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ABSTRACT

This paper presents a stochastic pricing model of a unique, path-dependent lease instrument common in the United Kingdom and numerous commonwealth countries, the upward-only adjusting lease. In this lease, the rental rate is fixed at lease commencement but will be reset to the market rate at predetermined intervals (usually every five years) if it exceeds the contract rent. Numerical results indicate how the initial coupon rate should be set relative to that on a symmetric up-and-downward adjusting “variable rate" lease under various economic conditions (level of real interest rates and expected drift and volatility of the underlying rental service flow). We also consider the calculation of effective rents when free rent periods are given during either a market collapse or a steady-state drift.

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1 Introduction

Valuing lease contracts from a real options perspective is well developed. For example, Grenadier (1995), using an endogenously derived term-structure for lease rates, determines the equilibrium lease rates for many different types of leases under various economic assumptions. In this paper, we utilize the concepts of market equilibrium relationships among lease rates in development of a model for valuing securities with path-dependent cash flows. We apply this model to an interesting commercial lease contract known as the upward-only adjustable lease.

The upward-only adjustable lease is a common commercial lease contract in the UK and other former commonwealth countries, such as Australia and New Zealand (Baum and Sams, 1990). In this lease, the rental rate is fixed for an extended term (formerly 25 years, but now 15 years) at lease commencement, but the lessor has the option to have the rent reset to the market rate at predetermined intervals (every five years). If market rents increase, then the contract rent will increase to the market level. If market rents decline, the rent remains at the previous level. Valuing this type of contract is quite complex because the contract rate at subsequent adjustment dates depends upon previous adjustments - introducing a path dependency problem. We derive an implicit equation for valuing the lease, and use it to compute the initial rents for a variety of leases with a given term that have the same values, as competition for space would require. The key is using the probability function of a stochastic variable to derive an explicit relationship between the expected market rents at the adjustment dates and the initial contract rent.

In Section 2, we utilize the Grenadier model to compute the lease rates on 5, 10, and 15 year fixed-rate leases at five year intervals 15 years into the future. These rental rates are based on assumptions regarding the service flow generated by the underlying asset. The model for pricing the upward-only adjusting lease relative to an up-or-downward adjusting lease is described in Section 3.

In Section 4, we use the results of Sections 2 and 3 to price the upward-only adjusting lease for different assumed levels of real interest rates and the expected drift (inflation rate) and volatility in the asset service flow. The impact of alternative adjustment indices is also illustrated. Section 5 introduces time dependency in the drift and illustrates this in the context of pricing at the onset of a market collapse where real rents are expected to decline for the first five years and then spike back up to the zero real growth trend line during the second five years. Section 6 analyzes the impact of free rent periods on the setting of contract rents and illustrates how one should compute effective rent from face rent when free rent periods are given. A summary concludes the paper.

2 Equilibrium Term-Structure of Lease Rents

We begin with the standard view that leasing is the process by which a property owner sells the use of an asset for a specified period to a lessee who promises to make payments over this lease term. This is the standard assumption for lease valuation following Miller and Upton (1976), McConnell and Schallheim (1983), Schallheim and McConnell (1985), and Grenadier (1995, 1996). As Grenadier (1996) points out, leasing separates property ownership from property use where the lessor receives the lease payments and the residual property value while the lessee receives the use of the property over the lease term. This economic model allows us to value the lease contract in an equilibrium context where the lease payments compensate the lessor for the loss of use of the property. The equilibrium context implies that, in an efficient market, all leases with the same maturity should provide the same present value to the lessor, irrespective of whether the rental rate is fixed, fully variable, or partially variable.

Rather than rely on traditional arbitrage arguments from option pricing models, we follow Grenadier (1996) and use a general equilibrium model, which relaxes the tradability assumptions required in arbitrage pricing. This assumption is appealing when modeling property leasing markets where the underlying asset is subject to significant transaction costs, is relatively indivisible, and cannot be sold short.

The demand for the use of a leased property is derived from its potential use as an economic input. As a result, the value of the lease depends upon the value of the asset service flow over the lease term. We assume that the value of an asset's service flow, $S(t, \omega)$ varies randomly in time t.[1] For simplicity we assume a risk–neutral equilibrium model where all assets are priced to yield the expected (deterministic) risk–free rate of return $r(t)$. Grenadier (1995) notes that this restriction can be relaxed by incorporating a risk premium into the drift rate in the manner of Cox and Ross (1976). We introduce the notation $\tilde{r}(t) = \int_0^t r(\tau)d\tau$. Because leasing an asset is equivalent to purchasing its use for a specified time, T, then, as Grenadier (1996) shows, the value of the lease contract given the value of the service flow at origination, S_0 is

$$Y(S_0, 0, T) = E_{S_0} \int_0^T e^{-\tilde{r}(t)} S_t \, dt. \tag{1}$$

Here E_{S_0} denotes the expectation given (conditioned on) S_0, where S_0 is the value of the asset's service flow at contract origination ($t = 0$). Below, this subscript is omitted, but when the value of the lease contract is calculated, the expected value is understood as in (1).

[1]The argument ω selects the random realization of the value function; ω is suppressed hereafter, so that we refer only to $S(t)$.

In this paper, we assume that the risk–free interest rate r is a constant, so that $\tilde{r}(t) = rt$. We also assume that the value of the asset's service flow obeys the stochastic differential equation

$$dS(t) = \alpha S(t)dt + \sigma S(t)dw(t), \tag{2}$$

where $w(t)$ denotes the standard Brownian motion, and the instantaneous drift coefficient, α, and the instantaneous volatility, σ, are given constants. (In Section 5 we generalize our discussion to the case when the drift is a deterministic function of time.)

The solution to (2) is the stochastic process given by

$$S(t) = S_0 \exp\left((\alpha - \frac{1}{2}\sigma^2)t + \sigma w(t)\right), \tag{3}$$

where $ES(t) = S_0 e^{\alpha t}$. (Eq.(3) can be verified by directly applying the Itô formula.) The service flow is the source of the asset's underlying economic value that evolves over time and based on (3) is log–normal distributed. The stochastic process used here has the flexibility to capture a variety of economic conditions, including a positive or negative drift in demand. The expected drift will depend on expected inflation, the expected growth rate in the driver of the demand for space (e.g., financial services employment for office) and expectations of how supply will adjust to demand growth.[2]

In an efficient rental market, the fixed level lease payments, set at origination, must equal the expected value of the asset's service flow over the lease term. Thus the contract rent, $R^f(0)$, set at origination is defined by the equation

$$Y(S_0, 0, T) = R^f(0) E \int_0^T e^{-\tilde{r}(t)}\, dt. \tag{4}$$

Using equations (1) – (4), the equilibrium rent on a fixed-rent lease at origination can be expressed as

$$R^f(0) = S_0 \frac{g(r-\alpha, 0, T)}{g(r, 0, T)}, \tag{5}$$

where $g(.,.,.)$ is the continuous time present value annuity function. The present value of a dollar stream between t_1 and t_2 is

$$g(a, t_1, t_2) \equiv \begin{cases} (e^{-at_1} - e^{-at_2})/a & \text{if } a \neq 0 \\ t_2 - t_1 & \text{if } a = 0 \end{cases}.$$

[2]For estimates of the relationship of U.K. rental rates to such factors, see Hendershott, MacGregor and White (2000).

Table 1 presents the contract rents of various term fixed-rent leases found using the standard Grenadier term-structure framework.[3] In these and all calculations, S_0 is normalized to unity. Rents are computed for two different real interest rate ($r - \alpha$) environments – 1% and 4% – and three different expected service flow growth rates (α) – 0%, 5%, and 10%. We present the fixed-rents for a standard 15-year lease assuming that the lease is originated in years 0, 5, and 10. To demonstrate the term-structure of lease rates, we also report the expected rent on a 10-year lease originated in year 5 and a series of 5-year leases originated in years 0, 5, and 10. Rents are higher for high expected service flow growth rates and decline as real interest rates increase. And the greater the expected service flow growth rate, the greater is the rent on a longer term fixed-rent lease relative to that on a shorter-term fixed-rent lease (compare the 5-, 10-, and 15-year lease rates at year five).

3 The Upward-Only Adjusting Lease

We now analyze the case of the upward-only adjusting lease contract. Consider a $3T$–year lease where at T and $2T$, the rent is adjusted to the then market rent, if and only if the market rent at T and $2T$ is higher than the previous contract rent.[4] We use $\rho^u(t)$ to denote the stochastic 'market' upward-only adjusting rent at time t and $R^u(0)$ to denote the equilibrium 'contract' rent at origination for an upward-only lease with two adjustments where the first adjustment occurs at T. Furthermore, we denote the 'contract' rents at the adjustment dates as $R^u(T)$ and $R^u(2T)$.[5] Thus, at origination $\rho^u(T)$ and $\rho^u(2T)$ are unknown.

The rent at each adjustment is set in line with the market rent on a new upward-only adjusting lease with the same term as the initial lease. Thus, the market rent at T is the expected equilibrium rent on a $3T$–year lease originated at T, which has two upward-only adjustments, and the market rent at $2T$ is the expected equilibrium rent on a $3T$–year upward-only adjusting lease also with two adjustments originated at $2T$.

Using the equilibrium condition that the present value of the expected rental stream over the term of the lease must equal the expected value of the service flow from the property

[3]This framework abstracts from such complications as vacancies or nonrenewals. Such complications would require that short-term leases rates be higher than long-term lease rates to compensate for down time and lease up expenses.

[4]Baum and Sams (1990) discuss the adjustment feature of the upward-only lease and illustrate a simplistic approach to valuing this lease by computing the present value of the expected adjusted rents. Baum, Mackmin and Nunnington (1989) illustrate the simplistic present value method for valuing the upward-only contract in a number of examples, but they do not discuss the rent review process.

[5]Note that we use the superscript, u, to denote the rent on the upward-only lease, the superscript, f, to denote the rent on a fixed-rent lease, and no superscript when referring to an up-or-downward adjusting lease.

over the lease term, the initial contract rent must balance the following equation:

$$Y(S_0, 0, 3T) = R^u(0) \int_0^T e^{-rt} dt + R^u(T) \int_T^{2T} e^{-rt} dt + R^u(2T) \int_{2T}^{3T} e^{-rt} dt \tag{6}$$

where

$$R^u(T) = E\left[\max[R^u(0), \rho^u(T)]\right] \tag{7}$$

and

$$R^u(2T) = E\left[\max[R^u(T), \rho^u(2T)]\right]. \tag{8}$$

Note the path dependency problem: the future rental rates needed to solve for the current rental rate themselves depend on the current rental rate.

The market rent, $\rho^u(T)$ in (7), is the equilibrium rent on a new $3T$–year upward-only lease at T and is determined by

$$Y(S_T, T, 4T) = \rho^u(T) \int_T^{2T} e^{-rt} dt + \hat{\rho}^u(2T) \int_{2T}^{3T} e^{-rt} dt + \hat{\rho}^u(3T) \int_{3T}^{4T} e^{-rt} dt \tag{9}$$

where $\hat{\rho}^u$ and $\hat{\hat{\rho}}^u$ denote the future upward-only adjusting rents conditional upon the market rent at T and as before,

$$\hat{\rho}^u(2T) = E\left[\max[\rho^u(T), \hat{\hat{\rho}}^u(2T)]\right] \tag{10}$$

and

$$\hat{\rho}^u(3T) = E\left[\max[\hat{\rho}^u(2T), \hat{\hat{\rho}}^u(3T)]\right]. \tag{11}$$

By the same logic, the market rent, $\rho^u(2T)$ in (8), is the equilibrium rent on an $3T$–year upward-only lease at $2T$ that has a value of $Y(S_{2T}, 2T, 5T)$. Because the contract rent adjusts to future market rents on upward-only contracts with the same length term, rather than a term equal to the remaining life of the lease, the solution requires solving an infinite system.

Baum, Beardsley, and Ward (1999) have suggested using Monte Carlo simulations of the market rental index to price the upward-only adjustments. It is not clear, however, how this method correctly accounts for the embedded adjustments in future rental rates. Furthermore, their model does not specify the relationship between market rents, benefits derived from utilizing the space (the service flow), and asset value.

We close the system by modeling the 'market-rent' index as a series of 'fixed-rent' contracts rather than upward-only adjusting contracts. Thus, consider a $3T$–year lease with upward–only adjustments at times T and $2T$ to the market 'fixed-rent' on a $3T$–year lease. The market rent at T, given the value of the asset's service flow $S(T)$, is given by

$$\rho(T) = S_0 e^{-\sigma^2 T/2+\sigma w(T)} g(r-\alpha, T, 4T)/g(r, T, 4T) \tag{12}$$

and the market rent at $2T$, given the value of the service flow $S(2T)$, is given by

$$\rho(2T) = S_0 e^{-\sigma^2 2T/2+\sigma w(2T)} g(r-\alpha, 2T, 5T)/g(r, 2T, 5T). \tag{13}$$

The initial equilibrium contract rent $R^u(0)$ is a constant that satisfies the equation

$$Y(S_0, 0, 3T) = R^u(0)g(r, 0, T) + R^u(T)g(r, T, 2T) + R^u(2T)g(r, 2T, 3T) \tag{14}$$

where

$$R^u(T) \equiv E\left[\max\left[R^u(0), \rho(T)\right]\right] \tag{15}$$

and

$$R^u(2T) \equiv E\left[\max\left[R^u(T), \rho(2T)\right]\right]. \tag{16}$$

The path dependency problem still exists: $R^u(2T)$ is dependent upon $R^u(T)$ and both are dependent upon $R^u(0)$, which is the unknown we solve for in equation (14). The Appendix shows, using the probability function of a stochastic variable, how to make $R^u(2T)$ and $R^u(T)$ explicit functions of $R^u(0)$, so that equation (14) fully accounts for these dependencies.[6]

4 Base Numerical Results

Using equations (12) and (13) we evaluate the expected values in (15) and (16), and then solve (14) numerically for the initial contract rent, $R^u(0)$, and the expected rents at the two adjustment dates for a 15-year upward-only adjusting lease with adjustments at years 5 and 10. These are given in columns (5) – (7) of Table 2. Column (4) shows the present value of the lease contract. For comparison, we calculate the expected rents for a 15-year, up-or-down adjusting lease (columns (8) – (10)). The up-or-down adjusting rents in years

[6]Equation (14) is readily generalized to a nT-year upward-only adjusting lease with $(n-1)$ adjustments at time intervals of length T.

5 and 10 are calculated as

$$E\rho(5) = S_0 \frac{g(r-\alpha, 5, 20)}{g(r, 5, 20)} \quad (17)$$

$$E\rho(10) = S_0 \frac{g(r-\alpha, 10, 25)}{g(r, 10, 25)}. \quad (18)$$

Note that adjustment rents are based on a constant 15-year term. The expected rent at origination is the constant that satisfies the equation:

$$Y(S_0, 0, 15) = R(0)g(r, 0, 5) + E\rho(5)g(r, 5, 10) + E\rho(10)g(r, 10, 15) \quad (19)$$

In columns (11) – (13), we report the difference between the upward-only rents and the up-or-down rents.

The numerical results show the impact of changes in the real risk-free interest rate, $r-\alpha$, the expected growth in the service flow, α, and the service flow volatility, σ. We examine the rental rates in low ($\sigma = 0.1$) and high ($\sigma = 0.2$) service flow volatility regimes. The differences between the initial rents for the two contracts indicates the discount necessary to compensate the tenant for the floor on rents caused by the upward-only adjustment clause. To offset this initial discount, the expected rental rate at the second adjustment period will be higher on the upward-only adjusting lease (owing to the rent floor). The relationship between the expected rental rates at the first adjustment date is uncertain: the rent floor will tend to make the rental rate on the upward-only lease higher, but the lower initial rental rate will make it lower. As is shown in column (12) of Table 2, for drifts of 5 to 10 percent these two forces precisely offset; for lower drifts, the rent-floor impact dominates the lower initial-rent impact.

4.1 Impact of Service Flow Drift (α) and Volatility (σ)

The initial rent discount on the upward-only lease will be greater the more likely the rent floor is to bind, and this likelihood is greater the lower the expected drift and the higher the expected volatility. Assuming low service flow volatility ($\sigma = 0.1$), the initial discount falls dramatically from about 12 cents per dollar of service flow at zero drift to only 1.5 cents at five percent drift, to 0.1 cents at a ten percent drift. As the drift increases to the double digit level, the odds of the rent floor binding becomes negligible. The offset to the low initial rental rate is a higher expected rental rate at the second adjustment date (and at the first date with zero expected drift).

With a doubling of expected volatility to 20 percent, the initial discount also roughly doubles at zero drift. At higher drifts, the increase in the initial discount, while smaller, is proportionately far greater, rising from only 0.1 cents to 3 to 4 cents at double digit drift. Even with high drift, high volatility creates a significant probability of the rent floor binding.

4.2 Impact of Real Risk-Free Interest Rate $(r - \alpha)$

An increase in the real risk-free rate from one to four percent reduces the upward-only initial rent discount by 15 to 35 percent. However, the premium by which the expected rent on the upward-only lease at the second adjustment exceeds that on the up-and-down lease remains unchanged. With the higher discount rate the unchanged premium is worth less in present value terms, resulting in the reduced initial rent discount.

4.3 Impact of an Alternative Adjustment Rule

Of course, there are many other possible adjustment indexes for the upward-only lease. For example, the rent could be adjusted only if the rent on five-year leases has risen since origination. This could be labeled the variable-rate adjustment. Or the rent at the adjustment dates could be adjusted to the rent on a lease of the same remaining maturity (and number of adjustments), rather than the same original lease term and number of adjustments. That is, the rent at the first adjustment period could be adjusted only if the rent on a 10-year lease with a single adjustment at five years had risen (the lease has only 10 years of remaining life at this point) and the rent at the second adjustment period could be increased only if the rent on a five year lease (the remaining term) has risen. This could be labeled the term-structure adjustment.

Table 3 compares the upward-only rents calculated under the UK method with the rents calculated using the term-structure method. As in Section 3, we examine the rental discounts in low and high drift and volatility environments. The UK and the term-structure methods produce dramatically different rent structures when growth in the rental service flow is positive because the rents on leases with different maturities – the rents to which the adjustment is occurring – differ so much. For example, when volatility is low ($\sigma = 10\%$) and growth is high ($\alpha = 10\%$), the initial term-structure rent is almost 10 times the UK rent. In comparison, the rents at the first and second adjustments calculated according to the term-structure rule are significantly lower than the rents found using the UK rule, only 83% and 67% at the first and second adjustment dates, respectively. Given that the rents at the adjustment dates are calculated over 15 years rather than over 10 or 5 years as in the term-structure method, it is not surprising that the future rents are higher. Because the model still solves for the equilibrium initial contract rent that equates to the value of the service flow over the lease term, the initial rent in the UK rule method must necessarily be set very low in order to balance the equation.

It is interesting to compare the results in Tables 2 and 3. In comparing the rents determined by the term-structure to the up-or-down adjusting model, we note that the upward-only rents are significantly different (i.e., the upward-only feature has value) when expected appreciation (α) is low or volatility (σ) is high. However, comparing rents calculated using the UK and the term-structure rules, we see that when expected appreciation (α) is low or volatility (σ) is high, differences between the rents decline. In fact, when $\alpha = 0$ the rents are equal. This suggests that during periods when the upward-only feature has the greatest impact on rent calculations ($\alpha = 0$), the differences between UK and the term-structure rules are minimal. However, during periods of high appreciation when the term-structure option pricing model indicates that the upward-only feature has little value over the fully indexed lease, the UK adjustment mechanism indicates that the upward-only feature has dramatic consequences on rent calculations.

We probably cannot over emphasize this point. Historically, UK (London) capitalization rates have been far lower than those on the continent and the US. The primary cause of this is the structure of the UK institutional lease, not so much the upward-only feature, but the adjustment to a rental rate based on a constant 15-year (formerly 25-year) term. Initial rents, and thus capitalization rates, must be extraordinarily low to compensate for this hyper drift.

5 Time Dependent Drift: Pricing When a Market is Expected to Collapse.

In this section we generalize the model to include time dependence in the instantaneous drift $\alpha(t)$. Real estate supply and demand often adjust in lumpy increments reflecting the completion of new buildings or firm expansion/contractions. While anticipated changes in real estate supply are readily observable and thus reflected in market rents, firm demand for real estate (either contraction or expansion) is not as readily observed. Moreover, one can reasonably project short-term periods of increasing or decreasing demand periodically during the cycle. By incorporating time dependence into the model, we are able to examine the pricing of a lease during a wide variety of potential economic conditions. We illustrate the sensitivity of this pricing when a market collapse is anticipated.

To incorporate time dependent drift, we assume that the value of the asset's service flow obeys the stochastic dynamics

$$dS(t) = \alpha(t)S(t)dt + \sigma S(t)dw(t) \tag{20}$$

with the solution to (20) given by

$$S(t) = S_0 \exp\left((A(t) - \frac{1}{2}\sigma^2)t + \sigma w(t)\right), \tag{21}$$

where $A(t) = \int_0^t \alpha(s)\,ds$. The equilibrium contract rent is calculated following the discussion in Sections 2 and 3. For example, in the case of two upward-only adjustments, we calculate the initial equilibrium rent $R^u(0)$ by (14), where $R^u(T)$ and $R^u(2T)$ are given by (15) and (16) with $\rho(T)$ and $\rho(2T)$ defined as

$$\rho(T) = S_0 e^{-\sigma^2 T/2 + \sigma w(T)} \int_T^{3T} e^{-rt + A(t)}\,dt / g(r, T, 3T) \tag{22}$$

and

$$\rho(2T) = S_0 e^{-\sigma^2 2T/2 + \sigma w(2T)} \int_{2T}^{3T} e^{-rt + A(t)}\,dt / g(r, 2T, 3T). \tag{23}$$

Thus, we can solve (14) (with (15) and (16) replaced by (22) and (23), respectively) numerically for alternative $\alpha(t)$ functions.

We utilize the time dependent drift feature to price a market collapse followed by a sharp rebound. We assume that the real service flow is expected to fall at a five percent rate during the first five years, rebound at a five percent real rate during the second five years, and be flat thereafter. The expected nominal service flow also depends on the underlying expected rate of inflation. We consider the case of five percent inflation. Thus we posit:

$$\alpha(t) = \begin{cases} 0 & \text{if } t \in [0, 5) \\ 0.1 & \text{if } t \in [5, 10) \\ 0.05 & \text{if } t \in [10, 25]. \end{cases}$$

As always, we normalize S_0 to unity.

Table 4 compares the rents when a market collapse is expected with those expected assuming a steady state five percent growth rate. Under either the one or four percent real interest rate and volatility of 10 or 20 percent, the value of the lease declines by eight percent in response to the expected temporary shortfall of the service flow. With the expected collapse, the initial contract rent declines by a third (one percent real rate) to a quarter (four percent real rate) irrespective of the volatility level. The expected rent in five years is about five percent less, and that in ten years is only one percent less. These results are not sensitive to either the level of real interest rates or volatility.

6 Computing Effective Rents When Free Rent Periods Are Given

During the early 1990s the London office market collapsed, and free rent periods rose from negligible amounts to three years (Hendershott, Lizieri and Matysiak, 1999). This created problems for time-series modeling of rent determination: just how does one make rents where free rent periods are given comparable to rents when free rent periods do not exist? Put another way, how should one calculate the "effective" rent (zero free rent period equivalent) from the quoted "face" rent and free rent periods?

Two simple conversion rules that have been advanced are illustrated in Figure 1, where rent is on the vertical axis and time is on the horizontal (Crosby and Murdoch, 1994). The quoted or face rent is A, but it is not paid until the fourth year. After the fifth year, the rent is uncertain because of the upward-only adjustment; rents may continue at A or may be higher if market rents have risen sufficiently.

The first rule for converting face rents to effective (zero free rent periods) rent is to compute the rent that will give the same present value of rents over the five years to the first adjustment. This is indicated by B in the figure. This rule almost certainly gives too low a rent. If rent at the adjustment date is less than A, the lease with free rent periods would earn greater rent during the second five year period than the lease without free rent periods. Thus the correct or indifferent effective rent is greater than B. The second rule is to select the rent that will give the same present value of rents over the entire 15-year life of the lease. This is indicated by C. This rule almost certainly gives too high a value. If market rent at either the first or second adjustment date exceeds C, the gap between the rent earned on the lease with free rent periods and that without will shrink relative to that in the figure, and the present value of rents on the lease without free periods will tend to be greater than that with free rent periods.

Given our α specification, we can compute the initial rent with or without free rent periods. We first compute the value of the service flow using equation (1). We then compute the initial rent without free rent, using equations (12)-(13). Finally, we modify equation (14), changing the first term to reflect the free rent period. The results are shown in Table 5 for five percent inflation during both steady state and collapsed markets. We note that the initial rent is greater with free rent periods the lower the volatility and the lower the real interest rate. The reverse is true for the rent at the adjustment periods. With free rent, the floor is higher and thus the rent in later years is likely to be higher. And the higher the floor, the more likely are the higher later rents. With the collapsed market, the lower is the initial rent and thus the less likely is the floor to bind and the lower is the rent in later years.

Table 6 compares the discount of the quoted face rent required to get the effective rent using our model and using the simple 5-year and 15-year rules. Given our assumptions, when the underlying inflation rate is five percent, the correct discount is about 60 percent (column 1). The discount is two to four percentage points less when the market is collapsing than when it is steady, and the discount is about three percentage points less at the higher real interest rate than the lower and at the higher volatility than the lower. With the collapsed market (zero expected service-flow growth during the first five years) or higher volatility, the more likely the higher face rent is to create a binding floor at the adjustment date. The higher discount rate lowers the initial rent discount because the impact of a binding floor in the out years is less.

The second and third columns give the discount that is implied by the simple 5 and 15 year amortization rules. Because of their simplicity, the discounts are independent of either expected volatility in future service flow or in the state of the market. Columns four and five show the errors caused by the simple rules. Column four is the ratio of column two to column one. It is unity plus the percentage overstatement in the discount. As can be seen, the percentage overstatement in the steady state market using the 5-year rule is five to 20 percent, with the overstatement being greater the larger is volatility and the higher the real discount rate. The overstatement of the discount caused by the five year rule is far less than the 40 to 60 percent understatement caused by the 15-year rule. The overstatement of the five year rule will be greater at lower underlying inflation rates, while the understatement of the 15-year rule will be less.

7 Conclusions

In this paper we develop an implicit equation for securities with path dependent cash flows and apply it to the upward-only adjusting lease. Our model is in the spirit of Grenadier (1996) and correctly calculates the expected rents at the first and second adjustment dates taking into account the asset service flow expected growth and volatility. Our findings indicate that the initial rent on upward-only leases should be significantly lower than that on a corresponding fully adjusting leases when the rent floor is likely to bind (when the expected drift in the service flow is low and volatility is high). This discount offsets the expected increase in rents at the adjustment dates. As expected, the higher the expected growth rate and the lower the volatility, the lower the discount.

We extend the model to examine the initial rent discount in different economic environments. Specifically, we compare the expected rents in a market expecting a recession to be followed by an expansion. Results indicate how equilibrium rents need to be set so as to reflect these expected conditions.

We also examine rent setting when free rent periods are given or show how to convert the face rents with free rent periods to equivalent effective rents without free rent periods.

We show the importance of allowing for the impact of the higher face rents accompanying the free rent periods on the rent floor at the future adjustment dates. The errors introduced by using two conversion rules, amortizing the free rent periods over the period to the first rent adjustment at the fifth year and over the full 15-year term of the lease. The former slightly overstates the effective rent calculation, whereas the latter greatly understates it.

While the model presented here captures the spirit of the upward-only adjusting process, the present model can be extended in a number of directions. First, the upward-only adjustment should be incorporated in the market rent index to which the contract adjusts. Second, other tenant options should be added, namely "break" or cancellation clauses as well as the option to default on the lease payments. Such additions will provide a much richer picture of the adjustment process and pricing associated with this contract.

Appendix

Recall, the expected contract rent at $2T$ is defined in (16) as

$$R^u(2T) = E\left[\max\left[R^u(T), \rho(2T)\right]\right]$$

where

$$\rho(T) = \psi_1 e^{\sigma w(T)}$$
$$\rho(2T) = \psi_2 e^{\sigma w(2T)}.$$

and

$$\begin{aligned} \psi_1 &\equiv e^{-\sigma^2 T/2} g(r-\alpha, T, 4T)/g(r, T, 4T) \\ \psi_2 &\equiv e^{-\sigma^2 2T/2} g(r-\alpha, 2T, 5T)/g(r, 2T, 5T). \end{aligned}$$

Because the joint probability density function, $p(x, y, T, 2T)$, of the random variable $(w(T), w(2T))$ is given by

$$p(x, y, T, 2T) = \frac{1}{\sqrt{2\pi T^2}} e^{-x^2/(2T)} e^{-(y-x)^2/(2T)},$$

we calculate $R_2^u(2T)$ as

$$\begin{aligned} R^u(2T) &= R^u(0) \int_{-\infty}^{\ln(R^u(0)/\psi_1)/\sigma} \int_{-\infty}^{\ln({}^u(0)R/\psi_2)/\sigma} p(x, y, T, 2T)\, dy dx + \\ &\quad \psi_2 \int_{-\infty}^{\ln(R^u(0)/\psi_1)/\sigma} \int_{\ln(R^u(0)/\psi_2)/\sigma}^{\infty} e^{\sigma y} p(x, y, T, 2T)\, dy dx + \\ &\quad \psi_1 \int_{\ln(R^u(0)/\psi_1)/\sigma}^{\infty} \int_{-\infty}^{\ln(\psi_1/\psi_2)/\sigma + x} e^{\sigma x} p(x, y, T, 2T)\, dy dx + \\ &\quad \psi_2 \int_{\ln(R^u(0)/\psi_1)/\sigma}^{\infty} \int_{\ln(\psi_1/\psi_2)/\sigma + x}^{\infty} e^{\sigma y} p(x, y, T, 2T)\, dy dx. \end{aligned}$$

Thus, using the error function $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$, it follows that the expected rent, $R^u(2T)$, is given by

$$R^u(2T) = I_1 + I_2 + I_3 + I_4$$

where

$$I_1 = \frac{R}{4}\left[1+\text{erf}\left(\frac{\ln(R^u(0)/\psi_1)}{\sqrt{2T}\sigma}\right) + \frac{2}{\sqrt{\pi}}\int_{-\infty}^{\ln(R^u(0)/\psi_1)/\sqrt{2T}/\sigma} e^{-u^2}\text{erf}\left(\frac{\ln(R^u(0)/\psi_2)/\sigma - \sqrt{2T}u}{\sqrt{2T}}\right) du\right],$$

$$I_2 = \frac{1}{\pi}\psi_2 e^{\sigma^2(2T)/2}\int_{-\infty}^{(\ln(R^u(0)/\psi_1)/\sigma - T\sigma)/\sqrt{2T}} e^{-u^2}\left(\int_{(\ln(R^u(0)/\psi_1)/\sigma-\sqrt{2T}u-2T\sigma)/\sqrt{2T}}^{\infty} e^{-z^2}dz\right) du,$$

$$I_3 = \frac{1}{4}\psi_1 e^{\sigma^2 T/2} \left[1-\text{erf}\left(\frac{\ln(R^u(0)/\psi_1)/\sigma - T\sigma}{\sqrt{2T}}\right)\right]\cdot \left[1+\text{erf}\left(\frac{\ln(\psi_1/\psi_2)}{\sqrt{2(T)}\sigma}\right)\right],$$

$$I_4 = \frac{1}{4}\psi_2 e^{\sigma^2(2T)/2} \left[1-\text{erf}\left(\frac{\ln(R^u(0)/\psi_1)/\sigma - T\sigma}{\sqrt{2T}}\right)\right]\cdot \left[1-\text{erf}\left(\frac{\ln(\psi_1/\psi_2)/\sigma - T\sigma}{\sqrt{2T}}\right)\right].$$

If we denote the right hand side of equation (14) in the text as $\Psi_2(R^u(0))$, then to have a positive solution we must have

$$\Psi_2(0) < g(r-\alpha, 0, 3T).$$

$\Psi_2(0)$ can be written explicitly as

$$\Psi_2(0) = g(r,T,2T)\frac{g(r-\alpha,T,4T)}{g(r,T,4T)} + \mathcal{E}g(r,2T,3T)$$

where

$$\mathcal{E} = \frac{g(r-\alpha,T,4T)}{g(r,T,4T)}\frac{1}{2}\left[1+\text{erf}\left(\frac{\ln(\psi_1/\psi_2)}{\sigma\sqrt{2T}}\right)\right] + \frac{g(r-\alpha,2T,5T)}{g(r,2T,5T)}\frac{1}{2}\left[1-\text{erf}\left(\frac{\ln(\psi_1/\psi_2)/\sigma - \sigma T}{\sigma\sqrt{2T}}\right)\right],$$

erf(x) is the error function, erf$(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$, and ψ_1 and ψ_2 are defined above. Thus, $\Psi_2(0)$ is a function of 6 variables, $(r, \alpha, \sigma, T, 2T, 3T)$, and the condition under which equation (14) has a positive solution for the value of the initial contract rent depends on values taken by these arguments.[7]

[7] Under certain conditions the general equilibrium framework underlying our model does not contain a solution for $R^u(0)$. Specifically, when α is much greater than r (when real interest rates are quite negative), no solution exists. For example, when $\sigma = 10\%$ and $r = 6\%$, α must be greater than 100% for no solution to exist.

References

Baum, Andrew, Colin Beardsley, and Charles Ward, "Derivatives Pricing Approaches to Valuation Models: Sensitivity Analysis of Underlying Factors," Presented at the Sixth Annual European Real Estate Society Conference, Athens, June 1999.

Baum, Andrew, David Mackmin, and Nick Nunnington, *The Income Approach to Property Valuation*, Thomson Business Press (3rd Ed.), 1989.

Baum, Andrew, and Gary Sams, *Statutory Valuations*, Routledge (2nd Ed.), 1990.

Cox, John C., and Stephen A. Ross. "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics*, 3, 1976, 145-166.

Crosby, Neil and Sandi Murdoch, "Capital Value Implications of Free Rent Periods," *Journal of Property Valuation & Investment*, 1994, 12, 51-64.

Grenadier, Steven R. "Leasing And Credit Risk," *Journal of Financial Economics*, 1996, 42(3,Nov), 333-364.

Grenadier, Steven R. "Valuing Lease Contracts: A Real-Options Approach," *Journal of Financial Economics*, 1995, 38(3), 297-331.

Hendershott, Patric H., Colin M. Lizieri and George A. Matysiah, "The Workings of the London Office Market," *Real Estate Economics*, 1999, 27, 365-387.

Hendershott, Patric H., Bryan MacGregor and Michael White, "A Panel Approach to Modeling Regional Rents," presented at the ASSA Meetings, Boston, January 2000.

McConnell, John J. and James S. Schallheim, "Valuation Of Asset Leasing Contracts," *Journal of Financial Economics*, 1983, 12(2), 237-261.

Miller, Merton H. and Charles W. Upton, "Leasing, Buying, and the Cost of Capital Services," *Journal of Finance*, 1976, 31, 761-786.

Schallheim, James S. and John J. McConnell, "A Model For The Determination Of "Fair" Premiums On Lease Cancellation Insurance Policies," *Journal of Finance*, 1985, 40(5), 1439-1457.

Table 1: **Term-Structure of Rents**

		15-year Leases at			**10-year Lease at**	**5-year Leases at**		
$r-\alpha$	α	Year 0	Year 5	Year 10	Year 5	Year 0	Year 5	Year 10
0.01	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.01	0.05	1.408	1.808	2.322	1.625	1.129	1.450	1.861
0.01	0.1	1.896	3.127	5.155	2.587	1.268	2.091	3.447
0.04	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.04	0.05	1.370	1.760	2.259	1.605	1.126	1.445	1.856
0.04	0.1	1.800	2.967	4.892	2.525	1.260	2.078	3.426

Table 2: **Equilibrium Rents Calculated for the Upward-Only and Up-or-Down Adjusting Leases**

				Upward-Only Rents			**Up-or-Down Rents**			**Upward-Only and Up-or-Down**		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$r-\alpha$	α	σ	Y	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(10)$	Year 0	Year 5	Year 10	Year 0	Year 5	Year 10
0.01	0	0.1	13.93	0.867	1.033	1.113	1.000	1.000	1.000	-0.133	0.033	0.113
0.01	0.05	0.1	13.93	0.594	1.808	2.352	0.611	1.808	2.322	-0.017	0.000	0.030
0.01	0.1	0.1	13.93	0.101	3.127	5.159	0.102	3.127	5.155	-0.001	0.000	0.004
0.04	0	0.1	11.28	0.889	1.040	1.118	1.000	1.000	1.000	-0.112	0.040	0.118
0.04	0.05	0.1	11.28	0.749	1.760	2.289	0.761	1.760	2.259	-0.012	0.000	0.029
0.04	0.1	0.1	11.28	0.456	2.967	4.895	0.457	2.967	4.892	-0.001	0.000	0.004
0.01	0	0.2	13.93	0.745	1.058	1.221	1.000	1.000	1.000	-0.255	0.058	0.221
0.01	0.05	0.2	13.93	0.521	1.809	2.485	0.611	1.808	2.322	-0.090	0.000	0.163
0.01	0.1	0.2	13.93	0.063	3.127	5.272	0.102	3.127	5.155	-0.039	0.000	0.117
0.04	0	0.2	11.28	0.785	1.073	1.232	1.000	1.000	1.000	-0.215	0.073	0.232
0.04	0.05	0.2	11.28	0.694	1.763	2.420	0.761	1.760	2.259	-0.067	0.003	0.160
0.04	0.1	0.2	11.28	0.430	2.967	5.003	0.457	2.967	4.892	-0.027	0.000	0.111

Note: Up-or-Down Rents are calculated using the Grenadier term structure approach (with contract values equal).

Table 3: **Comparison of Upward-only Adjusting Lease Rents Calculated According to the Term Structure and the UK Methods**

				Term-Structure Method			**Ratio of UK Method to Term-Structure Method**		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$r-\alpha$	α	σ	Y	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(10)$	Year 0	Year 5	Year 10
0.01	0	0.1	13.93	0.867	1.033	1.113	1	1	1
0.01	0.05	0.1	13.93	0.963	1.626	1.926	0.617	1.112	1.221
0.01	0.1	0.1	13.93	0.971	2.587	3.478	0.104	1.209	1.483
0.04	0	0.1	11.28	0.889	1.04	1.118	1	1	1
0.04	0.05	0.1	11.28	0.998	1.607	1.916	0.75	1.095	1.194
0.04	0.1	0.1	11.28	1.032	2.525	3.452	0.442	1.175	1.418
0.01	0	0.2	13.93	0.745	1.058	1.221	1	1	1
0.01	0.05	0.2	13.93	0.867	1.644	2.077	0.601	1.1	1.196
0.01	0.1	0.2	13.93	0.911	2.589	3.656	0.069	1.208	1.442
0.04	0	0.2	11.28	0.785	1.073	1.232	1	1	1
0.04	0.05	0.2	11.28	0.919	1.632	2.069	0.754	1.08	1.169
0.04	0.1	0.2	11.28	0.988	2.53	3.619	0.435	1.173	1.382

Table 4: **Equilibrium Rents Calculated for the Upward-Only Adjusting Lease Assuming an Expected Market Collapse**

			Steady-State Market				**Collapsed Market**				**Ratio of Collapsed Market to Steady-State Market**		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$r-\alpha$	α	σ	Y	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(10)$	Y	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(10)$	Year 0	Year 5	Year 10
0.01	0.05	0.1	13.93	0.594	1.808	2.352	12.833	0.4	1.735	2.342	0.674	0.959	0.996
0.04	0.05	0.1	11.28	0.749	1.76	2.289	10.333	0.572	1.676	2.278	0.764	0.952	0.995
0.01	0.05	0.2	13.93	0.521	1.809	2.485	12.833	0.336	1.735	2.459	0.645	0.959	0.989
0.04	0.05	0.2	11.28	0.694	1.763	2.42	10.333	0.526	1.676	2.389	0.759	0.951	0.987

Table 5: **Equilibrium Rents Calculated for the Upward-Only Adjusting Leases Assuming Free Rent Offered During the First 3 Years.**

				No Free Rent Period			**3-years Free Rent**			**Ratio of Free Rent and No Free Rent**		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$r-\alpha$	α	σ	Y	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(10)$	$R_2^u(0)$	$R_2^u(5)$	$R_2^u(0)$	Year 0	Year 5	Year 10
Panel A: Steady-State Market												
0.01	0.05	0.1	13.93	0.594	1.808	2.352	1.516	1.854	2.367	2.553	1.025	1.006
0.04	0.05	0.1	11.28	0.749	1.76	2.289	1.778	1.927	2.352	2.375	1.095	1.028
0.01	0.05	0.2	13.93	0.521	1.809	2.485	1.228	1.879	2.524	2.358	1.039	1.016
0.04	0.05	0.2	11.28	0.694	1.763	2.42	1.528	1.949	2.531	2.203	1.105	1.046
Panel B: Collapsed Market												
0.01	0.05	0.1	12.83	0.4	1.735	2.342	1.093	1.737	2.343	2.731	1.001	1
0.04	0.05	0.1	10.33	0.572	1.676	2.278	1.468	1.745	2.296	2.567	1.041	1.008
0.01	0.05	0.2	12.83	0.336	1.735	2.459	0.88	1.75	2.467	2.618	1.009	1.003
0.04	0.05	0.2	10.33	0.526	1.676	2.389	1.264	1.779	2.444	2.402	1.061	1.023

Table 6: **Comparison of Correct Discount of Quoted Face Rent to Obtain Effective Rent and Simple Conversion Rules**

$r-\alpha$	α	σ	Correct Discount*	Amortize over 5 Years**	Amortize over 15 Years***	Ratio of 5 Year Value to Correct Value****	Ratio of 15 Year Value to Correct Value*****
Panel A: Steady-State Market							
0.01	0.05	0.1	0.608	0.636	0.278	1.045	0.456
0.04	0.05	0.1	0.579	0.653	0.319	1.128	0.552
0.01	0.05	0.2	0.576	0.636	0.278	1.104	0.482
0.04	0.05	0.2	0.546	0.653	0.319	1.196	0.585
Panel B: Collapsed Market							
0.01	0.05	0.1	0.634	0.636	0.278	1.003	0.438
0.04	0.05	0.1	0.610	0.653	0.319	1.070	0.523
0.01	0.05	0.2	0.618	0.636	0.278	1.028	0.449
0.04	0.05	0.2	0.584	0.653	0.319	1.119	0.547

*Unity less the ratio of columns 5 and 8 in Table 5.

**Discount given by the 5 year rule. Note that it is independent of volatility and the state of the market.

***Discount given by the 15 year rule.

****Unity plus the percentage overstatement of the discount caused by using the 5 year rule.

*****Percentage understatement of the discount caused by using the 15 year rule.

Figure 1: Rules for Converting Face Rents (A) into Effective Rents (B and C)