

NBER WORKING PAPER SERIES

VOLATILE POLICY AND PRIVATE  
INFORMATION: THE CASE  
OF MONETARY POLICY

Larry E. Jones  
Rodolfo E. Manuelli

Working Paper 7072  
<http://www.nber.org/papers/w7072>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
April 1999

We would like to thank NSF for providing financial support for this research. Part of this work was done when both authors visited the Bank of Portugal, which provided a stimulating research environment. We also thank John Kennan, Bob Lucas, Tom Sargent, Nancy Stokey, Steve Williamson and seminar participants for useful conversations on these topics. We also thank the referees of the journal for their comments. The views expressed in this paper are those of the authors and do not reflect those of the National Bureau of Economic Research.

© 1999 by Larry E. Jones and Rodolfo E. Manuelli. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Volatile Policy and Private Information:  
The Case of Monetary Policy  
Larry E. Jones and Rodolfo E. Manuelli  
NBER Working Paper No. 7072  
April 1999

### **ABSTRACT**

In this paper we study how volatility in monetary policy affects economic performance in the presence of endogenously chosen information structures. To isolate the effects produced by the interaction of uncertainty in monetary policy and (possibly) asymmetric information, we consider a model in which in the absence of either one of these features the equilibrium would be efficient. The equilibria that we find, with volatility and asymmetry of information, are inefficient for two reasons: first, in some cases, economic agents fail to trade, even though it is always efficient to do so; second, to capture the rents associated with being informed, agents spend resources acquiring socially useless information. Thus, in addition to the more standard effects of volatile inflation, our model calls attention to two types of costs associated with monetary uncertainty: the cost of not trading, and the cost of allocating resources to wasteful activities. The model implies that if monetary policy is not volatile all agents are symmetrically informed and hence, the outcome is efficient. Alternatively, making policy “transparent,” i.e. guaranteeing that all agents share the same information, serves the same purpose.

Larry Jones  
MEDS - Kellogg School  
Northwestern University  
Evanston, IL, 60208  
and NBER  
lej@nwu.edu

Rody Manuelli  
Department of Economics  
University of Wisconsin  
Madison, WI, 53706  
and NBER  
manuelli@wisc.edu

## 1. Introduction

In this paper we study how volatility in monetary policy affects economic performance in the presence of endogenously chosen information structures. To isolate the effects produced by the interaction of uncertainty in monetary policy and (possibly) asymmetric information, we consider a model in which in the absence of either one of these features the equilibrium would be efficient. The equilibria that we find, with volatility and asymmetry of information, are inefficient for two reasons: first, in some cases, economic agents fail to trade, even though it is always efficient to do so; second, to capture the rents associated with being informed, agents spend resources acquiring socially useless information. Thus, in addition to the more standard effects of volatile inflation, our model calls attention to two types of costs associated with monetary uncertainty: the cost of not trading, and the cost of allocating resources to wasteful activities. The model implies that if monetary policy is not volatile all agents are symmetrically informed and hence, the outcome is efficient. Alternatively, making policy “transparent,” i.e. guaranteeing that all agents share the same information, serves the same purpose.

We study a bargaining model in which a buyer and a seller (both risk neutral) determine the terms of the transfer of one unit of an indivisible good. If the parties agree to transfer the good, the buyer makes a money payment to the seller. Residual cash balances are used to buy units of a divisible good. Thus, cash must be used to undertake exchange in both markets and, hence, the basic form of money demand in the model is of the cash-in-advance variety. We assume that the realization of the money supply follows a commonly known stochastic law and choose units of the divisible good so that its price equals the money supply.

We assume that the bargaining takes the form of a take it or leave it offer by the seller. The buyer then decides whether or not to purchase the good. (This mechanism can be shown to be efficient if either the two parties are symmetrically informed or the buyer is informed and the seller is not.) For this model (see Chwe (1995)), it can be shown that a natural monetary neutrality result holds: Proportional changes in the stochastic process determining the money supply are neutral. Further, it can be shown that if either both parties are informed or both parties are uninformed, there are no real effects of variability in the money supply. Because of this, there

are no output or welfare costs of inflation without the presence of private information.

That is, even though the model is of the cash in advance variety, if information were symmetric, the output and welfare effects of inflation would be zero. In contrast, when information is asymmetric, a “lemon’s problem” in money arises (cf. Akerlof (1970)). Since both parties to the agreement share a common, but uncertain, value of the good “money,” the uninformed trader must take into account that the informed trader will adjust his actions to the current realization of the money supply. For example, if the buyer is informed and the seller is not, the only time the offer is accepted is when the value of money is low (i.e., the money supply is large). Knowing this, sellers adjust nominal prices upward, lowering the probability of trade below its optimal level. Since this does not arise in the symmetric information approach, the effect we find here is over and above those of standard cash-in-advance models. Indeed, it can be quite large when the level of volatility is high-- the probability of trade is zero-- even though it is common knowledge among the parties that the buyer values the good more highly than the seller. (Note that this is outcome is informationally constrained optimal.) Thus, this is one model of a phenomenon common in hyperinflations, that some markets “close,” with participants’ accounts emphasizing the difficulty of forecasting future inflation as the major cause.<sup>2</sup> Moreover, since this effect is in addition to those of the standard, symmetric information, cash-in-advance approach, this type of model may eventually prove useful in resolving another empirical puzzle: the size of the relationship between inflation<sup>3</sup> and average growth. Regression estimates of this effect range from a low of a .25% per year drop in the growth rate for every 10% per year increase in the average inflation rate (Barro (1995)) to as high as .70% per year (Fischer (1991) and Roubini and Sala-i-Martin (1995)). This is a puzzle for standard theories since growth models in which the supply of money is deterministic, predict --roughly independently of how

---

<sup>2</sup> Heymann and Leijonhufvud (1995) observe that during the Argentine hyperinflation of the eighties many shop owners refused to trade and posted signs that indicated they were “ ‘Closed for Lack of Prices’ ” (page 104). This seems to be a recurring theme during periods of high degree of policy uncertainty. During the Russian high inflation period following the default on government debt in August 1998, newspaper reports indicate that stores shut down because of uncertainty. (“Stores are shutting down as they sell out of stock or pull items from the shelves because they do not know how much to charge...” Chicago Tribune, August 29, 1998. )

<sup>3</sup> Even though the literature has emphasized the impact of mean inflation on growth, it is almost impossible, in the absence of a tightly specified model, to estimate the separate effects of mean inflation and inflation uncertainty, since these two measures are highly correlated (the correlation coefficient is estimated to be around .98 in a cross section of countries.)

the demand for money is modeled-- effects that are qualitatively of the same sign but only one tenth as large (see, for example, Cooley and Hansen (1989) and Chari, Jones and Manuelli (1995)). Moreover, allowing the money supply to be stochastic, but with all individuals symmetrically informed, does not help improve the “fit” (see, Gomme (1993)), and, in some cases, implies that inflation variability and growth are positively correlated (Dotsey and Sarte (1997)). Since the results that we explore are over and above the usual effects, they are potentially much larger than those from the standard models.

The equilibrium of our model will feature, for some parameter values, different information pairs. This has an impact for our predictions of the nature of the output-inflation trade off. If the buyer is informed and the seller is not, high realizations of the money supply are expansionary, giving rise to a “standard” Phillips curve effect (as in Lucas (1972)). In the case of an uninformed buyer and an informed seller, the probability of trade is also less than one but, here, high realizations of the money supply decrease output, and hence, the Phillips curve is downward sloping. Thus, the slope of the inflation-output tradeoff depends on the distribution of information. Thus, this class of models shows some potential to explain the finding (see Gomme (1993)) that, in a cross section of countries, the sign of the Phillips curve varies, with a relatively large proportion, 62 out of 82, displaying a *negative* relationship between realized inflation and output. Thus, the evidence indicates the existence of both “standard,” upward sloping, and “backward,” downward sloping, Phillips curves, both of which are consistent with our model.

In the model that we study, the economy-wide slope of the Phillips curve will depend on the fractions of buyers and sellers that are informed. To understand the determination of these fractions, we consider a game in which, at the first stage, buyers and sellers can learn the realization of the money supply at some cost. In the second stage they are randomly paired. At low levels of variability the expected value of any given amount of money is close to its certainty value. Information is not very valuable. At the other end, high variance results in high prices and low probability of a transaction. In this case, information is not very valuable either. This intuition carries over to the equilibrium in which all agents choose their information structure as long as the parties' values for the indivisible good are not too far apart. Thus, for both low and high variances all individuals choose to be uninformed. For intermediate levels of volatility, the equilibrium is in mixed strategies with varying fractions of buyers and sellers being informed. In

this region, the inefficiency of equilibrium is due to two factors: first, trade occurs with probability less than one; second, real resources are used to acquire socially useless information. We show that the equilibrium shape of the Phillips curve depends on parameters, and is not necessarily monotone. For example, at some levels of volatility, as inflation increases, overall trade first falls and then rises as the effects of different match types become dominant. Thus, the Phillips curve first slopes down and then up. The opposite can also occur. More generally, the relationship between inflation and output is always non-linear, with the “shape” depending on aspects the full specification of monetary policy (e.g., both the mean and variance of the growth rate of the money supply).

The endogenous determination of the distribution of information implies that the model has rich implications for measures of cross sectional price variability, since different buyer seller pairs (as identified by the structure of information) trade at different prices. The prediction that prices of similar goods need not be the same across stores is shared by other theoretical models (e.g. the search models described in Sheshinski and Weiss (1993)), but, unlike the search models, our model does not imply a uniform distribution of prices driven by the costs of adjustments. Rather, price dispersion obtains as an equilibrium outcome even though there are no costs of changing prices, and it is affected by the variability of monetary policy. Moreover, the model in this paper has a sharp prediction: the relationship between the dispersion of prices across matches is a U-shaped function of the rate of inflation. This is precisely the form found in Tommasi’s study of the Argentine experience (1993). (See also the evidence discussed in see Lach and Tsiddon (1992), Eden (1994) and the studies in Sheshinski and Weiss (1993)).

The papers that are closest to this one are Casella and Feinstein (1990), Chwe (1995), and Wallace (1997). Casella and Feinstein consider a dynamic bargaining situation in which there is no asymmetric information and inflation acts as a tax. In their particular formulation, inflation reduces the effective discount rate and this, in turn, affects the outcome of the bargaining game. The key result in their paper is that producers will charge different prices depending on the age (or wealth since they are perfectly correlated) of the buyers that they face. Chwe (1995) considers a model in which buyers and sellers are asymmetrically informed about both the value of money and their private valuations of the indivisible good. He shows that there will be no welfare loss associated with private information about money as long as the value of money is common

knowledge or agents share the same priors and information partition. He also shows that when individuals have different priors, monetary uncertainty can increase the sum of expected utilities. Moreover, he shows that an equilibrium displays Phillips curve-like features in the case of an exogenously changing fraction of informed traders. Wallace (1997) studies a matching model in which agents are asymmetrically informed but, as in Chwe, this decision is exogenous. A set of buyers without cash reserves is chosen randomly and given money transfers (and hence they know that some transfers have occurred). He shows, in a setting in which the structure of information is not common knowledge, that a standard Phillips curve obtains with a short run adjustment in output with a long run adjustment of prices as all agents become informed. Our work emphasizes two aspects ignored by the literature; first, it considers the impact of different monetary regimes --identified with the degree of variability in monetary policy-- upon the equilibrium for a given structure of information; second, it emphasizes that the monetary regime determines the structure of information, and that this is a critical element in understanding the predictions of the model.

In section 2 we present the basic model for symmetric information structures. Section 3 explores the equilibrium under asymmetric information, and characterizes the effects of changes in policy uncertainty. In section 4 we endogenize the acquisition of information and characterize the nature of the equilibrium distribution of prices and Phillips curve. Finally, section 5 offers some concluding remarks.

## **2. The Basic Model and Equilibrium with Symmetric Information**

There is a large number of agents of two types: buyers and sellers. Both the buyer and the seller derive utility from two goods: an indivisible good which gives utility  $v^b$  to the buyer (if purchased) and  $v^s$  to the seller, and another which we call general consumption. We assume that utility is additive and given by,

$$u^b(x,c) = v^b x + c,$$

where  $x^b$ , restricted to be either 0 or 1, represents consumption of the indivisible good and  $c$  is

the level of general consumption. We treat the seller symmetrically and assume that his utility function is given by,

$$u^s(x,c) = v^s x + c.$$

We take the valuations of both buyers and sellers as common knowledge, and assume that  $v^b > v^s$ . This assumption implies that trading the indivisible good is always the ex post efficient outcome.

The buyer has an initial endowment of money  $m^b$ . If the seller announces the price  $p$  and the buyer purchases one unit of the indivisible good, the buyer's consumption of the general consumption good is  $c^b = (m^b - p)/M$ . Here we have imposed that the price of the consumption good is equal to the money supply  $M$ . This simply follows from our choice of units of the divisible good, and can be rationalized by imposing a cash-in-advance constraint on divisible consumption, the market for which opens after the value of  $M$  becomes common knowledge.<sup>4</sup> Let the seller's endowment be  $m^s$ . Then, if he sells the indivisible good, his level of general consumption is  $(m^s + p)/M$ . Without loss of generality assume that  $m^b = \lambda M$ , while  $m^s = (1 - \lambda)M$ , where  $0 \leq \lambda \leq 1$ . With these conventions it is possible to write the indirect utility function (with some abuse of notation) for each of the parties given that they know  $M$  and the price  $p$  has been announced by the seller as,

$$(2.1) \quad u^b(x^b, p, M) = v^b x^b + \lambda - x^b p / M,$$

$$(2.2) \quad u^s(x^b, p, M) = v^s (1 - x^b) + 1 - \lambda + x^b p / M,$$

where  $x^b = 1$  corresponds to the buyer consuming the indivisible good.

We restrict the seller's announced price to be measurable function of his information and assume that the distribution of the random variable  $M$  is common knowledge.

---

<sup>4</sup> It is possible to interpret  $M$  as the growth rate of the money supply, with last period's money supply normalized to one. In what follows we will usually use the "stock" interpretation of  $M$ , although the reader can, without any formal changes, think of  $M$  as a growth rate.



First, consider the case in which both parties are informed. Here, given a price,  $p$ , announced by the seller, the buyer's problem is to choose whether or not to buy the object. Since, by assumption, he knows the realization of the random variable,  $M$ , he must choose the action to maximize his conditional expected utility given  $M$ . In this case, he will buy the object as long as  $p \leq v^b M$ . Since the seller knows this and knows the value of  $M$ , the equilibrium has the seller charging the price  $p(M) = v^b M$ . The good is exchanged with probability one and the seller extracts all of the surplus from the buyer.

On the other hand, if neither party is informed, the announced price does not depend on  $M$ , and hence, the expected utility of the buyer if the price is  $p$  is his unconditional expected utility. If he purchases the good, utility is,

$$\lambda + v^b - pE(M^{-1}),$$

while the expected utility of not buying the good is  $\lambda$ . Thus, the rule the buyers uses is: buy if the price is less than or equal to  $v^b/E(M^{-1})$ . It is clear that --if the seller decides to sell at all-- he will always charge the price

$$(2.3) \quad p(M) = v^b/E(M^{-1}).$$

That selling at this price is also in the interest of the seller follows immediately from the assumption that  $v^b > v^s$ . Let  $\mu = E(M^{-1})$  and, to simplify notation, let the 'information state' be denoted by a pair,  $(I_1, I_2)$  where  $I_1$  denotes the informational status of the buyer and  $I_2$  is that of the seller. Thus,  $(U, U)$  means that both agents are uninformed, etc. We summarize this discussion in the following proposition.

*Proposition 2.1.*

*A:* In the case of an informed buyer meeting an informed seller,  $(I, I)$ , the equilibrium outcome is characterized by:

- (i) The indivisible good will be traded with probability one. [ $q(I, I) = 1$ ]
- (ii) The price announced by the seller is  $p(M) = v^b M$ .

- (iii) The expected utilities of the two parties are,
 
$$W^s(I,I) = 1-\lambda + v^b,$$

$$W^b(I,I) = \lambda .$$
- (iv) Monetary policy is neutral. The allocation and (both ex ante and ex post) utilities of two agents do not depend on the distribution for M.

*B:* In the case of an uninformed buyer meeting an uninformed seller, (U,U), the equilibrium outcome is characterized by the following,

- (i) The indivisible good is traded with probability one. [ $q(U,U)=1$ ]
- (ii) The price announced by the seller is  $p(M)= v^b/\mu$ .
- (iii) The expected utilities of the two parties are,
 
$$W^s(U,U) = 1-\lambda + v^b,$$

$$W^b(U,U) = \lambda .$$
- (iv) Monetary policy is neutral. The allocation and (ex ante, expected) utilities of two agents are the same as if the random variable  $1/M$  was equal to its mean value with probability one. Thus, they do not depend on the distribution for M. Ex post welfare does depend on the realization of M, however.

The neutrality results presented here (i.e., Proposition 2.1 A(iv) and B(iv)) are reminiscent Chwe's (1995) common knowledge results. There, it is shown that whenever both parties share the same prior beliefs (which is the only case we consider) and their information partitions are the same, money is neutral.

### 3. The Case with Asymmetric Information

In this section, we generalize the model presented above to consider cases of asymmetric information. Throughout we assume that the information structure is common knowledge.

#### 3.1 Uninformed Sellers and Informed Buyers

Consider the case in which an uninformed seller meets an informed buyer. We study the analogue of the game analyzed in section 2 in which the seller has all the monopoly power. More precisely, we consider a timing of moves in which the seller announces a price, and the buyer can either accept or reject. We do not allow any counteroffers. This description of the bargaining

environment may seem restrictive at first glance. However, following the arguments in Samuelson (1984) and Myerson (1985), it can be shown that this game implements the outcome of the mechanism that maximizes the utility of the seller given incentive compatibility and individual rationality on the part of the buyer when the buyer is informed and the seller is not. Formally,

*Proposition 3.1.1* Assume that distribution of  $M$  has compact support with a density which is positive on the interval  $[m^L, m^U]$ , and that the buyer is informed and the seller is not. Then, the outcome of the bargaining game is identical to that of the mechanism that maximizes the utility of the seller given incentive compatibility and individual rationality of the buyer.

*Proof:* All proofs are in the Appendix.

To analyze the bargaining game in this case, consider first the buyer's decision. Since he knows the realization of  $M$ , given the price  $p$ , his decision rule is,

$$(3.1.1) \quad \begin{cases} \text{buy if} & p \leq Mv^b \\ \text{do not buy if} & p > Mv^b. \end{cases}$$

The seller knows this rule (this is the sense in which he extracts all the surplus) but does not know the realization of  $M$ . Since the seller is restricted to announcing prices that are independent of the realization of  $M$  (this is where the measurability restriction is binding), the seller chooses  $p$  to maximize,

$$(3.1.2) \quad V^s(p) = \int \left[ \chi_{(p < Mv^b)} \left[ 1 - \lambda + \frac{p}{M} \right] + (1 - \chi_{(p < Mv^b)}) [v^s + 1 - \lambda] \right] dF_M$$

where  $F_M$  is the cdf of the random variable  $M$ . Note that if the seller charges the price  $p^L = m^L v^b$  the buyer will accept the offer with probability one. Thus, the seller will always choose prices such that  $p \geq p^L$ . At the other extreme, any price exceeding  $p^U = m^U v^b$  will result in a zero probability of trade. Let  $p^*$  denote the solution to this maximization problem.

With this investment in notation, it is possible to give two types of neutrality results for this model. First, notice that if the distribution of  $M$  is a point mass at some value  $m^*$ , it follows immediately that the seller will announce the price  $p=m^*v^b$ . Given this, the buyer will always buy the object and trade will occur with probability one. Note that in this case, the outcome is ex-post efficient. This is one sense in which without variability in the money supply, there are no inefficiencies in this model. A second type of neutrality result holds even in cases in which there may be welfare losses from inflation. This is that the equilibrium outcome of the model is identical for any linear transformation of the money supply process. That is, inspection of the decision rule for the buyer above shows that for the random variable  $\alpha M$  with  $\alpha > 0$ , the optimal decision rule for the buyer has a reservation price which is  $\alpha$  times as high as the reservation price with the random variable  $M$ , and, hence, the equilibrium nominal price is also  $\alpha$  times as high, etc.

It turns out that it is more convenient to parameterize the distribution of the inverse of the money supply:

*Assumption 1: The Probability Model for M.* Assume that  $1/M$  has a uniform distribution on  $[\mu - k, \mu + k]$ , where  $0 \leq k < \mu$ .

Note that given this specification  $m^L = 1/[\mu + k]$ , and  $m^U = 1/[\mu - k]$ . Finally, to simplify notation, it is useful to express  $v^s$  as a fraction of  $v^b$ . More specifically, define  $\epsilon$  by:

$$v^s = (1 - \epsilon)v^b.$$

Then,  $\epsilon$  measures how far apart the values of the buyer and seller are. This, in turn, measures the potential gains from trade between the two parties. In this paper we only consider the case in which the differences in valuation are small (see the working paper version for the other case). Formally, we restrict ourselves to the case  $\epsilon \leq 1/2$ . We need one last bit of notation before describing the equilibrium behavior in this environment. Let  $k(\mu, \epsilon)$  be defined by

$$k(\mu, \epsilon) = \mu \frac{[1 - (1 - 2\epsilon)^{1/2}]}{[1 + (1 - 2\epsilon)^{1/2}]}$$

### *The Equilibrium of the Bargaining Game*

The seller maximizes (3.1.2) given its expectations about the realization of the money supply. Optimal behavior on the part of the buyer is completely summarized by the “purchasing rule” (3.1.1). We are now ready to describe the equilibrium of the bargaining game.

*Proposition 3.1.2.* Let  $\hat{p}(\mu, k, \epsilon)$  be the solution to the seller's problem (maximization of (3.1.2) subject to bounds constraints) and let  $\hat{q}(\mu, k, \epsilon)$  denote the resulting probability of the object being traded. Then, the equilibrium is characterized by,

- i) If  $k \leq k(\mu, \epsilon)$ ,  $\hat{p}(\mu, k, \epsilon) = p^L$  and  $\hat{q}(\mu, k, \epsilon) = 1$ ,
- ii) If  $k > k(\mu, \epsilon)$ , but  $k < \mu$ ,  $p = v^b(1 - 2\epsilon)^{1/2} / (\mu - k)$  and  $\hat{q}(\mu, k, \epsilon) = (\mu - k)[(1 - 2\epsilon)^{-1/2} - 1] / 2k < 1$ .  
In this case, trade occurs whenever  $M$  is large, i.e., whenever  $M \geq M_{1,U} \equiv p/v^b = (1 - 2\epsilon)^{1/2} m^U$ .

This simple model delivers the standard result in many fixed (or predetermined) (see Lucas (1989) and Lucas and Woodford (1993) for examples with explicit micro foundations) price models: an expansionary monetary policy reduces the real cost of goods for sale and it increases output. In our case, the result is extreme due to the indivisibility of the good; for low values of the money supply there is no trade (and output is low), while for high values, output is efficient. The model also suggests an asymmetric information interpretation of the reluctance to trade on the part of store owners documented by Heymann and Leijonhufvud (1995). They describe the following scene that took place in December 1989 when the inflation rate reached suddenly increased from a few percentage points per month to over 20% per week in a short period.

“ A customer finds a good inside a shop, with a clearly marked price, and decides to buy it. The shopkeeper refuses; he explains that the posted price has no significance, because he cannot be sure that the wholesaler will not double his own price the next day. When asked what he would do if someone offered to pay double the marked price, the

shopkeeper answers that he would not sell anyway, for what if the wholesale price tripled before he replaced the good?" (page 106, footnote 19)

This shows the effect of uncertainty about the value of money when deciding the terms of trade.

### *Changing the volatility of M*

Next, we will discuss the consequences of changing the volatility of the money supply upon the equilibrium. Proposition 3.1.2 gives us a partial, but incomplete accounting for what happens when uncertainty about policy is increased. There, holding  $\mu$  fixed, as  $k$  is increased, the probability of trade decreases toward zero. Notice however that we are simultaneously changing both the average value of  $M$  (it goes to infinity) and its variance (we are holding  $\mu = E(1/M)$  fixed). The problem is that changes in  $k$  holding  $\mu$  constant are **not** equivalent to increases in the variance of  $M$ , holding its mean constant. For this reason, we will be interested in **simultaneously** changing  $k$  and  $\mu$  so that  $E(M) = m^*$  is fixed. It is easy to check that to keep  $E(M) = m^*$  constant as we change  $k$ ,  $\mu$  must obey,

$$\mu(k) = k(e^{2m^*k} + 1) / (e^{2m^*k} - 1).$$

It is straightforward to check that increases in  $k$  correspond to increases in the variance of  $M$ , holding the mean constant.

In section 3.1 we described the constant  $k(\mu, \epsilon)$ , which played a critical role in determining the efficiency of an allocation. Since we are interested in changes in the variance of the money supply holding its mean ( $m^*$ ) constant, it is necessary to describe the value of  $k$  that corresponds to the threshold effect associated with  $k(\mu, \epsilon)$ . Let  $k_1^*$  be the unique value of  $k$  that satisfies,  $k_1^* = k(\mu(k_1^*), \epsilon)$ . A simple calculation shows that  $k_1^* = \ln[(1 - 2\epsilon)^{-1/2}] / 2m^*$ . It follows from Proposition 3.1.2 that, for  $k \leq k_1^*$ , the probability of trade is one while for  $k > k_1^*$  it is strictly less than one. From now on, to simplify notation, we will ignore the dependence of the equilibrium variables on  $\epsilon$  and  $m^*$ .

Calculating the effects of the equilibrium price and probability of trade when changing  $k$  with  $\mu = \mu(k)$  follows from a straightforward application of Proposition 3.1.2:

*Proposition 3.1.3.*

(i) The equilibrium price announced by an uninformed seller who meets an informed buyer --  $p(I,U,k)$  -- is

$$(3.1.3) \quad p(I,U,k) = v^b m^* \max\{(e^{2m^*k}-1)/2m^*ke^{2m^*k}, (1-2\epsilon)^{1/2} (e^{2m^*k}-1)/2m^*k\}.$$

(ii) The resulting equilibrium probability of trade is given by:

- a) For  $k \leq k_1^*$ ,  $q(I,U,k) = 1$ , while
- b) For  $k > k_1^*$ ,  $q(I,U,k)$  is given by:

$$(3.1.4) \quad q(I,U,k) = \frac{1}{2} + (e^{2m^*k}-1)^{-1} [(1-2\epsilon)^{-1/2} - (\frac{1}{2})(e^{2m^*k}+1)].$$

(iii)  $q(I,U,k)$  is decreasing in  $k$  and  $\lim_{k \rightarrow \infty} q(I,U,k) = 0$ .

Direct calculations show that  $p(I,U,k)$  is decreasing for  $k < k_1^*$ , and increasing for  $k > k_1^*$ . Thus, in this case, prices are a V-shaped function of the variance of the money supply. In the region  $(0, k_1^*]$  -- in which trade occurs with probability one -- an increase in the variance of the money supply forces the seller to lower prices to guarantee a sale, so that even in the lowest realization of the money supply the buyer finds advantageous to purchase the indivisible good. Thus, in this region, an increase in variance reduces the seller's monopoly position. Of course, lowering the price reduces consumption of the divisible good on the part of the seller. Thus, in this region, utility is redistributed from the seller to the buyer, but there are no efficiency costs of volatility. At  $k = k_1^*$ , this decrease in consumption is sufficiently high that the seller decides to charge slightly higher prices and face the possibility of trading with probability less than one. Thus, in this second region increases in the variance are accompanied by price increases and reductions in the transaction volume. Here, increases in volatility lower the utility of both the buyer and the seller. Finally, the essence of part iii) is that when volatility is high markets close. Thus, this gives a model of "Closed for Lack of Prices."

Even though Proposition 3.1.3 uses the assumption that  $1/M$  is uniform, the finding that the probability of trade is decreasing in the volatility of the policy can be considerably generalized. More precisely, for any distribution of  $M$ , consider increases in risk as affected by scale changes in the statistical sense. Thus, the  $M_k$  is "riskier" than  $M$  if there exists a  $k > 1$  such

that  $M_k = k (M - E(M)) + E(M)$ . Then the mean of  $M_k$  is independent of  $k$  but its variance is increasing in  $k$ . If the density of  $M$  is zero at both end points of its support, and is unimodal with mean equal to the mode, it can be shown that the probability of trade is a decreasing function of  $k$ . (Details available from the authors upon request.)

### 3.2 Asymmetric Information: Informed Sellers and Uninformed Buyers

Finally, we study the bargaining problem between an uninformed buyer and an informed seller, (U,I). In this case the seller knows  $M$  and can announce prices which are contingent on  $M$ . We will analyze perfect Bayesian equilibria of the game in which sellers are first informed about the value of  $M$ , second, sellers announce a price and finally, buyers, having seen the price, but not  $M$ , decide whether or not to buy the object. A perfect Bayesian equilibrium is then a choice of price by the seller for every outcome of  $M$ ,  $p(M)$ , an accept/reject rule for the buyer given the price (but not  $M$ ),  $X(p) = 0$  or  $1$ , and a set of beliefs about the true state of nature for the buyer given that the seller has offered the price  $p(M)$ . Of course,  $X(p)$  must maximize the buyer's utility given his beliefs and the price rule,  $p(M)$ , must (for each  $M$ ) maximize the seller's utility. Finally, the beliefs of the buyer must be credible.

To analyze the model, first consider the problem of a buyer. Since we will only be interested in pure strategy equilibria, a buyer's strategy can equally well be summarized by an 'acceptance set.' Accordingly, let  $A$  be the set of prices for which the buyer will accept and trade will occur. For simplicity, we will assume that the buyer can only choose among  $A$ 's which contain their supremum. This is a restriction on the strategy space of the buyer. Note further that given any acceptance set,  $A$ , the only relevant price from this set is its supremum,  $\sup A$ , since if the seller wants to sell the object to the buyer given what he knows about  $M$ , it is always in his interest to charge the maximal acceptable price. If he doesn't want to sell to the buyer, he can charge any price above  $\sup A$ , but  $p^U$  is a natural candidate. Given this, we can equivalently represent the problem of the buyer as choosing a reservation price, which we will denote by  $p^f$ .

This version of the bargaining game is a standard signaling game where the seller knows the value of  $M$  and signals it through his choice of prices. Not surprisingly, it is not rational (i.e., an equilibrium strategy) for the seller to completely reveal his information. Thus, he chooses



only two prices, one (low) to signal that he wants to sell, and another ( $p^U$ ) to signal that he wants to buy. As in the (I,U) case, for low levels of volatility the probability of trade is one, and it falls for high values of  $k$ . Accordingly, define  $k_2^* = \ln[(1+\epsilon)/(1-\epsilon)]/2m^*$ .

*Proposition 3.2.1.* In the case of an uninformed buyer meeting an informed seller, (U,I), the equilibrium outcome is characterized by the following:

- (i) If  $k \leq k_2^*$ , there are a continuum of equilibria indexed by the buyer's reservation price,  $p^r$  where  $p^r \leq v^b/\mu(k)$ . These equilibria have a probability of trade  $q(U,I,k)$  equal to one for  $m^U v^s \leq p^r \leq v^b/\mu(k)$ . The equilibrium welfare of the agents (in the equilibrium that maximizes the welfare of the seller) is given by:

$$W^b(U,I,k) = \lambda, \text{ and,}$$

$$W^s(U,I,k) = 1 - \lambda + v^b.$$

- (ii) If  $k > k_2^*$ , there are a continuum of equilibria indexed by the buyer's reservation price,  $p^r$  where  $p^r \leq m^L v^b (1+\epsilon)$ . The indivisible good will be traded with probability  $q(U,I,k) = [\epsilon/(1+\epsilon)][2e^{2m^*k}/(e^{2m^*k}-1)] < 1$ . Trade occurs when  $M \leq m^L(k)(1+\epsilon)/(1-\epsilon) \equiv M_{U,I}(k)$ , i.e., when  $M$  is low. When  $M > M_{U,I}(k)$ , the price charged is  $p^U$  and the good is not traded. The equilibrium welfare of the agents (in the equilibrium that maximizes the welfare of the seller) is given by:

$$W^b(U,I,k) = \lambda, \text{ and,}$$

$$W^s(U,I,k) = 1 - \lambda + v^b [1 - \epsilon + \epsilon q(U,I,k)].$$

- (iii) As  $k \rightarrow \infty$ , the maximum probability of trade converges to  $2\epsilon/(1+\epsilon)$ .

Here, since the seller has a monopoly on market power (by virtue of moving first in the bargaining game), and information, the buyer is pushed to his reservation utility for all values of  $k$ . Note that in this informational setting, the Phillips curve goes the opposite way from the previous case. That is, here trade occurs when  $M$  is small. Finally, note that the equilibrium price in this setting does not depend on  $M$  (other than to signal that  $M$  is low enough so that trade should occur), even though it could (i.e., the seller knows  $M$ ).

In what follows we will restrict attention to the equilibrium that maximizes the welfare of the seller which, in this case, also maximizes the probability of trade.

#### 4. Endogenous Information Structures

So far we have taken the information structure of the buyer-seller pairs as given. In this section we derive the distribution of information across trading pairs as the equilibrium of an information acquisition game. There are three reasons for this. First, the results in section 3 were predicated on the assumption that the parties to a match were differentially informed. Although it is straightforward to show that when  $k$  is low it does not pay for anybody to be informed, it is less obvious that when volatility is high some parties will still choose to be uninformed. This is shown in Theorem 4.1. Second, an implication of the results of section 3, is that the equilibrium slope of the Phillips curve depends on the equilibrium distribution of information. Third, the model's predictions for the relationship between the level of inflation and the cross-sectional distribution of prices --the type of evidence gathered in studies of high inflations-- also depends on the distribution of information. To analyze these last two questions, it is essential to characterize the equilibrium distribution of information and the volatility of monetary policy. This is also done in this section.

To model endogenous information, we study a two stage game. In the first stage, individuals choose whether to become informed or not. This decision is made simultaneously by all players (buyers and sellers). In the second stage --given the information structure-- buyers and sellers are randomly matched, and they engage in the bargaining problem described in the previous two sections. We assume that in the second stage the information structure is common knowledge; in other words, both parties to a match know whether the other is informed or not. Thus, the equilibrium strategies for the second stage are those described in sections 2 and 3. A second stage equilibrium always exists and, given our selection in the (U,I) case, it is unique. Thus, we study the equilibrium of a one period game in information strategies that has as its payoffs the expected utilities that were calculated in the previous section. The notion of equilibrium that we use is Nash equilibrium.

We assume that the cost of becoming informed is given by  $\phi$  (measured in units of the divisible consumption good). This cost of acquiring information can be interpreted broadly as encompassing private time and expenditure costs which can be affected by the government. As an example, secrecy rules associated with the meetings of the FOMC have helped create a sizable

industry of Fed watchers who try to estimate --given the fragmentary information available-- the Fed's actual policy. If the Fed plainly announced in unequivocal terms its intentions and policies well in advance, it is likely that this industry would greatly shrink. This is an example in which  $\phi$  would correspond to the cost of hiring Fed watchers as consultants. Thus, in this sense,  $\phi$  is a policy variable, determined by the "transparency" of the monetary authority. Alternatively, many accounts of hyperinflations emphasize that individuals spend a great deal of time trying to determine the real value of alternative transactions. This, in our model, corresponds to finding the actual value of  $M$ , since this is all that is required to evaluate a proposed transaction.

We will consider only symmetric equilibria in pure strategies extended to the natural asymmetric situation when the unique equilibrium is in mixed strategies. The implied first stage payoffs are given by,

Table I: Payoffs in the First Stage ( $\epsilon < 1/2$ )

Seller	Uninformed (U)	Informed (I)
Buyer		
Uninformed (U)	$1 - \lambda + v^b$ $\lambda$	$1 - \lambda + v^b \Upsilon^s - \phi$ $\lambda$
Informed (I)	$1 - \lambda + v^b \Upsilon^s$ $\lambda + v^b \Upsilon^b - \phi$	$1 - \lambda + v^b - \phi$ $\lambda - \phi$

The functions  $\Upsilon^j$   $j = b, s$  are defined in the Appendix. Since this is a  $2 \times 2$  game with known payoffs, it follows that an equilibrium always exists. Since the maximum amount of surplus available to the two players is  $v^b - v^s = \epsilon v^b$ , it follows that it is a dominant strategy for each player to not purchase information if  $\phi \geq \epsilon v^b$ . In this case then, the unique equilibrium is (U,U) independent of the volatility of  $M$ . Thus, we assume  $\phi < \epsilon v^b$ .

*Theorem 4.1.* Let  $\phi < \epsilon v^b$ . For all values of  $k$  a Nash equilibrium exists. There exist values of  $k$ , denoted  $k_1(\phi)$  and  $k_2(\phi)$  such that,

- i)  $k_1(\phi) < k_1^* < k_2(\phi)$ ;  $k_1(\phi)$  is decreasing in  $\phi$  and converges to 0 as  $\phi$  goes to zero;  $k_2(\phi)$  is increasing in  $\phi$  and converges to  $\infty$  as  $\phi$  goes to zero.
- ii) For  $k \in [0, k_1(\phi))$ , the unique symmetric Nash equilibrium strategies are (U,U).
- iii) For  $k = k_1(\phi)$  there are two pure strategy Nash equilibria. The Nash equilibrium strategies are given by (U,U) and (I,U). In addition, there are infinitely many mixed strategy Nash equilibria. The equilibrium strategies are given by any mixture over {I,U} on the part of the buyer, and the mixture that puts mass one on U on the part of the seller.
- iv) For  $k \in (k_1(\phi), k_2(\phi))$  there are no pure strategy Nash equilibria. There exists a unique mixed strategy equilibrium, and the equilibrium strategies are characterized by the following probabilities that buyers ( $\pi^b$ ) and sellers ( $\pi^s$ ) are informed (i.e. they choose I),

$$\pi^b(k) = [(\phi/v^b) + (1 - \Upsilon^s(U, I, k))] / [(1 - \Upsilon^s(I, U, k)) + (1 - \Upsilon^s(U, I, k))].$$

$$\pi^s(k) = [v^b \Upsilon^b(I, U, k) - \phi] / v^b \Upsilon^b(I, U, k).$$

- v) For  $k \geq k_2(\phi)$  there is a unique Nash equilibrium. The equilibrium strategies are (U,U).

As can be seen from the payoff matrix, buyers gain from being informed only if they are paired with an uninformed seller, and sellers benefit only if they are paired with an informed buyer. In the region in which monetary policy is not too volatile even uninformed sellers extract most of the surplus from the arrangement when facing informed buyers. Hence, in this region, the cost of information exceeds the gains to a potential buyer even if he is sure to meet an uninformed seller. It follows that it does not pay for buyers to become informed and hence, sellers will also not be informed.

When the level of volatility is high, uninformed sellers set prices high when facing an informed buyer driving the probability of trade to zero. Given this, buyers have little incentive for becoming informed. Since there are few informed buyers, it does not pay for sellers to be informed either. Again, the unique Nash equilibrium is (U,U).

For intermediate values of  $k$  --more specifically for  $k_1(\phi) < k < k_2(\phi)$ -- there are only mixed strategy equilibria. At the lower end of this region (low volatility) a large fraction of the buyers are informed (how large depends on  $k_1^*$ ; if  $k_1^* > k_1(\phi)$  large is 100%), since the gains from being informed when meeting an uninformed seller exceed the cost. On the other hand, almost none of the sellers is informed. The reason is that, for  $k$  close to  $k_1(\phi)$ , sellers do not stand to gain at all

from acquiring information: the potential gain is equal to the cost since they are going to meet informed buyers with probability one. As  $k$  increases, the fraction of buyers who are informed decreases. The fraction of informed sellers increases up to the point  $k=k_1^*$ , and then decreases. For  $k$ 's less than  $k_1^*$ , as  $k$  is adjusted, the only change in the payoff matrix is a redistribution from sellers to buyers in (I,U) matches. This increases the incentives for sellers to become informed hence lowering the incentives to buyers. Since this is a pure redistribution, these effects exactly offset, keeping the fraction of agents who are informed unchanged. For values of  $k$  greater than  $k_1^*$ , there is, in aggregate, a loss in welfare as the probability of trade falls below 1 for (I,U) pairs with aggregate information gathering falling as well.

The exact form of the the mixed strategy equilibrium in the region  $k_1(\phi) < k < k_2(\phi)$  depends on where  $k_2^*$  lies in the interval  $(k_1(\phi), k_2(\phi))$ . In all cases,  $k_2^* < k_2(\phi)$ . Since our interest is to show how monetary policy can lower the probability of trade, we will concentrate on the case in which the probability of trade is highest. This corresponds to a high  $k_2^*$  (see Proposition 3.2.1). It turns out that depending on the value of  $\epsilon$ ,  $k_2^*$  can lie on either side of  $k_1^*$ . It is possible to show that  $k_2^* > k_1^*$  if  $(1+\epsilon) \geq (1-\epsilon)(1-2\epsilon)^{-1/2}$ . It is easy to check that this inequality is satisfied for small values of  $\epsilon$ . In this case, the aggregate fraction of informed individuals, given by  $\bar{\pi}(k) \equiv (\pi^b(k) + \pi^s(k))/2$ , is zero for small levels of  $k$ , rises to half the population, and remains at that level as  $k$  rises up to  $k_1^*$ . Further increases in the variability of the money supply result in decreases in  $\bar{\pi}(k)$ .<sup>5</sup>

The “identity” of who is informed also changes with  $k$ . This is important in this model, since it is the distribution of information across buyers and sellers that determines both the equilibrium slope of the Phillips curve and the cross sectional variability of prices. Neither buyers nor sellers are informed at either extremely high or extremely low levels of  $k$ . At the low end of the region in which the equilibrium is in mixed strategies, the fraction of buyers who are informed,  $\pi^b(k)$ , is high (in fact,  $\pi^b(k_1(\phi)) = 1$ ) and the fraction of sellers that are informed,  $\pi^s(k)$ ,

---

<sup>5</sup> Formally, aggregate information  $\bar{\pi}(k) \equiv (\pi^b(k) + \pi^s(k))/2$  satisfies,

- a) It is zero for either  $k < k_1(\phi)$ , or  $k > k_2(\phi)$
- a)  $\bar{\pi}(k) \in [0, 1/2]$  for  $k = k_1(\phi)$ ,
- b)  $\bar{\pi}(k) = 1/2$  for  $k_1(\phi) \leq k \leq k_1^*$ ,
- c)  $\bar{\pi}(k)$  is less than 1/2, for  $k_1^* < k < k_2(\phi)$ , and is decreasing in this region.

is low ( $\pi^s(k_1(\phi)) = 0$ ). As variability increases,  $\pi^b(k)$  falls, while  $\pi^s(k)$  first rises and falls.<sup>6</sup> The behavior of the different measures of information is illustrated in Figure 1.

### *The Behavior of Prices*

The model has implications for the effects of monetary shocks on the cross sectional distribution of prices, and, hence, on the responsiveness of both mean and variance of prices to changes in  $M$ . For both small and large values of  $k$  ( $k < k_1(\phi)$ , and  $k > k_2(\phi)$ ), all agents are uninformed, and the price of the indivisible good is equal to  $v^b/E[1/M]$  independent of the realization of  $M$ . For values of  $k$  in the interval  $[k_1(\phi), k_2(\phi))$ , different information pairs trade at different prices. The distribution of prices has the same qualitative characteristics for all values of  $k$  in this region, but, the actual prices charged in a (U,I) match depend on the location of  $k$  relative to  $k_2^*$ . To illustrate this we consider the case  $k \geq k_2^*$ . The distribution of prices across information pairs is given by:<sup>7</sup>

Information state	Probability	Price Charged
(U, U)	$(1-\pi^b)(1-\pi^s)$	$v^b/E[1/M]$
(I, I)	$\pi^b \pi^s$	$v^b M$
(I, U)	$\pi^b(1-\pi^s)$	$v^b(1 - 2\epsilon)^{1/2} m^U(k)$
(U, I)	$(1-\pi^b) \pi^s$	$v^b(1 + \epsilon) m^L(k)$ if $M \leq M_{U,I}(k)$ $v^b m^U(k)$ if $M > M_{U,I}(k)$

Note that, given  $k$ , there are only two information pairs for which the price charged

<sup>6</sup> Formally,

a)  $\pi^b(k_1(\phi))=1$ ,  $0 < \pi^b(k_2(\phi)) < 1$ , and  $\pi^b(k)$  is decreasing in  $k$ , if  $[(\phi/\epsilon v^b)(1-\epsilon) + \epsilon(1+\epsilon)] \leq [(\phi/\epsilon v^b)(1+\epsilon) + 1 - \epsilon][1 - (1-\epsilon)^{1/2}]$ .  
b)  $\pi^s(k_1(\phi)) = \pi^s(k_2(\phi)) = 0$ , and  $\pi^s(k)$  is increasing for  $k < k_1^*$  and decreasing for  $k > k_1^*$ .

<sup>7</sup> Recall that (U,I) pairs trade whenever  $M$  is below  $M_{U,I}(k)$ , and (I,U) pairs trade whenever  $M$  is above  $M_{I,U}(k)$ .

depends on  $M$ . The first is the (I,I) pair, where changes in  $M$  are matched perfectly. The other is the case of the informed seller and uninformed buyer where the price jumps discontinuously up when that  $M$  is reached where the equilibrium calls for the seller to choke off all trade. Given this property, it follows that the conditional mean and variance of prices given  $M$  have simple forms. The average value of prices is given by:

$$\begin{aligned}\bar{p}(M;k) &= \alpha_0 + v^b \pi^b \pi^s M && \text{if } M \leq M_{U,I}(k), \text{ and} \\ \bar{p}(M;k) &= \alpha_1 + v^b \pi^b \pi^s M && \text{if } M > M_{U,I}(k),\end{aligned}$$

for some constants  $\alpha_0$  and  $\alpha_1$ . Thus, even though prices are responsive to monetary shocks in this region, they are sluggish in the sense that they are less responsive than would be predicted by a full information flexible price model ( $p=v^b M$ ).

Next, let  $\sigma_p^2(M;k)$  denote the cross sectional variance of prices (again taken across all pairs of traders). For low and high values of  $k$ ,  $\sigma_p^2(M;k) = 0$ . For  $k \geq k_2^*$ ,

$$\begin{aligned}\sigma_p^2(M;k) &= \beta_0 + \pi^b \pi^s (v^b)^2 M^2 - (\alpha_0 + v^b \pi^b \pi^s M)^2 && \text{if } M \leq M_{U,I}(k), \text{ and} \\ \sigma_p^2(M;k) &= \beta_1 + \pi^b \pi^s (v^b)^2 M^2 - (\alpha_1 + v^b \pi^b \pi^s M)^2 && \text{if } M > M_{U,I}(k),\end{aligned}$$

where, again,  $\beta_0$  and  $\beta_1$  are constants. Thus,  $\sigma_p^2$  follows two quadratics, jumping from one to the other at  $M = M_{U,I}(k)$ . Since, the coefficient on the  $M^2$  term in both of these expressions is  $(v^b)^2[\pi^b \pi^s - (\pi^b \pi^s)^2] > 0$ , they are both U-shaped. Moreover, it can be shown that if  $\epsilon$  is small, they are both minimized within the range  $[m^L(k), m^U(k)]$ . Thus, for a given  $k$ ,  $\sigma_p^2(M;k)$  is a non-monotone function of  $M$ , first falling and then rising. This matches a common finding in the empirical literature on the effects of high inflations. In particular, in his study of the Argentine hyperinflation, Tommasi (1993) analyzes data on the relationship between the cross sectional variance of prices in a period and the level of inflation in that period and finds that it is non-monotonic. He finds that this dispersion is decreasing in the level of inflation for low levels and

increasing for high levels.<sup>8</sup>

### *The Equilibrium Phillips Curve*

Is the model consistent with both upward and downward bending Phillips curves? To show that this is possible consider two cases. If  $k_1^* < k_2^*$  and  $k \in (k_1^*, k_2^*)$  the only pair that fails to trade for any realization of the monetary shock is (I,U). This pair does not trade only when the value of  $M$  is low (see Proposition 3.1.2). Thus, in this region, the Phillips curve slopes upward. On the other hand, if  $k_2^* < k_1^*$  and  $k \in (k_2^*, k_1^*)$ , then the only pairs that do not trade all the time are uninformed buyers and informed sellers. In this case, Proposition 3.2.1 shows that the Phillips curve slopes downward.

In addition to these two possibilities, the model has rich and complex implications for the shape of the inflation-output tradeoff. Since, the particular details depend on parameter values, here we restrict attention to the case  $k_1^* < k_2^*$ , and describe how the Phillips curve changes as a function of  $k$ . For  $k \leq k_1^*$ , all information pairs trade with probability one; thus the Phillips curve is perfectly flat. As noted above, for  $k \in (k_1^*, k_2^*)$ , it is upward sloping. The region  $[k_2^*, k_2(\phi))$  is more interesting because both the (I,U) pairs and the (U,I) pairs may not trade depending on the realization of  $M$ . The pairs (I,U) trade whenever the realization of  $M$  exceeds the threshold  $M_{I,U}(k)$  while the (U,I) pairs trade whenever  $M$  falls short of  $M_{U,I}(k)$ . There are two cases which correspond to  $M_{I,U}(k) < M_{U,I}(k)$  and  $M_{I,U}(k) > M_{U,I}(k)$ . Which one of these obtains depends on the value of  $k$ .

*Case I:*  $M_{I,U}(k) > M_{U,I}(k)$

This case corresponds to a low value of  $k$ . In the region  $[m^L(k), M_{U,I}(k)]$  all information pairs but (I,U) trade; output is “moderately” low. In the region  $[M_{U,I}(k), M_{I,U}(k)]$  neither the (I,U) or the (U,I) information pairs trade, and output is low. Finally, in the region  $[M_{I,U}(k), m^U(k)]$  the only pair that fails to trade is (U,I); hence, output is again “moderately” low. Overall, the relationship between  $M$  and output has a U-shape.

---

<sup>8</sup> In Tommasi's empirical study, he finds that price dispersion is minimized at zero inflation. However, the functional forms that he considers only allow the minimum to be at zero.



*Case II:*  $M_{I,U}(k) < M_{U,I}(k)$ .

This case corresponds to a high value of  $k$ . In the region  $[m^L(k), M_{I,U}(k)]$  all information pairs but (I,U) trade; output is “moderately” low. In the region  $[M_{I,U}(k), M_{U,I}(k)]$  all information pairs trade, and output is high. Finally, in the region  $[M_{U,I}(k), m^U(k)]$  the only pair that fails to trade is (U,I); hence, output is again “moderately” low. Overall, the relationship between  $M$  and output resembles an inverted-U.

Thus, even in a simple model like the one in this paper the sign of the slope of the Phillips curve depends on both the parameters of the model (e.g.  $k, \epsilon, \phi$ ) and the realization of  $M$ . Hence, any econometric study should allow for the non-linearities associated with these different regimes.

## 5. Concluding Comments and Extensions

In this paper we have presented a model in which perfectly anticipated inflation is superneutral: if the variance of the money supply (or the growth rate of the money supply) is zero, the real equilibrium is independent of the mean of the money supply (or growth rate). On the other hand, it was shown that increases in the variability of the money supply holding its mean constant can have substantial real effects. The key feature is that --even when the decision to become informed is endogenous-- there is ex-post asymmetric information. This gives rise to efficiency effects of volatility for two reasons. First, given an information state, the probability of trade is decreasing in the level of volatility. Thus, there are states in which the good should be traded, but it is not. In addition to this, when information gathering is endogenous, the decision to become informed costs real resources. In this model, the social value of information is zero (it only redistributes bargaining power between the buyer and seller) and hence, any such activity is a loss in efficiency. This gives rise to a clear policy recommendation from the model: Set volatility at zero. In this case, the outcome is efficient. Moreover, in contrast to the standard cash goods/credit goods treatment of the impact of inflation, the welfare cost that we find extends to **all** nominally denominated transactions, and not only to those in which cash is the medium of exchange. This effect, increases the cost of inflation relative to the one derived in that cash/credit

good calculation.

The model can account, qualitatively, for several puzzling observations: First, it implies that the cross-sectional distribution of prices is not degenerate even when all agents are symmetric *ex ante*. Indeed the variance of the cross sectional distribution of prices is not a monotonic function of the realization of the inflation: At low levels, an increase in the rate of inflation decreases the dispersion of prices while it increases it for high levels. This agrees with the evidence during high inflations. Second, it is consistent with both upward and downward sloping Phillips curves. This is due to the effect that different monetary regimes have on both the fraction and identity of the agents that are informed. Finally, if we allow for different costs of acquiring information, the model can explain while, in the face of extremely high uncertainty, sellers would rather close than sell.

The model can be extended along several dimensions. First, even though we discussed a “static” version, it is possible to show that the same decision rules apply to a dynamic infinite horizon version in which each family is composed of a buyer-seller pair. In such a dynamic setting, the model also delivers the implication that nominal interest rates are “sluggish,” in the sense that they do not increase one for one with mean inflation. Moreover, increases in the variance of the monetary shock, holding average inflation constant, reduce nominal interest rates, leaving real rates unchanged. (For details see the working paper version.). Second, the model suggests a role for inventories in periods of high inflation. If the good is storable, the breakdown of trade will occur at even more moderate levels of inflation.

Finally, the mechanism that we capture, uncertainty about government policies with costs of learning the true policy, is more general than our monetary interpretation. The basic intuition, that in these cases the lemon's nature of the game between asymmetrically informed agents can exacerbate the effects of uncertainty, should apply to other policies as well. Interesting possibilities include exchange rate policy and debt repayment policy which we hope to study in future work.

## BIBLIOGRAPHY

- Akerlof, G., (1970), "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 89, pp:488-500
- Barro, R., (1995), "Inflation and Economic Growth," *Bank of England Quarterly Bulletin*, pp:166-176, May. ( Another version is in NBER Working Paper 5326, October 1995)
- Casella, A. and J. Feinstein, (1990), "Economic Exchange During Hyperinflations," *Journal of Political Economy*, Vol. 98, No. 1, pp:1-27.
- Chari, V. V., L. E. Jones and R. E. Manuelli, (1995), "The Growth Effects of Monetary Policy," *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 19, No. 4, Fall, pp: 19-32.
- Chwe, M., (1995), "Money Neutrality is Equivalent to Common Knowledge and Game Theoretic Bargaining Foundations of the Phillips Curve," Department of Economics, University of Chicago.
- Cooley, T. F. and G. D. Hansen, (1989), "The Inflation Tax in a Real Business Cycle Model," *American Economic Review*, 79, September, pp:733-48.
- Dotsey, M. and P-D. Sarte, (1997), "Inflation Uncertainty and Growth in a Simple Monetary Model," Federal Reserve Bank of Richmond Working Paper.
- Eden, B., (1994), "Inflation and Price Dispersion," Bank of Israel Research Department Working Paper No. 94.13, August.
- Fischer, S., (1991), "Growth, Macroeconomics and Development," *NBER Macroeconomics Annual 1991*, Vol. 6, ed. Olivier Jean Blanchard and Stanley Fischer, pp. 329-64. Cambridge, Mass.: MIT Press/National Bureau of Economic Research.
- Gomme, P., (1993), "Money and Growth Revisited: Measuring the Costs of Inflation in an Endogenous Growth Model," *Journal of Monetary Economics*, 32(1), August, pp:51-77.

- Heymann, D and A Leijonhufvud, (1995), *High Inflation*, Clarendon Press, Oxford.
- Lach, S. and D. Tsiddon, (1992), "The Behavior of Prices and Inflation: An Empirical Analysis of Disaggregated Price Data," *Journal of Political Economy*, Vol. 100, No. 2, pp:349-389, April.
- Lucas, R. E. Jr, (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, pp:103-124, April.
- \_\_\_\_\_ (1989), "The Effects of Monetary Shocks When Prices are Set in Advance, Working Paper, November.
- Lucas, R. E. Jr, and M. Woodford (1993), "Real Effects of Monetary Shocks in an Economy with Sequential Purchases," NBER Working Paper No. 4250, January.
- Myerson, R., (1985), "Analysis of Two Bargaining Problems with Incomplete Information," in *Game Theoretic Models of Bargaining*, A. Roth, Ed., Cambridge University Press, Cambridge.
- Roubini, N. and X. Sala-i-Martin, (1995), "A Growth Model of Inflation, Tax Evasion, and Financial Repression," *Journal of Monetary Economics*, 35(2), April, pp: 275-301.
- Samuelson, W., (1984), "Bargaining Under Asymmetric Information," *Econometrica*, Vol. 52, No. 4, July, 995-1006.
- Sheshinski, E., and Y. Weiss, (1993), *Optimal Pricing, Inflation, and the Cost of Price Adjustment*. MIT Press, Cambridge.
- Tommasi, M. , (1993), "Inflation and Relative Prices: Evidence from Argentina," in Sheshinski, E., and Y. Weiss, (eds.), *Optimal Pricing, Inflation, and the Cost of Price Adjustment*. MIT Press, Cambridge, pp:485-512.
- Wallace, N., (1997), "Short-Run and Long-Run Effects of Changes in Money in a Random-Matching Model," *Journal of Political Economy*, 105(6), December, pp: 1293-1307.

## Appendix

*Proof of Proposition 3.1.1:* Although not identical, the proof follows the arguments in Myerson (1985) and Samuelson (1984) very closely. A mechanism is described by a probability of trading the indivisible good as a function of the state,  $M$ ,  $q(M)$  and a state contingent transfer of the divisible good,  $T(M)$  from the buyer to the seller. With this notation, the utility of the buyer is given by:

$$U^b(M) = v^b q(M) + (m^b - T(M))/M = v^b q(M) + \lambda - T(M)/M$$

while that of the seller is given by:

$$U^s(M) = v^s (1 - q(M)) + (m^s + T(M))/M = v^s - v^s q(M) + 1 - \lambda + T(M)/M.$$

The mechanism,  $(q(M), T(M))$  is feasible if:

1.  $0 \leq q(M) \leq 1$  for all  $M$ .
2. For all  $M, M'$ ,  $v^b q(M) + \lambda - T(M)/M \geq v^b q(M') + \lambda - T(M')/M$ .
3. For all  $M$ ,  $v^b q(M) + \lambda - T(M)/M \geq v^b 0 + \lambda - 0/M = \lambda$ .

Here, 2 invokes the revelation principle to get that truth telling is an equilibrium (i.e., incentive compatibility holds) and 3 captures individual rationality. Note that feasibility implies that a mechanism is feasible if and only if:

1.  $0 \leq q(M) \leq 1$  for all  $M$ .
2. For all  $M, M'$ ,  $M v^b q(M) - T(M) \geq M v^b q(M') - T(M')$ .
3. For all  $M$ ,  $M v^b q(M) - T(M) \geq 0$ .

Now, let  $\pi(M, M') = M v^b q(M') - T(M')$ . This is the utility that the buyer will receive if the true state is  $M$  and he announces that the state is  $M'$ . Then, one obtains that for any feasible mechanism:

4.  $v^b (M - M') q(M') \leq \pi(M, M) - \pi(M', M') \leq v^b (M - M') q(M)$ .
5.  $\pi(M) \equiv \pi(M, M)$  is Lipschitz continuous with Lipschitz constant  $v^b$  (and hence differentiable almost everywhere).
6.  $q$  is weakly increasing in  $M$ .
7.  $d\pi(M)/dM = v^b q(M)$  everywhere  $\pi$  is differentiable.

Note that 4 follows from applying incentive compatibility. 5 follows immediately from 4. 6 follows from 4 with  $M \geq M'$ . 7 follows from 4 by taking the limit as  $(M - M')$  goes to zero. It follows that the mechanism is individually rational for the buyer if and only if  $\pi(m^L) \geq 0$ . (This follows from 2 and 7).

Our next goal is to characterize the utility of the seller for any feasible mechanism. Recall that this is given by:

$$U^s = 1 - \lambda + \int_{m^L}^{m^U} \left[ v^s - v^s q(m) + \frac{T(m)}{m} \right] f_M(m) dm = 1 - \lambda + v^s + \int_{m^L}^{m^U} \left[ \frac{T(m)}{m} - v^s q(m) \right] f_M(m) dm$$

The idea here is to get rid of  $T(m)/m$  in this expression by using what we have already shown about the buyers utility. To do this, note that we can write and,

$$U^b + U^s = 1 + v^s + \int_{m^L}^{m^U} [v^b - v^s] q(m) f_M(m) dm$$

$$U^b = \int_{m^L}^{m^U} \left[ v^b q(m) + \lambda - \frac{T(m)}{m} \right] f_M(m) dm = \lambda + \int_{m^L}^{m^U} \pi(m) \frac{f_M(m)}{m} dm$$

Using a change of variables and the fact that  $\pi$  is monotone, we can rewrite this as:

$$U^b = \lambda + \pi(m^L) \int_{m^L}^{m^U} \frac{f_M(m)}{m} dm + \int_{m^L}^{m^U} v^b R(s) q(s) ds$$

where  $R(s)$  is defined by:

$$R(s) = \int_s^{m^U} \frac{f_M(m)}{m} dm.$$

Given this, we can write:

$$U^s = v^s + 1 - \lambda - \pi(m^L) \int_{m^L}^{m^U} \frac{f_M(m)}{m} dm + \int_{m^L}^{m^U} \left[ v^b - v^s - v^b \frac{R(m)}{f_M(m)} \right] f_M(m) q(m) dm.$$

In order to find the mechanism that maximizes the utility of the seller, it is clear that  $\pi(m^L) = 0$ . Thus, it follows that the mechanism that maximizes the seller's utility is that which maximizes

$$U^s = v^s + 1 - \lambda + \int_{m^L}^{m^U} \left[ v^b - v^s - v^b \frac{R(m)}{f_M(m)} \right] f_M(m) q(m) dm.$$

where  $q$  must lie between 0 and 1 and must be weakly increasing. Since  $U^s$  is linear in  $q$  it follows that the solution to this problem is to set  $q$  equal to either 0 or 1 only. Thus, we have that the mechanism that maximizes the utility of the seller is of the form  $q(m) = 0$  for all  $m$  in  $[m^L, m']$  and  $q(m) = 1$  for all  $m$  in  $(m', m^U]$  for some choice of  $m'$ . Thus, the outcome of this mechanism is the same as that of the bargaining game analyzed in section 3.1. That the mechanism is as described follows from the argument given above.

To see that this coincides with the bargaining game, note that incentive compatibility requires that the transfer  $T(M)$  be the same for all  $m$ 's with the same probability of exchanging the object. Thus,  $T(M)$  can take two possible values:  $T_1$  when the probability of trade is one, and  $T_0$  when the probability of trade is zero. We now argue that  $T_0$  must equal zero. To see this recall that if  $q(M)$  is ever zero --the only relevant case as otherwise it follows that  $T(M)$  is constant-- it must be zero at  $M=m^L$ . However, at this point  $\pi(m^L) = -T(m^L) = -T_0$ . Since we have shown that  $\pi(m^L) = 0$ , it follows that  $T_0 = 0$ . Hence, the outcome of this mechanism is the same as one which transfers  $T_1$  from the buyer to the seller and charges the price,  $p = T_1$  anytime the object is traded. If  $q(m^L) = 1$ , it follows that  $q(M) = 1$  for all  $M$ , and that in this case the object is always traded and the seller receives a payment equal to  $T_1 = p$ . ■

*Proof of Proposition 3.1.2:* It is convenient to describe (3.1.2) in terms of the distribution of the inverse of the money supply. Thus, let  $F$  be the cdf of the random variable  $1/M$ , and assume that it has a density  $f$ . The random variable  $M$  is assumed bounded,  $0 < m^L \leq M \leq m^U < \infty$ . Note that an equivalent description of the optimal decision rule by the buyer is that he buys the indivisible good if and only if  $1/M < v^b/p$ . Thus, the seller's indirect utility over prices is given by,

$$V^s(p) = 1 - \lambda + p \int_{1/m^u}^{v^b/p} x f(x) dx + (1 - F(\frac{v^b}{p}))v^s.$$

This objective function is continuous but not differentiable in the interior of  $\mathfrak{R}_+$ . The interval  $[0, p^L]$  is easy to handle. In this region, the good will be sold with probability one, and the function  $V^s(p)$  is increasing in  $p$ . It follows that the seller will never announce a price less than  $p^L$ . Let  $p^U$  be the lowest price such that the buyer does not purchase the good for any realization of  $M$ . In our setting  $p^U$  is given by  $v^b/(\mu - k)$ . For prices  $p \geq p^U$ , the function  $V^s(p)$  is just  $1 - \lambda + (1 - \epsilon)v^b$ , a constant. It is immediate to check that in the interval  $(p^L, p^U)$  function is strictly concave if  $\epsilon < 1/2$ , and linear if  $\epsilon = 1/2$ . If  $\epsilon < 1/2$ , the first order condition for the seller's maximization problem with the additional requirements that  $p \geq p^L$  (with Lagrange multiplier  $\gamma$ ) is,

$$\int_y^z x f(x) dx - (1/p)^2 f(v^b/p) [(v^b)^2 - v^b v^s] + \gamma = 0,$$

or,

$$\int_y^z x f(x) dx - (v^b/p)^2 f(v^b/p) \epsilon + \gamma = 0,$$

where  $z = v^b/p$  and  $y = 1/m^u$ . Using the specific form for  $f(x)$  we get,

$$\gamma = (v^b/p)^2 (1/2k) \epsilon - (1/2k)(1/2) [(v^b/p)^2 - (\mu - k)^2].$$

Alternatively, this condition is,

$$(*) \quad 4k\gamma = (v^b/p)^2 (2\epsilon - 1) + (\mu - k)^2.$$

Depending on the size of  $\mu - k$ , it is possible that even when  $p = p^L$ , the Lagrange multiplier is positive. Define  $k_L$  as the lowest value of  $k$  such that (\*) is satisfied with  $p = p^L$  and  $\gamma = 0$ . Thus,  $k_L$  must satisfy,

$$(\mu + k_L)^2 (1 - 2\epsilon) = (\mu - k_L)^2,$$

or,

$$(1 - 2\epsilon) = [(\mu - k_L)/(\mu + k_L)]^2,$$

or, recalling the definition of  $k(\mu, \epsilon)$ ,

$$k_L = k(\mu, \epsilon).$$

From the definition of  $k(\mu, \epsilon)$  it follows that  $k_L$  is unique. We now show that for  $k > k_L$  the unique  $p$  that solves the first order condition assuming  $\gamma = 0$  is strictly greater than  $p^L$ . To see this impose  $\gamma = 0$  and solve for  $p$ . The solution is,

$$\hat{p}(\mu, k, \epsilon) = v^b(1-2\epsilon)^{1/2}/(\mu-k).$$

To check that  $p > p^L$  we need to check that,

$$v^b(1-2\epsilon)^{1/2}/(\mu-k) > v^b/(\mu+k),$$

which corresponds to,

$$(1-2\epsilon)^{1/2} > (\mu-k)/(\mu+k), \text{ or equivalently, } k > k(\mu, \epsilon).$$

The candidate solution automatically satisfies  $p < p^U = v^b/(\mu-k)$ . Finally, note that the probability of trade is given by  $q(\mu, k, \epsilon) = \Pr(p < Mv^b) = \Pr(1/M < v^b/p)$ . Using the expression for  $p(\mu, k, \epsilon)$  and the form for  $F$  shows that

$$\hat{q}(\mu, k, \epsilon) = (\mu-k)[(1-2\epsilon)^{1/2} - 1]/2k$$

as desired.

If  $\epsilon = 1/2$ , it is easy to check that the derivative of  $V^s(p)$  is negative and given by  $-(\mu-k)^2/2k$  on  $(p^L, p^U)$ . It follows that the optimal policy is to charge  $p^L$ , which results in a sale with probability one. ■

*Proof of Proposition 3.2.1:* To characterize equilibrium in this setting, we will first treat  $p^f$  parametrically and impose rationality on the part of the buyer below. Given a level of  $p^f$ , we can characterize the optimal behavior of the seller. If the seller chooses the price  $p^f$ , he sells the object and hence receives utility  $1-\lambda + p^f/M$ , while if he charges any price higher than  $p^f$ , he does not sell the object and hence receives utility  $1-\lambda + v^s$ . Thus, he will charge the price  $p^f$  if and only if  $M \leq \min(m^U, p^f/v^s) \equiv M^*$ . If, on the other hand,  $M > M^*$ , he will charge some price higher than  $p^f$ . Without loss of generality, we will assume that he charges the highest possible price,  $p^U$ , in this case.

This description of the optimal behavior of the seller given any reservation price rule determines the beliefs of the buyer using Bayes Rule. If the buyer sees the price  $p^f$ , the conditional distribution on  $1/M$  given  $p=p^f$  is uniform on the interval  $[1/M^*, \mu+k]$ , while if the buyer sees the price  $p^U$ , the conditional distribution on  $1/M$  given  $p=p^U$  is uniform on the interval  $[\mu-k, 1/M^*]$ . As is standard in these models, beliefs are not pinned down for any other values of  $p$  without making further assumptions.

Given these restrictions on beliefs, we will now derive the restrictions on buyers' behavior that results from optimal decision making on his part. First, the utility received by the buyer if he buys and the price is  $p^f$  is given by:

$$U = \lambda + \int_{m^L}^{M^*} [v_b - \frac{p^f}{M}] dF_{M|p^f} = \lambda + v_b - \frac{p^f}{2} \left( \frac{1}{M^*} + \frac{1}{m^L} \right).$$

On the other hand, if he does not buy, his utility is  $\lambda$ . Thus, the restrictions imposed by rationality by the buyer reduces to two cases. The first is if  $p^f/v^s \leq m^U$ . In this case, optimality of the buyers decision rule can be simplified to:

$$p^f \leq m^L v^b (1+\epsilon).$$



It is straightforward to check that these strategies (with the price of the seller equal to  $p^U$  whenever  $M > M^*$ ) do in fact constitute an equilibrium as long as  $p^r \leq \min(m^L v^b (1+\epsilon), m^U v^s)$ .

Notice that  $\min(m^L v^b (1+\epsilon), m^U v^s) = m^L v^b (1+\epsilon)$  if and only if  $k \geq \epsilon \mu$ , and  $\min(m^L v^b (1+\epsilon), m^U v^s) = m^U v^s$  if and only if  $k \leq \epsilon \mu$ . Thus, if  $k \leq \epsilon \mu$ , there are equilibria of this type with  $p^r$  up to  $m^U v^s$ . The relevance of this is that if  $p^r = m^U v^s$ , the probability of trade is equal to one and the outcome is efficient.

On the other hand, if  $p^r \geq m^U v^s$ , note that this implies a probability of trade equal to one given optimal behavior on the seller's part and, optimality on the buyer's part reduces to  $p^r \leq v^b \mu$ . These two (i.e.,  $p^r \geq m^U v^s$  and  $p^r \leq v^b \mu$ ) are mutually consistent if and only if  $k \leq \epsilon \mu$ .

The equilibrium that maximizes the utility of the seller is the one in which  $p^r$  is at its maximal level. This is given by  $p^r = v^b \mu$  when  $k \leq \epsilon \mu$  and  $p^r = m^L v^b (1+\epsilon)$  when  $k > \epsilon \mu$ .

In this equilibrium, when  $k > \epsilon \mu$ , trade occurs whenever  $m^L \leq M \leq M^* = m^L(1+\epsilon)/(1-\epsilon)$ . Thus, the probability of trade is given by:

$$P\left(\frac{m^L \leq M \leq m^L(1+\epsilon)}{(1-\epsilon)}\right) = P\left(\frac{(1-\epsilon)}{m^L(1+\epsilon)} \leq 1/M \leq 1/m^L\right) = P\left(\frac{(\mu+k)(1-\epsilon)}{(1+\epsilon)} \leq 1/M \leq \mu+k\right) = \frac{\epsilon(\mu+k)}{(1+\epsilon)k},$$

and converges to  $2\epsilon/(1+\epsilon)$  when  $k \rightarrow \infty$ , with  $\mu = \mu(k)$ .

In the case that  $k \leq \epsilon \mu$ , this maximum probability of trade is given by one. Given the definition of  $k_2^*$ , it follows that  $k \leq \epsilon \mu$ , if and only if  $k \leq k_2^*$ ; and  $k > \epsilon \mu$ , if and only if  $k > k_2^*$ . ■

#### Definition of $\Upsilon$ functions

Using the results from Proposition 3.1.3 the seller's payoff, in the (I,U) case, can be written as,  $W^s(I,U,k) = 1 - \lambda + v^b \Upsilon^s(I,U,k)$ , where

$$\Upsilon^s(I,U,k) \begin{cases} \Upsilon_1^s(I,U,k) = (e^{2m^*k} + 1)/2e^{2m^*k}, & \text{if } k \leq k_1^* \\ \Upsilon_2^s(I,U,k) = (e^{2m^*k} - 1)^{-1} \{(1-\epsilon)e^{2m^*k} - (1-2\epsilon)^{1/2}\} & \text{if } k > k_1^*, \end{cases}$$

It follows that,

- (i)  $\Upsilon^s(I,U,k)$  is a continuous function of  $k$ ,
- (ii)  $\Upsilon^s(I,U,k)$  is a decreasing function of  $k$ ,
- (iii)  $\lim_{k \rightarrow 0} \Upsilon^s(I,U,k) = 1$ ,  $\lim_{k \rightarrow \infty} \Upsilon^s(I,U,k) = (1-\epsilon)$ ,
- (iv)  $\Upsilon^s(I,U,k)$  is a differentiable function of  $k$ , except at  $k_1^*$ .

Similar expressions hold for the buyer. Denote the buyer's expected utility by  $W^b(I,U,k) = \lambda + v^b \Upsilon^b(I,U,k)$  where,

$$\Upsilon^b(I,U,k) \begin{cases} \Upsilon_1^b(I,U,k) = [1 - (e^{2m^*k} + 1)/2e^{2m^*k}], & \text{if } k \leq k_1^* \\ \Upsilon_2^b(I,U,k) = (e^{2m^*k} - 1)^{-1} [((1-2\epsilon)^{-1/2} - (1-2\epsilon)^{1/2})/2 + ((1-2\epsilon)^{-1/2} - 1)], & \text{if } k > k_1^*. \end{cases}$$

Direct calculations show that,

- (i)  $\Upsilon^b(I,U,k)$  is a continuous function of  $k$ ,
- (ii)  $\Upsilon^b(I,U,k)$  is increasing for  $k \leq k_1^*$  and decreasing for  $k > k_1^*$ ,
- (iii)  $\lim_{k \rightarrow 0} \Upsilon^b(I,U,k) = 0$ ,  $\lim_{k \rightarrow \infty} \Upsilon^b(I,U,k) = 0$ ,
- (iv)  $\Upsilon^b(I,U,k)$  is a differentiable function of  $k$ , except at  $k = k_1^*$ .

Consider next the case in which an uninformed buyer meets an informed seller (U,I) (section 3.2). From Proposition 3.2.1, the seller's utility can be written as  $W^s(U,I,k)=1-\lambda + v^b\Upsilon^s(U,I,k)$ , where,

$$\Upsilon^s(U,I,k) \begin{cases} 1 & \text{if } k \leq k_2^* \\ \Upsilon_2^s(U,I,k) = 1-\epsilon + 2\epsilon^2 e^{2m^*k}/[(1+\epsilon)(e^{2m^*k}-1)] & \text{if } k \geq k_2^*, \end{cases}$$

Note that  $\Upsilon^s(U,I,k)$  satisfies:

- (i)  $\Upsilon^{sb}(U,I,k)$  is a continuous function of  $k$ ,
- (ii)  $\Upsilon^s(U,I,k)$  is decreasing for  $k \geq k_2^*$ ,
- (iii)  $\lim_{k \rightarrow 0} \Upsilon^s(U,I,k)=1$ ,  $\lim_{k \rightarrow \infty} \Upsilon^s(U,I,k)=1-\epsilon + 2\epsilon^2/(1+\epsilon)$ ,
- (iv)  $\Upsilon^s(U,I,k)$  is a differentiable function of  $k$ , except at  $k=k_2^*$ .

*Proof of Theorem 4.1:* It is convenient to prove first a series of auxiliary results in the form of lemmas.

*Lemma 1* Assume  $\phi < \epsilon v^b$ . For each  $\phi$ , there is a unique value of  $k$ , denoted  $k_1(\phi)$ , such that,

- i)  $v^b\Upsilon^s(I,U,k_1(\phi)) = v^b - \phi$ .
- ii)  $v^b\Upsilon^s(I,U,k) > v^b - \phi$ , for  $k < k_1(\phi)$ , and  $v^b\Upsilon^s(I,U,k) < v^b - \phi$  for  $k > k_1(\phi)$ .
- iii)  $k_1(\phi)$  is decreasing in  $\phi$  and converges to 0 as  $\phi$  goes to zero.

*Proof:* The results trivially follow from the properties of the function  $\Upsilon^s(I,U,k)$  as given in Appendix B. There it is shown that  $\Upsilon^s(I,U,k)$  is a decreasing function of  $k$  and that  $\Upsilon^s(I,U,0)=1$ . These properties imply our results. ■

*Lemma 2* Assume that  $\phi < \epsilon v^b$ . For each  $\phi$ , there exist two distinct values of  $k$ , denoted  $k_2(\phi)$  and  $k_3(\phi)$  (without loss of generality assume that  $k_3 < k_2$ ) such that,

- i)  $v^b\Upsilon^b(I,U,k_2(\phi)) = \phi$ .
- ii)  $v^b\Upsilon^b(I,U,k) < \phi$ , for  $k < k_3(\phi)$  and  $k > k_2(\phi)$ , and  $v^b\Upsilon^b(I,U,k) > \phi$  for  $k \in (k_3(\phi), k_2(\phi))$ .
- iii)  $k_2(\phi)$  is increasing in  $\phi$  and converges to  $\infty$  as  $\phi$  goes to zero.

*Proof:* It follows trivially from the properties of the function  $\Upsilon^b(I,U,k)$  described in Appendix B. The key properties are that  $\Upsilon^b(I,U,k)$  is a continuous function of  $k$ ,  $\Upsilon^b(I,U,0)=0$ ,  $\lim_{k \rightarrow \infty} \Upsilon^b(I,U,k)=0$ , and that  $\Upsilon^b(I,U,k)$  is increasing for  $k \leq k_1^*$  and decreasing for  $k > k_1^*$ . Finally, the assumption that  $\phi$  is small implies  $\Upsilon^b(I,U,k_1^*) > \phi$ . The convergence of  $k_2(\phi)$  to zero as  $\phi$  goes to  $\infty$  follows from  $k_2(\phi) > k_1^*$ , and from  $\lim_{k \rightarrow \infty} \Upsilon^b(I,U,k)=0$ , and  $\Upsilon^b(I,U,k)$  decreasing for  $k > k_1^*$ . ■

*Lemma 3* Assume that  $\phi < \epsilon v^b$ . Then,  $k_1(\phi)=k_3(\phi) < k_1^* < k_2(\phi)$ .

*Proof:* The two inequalities follow trivially from the previous result. We next prove that  $k_1(\phi)=k_3(\phi)$ . For  $k < k_1^*$ , the calculations in Appendix B show that  $v^b\Upsilon^s(I,U,k) + v^b\Upsilon^b(I,U,k)=v^b$ . It follows that if  $k=k_1(\phi)$  we have  $v^b\Upsilon^s(I,U,k_1(\phi)) + v^b\Upsilon^b(I,U,k_1(\phi))=v^b$ . This equality is just,  $-\phi + v^b\Upsilon^b(I,U,k_1(\phi),\epsilon)=0$ . However, for  $k < k_1^*$  this is the definition of  $k_3(\phi)$ . ■

*Proof of the Theorem:*

- i) It follows from Lemma 3.

ii) For  $k \in (0, k_1(\phi))$ , it follows that  $v^b \Upsilon^s(I, U, k) > v^b - \phi$ , and  $v^b \Upsilon^b(I, U, k) < \phi$ . Inspection of the payoffs in Table 1 show that playing (U,U) is an equilibrium in dominant strategies, as both players are better off not acquiring information.

iii) At  $k = k_1(\phi)$ , we have that  $v^b \Upsilon^s(I, U, k) = v^b - \phi$ , and  $v^b \Upsilon^b(I, U, k) = \phi$ . First, note that in all cases if the buyer is uninformed, the seller's best response is to be uninformed. This is because in the (U,U) case the seller extracts all the surplus and it does not have to pay for the information. In this region (U,U) is an equilibrium as well. If the buyer is informed, the seller is indifferent between being informed or uninformed (this follows from  $v^b \Upsilon^s(I, U, k) = v^b - \phi$ ). Thus, the seller's best response to the buyer being informed is either U or I. On the other hand, if the seller chooses U, the buyer's best response is either U or I. However if the seller selects I, the buyer's best response is U. It then follows that the outcome (I,U) is another equilibrium outcome. These arguments also show that a mixed strategy equilibrium of the type describe always exist, since buyers are indifferent between I and U, whenever sellers play U.

iv) The region  $k \in (k_1(\phi), k_2(\phi))$  corresponds to the case  $v^b \Upsilon^s(I, U, k) < v^b - \phi$ , and  $v^b \Upsilon^b(I, U, k) > \phi$ . Let  $\pi^b(k)$  and  $\pi^s(k)$  be, respectively, the probability that buyers and sellers play I. Consider first the payoff of the buyers given the strategy of the sellers. They are given by,

$$\begin{aligned} \text{If the buyer chooses U, he/she gets} & \lambda. \\ \text{If the buyer choose I, he/she gets} & \pi^s(k)(\lambda - \phi) + (1 - \pi^s(k))(\lambda + v^b \Upsilon^b - \phi). \end{aligned}$$

For the buyer to be indifferent between the two, it must be the case that  $-\pi^s(k)\phi + (1 - \pi^s(k))(v^b \Upsilon^b - \phi) = 0$ , or  $\pi^s(k) = (v^b \Upsilon^b - \phi) / v^b \Upsilon^b$ . Now, consider the sellers' best response. The seller's payoffs are given by,

$$\begin{aligned} \text{If the seller chooses U, he/she gets} & \pi^b(k)(1 - \lambda + v^b \Upsilon^s(I, U)) + (1 - \pi^b(k))(1 - \lambda + v^b). \\ \text{If the seller chooses I, he/she gets} & \pi^b(k)(1 - \lambda + v^b) + (1 - \pi^b(k))(1 - \lambda + v^b \Upsilon^s(U, I)) - \phi. \end{aligned}$$

Thus, for sellers to be indifferent between the two actions it must be the case that,  
 $\pi^b(k) = [(1 - \lambda + v^b) + (1 - \lambda + v^b \Upsilon^s(U, I))] / [(1 - \lambda + v^b \Upsilon^s(I, U)) + (1 - \lambda + v^b)]$ .

It is straightforward to check that these probabilities are between zero and one, and hence that these are equilibrium mixed strategies.

v) At the point  $k = k_2(\phi)$ , we have shown that  $v^b \Upsilon^s(I, U, k) < v^b - \phi$ , and  $v^b \Upsilon^b(I, U, k) = \phi$ . This is the region where, if they knew they would meet an uninformed seller, buyers would be indifferent between acquiring information or not. It turns out that the value of information is zero in this case. Thus, the unique equilibrium is (U,U). Finally, for  $k > k_2(\phi)$ , the value of information is too small and U is a dominant strategy for both players. Thus, the unique Nash equilibrium is (U,U). ■

