NBER WORKING PAPER SERIES

CAN MARKET AND VOTING INSTITUTIONS GENERATE OPTIMAL INTERGENERATIONAL RISK SHARING?

Antonio Rangel
Richard Zeckhauser

Working Paper 6949
http://www.nber.org/papers/w6949

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 1999

Prepared for the NBER Conference on Risk Aspects of Investment Based Social Security Reform, January 15 -16, 1999. We want to thank Peter Boasserts, David Cutler, Peter Diamond, Martin Feldstein, Peter Hammond, Matthew Jackson, Chad Jones, John Ledyard, Jeffrey Liebman, Marcos Lisboa, Greg Mankiw, Stephen Morris, Giussepe Moscarini, Thomas Nechyba, Ben Pollack, Thomas Sargent, Chris Sims, Steve Tadelis, and the participants in the Macro seminar at Harvard and Yale, the Public Economics seminar at Harvard, and the NBER Pre-Conference on Risk Aspects of Investment Based Social Security Reform for their helpful comments. The views expressed here are those of the author and do not reflect those of the National Bureau of Economic Research.

Can Market and Voting Institutions Generate
Optimal Intergenerational Risk Sharing?
Antonio Rangel and Richard Zeckhauser
NBER Working Paper No. 6949
February 1999

ABSTRACT

Are market and voting institutions capable of producing optimal intergenerational risk-sharing? To study this question, we consider a simple endowment economy with uncertainty and overlapping generations. Endowments are stochastic; thus it is possible to increase the welfare of every generation using intergenerational transfers that might depend on the state of the world. We characterize the transfers that are necessary to restore efficiency and compare them to the transfers that take place in markets and voting institutions. Unlike most of that literature, we study both ex-ante and interim risk-sharing.

Our main conclusion is that both types of institutions have serious problems. Markets cannot generate ex-ante risk-sharing because agents can trade only after they are born. Furthermore, markets generate interim efficient insurance in some but not all economies because they cannot generate forward (old to young) intergenerational transfers.

This market failure, in theory, could be corrected by government intervention. However, as long as government policy is determined by voting, intergenerational transfers might by driven more by redistributive politics than by risk sharing considerations. Successful government intervention can arise, even though agents can only vote after they are born, but only if the young determine policy in every election.

Antonio Rangel
Department of Economics
Stanford University
Stanford, CA 94305-6072
rangel@leland.stanford.edu

Richard Zeckhauser
J.F.K. School of Government
Harvard University
79 JFK St.
Cambridge, MA 02138
and NBER
richardz@ksg1.harvard.edu

1 Introduction

We study economies confronted with generational risks. Such risks are shocks that affect all the members of a generation, but have a smaller effect on other generations. They include the risk of having a bear market during the years one saves for retirement, being the age cohort that goes off to an extended war, or having one's prime employment years during a recession. With significant shocks of this sort, there are substantial benefits to intergenerational risk-sharing. For example, young generations could insure the elderly against inadequate stock market performance. Indeed, precisely this risk-sharing measure has been discussed in the current debate on Social Security reform.

The central question of this paper is: Can market institutions or government actions generate efficient intergenerational risk-sharing? Both markets and governments have an effect on risk-sharing, markets through the trade of financial instruments, governments through social insurance programs. Each has potential on the intergenerational front, since there are financial instruments that last for many generations, and some social insurance programs, such as Social Security or subsidized education, transfer resources across generations. However, our analysis dashes hopes. We show that markets have problems generating optimal insurance, and that self-interested voters may defeat government efforts to overcome market failures.

We use a simple endowment economy with uncertainty and overlapping generations to study the problem. Each period there is uncertainty about the size of the aggregate endowment and its distribution between young and old. To see the role for intergenerational risk-sharing, consider an extreme example in which the entire endowment goes either to the young or to the old. Absent a risk-sharing arrangement, agents consume nothing half the time, a woefully bad outcome. By contrast, a contingent claim or government program that transfers resources from young to old when the old get nothing, and vice versa, increases the welfare of every generation.

Optimality is a straightforward concept in deterministic economies, but not in stochastic economies with overlapping generations. In the latter at least two notions of optimality are frequently used: ex-ante and interim efficiency. Both define Pareto improvements in the standard way, "a policy is Pareto improving if, and only if, it improves someone's welfare without hurting anyone else," but differ on what the word anyone means. For the proponents of ex-ante efficiency, identity is given solely by the time of birth. Thus, an individual born in a given period, but in two different states of the

world, is the same person. The advocates of interim efficiency disagree. For them identity is given by the time and state of the world at birth. In the interim view every generation has many different incarnations and a policy can be Pareto improving only if it does not decrease the welfare of any of them. Many results in the literature on intergenerational risk sharing hold for one notion of optimality, but not the other. Not surprisingly, there is spirited intellectual debate about which criterion is appropriate. We sidestep this debate, and like the rabbi in many a story, declare both sides correct. In particular, we show that both markets and voting institutions have difficulties generating desirable outcomes whether judged by ex-ante or interim efficiency.

We start the analysis by establishing, as a benchmark, the intergenerational transfers that yield efficient outcomes. Clearly, efficient risk sharing would be achieved if a disinterested social planner orchestrated the transfers. But such a figure, however often invoked in economics, is chimerical. Efficient risk sharing will only emerge if it is generated endogenously by the institutions in our society. In other words, to determine what risk spreading will take place, we need to examine the intergenerational transfers that will be generated by markets and governments.

Among the market's finer achievements is its ability to spread risks in static economies. Financial assets, derivatives, and contingent claims, are among the instruments markets deploy as they reallocate risks to those who can bear them most cheaply. Shouldn't such instruments, at least if the asset structure is rich enough, be able to spread risks optimally across generations? Alas not. First, since agents can not trade before they are born they can not buy ex-ante insurance. The aspiration level for markets is at best interim efficiency. Unfortunately, even this far lower hurdle presents problems. To understand why, it is useful to study markets with varying asset structures.

First consider an economy with contingent commodities. Within it, agents facing the risk of a stock-market collapse could insure themselves by purchasing contingent claims that pay off only given that eventuality. They could finance these purchases by selling contingent claims that pay off when the stock market booms. These trades provide valuable insurance because they shift consumption from high to low consumption states. However, with overlapping generations, trade takes place sequentially and agents can trade only with co-existing generations, who face similar risks. As a result these valuable trades do not take place.

Next add an infinitely lived asset to this economy. The asset, call it

money, pays no dividends and offers no consumption value. Infinitely lived assets are important because they generate intergenerational transfers when traded at positive prices. If prices fluctuate in the right way, these transfers even have the potential to generate efficient risk-sharing. In fact, money generates interim efficient insurance in some, though not all economies. Positively priced money generates backward transfers, those from young to old. Forward transfers can arise only if money trades at a negative price. But in equilibrium, the price of money can never be negative, since the older generation could simply destroy it rather than sell it. Thus, money can provide efficient risk-sharing only when backward transfers are what is required. Unfortunately, many interesting generational risks, such as stock market risk, sometimes require transfers from old to young. Here efficient risk-sharing has the young help the old when the market drops, but lets the young participate in the gains when the market booms.

Finally, consider an economy with a market mechanism that can generate forward transfers. This mechanism is a voluntary scheme in which each generation makes voluntary contributions during its youth and receives, as payoffs in old age, the contributions of the next young generation.[1] Such an asset could be run by any infinitely lived institution and is in spirit an intergenerational Ponzi scheme. Here, unlike the case of money, nothing "real" is exchanged. Yet the two assets are extremely similar. Agents are willing to provide (invest in) them only if they believe that future generations will do the same. Since money pays no dividends and has no consumption value, agents will buy it only when they believe that their successors will do the same. Thus, there is always an equilibrium in which money has no value. Similarly, in the voluntary scheme there is an equilibrium with zero contributions. In both cases the value of the asset depends on self-fulfilling expectations. The only difference between them is that the voluntary system can generate negative contributions and thus forward transfers. As a result, with a voluntary pay-as-you-go mechanism there is always an equilibrium that yields interim efficient risk-sharing. However, as we argue later in the paper, this equilibrium is unlikely to arise.

These three institutions - contingent commodities, infinitely lived assets, and voluntary transfer mechanisms - represent the archetypal market structures that might generate efficient risk-sharing. Since none of them works

[1]As far as we know, Demange and Laroque (98) have been the first to describe this mechanism.

in all cases, it is natural to ask whether government policy can ride to the rescue. Obviously, a government acting as social planner could help by carrying out the transfers required for optimality. The government, after all, does have the power to transfer income across generations, and thereby share risks efficiently. But the government's objectives are determined by political pressures, not by an innate desire to correct inefficiencies. Thus, the only interventions that are feasible are those that are desired by a majority of the voters.

A government with the power to carry out transfers between generations could use this power to implement optimal generational risk-sharing. But it could also use it to carry out a selfish, purely redistributional policy. Voters, young and old, might choose to stick with the efficient risk-sharing rule, but they might vote to expropriate the other generation instead. Since old citizens die at the end of the period, they always favor imposing a big redistributive tax on the young. For example, even if the stock market boomed, they might favor taxing the young, and if in the majority might vote that policy.

The young have different incentives, which may lead them to behavior that at least has a nobler cast. There are equilibria in which the young, even though they are in political control, implement an optimal risk-sharing rule that requires transfers to the elderly. They ascribe to what we label the Golden Transfer Policy: transfer to your predecessors as you would like your successors to transfer unto you. Not only do the young refrain from expropriating the elderly, they give actual transfers, because they want to get equivalent transfers from the next generation of young voters. Thus, if the median voter if always young, as they might be with consistent population growth, equilibria can readily emerge that offer efficient ex-ante risk sharing, even when the immediate interest of the median voter would be to defect, indeed to impose transfers in precisely the opposite direction.

Our applause should be restrained, however, since there is always another equilibrium at which the generations ignore risk-sharing considerations, and expropriate each other to the extent that voting power and the constitution allow. In this case, every generation is worse off than it would be with generational autarchy, where the initial allocation prevailed and no transfers were possible.

The formal conclusions of this paper are rather dreary. The real world offers a somewhat brighter picture, since the story of intergenerational overlap we rely on is extreme, with lives lasting but two periods. If agents were to live

for say 75+ periods both market and voting institutions would accomplish considerably more risk-sharing. With just three-period lives, the young and middle-aged generations can insure each other against the next period's risk, since they will then be alive together. For example, the young might sell contingent claims against a stock market plunge, receiving in return some claims in case of boom. As overlaps expand, the market's performance improves, though full efficiency remains out of reach. Longer life spans will also tend to lure voters more toward cooperative behavior, and less toward expropriation. Other features of the political landscape, e.g., nongenerational concerns or risk-sharing within generations, also have the ability to temper generational ruthlessness.

We conclude this introduction by relating our work to the existing literature. The main contribution of this paper is to take both market and political institutions seriously. There is a series of papers (see Bohn (98), Enders and Lapan (82), Fisher (83), Gale (90), Green (77), Gordon and Varian (88) and Smith (82)) that describe a range of circumstances where market institutions produce insufficient risk-sharing, hence lead to inefficiency, and thus leave open a role for Pareto improving government interventions. Some of these papers use more realistic models of the economy but, unlike us, they do not study whether institutions that might overcome the inefficiency are feasible.

Obviously, our work is also related to the literature on monetary equilibria in stochastic economies (see, for example, Demange and Laroque (98a,b), Lucas (72), Manuelli(90), Muench(77) and Peled(82,84).) However, there are important differences between the two approaches. We are interested in money as one of several mechanisms that might generate an optimal sequence of intergenerational transfers. In some sense, we follow a top-to-bottom approach. We first characterize the optimal transfers and then concentrate on specific institutions that might be able to generate them. By contrast, this literature follows a bottom-to-top approach. They consider the specific case of markets with money and show that, whenever monetary equilibria in which money trades at positive prices exists, it yields interim, but not ex-ante efficient risk-sharing. Using our different approach, we show that money can fail to produce sufficient risk-sharing, even when interim efficiency is the measuring rod.

The next section presents a model of the economy. Section 3 explores what might be meant by efficient risk sharing between generations. Section 4 introduces retirement risk as a specific form of generational risk, setting the

stage to examine the risk spreading performance of alternative institutions. Section 5 spotlights the strengths and vulnerabilities of market institutions as risk-sharing instruments. Section 6 gives parallel treatment to voting institutions. Section 7 concludes.

2 A Model of the Economy

Consider a simple overlapping generations economy in which every period $t = 0, \pm 1, \pm 2, ...$ a new generation t is born and lives for two periods, t and $t + 1$. To focus on intergenerational risk sharing, suppose that there is only one agent per generation. Every period agents receive an endowment that depends on the state of the world θ. This is a pure exchange economy and the endowments cannot be stored. Thus, every period aggregate consumption equals the aggregate endowment. Let $e^y(\theta_t)$ denote the endowment of the young in period t, $e^o(\theta_t)$ the endowment of the old, and $E(\theta_t)$ the aggregate endowment. The set of states is finite and the endowment process is IID, with the probability of θ given by $\pi(\theta)$.

Each generation is born after that period's uncertainty is resolved. This assumption is important because it implies that young and old cannot enter into mutually beneficial risk sharing agreements. After its birth, a generation faces uncertainty only in old age and, since only future generations can insure them against this risk, there are no gains from trading with the elderly alive at the time.

The preferences of generation t are given by

$$u(c_t^y, c_{t+1}^o) = f(c_t^y) + f(c_{t+1}^o);$$

where c_t^y and c_{t+1}^o denote, respectively, its consumption in young and old age. We assume that f is continuously differentiable, strictly concave, monotonic and satisfies $\lim_{x\to 0} f'(x) = \infty$ and $\lim_{x\to 0} f(x) = -\infty$. These standard properties guarantee interior solutions to the maximization problems described below. The last two properties indicate a very strong desire for consumption smoothing since they imply that any amount of positive consumption in both periods is preferred to an allocation with zero consumption in either young or old age.

An allocation is a function that specifies the consumption of young and old for any possible history of shocks. Given the purpose of our analysis and the nature of the model, we focus on stationary allocations in which

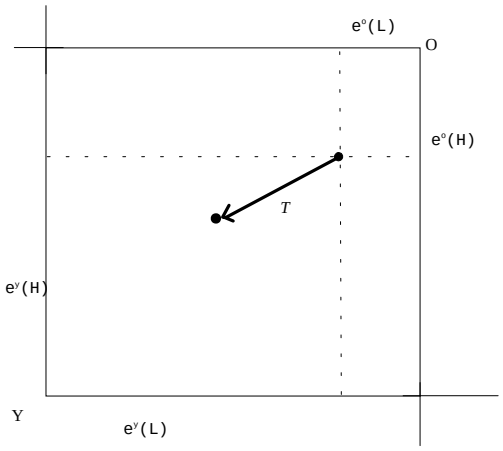

Figure 1:

consumption depends only on the current state of the world. In addition, feasibility requires that

$$c^y(\theta) + c^o(\theta) \leq E(\theta) \text{ for all } \theta.$$

Optimal intergenerational risk sharing, as we will see below, requires carrying out intergenerational transfers between young and old that might depend on the state of the world. A risk-sharing rule is a function $T(\theta)$ that specifies the transfer from young to the old in state θ; negative values indicate transfers in the opposite direction. Clearly, a sharing rule generates allocations equal to

$$c^y(\theta) = e^y(\theta) - T(\theta) \text{ and } c^o(\theta) = e^o(\theta) + T(\theta).$$

Figure 1 gives a graphical representation of the economy that will be useful later on. The diagram is a variation of the well known Edgeworth-box. Consider an economy with only two states, high and low. The dimensions of the box are given by the size of the aggregate endowment, the low state on the horizontal axis and the high state on the vertical axis. A point in the box denotes how the endowment is distributed between young and old. For example, the central point represents a sharing rule in which young and old always consume the same amounts. Endowments and final allocations are denoted by points, risk-sharing rules are denoted by vectors.

3 What is Efficient Risk Sharing?

The goal of this paper is to understand if market and political institutions can generate optimal risk sharing. Hence, the analysis starts with a discussion of optimality. This notion is so widely used in economics that the reader might find it surprising that a discussion is needed at all. The problem is that, while the concept of optimality is straightforward in deterministic economies, the same is not true in stochastic models with overlapping generations. In fact, at least two alternative notions are frequently (and passionately) used: ex-ante and interim efficiency. Both define Pareto improvements in the standard way, "a policy is Pareto improving if, and only if, it improves someone's welfare without hurting anyone else," but differ on what the word anyone means. The disagreement is based on a different conception of identity. Is identity given solely by the time of birth or is it given by the time and the state of the world at birth? In other words, is an agent born in a given period, but in different states of the world the same agent? For the proponents of ex-ante efficiency, he is. The advocates of interim optimality disagree.

The disagreement carries over to the definition of optimality:

EX-ANTE OPTIMALITY: *A stationary allocation* $c()$ is **ex-ante Pareto Optimal** *if there doesn't exist another feasible stationary allocation* $\overline{c}()$ *such that for every generation* t

$$\sum_{\theta}\sum_{\widehat{\theta}}\pi(\theta)\ \pi(\widehat{\theta})u(\overline{c}_t^y(\theta),\overline{c}_t^o(\widehat{\theta}))\geq\sum_{\theta}\sum_{\widehat{\theta}}\pi(\theta)\ \pi(\widehat{\theta})u(c_t^y(\theta),c_t^o(\widehat{\theta})),$$

with strict inequality for at least one generation.

INTERIM OPTIMALITY: *A stationary allocation* $c()$ is **interim Pareto Optimal** *if there doesn't exist another feasible stationary allocation* $\overline{c}()$ *such that for every generation* t *and every state* θ

$$\sum_{\widehat{\theta}}\ \pi(\widehat{\theta})u(\overline{c}_t^y(\theta),\overline{c}_t^o(\widehat{\theta}))\geq\sum_{\widehat{\theta}}\pi(\widehat{\theta})u(c_t^y(\theta),c_t^o(\widehat{\theta})),$$

with strict inequality for at least one pair (t,θ).

In the ex-ante view, since agents are able to compute expected utility before their birth, well-being is determined by the average welfare of their different incarnations. These averages have no meaning for the proponents

of interim optimality. For them, consciousness and identity cannot precede birth and thus, when an agent is able to evaluate utility for the first time, he incorporates in his calculations all the information that he has at birth. As a result, he dislikes a policy that may have increased his welfare ex-ante, but decreases it in the particular state in which he is born.

An interim Pareto improvements is less likely to exist than an ex-ante one. In the ex-ante view, a Pareto improvement can occur when a generation wins in some incarnations, looses in others, but is better off on average. This is not interim Pareto improving, however, because it hurts some incarnations. As a result, every ex-ante efficient allocation is also interim efficient, but not vice versa.

Which notion is the correct one?[2] Clearly, the answer to this question must be based on the merits of the notion and not on whether market or political institutions are able to generate that type of optimality. One must first select the optimality criteria, and only then study which institutions are able to generate it.

Peled (1982) argues for interim efficiency, because it mirrors the informational structure of the economy. In particular, it assumes that agents evaluate policy with the same information that they have when they trade in the market or when they vote: the young generation t knows θ_t but not θ_{t+1}. By contrast, the informational structure in the ex-ante view has the flavor of a Rawlsian veil of ignorance in which agents evaluate policy before their birth.

This is a sound argument but it is not sufficient to rule in favor of interim efficiency. If agents could travel out of time and trade before their birth, with knowledge about when they will be born but not about how uncertainty will be resolved, they would buy ex-ante insurance. Thus, agents do not insure ex-ante because they are technologically constrained, since time travel has not been invented. But they would remove the constraint if they could find a way. In this view, the ex-ante transfers can be seen as a way of removing

[2]This question has been the subject of a long debate that started, at least, with the publication of Lucas (72). He proposed an ex-post notion of optimality in which agents know how uncertainty is resolved throughout their lives. This notion is unsatisfactory because it rules out, by assumption, any gains from insurance. Muench (77) pointed out these problems – that Lucas acknowledged in a reply (see Lucas (77)) – and proposed an ex-ante notion as an alternative. In a classic paper, Peled (82) responded to Muench by proposing and forcefully defending an interim notion of optimality that lies between the previous two criteria. The literature has used ex-ante and interim notions ever since.

the technological constraint. Agents would experience the technology as an improvement, and as a result, they also view the ex-ante transfers as Pareto improving.

Since both efficienty notions have significant standing and the goal of this paper is not to advocate for one or the other, we side step the debate by analyzing the risk-sharing properties of institutions under both points of view. The reader might find, as an added benefit, that the analysis sheds light on the differences between them.

In the rest of this section we characterize the set of optimal risk-sharing rules. Let's look first at ex-ante optimality. The set of ex-ante efficient stationary allocations is given

$$\max \sum_{\theta} \sum_{\widehat{\theta}} \pi(\theta)\pi(\widehat{\theta})u(c^y(\theta), c^o(\widehat{\theta})),$$

subject to

$$c^y(\theta) + c^o(\theta) \leq E(\theta) \text{ for all } \theta.$$

The properties of $f()$ imply that the optimal allocation is uniquely determined by the first order conditions

$$f'(c^y_{EA}(\theta)) = f'(E(\theta) - c^y_{EA}(\theta)), \tag{1}$$

which imply the simple risk-sharing rule

$$c^y_{EA}(\theta) = c^o_{EA}(\theta) = \frac{1}{2}E(\theta).$$

Thus, the ex-ante optimal allocation is independent of how the endowment is distributed between young and old. As we will see in a moment, the same is true for interim efficient allocations.

Now look at interim optimality. Let $\gamma(\theta_1)$, ...,$\gamma(\theta_n)$ be strictly positive weights satisfying $\sum \gamma(\theta) = 1$. The set of interim efficient stationary allocations is given by the solution, for all possible weights, to[3]

$$\max \sum_{\theta} \sum_{\widehat{\theta}} \gamma(\theta)\pi(\widehat{\theta})u(c^y(\theta), c^o(\widehat{\theta})), \tag{2}$$

[3]This program yields an ex-ante optimal allocation if, and only if, $\gamma(\theta) = \pi(\theta)$. This is another way of seeing that ex-ante efficiency implies interim efficiency, but the opposite is not true.

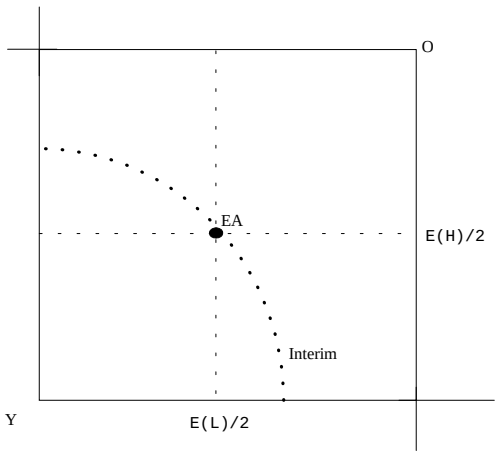

Figure 2:

subject to

$$c^y(\theta) + c^o(\theta) \leq E(\theta) \text{ for all } \theta.$$

Once more, we get that for each vector of weights $\gamma()$ the solution is fully and uniquely characterized by the first order conditions

$$\gamma(\theta) f'(c^y_{IN}(\theta)) = \pi(\theta) f'(E(\theta) - c^y_{IN}(\theta)). \tag{3}$$

The difference between the two sharing rules can be seen in Figure 2. The curve in the southwest part of the diagram denotes the locus of interim efficient allocations. By contrast, the unique ex-ante allocation is the point EA lying at the center of the box. Thus, for every endowment point there is a unique ex-ante efficient risk-sharing rule, but many interim efficient ones.

We will see below that institutions have a harder time generating forward transfers, that go from old to young, than backward transfers, that go from young to old. Thus, it is important to identify the cases in which forward transfers are needed to restore efficiency. Figures 3 and 4 provide an answer to this question. The diagram in Figure 3 can be used to describe both the economy with endowment e and, by moving the endowment point around, the class of economies with that aggregate endowment. As Figure 3 illustrates, the optimal ex-ante risk-sharing rule requires a transfer towards EA, regardless of the endowment's location. Southwest movements indicate transfers from young to old, whereas northeast movements indicate forward

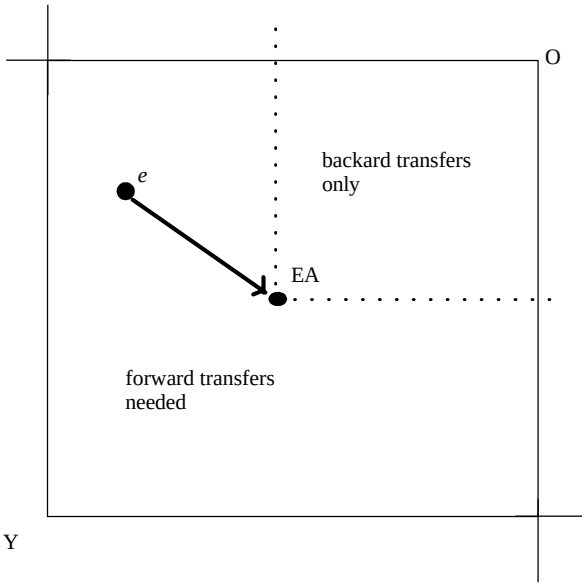

Figure 3:

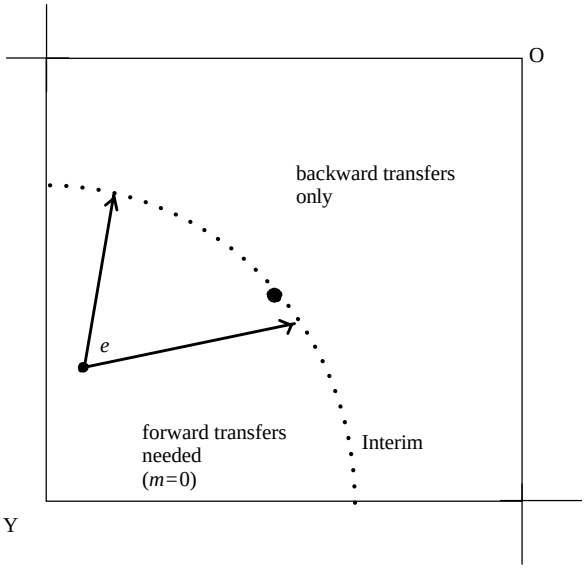

Figure 4:

transfers. We can see that the only economies in which ex-ante efficiency can be restored using only backward transfers are those to the northeast of EA. In other words, for a large class of economies, perhaps more than half, optimal ex-ante risk sharing requires the use of transfers from old to young in at least one state of the world.

In Figure 4 we repeat the analysis for the case of interim efficiency. Since the set of efficient allocations has now increased, the problem is less severe. However, it is still the case that forward transfers must be used in any economy to the southwest of the optimal locus. Thus, as the diagram shows, forward transfers are needed under either criterion.

4 Example: Retirement Risk

The possibility of a stock market collapse and its effects on the welfare of the elderly is an important issue before this conference and a prominent example of generational risks. For this reason it is useful to look at an example that focuses precisely on this issue. Suppose that agents have a constant endowment when young, but face uncertainty about their retirement income. For concreteness, the stock market either collapses (L) or booms (H), with equal probability, and that $e^y(L) = e^y(H) = 2$, $e^o(L) = 1$, and $e^o(H) = 4$. Agents' preferences are logarithmic.

Aggregate output fluctuates widely; it is 3 in the low state and 6 in the high one. Furthermore, all the gains during booms accrue to the elderly, but they also take all the losses during downturns. The efficient ex-ante risk-sharing rule,

$$T^{EA}(L) = \frac{1}{2} \text{ and } T^{EA}(H) = -1,$$

is an intuitive arrangement in which the young insure the elderly against the downturn, for example if the stock market collapses, and in return the elderly share the windfall from bull markets.

It is worth emphasizing that this insurance arrangement differs significantly from the minimum-pension guarantees that are often discussed in the social security literature. First, this risk-sharing rule is two sided. Retirees are protected against retirement risk, but they have to share the gains during booms. Second, the size of transfers depends on the entire state of the economy and not only on the performance of the stock market. This is an important feature because the cost of insuring the elderly depends on the

income of the young. The young can insure the elderly only to the extent that their income remains relatively high during stock market collapses. In our example, this condition is satisfied; it is reflected in the relative size of the transfers.

This example also illustrates the differences between ex-ante and interim efficiency. In the absence of any insurance arrangements, the expected utility at birth is 1.38 regardless of the state of the economy. The ex-ante risk sharing rule yields an average expected utility at birth, calculated of course before the agent is born, equal to 1.50. But agents born in the low state have only 1.15 units of expected utility. Thus, they are happy to renege on the insurance arrangement.

5 Market Institutions

We have shown that efficient risk sharing requires intergenerational transfers and characterized the transfers that restore efficiency. But a social planner who could carry them out exists only in the scribblings of economists. Thus, efficient risk-sharing can occur only if the transfers are generated endogenously by the institutions in the economy. This leads to the central question of this paper: Are there institutions capable of generating efficient intergenerational risk sharing? Two types of institutions are often suggested as possible solutions: markets and government. We study market institutions in this section and government intervention in the next, and show that both of them have shortcomings as risk-sharing mechanisms.

Intuitively, one would expect the market to provide optimal insurance as long as the asset structure of the economy is rich enough. Although this intuition holds for static economies, and even for dynamic economies with infinitely lived agents, it fails in the overlapping generations model. There are two reasons for this failure, the first is straightforward, the second more subtle. The first problem is that agents cannot buy ex-ante insurance since they can trade only after they are born. Thus, the market can provide at best interim insurance. The second problem presents a challenge even for this weaker notion of insurance. In an overlapping generations economy trade takes place sequentially. Each period only the generations alive at the time can trade with each other. As we will see below, this restricts dramatically the amount of intergenerational risk sharing that takes place, even in economies with a rich asset structure.

To understand better the restrictions imposed by sequential trading, and the role that different assets play in intergenerational risk-sharing, this section studies three markets with different asset structures. First we study markets with contingent claims. Next we add an infinitely lived asset, like money, that pays no dividends; but is sold from one generation to the next. Finally we consider a new type of mechanism that resembles a voluntary pay-as-you-go social security system.

5.1 Contingent Claims

Consider an economy in which two types of commodities are exchanged every period: (1) that period's consumption good, and (2) contingent claims for next period. Since trading takes place after the uncertainty has been resolved, prices might depend on the state of the world. Let $p(.|\theta)$ denote the price of contingent commodities in state θ, where $p(\widehat{\theta}|\theta)$ buys one unit of next period's consumption if the state of the world turns out to be $\widehat{\theta}$. We normalize the price of this period's commodity to 1 so that all contingent prices are real prices.

The equilibrium in this market is easily characterized. At any positive price, the elderly demand zero contingent commodities. If they could, they would sell short, since they die at the end of the period and thus can default on their obligations. But we assume that short-selling is not allowed.[4] As a result, there are no intergenerational transfers in equilibrium. The price of the contingent claims adjust so that the young demand exactly their endowment, no claims are exchanged, and no risk-sharing takes place.

A young agent born in state θ faces the following market problem:

$$\max \ f(c^y(\theta)) + \sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(c^o(\widehat{\theta})),$$

subject to

$$c^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) c^o(\widehat{\theta}) \leq \quad e^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) e^o(\widehat{\theta}).$$

Thus, the equilibrium vector of prices is given by

$$p(\widehat{\theta}|\theta) = \frac{\pi(\widehat{\theta}) f'(e^o(\widehat{\theta}))}{f'(e^y(\theta))} \text{for all } \widehat{\theta}.$$

[4]This assumption is necessary not only to get results that make sense, but also to get existence of equilibrium.

Note that the price of contingent commodities is high for states in which the agent gets a poor outcome, and low when he does well. For example, in the case of retirement risk we get $p(L|H) = p(L|L) = 1$ and $p(H|H) = p(H|L) = 0.25$. At a price of four to one, agents facing the possibility of a stock market crash would choose no insurance. Thus, we have obtained the following result:[5]

PROPOSITION: *The market with contingent claims has a unique equilibrium in which no intergenerational transfers, and thus no risk-sharing, takes place.*

One could think that the problem is due to market incompleteness and not the structure of sequential trading. After all, in our discussion we only included one contingent commodity whereas Arrow-Debreu markets have one contingent claim for every possible state and period. That would be incorrect. The result remains unchanged even if we introduce the entire set of contingent commodities.

The no trade result, however, depends crucially on two other assumptions of the model. First is the assumption of one (representative) agent per generation. In a more realistic model with many agents and heterogeneity within generations, some trade would occur. However, all the trades would be between the members of the same generation. No intergenerational trade, and thus no intergenerational risk sharing, would take place. For this reason, and given the objectives of this paper, studying the one agent case imposes no conceptual loss.

A more problematic assumption is that lives last for two periods. In a more realistic model in which agents live for 75 periods, and with almost an equal number of overlaps, the trading of contingent commodities generates intergenerational transfers and thus risk-sharing. To see why, consider the natural extension to three period lives. Here young and middle-age agents face risks during the next period of their lives and can insure each other using contingent claims. It is natural to conjecture that the market efficiency increases with the number of overlaps, and that it reaches a 100% in the limit case of infinitely lived agents. As far as we know, however, this problem has not been solved.

[5]Cass and Shell (83) obtained a similar result and Baxter (89) contains an identical result.

5.2 Contingent Claims and Infinitely Lived Assets

Now consider an economy in which, in addition to the contingent claims described above, there is an infinitely lived asset that pays no dividends and has no consumption value. Following Samuelson's (1958) classic paper, we refer to this asset as money. The introduction of money is valuable because it generates intergenerational transfers when it trades at a positive price and, as long as the prices fluctuate in appropriate fashion, it even has the potential to provide some intergenerational risk sharing.

Let M be the amount of money in the economy and $m(\theta)$ its price in state θ. Consider the market problem of an agent born in state θ:

$$\max_{z(),\mu} \; f(c^y(\theta)) + \sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(c^o(\widehat{\theta})),$$

subject to

$$c^y(\theta) \leq e^y(\theta) - m(\theta)\mu - \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) z(\widehat{\theta}),$$

and, for all $\widehat{\theta}$,

$$c^o(\widehat{\theta}) \leq e^o(\widehat{\theta}) + m(\theta)\mu + z(\widehat{\theta}).$$

These budget constraints can be rewritten as

$$c^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) c^o(\widehat{\theta}) \leq [e^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) e^o(\widehat{\theta})] + [\sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) m(\widehat{\theta}) - m(\theta)]\mu. \tag{4}$$

In equilibrium, the following no arbitrage condition holds in every state θ

$$\sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) m(\widehat{\theta}) \leq m(\theta).$$

Otherwise agents could make a "profit" by selling $m(\widehat{\theta})$ units of contingent claims in each market, using the proceeds to buy money that is used to service the claims, and keeping the difference. Thus, in equilibrium the budget constraint reduces to

$$c^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) c^o(\widehat{\theta}) \leq [e^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) e^o(\widehat{\theta})], \tag{5}$$

and agents do not have a strict incentive to buy money. Agents can achieve the same amount of risk sharing buying contingent commodities. We will return to this issue later in the section.

Can money generate optimal risk sharing? It has long been recognized that, since agents can only trade after they are born, money cannot generate ex-ante risk-sharing. However, there exists a large literature in macroeconomics (see, among others, Peled(82,84), Manuelli (90), Demange and Laroque (98a,b)) that studies the interim efficiency of monetary economies. These papers study conditions under which an interim Weak First Welfare Theorem (WFWT) can be established. The First Welfare Theorem (FWT) states that every market equilibrium is efficient. By contrast, the Weak FWT holds if there is always a market equilibrium that is efficient, even when there are other inefficient equilibria.

It is easily seen that this market institution cannot satisfy the FWT because there is always an equilibrium with $m(\theta) = 0$ for all θ. In this case money might change hands, but no real resources are transferred and thus no risk-sharing takes place. The problem is that monetary equilibria depend on expectations. Since money pays no dividends and has no consumption value, agents are willing to buy it only if they believe that they will subsequently be able to sell it for a positive and profitable price.

However, the problem is even worse:

THEOREM: *Both the ex-ante and the interim Weak First Welfare Theorem fail for a market with contingent claims and money as its risk-sharing instruments.*

This result derives from a very simple fact. In equilibrium the price of money can never be negative. In a state with negative prices the old would burn their money rather than sell it. Also, an equilibrium in which $m(\theta) = 0$ for some states and $m(\widehat{\theta}) > 0$ for some other states cannot exist; that is because the young would have an infinite demand for money in the first class of states and markets would not clear. We can thus conclude that money trades at positive prices in any equilibrium that generates intergenerational transfers. But this implies that although money can generate either no transfers or backward transfers, it can never produce transfers from old to young. As a result, as Figure 4 shows, the interim WFWT fails because all the economies in the southwest part of the box need forward transfers to restore efficiency. In fact, these economies have a unique equilibrium in which money has no value. Since ex-ante efficiency implies interim efficiency, this argument also establishes the failure of the ex-ante WFWT.

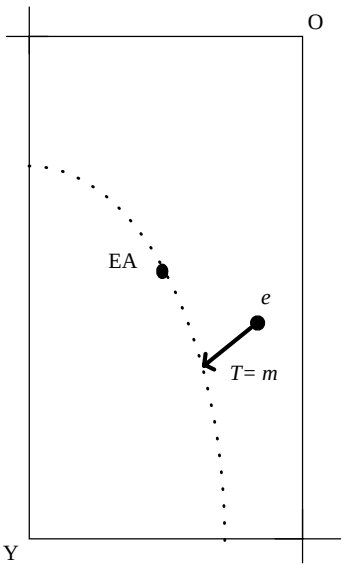

Figure 5:

At face value, this theorem seems to contradict results in the previous literature that prove that any equilibrium in which money trades at non-zero prices is interim efficient. But there is no contradiction. The problem, as Figure 4 shows, is that equilibria with positive money prices might not exist. In those economies, the market cannot generate even efficient interim risk-sharing.[6]

To understand better the differences between ex-ante and interim insurance, it is useful to revisit the example of retirement risk. Figure 5 depicts the non-trivial monetary equilibrium that arises in this case. Money prices are $m(L) = m(H) = m = 0.4435$. Thus, the trading of money generates constant backward intergenerational transfers and restores interim efficiency. However, from the ex-ante point of view, this arrangement is quite counter-intuitive. With money the young always make a transfer to the old, which insures the old against the possibility of a stock market collapse. But, in contrast to ex-ante risk-sharing, the old do not share the gains of a stock

[6]Demange and Laroque (98) allow money to take negative prices and thus they are able to obtain a WFWT. The explanation is that their "negative money" resembles more the voluntary pay-as-you-go system studied in the next subsection than standard fiat money.

market boom. In fact, since

$$c^o(L) = \frac{1}{2} + m \text{ and } c^o(H) = 4 + m$$

the old end up consuming more in both states of the world.

5.3 Voluntary Pay-As-You-Go Mechanisms

The main problem with money is that it cannot generate forward transfers. In this section we study a market mechanism that can. A voluntary pay-as-you-go system is basically an intergenerational Ponzi scheme in which each generation makes voluntary contributions during its youth and receives, as payoffs in old age, the contributions of the next generation. Such a mechanism could be run by any infinitely lived institution.

The return from investing in this asset is proportional to the aggregate contributions of the next generation and inversely proportional to the aggregate contributions of the present generation. Consider the market problem of an agent born in state θ. If we let $\tau(\theta)$ denote aggregate contributions, the return on investing one unit is $\tau(\widehat{\theta})/\tau(\theta)$ when tomorrow's state is $\widehat{\theta}$. Thus, the agent solves

$$\max_{z(),\alpha} \; f(c^y(\theta)) + \sum_{\widehat{\theta}} \pi(\widehat{\theta}) g(c^o(\widehat{\theta})),$$

subject to

$$c^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) c^o(\widehat{\theta}) \leq [e^y(\theta) + \sum_{\widehat{\theta}} p(\widehat{\theta}|\theta) e^o(\widehat{\theta})] + [\sum_{\widehat{\theta}} p(\widehat{\theta}|\theta)\tau(\widehat{\theta}) - \tau(\theta)]\frac{\alpha}{\tau(\theta)}; \tag{6}$$

where α is his contribution to the asset. The agent is a price taker and thus he takes the "price" of the asset, or more precisely the aggregate contributions, as given.

As in the case of money, agents are willing to invest only if they believe that future generations will do the same. Thus, there is always an equilibrium in which $\tau(\theta) = 0$ for all θ, and no risk-sharing takes place. This suggests that money and voluntary pay-as-you-go systems are very similar assets, even though in the later case no "commodity" or piece-of-paper is exchanged. This is not surprising because the value of both assets depends on self-fulfilling expectations.

A careful examination of the budget constraints (6) and (4) shows that the two assets are almost identical. To see why suppose that $M = 1$, so that there is only one unit of money in the economy. In this case μ denotes the share of the money supply that the agent is willing to hold and in equilibrium we must have $\mu = 1$. Similarly, in (6) $\alpha/\tau(\theta)$ denotes the share of contributions that the agent is willing to make and in equilibrium $\alpha = \tau(\theta)$. But this implies that, as long as $m(\theta) = \tau(\theta)$ for all θ, the two assets are identical and we have the following result:

PROPOSTION: *If contributions cannot be negative, then there is a one-to-one correspondence between the equilibria in the market with money and the equilibria in the market with a voluntary pay-as-you-go mechanism.*

The equivalence breaks down if we allow contributions to the voluntary pay-as-you-go system to be negative. In particular, the unrestricted pay-as-you-go system is able to generate the forward transfers that are needed to restore interim efficiency.

THEOREM: *If negative contributions are possible, the market with a voluntary-pay-as-you-go mechanism satisfies the interim Weak First Welfare Theorem, but not the ex-ante one.*[7]

What is a negative contribution? Strictly speaking, it is a loan in which the cost of borrowing is determined by the actions of the next generation. Suppose that today's young generation contributes $-T$ to the system and the next generation also contributes $-T$. Then the asset amounts to a loan with an interest rate equal to one. On the other hand, the loan is free if the next generation decides to contribute 0.

It is important to emphasize that with unrestricted contributions a generation that buys the asset could find itself with an arbitrarily large liability. In a voluntary pay-as-you-go system, the "dividend" that the asset pays is determined by the actions of the next generation, and they could decide to borrow an arbitrary large amount. Of course, such behavior does not take place in equilibrium because the next generation would do the same to them, but such an action is a possibility. This is particularly problematic since, as we saw in (5), in equilibrium the price of the contingent commodities is such

[7]The proof of this result follows from the analysis of Demange and Laroque (98b).

that agents do not have an strict incentive to buy the asset. So, why should they take the risk? Thus, the equilibrium with $\tau(\theta) = 0$, and no risk-sharing, is more plausible than the equilibrium with negative prices.

We can conclude that these archetypal market institutions cannot generate ex-ante efficient risk-sharing and are unlikely to generate interim efficiency, at least on a large class of economies. Can government policy come to the rescue?

6 Voting Institutions

The government has the power to transfer income across generations, and thus to provide risk-sharing. However, its objectives are determined by political pressures and not by an innate desire to correct inefficiencies. In this section we characterize the risk-sharing rules that arise as the result of electoral competition.

Each period there is an election in which all the agents alive at the time vote. Although many issues are decided in a typical election, here we only model the choice of intergenerational redistribution. The policy space in state θ is given by

$$P(\theta) = \{T : -e^o(\theta) \leq -\underline{R}(\theta) \leq T \leq \overline{R}(\theta) \leq e^y(\theta)\},$$

where T is the transfer from young to old that is implemented in the period and $[-\underline{R}(\theta), \overline{R}(\theta)]$ are exogenously given constitutional constraints on intergenerational expropriation. We assume that $-\underline{R}(\theta) < 0 < \overline{R}(\theta)$ so that both forward and backward transfers are possible. Clearly, efficient risk-sharing arises only if voters always choose the optimal transfer $T^*(\theta)$ that we characterized in section 3.

Every period the young choose policy with probability λ and the old with probability $1 - \lambda$. We refer to the generation choosing policy as the median voter. If $\lambda = 1$ the young always choose policy, for example in an economy with a rapidly growing population, whereas if $\lambda = 0$ the old always determine policy, for example when the old are well organized politically, e.g., through AARP. The model implicitly assumes that voters' preferences are determined by age. Although this assumption may not describe voting behavior for general issues, like abortion or school prayer, it is justified for intergenerational redistribution where the impact of the policy on an agent depends only on his age (see Poterba (97) for evidence in this regard.)

Under these assumptions the voting institution can be modelled as the following infinitely repeated game:

- Every period t, Nature selects θ_t and chooses the identity of the median voter.
- After observing these outcomes, the median voter chooses a policy $p_t \in P(\theta_t)$.
- There is complete information about the history of previous policies and states. History at time t is denoted by

$$h_t = (..., \theta_{t-2}, \theta_{t-1}; ..., p_{t-2}, p_{t-1}).$$

- The set of political equilibria is given by the set of sequential equilibria of this game.

Let $p^o(\theta, h)$ denote the policy that an old median voter selects in state θ, given history h. Define $p^y(\theta, h)$ similarly. Strictly speaking, voters only have one decision to make: how much to transfer between young and old. However, in order to understand better the politics of intergenerational risk-sharing, it is useful to think of the policy space as having two dimensions

$$P(\theta) \subseteq \{I : I = T^*(\theta), 0\} \times \{R : -\underline{R}(\theta) \leq R \leq \overline{R}(\theta)\};$$

i.e.,

$$p(\theta) = \{T^*(\theta), 0\} + R(\theta).$$

The first dimension, $I \in \{T^*(\theta), 0\}$, measures whether or not the efficient risk-sharing transfer is taking place. The second dimension, $R \in [-\underline{R}(\theta), \overline{R}(\theta)]$, measures the amount of pure intergenerational redistribution. This representation emphasizes an important feature of the political process: once the door is open to carry out transfers between the generations, voters can choose optimal risk sharing, but they can also choose expropriation. Take, as an example, the case of retirement risk and suppose that the median voter is always old. When the stock market collapses they can tax the young and vote a minimum pension to themselves, as efficiency considerations demand. But they can also raise the taxes of the young in the case of a boom where, according to ex-ante risk-sharing, they are supposed to tax themselves.

The first step in analyzing the model is to note that an old median voter always chooses no risk-sharing and maximum redistribution:

$$p^o(\theta, h_t) = \overline{R}(\theta). \tag{7}$$

Since the elderly are not affected by future policy, their best option is to maximize current consumption and, given the opportunity, they expropriate the young. From their point view, the choice of taxes is purely about redistribution and not about risk-sharing; after all, they do not face any more risks. Thus, we can interpret the vote of the elderly as a decision not to carry out any risk-sharing and expropriate the young as much as possible. In the expanded version of the game this can be written as:

$$\widetilde{p}^o(\theta, h_t) = (0, \overline{R}(\theta)).$$

What about young voters? As was the case with the market, the equilibria of this model depends on the beliefs that present generations have about the behavior of future generations. There are, thus, two types of outcomes: Markovian equilibria, in which no risk-sharing takes place, and non-Markovian equilibria, with optimal risk-sharing.

In a Markovian equilibrium agents maximize their pay-off for the period because they believe that their actions do not affect the behavior of future voters. In this case the young also expropriate as much as possible:

$$p^y(\theta, h_t) = -\underline{R}(\theta).$$

As before, we can interpret their vote as a decision to focus purely in redistributive politics, without any concern for risk sharing ($\widetilde{p}(\theta, h_t) = (0, -\underline{R}(\theta))$.)

We can conclude that in a Markovian equilibrium every generation expropriates the others as much as possible, and the direction of expropriation depends on who ends up as the median voter. It is important to emphasize that when there are few constraints on expropriation, for example if $\underline{R}(\theta) = e^o(\theta)$ and $\overline{R}(\theta) = e^y(\theta)$, agents are worse off here than with generational autarchy, where no risk-sharing takes place.

There are also non-Markovian equilibria in which the young voters implement the optimal risk-sharing policy and resist the temptation of expropriating the elderly. Consider, for example, the following strategy:

$$p^y(\theta, h) = \begin{cases} T^*(\theta) & \text{if, at } h, \text{ all the previous young median voters chose } T^*(\theta) \\ -\underline{R}(\theta) & \text{otherwise} \end{cases}. \tag{8}$$

Here, optimal risk-sharing takes place as long as every previous young generation has done the same. But if there is any deviation from this behavior, they revert to the short-sighted expropriation strategy $(0, -\underline{R}(\theta))$. Cooperation is not conditioned on the behavior of the old since everyone knows that they always vote for expropriation. Using strategies of this type we can establish the following result:

THEOREM: (1) *If there are no limits to intergenerational expropriation ($\underline{R}(\theta) = e^o(\theta)$ and $\overline{R}(\theta) = e^y(\theta)$), then there are political equilibria in which the optimal (ex-ante or interim) risk-sharing rule is implemented whenever the median voter is young.*

(2) Thus, if the median voter is always young ($\lambda = 1$) there are equilibria that generate both interim and ex-ante efficient risk-sharing.

(3) However, regardless of who is the median voter, there is always an equilibrium in which there is no risk-sharing, and full expropriation takes place.

The proof of this theorem describes the essence of the problem. As we discussed above, the elderly always expropriate as much as possible. Thus, to check that the strategies (7) and (8) are a political equilibrium we only need to verify the incentives of the young, who make decisions in two types of situations: (1) histories in which every previous young generation followed the efficient risk-sharing policy, and (2) histories in which there was a deviation from this code of behavior.

Consider the first case and suppose that the state of the world is θ. The young median voter knows that if the next median voter is old he will choose $\overline{R}(\theta)$. On the other hand, if the next median voter is young he will copy the behavior that the young voter chooses today; i.e., expropriation will trigger expropriation, whereas optimal risk-sharing will maintain optimal risk-sharing. Thus, if the agent chooses $T^*(\theta)$ his life-time payoff becomes

$$f(e^y(\theta) - T^*(\theta)) + \lambda(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) + T^*(\widehat{\theta})))$$
$$+(1-\lambda)(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) + \overline{R}(\widehat{\theta}))).$$

On the other hand, if he decides not to cooperate with the risk sharing rule, he is better off expropriating the elderly as much as possible ($T = -\underline{R}(\theta)$),

reaping a payoff equal to

$$f(e^y(\theta) + \underline{R}(\theta)) + \lambda(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) - \underline{R}(\widehat{\theta}))$$
$$+(1-\lambda)(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) + \overline{R}(\widehat{\theta}))).$$

Thus, he cooperates as long as

$$f(e^y(\theta) - T^*(\theta)) + \lambda(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) + T^*(\widehat{\theta})))$$

is greater than

$$f(e^y(\theta) + \underline{R}(\theta)) + \lambda(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(e^o(\widehat{\theta}) - \underline{R}(\widehat{\theta})).$$

For λ sufficiently large the left-hand side is approximately equal to the interim expected utility[8] generated by the risk-sharing policy $T^*(\theta)$. By contrast, the right-hand side gives the payoff, in expected utility terms, of the Markovian equilibrium in which every generation expropriates as much as it can. Without bounds of expropriation, $\underline{R}(\theta) = e^o(\theta)$ and $\overline{R}(\theta) = e^y(\theta)$, this payoff becomes

$$f(E(\theta)) + \lambda(\sum_{\widehat{\theta}} \pi(\widehat{\theta}) f(0)).$$

And so, we finally conclude that the young choose optimal risk-sharing over expropriation since they have a strong desire for consumption smoothing ($\lim_{x\to 0} f(x) = -\infty$.)

Now consider the second type of histories. According to (7) and (8), future voters will ignore the risk-sharing rule and expropriate the other generation as much as possible. As a result, present voters have no incentive to refrain from expropriation and they also choose $T = -\underline{R}(\theta)$.

An interesting property of the previous equilibrium is that young agents are willing to implement the optimal ex-ante transfer even though they vote after they are born. Since part of the uncertainty has been revealed, agents might have an incentive to default from the risk-sharing rule. For example, in the case of retirement risk, agents born in the low state get a pay-off of 1.15

[8]That is, the expected utility conditional on knowing that the state of the world at birth is θ.

with insurance and 1.38 without insurance. And yet, the previous analysis shows that they are willing to give a transfer to the elderly when the stock market collapses.

In this example, the only reason why young voters are willing to help the elderly is that failure to do so would trigger an expropriation war that could hurt them in old age. A risk-sharing rule is interim individually rational if, regardless of the state in which the agent is born, it is better than the initial endowment. Since the ex-ante insurance arrangement is not interim individually rational, the promise of future insurance is not enough incentive to go along with it. The required additional incentive is provided by the possibility of expropriation.

Consider the problem of designing a constitution that specifies the rules of the political process. Can we modify the rules in a way that makes optimal risk-sharing more likely and rules out the possibility of intergenerational expropriation? One important lesson from the previous analysis is that risk-sharing is more likely if the young always choose policy. However, youth dominance is not enough. Given that the Markovian equilibria always exist, we are faced with a rather perverse trade-off. In order to avoid short-sighted expropriation, a restricted policy space like

$$P(\theta) = \{T^*(\theta), 0\},$$

where expropriation might be prohibited through say a constitutional amendment, seems desirable. In this case each generation can only choose to implement the optimal transfer, or do nothing, but cannot expropriate. However, ruling out expropriation might be detrimental..

PROPOSITION: *The voting institution with a restricted policy space* $\{T^*(\theta), 0\}$ *and in which the median voter is always young can only support risk-sharing rules that are interim individually rational. Thus, in general it can generate interim but not ex-ante efficient risk-sharing.*

The problem with restricting the policy space is that it only supports risk-sharing rules that are interim individually rational. A risk sharing policy is interim individually rational if it does not decrease the expected utility of an agent, conditional on knowing the state of the world at birth. As the case of retirement risk illustrates, in most cases of interest this condition is not satisfied. Thus, we are left with a Hobson's choice: we can either go with a restricted policy space, that rules out extreme expropriation but

also sacrifices ex-ante insurance, or with the unrestricted policy space, that generates optimal risk-sharing and no expropriation in the good equilibrium, but produces disastrous outcomes in the bad one.[9]

Can government intervention generate efficient intergenerational risk-sharing? The government certainly has the power on paper to carry out the efficient policy. For example, in the case of retirement risk it could help retirees by taxing the young when the stock market crashes and similarly tax the elderly when the market booms. However, our analysis suggests that, as long as government policy is determined by citizen's votes, intergenerational transfers might be driven more by redistributive politics than by risk-sharing considerations.

7 Conclusions

Our conclusions are not happy: neither market nor political institutions can be counted on to generate efficient intergenerational risk-sharing. Reality may be less dreary, since our stark results follow from the assumption of two period lives. As lives extend, overlaps increase and trade in contingent commodities generates useful risk sharing. Similarly, the incentives to expropriate diminish with longer lives. Thus, market and voting institutions may do better than in our models, but our main conclusion holds: neither can be relied upon to restore full efficiency.

Our analysis here is positive. The goal is to understand whether and when existing institutional arrangements generate efficient risk-sharing. Though our analysis applies to all generational risks, including wars and economic declines, in the context of this volume it is important to consider the implications of our analysis for the current debate on social security reform.

The first implication is that, since markets cannot generate optimal risk-sharing, a system of individual accounts can not produce an efficient allocation. In a market with a rich enough asset structure, agents can buy a lot of insurance against events like a stock market crash, but not perfect insurance. The size of the efficiency loss is an open empirical question.

This inefficiency can only be corrected by a government policy that carries out transfers between young and old, with the size and direction of the

[9]The political economy of ex-ante insurance is very similar to the political economy of public investment in future generations. The necessary and sufficient conditions to get optimal investment in future generations are explored in detail in Rangel (98).

transfers depending on the state of the world. Thus, a combination of private accounts and a contingent pay-as-you-go system, that transfers income in both directions, in principle dominates the pure private system.

But the second major lesson of this paper casts doubt on turning that potential to reality. The political economy of intergenerational transfers – i.e., the fact that generations have to vote for the transfers that are made – may prevent a politically responsive government from intervening successfully. In some extreme cases, it may impose expropriating transfers that reduce the welfare of every generation.

It is important to emphasize, however, that the possibility of political failure does not rule in favor of a purely private approach to social security. There are many government policies, besides risk-sharing, that redistribute income between generations. Government debt and the choice of the tax base are two prominent examples. Medicare most directly transfers to the elderly. These policies are subject to the same redistributive politics that we study in this paper. Thus, intergenerational expropriation can take place even if the government is not involved in social security. The same elderly that would use the pretense of risk-sharing to expropriate the young during a stock market boom can use debt to achieve an identical goal. This paper shows that government policy may not be able to provide optimal insurance. It does not show, however, that intergenerational redistribution is higher when the government is involved in social security.

The lessons of our overlapping generations model for efficiency can be distilled briefly. The risk-sharing brought by market institutions is always welcomed, but never sufficient. The government has the power to generate efficient risk-sharing, but sometimes that power is deployed insufficiently or perversely.

References

[1] Baxter, M. (1989) "Money and Market Incompleteness in Overlapping Generations Models," *Journal of Monetary Economics,* 24: 69-91.

[2] Bohn, H. (1988) "Risk Sharing in a Stochastic Overlapping Generations Economy," mimeo UC-Santa Barbara.

[3] Cass, D. and K. Shell, (1983) "Do Sunspots Matter?," *Journal of Political Economy 91(2): 193-227.*

[4] Demange, G and G. Laroque (1998a) "Optimal Intergenerational Risk Sharing and Capital Accumulation," Delta manuscript.

[5] Demange, G and G. Laroque (1998b) "Social Security and Demographic Shocks," forthcoming in *Econometrica.*

[6] Enders, W. and H. Lapan, (1982) "Social Security Taxation and Intergenerational Risk Sharing," *International Economics Review (23): 647-658.*

[7] Fisher, S. (1983) "Welfare Aspects of Government Issue of Indexed Bonds," in Dornbusch and Simonsen (eds.) "Inflation, Debt and Indexation," MIT Press, 233-246.

[8] Gale, D. (1994) "The Efficient Design of Public Debt," Chapter 9 in Allen and Gale's *Financial Innovation and Riks Sharing*, MIT Press.

[9] Gordon, R. and H. Varian (1988) "Intergenerational Risk Sharing," *Journal of Public Economics (37): 180-202.*

[10] Green, J. (1977) "Mitigating Demographic Risk through Social Insurance," *NBER Working Paper no 215.*

[11] Lucas, R. (1972) "Expectations and the Neutrality of Money," *Journal of Economic Theory 4: 103-24.*

[12] Lucas, R. (1977) "Repply to Muenc, Polemarchakis and Weiss," *Journal of Economic Theory 15: 351-52.*

[13] Manuelli, R. (1990) "Existence and Optimality of Currency Equilibrium in Stochastic Overlapping Generations Models: The Pure Endowment Case," *Journal of Economic Theory* 51: 268-94.

[14] Muench, T. (1977) "Efficiency in a Monetary Economy," *Journal of Economic Theory*, 15: 325-44.

[15] Peled, D. (1982) "Informational Diversity over Time and the Optimality of Monetary Equilibria," *Journal of Economic Theory 28: 255-74.*

[16] Peled, D. (1984) "Stationary Pareto Optimality of Stochastic Asset Equilibria with Overlapping Generations," *Journal of Economic Theory 34: 396-403.*

[17] Poterba, J. (1997) "Demographic Structure and the Political Economy of Public Education," *Journal of Policy Analysis and Management*, 16: 48-66.

[18] Rangel, A. (1998) "Forward and Backward Intergenerational Goods," mimeo Stanford University.

[19] Samuelson, P. (1958) "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy 66(6):467-82.*

[20] Smith, A. (1982) "Intergenerational Transfers as Social Insurance," *Journal of Public Economics (19): 97-106.*