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# EFFICIENT COMPETITION WITH SMALL NUMBERS -- WITH APPLICATIONS TO PRIVATISATION AND MERGERS

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## ABSTRACT

This paper studies competition between a small number of suppliers and a single buyer (or an auction with a small number of bidders and a single seller), when total demand (supply) is uncertain. It is well known that when a small number of suppliers compete in supply functions the service is not provided efficiently. We show that production efficiency is obtained if suppliers compete in simple two-part bid functions. However, profits are not eliminated. Moreover, the buyers' (sellers') decision regarding how much to buy is not efficient. We also show that suppliers (bidders in an auction) always have an incentive to merge (form bidding rings) in this setting.

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# 1 Introduction

Privatization of public assets and responsibilities has gained considerable momentum in a number of countries in recent years. The allocation of tasks among potential suppliers is usually resolved by ways of some sort of competition. In many important cases, demand is stochastic and suppliers have to commit to bids that covers all possible realizations of total demand. There has also been renewed interest in auctions, in particular in multi-unit auctions. Much of the early work in the literature focused on single-unit auctions, while many auctions of interest are multi unit ones.[1] It turns out that these two areas are very closely related. In the model developed below, the results for one setting can be reinterpreted for the other.

One example of privatization is deregulation of electricity generation. Here, the generating units have to commit to a single bid that covers a given number of time periods when the per period (say hourly) demand for electricity varies with conditions exogenous to the industry[2] (such as weather or time of day). Another example is privatization of public responsibilities, for instance, for the elderly. Suppose a local government is responsible for providing a service to people over the age of 65 and wishes to subcontract to private firms. As the number of elderly will vary over time, private firms will need to commit to a price for all likely quantities the authorities might wish to demand. Another area where privatization has occurred is trash collection and the privatization of prisons in the U.S.[3]

[1] Auctions of the airwaves, of quota licenses and treasury bills are examples.

[2] We will not go into the enormous literature in this area. Deregulation of electricity generation has occured in countries such as England and Wales, Norway, New Zealand, and is gaining ground in the U.S. where a number of states, including Pennsylvania, are following the lead of California. Although the minimum efficient scale is considerably below market size, there are often a small number of firms in the generation of electricity with substantial market power.

[3] Hart, Shleifer and Vishny (1996) point out that privatization may not be desirable when contracts are incomplete as private provider may under supply factors, such as

Situations like these have been studied in models where firms bid supply functions and the buyer chooses the suppliers and the quantities they supply. These allocation games are essentially multi-unit auctions and share the problem of multiple equilibria common to such auctions in a complete information setting. The basic reason is evident from looking at Figure 1. Suppose that the buyer wants to buy $n$ units, which is the size of the base in Figure 1, and there are two sellers. The left hand origin is that of Seller 1 and the right hand one is that of Seller 2. If the supply function bid by Seller 2 is given by the curve $DD$, Seller 1 faces a downward sloping residual demand which is identical to $DD$. In exercising his market power he will be best off by trying to get to supply $O_1R$ where his marginal revenue curve, given by $DV$, and his marginal cost curve, given by $CC$, intersect each other. If the buyer, for example, buys at a uniform price at which demand equals supply, Seller 1 can ensure that he is awarded $O_1R$ units by bidding *any* upward sloping supply curve through the point $F$. However, the position of the supply curve offered by Seller 2 depends on the position and *shape* of the supply curve Seller 1 offers. Thus, with complete information and no uncertainty, there are a multiplicity of equilibria in such games[4].

The selection problem in the supply competition case has been resolved by introducing uncertainty[5]. As different supplies will be desired for different states of the world, under certain conditions these can all be attained in equilibrium by bidding a single supply function. This gives a natural way

quality, which are hard to contract.

[4]The analysis of multi unit auctions is analogous. Just reinterpret $n$ as the number of units for sale and $CC$ as the demand curve offered by Buyer 1. Then $CC$ is also the residual supply curve facing Buyer 1 who then chooses the best point for himself along $CC$ and offers some demand curve through this point. This exercise of market power results in what is called demand reduction in the auction setting. Again there will be many equilibria. See Krishna and Tranæs [10] for more on this.

[5]In a related branch, the menu auction literature, the selection problem has been handled (avoided) by focusing on truthful equilibria as suggested by Bernheim and Whinston [2].

of pinning down the shape of the supply function in equilibrium. Conceptually this goes back to Robson [12], considerably generalized in Klemperer and Meyer [9][6]. It has recently been applied to electricity spot markets, foreign direct investments, and strategic trade (see Green and Newbery [6], Haaparanta [7], and Grant and Quiggin [5]) respectively.

The Robson-Klemperer-Meyer ($RKM$) approach of looking at supply function equilibrium ($SFE$), has been used lately in designing the extensive privatization programs within utilities both in the US and in Europe. For example, the setup used in the U.K. in the electricity market is essentially the one studied in $RKM$. However, equilibria in supply functions along the lines of $RKM$ have a drawback from the point of view of the economy *in toto* as they are not efficient! In equilibrium, suppliers bid above their marginal cost functions and marginal costs of production are not equalized across suppliers. That the suppliers increase their bids strategically are supported by a resent empirical study for the U.K. (see Wolfram [16]). The potentially widespread use of supply function bidding makes this all the more critical.

In Section 2 we propose a simple contract design to regulate the payment and allocation between the supplying firms (the generating companies or sellers) and the buying body (the dispatcher or buyer). In contrast to the $RKM$ assumption that the market price is determined by demand equalling supply, we assume that the buyer is also a player, in that he chooses his suppliers to minimize his cost.[7] This seems a very natural assumption when there is a single buyer of the $n$ units. We suggest that firms be allowed to bid the following type of non-linear bid functions: a *marginal price* for each additional unit supplied, as well as a fixed payment, a bonus, as a function of the total demand. The first is of course just another way of specifying a *standard supply schedule*, allowing the supplier to condition total supply

[6]Their approach is intimately related to the literature on consistent conjectures (See Bresnahan [3] and Turnbull [14]).

[7]See Baliga, Corchon, and Sjostrom [1] for more on the topic of the planner as a player.

on the price. The second part, the bonus, is allowed to depend on whether demand is high or low, i.e., whether times are good or bad. In Section 3 we show that in equilibrium, suppliers use only the bonus strategically to raise profit, not the variable payment, so that the allocation of the quantities is efficient even with a small number of suppliers.

The motivation behind privatization is to encourage competition resulting in enhanced efficiency, lower costs, lower prices and profits. When the suppliers compete in standard supply functions neither efficiency nor zero profits is achieved. When they compete in the bid functions suggested here, we get efficiency, but while marginal costs of production are equalized across suppliers, the firms still receive positive profits.[8] This is the subject of Section 4.

Our result relates to Vickrey [15]. For each realization of the random demand, each supplier delivers the same quantity and receives the same total payment as in the mechanism suggested by Vickrey [15]. The Vickrey scheme can be thought of as having the suppliers offer upward sloping marginal price functions. Given the marginal price functions offered, the buyer must allocate output to the supplier with the lower marginal price, and for each unit, pay the supplier the marginal price asked for by the other supplier. This makes it a dominant strategy for each supplier to bid his true marginal cost as his marginal price. The outcome and total payments (which equal the marginal cost plus what we call the Vickrey bonus) to suppliers under this implementation scheme are the same as those in the game we propose. However, there is no reason for the buyer to want to follow these rules so that there is a credibility problem. In our setting in contrast, the buyer merely acts in his own interest as he is also a player. In Vickrey's work, the suppliers

[8] If considered excessive, we argue later, these profits can be reduced by asking for an entry fee from the suppliers. However, to the extent that there are likely to be large fixed costs involved in the generation industry, they need to be covered by profits at this stage to keep investment from drying up.

receive a price per unit equal to their true marginal costs, plus a bonus, whose computation is *part of the mechanism design* and hence exogenous to the strategic interaction. In our case, in contrast, the suppliers bid is made up of a bonus request function and a supply function. We show that *if the buyer minimizes the cost of his purchases*, then the equilibrium bonuses in this game are exactly the Vickrey bonuses for each seller.

Our results translate directly to the multi-unit auction where there is a single seller of $n$ (which is state dependent) units and many buyers; where the seller chooses the allocations, given the bids, to maximize his revenue. Introducing uncertainty about the size of $n$, in this setting, and allowing bidders to ask for a fixed fee which depends on the state and offer a marginal price for each additional unit results in bidders bidding their true marginal valuations and asking for their Vickrey bonus for each value of $n$. The initiated reader will recall that this is the same outcome as that given by using the truthful equilibrium refinement ($TER$) proposed by Bernheim and Whinston [2] to pick among the multiplicity of Nash equilibria that exist without uncertainty. The $TER$ requires each bidder to bid in a manner that makes him indifferent to the actual allocation made to him. In contrast, our results imply that this same outcome obtains without needing any such refinement for each value of $n$ in the unique Nash equilibrium if bidders are allowed to bid the two part functions we suggest.

Finally, we show that the production efficiency result remains when we assume that the buyer (seller) can decide how much to buy and wishes to maximize surplus (sell) or when we let the suppliers (bidders in an auction) merge (form bidding rings). However, the buyer's (seller's) decision regarding how much to buy (sell) is not efficient as discussed in Section 5. Both a subsidy and price regulation are needed to correct this distortion. In Section 6 we look at mergers (or the formation of bidding rings) and show that a merger raises the cost facing the buyer and this tends to reduce sales when the demand is price dependent. Section 7 contains some final thoughts.

## 2 The Supply Bidding Game

There is one buyer with a state dependent demand, $n(\theta)$, where $\theta$ denotes the state. The buyer's valuation is not important at this stage, as long as it is high enough for him to want the units he is bidding for in spite of the price he pays[9]. The set of suppliers are denoted by $M$ and we assume there are $m$ of them, each of whom face production costs $C_i(q)$; $C_i(\cdot)$ is twice continuously differentiable, with $C_i'(\cdot) > 0$ and $C_i''(\cdot) > 0$. Each supplier offers a bid function over $[0, n]$ where $n \equiv n(\theta)$ is a random variable with strictly positive density everywhere on the support $N$. We restrict a bid to consist of a payment $T_i(q_i)$, solely depending on the quantity supplied $q_i$, and an additional payment $S_i(n)$, independent of the quantity supplied, but contingent on the total purchase, $n$, by the buyer. So $B_i(q_i, n) = T_i(q_i) + S_i(n)$ specifies the total price as a function of $q_i$ and each $n$ in $N$. Of course, if nothing is demanded from them they obtain no bonus or output contingent revenue, so we require that $B_i(0, n) = 0$.

The timing of the game is depicted in the time line given in Figure 2. First the buyer decides on how much it will buy in each state. We take this as exogenous until later on. Then the $m$ suppliers submit their bid functions simultaneously. After this the state is realized and finally, the buyer decides how much to buy from whom, so as to minimize his total payment for the $n(\theta)$ units he wants to buy. Throughout we make the following assumption to ensure uniqueness of the equilibrium.

**Assumption 1:** The distribution of shocks is such that $n(\theta)$ is distributed over $[0, \infty]$.

[9]This may sound strange as his valuation determines how many units he wants and we address this later on.

## 3 Supply Equilibrium

The supply bidding game just described has some attractive properties. We will first provide an intuitive argument based on a diagram, for why a particular strategy is a Nash Equilibrium and then argue more formally that it is the only Nash Equilibrium. The argument is made using Figure 1. For simplicity say there are just two firms. The origin for Firm 1 is at the left and for Firm 2 is on the right as depicted. The length of the base gives the number of units the dispatcher demands.

We depict offers of the kind $T_i(q_i)+S_i(n)$ as follows; $T_1(q_1)$ is total variable bid by Firm 1 and so equals the area under the curve $CC$, which gives the marginal price. Suppose that Firm 2 has offered the marginal price curve $DD$.[10] What is optimal for Firm 1? First notice that if Firm 2 has offered the curve $DD$ as its *marginal price*, $DD$ acts like the residual marginal revenue curve facing Firm 1. Only by charging a price below Firm $2's$ marginal price will Firm 1 be able to sell the unit in question to the buyer as he is minimizing the cost of acquiring $n$ units. If Firm $1's$ marginal cost curve is $CC$, then Firm 1 will only want to sell $O_1K$ units since at this point its marginal cost equals the marginal price offered by Firm 2. More than this would involve selling the extra units at a loss. Certainly, there are an infinite number of curves which ensure this: namely all curves which are upward sloping and going through the point $E$. It can raise its asking price as far as possible and sell $K$ units by offering a curve just under $DD$ up to $K$ units and above $DD$ for more than $K$ units. That is, it only has to offer a marginal price slightly below the one offered by Firm 2 to get to sell its desired output level. Hence, in response to an offer by Firm 2 of $DD$, Firm 1 can make profits equal to the area between $DD$ and its marginal cost curve $CC$, up to the output level $K$. These profits equal the area $S_1$. Note that Firm 1 can make exactly profits

[10] In general, we only need to ensure that there is a single crossing for all values of $N$.

of $S_1$ if it offers a marginal price equal to its marginal costs and asks for a bonus of $S_1$.

However, $n$ is random. If Firm 1 offers any curve other than its true marginal cost curve, it will not be selling its optimal output level for every realization of $n$. Hence offering the true marginal costs as the marginal price and asking for a bonus corresponding to $S_1(n)$ for each realization of $n$ enables him to attain this maximum profit for each realization.[11] Note that maximizing expected profit requires maximizing profits state by state here.

Firm $1's$ marginal contribution to the problem in equilibrium is its Vickrey bonus. Our main result is that in equilibrium each firm will offer its true marginal costs as its marginal price and ask for its Vickrey bonus. It will not ask for less as it can do better by asking for more. It will not ask for more, since if it does, it will be eliminated from consideration by the buyer and make nothing. If one firm asks for more, or asks for a greater increment than the other, then only its rival will serve the market and this firm will make no profits so that this is dominated. What if both firms ask for an equal increment? If Firm 2 asks for its marginal costs and as a bonus asks for $S_2(n) + \epsilon$, and firm 1 asks for its marginal costs and a bonus of $S_1(n)+ \epsilon$, then if the buyer buys from *only* one firm he pays less than if he buys from both. Hence, the buyer or dispatcher only buys from one of the firms. This, however, means that the other will cut his bonus request and be the chosen one, and so no positive $\epsilon$ can be maintained in equilibrium. Thus, by restricting firms to bidding a two part price function as done above, we ensure efficiency. Note that the dispatcher buys from the two firms so that their marginal prices are equalized. As these are also their marginal costs, we have efficiency. Note that this gives a firm the same profits as it could

[11]The role of Assumption 1 becomes apparent. If all $n$ in $N$ are not optimal for a supplier for some state $\theta$, no matter how unlikely the state, then he would not care about the shape of the function he offered in such regions. This in turn could cause multiple equilibria to exist.

make if it was allowed to offer a state contingent marginal price function.

**Theorem 1** *If Assumption* 1 *holds then the supply bidding game has a unique Nash equilibrium. This consists of each supplier asking for a variable payment which equals his production costs, and a fixed one which equals his Vickrey bonus. As* $B_i(q_i, n) = C_i(q_i) + S_i(n)$ *in equilibrium, the buyer allocates orders so that marginal costs are equalized and thus the service is provided efficiently.*

*Proof*: For each realization of the random variable $\theta$, the buyer buys $n(\theta)$, and puts together his total purchase from the different suppliers so as to minimize his total payment. The buyer solves the problem:

$$\underset{q}{Min} \sum_i (B_i(q_i) + S_i(n))$$

$$\sum_i q_i = n,\ q_i \geq 0,\ i = 1, .., m.$$

We first derive the highest *total price* supplier $j$ can obtain for delivering a given amount $q_j$, given bids offered by all other suppliers, $M_{-j}$. Again the buyer puts together the purchase of $n - q_j$ from all suppliers but $j$, so as to minimize his payment for the $n - q_j$ units. Thus, the buyer solves the problem

$$\underset{q_{-j}}{Min} \sum_{i \neq j} (B_i(q_i) + S_i(n))$$

$$\sum_{i \neq j} q_i + q_j = n,\ q_i \geq 0,\ i = 1, .., m$$

where $q_{-j}$ is the allocation vector for all suppliers but $j$. Let the value function for this problem be denoted $R_{-j}(n - q_j)$, which then defines the minimized total cost of obtaining the $n - q_j$ units delivered by the suppliers in $M_{-j}$. By the Berge maximum theorem $R_{-j}(.)$ is continuous. Similarly, $R_{-j}(n)$ gives the minimized cost of obtaining $n$ units when all $n$ units to be obtained must come from the suppliers in $M_{-j}$. Let $P_j(q_j) = R_{-j}(n) - R_{-j}(n - q_j)$. Note that from the envelope theorem, $R'_{-j}(n - q_j)$, the change in $R_{-j}(n - q_j)$ as $q_j$ falls, is the shadow cost of obtaining an additional unit from the sellers in $M_{-j}$. This equals the marginal price asked for by each of the suppliers from whom some units are purchased due to the necessary conditions for minimization of the buyers payments. Of course, second order conditions for a minimum require that this marginal price be rising so that $R''_{-j}(n-q_j) > 0$. Thus, as $P'_j(q_j) = R'_{-j}(n - q_j)$, and $P''_j(q_j) = -R''_{-j}(n - q_j) < 0$, $P_j(q_j)$ is concave.

Now by offering a bid just below $P_j(q_j)$ for any particular value taken by $q_j$, and some bid above $P_j(q_j)$ otherwise, $j$ can ensure that he supplies $q_j$ units and makes as much as possible doing so. Hence $P_j(q_j)$ can be thought of as the best total price supplier $j$ can get from selling $q_j$ units.

Next we ask how many units $j$ should aim for. In effect, supplier $j$ maximizes his profit

$$\Pi_j(q_j) = P_j(q_j) - C_j(q_j).$$

Let $q_j^*(n)$ denote the value of $q_j$ that maximizes $\Pi_j(q_j)$, given the bids of the suppliers $M_{-j}$ and given that the state of the world is $n$. We suppress these arguments in our notation for convenience. $\Pi_j^*(n) = \Pi_j(q_j^*(n))$ is therefore the maximized value of the surplus available to buyer $j$. $\Pi_j(q_j^*(n))$ is conditional on the bids offered by the other suppliers and is called the marginal contribution of seller $j$[12].

[12]It is worth making clear the difference between the "marginal contribution" of supplier $j$ just defined and his "Vickrey bonus". The latter is his marginal contribution when all other suppliers are bidding their marginal cost as their marginal price. We show below that for every $n$, and any given bids by the other suppliers it is a dominant strategy to ask for the true marginal cost as the marginal price and to ask for the marginal contribution

As $P_j(q_j)$ is concave and $C_j(q_j)$ is assumed to be strictly convex, $\Pi_j(q_j)$ is strictly concave. Hence for each realization of $n$, (and bids by others) there is a unique $q_j^*(n)$ that maximizes $\Pi(q_j)$. So in equilibrium $j$ sets $S_j(n) = \Pi(q_j^*(n))$ for all $n \in N$. Thus, a best reply (to given bids by others) of $j$ would be to bid $B_j^*(q_j, n) = C_j(q_j) + \Pi_j(q_j^*(n))$; $B_j^*(q_j, n)$ involves pricing at costs and asking for a bonus which is exactly equal to the suppliers' marginal contribution. However, in equilibrium, all suppliers bid in this manner, namely bidding their true marginal costs as their marginal price. Given these bids of the suppliers $M_{-j}$, supplier $j$ cannot do better than $\Pi_j^*(n)$ which is his Vickrey bonus given the bids by all others.

Now, could $j$ do equally well by offering some bid other than $B_j^*(q_j, n)$? The answer is no. We show this in two steps. First we show that, given what others are asking, if the supplier offers anything but his true marginal costs as the marginal price, he could do better. Thus, it is a dominant strategy to always set the marginal price at marginal costs. Second, we check that there are no equilibria where all suppliers ask for more than their Vickrey bonuses while bidding their marginal costs as their marginal price.

1. Suppose by way of contradiction that Supplier 1 has a best reply bid function where $T_1'(\cdot) \neq C_1'(\cdot)$ in a small neighborhood.[13] This is depicted in Figure 1 where in a neighborhood around $K$, the marginal price bid by Supplier 1 falls below $CC$, which depicts the marginal cost of Supplier 1. Given the bids of all other suppliers, we have the marginal price of buying from them being $P_1'(q_1)$ depicted by $DD$.[14] Thus, Supplier 1 will be asked

(which depends on these other bids) as the fixed fee. The fixed fee asked for in equilibrium is the Vickrey bonus.

[13]If the region around $K$, is never the chosen allocation for any random shock then this deviation makes no difference to the payoffs as what is lost in one component of the bid is exactly gained in the other. However, we assume that the distribution of shocks is such that all points are a chosen allocation for some state of nature.

[14]This is the horizontal sum of the marginal bids of the other suppliers considered by the buyer.

to supply $O_1T$ units. This of course raises the fixed payment demanded in this state by Supplier 1 to the area $S_1$+ $ZEW$. However, note that the *total revenue* (fixed and marginal components) Supplier 1 can obtain remains the same, it is exactly the area under $DD$ from $O_1$ to $T$. Thus, his marginal revenue is given by $DD$. However, for the units between $K$ and $T$, marginal cost, along $CC$, exceeds his marginal revenue and so he could do better, to the tune of the shaded region in Figure 1, by reporting his true marginal cost and adjusting his fixed fee accordingly. Analogous arguments for using a marginal price above true marginal costs can also be made.

2. Could it be an equilibrium that for some realization all suppliers demand more than their respective $\Pi_i^*(n')$ as the fixed component? Assume there exist $S_1(n') > \Pi_1^*(n'), S_2(n') > \Pi_2^*(n')..., S_m(n') > \Pi_m^*(n')$ such that it is an equilibrium that all suppliers bid $C_i(q) + S_i(n')$. However, $q_1^*, ..., q_m^*$ is no longer the allocation of supply that minimizes the buyer's total payment for the $n'$ units. Suppose the buyer excludes one of the suppliers, say $j$, and demands the $q_j^*$ units from the suppliers $M_{-j}$. Recall that supplier $j$ demands the price $C_j(q_j^*) + S_j(n')$ for the $q_j^*$ units, while the $q_j^*$ units can be purchased from the suppliers $M_{-j}$ for $P_j(q_j^*) = C_j(q_j^*) + \Pi_j^*(n') < C_j(q_j^*) + S_j(n')$. So $C_j(q) + S_j(n')$ is not a best reply for $j$; Supplier $j$ would wish to reduce $S_j(n')$ in order to be included and get to supply a positive amount.

Finally, could it be an equilibrium for some realization $n'$ that all suppliers demand less than their respective $\Pi_i^*(n')$? Again, the answer is no; in that case a supplier, say $j$, could raise his bonus slightly and still be delivering $q_j^*$ because the buyers best alternative is to buy $q_j^*$ at the price $\Pi_j^*(n')$.

Thus, the unique equilibrium consists of each supplier, indexed by $j$, bidding $B_j^*(q_j, n) = C_j(q_j) + \Pi_j^*(n)$, which concludes our proof.

*Q.E.D.*

Thus, in the unique equilibrium all suppliers offer their marginal costs as their marginal price and demand their Vickrey bonus for each realization of

$n$. Notice that this is not a dominant strategy equilibrium: If, for example, the only other firm asks for a huge bonus in each state and bids his marginal cost as his marginal price, it will be optimal to ask for a huge bonus. However, this could not be an equilibrium as some firm will be eliminated from consideration by the buyer and the eliminated firm will find it in its interest to reduce its bid to be considered.

Since all sellers only ask for a bonus equal to their marginal contribution, in equilibrium if price at which the horizontal sum of marginal costs equals $n$ involves a positive allocation for firm $k$, then its bonus will be positive. Otherwise it will be zero and the firm will not be competitive. Since bids are true costs plus a constant and the buyer chooses the suppliers so as to minimize his total payment, he must be equating these marginal bids and hence minimizing the costs of providing the $n$ units in each state.

Our result relates to Vickrey [15]. For each $n$, each supplier delivers the same quantity and receives the same total payment as in the mechanism suggested by Vickrey. The quantity being the same follows from both the Vickrey allocation and the equilibrium allocation in our game being efficient. In our game, the equilibrium profit $\Pi_i(q_i^*)$, obtained by supplier $i$, is equal to the buyers' best alternative, that is, the price the buyer would have had to pay for $i's$ supply had he bought it from all buyers but $i$. This is exactly the definition of the bonus payment in Vickrey's article. Note that the Vickrey bonus to a firm increases as $n$ rises and falls as $m$ rises.

# 4 Revenue and Cost

We have suggested a simple bidding game which ensures efficiency, but what about average prices? Depending on the shape of the suppliers cost functions the profits can be quite substantial. For a given $n$ the average price per unit is

$$p^a = \frac{\sum_{i \in M} (C_i(q_i^*) + \Pi_i^*(n))}{n}.$$

This is depicted as $P_1^a(q_1)$ in Figure 3, which lies above the marginal price $P_1'(q_1)$. The equilibrium allocation $q_1^*$ is where this marginal price equals its marginal cost of production depicted by $C'(q_1)$ in Figure 3. Since marginal costs are increasing, average costs of $ac_1(q_1)$ lie below them. The average price relates to the competitive price $p^c$ and average costs as follows

$$p^a \geq p^c = C_i'(q_i^*) \geq \frac{\sum_{i \in M} C_i(q_i^*)}{n}.$$

Thus, the average price can be higher than the market clearing price and even further above average costs. If there are only a few suppliers and marginal costs are steep, then the average price can be much higher than average costs.

Although there is efficiency in production, profits remain in the hands of suppliers. These will be high if marginal costs of all other firms put together are sharply increasing and could be low otherwise. If the profits earned are considered excessive, they can be reduced by asking for an entry fee from the suppliers. However, recall that there are large fixed costs involved in the generation industry. These costs need to be covered by profits in order to keep investment from drying up. Thus, a way of preventing inordinately large profits would be for the Government to set an entry fee just large enough to keep profits from being excessive.

Despite production efficiency, we will see that the buyers choice of $n$ in each state is unlikely to be efficient.

# 5 Demand with Regulated Monopsony

First we will look at the case where there is only one dispatcher who purchases from the generating firms and sells to competitive final users of electricity. The demand of these final users is a random variable, say it varies with macroeconomic conditions. Each realization of demand on the part of these final users gives a demand curve facing the monopolist dispatcher. Associated with each demand curve is a marginal revenue curve as depicted in Figure 4.

What are the costs to the dispatcher of purchasing $n$ units? Well, we just calculated them! For each $n$, the costs are

$$\Gamma(n) = \sum_{i=1}^{m} S_i(n) + C(n)$$

where $C(n)$ represents the minimized total costs of $n$ and equals the total cost of production by all suppliers and hence equals the area under their marginal cost curves; let $\sum_{i=1}^{m} S_i(n) = \Pi^*(n)$. Marginal costs corresponding to these total costs are depicted in Figure 4 by $C'(n)$. The dispatcher wishes to maximize his profits and so chooses $n$ to equate marginal revenue to marginal costs.

$$MR(n) = \sum_{i=1}^{m} S_i'(n) + C'(n) = \Pi'^*(n) + C'(n).$$

In other words he chooses $n$ where the two curves intersect at $n^M(\theta)$ in Figure 4. Notice that this is socially suboptimal. First there is the monopoly distortion from having a single seller in the final product market which causes a divergence between social and private benefit as demand exceeds marginal revenue. Second, as $S_i'(n)$ is positive, private marginal costs exceed social marginal costs of $C'(n)$. Hence, although electricity is being produced efficiently (despite small numbers in the generating market), the distortion arising from the seller having to pay $\sum_{i=1}^{m} S_i'(n)$ more than the marginal production costs causes him to demand too little.

If we regulated the monopolist to take price as given, we could by setting the price at $P^{RM}(\theta)$, get him to buy $n^{RM}(\theta)$. However, there would still be the distortion due to the seller having to pay $\sum_{i=1}^{m} S_i'(n)$ more than the marginal production costs. Thus, only if a per unit subsidy of $s(\theta)$ were given in addition to regulation, or the buyer's fixed payments were paid for by the government, would the first best output of $n^E(\theta)$ occur. Regulating the price in the final good market is not enough to get the first best.

# 6 Mergers

In this section we look at the incentives for mergers and their welfare consequences. We show that all mergers raise the joint profits of the merged firms so that there will be incentives for mergers to occur. However, although the allocation of output between firms remains efficient, the cost facing the buyer rises and this tends to reduce sales, which are already too low, even further. Thus, in the absence of synergies, mergers are harmful in our setting. Conversely, breaking up firms, as long as this does not adversely affect costs, is beneficial.

A natural definition of what occurs with a merger exists in our setting. When a group of firms merge, they have access to their joint production capability. Hence, the marginal costs of a group of merged firms is given by the horizontal sum of their marginal costs. Thus, our setup does not have the strange results associated with mergers in simple oligopoly settings studied in early work on this subject.[15] We assume that there are no cost reducing synergies from the merger. Now consider the effects of such a merger, say between firms $j$ and $k$, on their profits in equilibrium. Now even with fewer firms, the allocation of output between the firms in equilibrium is unchanged as it remains efficient: the merged firm merely produces the combined output

[15] In Salant, Switzer and Reynolds [13] for example, a merger is seen as simply a reduction in the number of firms. This makes mergers unattractive if there are more than two firms as a reduction in the number of firms raises total profits, but reduces those of the merged firms unless it results in a monopoly. Later work, see Perry and Porter [11], uses a conjectural variations setup to examine the conditions under which competitive firms coalesce into dominant oligopolists. They treats mergers as raising the capital of the merged firm, thereby creating a larger firm. Farrell and Shapiro [4], look at mergers in a Cournot setup and allow for three sources of efficiency from mergers, reallocation of output, of capital, and learning.

of firms $j$ and $k$ prior to the merger. This is depicted in Figure 5. The curve $OC$ depicts the horizontal sum of all the firms marginal cost curves. This intersects the vertical line at $n$, the number of units purchased by the buyer, at $C$. This gives the marginal costs of each firm in equilibrium and hence its output as marginal costs are equalized across firms in equilibrium. The curves $OE$ and $OD$ give the horizontal sum of marginal costs without firm $j$, and without firm $j$ and $k$, respectively. Thus, the distance $AB$ and $BC$ give the equilibrium outputs of firm $j$ and $k$ respectively.

Now recall that the profits retained by firm $i$ prior to the merger equals

$$\begin{aligned} \Pi_i(q_i^*(n)) &= P_i(q_i^*(n)) - C_i(q_i^*(n)) \\ &= R_{-i}(n) - R_{-i}(n - q_i^*(n)) - C_i(q_i^*(n)). \end{aligned}$$

The profits retained by firm $j$ and $k$ after their merger equals

$$\begin{aligned} \Pi_{j+k}(q_j^*(n) + q_k^*(n)) &= P_{j+k}(q_j^*(n) + q_k^*(n)) - C_j(q_j^*(n)) - C_k(q_k^*(n)) \\ &= R_{-(j+k)}(n) - R_{-(j+k)}(n - q_j^*(n) - q_k^*(n)) \\ &\quad - C_j(q_j^*(n)) - C_k(q_k^*(n)). \end{aligned}$$

The difference between the profits earned by firms $j$ and $k$ before and after the merger is thus:

$$\begin{aligned} \Delta\Pi_{j+k} &= \Pi_{j+k}(q_j^*(n) + q_k^*(n)) - \Pi_j(q_j^*(n)) - \Pi_k(q_k^*(n)) \\ &= R_{-(j+k)}(n) - R_{-(j+k)}(n - q_j^*(n) - q_k^*(n)) \\ &\quad - \left[\left(R_{-j}(n) - R_{-j}(n - q_j^*(n))\right) + (R_{-k}(n) - R_{-k}(n - q_k^*(n)))\right] \\ &= IADn - FBEn - IAGF > 0. \end{aligned}$$

Note that the line $AG$ is flatter than $AD$ as it has the slope of the horizontal sum of the marginal cost curves when only firm $k$ is out of the picture, not both firm $j$ and $k$. It is easy to see that as a result of this construction, the area $IADn$ is larger than the sum of the areas $FBEn$ and $IAGF$. The difference is the shaded area in Figure 5. Thus, mergers are always profitable. Also note that as the horizontal sum of marginal cost curves in the absence of firm $s \neq j, k$ is unaltered by the merger, as is the horizontal sum of all marginal cost curves, excluded firms are unaffected by a merger for given $n$. Note, however, that when it comes to choosing $n$, the higher profits with a merger result in a higher level of costs and costs per additional unit faced by the buyer. This leads to a lower choice of $n$ in every state. As demand is already too low, one would expect adverse consequences for welfare, and on the profits of firms excluded from the merger as a result of the reduction in $n$ (since the Vickrey bonus is increasing in $n$). This occurs despite the absence of any inefficiency in the allocation of output across firm before or after the merger.

Translating this result into the analogous one for multi-unit auctions shows that forming bidding rings, where a group of buyers bids jointly for all the units they need, will always be beneficial for the bidders, without harming or helping other bidders, but at the cost of the seller who earns less revenue and so may choose to auction fewer units. The reduction in the number of units sold will reduce the surplus retained by the excluded bidders and hence their welfare.

# 7 Conclusion

In this paper we have offered a simple device by which a random demand can be matched (ex post) efficiently. However, this could be quite expensive. The expense is relatively small if marginal costs do not increase sharply with output and could be quite significant if costs rise sharply. In either case, some of the additional cost could be recovered by having an entry fee. We also show that in our setting, mergers are always profitable and while they do not affect the profits of non merging firms, or cause any inefficiency in production, they do raise costs to the buyer and so reduce demand which is already too low. Finally, we show that our model can be reinterpreted as a multi-unit auction with many bidders with analogous results.

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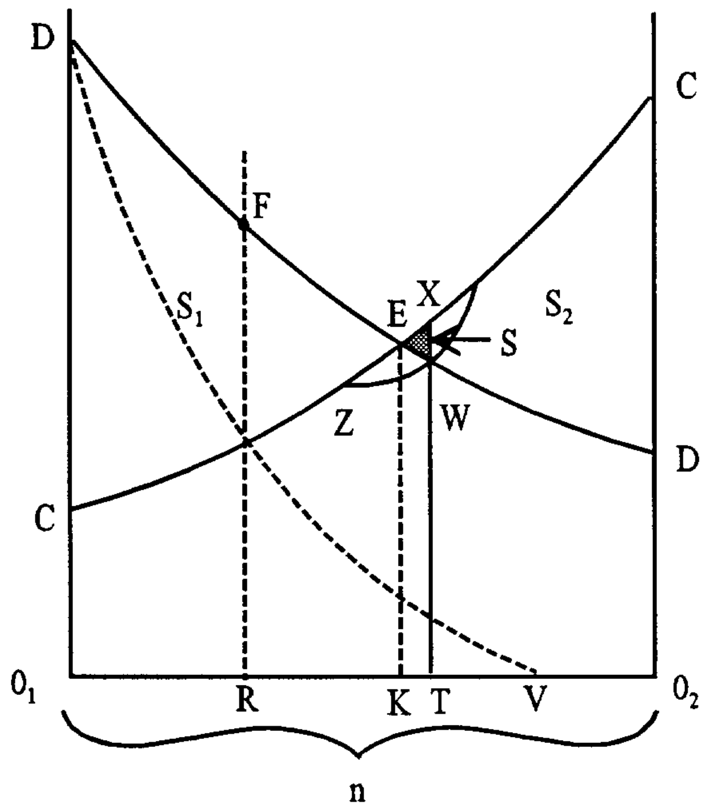


Figure 1
Supply Function Equilibria with Fixed Random Purchases

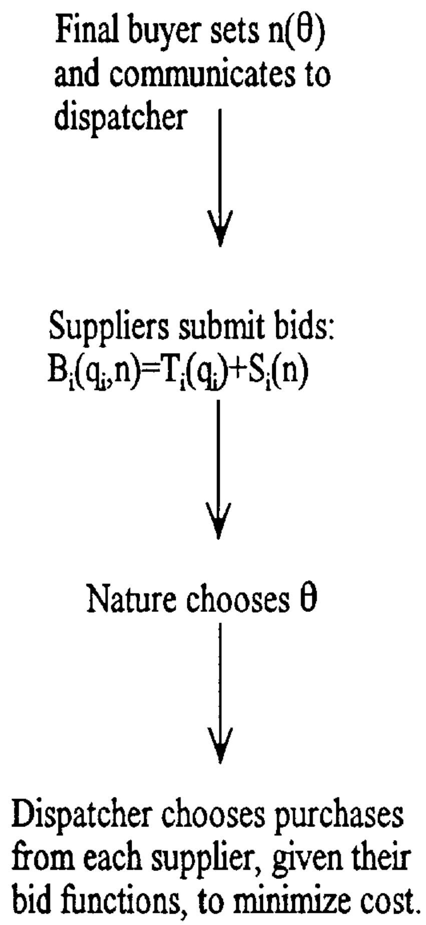


Figure 2
The Time Line

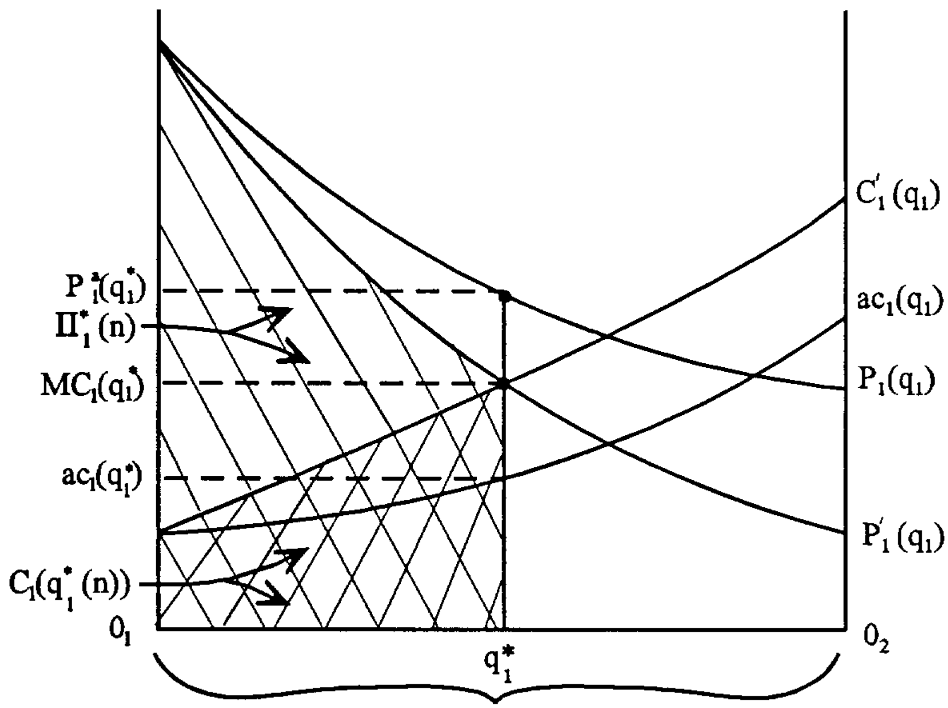


Figure 3
Revenue and Prices in Equilibrium

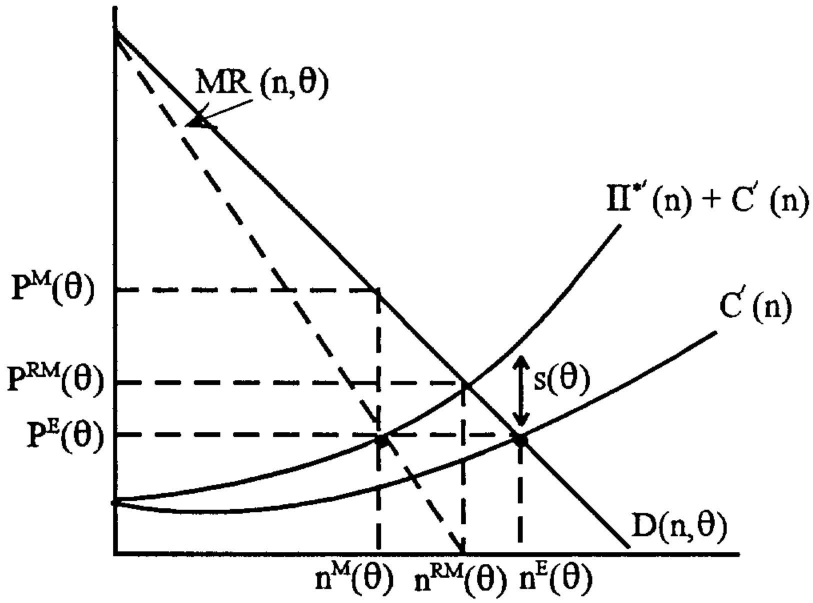


Figure 4
The Choice of $n(\theta)$ under Monopoly

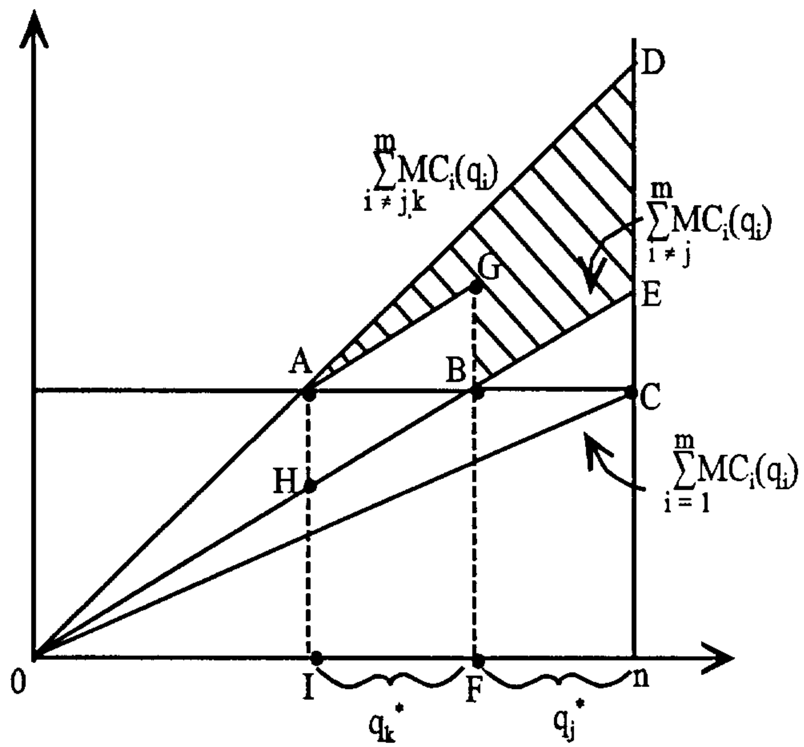


Figure 5
The Effect of a Merger