district boundaries bring with them their own jurisdiction's locally determined per student expenditure. This policy leads to a nonstratified outcome and homogeneous schools, but with a severe free-rider problem in school finance: Voting to raise one's local tax would attract outsiders (or reduce exit), this externality leading to substantially lower schooling expenditure. Anticipating this, an inter-jurisdictional choice policy might require that a household exercising choice become a member of their chosen school's jurisdiction for purposes of school finance. That is, they pay their chosen district's tax rate while being allowed to vote there on the school budget.[35] We show in this paper's appendix that this policy would "frequently" lead to the same outcome as does choice in a single jurisdiction if there are no frictions (i.e., non-tax costs) to exercising choice. However, as we discuss more fully below, potential recipient districts generally have an incentive to resist accepting students from outside the district, casting doubt on the extent to which choice is likely to be frictionless. The logic is that housing prices must be equalized, and potential differences in tax rates will "frequently" not alone be enough to support an equilibrium with stratified schools.

4. Computational Analysis and Welfare Effects.

We begin with a general discussion of welfare issues concerning our model. This facilitates the interpretation of our computational results which we go on to present.

While much of our analysis concerns traditional efficiency measures, this is presented with serious caveats. First, education is regarded by many as a primary means to lessen equity problems, and we are not unsympathetic to this view. Second, equity aside, long-term externalities associated with low educational achievement or wide variance in educational achievement may exist, e.g., crime and resentment. For both these reasons, it is important to also consider the distribution of educational achievement.

A third caveat concerns education as an investment rather than consumption good. If education is an investment good, then our model implicitly assumes imperfect opportunities for borrowing on future earnings which constrains all households. This follows from our assumption that household demand for educational quality (equivalently, student achievement) increases with income.[36] The standard static analysis does not properly measure welfare changes under the investment interpretation; one must measure and value changes in aggregate achievement and factor this into the welfare measure.

With these reservations in mind, we turn to (standard) efficiency analysis. Understanding properties of Pareto-efficient allocations provides perspective for understanding the variation in welfare (producer surplus plus compensating variation) across the policy regimes analyzed below. We assume that there are at most two schools and that an allocation entails an assignment of all students to a school (i.e., no schooling is not an option). Let $A(b,y) \in [0,1]$ denote the proportion of students of type (b,y) assigned to school 1, so $1 - A(b,y)$ is the proportion assigned to school 2. For the applications we study, A will equal 0 or 1 for almost all types, i.e., efficient student bodies entail virtually no overlap of types. We assume no transportation costs so that neighborhood residence is irrelevant to efficiency. Set $p_i = c$, $i = 1,2$, giving the anonymous land owners no rents. The next proposition is the main result in Epple and Romano (2000). It includes a description of the "social marginal cost" of a student attending school i which we denote SMC_i. Also, let $r_i(b,y)$ denote the "regulated price" that a social planner charges type (b,y) to attend school i. ($r_i(b,y)$ will only be a function of b at the optimum as we will see.)

Proposition 10 is a variant of the Second Fundamental Welfare Theorem.

<u>Proposition 10</u>: If appropriate lump-sum transfers of income are arranged, then every Pareto efficient allocation can be achieved by utility-maximizing school choices, with, for all (b,y), students paying prices:

$$r_i(b,y) \ = \ SMC_i \ \equiv \ X_i \ + \ \frac{q_\theta(X_i,\theta_i)}{q_X(X_i,\theta_i)}(\theta_i - b), \ \ i=1,2; \tag{21}$$

with X_1 satisfying:

$$1 \ = \ \frac{\int\int_S A(b,y)\dfrac{\partial U^1(b,y)/X_1}{\partial U^1(b,y)/\partial y}f(b,y)dbdy}{\int\int_S A(b,y)f(b,y)dbdy}; \tag{22}$$

X_2 satisfying the analogous Samuelsonian condition for school 2;

$$U^i(b,y) \equiv U[y-c+R(b,y)-r_i(b,y),a(q(X_i,E_i),b)],\ i=1,2; \tag{23}$$

R(b,y) denoting the lump-sum transfer function; and, finally, θ_i, i = 1,2, and A(b,y) those implied by utility-maximizing choices of schools.

Proof: See Epple and Romano (2000). ■

Here we just provide intuition for this result, with formal proof in the paper cited. With prices that reflect the peer externality in schools (and efficiently chosen expenditure levels), individual school choice will yield an efficient allocation. The social cost of type (b,y) entering school i, SMC_i, equals the per student expenditure plus the dollar value of the peer-group externality. The value of the peer externality is the last term in (21). q_θ/q_X equals the cost of maintaining quality as θ changes, which is multiplied by (θ_i-b), the change in θ_i that results due to student type b's attendance at i. Note that the peer-group "cost" of attendance is negative for students having ability higher than the student body's mean, and their SMC can then be negative. Note, too, that the social cost depends on b, but not y. Hence, efficient prices are non-anonymous with respect to ability but not income.

The efficient expenditure levels satisfy the usual Samuelsonian conditions. Note that school budgets balance: integrating r_i over the student body in i yields total expenditure in school i. The lump-sum transfers that are considered must also satisfy budget balance.

We next consider specifics in the Cobb-Douglas case. A natural benchmark allocation presumes no income transfers, so set R(b,y) = 0 for all (b,y). We have:

<u>Proposition 11</u>: For Cobb-Douglas utility/achievement, the no-transfer P.E. allocation has:

a) $q_2 > q_1$;

b) stratification by income with linear boundary locus:

$$y = [c + \frac{(X_2+\eta_2\theta_2)q_2-(X_1+\eta_1\theta_1)q_1}{q_2-q_1}] - [\frac{\eta_2 q_2-\eta_1 q_1}{q_2-q_1}]b, \tag{24}$$

where $\eta_i \equiv (q_\theta/q_X)_i$, $i=1,2$;

c)

$$X_i = \frac{\alpha}{1+\alpha}[\bar{y}_i - c],\ i = 1,2; \text{ and} \tag{25}$$

$$\eta_i = \frac{\gamma}{(1+\alpha)}\frac{(\bar{y}_i-c)}{\theta_i},\ i = 1,2; \tag{26}$$

where $\bar{y}_i$ and θ_i are the school means implied by the efficient allocation. Further, $E[b|y]$ invariant to y is sufficient for:

d) $\theta_2 > \theta_1$; and

e) stratification by ability (hence, a downward sloping boundary locus).

If, also, $\gamma < 1$, then

f) $X_2 > X_1$.

Proof: See the appendix. ■

Figure 5 illustrates a typical P.E. allocation, calculated for the baseline case of our computational model (parameter values match those in Table 1, discussed below). It is notable that a strict hierarchy ($q_2 > q_1$) is efficient even if expenditure is not permitted to vary (see Epple and Romano, 1998a). This results because relatively low-ability and high-income types are willing to subsidize relatively low-income and high-ability types to attend the same school. This necessarily leads to a downward sloping boundary between the student bodies if b and y are independently distributed (results (d) and (e) in Proposition 11). In all our computational analysis, including with $E[b|y]$ increasing in y (and including computations for other related papers), we have found downward sloping boundary loci in efficient allocations (e.g., Figure 5 has $E[b|y]$ increasing in y). However, finding general and meaningful restrictions on the distribution of types that ensures this has been elusive. Similarly, the condition in part f of Proposition 11 that $\gamma < 1$ for $X_2 > X_1$ is not necessary, but is not very restrictive.

This P.E. benchmark reveals sources of inefficiency in the equilibrium outcomes above. Welfare gains result from partitioning the population into schools as in Figure 5. We then expect that the introduction of school choice will tend to reduce aggregate welfare, because the neighborhood-school partition is a better approximation to the efficient partition when $E[b|y]$ is increasing. The neighborhood-equilibrium partition is not, however, efficient in general. The implicit pricing of schools in neighborhood equilibrium is independent of ability, hence, incorrectly accounts for the peer-group effect. As the correlation in (b,y) increases, partitioning only according to income as in the neighborhood equilibrium becomes a perfect substitute for partitioning by income and ability. While the point of partition in the neighborhood equilibrium will not generally be the efficient one since it is driven by neighborhood line and their housing supplies, this line of argument suggests that welfare losses from choice will rise with the correlation in (b,y).

We have shown that expenditure "typically" rises with school quality in the efficient allocation (Proposition 11-f). Consider centralized versus de-centralized finance (Tiebout equilibrium) without school choice. Comparing equations (14)-(15) to (25), we see de-centralized finance provides a first approximation to the efficient outcome. The usual voting bias from median (neighborhood) income differing from mean income arises, as does another bias from the distorted housing price in neighborhood 2. Nevertheless, we expect that centralization of finance (absent choice) will lower welfare.

Tables 1 - 7 present representative results from our simulations, with Table 1 the "baseline case." Throughout we set $b_m = 0$ and assume $\begin{bmatrix} \ln b \\ \ln(y-y_m) \end{bmatrix}$ is distributed bi-variate normal with covariance matrix $\begin{bmatrix} \sigma_b^2 & \rho\sigma_b\sigma_y \\ \rho\sigma_b\sigma_y & \sigma_y^2 \end{bmatrix}$. We set y_m = \$5000 and use 1989 U.S. annual mean (\$36,360) and median (\$28,860) income to set the mean and variance of ln $(y-y_m)$.

We use the Cobb-Douglas utility-achievement function, which implies that the mean of ability is irrelevant to our calculations.

We calibrate the distribution of ability so that it has the same median and mean as income. This may be motivated as follows. Consider a steady state and suppose that income is proportional to achievement. This provides a cardinalization of achievement, and this coupled with the educational production function induces a distribution on ability. For simplicity we calibrate ability for the case in which all students receive the same educational quality. In this case, the Cobb-Douglas achievement function implies that the logarithm of achievement is a linear function of the logarithm of ability. Hence, in this case, the steady state lognormal distribution of income and the assumption that income is proportional to achievement imply that ability has a lognormal distribution as well. It is then convenient to choose the unit of measurement of ability so that the mean and variance of ability for this case of equal school qualities equals the mean and variance of income.

Two papers (Solon, 1992; Zimmerman, 1992) provide evidence on the correlation between father's and son's income, and they are in agreement that the best point estimate of this correlation is approximately .4. Hence, we set $\rho = .4$ in the baseline case. This completes the calibration of f(b,y).

We have set $\alpha = .06$ because this implies a household purchasing educational expenditure would spend approximately 5.6% = $\alpha/(1+\alpha)$ percent of their income, the actual U.S. educational percentage expenditure in 1989. Lacking evidence on the relative importance of peer group and expenditure, we also set $\gamma = .06$ in the baseline case. The value of β is irrelevant to our calculations, again due to the Cobb-Douglas specification.

We set the annual amortized construction cost of a house, c, equal to \$2,500. Last, we set the transportation cost of exercising inter-neighborhood choice equal to \$300 for the case of choice with friction. We have computed equilibria with all parameters varied and report representative results here.

In addition to the equilibrium values in the four equilibria, each table presents welfare changes relative to the neighborhood- school, one-jurisdiction equilibrium. CV_i, i=1,2, denotes the mean compensating variation resulting from the policy change of those who reside in neighborhood i in the neighborhood-school, one-jurisdiction equilibrium, averaged over the entire population. (Multiply CV_i by two to get the mean CV_i for the given subset of the population.) Adding to $(CV_1 + CV_2)$ the per capita change in the housing price in neighborhood 2 ($p_1 = c$ always), one obtains the per capita welfare change, equal to per capita producer surplus plus compensating variation. We have calculated equilibrium without redistributing the land rents for simplicity and because it is not likely to much affect equilibrium. In so doing, we also avoid having to specify land ownership. In our computations, if all land is owned by those with income above the third quartile, our equilibrium calculations would be unchanged. Skewing the top end of the net income distribution has no effect on equilibrium because no such households are ever pivotal decision makers in our computations.

Frictionless choice lowers aggregate welfare in every simulation (including all unreported ones), i.e., ΔW is consistently negative. Noting that per student expenditure changes little from the benchmark (one-jurisdiction, neighborhood) equilibrium, in fact rises (because of the positive income effect on voting from the reduction in p_2), it is clear the welfare loss is explained by the homogenization of peer groups. Comparing Table 1 to 2, the latter having a higher (lower) γ (α), one sees that the welfare loss rises with increased weight placed on the peer group in educational achievement. (See also Table 3.) The reason that CV_2 is usually positive when choice is introduced is because the housing price p_2 declines -- but someone bears this loss.

While choice causes an overall welfare loss, those residing initially in neighborhood 1 consistently gain on average from its introduction. This holds when we assign to this group a proportion of the loss in producer surplus in the housing market from choice equal to their income share (these calculations not shown in the tables).

Absent choice, going from one jurisdiction to two consistently increases welfare, but also increases the dispersion of school qualities. Since peer groups in schools are unchanged, these result because of changes in educational expenditures. Not surprisingly, the housing price in neighborhood 2 rises substantially. Those initially residing in neighborhood 1 lose out on average, and this persists if they are assigned their (income) proportional share of the increased producer surplus in the housing market (not shown).

For the case of choice with a transport cost, we set T = 300 in the computations, letting it vary once (T = 500 in Table 4). With one exception (Table 5), we find T = 300 (or 500) induces a large percentage of those with income below the median to exercise choice and attend school in neighborhood 2, that percentage ranging from about 72 to 99.7. Compare this equilibrium to frictionless choice with one jurisdiction (ignoring Table 5 for the moment). The voting equilibria are not much different so expenditures vary little. The welfare loss from choice with transport costs exceeds that from frictionless choice by about the amount of the transportation costs expended. Assigning the losses in producer surplus again by income shares (not shown in the tables), we find that the greater welfare loss under choice with transport costs is borne largely by those with below median income. Those that exercise choice obviously pay the transportation costs. Those that choose not to commute face a substantially diminished peer group.

The exceptional case of Table 5 has low correlation of income and ability. The per capita welfare loss relative to the benchmark equilibrium equals only $17.65. Because the peer group difference is small in the neighborhood equilibrium, little incentive to exercise choice is present and less than 9% of those with below median income do so. Note, too, that the price difference between neighborhoods (p_2-c) is only $322 in the benchmark equilibrium for the parameter settings in Table 5. This is another manifestation of the limited value of the peer quality gain to migration in this case. By contrast, for the other cases we report in the tables, the price differential in the benchmark case is much larger, ranging between $530 and $960.

Table 6 has a higher construction cost of housing than in the baseline case. This has a negative income effect, manifest in lower schooling expenditures in all equilibria and less exercise of choice when there is a transportation cost.

Table 7 has a more right-skewed income distribution, holding constant the median income. This is associated with greater inframarginal demand for segregation by the relatively wealthy. This amplifies the aggregate welfare loss from frictionless choice and the aggregate welfare gain from neighborhoods becoming jurisdictions.

5. Discussion and Conclusions.

Many models of multi-jurisdictional equilibrium are structured so that differences in tax and expenditure policies across jurisdictions are the only force leading to stratification of population across jurisdictions. These models have been quite fruitful in studying a variety of policy issues related to state and local government finance. In investigating school finance policy issues, it is natural to turn to those models to understand the effects of changing the structure of school finance. In geographic areas in which students are served by a single school in each district, these models should give reasonable guidance, particularly in studying finance policies that entail only partial equalization across jurisdictions.

Unfortunately, these conditions are almost never met. As we noted in the Introduction, the vast majority of children in the U.S. go to school in multi-school districts. Central-city districts are virtually all multi-school districts. Thus many students go to school in districts that have dozens of schools. The second difficulty with using the traditional model to study school policy is that it gives either no predictions or incorrect predictions in cases where there is full equalization of expenditure per student or in the study of intra-district public school choice. The difficulty lies in the common assumption that stratification of the population across schools is driven by expenditure differences and that stratification does not arise when expenditures are equalized.

Even a cursory look at the stratification of households across neighborhood schools in large urban districts is sufficient to put the rest the notion that there is no stratification among schools in a district when expenditures are equalized. We have shown in this paper that there is likely be little or no change in stratification when expenditures are equalized in neighborhood school systems. School quality differences arising from peer effects give rise to housing price differentials across neighborhoods that are sufficient to sustain stratification. Thus, while expenditure equalization may lead to some reduction of school quality differences, equalization of school quality will generally not arise when expenditures per student are equalized.

Our paper provides insights into public school choice programs and offers a foundation for addressing many issues related to public school choice. We noted earlier that, while many states have inter-district school choice programs, few students participate in such programs. This is as our model would predict, because districts that are prospective recipients of choice students will resist accepting such students. To see why, consider two districts D1 and D2 and a student from D1 wishing to attend school in D2. For simplicity, suppose that each district has only one school. It is easy to see that D2 will resist if the funding the student brings from D1 is less than the expenditure per student in D2. Suppose then that the choice program compensates for any such funding disparities. Our model predicts that D2 will nonetheless resist. In the absence of funding disparities, students will wish to transfer from schools with low average peer ability to those with high average peer ability. The clientele of the prospective recipient school will resist accepting such students since, on average, they will be of lower peer ability than the incumbent students. In practice, in states where district participation in inter-district choice programs is voluntary, high income districts typically opt out. In states where such formal opting out is not permitted, *de facto* opting out is nonetheless likely to occur as prospective recipient districts give priority to local residents and then "find" that they have little or no excess capacity to serve students from outside the district.

Of course, the same incentives to resist choice students arise within districts. Within-district programs often tackle this resistance by requiring schools to select at random from their applicant list if they are over subscribed. Of course, a recipient school may still be less welcoming to students from outside the neighborhood than to students from inside the neighborhood, but such informal resistance is likely to be muted if district administrators are sufficiently committed to the choice program. Students exercising choice may then find that the gain in school quality is sufficient to justify living with any residual resistance. Such resistance may, however, be similar to the role of our transportation cost variable, T, in discouraging students from the lowest-income households from attending a higher quality choice school.

Our model points to potential unintended consequences of public school choice programs. For example, suppose, again, that there are two districts, D1 and D2. Suppose now that D1 has two neighborhood schools, A and B, while D2 has one school, C. For simplicity, let the three schools be of equal size. Suppose that low- and high-income students are in district D1 while middle-income districts are in D2. Specifically, let the incomes of attendees of A be low-income students, with B serving high-income students, and C serving middle-income students. Thus, the ordering of school qualities is A, C, B. How could such an allocation be an equilibrium? One could easily construct realistic examples in which tax base per student in D1 would be comparable to tax base per student in D2, so that equilibrium would be characterized by little difference in spending per student in the two districts. With $E[b|y]$ increasing in y, the ordering of school qualities A, C, B would then primarily reflect differences in peer qualities.

Beginning with such an equilibrium, consider the effect of introducing a frictionless intra-district choice program in D1. What are the possible equilibria with this choice program? The equalization of peer qualities in D1 means that there will no longer be stratification within districts. There will, however, be stratification across districts. Thus, one possible equilibrium is that the high-income households will remain in D1, middle-income households will move to D1, and the poor will move to D2. The other possible equilibrium is that the high-income households will move to D2 and the middle-income households will move to D1.[37] What are the effects on school quality? In the first case, students in poor households receive a worse education after introduction of the choice program. They face the same peer quality as before the change and reside in a district where the tax base is lower than in the original equilibrium. In addition, the pivotal voter is poorer than in the original equilibrium, so there will be a decline in spending due both to the lower tax base and to the lower willingness of the pivotal voter to tax in support of education. Middle-income students will likely gain since they move to a district with higher average tax base and higher average peer quality. The effect on high-income students is ambiguous since their district's tax base per student rises but choice causes the quality of peer students to fall. In the second case, the effect of the change on students in low-income households is ambiguous, depending on whether peer effects or expenditure effects are more important. Students in high-income households will receive higher quality education than before the change and students in middle-income households will receive lower-quality education.

The preceding illustrates the potential of our model for anticipating consequences of public school choice programs.[38] Since choice programs have been undertaken by a number of central-city districts, the possibilities raised in this example cannot be easily dismissed. Metropolitan areas typically contain suburban districts that have lower average income than average income in the wealthiest central city neighborhoods. Thus, there is a very real possibility that introduction of choice programs in central-city districts will induce exodus from the city by the high-income households and entry by middle-income households. Of course, a central city district may nonetheless decide that the choice program should be undertaken. The point of the example is to illustrate that our model provides a vehicle for thinking through the likely consequences of such policy changes.

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Footnotes.

1. The first nine states are Hawaii, Maryland, West Virginia, Nevada, Florida, Louisiana, Georgia, Virginia, and North Carolina, and the second four states are Tennessee, Utah, Alabama, and Mississippi. These data are from the Common Core of Data for 1998.

2. These calculations use data from the Census of Governments for 1972 and the Common Core of Data for 1994. The ratios for these years have been calculated for 45 states, excluding Hawaii (whose ratio is one), Alaska, Arkansas, Montana, and Vermont. Districts are nonunified in the latter three states and were exempted from the calculations. Special thanks to David Figlio for providing access to his data base.

3. See *Public School Finance Programs of the U.S. and Canada 1993-94* for state-by-state description of their school finance their school finance system and summary of reforms and initiatives.

4. Among the 45 states for which calculations were made (see footnote 2), the change from 1972 to 1994 of the 95 percentile to 5 percentile district expenditure ratio exceeded 10 percent (of the 1972 ratio) in 18 states. In 16 of these states the ratio declined. The exceptions are Maine and Missouri. See Murray, Evans, and Schwab (1998) for evidence that court cases have substantially reduced expenditure inequality.

5. Probably the biggest story is Florida's statewide voucher program for students in failing public schools introduced in the 99-00 academic year. But there are a myriad of choice programs and initiatives. See *School Choice: What's Happening in the States 2000* for a recent state-by-state summary.

6. The National Center for Educational Statistics (1996) estimated that 13.8 percent of U.S. school districts had intra-district open enrollment policies and 28.6 percent of districts were subject to inter-district open enrollment policies in 93-94. Participation of students subjected to the respective policies was 24.5 percent and 1.6 percent respectively. The estimated rate of "participation" in intra-district policies must be interpreted with care since some programs require households to choose their school, with then 100 percent participation rate. More generally, programs differ in limitations and requirements. See *School Choice: What's Happening in the States 2000* for details and sources for more information.

7. See Ellickson (1971), Westhoff (1977), Epple, Filimon, and Romer (1984), Goodspeed (1989), Epple and Romer (1991), Fernandez (1997), Brueckner (2000),and the papers that focus on schooling discussed next.

8. Nechyba's jurisdictions are divided into neighborhoods that differ by their qualities of housing, but public-school students in all neighborhoods within a jurisdiction attend the same school. The primary effect of neighborhoods in his analysis is to increase mobility of households among jurisdictions that send their children to private schools.

9. Note that there is no loss in generality in specifying exponents on the numeraire and q to both be unity. Any specification: $\tilde{U} = [y(1-t)-p]^{z_1} q^{z_2} b^{z_3}$ can be rewritten as (1) by letting $U \equiv \tilde{U}^{1/z_1}$ and by redefining what is called "quality."

10. We will describe neighborhoods as having one school. If neighborhoods have more than one school for students at a given level, then they must be of homogeneous quality. Hence, within neighborhood policy (e.g. within neighborhood choice) and the economic environment (e.g. frictionless within neighborhood choice) imply homogeneity.

11. The model applies precisely to statewide finance if households are exogenously assigned to regions (e.g., by employment opportunities) and regions are homogeneous with regard to their distributions of household types and their neighborhoods' housing supplies. Alternatively, the model applies precisely if choice of neighborhood in the entire state is determined by schooling. These are obviously very strong assumptions, but are needed only to be consistent with the model's determination of voting equilibrium. Key results like stratification across neighborhoods use only that expenditure is constant across neighborhoods, and then would

hold for appropriately modified determination of voting equilibrium.

12. Perhaps surprisingly, if voting occurs in the second stage and school choice is last, all results below are correct so long as equilibrium exists under this alternative. This is because, for one subset of cases we study, the exercise of choice does not vary with the tax; and, for the remaining cases, the preferred tax of voters is (locally) independent of anticipated variation in the exercise of choice. The problem with the alternative timing is that existence of voting equilibrium is not guaranteed in the latter cases, leading us to adopt the timing in Figure 1. We developed the results with the alternative timing in an earlier version of this paper (Epple and Romano, 1995) while simply assuming existence.

13. The influence of ability on own educational achievement is well documented and not controversial (see Hanushek, 1986). In the economics literature, Henderson, Mieszkowski, and Sauvageau (1978) and Summers and Wolfe (1977), Toma (1996), and Zimmer and Toma (1997) find significant peer group effects. Evans, Oates, and Schwab (1992) adjust for selection bias in the formation of peer groups and show that it eliminates the significance of the peer group in explaining teenage pregnancy and dropping out of school. They are careful to point out that their results should not be interpreted as suggesting that peer-group effects do not exist, but that scientific demonstration of those effects is inadequate. Note, too, that their work supports the notion that peer group variables enter the utility function since a selection process does take place. The psychology literature on peer group effects in education also contains some controversy. See Moreland and Levine (1992) for a survey that concludes:

> "The fact that good students benefit from ability grouping, whereas poor students are harmed by it, suggests that the mean level of ability among classmates, as well as variability in their ability levels, could be an important factor. The results from several recent studies ... support this notion."

Theoretical models of education that incorporate peer-group effects include Arnott and Rowse (1987), Manski (1992), Eden (1992), Rothschild and White (1995), Epple, Newlon, and Romano (1997), Epple and Romano (1998a,1998b), and Caucutt (forthcoming), in addition to the papers discussed in the Introduction.

14. In fact, many of the results are independent of b's positive impact on the child's own achievement. This is so when assumption (A2-1) presented below holds.

15. The monitoring interpretation of the public school input suggests that teacher-administrator contracts should reflect school quality. This interpretation of our model embraces the belief that in fact users of a school are the primary enforcers of (implicit) contracts, and they vary in their ability and willingness to do so. McMillan (1999) develops a related theoretical model. For a study of the effects of centralized versus decentralized school finance systems on the effectiveness of explicit incentive contracts with school administrators see Hoxby (1995).

16. For any <u>given</u> income, only higher ability students attend higher quality schools.

17. We continue to assume everyone can afford a house, again justified below. Specifically, it must be that $y_m(1-t) > c$.

18. The argument that an equilibrium that is not maximally stratified will tend to be unstable is as follows. An arbitrary finite perturbation of the residences across two neighborhoods that begin with $\theta_1 = \theta_2$ would generally imply differences in the peer measures. Households would relocate toward the higher-quality neighborhood and bid up its relative housing price, implying the relocation pattern would satisfy income stratification (as in Proposition 1b). In turn, the relocations and assumption (A1) would imply a greater quality differential. And so on. To formalize this argument, one needs to make assumptions about the rates at which types can relocate and what they anticipate, if anything, would change. Consider an example. Suppose there are two neighborhoods, initially homogenous with $p_1 = p_2 = c$. Perturb their residences so that θ_2 is a little higher than θ_1. Now let an arbitrary selected positive measure of types relocate and suppose that they anticipate no changes in variables due to their own relocations. The housing market price that clears their housing exchanges must have $p_2 > p_1$ (the latter price might be anchored at c), and the relocation pattern must

satisfy income stratification among those moving. But then θ_2 would rise further and θ_1 would decline further, these by (A1) and that the measure permitted to move was selected arbitrarily (i.e., the θ's would change as stated with probability one).

19. Voting equilibrium can be shown to exist much more generally. See Roberts (1977), Epple and Romer (1991), and Gans and Smart (1996).

20. The argument applies if point (X^*,t^*) is at a boundary of the choice space, e.g., if $t^* = (y_m-c)/y_m$. Then one of the regions identified in what follows is simply empty.

21. The reader may note that in our initial application of Lemmas 1 and 2 below, because the feasible choice set is convex, voting preferences are single peaked. One reason that we develop the results relying instead on Lemmas 1 and 2 is because the technique can be applied in variations of the model where single peakedness may not be present.

22. Absent satisfaction of (A3), households with income y_x (who live in neighborhood 2) and those with income y_m (who live in neighborhood 1) both have median preferences. Their preferences are distinct and multiple majority voting equilibria exist, made up of the preferred points of the two types and, generally, some points "in between."

23. This requires $\alpha(1-c/y_{p1})/(1+\alpha) < (y_m-c)/y_m$. One set of sufficient conditions is $\alpha < 1$ and $y_m > 2c$.

24. The same results obtain if there are capacity constraints but every applicant, independent of residence, has the same probability of admittance. Aggregate uncertainties disappear because of the atomism of households.

25. We also ignore any possible changes in transportation costs born by tax payers. Explicit consideration of transportation costs born by tax payers would not effect the set of equilibrium residential and school choices, since these costs are invariant to individual choices. However, equilibria which will be seen to be economically equivalent in the text's model, would no longer be equivalent because gross and net (of transportation costs) per student expenditure would vary with the equilibrium allocation. This is easy to model, but, we think, not very important.

26. The parameter restrictions provided in footnote 23 continue to be sufficient.

27. The parameter restrictions in footnote 23 remain sufficient.

28. Multiple voting equilibria cannot arise in either neighborhood.

29. The sufficient conditions in footnote 23 continue to be enough assuming $p_1 = c$. These conditions are sufficient for an interior solution in neighborhood 1. Given an interior solution in neighborhood 1, the solution in neighborhood 2 must permit the median-income household to purchase a home there, since that household is indifferent to residing in neighborhood 1.

30. Given existence, neither can we show uniqueness in this case.

31. There is an income and substitution effect on the preference for taxes of changing the tax base ($\bar{y}_i$). "Consuming" a lower tax rate at the same E is feasible when $\bar{y}_i$ rises, but the marginal sacrifice of X from reducing t also increases. These offset exactly with the Cobb-Douglas preferences.

32. As discussed in the Introduction, an alternative version of this result can be found in Benabou (1993,1998).

33. It is correct to observe that even for small α in the Cobb-Douglas case stratification may result with multiple jurisdictions if $E[b|y]$ is invariant to y, while equilibrium will not be stratified in the single-jurisdiction model. Note, however, that only slightly rising $E[b|y]$ gets back stratification in the latter model. Moreover, for utility specifications satisfying (A2-2) and $E[b|y]$ flat, near equivalence holds as expenditure effects disappear.

34. In 1993-94, 28.6 percent of school districts had inter-district choice policies (National Center for Education Statistics, 1996). However, only 1.6% of public-school students residing in these districts attended school outside of the district where they resided.

35. Whether outsiders are allowed to vote or note does not matter to equilibrium.

36. Actually, our equilibrium results do not require a binding borrowing constraint on "high-income households," specifically those with income above the maximum income of any pivotal voter. If demand ceases to increase with income above this threshold, all our equilibrium results continue to hold.

37. If one thinks of the multi-school district as a central city, then the second is the more likely outcome since forces not in our model lead poor to live in citeis (Glaeser, Kahn, and Rappoport, 2000).

38. Incorporating a private schooling sector is obviously of interest, an extension that we are currently pursuing.

TABLE 1

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_1				10,434
θ_1	28,565	36,360	28,565	18,450
θ_2	44,154	36,360	44,155	36,777
t_1	.051	.052	.049	.051
t_2	.051	.052	.051	.051
X_1	1,858	1,880	958	1,865
X_2	1,858	1,880	2,706	1,865
p_2	3,142	2,500	4,569	2,800
q_1	2.91	2.95	2.79	2.83
q_2	2.98	2.95	3.05	2.95
CV_1		114	-306	-29.5
CV_2		43.6	-176	-90.5
$\Delta p_2/2$		-321	714	-171
ΔW		-163	232	-291

Parameters Utility/Cost Function: α = .06 γ = .06 c = 2,500

Parameters Distribution Function: ρ = .4 y_m = 5000 $\bar{y}$ = 36,360

y_{q1} = 19,480 y_{med} = 28,860 y_{q3} = 44,290

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = .7397

$\sigma_{\ln b}$ = .68

TABLE 2

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_I				7598
θ_1	28,565	36,360	28,565	14,392
θ_2	44,155	36,360	44,155	36,390
t_1	.043	.043	.042	.043
t_2	.043	.043	.043	.043
X_1	1,561	1,581	806	1,564
X_2	1,561	1,581	2,283	1,564
p_2	3,254	2,500	4,456	2,800
q_1	2.96	3.01	2.87	2.82
q_2	3.05	3.01	3.11	3.01
CV_1		134	-256	-14.9
CV_2		50.9	-148	-98.3
$\Delta p_2/2$		-377	601	-277
ΔW		-192	196	-340

Parameters Utility/Cost Function: α = .05 γ = .07 c = 2,500

Parameters Distribution Function: ρ = .40 y_m = 5000 $\bar{y}$ = 36,360

y_{q1} = 19,480 y_{med} = 28,860 y_{q3} = 44,290

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = .7397

$\sigma_{\ln b}$ = .68

Change From Table 1: α lower; γ higher

TABLE 3

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_I				14,528
θ_1	28,565	36,360	28,565	22,074
θ_2	44,155	36,360	44,155	38,078
t_1	.059	.060	.057	.060
t_2	.059	.060	.059	.060
X_1	2,152	2,173	1,107	2,184
X_2	2,152	2,173	3,119	2,184
p_2	3,031	2,500	4,681	2,800
q_1	2.86	2.89	2.73	2.82
q_2	2.92	2.89	3.00	2.90
CV_1		94.2	-354	-31.4
CV_2		36.3	-203	-58.6
$\Delta p_2/2$		-265	825	-115
ΔW		-135	268	-205

Parameters Utility/Cost Function: α = .07 γ = .05 c = 2,500

Parameters Distribution Function: ρ = .40 y_m = 5000 $\bar{y}$ = 36,360

y_{q1} = 19,480 y_{med} = 28,860 y_{q3} = 44,290

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = .7397

$\sigma_{\ln b}$ = .68

Change From Table 1: α higher; γ lower

TABLE 4

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 500) 1 Jurs.
y_I				12,352
θ_1	28,844	36,360	28,844	14,857
θ_2	47,876	36,360	47,876	37,630
t_1	.051	.052	.049	.051
t_2	.051	.052	.050	.051
X_1	1,849	1,880	958	1,862
X_2	1,849	1,880	2,686	1,862
p_2	3,461	2,500	4,870	3,000
q_1	2.88	2.95	2.77	2.80
q_2	3.00	2.95	3.07	2.96
CV_1		179	-306	-49.9
CV_2		89.1	-170	-112
$\Delta p_2/2$		-480	705	-230
ΔW		-212	229	-392

Parameters Utility/Cost Function: $\alpha = .06$ $\gamma = .06$ $c = 2{,}500$

Parameters Distribution Function: $\rho = .60$ $y_m = 5000$ $\bar{y} = 36{,}360$

$y_{q1} = 19{,}480$ $y_{med} = 28{,}860$ $y_{q3} = 44{,}290$

$\bar{b} = 36{,}360$ $b_{med} = 28{,}860$ $\sigma_{\ln y} = .7396$

$\sigma_{\ln b} = .68$

Change From Table 1: ρ higher; T higher

TABLE 5

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_I				26,966
θ_1	32,427	36,360	32,427	32,102
θ_2	40,293	36,360	40,293	39,923
t_1	.051	.052	.049	.051
t_2	.051	.052	.051	.051
X_1	1,869	1,880	958	1,863
X_2	1,869	1,880	2,727	1,863
p_2	2,822	2,500	4,268	2,800
q_1	2.93	2.95	2.82	2.93
q_2	2.97	2.95	3.04	2.97
CV_1		54.3	-306	-4.22
CV_2		13.8	-181	-2.42
$\Delta p_2/2$		-161	723	-11.0
ΔW		-92.9	236	-17.7

Parameters Utility/Cost Function: α = .06 γ = .06 c = 2,500

Parameters Distribution Function: ρ = .20 y_m = 5000 $\bar{y}$ = 36,360

y_{q1} = 19,480 y_{med} = 28,860 y_{q3} = 44,290

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = .7396

$\sigma_{\ln b}$ = .68

Change From Table 1: ρ lower

TABLE 6

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_I				13,644
θ_1	28,565	36,360	28,565	21,416
θ_2	44,155	36,360	44,155	37,748
t_1	.048	.049	.045	.049
t_2	.048	.049	.049	.049
X_1	1,752	1,773	873	1,790
X_2	1,752	1,773	2,612	1,790
p_2	4,605	4,000	5,952	4,300
q_1	2.90	2.94	2.78	2.85
q_2	2.97	2.94	3.05	2.95
CV_1		104	-278	-25.2
CV_2		23.5	-149	-63.3
$\Delta p_2/2$		-303	673	-153
ΔW		-166	247	-241

Parameters Utility/Cost Function: α = .06 γ = .06 c = 4,000

Parameters Distribution Function: ρ = .40 y_m = 5000 $\bar{y}$ = 36,360

y_{q1} = 19,480 y_{med} = 28,860 y_{q3} = 44,290

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = .7397

$\sigma_{\ln b}$ = .68

Change From Table 1: c higher

TABLE 7

	Neigh. Sch. 1 Jurs.	Choice (T = 0) 1 Jurs.	Neigh. Sch. 2 Jurs.	Choice (T = 300) 1 Jurs.
y_I				12,905
θ_1	28,565	36,360	28,565	22,860
θ_2	44,155	36,360	44,155	38,534
t_1	.051	.052	.048	.053
t_2	.051	.052	.051	.053
X_1	2,300	2,326	839	2,365
X_2	2,300	2,326	3,708	2,365
p_2	3,142	2,500	5,139	2,800
q_1	2.94	2.99	2.77	2.91
q_2	3.02	2.99	3.11	3.00
CV_1		100	-412	-15.7
CV_2		-63.2	-67.4	-96.0
$\Delta p_2/2$		-321	998	-171
ΔW		-284	519	-283

Parameters Utility/Cost Function: α = .06 γ = .06 c = 2,500

Parameters Distribution Function: ρ = .40 y_m = 5000 $\bar{y}$ = 45,000

y_{q1} = 17,020 y_{med} = 28,860 y_{q3} = 52,360

$\bar{b}$ = 36,360 b_{med} = 28,860 $\sigma_{\ln y}$ = 1.017

$\sigma_{\ln b}$ = .68

Change From Table 1: $\sigma_{\ln y}$ higher

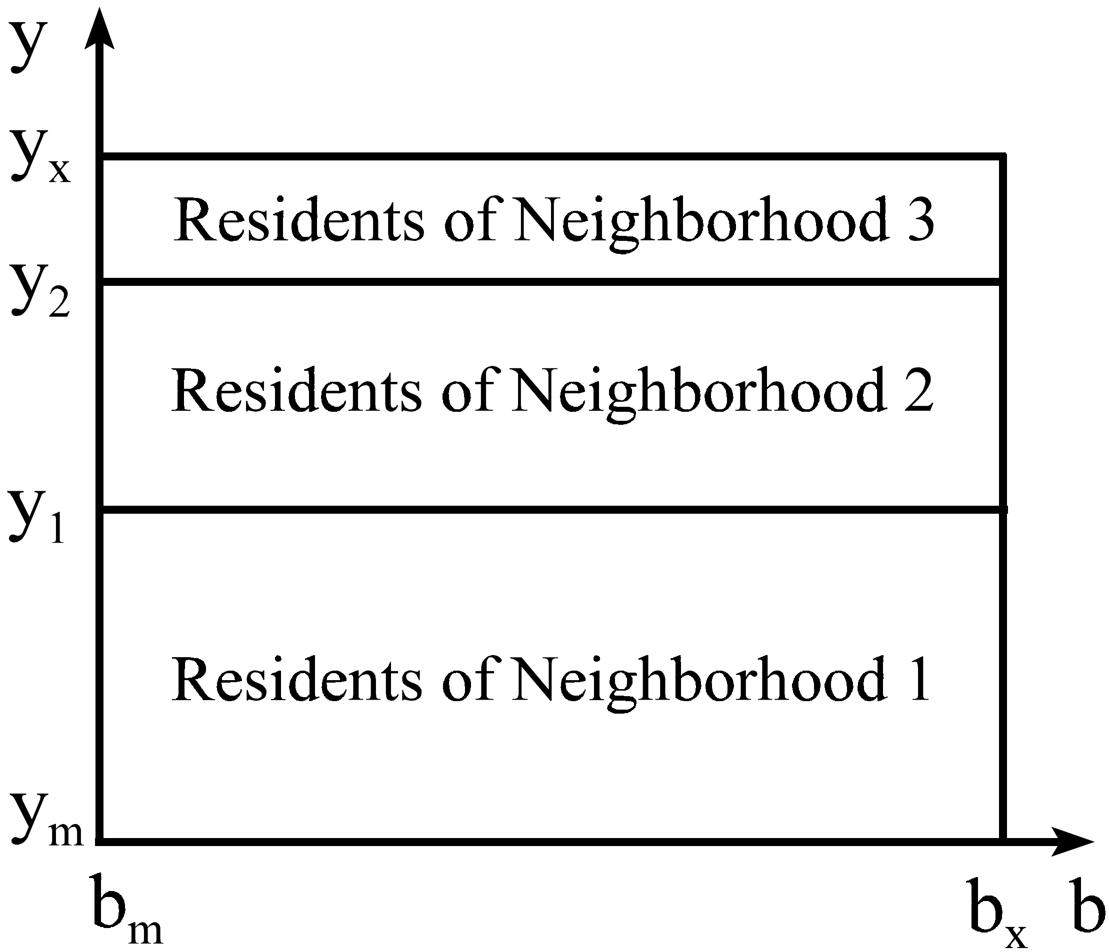
y
y_x
Residents of Neighborhood 3
y_2
Residents of Neighborhood 2
y_1
Residents of Neighborhood 1
y_m
b_m
b_x
b

Figure 2

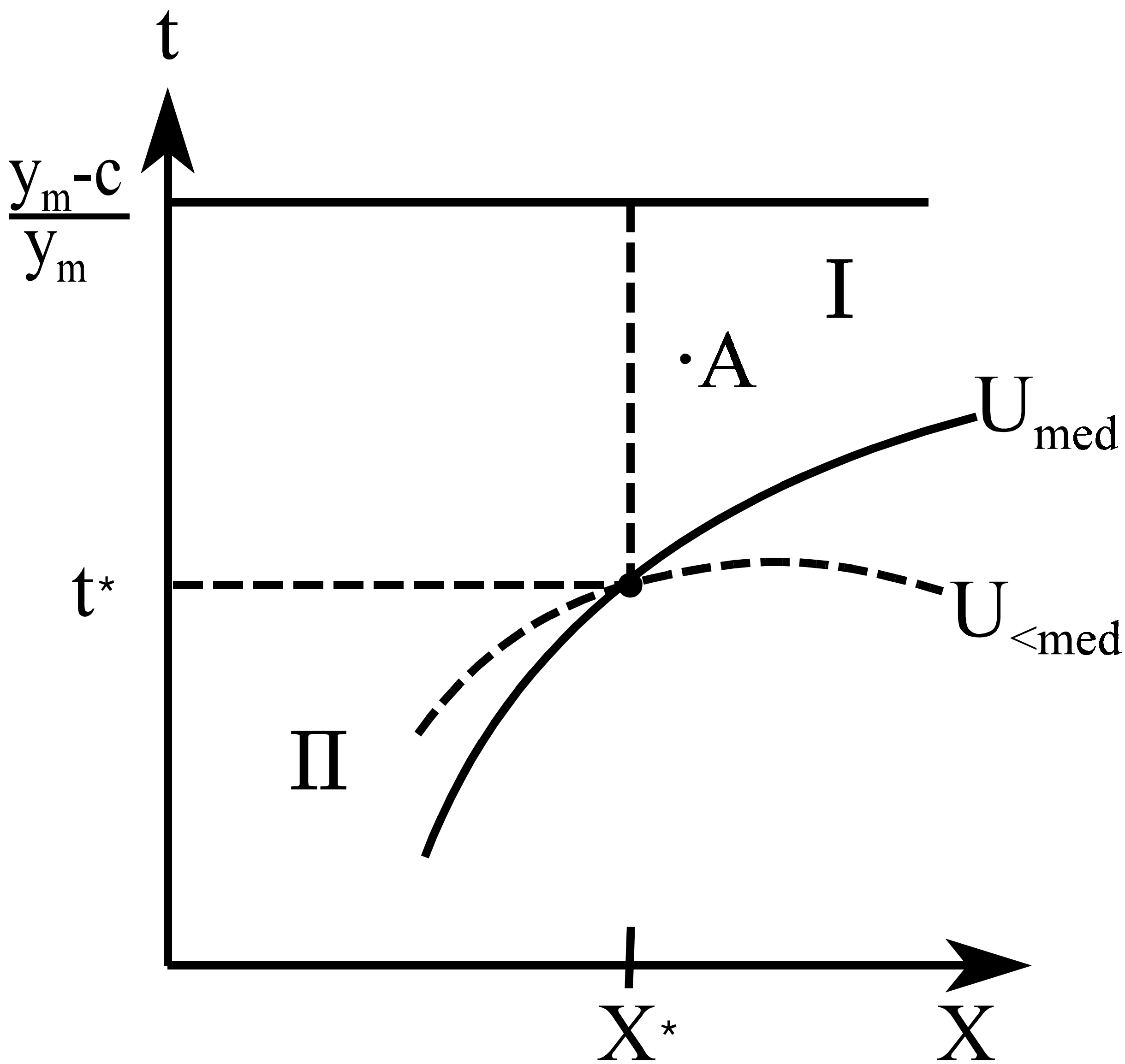

Figure 3

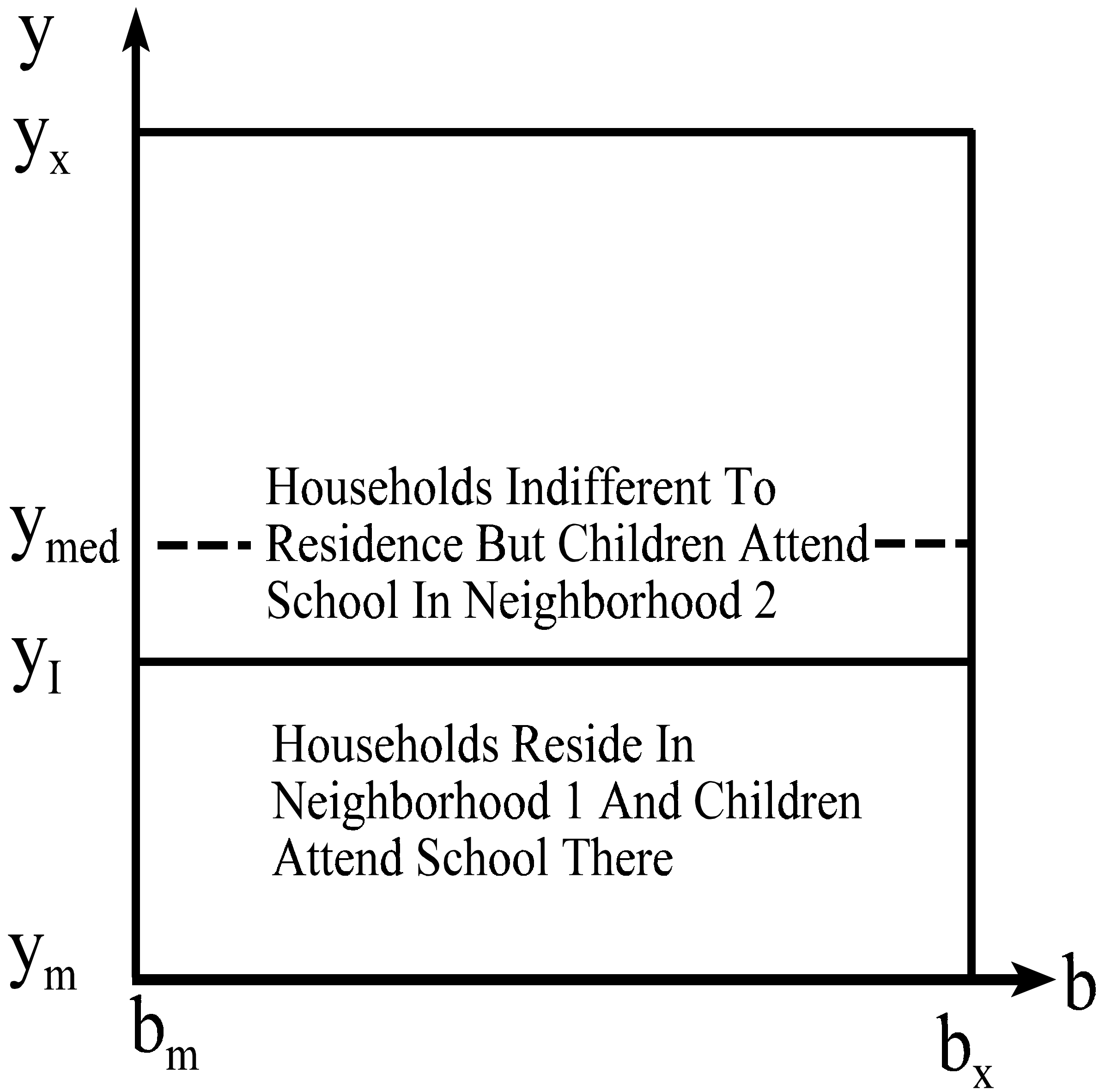
y
y_x
y_{med}
y_I
y_m
Households Indifferent To Residence But Children Attend School In Neighborhood 2
Households Reside In Neighborhood 1 And Children Attend School There
b_m
b_x
b

Figure 4

BOUNDARY LOCUS FOR EFFICIENT ALLOCATION

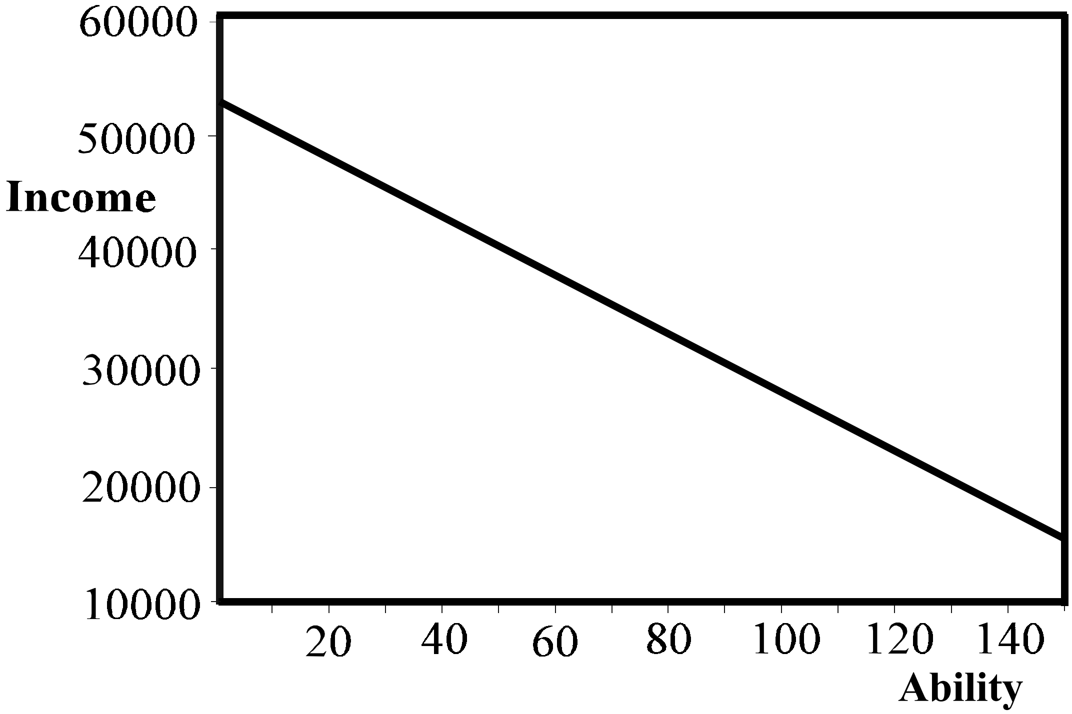

Figure 5
(Parameter values are as in Table 1.)

Allocation Values:

	Neighborhood 1	Neighborhood 2
q_i:	2.84	3.15
θ_i:	28,481	58,081
x_i:	1,234	3,623
$\left(\frac{q_\theta}{q_x}\right)_i$:	.044	.065
% population:	72	28

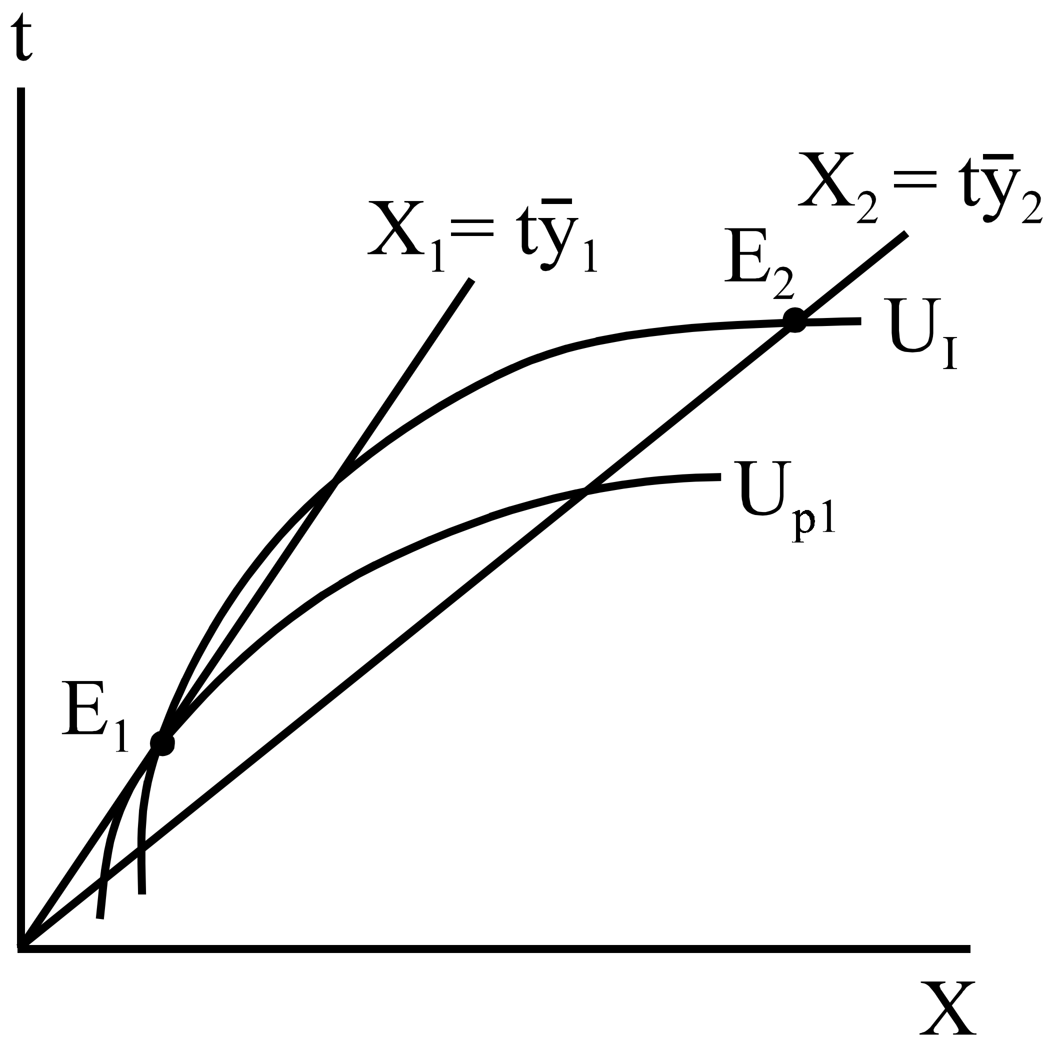

Figure 1A

Appendix for "Neighborhood Schools, Choice, and the Distribution of Educational Benefits"

<u>Derivation of (8-1) - (8-3)</u>

Let $R \equiv \theta_1/\theta_2$ and write the equilibrium conditions (4), (5) and (7):

$$p_2 = c + [y_{med}(1 - t) - c][1 - R^{\gamma}]; \tag{a1}$$

$$F_y(y_{p1}) + F_y(\frac{p_2}{c}y_{p1}) = 1 \quad ; \tag{a2}$$

and

$$t = \frac{\alpha}{1+\alpha}(1 - \frac{c}{y_{p1}}); \quad x = t\bar{y}. \tag{a3}$$

To confirm (8-1), differentiate (a1) - (a3) letting R change:

$$dp_2 = -y_{med}[1 - R^{\gamma}]dt - \gamma R^{\gamma-1}[y_{med}(1 - t) - c]dR; \tag{a4}$$

$$[f_y^1 + f_y^2 \frac{p_2}{c}]dy_{p1} + f_y^2 \frac{y_{p1}}{c} dp_2 = 0; \tag{a5}$$

and

$$dt = \frac{\alpha}{1+\alpha} \frac{c}{y_{p1}^2} dy_{p1} \; ; \quad dx = dt \cdot \bar{y}; \tag{a6}$$

where $f_y^1 \equiv f_y(y_{p1})$ and $f_y^2 \equiv f_y(\frac{p_2}{c}y_{p1})$. Solving (a4) - (a6) for dp_2/dR yields:

$$\frac{dp_2}{dR} = -\frac{\gamma R^{\gamma-1}[y_{med}(1 - t) - c]}{1 - [1 - R^{\gamma}]\frac{\alpha}{1+\alpha}\frac{y_{med}}{y_{p1}}\frac{f_y^2}{f_y^1 + f_y^2\frac{p_2}{c}}}. \tag{a7}$$

Since the numerator of (a7) is positive, the sign of $\frac{dp_2}{dR}$ is negative if the denominator of (a7) is positive. One can see by inspection of the denominator that it must be positive if

$y_{med}f_y^2 < y_{p1}f_y^1 + y_{p1}f_y^2\frac{p_2}{c}$. The latter holds since $y_{p1}\frac{p_2}{c} = y_{p2}$ and $y_{p2} > y_{med}$. Since $\frac{\theta_2}{\theta_1} = R^{-1}$, $\frac{dp_2}{dR} < 0$ implies the first result in (8-1). Since $\frac{dp_2}{dR} < 0$, $\frac{dy_{p1}}{dR} > 0$ by (a5). Then the second result in (8-1) follows from (a6) (keeping in mind that $\theta_2/\theta_1 \equiv R^{-1}$).

Observing that γ appears only as an exponent on R in (a1) - (a3) and that $R = \theta_1/\theta_2 < 1$, the results in (8-2) follow from those in (8-1).

To confirm (8-3), differentiate (a1) - (a3) letting α change, yielding:

$$dp_2 = -y_{med}[1 - R^{\gamma}]dt \quad ; \tag{a8}$$

$$dt = \frac{1 - c/y_{p1}}{(1 + \alpha)^2}d\alpha - \frac{\alpha}{1 + \alpha}\frac{c}{y_{p1}^2}dy_{p1} \quad ; \quad dx = dt \cdot \bar{y} \quad ; \tag{a9}$$

and, again, (a2). Solving for $\frac{dt}{d\alpha}$ yields an obviously positive expression, then implying $\frac{dx}{d\alpha} > 0$ by (a9). The first result in (8 - 3) follows from (a8).

<u>An Inter-Jurisdictional Choice Policy</u>.

The model is the same as in Section 3C of the text except for the school-choice policy. Following residential choice, households commit to attend school in their own neighborhood or the other one. Every household then votes for the tax-expenditure pair with those committed to the same school, and with tax base consisting of that school's households. Hence, those that attend a school comprise a jurisdiction independent of their residences. We assume transportation costs are negligible (zero).

We show that equilibrium will "likely" be the same as when there is frictionless choice across two neighborhoods of one jurisdiction. Since school and thus jurisdictional membership is independent of the first-stage residential choice, housing prices must be the same in the two neighborhoods. Hence, $p_2 = p_1 = c$, as we have argued earlier that $p_1 = c$ is a sensible convention.

The difference in utility if school and jurisdiction 2 are selected from that if school and jurisdiction 1 are selected is given by:

$$\Delta \equiv b^{\beta}\{[y(1 - t_2) - c]q_2 - [y(1 - t_1) - c]q_1\}$$
$$= b^{\beta}\{[(1 - t_2)q_2 - (1 - t_1)q_1]y - c(q_2 - q_1)\}. \qquad (a10)$$

Assuming $q_2 \geq q_1$ with no loss in generality, we see using (a10) that equilibrium has either:

(a) $q_2 > q_1$; $t_2 > t_1$; and income stratification;

or

(b) $q_2 = q_1$; $t_2 = t_1$; and all households indifferent to their school/jurisdictional choice.

If $q_2 > q_1$, then income stratification is implied by the linearity of Δ in y. For this case, if not $t_2 > t_1$, then school/jurisdiction 2 would be preferred by all. We emphasize that the conditions in (a) are just selected necessary conditions for a stratified equilibrium; they are not sufficient. If $q_2 = q_1$ and $t_2 \neq t_1$, then (a10) would imply everyone prefers the school/jurisdiction with lower tax rate. (If everyone were in one school, then a rich type with a bright child would be better off attending his own school.)

For realistic parameterizations, stratified equilibrium will not exist. We show why with some intuitive arguments, in lieu of a (more lengthy) computational analysis. Assume that there is a stratified equilibrium. Then $q_2 > q_1$ both because the rich school/jurisdiction has a better peer group and a wealthier tax base ($t_2 > t_1$ is a necessary condition recall). Such an equilibrium would have an indifferent household (income) as well, for whom (a10) would vanish. But it is very difficult to satisfy all these conditions. The reason is that the tax rate that will be selected in equilibrium by the rich will not typically be high enough to keep out the poorer types, and there is no longer a housing price differential that can serve as a deterrent.

To see this, first suppose that c is small. Then the equilibrium tax rates will hardly differ. (a3) describes the tax rate in each jurisdiction if y_{p1} is replaced by the median income in the jurisdiction. As $c \to 0$, $t_1 \to t_2$ for any allocation, and no indifferent household can exist. An analogous argument precludes stratified equilibrium as $\alpha \to 0$.

Another way to see the difficulty in obtaining a stratified equilibrium is by a graphic analysis. Assume initially that $E[b|y]$ is invariant to y. We will show that it is quite difficult to obtain a stratified equilibrium and then show $E[b|y]$ that increases in y makes it "more difficult."

Assuming a stratified equilibrium, Figure 1A depicts in the (X, t) plane a "voting indifference curve" of the pivotal voter in the poor school/jurisdiction (U_{p1}), an indifference curve of the type indifferent between schools (U_I), and the budget constraints of the two school/jurisdictions. (The indifference curves are those discussed in and preceding Lemma 1.) E_1 shows the equilibrium expenditure per student and tax rate in school/jurisdiction 1. The indifferent household has indifference curve through E_1 that is steeper than is the pivotal voter's because the former type has higher income. (See Lemma 1.) (Note that in the extreme of $c = 0$, the indifference mappings of all types would be the same, again precluding equilibrium as we will see momentarily.) Equilibrium in school/jurisdiction 2 would have to be at point E_2 so that "type I" is indifferent. Hence, the indifference curve of the pivotal voter in school/jurisdiction 2 (i.e., the median-income type there) would need to be tangent to $X_2 = t\bar{y}_2$ at E_2. This indicates that preferences for X needs to rise precipitously with income to obtain such an equilibrium. If, for example, α is small, then this will not occur.

If we now let $E[b|y]$ increase with y, then the difficulty is exacerbated. The preference mappings are unchanged (due to the Cobb-Douglas specification), but the values of utility are higher in jurisdiction 2 than in jurisdiction 1. Utility at E_2 in school 2 is then higher than utility at E_1 in school 1 for type I. This implies the equilibrium point in jurisdiction 2 is higher up $X_2 = t_2\bar{y}_2$ than E_2.

Given equilibrium of type (a) above does not exist, then equilibrium is of type (b). In such an equilibrium, everyone is indifferent to their residential and school/jurisdictional choice. Hence, assume types randomize over their choices, all with the same probabilities. This implies schools and jurisdictions that are homogeneous, i.e., each school's distribution of types is the same as the population distribution. The outcome is the same as with frictionless choice and one jurisdiction.

Proof of Proposition 11.

Proposition 11 is essentially an application of results in Epple and Romano (1998a, 1998b). Here we sketch proofs for convenience.

a. This is the "strict hierarchy result" in the papers just cited, developed assuming fixed expenditures across schools in Epple and Romano (1998a) and extended to variation in expenditure in Epple and Romano (1998b). The proof proceeds as follows. Assume $q_1 = q_2$ and show a Pareto improvement is feasible. First show that $q_1 = q_2$ implies $X_1 = X_2$ and $\theta_1 = \theta_2$ using quasiconcavity of $q(X,\theta)$. If say $\theta_2 > \theta_1$, then $X_2 < X_1$ and q_θ/q_X is higher in school 1 than in school 2. Using Proposition 10 and the definition of SMC_i, it is implied that there is an ability threshold B, such that all types with $b >$ (<) B would choose to attend school 1(2). This contradicts $\theta_2 > \theta_1$.

Having established $q_1 = q_2$ implies $X_1 = X_2$ and $\theta_1 = \theta_2$, the schools can be regarded as having homogeneous student bodies (with respect to both b and y). Then it is shown that one can engender a Pareto improvement by having the schools exchange students in a particular way that leads one school to be of higher quality, with more able and also richer students, and the opposite for the other school. The Pareto improvement does not require changes in X_1 or X_2. Mathematically, this is somewhat involved since it relies on second-order effects (as first-order effects vanish), so we refer the reader to the Proof of Proposition 1 in Epple and Romano (1998a). The intuition is, however, not too complicated: Those in the improved school are obviously better off. Those in the school that has deteriorated are better off because the contribution to costs of the departed students are relatively low due to their high abilities and thus low SMC, and the reduced quality is of relatively low "cost" since the student body becomes relatively poor and cares less about quality.

b. Given $q_2 > q_1$, income stratification follows by (A - 1) and that prices depend only on student ability (see (21)). That is, for any given ability, if there is a household indifferent to the schools when $r_i = SMC_i$, $i = 1,2$, then all types having higher (lower) income strictly prefer school 2(1). (If there is no indifferent type, then all types with that ability attend one of the two schools.)

The indifferent set in (24) is found simply by equating utilities given $r_i = SMC_i$ and using the definition of η_i.

c. Equation (25) is found from (22) (and the analogue for X_2), again using $r_i = SMC_i$, $i = 1,2$. Specifically, for $i = 1$:

$$\frac{\partial U^1(b,y)/\partial X}{\partial U^1(b,y)/\partial y} = \frac{\alpha}{X_1}(y - c - r_1) \tag{a11}$$

$$= \frac{\alpha}{X_1}\left(y - c - X_1 - \frac{q_\theta}{q_X}(\theta_1 - b)\right)$$

$$= \frac{\alpha}{X_1}\left(y - c - X_1 - \frac{\gamma}{\alpha}\frac{X_1}{\theta_1}(\theta_1 - b)\right)$$

$$= \frac{\alpha}{X_1}(y - c) - \alpha - \frac{\gamma}{\theta_1}(\theta_1 - b);$$

the second-to-last equality using:

$$\frac{q_\theta}{q_x} = \frac{\gamma X}{\alpha\theta}. \tag{a12}$$

Then substitute (a11) into (22) and rearrange, while noting that the density of types in school 1 is given by $Af/\int\int Afdbdy$. X_2 is found analogously.

The result in (26) is found by substituting (25) into (a12).

d. & e. These are efficiently proved together. Note from (24) that the boundary locus in the (b,y) plane is linear (see Figure 5 for an example), so that, given we have established income stratification, ability stratification corresponds to a downward sloping boundary locus. Suppose this locus is not downward sloping. Then, for $E[b|y]$ constant in y, $\theta_2 \le \theta_1$. Hence, $q_2 > q_1$ implies $X_2 > X_1$. This implies $\eta_2 > \eta_1$, which by (24) implies a downward sloping boundary locus -- a contradiction.

Hence, the boundary locus *is* downward sloping, obviously also implying $\theta_2 > \theta_1$.

f. $\eta_i q_i = \frac{\gamma}{\alpha} X_i^{1+\alpha}\theta_i^{\gamma-1}$. By part e of this proposition and (24), $\eta_2 q_2 > \eta_1 q_1$. Using part d of this proposition then, $X_2 > X_1$ whenever $\gamma < 1$. ■