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THE TIMING OF PURCHASES AND AGGREGATE FLUCTUATIONS

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ABSTRACT

This paper analyzes how the decision of when to buy a durable good affects both non-durable consumption and business cycle dynamics. At the individual level, we show that the timing of durable goods purchases plays an important role in smoothing consumption over time. In the benchmark case, the time at which the agent purchases the durable good is the only variable that reacts to changes in wealth, while other variables, such as the consumption of non-durables or the amount of the durable that the individual purchases, remain unchanged. At the aggregate level, we show that timing decisions can serve as a mechanism for the amplification and propagation of aggregate shocks. A decline in wealth causes individuals to delay their durable goods purchases which reduces demand dramatically for some time.

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1 Introduction

Many durable goods are purchased by consumers in discrete amounts at specific points in time. This paper analyzes how these timing decisions affect both non-durable consumption and business cycle dynamics. At the individual level, we show that the timing of durable goods purchases plays an important role in smoothing consumption over time. In the benchmark case, the time at which the agent purchases the durable good is the only variable that reacts to changes in wealth, while other variables, such as the consumption of non-durables or the amount of the durable that the individual purchases, remain unchanged. We call this result the "insulation effect." At the aggregate level, we show that timing decisions can serve as a mechanism for the amplification and propagation of aggregate shocks. A decline in wealth causes individuals to delay their durable goods purchases. This delay reduces demand dramatically for some time. The decline, however, is only temporary. As agents rebuild their wealth, demand for the durable returns to its pre-shock level.

The microeconomic framework of the paper consists of an infinitely lived individual, who consumes two goods, one is non-durable and the other is durable and non-depreciating. Utility is additively separable across time and depends on the flow of non-durable consumption and on the size of the durable. Due to some non-concavity in tastes or to the presence of a transaction cost, the consumer prefers not to purchase small amounts of the durable early in life, but rather waits, accumulates wealth, and makes a discrete purchase later in life. Utility maximization occurs on three margins: the amount of non-durable consumption in each period, the size of durable purchased, and the time of purchase. We show that there is a range of wealth levels, for which the individual chooses to purchase the durable at some positive and finite time. Within this range, changes in wealth affect only the timing of the purchase and not the size of durable or the consumption of the non-durable. Whereas complete insulation only occurs under special conditions, such as a constant relative price of the durable good, we show that there is always partial insulation. In more general settings, consumers still alter the timing of the durable purchase in response to changes in wealth, but the responses of the size of the durable and non-durable consumption are ambiguous. These responses depend on how the timing decision interacts with the price process.

The simplicity of the microeconomic model allows us to embed discrete purchases in an aggregate framework. We consider a small open economy in which infinitely lived individuals are born continuously and population is increasing in a fixed rate. Each individual produces a fixed amount of one of the two goods with a constant returns to scale technology and mobility between the two sectors is free. Hence, the relative price of the durable good is fixed and individuals are as in the benchmark model above. We consider two types of shocks: a sudden decline of financial wealth, and a sudden temporary decline in productivity. Each shock creates a decline in lifetime discounted wealth, and as a result the individuals who were going to purchase the durable postpone their purchase and the demand for the durable good drops to zero. After some time individuals rebuild their wealth and durable good purchases resume.

The basic model has a number of implications for business cycle theory. One is that the delay in the purchase of durables can propagate wealth and income shocks. In the model, these shocks have long lasting effects on the demand for durables which disappear endogenously after some time. The second implication is that durable purchases are much more volatile than non-durable purchases. Shocks reduce demand for the durable drastically, while non-durable consumption is unaffected. The third implication concerns the distinction between shocks that affect all future generations, such as a permanent productivity shock, or shocks that affect only certain age cohorts, such as wealth shocks and temporary productivity shocks. In the later case, shocks lead to a bust that is followed by a boom. Agents who were about to purchase the durable at the time of the shock delay, but agents planning to purchase further in the future do not. Shocks that affect all cohorts, on the other hand, cause agents to delay at all time horizons and therefore do not cause any bunching. The final implication is also related to the case of a temporary productivity shock and is relevant for international business cycle theory. In standard RBC models with domestic productivity shocks the trade balance is acyclical or mildly procyclical. This contradicts the empirical evidence. In our model a temporary adverse productivity shock delays purchases of durable and thus reduces demand for durable considerably, which leads to a surplus in the trade balance. Hence the trade balance is countercyclical here.

Next we introduce uncertainty by extending the benchmark model to an environment with occasional shocks to wealth. In this case, individuals anticipate the distribution of shocks and take this risk into account when making their decisions. Again we find that individuals delay their purchases of the durable good, though not to a specific future period, but until their wealth reaches some threshold level. We show that the insulation effect still holds and that the aggregate dynamics are similar to the case of a one time unanticipated shock.

In most of the paper we assume that the relative price of durable is fixed. In the last section, we relax this assumption and examine how the economy reacts to wealth shocks when labor mobility between the two sectors is imperfect. If workers in the durable goods sector cannot move to the non-durable sector when demand falls, supply becomes inelastic and the relative price of durables and non-durables must adjust to clear the market. This analysis carries three main messages. First, it introduces a feedback into the economy. Durable producers lose not only from the wealth shock but also from the decline in the price of the durable good. This leads them to delay their purchase of the durable even further, which enhances the propagation mechanism. Second, this case shows that sometimes price fluctuations do not necessarily reflect flexibility of markets, and can even lengthen recessions. Third, it demonstrates that our model is rather tractable and can be solved even when prices are flexible.

There is by now a large literature which considers the infrequent purchase of durable goods. Grossman and Laroque (1990) pioneered this literature by showing that an (S,s) policy is optimal in the presence of fixed transactions costs. An number of papers have studied the aggregate effects of (S,s) policies, a few of which touch on aspects of the current paper. [1] Bar-Ilan and Blinder (1996) demonstrate empirically the importance of distinguishing between the number of durables purchased within a period and the average size of purchase, and thereby the importance of the timing of discrete purchases. Caballero (1993) shows how shocks can have persistent effects when agents follow (S,s) policies. Persistence in his model, however, comes at the cost of reducing the impact of the shock. Hassler (1996) and Carroll and Dunn (1998) show that shocks which shift the (S,s) bands can have large effects on the demand for durables. De Gregorio *et al.* (1996), Carroll and Dunn, and Adda and Cooper (1999) also discuss the possibility of boom-bust cycles in demand.

Because (S,s) policies are so difficult to place in equilibrium settings, there are few equilibrium models of durable demand with discrete purchases.[2] Parker (1996) considers a setting in which depreciation causes consumers to enter the market in waves. He analyzes the effect of intertemporal substitution on the markup of price over marginal cost. Caplin and Leahy (1997) approximate the distributional dynamics to obtain a tractable model of durable goods cycles. They illustrate the dynamic effects of various types of shocks. Neither of these papers, however, considers the feedback between the durable goods market and the rest of the economy.

[1] See, for example, Bertola and Caballero (1990), Caballero (1993), Eberly (1994), Bar-Ilan and Blinder (1996), Hassler (1996), De Gregorio *et al.* (1996), Parker (1996), Caplin and Leahy (1997) Carroll and Dunn (1998), and Adda and Cooper (1999).

[2] For recent efforts to fit (S,s) policies into general equilibrium settings see Dotsey, King and Wolman (1999) on pricing and Thomas (1999) on investment.

This paper differs from the (S,s) literature in two main aspects. First, it focuses on the interaction between durable and non-durable goods and discusses the insulation effect of the timing of purchases. Second, and more technically, we consider a life cycle model in which the durable is only purchased once. This focuses attention on the timing of purchases as a propagation mechanism for aggregate shocks. Instead of tracking the distribution of holdings, we only need to keep track of the fraction of the population that have purchased the durable. This makes aggregation and comparative statics comparatively simple. It also allows us to analyze mean reverting shocks, something that the (S,s) literature tends to avoid because they expand the state space.

In section 2, we present the benchmark microeconomic model with variations. This section is devoted to the "insulation effect." In section 3, we place the benchmark model in an aggregate setting and discuss the response of the economy to wealth shocks. Section 4 presents three extensions to the aggregate model: temporary and permanent productivity shocks, uncertainty, and feedback. Section 5 concludes.

2 The Timing of Purchases and the Insulation of Non-Durable Consumption

Our aim is to develop a simple model in which we can investigate the timing of purchases. We consider a setting in which it is optimal to make a single discrete purchase of a durable good. We motivate this discrete purchase by assuming that there is a minimum size of the durable that provides utility. This assumption makes utility convex near zero and makes small purchases unattractive to the consumer, who then must accumulate enough wealth to afford both the durable and the non-durable. Purchasing the durable earlier yields more pleasure from the durable good, but puts pressure on non-durable consumption.

While we motivate discrete purchases with an assumption on utility, anything that discourages small purchases will do. It might be the case that for technological reasons firms cannot produce the durable in small sizes. This will most likely be the case when many parts of the durable are complementary, much as the engine, wheels and chassis of a car are complementary.[3] In other cases cheap access to a quantity of the durable

[3]The existence of a minimal scale is a tricky issue. Goods such as new cars and new houses may not come in small sizes, but, if we broaden the definition of the good, it is clear that agents can purchase a used car at a fraction of the price of the cheapest new car. If we broaden the definition further, goods such as bicyles are often available at an even lower cost.

may discourage small purchases. For example, public transportation may dominate the purchase of a cheap auto or the option to live at home with one's parents may dominate a small home or apartment. Finally, we show in Appendix A that fixed costs to the purchase of the durable have the same effect as a convexity in utility.

The Basic Model

Time is continuous and there is no uncertainty.[4] An infinitely lived consumer derives utility from both the consumption of a durable good and a non-durable good. Utility is additively separable across time and between the two goods. The consumer discounts future utility at a rate ρ.

The durable comes in a variety of sizes and does not depreciate. Let $v(s)$ denote the flow of utility from a durable of size s. We assume that there is some minimum size of the durable that provides utility. Given some $\hat{s} > 0$, let $v(s) = 0$ for $0 \leq s \leq \hat{s}$, and let $v(s)$ be increasing, differentiable, and concave for $s > \hat{s}$. The flow of utility from the non-durable is $u(c)$, where c is non-durable consumption, $u'(c) > 0$, and $u''(c) < 0$.

Let T denote the time at which the consumer purchases the durable good. To abstract from issues of resale, we assume that this purchase is irreversible. The consumer's utility U is therefore:

$$U = \int_0^\infty e^{-\rho t} u(c_t) dt + \int_T^\infty e^{-\rho t} v(s) dt. \tag{1}$$

Without loss of generality we assume that the consumer begins life in period $t = 0$ with initial wealth W. We assume that the interest rate r is equal to the discount rate, $r = \rho$.[5] Consumption smoothing therefore implies that c is constant and equal to the annuity value of financial wealth less the prospective cost of the durable. Letting P_t denote the unit price of the durable at date t and normalizing the price of the non-durable to one, we have:

$$c = rW - e^{-rT} r s P_T. \tag{2}$$

The consumer therefore chooses the date of purchase T and the size of the purchase s in order to maximize

$$U = \frac{1}{r} u\left(rW - e^{-rT} r s P_T\right) + \frac{1}{r} e^{-rT} v(s). \tag{3}$$

[4]We incorporate uncertainty into the model in section 4.

[5]The case in which the interest rate and the discount rate differ is presented in Appendix B.

The first order conditions follow immediately. Differentiating with respect to T, we have:

$$\frac{v(s)/s}{u'(c)} = rP_T - \dot{P}_T \tag{4}$$

Differentiating with respect to s yields:

$$\frac{v'(s)}{u'(c)} = rP_T. \tag{5}$$

Each condition equates, in its own way, the ratio of marginal utility to relative price. Condition (5) is a static first order condition. It treats T as fixed. The relevant price of the durable is therefore the annuity value of the price at the date of purchase. Given that the price of the non-durable is normalized to one, this is also the relevant relative price. Condition (4) is a dynamic first order condition. It treats s as fixed. Given the discrete nature of the durable purchase, the marginal utility of durable consumption is equal to the average utility. The left hand side is therefore the ratio of the marginal utilities. The right hand side is the user cost of the durable.

Fixed Price

To gain intuition it is easiest to begin with the case in which the price of the durable is fixed at P. In this case, we can eliminate P from (4) and (5) to arrive at an expression for the optimal purchase size s^*

$$\frac{v(s^*)/s^*}{v'(s^*)} = 1 \tag{6}$$

Note that s^* is independent of both the price of the durable and wealth.

Given s^* and the fixed price, equation (5) pins down the optimal level of non-durable consumption at a level c^*. Note that c^* is independent of W. Finally, given c^*, s^*, and W, the optimal time of purchase T^* is determined by (2).

This case provides a stark example of how the timing of the purchase of the durable can insulate non-durable consumption from shocks, and for this reason we refer to it as the "insulation effect." Changes in wealth affect T^*, but not c^* or s^*. The consumer always purchases the durable when wealth reaches a level of $\overline{W} \equiv c^*/r + s^*P$. Changes in initial wealth merely alter the date at which this level of wealth is reached.

The insulation effect requires an interior solution for T^*. There are three cases to consider. At one extreme, if $W \leq \underline{W} \equiv c^*/r$, then the consumer does not have enough wealth to consume c^*. In this case, $T^* = \infty$. The consumer spends the annuity value of

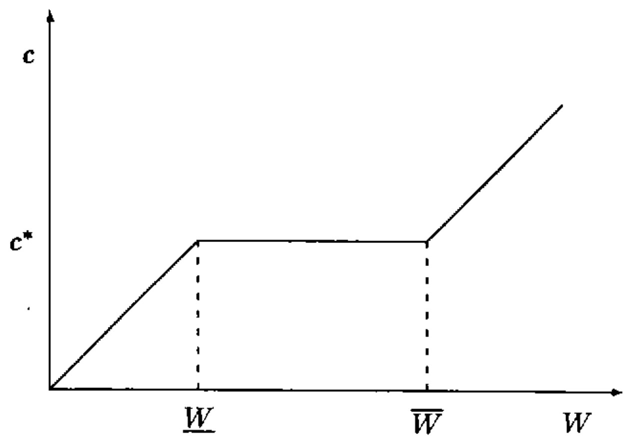

Figure 1:

wealth on the non-durable and never accumulates enough wealth to purchase the durable. At the other extreme, if $W \geq \overline{W} \equiv c^*/r + s^*P$, then the consumer has more wealth than needed to consume c^* and purchase the durable at date $T^* = 0$. In this case, the consumer purchases the durable immediately and increases both non-durable consumption and the size of the durable s according to the first-order condition (5) and the budget constraint (2). Finally, if $W \in (\underline{W}, \overline{W})$, we have $T^* \in (0, \infty)$ and consumption is equal to c^* regardless of wealth. It is over this range that the insulation effect is operative and wealth affects only the timing of the purchase of the durable. Figure 1 illustrates the effect of wealth on the consumption of the non-durable.

Discussion

The *complete* insulation of non-durable goods purchases is a very special result. It rests on three assumptions: the fixed price of the durable, the equality of the interest rate and the discount rate, and the time-additivity of preferences. The first of these ensures that changes in the timing of the purchase of the durable do not affect the marginal utility of consumption. The second ensures that the marginal utility of consumption is constant over time. The third is the source of linearity in the model. With these assumptions, adjustments in the date of purchase maintain a constant marginal utility of wealth, whereas increases in non-durable consumption or the size of the durable yield decreasing returns.

It is easy to show that the separability between durables and non-durables is not important for the insulation result. Separability merely simplifies the algebra. With non-separability, increases in c or s still involve diminishing returns, whereas changes in T^* affect utility linearly. Again, the timing of purchase is the preferred margin of adjustment.

The fixed price of the durable is clearly the most stringent of the assumptions. We therefore analyze the effects of relaxing this assumption below. We analyze the effect of choosing a discount rate that differs from the interest rate in Appendix B. In both cases the effects of a change in wealth on non-durable consumption is ambiguous and depends on the properties of the price process.

Flexible Price

We now consider arbitrary price processes. We assume throughout that the expected future price path is not too volatile, in order not to violate the concavity of (3) with respect to T.

Using the first and the second-order condition for the optimal choice of T reveals that T is decreasing in W. Increases in wealth cause the agent to purchase the durable earlier, regardless of the properties of the price path. Even if insulation is not complete, the adjustments in the timing of purchase provide some insulation.

Combining (4) and (5) determines the optimal purchase size:

$$\frac{v(s)/s}{v'(s)} = 1 - \frac{\hat{P}}{r} \tag{7}$$

where $\hat{P}$ is the rate of change of the price of the durable. The second order conditions ensure that $\hat{P} < r$, so that increases in $\hat{P}/r$ reduce s. A change in the timing of purchase only affects the size of the durable if it affects the growth rate of the price process. Surprisingly, s is independent of the level of the price of the durable.[6]

The effect of a change in wealth on non-durable consumption follows from differentiating (5):

$$\frac{dc}{dW} = \frac{-v''(s)\frac{ds}{dT} + ru'(c)\dot{P}}{-rPu''(c)}\frac{dT}{dW}$$

Given that c was independent of W in the fixed-price case, it is not surprising that in the general case the effect of a change in wealth on the level of consumption is ambiguous. The first term in the numerator has the sign of $d\hat{P}/dT$ while the second term has the sign of $-\hat{P}$.

[6]Increases in P cause consumption to rise by (5) and, the combined effect on wealth, causes the optimal date of purchase to rise.

The sign of dc/dW therefore depends on the properties of the price process. If the price is increasing, then an increase in wealth, which causes the durable to be purchased earlier, reduces the relative price of the durable, which tends to depress non-durable consumption. If the price is accelerating, then an increase in wealth by causing the durable to be purchased earlier reduces $\hat{P}$, which raises s; c must rise so that (5) continues to hold.

While the stark result of complete insulation is special it serves three purposes. First, the lesson is general. In response to changes in their environment, agents will alter the timing of their durable goods purchases and this will provide some insulation for non-durable consumption. Second, given that the response of non-durable consumption to a change in wealth is ambiguous, it serves as a benchmark case. Third, the stark result simplifies consumer behavior and provides a means of developing aggregate models with discrete purchases. It is to this task that we now turn.

3 Aggregate Implications

Consider a small open economy with a continuum of infinitely lived agents who are identical except for their date of birth and hence the wealth that they have accumulated over their lifetime. Each of these agents solves a problem similar to that of the previous section except that instead of being born with an endowment, individuals begin life with zero wealth and earn income throughout their lives. Agents are self-employed and can choose to produce either a unit flow of the non-durable or a flow D of the durable.

We assume that the labor market is flexible. Workers are free to switch sectors, so that in equilibrium the returns to working in each sector must be equal. This mobility between sectors also fixes the price of the durable.

$$P = 1/D.$$

We assume that the non-durable good is tradable, whereas the durable is non-traded. This makes most sense if we think of the durable as something like housing and the non-durable as something like agriculture. Asset trade with the rest of the world pins down the interest rate at r equal to the discount rate.

Let N_t denote the size of the cohort born at date t. Agents live forever and the population grows at a rate g.

Solution to the Individual's Problem in Steady State

Let W_t^b represent the wealth at time t of an agent born at date b. Wealth at birth is equal to the present value of labor income:

$$W_b^b = \int_b^\infty e^{-r(t-b)}\,dt = 1/r.$$

We assume that W_b^b is in the range that yields an interior solution for the age at which the durable is purchased. We therefore adopt the results of the fixed-price section directly. Equation (6) determines the optimal size of the durable purchase, s^*, and, given s^*, Equation (4) determines the optimal level of consumption of the non-durable, c^*. The age at which the agent purchases the durable, T^*, follows from the budget constraint (2):

$$e^{-rT^*} = \frac{1-c^*}{rPs^*}$$

It will be useful for what follows to calculate the individual's financial wealth at age x, F_x. If $x < T^*$, this is simply,

$$F_x = \frac{1-c^*}{r}\left(e^{rx}-1\right).$$

At age T^* the agent purchases the durable and financial wealth falls to,

$$-\frac{1-c^*}{r}$$

where it remains forever.

Aggregate Steady State

In steady state, individuals born at date b purchase the durable at date $b+T^*$. As the population grows at rate g, purchases of the durable good grow at rate g as well.

The current account may be either positive or negative in steady state. The young accumulate wealth as they wait to purchase the durable. Upon purchasing the durable agents go into debt. The aggregate net assets depend on the relative sizes of r, g, and T^*. Integrating over the population shows that the stock of foreign assets in period t is equal to

$$N_t\left(\frac{1-c^*}{r}\right)\left[\frac{e^{(r-g)T^*}-1}{r-g}-\frac{1}{g}\right]. \tag{8}$$

This grows with N_t if the term in square brackets is positive, in which case the current account is positive. Intuitively, if population growth is high, there are more young savers, the current account is positive. Also asset accumulation increases the longer agents' wait to purchase the durable. The effect of an increase in the interest rate is ambiguous.

A Wealth Shock

Suppose that at date $t = 0$ a shock hits the economy that unexpectedly erodes all financial assets so that agents are left with a fraction λ of their accumulated wealth. This may be due to a stock market crash or a decline in bond prices. For simplicity, we assume that the shock affects only those with accumulated assets. Those with debts do not see their debts reduced.

If labor is perfectly mobile across sectors and there are no impediments to trade with the rest of the world, the shock will have no effect on agents' income. The shock affects the demand for durable goods, but agents always have the option of producing non-durable goods and shipping them abroad. Note also that the shock will not affect the consumption of non-durables by those who do not own durables. These agents will simply adjust the timing of their purchase of the durable good. Those who own durables do not alter their consumption since they are indebted and the shock does not alter their debt. The only effect of the shock is on the demand for durables.

To solve for this effect, consider an individual born at date $b \in (-T^*, 0)$. This agent is born prior to the shock and has not yet purchased a durable. Immediately after the shock, the agent has financial wealth in the amount of:

$$\lambda \left(\frac{1-c^*}{r}\right)\left(e^{-rb}-1\right).$$

The agent will wait until total assets reach $\overline{W} = \frac{c^*}{r} + s^*P$ before purchasing the durable. Substituting financial and human capital into the budget constraint (??), we see that the agent waits until date $T(b)$ where:

$$e^{-rT(b)} = e^{-rT^*}(1-\lambda+\lambda e^{-rb}).$$

Figure 2 plots T as a function of b. $T'(b) > 0$ since older agents have more assets and purchase at an earlier date. $T(0) = T^*$ since an agent born at date zero has no wealth at the time of the shock and is therefore unaffected by the shock. Aggregate purchases of durables therefore return to trend after date T^*. Let T_0 denote the date at which the cohort that was about to purchase the durable at the time of the shock reenters the market.

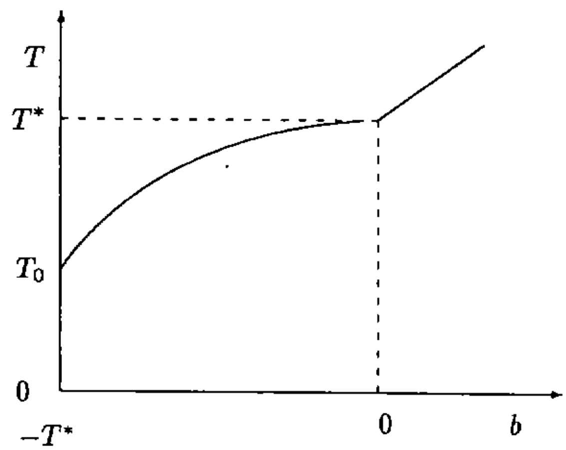

Figure 2: Date of purchase as a function of the birth date.

$T_0 = T(-T^*)$ and is strictly greater than zero. This means that the people that were about to purchase the durable at the time of the shock, those of age T^*, delay their purchases for a discrete period of time. Purchases of the durable dry up during the period $(0, T_0)$. At dates $t \in (T_0, T^*)$ purchases are equal to

$$e^{gb(t)} b'(t) N_0,$$

where $b(t) = T^{-1}(t)$ is the inverse of T; it maps $T(b)$ to b and gives the birth date of those who purchase at date T. Since T is monotonic, this inverse is well defined.

There are two effects on the number of agents purchasing the durable good once purchases resume after date T_0. First, the cohort that delays until $t \in [T_0, T^*)$ is smaller than the cohort that would have purchased at this date in the absence of the shock. N_0 is the size of the cohort born at date zero, so $e^{gb(t)} N_0$ is the size of the cohort that purchases at date t. This is less than $e^{g(t-T^*)} N_0$, which is the size of the cohort that would have purchased at date t in the absence of a shock. Second, older agents lose more wealth and therefore delay longer. This leads to a bunching of purchases that tends to make the number of purchases greater than they would have been in the absence of the shock. The term $b'(t) = 1/[1 - (1 - \lambda)e^{r(t-T^*)}]$ reflects this bunching. These two forces are ambiguous at date T_0, but the second force becomes progressively more important and dominates by date T^*. Since purchases return to trend by date T^*, the number of purchases between dates 0

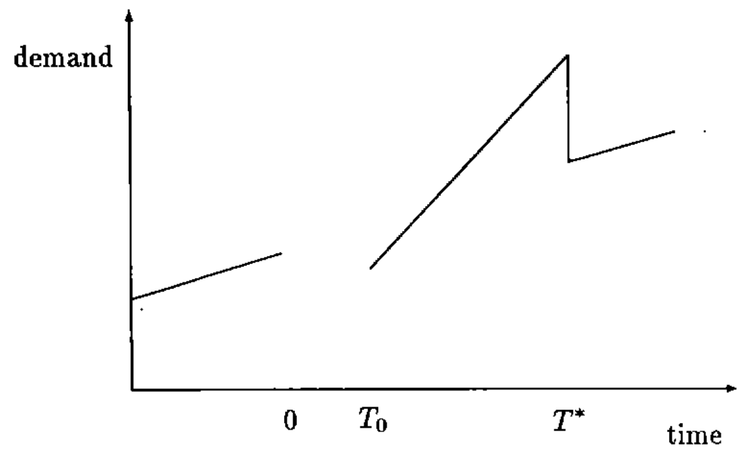

Figure 3: Log demand as a function of time.

and T^* is unaffected by the shock. The shock simply shifts purchases toward date T^*.

Figure 3 shows the demand for the durable following the shock. The model predicts a slump in durables purchases followed by a boom and a return to trend. This pattern occurs because the shock only affects cohorts between 0 and T^*. These cohorts delay purchases in order to rebuild their assets. Although they delay, they still purchase the durable before the cohorts born after the shock. This causes the initial steep fall in demand, the bunching of demand near date T^*, and the return to trend after date T^*.

We can describe the evolution of the current account and the trade balance in response to the wealth shock. Individuals that have durables consume their income minus the interest payments that they pay on their debt. They contribute nothing to the current account dynamics. Those born after the shock save and purchase durables as they would have in the absence of the shock. They too contribute nothing to the current account dynamics. After T^* the current account returns to its steady state path. Those of age 0 to T^* account for all of the dynamics in the current account. They continue to consume c^* and they continue to earn 1 from production. Two things change: their interest income falls by $r(1-\lambda)W_0^b$ and they delay their purchases of the durable good. These forces work in opposite ways on the current account. The fall in income tends to reduce saving, whereas the reduction in consumption of durables tends to improve it. Since this group rebuilds their assets to $\overline{W}$ before purchasing the durable, their lifetime saving must increase and the second effect must prevail on average. The decline in wealth reduces the consumption of durables which leads to an improvement in the current account.[7]

Note that the behavior of the trade balance is more straightforward. Consumption of non-durables is not affected by the shock. Production of non-durables, however, rises because agents who had been producing the durable switch to producing non-durables for export. The trade balance reflects the demand for the durable. It rises immediately after the shock, falls below trend as date T^* approaches and returns to trend.

4 Extensions

4.1 Temporary and Permanent Shocks

We now analyze how the economy responds to temporary and permanent shocks. Consider a common shock to productivity in both sectors. Suppose that income has been constant at 1 for a long time, but that at date zero it falls to λ where $\lambda \in (0,1)$. Thereafter income returns to 1 at a rate α. α is a measure of the persistence of the shock. At date t income is equal to:

$$y_t = (\lambda - 1)e^{-\alpha t} + 1.$$

The shock causes the human capital component of wealth to fall by $(1-\lambda)/(r+\alpha)$ at date zero for all individuals alive at date zero and by $(1-\lambda)e^{-\alpha b}/(r+\alpha)$ at birth for all individuals born at date $b > 0$.

The effect of the shock on the time at which agents born before the shock purchase the durable is easily seen from the budget constraint

$$c^* + e^{-rT} r P s^* = r W_0^b$$

where T is the time at which an agent purchases the durable. Since the shock does not affect c^* or s^* the budget constraint gives a relationship between the time of purchase and wealth after the shock. Substituting for wealth and rewriting the equation in terms of the age of purchase $x = T - b$ yields

$$e^{-rx} = e^{-rT^*} - \frac{1-\lambda}{(r+\alpha)\, s^* P} r e^{rb}$$

[7]This is similar to Blanchard's (1985) model with finite lifetimes.

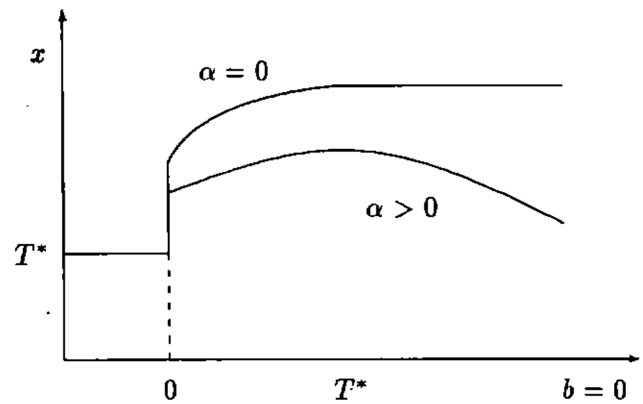

Figure 4: Age of purchase as a function of age at time of shock.

where T^* is the age of purchase in steady state and $-T^* \leq b \leq 0$. It follows that the shock hits the young hardest and that they delay more. This is the opposite of what we saw in the case of the wealth shock. The reason is that the young have accumulated less financial wealth at the time of the shock. For those born after the shock, $b > 0$, the age of purchase is given by

$$e^{-rx} = e^{-rT^*} - \frac{1-\lambda}{(r+\alpha)\, s^* P} r e^{-\alpha b}$$

Note that if $\alpha < 1$ then the later agents are born the smaller the lost income and the shorter the delay in purchasing the durable.

The implications for durable consumption are straightforward. Figure 5 illustrates how the income shock causes various cohorts to delay their purchases. Let's assume first that $\alpha = 0$ so that the shock is permanent. The fall in labor income leads everyone to delay their purchase of the durable. Those who are between ages zero and T^* at the time of the shock, however, have accumulated some financial wealth and so do not delay as much. If $\alpha > 0$, two effects are present. Among those who are alive at the time of the shock, older agents have accumulated more assets and therefore delay less. Among those who are born after the shock, those born later experience less loss of income and therefore delay less. Since the shock is mean reverting, the delay in purchases eventually disappears. This gives rise to the hump shaped pattern in Figure 4.

The evolution of the demand for durables follows from this purchase behavior. Initially, demand falls to zero as all agents delay. After purchases begin, a delay that is increasing in b tends to spread out purchases and leave demand below trend, whereas a delay that is decreasing in b bunches purchases thereby increasing demand. We see that the permanent shock leads to a period of low demand and a gradual return to trend. The temporary shock begins with a period of weak demand, followed by a period of above average demand before returning to trend. Hence permanent shocks lead to busts and temporary shocks lead to a bust-boom cycle.[8]

Let us now consider the effect of this productivity shock on the trade balance. While in the standard RBC models the trade balance is acyclical or mildly procyclical.[9] The smoothing of non-durable consumption effectively cancels any effect of investment on the trade balance. Here a productivity shock might be to a countercyclical trade balance, as in the data. Note, that the trade balance prior to the shock is:

$$TB_0^- = n\frac{N_0}{g} - c^*\frac{N_0}{g},$$

where n is the share of non-durable producers in steady-state; $n = 1 - \gamma g e^{-gT^*}P$. After the shock demand for the durable dries up and everyone produces the non-durable. The trade balance following the shock is

$$TB_0^+ = \lambda\frac{N_0}{g} - c^*\frac{N_0}{g} + e^{-rT^*}\frac{N_0}{g}\frac{r(1-\lambda)}{\alpha + r}.$$

The first term on the right-hand side is the production of the non-durable. The last two terms represent non-durable consumption. The last term captures the decline in consumption by those who have already purchased the durable. It is clear that $\lambda > n$ is a sufficient condition for the trade balance to be countercyclical. Only if the productivity shock is very large does the trade balance fall as a result of the decline in productivity.

4.2 Recurrent Shocks

So far we have considered only one-time unanticipated shocks. The purpose of this section is to incorporate recurrent shocks into the model. Naturally, if the economy is hit by occasional shocks, they can no longer be completely unanticipated. We extend our analysis of wealth shocks and assume that there are periodic collapses, in which financial wealth F

[8]The wealth shock in the last subsection also led to a bust-boom cycle, since it only affected the cohorts alive at the time of the shock, and these cohorts rebuilt their assets.

[9]See, for example, Kehoe, Kydland and Prescott (1995).

falls to a fraction λ of its initial level, and that these collapses are Poisson with an arrival rate equal to β.

Given that these shocks are anticipated, the expected rate of return is no longer equal to the interest rate r, but rather to $r - \beta(1-\lambda)$. We therefore assume, that the discount rate ρ is equal to $r - \beta(1-\lambda)$. As before, we assume that there are no shocks to debt and that borrowers pay an interest rate equal to the expected return on assets. All other aspects of the individual's problem are the same. We assume that the price of the durable is constant and that the wage rate is equal to 1.

For the individual, the state of the economy is characterized by the level of financial wealth F and whether or not they have purchased a durable. Let $V(F)$ represent the present value of utility under the optimal purchase policy for an agent who has yet to purchase the durable. The Bellman equation for this agent's problem is:

$$\rho V(F) = \max\left\{\max_s u\left(\rho F + 1 - \rho P s\right) + v(s), \max_c u(c) + V'(F)[rF + 1 - c] + \beta[V(\lambda F) - V(F)]\right\}. \tag{9}$$

Here the two terms in brackets reflect the two options available to the agent: purchase the durable and wait respectively. When the agent purchases a durable of size s, an amount $F + \frac{1}{\rho} - Ps$ is left over for non-duable consumption. When the agent waits, financial assets grow at rate $rF + 1 - c$ and financial shocks arrive at rate β.

We conjecture that the value function takes the form $V(F) = a + bF$. Substituting this solution into the first order condition for consumption when waiting is optimal yields

$$u'(c) = V'(F) = b.$$

It follows that in the period prior to the purchase of the durable, consumption is constant at some level $\bar{c}$. Substituting the conjectured solution into (9), we arrive at the following relationship between the parameters:

$$\rho a = u(\bar{c}) + b(1 - \bar{c}) \tag{10}$$

There are two boundary conditions. The first is a value matching condition. At the level of financial wealth that the durable is purchased $\bar{F}$, $V(F)$ must equal the present value of utility from the durable and the non-durable

$$\frac{1}{\rho}u(\rho\bar{F} + 1 - \rho P s) + \frac{v(s)}{\rho} = V(\bar{F}) = a + b\bar{F}. \tag{11}$$

The second is a smooth pasting condition which relates the marginal utility of consumption before and after the purchase of the durable:

$$u'(\rho\bar{F} + 1 - \rho Ps) = V'(\bar{F}) = b = u'(\bar{c}).$$

According to this condition, marginal utility cannot jump at the date of purchase, and therefore consumption is constant over the individual's lifetime.

Combining equations (10) and (11) and noting $\bar{c} = \rho\bar{F} + 1 - \rho Ps$, we arrive at

$$\frac{v(s)}{\rho Ps} = b = u'(\bar{c})$$

This is the same first order condition that we had before in equation (4).

Finally, solving for a and b, we find that the choice of s that maximizes V is given by

$$v(s)/s = v'(s).$$

This condition is the same as equation (10) in the certainty case.

It follows that the optimal choices of consumption and the size of the durable are the same in this model with periodic shocks to wealth as they were in the benchmark model with certainty. In a sense, this is not surprising given the insulation effect. Shocks to wealth affect only the timing of purchases.[10] Individuals accumulate wealth and pruchase the durable only when their financial wealth reaches a level $\bar{F}$, where,

$$\bar{F} = \frac{c^*}{\rho} + Ps^* - \frac{1}{\rho}$$

The aggregate dynamics of the model with uncertainty are similar to those of section 3. Agents buy the durable when their wealth reaches a certain threshold. Shocks to financial assets shift agents away from this threshold. As a result agents delay their purchases of the durable as they rebuild their assets, and the aggregate demand for durables falls to zero for some time. The only difference is that the economy may occasionally be hit by a new shock before it has recovered from the last one. In this case, demand may remain low for a much longer period.

[10] We were careful to choose our shocks so that they maintained wealth in the interval $(\underline{W}, \bar{W})$ in order to take full advantage of the insulation result. If the shock had taken wealth outside of this range, insulation would not have been complete.

4.3 Interactions and Feedback

Thus far the only effect of the shock from the individual's perspective was the direct effect on the individual's wealth or income. One agent's actions had no effect on others. Aggregation was easy because the aggregate dynamics were simply the sum of the individual dynamics. In many cases, however, we might expect interactions among agents. The decline in the demand for the durable good may further reduce the income of durable goods producers if they cannot costlessly shift into non-durable goods production. They see their income fall for two reasons: the direct effect of the shock and the indirect effect through the demand for their output.

At the same time, other effects work to dampen the effect of the shock. If durable producers cannot costlessly shift to non-durable production, supply of the durable will remain high. As demand falls, the price must fall in order to equate supply and demand. This decline in price lessens the impact of the shock on non-durable goods producers.

In order to model these interactions, we introduce a friction in the labor market so that agents do not always have the option of producing the non-durable good for export. One simple friction is to assume that a fraction γ of the agents who choose to produce durable goods cannot switch jobs. It may be the case that durable goods industries require human capital that is more specific to the industry and less general to other occupations, or simply that it is more difficult for suppliers to alter their plans than for consumers to alter the timing of durable consumption. We assume that each age cohort is divided in the same proportions between durable and non-durable goods producers and that the fraction if immobile durable producers is likewise spread evenly over age cohorts in proportion to their size.

As before we assume that financial wealth F falls to λF at date zero. When wealth falls, demand for the durable would tend to fall to zero as agents delay their purchases of the durable. In this case, however, production does not fall to zero due to the friction in the labor market. In order to balance supply and demand the price of the durable must fall. This leads to the feedback. Producers of the durable not only take a direct hit from the wealth shock, they take an indirect hit from the fall in demand for the durable good. For simplicity we focus on the timing of purchases and fix the size of the durable to 1.[11]

The recession can be divided into two phases. In the first phase the price of the durable P_t is less than the steady state price. Only agents who are constrained to produce the

[11] An analysis of the case with flexible size is available from the authors upon request.

durable will willingly choose to do so during this phase. Supply is fixed by the number of agents attached to durable goods production. At some date $\bar{T}$ the price of the durable returns to its steady state level, and the second phase begins. After this date, the price of the durable is fixed at its steady state level, so that agents are indifferent as to which good they produce, and supply adjusts to meet demand. Since the second phase is essentially the same as the case that we studied in section 3, we concentrate on the first phase in what follows.

Individuals

We begin with the consumption decision of the non-durable good producer. Let $W_0^{b_n}$ denote total wealth (financial and human) in period zero of an individual who is born at date b_n where $-T^* \leq b_n \leq 0$ and who produces non-durables. Recall that T^* is the age at which agents purchase the durable in steady state and c^* consumption in steady state.

After the crash, this individual's total wealth, financial and human, is equal to:

$$W_0^{b_n} = \lambda \frac{1-c^*}{r}\left(e^{-rb_n}-1\right)+\frac{1}{r}.$$

Note that these workers do not lose income, only wealth.

Let T denote the date of purchase and let v denote the utility from the durable purchase. Total utility of a non-durable producer born at b_n is

$$U_{b_n} = \frac{u(rW_0^{b_n} - re^{-rT}P_T)}{r} + \frac{v}{r}e^{-rT},$$

and maximization with respect to T yields,

$$u'(rW_0^{b_n} - re^{-rT}P_T)(rP_T - \dot{P}_T) = v. \qquad (12)$$

This is the same first-order condition as we had before in the flexible price case in section 2 except that size is fixed. Recall from section 3 that increases in $W_0^{b_n}$ reduce the date of purchase T. The older non-durable producers therefore purchase first.

The solution to the durable goods producers problem is similar except that wealth falls by the additional amount Δ which reflects the decline in their income due to the decline in the price of the durable. Let $W_0^{b_d}$ denote the wealth at date zero (after the shock) of a producer of durables born at date b_d where $-T^* \leq b_d \leq 0$. Then $W_0^{b_d} = W_0^{b_n} - \Delta$ and the first order condition for the optimal purchase time is:

$$u'(rW_0^{b_d} - re^{-rT}P_T)(rP_T - \dot{P}_T) = v. \tag{13}$$

It is convenient to solve for the birth date as a function of the time of purchase. Let the function $b_n(t)$ denote the birth date of those non-durable goods producers who purchase at date t, and let $b_d(t)$ denote the same for durable goods producers. It is easy to see that both functions are monotonicly increasing. In addition, equations (12) and (13) imply a relationship between the birth dates of durable and non-durable goods producers that purchase at the same date. In particular, if both make purchases then they both have the same wealth, $W_0^{b_n(t)} = W_0^{b_d(t)}$. Substituting the expressions for initial wealth and rearranging:

$$e^{-rb_d(t)} = e^{-rb_n(t)} + \frac{r\Delta}{\lambda(1+c^*)} \tag{14}$$

An implication of this relationship is that the non-durable producers who buy at t are younger: $b_d(t) < b_n(t)$.

Equation (12) characterizes the price process during the first phase. To complete this description we need three pieces of information. We need to know the function $b_n(t)$ and the boundary condition on the price path. The boundary condition is the date $\tilde{T}$ at which price returns to its steady state value. We also need to solve for Δ. We address these issues in turn.

Supply and Demand

During the period prior to $\tilde{T}$ supply is fixed at a fraction of the level that prevailed at the time of the shock. No newly born agents will choose to produce durables so long as their price is below steady state, and all durable producers that have the opportunity to switch will switch to producing non-durables. Only those that cannot switch will continue to produce durables.

Immediately prior to the shock, the economy is in steady-state and the supply of the durable good is equal to the steady state demand, $N_0e^{-gT^*}$. After the shock supply falls to $\gamma N_0e^{-gT^*}$. Hence the supply of durables over the period 0 to t is:

$$\int_0^t \gamma N_0 e^{-gT^*} ds = t\gamma N_0 e^{-gT^*} \tag{15}$$

$b_n(t)$ must be such that demand is equal to this supply. We know that initially there is a period of time in which only non-durable goods producers who were born prior to the shock purchase the durable. As time passes durable goods producers can re-enter the market.[12] Let T' be the date that durable goods producers begin purchasing, $b_d(T') = -T^*$.

Consider first the period when only non-durable producers purchase. The fraction of each cohort that belongs to this group is equal to $n = \left(1 - \gamma g e^{-gT^*} P\right)$. Recall that we assume the same fraction of each age cohort finds itself constrained. Hence demand over the period 0 to t is:

$$\int_{-T^*}^{b_n(t)} n N_0 e^{gb} db = \frac{n}{g} N_0 (e^{g b_n(t)} - e^{-gT^*}).$$

Putting together supply and demand and solving for t yields:

$$t = \frac{n}{\gamma g}(e^{g(b_n(t)+T^*)} - 1) \tag{16}$$

This equation implicitly defines the function $b_n(t)$ over $[0, T']$.

We next define b_n for $t \geq T'$. Demand from 0 to t is equal to the sum of the demand of both durable and non-durable goods producers:

$$\int_{-T^*}^{b_n(t)} n N_0 e^{gb} db + \int_{-T^*}^{b_d(t)} (1-n) N_0 e^{gb} db$$

implying

$$t = \frac{1}{\gamma g}\left[n e^{g(b_n(t)+T^*)} + (1-n) e^{g(b_d(t)+T^*)} - 1\right]$$

Using (14) to eliminate b_d yields the expression for $b_n(t)$.

The boundary condition

We now introduce the boundary conditions that pins down the price process. At $\tilde{T}$ price has returned to its steady state level, $P_{\tilde{T}} = P$. Therefore at $\tilde{T}$ agents must be wealthy enough to want to purchase the durable at its steady-state price. Hence

$$c^* = r W_0^{b_n(\tilde{T})} - r e^{-r\tilde{T}} P \tag{17}$$

where c^* is the steady state level of consumption.

[12]For sufficiently large shocks it might be the case that durable goods producers reenter after the price returns to steady state or even after non-durable goods producers born after the shock pruchase. We shall assume that γ is sufficiently small to avoid these scenarios. It is straightforward to amend the analysis to include these cases.

Given the function b_n, the end of the recession $\tilde{T}$ in equation (17), and the differential equation (12), we can solve for the price path P_t, $0 \leq t \leq \tilde{T}$.

The loss of the durable goods producers.

Everything to this point was contingent on a value for Δ. We pin down Δ by a fixed point argument. Given a conjecture for Δ we get a price path, from which we can define the actual loss by

$$\psi(\Delta) = \int_0^{\tilde{T}} \frac{P - P_T}{P} e^{-rT} dT$$

In equilibrium the conjectured and actual losses are the same:

$$\psi(\Delta) = \Delta$$

A simple argument shows that an equilibrium exists. First, note that if Δ were zero, then price would still have to fall in order to induce consumers to purchase the durable at date zero. Hence $\psi(0) > 0$. On the other hand, even if Δ is equal to infinity, the price of the durable can not fall below zero so ψ is bounded above by $1/r$. Combining these statements with the continuity of ψ shows that a fixed point exists.[13]

Discussion

In earlier sections of the paper, changes in wealth led agents to alter the timing of their purchases causing large fluctuations in aggregate demand. In this section, price adjusts to smooth demand for the durable. The recession, however, lasts longer. The fall in the price of the durable hurts durable goods producers by reducing their income. This causes them to delay their return to the market by more than they did in the flexible price case. It is interesting to note that in this case price flexibility is correlated with a longer recession; due to the friction price adjustment is forced to substitute for quantity adjustment.

Given the fluctuations in the price of the durable, non-durables are not completely insulated from the shock. It is easy to see from (12) that the low and increasing price of the durable reduces non-durable demand. By reducing agents incentive to intertemporally

[13]It is also the case that the slope of ψ is positive. As Δ rises, durable producers become less wealthy and hence less willing to buy. $b_d(t)$ falls and T' rises. This reduces demand and the price must fall in order to encourage non-durable producers to fill the gap. The fall in price reduces ψ. This positive slope suggests that the model might have multiple equilibria. We have not been able to construct examples of multiple equilibria to date.

substitute, the price fluctuation smooths durable demand at the expense of non-durable demand.

To get an idea of the importance of the equilibrium feedback, we simulated the model. The baseline parameterization included a logrithmic utility funciton for non-durable consumption together with $P = 5$, $r = .05$, $v = .28$, $\lambda = .7$, $\gamma = 1$and $g = .01$. In most cases the price movements were small; price fell initially by about 1% and returned smoothly to its steady state level. Table 1 presents the values of $\tilde{T}$ and Δ for various choices of γ and λ. Because the decline in price is slight, the additional loss to the durable producers is slight. To interpret the size of Δ note that the present value of income absent the shock is 100, so the additional loss to the durable producers is less than 1/1000 of their lifetime income. The effect on the length of the recession, however, tends to be quite long.

	$\tilde{T}$	Δ
$\gamma = 0$	3.76	0
$\gamma = .4$	5.62	.0099
$\gamma = .7$	8.56	.0205
$\gamma = 1.0$	14.56	.0505
$\lambda = .6$	15.30	.0714
$\lambda = .8$	13.35	.0303

Table 1.

As predicted, increasing the fraction γ tied to the non-durable section, increases the loss to durable producers and the duration of the recession. Increasing the size of the shock (reducing λ) increases the loss to the durable producers and the duration of the recession.

5 Concluding Remarks

We have presented a simple model of durable goods purchases. The model emphasizes the timing of durable goods purchases. When agents can delay the timing of purchases, they can use this flexibility to insulate their consumption of non-durables from shocks. We investigate conditions under which this insulation is complete.

We embed this simple model in an aggregate context and show how shifts in the timing of purchases can lead to large fluctuations in demand. We show how temporary shocks lead to a boom-bust pattern that may help to explain the amplification and propagation of business cycles. We show that the current account is naturally counter-cyclical because recessions are times in which agents build up their assets in anticipation of making purchases.

6 Appendix A

Fixed Costs

The present value of utility is as before

$$\frac{u(c)}{r} + e^{-rT}\frac{v(s)}{r}$$

except that now $v(s)$ is everywhere concave. We assume that the cost of purchasing s units of the durable good is $k + sP$, where k is a fixed cost.[14]

Consumption smoothing implies that

$$c = r[W - e^{-rT}(k + sP)].$$

Assuming an interior solution for the time of purchase, the first order conditions for T and s when price is fixed are respectively,

$$\begin{aligned} v(s) &= ru'(c)(k + sP). \\ v'(s) &= ru'(c)P. \end{aligned}$$

Combining these two relations to eliminate the marginal utility of consumption, yields:

$$\frac{v(s^*)}{k + s^*P} = \frac{v'(s^*)}{P}$$

The optimal purchase size is such that the average utility that a consumer gets per dollar spent is equal to the marginal utility that a dollar purchases. Note that at s^* the elasticity of utility with respect to the size of the durable is less than one:

$$\frac{s^*}{v(s^*)}v'(s^*) = \frac{Ps^*}{k + Ps^*} < 1.$$

When $k = 0$ this reduces to our earlier result. Another rearrangement of this equation yields,

$$v'(s^*) = \frac{v(s^*)}{s^* + k/P}.$$

[14] We have added the fixed cost to the purchase price of the durable. It therefore represents a volume discount, a delivery cost, or an agency fee. We would obtain similar results if we had added the fixed cost to the utility function, in which case the fixed cost would represent a decision cost or search effort.

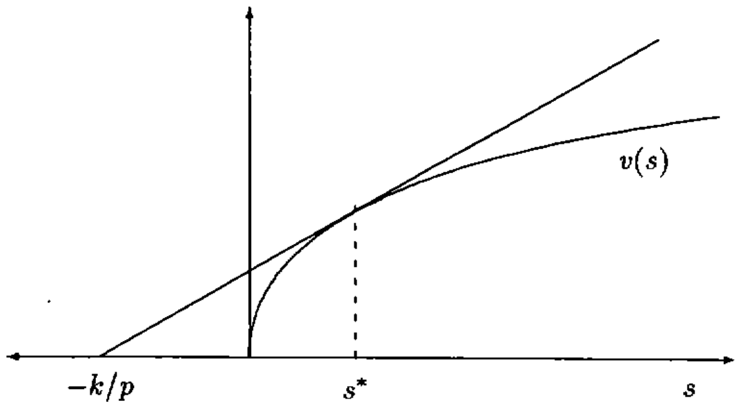

Figure 5:

This shows that s^* is the point at which a ray from $(-k/P, 0)$ to (v, s) is tangent to v. As seen from figure A1, if v is concave this point is unique.[15]

Again, there is a range of wealth levels for which c^* and s^* are independent of wealth. If there is not enough wealth to consume c^* forever, the durable is never purchased. If there is wealth left over when the durable is purchased at date $t = 0$, the consumer chooses larger consumption of both goods.

7 Appendix B

In this appendix we relax the requirement that the discount rate is equal to the interest rate. Let r denote the interest rate and ρ the discount rate. We assume a constant elasticity flow utility function for non-durables: $u(c) = c^{1-\theta}/(1-\theta)$.

After a little manipulation the maximization problem becomes

$$\max_{s,T} \Omega \frac{c_0^{1-\theta}}{1-\theta} + e^{-\rho T} \frac{v(s)}{\rho}$$

[15] If we had placed the fixed cost in the utility function, the present value of utility from purchasing the durable would have equaled zero for $s = 0$ and then dropped discretely to $-k$ before rising monotonically. It is as if the origin of figure 1 were shifted to $\left(0, \frac{kv'(s^*)}{P}\right)$. In this sense adding the fixed cost is essentially the same as adding a convex segment to the utilitity function.

subject to the budget constraint

$$W_0 = \Omega c_0 + e^{-rT} P_T s$$

where $\Omega = \theta/[\rho - r(1-\theta)] > 0$.

The first order conditions become:

$$\begin{aligned} \frac{v'(s)}{u'(c_0)} &= e^{(\rho-r)T}\rho P_T \\ \frac{v(s)/s}{u'(c_0)} &= e^{(\rho-r)T} P_T \left(r - \frac{\dot{P}_T}{P_T}\right). \end{aligned}$$

Again $dT/dW < 0$ follows from the second order condition for maximization with respect to T. Combining the first order conditions reveals that s depends only on $\dot{P}_T/P_T$, ρ, and r. Its derivative with respect to W is therefore ambiguous. Finally, dc_0/dW is also ambiguous and depends among other things on $\rho - r$.

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