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ABSTRACT

We present a model where long-run growth and industrial location are jointly endogenous by introducing Romerian growth into a Krugmanesque economic geography model. We show that growth is a powerful destabilising force, but inter-regional learning spillovers are a stabilising force. Moreover, including endogenous growth allows a broader view of integration. While traditionally seen only in terms of trade costs, many aspects of economic integration are more naturally viewed as lowering the cost of trading information rather than goods, i.e. as reducing the extent to which learning externalities are localised. Raising learning spillovers is stabilising, so integration may encourage geographic dispersion (the traditional result is that integration tends to encourage agglomeration). This may be useful for evaluating real-world regional policies—e.g. subsidisation of universities, technical colleges and high-technology industrial parks in disadvantaged regions—that are aimed at combating the localisation of learning externalities. Finally we show that agglomeration of industry is favourable to growth and that this growth effect can mitigate, but not reverse, losses suffered by residents of the periphery when catastrophic agglomeration occurs.

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1. Introduction

The primary impact of most international market opening initiatives is to increase efficiency and specialisation of industry within nations. This is not, however, the only possibility. Integration can affect the international location of industrial activity. In particular, the recent literature on economic geography -- e.g., Krugman (1991a, b), Venables (1996), Krugman and Venables (1995), and Puga (1997) -- suggests that close but imperfect integration may create regional winners and losers.

By advancing our understanding of the links between agglomeration and economic integration, the economic geography literature has made an important contribution to analysis of trade liberalisation. However, because the leading models focus on essentially static frameworks (i.e., models in which the long-run growth rate is zero by assumption), the literature is unable to evaluate the interaction between economic integration, the location of industry, and long-run growth. Four reasons suggest that it is important to remedy this omission.

First, there are clear theoretical grounds for believing that growth affects location and location affects growth. For instance, virtually all endogenous-growth models rely on technical externalities such as knowledge spillovers or production externalities. Empirical studies, such as Eaton and Kortum (1996), show that these externalities are related to the geographic distribution of industry and/or R&D activity. The natural presumption, therefore, would be that concentrating industry and/or R&D activities would be beneficial to growth. Moreover, due to these same externalities, the natural presumption would be that rapid growth would promote industrial agglomeration for standard Marshallian reasons. (In economic geography terminology, growth-inducing externalities are an additional centripetal force.) Given this, policy makers may face a dilemma between policies that promote growth and policies that promote the geographic dispersion of

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industry. Evaluating this possibility requires a model in which growth and location are endogenous.

Second, policy makers often justify regional policies on the assumption that there is a connection between regional growth rates and possession of an industrial base. In order to evaluate the static and dynamic benefits of policies aimed at creating a ‘vibrant regional economy,’ it is necessary to precisely model the links between policies, location and growth.

Third, the traditional axis of investigation in economic geography models focuses on the cost of selling goods at a distance. Lowering the cost of trade in goods, however, is only one aspect of integration. Another important aspect of regional integration is its effect on the cost of sharing knowledge. In Europe, for instance, integration means that people now know much more about each other’s nations, cultures, businesses, and technology. Two facets seem to stand out here, business and personal travel, and cross-border merger and acquisitions (M&A) activity. The volume of both travel and M&A activity has expanded rapidly due to European integration and this has clearly reduce the localisation of commercially relevant knowledge (e.g. product and process innovations). Moreover, many governments seem to act on the belief that increasing knowledge flows can help peripheral regions. The promotion of regional universities and high-technology industrial parks in disadvantaged regions provide some examples. This suggests that the traditional focus on the cost of trading goods should be augmented by a focus on the cost of trading ideas. Of course, evaluating the impact of this sort of integration requires a model in which knowledge spillovers matter. Again, augmenting the standard core-periphery model to allow for endogenous is the natural way of doing this.

The final reason is purely academic. The underlying structures of geography and growth models are remarkably similar (e.g., both are based on Dixit-Stiglitz monopolistic competition). It seems important to theoretically explore the connections and similarities between the two literatures that have developed independently.

This paper extends the well-known core-periphery model of Krugman (1991) by introducing endogenous growth of the Romer (1990) type. There are certainly many issues to explore in models that allow location and growth to be jointly determined. As a first step, this paper focuses primarily on stability issues, that is to say, on how the introduction of endogenous growth affects the stability of the symmetric equilibrium. We also briefly consider how agglomeration affects long-run growth. What we show is that in addition to the usual demand-linked and cost-linked circular causality cycles (so-called backward and forward linkages), endogenous growth introduces a growth-linked cycle of circular causality. In the absence of inter-regional knowledge spillovers, this extra centripetal force is strong enough to make the symmetric equilibrium unstable at any level of trade costs. We further investigate the impact of endogenous growth on stability by showing that international knowledge spillovers are a stabilising force. Finally, we show that in our model, geographic agglomeration speeds real income growth in all regions. This opens the door to a possibility that dynamic gains from agglomeration may help offset the well-known static income losses in regions that lose industry.

Our study is related to, but quite distinct from, Martin and Ottaviano (1996). That paper introduces Romer-type endogenous growth into an economic geography model based on the Venables (1996), and Krugman and Venables (1995) models of agglomeration with vertically linked industry. In contrast, our paper works with the Krugman (1991) model with footloose labour. Furthermore, Martin and Ottaviano (1996) focus on illustrating how growth affects location and location affects growth. They do not address stability issues in detail.

The rest of this paper is organised into six sections. Section 2 presents and solves the core-periphery model with endogenous growth using a q-theory approach to trade and endogenous growth models. Section 3 presents the formal analysis necessary to evaluate the stability of the symmetric and core-periphery outcomes. Section 4 considers stabilising and de-stabilising integration. Section 5 looks at the growth effects of agglomeration and its welfare effects. The last section contains a summary and our concluding remarks.

2. Core-Periphery Model with Endogenous Growth

Our dynamic model finds its foundations in the standard core-periphery model of Krugman (1991). To bolster intuition and introduce notation, we briefly review that model without explicitly reporting its equations (our functional form assumptions are made below when growth is introduced).

2.1 The Static Core-Periphery Model

The static model assumes two initially symmetric regions (north and south), two factors of production (workers L and agriculturists A) and two sectors (manufactures X and agriculture Z). Regional supplies of A as well as the global supply of L are fixed, but the inter-regional distribution of L is endogenous with L flowing in response to real wage differences. The monopolistically competitive, increasing returns X-sector employs only L to produce output. Z is a homogenous good produced under perfect competition and constant returns using only A. Units of Z are chosen such that Z's unit input coefficient is unity. Z and X are traded with Z-trade being cost-less, but X-trade being inhibited by frictional (i.e. iceberg) trade costs such that $\tau \geq 1$ units must be shipped to sell one unit in the other region.* The preferences of all citizens are identical; namely upper tier preferences of the representative consumer (in each region) are Cobb-Douglas with μ (a mnemonic for manufactures) as the expenditure share on the X-sector composite. Preferences over X-varieties are given by the standard CES sub-utility function with σ as the constant elasticity of substitution.

* τ-1 is the tariff equivalent of all natural and man-made barriers to trade in goods.

2.2 Adding Growth

Ceaseless accumulation of human, knowledge and/or physical capital is the source of all long-run growth. We must therefore implicitly or explicitly add capital to get endogenous growth. We choose the explicit route by assuming that making each X-variety entails a one-time fixed cost--consisting of one unit of capital K--in addition to the usual variable cost involving only L. The specific (flow) cost function is $\pi + wa_X x_i$, where π is K's rental rate, w is the wage, a_X is the unit labour requirement and x_i is variety i output.

We view capital as new knowledge embedded in a manufacturing facility that is immobile across regions. Using the terminology of the growth literature, our capital is putty-clay as far as location is concerned.

Adding growth also requires introduction of a capital-producing sector--the I-sector, where I is a mnemonic for 'investment goods' (and innovation). K is the output of the perfectly competitive I-sector and we assume that a new unit of capital is made with 'a_I' units of L. To individual I-firms a_I is a parameter, however following Lucas (1988), Romer (1990), and Grossman and Helpman (1991), the I-sector is assumed to be subject to technological externalities. That is, a_I falls as the I-sector's level of production rises.

Formally, the I-sector production function is*:

$$Q_K = \frac{L_I}{a_I}; \quad a_I = \frac{1}{K_{-1} + lK^*_{-1}}, \tag{1}$$

where Q_K is the flow of new capital and L_I is I-sector employment; variables without time subscripts are contemporaneous and those with the subscript "-1" are lagged one period. Given (1), the difference equation for K is:

$$K_t = (1-d)K_{t-1} + Q_K \tag{2}$$

where the amount of depreciation depends upon capital's life span, T.

Adding endogenous growth also requires intertemporal preferences. With ρ>0 as the discount rate, preferences of the infinitely lived representative consumer are:

$$U = \sum_{t=0}^{\infty} b^t \ln C_t; \; C = C_X^m C_Z^{1-m}, \; C_X = \left(\int_{i=0}^{K+K^*} c_i^{1-1/s} \right)^{\frac{1}{1-1/s}}; b = \frac{1}{1+r}; s > 1, 0 < m < 1 \tag{3}$$

where C_Z and C_X are per capita consumption of Z and the CES X composite, and c_i is per capita consumption of X-variety i. Full employment (with one-unit of capital per variety) implies that K+K* is the global number (mass) of varieties, and '*' denotes southern variables. The mass of northern-based workers is L, so regional income is $w_A A + wL + \pi K$, where w_A is the wage of A.

* This functional form, which is from Baldwin and Forslid (1996), is similar to that of Van de Klundert and Smulders (1996)

Following Krugman (1991) and Fujita, Krugman and Venables (1998), we assume that the flow of interregional migration of workers is proportional to 'wage pressure'. Wage pressure in the static framework of Krugman and others is simply the contemporaneous difference in real wages. This, of course, requires workers to have static expectations about future wages, or to ignore future wage changes altogether. In our model we allow for forward looking behaviour in the sense that wage pressure is related to the (log) difference in the present values of real wages.[*] Finally, workers are assumed to migrate in response to the difference in the present value of utility (call this ratio W). The specific law of motion assumed is[†1]:

$$L - L_{-1} = W(L^w - L_{-1}) \tag{4}$$

where W is the shadow value of migration and L^w is the total labour supply. In steady states, W is the present value of the log real wage difference.

Finally, consider way in which θ_K evolves. Given (1) and (2):

$$q_{K,+1} = \frac{(1-d)K_{t-1} + Q_K}{(1-d)K_{t-1} + Q_K + (1-d)K^*_{t-1} + Q^*_K} \tag{5}$$

This expression is significantly simpler when T=∞ or T=1.

2.3 Important Intermediate Results

Utility optimisation yields a constant division of expenditure between X and Z, and CES demand functions for X-varieties. The latter may be written as:

$$c_j = \frac{s_j mE}{p_j}; \quad s_j \equiv \frac{p_j^{1-s}}{\int_{i=0}^{K+K^*} p_i^{1-s}\, di} \tag{6}$$

where s_j is variety-j's market share and E is region-specific consumer expenditure. Optimisation also implies a transversality condition and the Euler equation, $E/E_{-1} = (1+r)/(1+r)$, where r is the rate of return on savings.

On the supply side, free trade in Z equalises northern and southern agriculturists' wage rates (both countries always produce Z). Thus, taking Z as numeraire $p_Z=w_A=w_A^*=1$. As usual, 'milling pricing' is optimal for X-firms, so measuring units such that $a_X=(1-1/\sigma)$ implies that the northern local and export consumer prices are:

$$p = w, \qquad p^* = wt \tag{7}$$

Similar pricing rules hold for southern firms. For convenience, we follow Krugman (1991) and choose units such that $L^w=\chi\mu$ and $2A=\chi(1-\mu)$, so w=w*=1 in the symmetric

[*] There are, of course, other considerations involved in the migration decision that may play a role. Here we stick to a simple formulation in order to focus on essentials.
[†] Numbered notes refer to endnotes that contain more extensive, technical discussions and derivations.

equilibrium and w=1 in the core-periphery outcome. The parameter χ is a scaling factor that permits calibration to an arbitrary growth rate.

Since K is variety specific, K's reward is the operating profit of a typical X-firm. Due to mill pricing, operating profit π is the value of sales divided by σ, where the value of sales is defined to be production at producer prices, or consumption as consumer prices. Thus π can be written in two ways:

$$p = \frac{wL_X}{(s-1)K} = \frac{m}{s}(sE + s^* E^*) \qquad \textbf{(8)}$$

The first and second expressions define sales in terms of production and consumption, s is a typical northern firm's share in its local market, and s* is its share in its export market. Analogous expressions hold for π^*.

Turning to the I-sector, we note that competition implies that K is priced at wa_I.

The market for northern X-varieties must clear at all moments. By symmetry, output per variety is $L_X/a_X K$ and this must equal $s\mu E + s^*\mu E^*$. Exploiting symmetry of varieties and rearranging, the north's aggregate market clearing condition is:

$$\frac{wL_X}{1-1/s} = \frac{w^{1-s}\, mEq_K}{q_K w^{1-s} + f(1-q_K)w^{*1-s}} + \frac{fw^{1-s}\, mE^* q_K}{fq_K w^{1-s} + (1-q_K)w^{*1-s}}; \quad f \equiv t^{1-s} \qquad \textbf{(9)}$$

where $\theta_K \equiv K/(K+K^*)$ is Jonesian share notation for north's share of world K, and ϕ measures the 'free-ness' of trade (ϕ equals zero when $\tau=\infty$ and equals unity when $\tau=1$). This expression, which is an excess-supply-equals-zero condition, is often referred to as the 'wage equation' since it--together with its analogue for the south--determines the market clearing w and w* for any given θ_K.

The north's price index, denoted as P, equals $w_A^{1-\mu}(Kw^{1-\sigma} + \phi K^* w^{*1-\sigma})^{\mu/(1-\sigma)}$ and the south's is analogous.

2.4 Solving for the Long-Run Growth Equilibrium

As a generic property, simple endogenous growth models are marked by a long-run (i.e. steady state) equilibrium in which the sectoral and international division of factors is time-invariant.[*] Exploiting this notion of a 'static economy representation', Baldwin and Forslid (1999) show that solving for the steady-state growth path is very much like solving for the equilibrium resource allocation in a static economy. We employ the same approach here. In particular, characterising the long-run equilibrium requires us to solve for the steady-state allocation of A and L within each region as well as the allocation of L between the two regions. The allocation of A is trivial, due to the core-periphery model's assumptions that A is internationally immobile and sector-specific. The long-run allocation of L is a more challenging problem.

[*] We say 'time-invariant' rather than 'constant' to avoid the impression that the allocation is exogenous.

2.4.1 Two-step Solution Technique

The inter-regional allocation of L depends upon migration and migration is driven by the discounted real wage difference. This difference itself depends upon the division of each region's L-supply between its X and I sectors. The L_I's, in turn, affect θ_K and the L_X's and thus they affect real wages via the w's and the perfect price indices. Given this simultaneity, it proves convenient to solve for the long-run equilibrium in two steps. First, taking L and L* as given, we characterise how the other endogenous variables depend upon L and L*. Second, we identify the L and L* that are consistent with equilibrium in the inter-regional labour allocation, i.e. no migration.

As we shall see, the state of the dynamic system is fully characterised by L, W and θ_K, so we take these as our state variables.*

Given the North's as-yet undetermined level of L, the sectoral division of L is pinned down by finding the equilibrium level of L_I (full employment implies L_X=L-L_I). L_I is the amount of labour devoted to the creation of new K, so it is really nothing more than the level of real investment. Moreover, given that L_I drives the accumulation of the K, we see that characterising the endogenous growth rate boils down to characterising the level of investment in a general equilibrium model. While there may be many ways of determining investment, Tobin's q-approach, introduced by Tobin (1969), is a powerful, intuitive, and well-known method for characterising investment in a general equilibrium model. The essence of Tobin's approach is to assert that the equilibrium level of investment is characterised by the equality of the stock market value of a unit of capital--which we denote with the symbol V--and the replacement cost of capital, which is wa_I in our model. Tobin took the ratio of these, so what might be called the X-sector free-entry condition becomes Tobin's famous steady-state condition $q \equiv V/wa_I=1$.†

Specifically, to characterise the long-run equilibrium, we find the time-invariant L_I and θ_K consistent with q=1. Whatever levels these turn out to be, their time invariance implies two important facts. From (1) and (2), the growth rates of capital--g and g*--are time-invariant, and from the full employment condition the L_X's are time-invariant. Furthermore, the second fact and the constancy of the (yet to be determined) θ_K imply (via the wage equations) that w and w* will be time-invariant.‡ We turn now to q's numerator.

V is, by definition, the present value of the income stream accruing to one unit of capital. Steady state V's are easy to solve for in two cases, T=1 and T=∞. Neither is particularly realistic, but for reasons of algebraic elegance, the latter has been the focus of most of the endogenous growth literature. When T=∞, $V_t=\pi_t/(1-\beta(1+g))$ and the law of

* Using a more elaborate terminology, L and θ_K are state variables while W is a co-state variable.

† Note that the steady-state investment level can also be found by assuming I-sector and X-sector activities are integrated in each firm as in Peretto (1996).

‡ Given the non-linearity of the market-clearing conditions (8), the exact dependency of the w's on θ_K, ϕ and L_X is too complex to be revealing. Be that as it may, the only relevant fact for our analysis is that the resulting w and w* are time-invariant.

motion for K is that $K-K_{-1}$ equals $L_I(K_{-1}+\lambda K^*_{-1})$. For our purposes, however, it turns out simpler to take T=1 so that $V_t=\pi_t$ and K is given by $L_I(K_{-1}+\lambda K^*_{-1})$.*

Utilising (1) to write the replacement cost of capital as wa_I in terms of L_I and (7) to write out $V_t=\pi_t$ in terms of L_I, q=1 implies[2]:

$$L_I = L/\sigma \tag{10}$$

Full employment implies $L_I^*=(L^w-L)/\sigma$.

The second step is to return to the issue of which L and θ_K are logically consistent with a time-invariant inter-regional and inter-sectoral division of labour. W drives changes in L, so we first turn to the expression for W. W is the shadow value of migrating versus not migrating. As usual this is governed by an asset-pricing-like condition. In discrete time, this is:

$$W_{+1} = (1/\beta)W - (1/\beta)\ln\Omega \tag{11}$$

where Ω is the ratio of contemporaneous real wages. Consider next θ_K's law of motion. When T=1—i.e. capital lasts only one period (we think of a period as 10 years each)—then (1), (2), (10) and full employment imply:

$$\theta_{K,+1} = \frac{L(\theta_K + \lambda(1-\theta_K))}{L(\theta_K + \lambda(1-\theta_K)) + (L^w - L)(\lambda\theta_K + 1 - \theta_K)} \tag{12}$$

By definition of a long-run equilibrium, the equilibrium values of L, W and θ_K must be such that L, W and θ_K stop evolving. Solving (11), we see that the stationary value for W is $\ln\Omega/(\beta-1)$. Consequently, migration stops only if W=0 or the core-periphery outcome is reached. W=0, however, when $\ln\Omega=0$ for all future periods. This tells us that any interior long-run equilibrium must be marked by equal real wages. Solving (12) for the steady-state θ_K (i.e., when $\theta_{K,+1}=\theta_K$), implies $(g-g^*)(1-\theta_K)\theta_K/(1+(g-g^*)\theta_K+g^*)=0$. By inspection this implies that any long run equilibrium must be a core-periphery outcome, or an interior equilibrium with equal growth rates. In short, there are only two types of steady states: (i) the core-periphery outcome or (ii) interior steady states with equal real wages and equal growth rates. The symmetric equilibrium plainly satisfies the equal wage and growth condition. There are other interior equilibria that satisfy these requirements but, as we shall see, they are always unstable.

3. Stability Analysis

Implicitly or explicitly, all economic geography models are dynamic models and a key question is the stability of various long-run equilibria. One particularly important issue is the way in which changing trade costs can cause spatial structures to emerge or change.

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* Assuming capital lasts only one period, reduces the number of state variables from five to three. And by taking periods to be rather long, e.g. ten years, the T=1 assumption is no less unrealistic than $T=\infty$.

As we shall see, an entirely distinct issue arises naturally in our model, namely is the way in which changing the cost of trading ideas can affect spatial structures.

3.1 Formal Stability Analysis

Standard stability analysis in the core-periphery model (introduced by Krugman 1991) has a strong intuitive appeal. Starting from the symmetric equilibrium in the canonical Krugman-Venables model, one considers the contemporaneous real wage impact of moving a small amount of labour between the two regions. If this perturbation raises the real wage in the receiving region, the location equilibrium is considered unstable. Otherwise, it is stable. While this intuitive approach has no obvious basis in formal stability analysis, it turns out to be exactly right in simple models.

Unfortunately, in more complex model, such as ours, it is not sufficient in general, so we must rely on a more formal, more elaborate approach (see the appendix of Barro and Sala-i-Martin 1995 for an excellent introduction to these methods).

3.2 Stability of the Symmetric Equilibrium

The three difference-equations (4), (11) and (12) control the dynamics—and therefore the stability—of our model. Since these are non-linear in the state variables L, W and θ_K, we linearise the system about the symmetric steady state and study the stability properties of the resulting equations. The linearised system is:

$$x_{t+1} = J\, x_t, \quad x_t \equiv (L_t - L^W/2, W_t, q_K - 1/2)^T \qquad \textbf{(13)}$$

where J is the Jacobian matrix (i.e. matrix of own and cross partials) evaluated at the symmetric steady state. Specifically, J evaluated at the symmetric outcome is:

$$J = \begin{bmatrix} 1 & m/2 & 0 \\ J_{21} & 1/b & J_{23} \\ 1/m & 0 & (1-l)/(1+l) \end{bmatrix} \qquad \textbf{(14)}$$

where J_{21} equals d(lnΩ)/dL times (-1/β) and J_{23} equals d(lnΩ)/dθ_K times (-1/β); recall that Ω is the ratio of the real wages.

Evaluating the derivatives in J that involve Ω is something of a challenge. Although Ω is clearly a function of L and θ_K, the function cannot be explicitly defined (the wages equations cannot be solved for w and w*, since they involve the power 1-σ, which is, in general, not an integer). For the purposes at hand, however, we only need the derivatives of the function evaluated at a specific point. To get these, we write the excess-supply (i.e. wage) equations and perfect price indices in implicit form, namely, as $XS^i[L, \theta_K]$ and $P^j[w,w^*,\theta_K]$ (j=1,2), where XS is a mnemonic for excess supply. Using these, and noting that Ω≡(w/P)/(w*/P*), we have:

$$\frac{d(\ln\Omega)}{dL} = \frac{1}{w}\frac{dw}{dL} - \frac{1}{w^*}\frac{dw^*}{dL} - \frac{1}{P}\frac{\partial P}{\partial w}\frac{dw}{dL} - \frac{\partial P}{\partial w^*}\frac{dw^*}{dL} + \frac{1}{P}\frac{\partial P^*}{\partial w}\frac{dw}{dL} + \frac{\partial P^*}{\partial w^*}\frac{dw^*}{dL} \qquad \textbf{(15)}$$

where all derivatives are evaluated at the symmetric steady state. We can find dw/dL and dw*/dL by totally differentiating the two wage equations, evaluating the derivatives at the symmetric steady state and then solving for the desired derivatives. Since both P and P* can be written as explicit functions of the w's and θ_K, direct calculation establishes the partials involving the P's. An analogous expression is used to establish $d(\ln\Omega)/d\theta_K$.

Four features of J_{21} and J_{23} are worth noting. First, the major components $d(\ln\Omega)/dL$ and $d(\ln\Omega)/d\theta_K$ are functions of the free-ness of trade, ϕ. Second, $d(\ln\Omega)/dL$ is negative for all ϕ. Migration therefore always has a stabilising effect on the relative nominal wages (i.e. the wages measured in terms of the numeraire). This differs from the static core-periphery model, where d(w/w)/dL switches sign from negative (for high trade barriers) to positive (for low barriers). This difference is easily accounted for. In the static model, changes in L are automatically accompanied by proportional changes in the number of firms (since firms are assumed to instantaneously enter and exit and the cost function in homogenous). In our model, however, L and θ_K can vary independently. The third fact is that $d(\ln\Omega)/d\theta_K$ is positive for all ϕ. Thus we see that raising θ_K always tends to raise the real wage difference and this acts as a destabilising mechanism. The fourth fact is that the strengths of the two effects are affected in opposite ways by a lowering of trade costs. The derivative $d(\ln\Omega)/dL$ tends to approach zero (i.e. gets less negative) as ϕ approaches unity, but $d(\ln\Omega)/d\theta_K$ gets more positive as ϕ approaches unity.

Finding the Eigenvalues and the Stability Test

It is straightforward to analytically find the three eigenvalues of the J matrix. They are, however, the solutions to a third-order polynomial and are therefore quite complex (in both sense of the word). Indeed, even in simple cases such as λ=0 or 1, they are too unwieldy to report (a MAPLE spreadsheet that derives them is available from the authors upon request).

The eigenvalues can, however, be readily evaluated for various values of the underlying parameters (σ, μ and β) and then plotted for all possible policy parameters $0\leq\lambda\leq1$ and $0\leq\phi\leq1$.[*] In our calculations, we take σ=5 μ=1/4 and β=1/2 (this discount factor implies an annual discount rate of about 7% when periods are 10 years); extensive sensitivity analysis (not reported here) shows that the gist of results hold for all reasonable values of the underlying parameters.

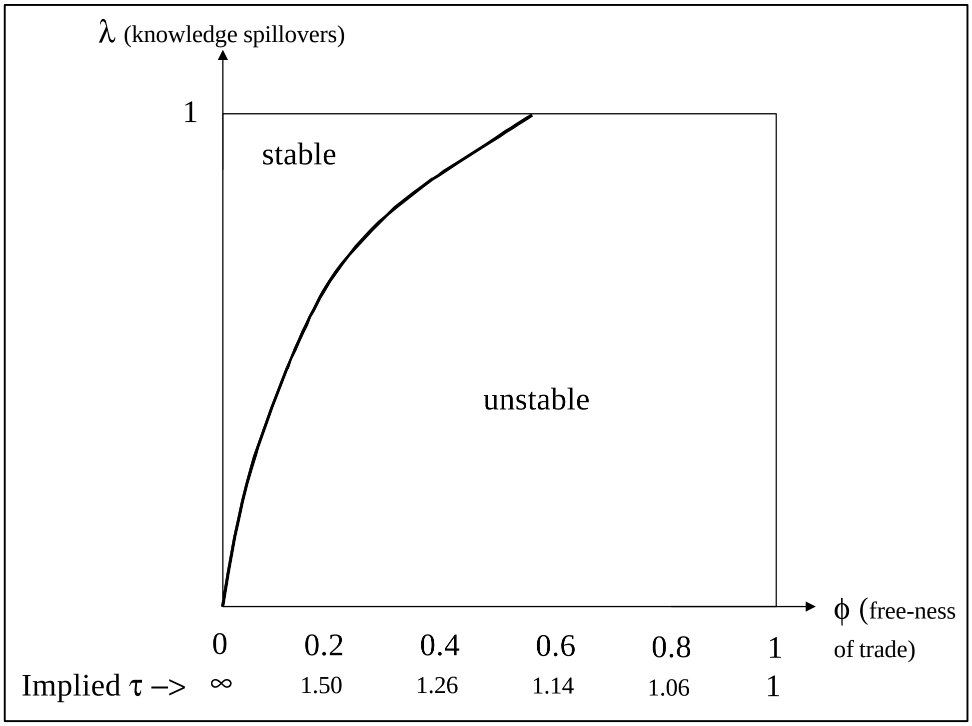
Since only one of our three state variables can jump, stability requires that the real part of at least two eigenvalues are less than unity. In particular, the system is saddle-path stable when two of the three real parts are less than unity (with two real parts less than unity, the system can jump on to the saddle 'path', i.e. stable manifold, from any arbitrary initial condition on the non-jumpers).

Calculation shows the first eigenvalue always exceeds unity, while the real part of the second is always less than unity. The real part of third switches between greater than and less than unity depending upon the policy parameters λ and ϕ.

[*] One advantage to working with ϕ instead of τ is that it yields a compact parameter space.

Since the stability of the system depends only upon the magnitude of the real part of the third eigenvalue, we can fully describe the stability of the symmetric equilibrium by studying the level of the third eigenvalue's real part. This is accomplished in Figure 1 by numerically evaluating the eigenvalue's real part for a fine grid of λ and ϕ values. Given the stability test, we need only show the level curve that corresponds to the knife-edge case where the real part of the third eigenvalue is equal to unity.

Figure 1: Symmetric Equilibrium Stability Map



The first point to note is that endogenous growth is *per se* a de-stabilising force. When $\lambda=0$ (no learning spillovers between regions), the system is always unstable, regardless of the level of trade free-ness.

Intuition for this result is simple. As Grossman and Helpman (1991) show in their 'hysteresis in growth' model, the case of no spillovers implies that any perturbation will cause the relative capital stocks (of initially symmetric nations) to diverge forever. The reason is that the nation with the slight head-start finds that it accumulates I-sector experience faster than the other nation. This lowers the replacement cost of capital--denominator of its Tobin's q--faster and this in turn attracts more resources to the I-sector of the fast-accumulating nation. With love-of-variety preferences, the difference in the rate of K accumulation implies a continual increase in the real wage gaps (in favour of the nation with faster capital growth). Thus, if one allows labour mobility (as in our model), labour would move continuously to the fast-growth region. Moreover, this shift in labour forces would exaggerate the growth rate difference. In short, a growth-linked chain of circular causality would imply that the symmetric equilibrium is unstable for any level of trade costs.

The second point is that knowledge spillovers are a stabilising force. There are two ways to see this. With λ=1 (perfect knowledge spillovers between regions), the system is stable for sufficient low levels of trade free-ness (i.e. for high levels of trade costs). Our findings show that the critical level of ϕ does not vary much with the underlying parameters. For the parameters used in Figure 1, the critical level is about 0.6. With σ=5, this implies at trade cost of about 1.14. Thus, when knowledge spillovers are perfect, the symmetric outcome is stable for even quite low trade costs (recall that τ reflects all costs of selling goods at a distance, not just transport costs). Second, as is clear from the figure, the range of ϕs for which the system is stable expands as λ rises. In this sense, knowledge spillover is a stabilising force that goes a long way to countering the de-stabilising effects of growth.

The intuition for this result is also uncomplicated. The production externalities in the I-sector (which are necessary for growth) create their own circular causality encouraging agglomeration. The strength of this force, however, depends upon the extent to which the externalities are localised. As λ rises to unity, the growth-linked agglomeration force disappears.

3.3 Stability of the Core-Periphery Equilibrium

It is simpler to study the stability of the core-periphery (CP) outcome since the perturbation method is perfectly valid in this case. This validity is easily seen. With some work, we find that the three eigenvalues for the CP equilibrium are (1-W, 1/β, 0).[*] Clearly, the equilibrium is stable only if W>0 (recall that stability requires two eigenvalues to be less than unity). Since W>0 if and only if the relative real wage is greater than unity, we can conduct our analysis by asking how a small migration shock would alter the relative real wage.

If the CP equilibrium is to be unstable then it must be that a small group of migrating workers will find it worth their while to build new capital in the periphery. Using the Section-2 reasoning, we know that of the dL* migrants, dL*/σ will work in the I-sector and the rest in the X-sector. As usual, q*=1, i.e. $\pi^*=w^*a_I^*$, must hold if they are optimising. Employing (6), this means that $w^*/\lambda K_{-1}$ equals $(\mu w^{*1-\sigma}/\sigma K)(\phi E/\Delta+E^*/\Delta^*)$ where Δ is defined as $(1+(K^*/K)\phi\, w^{*1-\sigma})$, Δ^* is defined as $(\phi+(K^*/K)\, w^{*1-\sigma})$, and K* and K are given by $(dL^*/\sigma)(\lambda K_{-1})$ and $(L/\sigma)(\lambda K_{-1})$, respectively. Taking the limit as dL* approaches zero and rearranging, the incipient w* in the CP equilibrium is $w^*_{CP} = ((lm/L^w)(f(L^w+A)+A/f))^{1/s}$. Furthermore, if the CP equilibrium is unstable it must be that the small vanguard of migrants earns a higher real wages than their core-based colleagues. This real-wage condition is $w^*_{cp}>\phi^{\mu/(1-\sigma)}$, so with some manipulation our stability test is:

$$l\,(f^2(1+m)+1-m) \le 2f^{1-m/(1-1/s)} \qquad \textbf{(16)}$$

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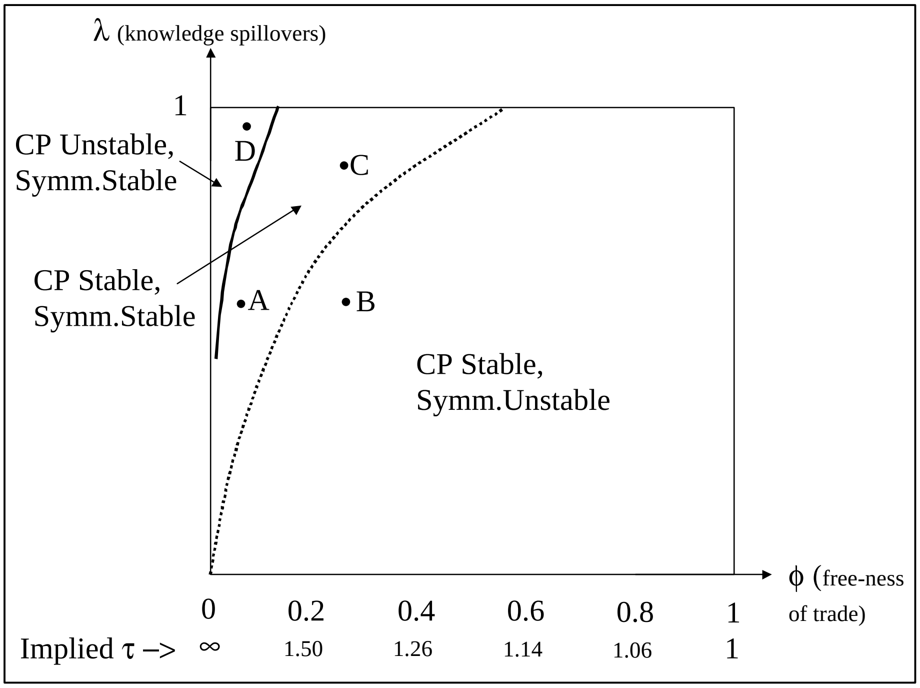
[*] The MAPLE worksheet stab_CP.mws showing this is available from the authors.

As with the symmetric equilibrium, we can map this stability condition in (λ,ϕ) space (as noted above, this is identical to mapping the third eigenvalue of the Jacobian

Figure 2: Core-Periphery Stability Map



evaluated at the CP equilibrium). The result is Figure 2 (this assumes the same values of the underlying parameters as the previous figure). The dividing line between the stable and unstable regions is defined by the combinations of λ and ϕ where the real-wage condition is zero. The heavy solid line shows this by plotting the zero-level curve of the w^*_{cp}-$\phi^{\mu/(1-\sigma)}$ function. What we find is that for a large portion of the (λ,ϕ) space, the core-periphery outcome is stable. For instance, consider λ=0. Here, starting up K production in the periphery is prohibitively expensive, so the CP outcome is stable for all level of trade free-ness. At the other extreme, when λ=1 the CP outcome can be unstable, but only for very high level of trade costs.

Note that increasing λ expands the range of instability (in terms of trade free-ness). Thus, spillovers increase the range of stability for the symmetric equilibrium, but widens the range of instability for the CP outcome. In this sense, we can say that endogenous growth is an agglomeration force, but knowledge spillovers are a dispersion force. Intuitively, spillovers are de-stabilising for the CP outcome since mitigate the force of 'growth-linked' circular causality discussed above.

For comparison, we have reproduced the knife-edge level curve for the symmetric equilibrium in Figure 2 (shown by the dotted locus). The two level curves partition the policy space, viz. λ and ϕ space, into three regions. In small, northwest region, where λ is high and ϕ is low, the CP outcome is unstable but the symmetric equilibrium is stable. In the middle region both equilibria are stable and in the large, southeast region, only the CP steady state is stable.

Plainly, the middle region corresponds to what FKV call the ‘tomahawk diagram’—i.e. a situation of overlapping stability. That is, if we took the level of λ as given and investigated only how changes in trade costs affect stability, we would find that for some levels of ϕ, both the symmetric and CP outcomes were stable.

4. Stabilising and Destabilising Integration

The traditional axis of investigation in economic geography models focuses on the cost of selling goods at a distance. Lowering the cost of trade in goods, however, is only one aspect of integration.

Europe, for instance, is much more closely integrated now than it was twenty years ago. Some part of the this is due to greater trade in goods (fostered by lower trade costs), but much of this comes in the form of what European know about each others’ cultures, businesses and technology. Examples abound. Many more Europeans are able to receive foreign TV channels and speak a foreign language. Business and personal travel has increased enormously and cross-country educational exchanges are common. More directly relevant, however, has been the rapid expansion of intra-European merger and acquisitions and the remarkable rise in intra-European foreign direct investment FDI. Such undertakings clearly reduce the localisation of commercially relevant knowledge, such as product and process innovations. There is some evidence that this knowledge-sharing aspect of integration has outpaced the increase in goods trade. The European Commission, for instance, reports that the Single Market Programme induced intra-EU FDI to expand seven times faster than intra-EU trade.

Finally, encouraging knowledge spillovers are frequently the object of explicit policy. Much of EU research spending, for instance, is explicitly linked to the goal of fostering intra-European knowledge exchanges. The EU’s Human Capital and Mobility and Training and Research Mobility programmes, for example, funded research networks the involved institutions in a several EU nations and paid for EU post-graduate students to work in other EU nations). Additionally, national and EU regional policies are often aimed at encouraging the inflow of knowledge to disadvantaged regions (e.g. via regional universities) and the inflow of people with knowledge (e.g. via high-technology industrial parks).

In short, the cost of trading ideas has also fallen along with the cost of trading goods. This suggests that the traditional focus on goods costs should be augmented by a focus on the cost of trading ideas. In our model we take ϕ as a measure of the former and λ as a measure of the latter.

Given Figure 2 and this mapping between real-world integration and our policy variables, the assertion that integration may be stabilising or destabilising is straightforward. A purely trade-cost reducing integration policy encourages agglomeration. A policy that moved the world from, say point A in Figure 2 to point B would result in extreme agglomeration. By contrast, a policy that lowered the cost of trading both goods and ideas reduces the risk of extreme agglomeration. For example, a policy combination that raised both ϕ and λ, taking the world from point A to point C would result in gains from trade without delocalisation. Indeed, an integration policy that raises learning spillovers sufficiently can, in some case, lead to a dispersion of economic activity. For example, if the world started out in a CP situation at A (recall A is consist with either a CP or a symmetric equilibrium) and λ were raised enough to move the economy to D, integration would result in a radical dispersion of industrial activity.

5. Growth, Agglomeration and Welfare

Due to the localised nature of learning externalities, agglomeration is pro-growth. Moreover, the presence of I-sector externalities also ensures that laissez-faire growth is sub-optimal. This raises the interesting possibility that the pro-growth aspect of agglomeration will mitigate and might entirely offset the static welfare loss experienced by periphery agriculturists.[*]

More specifically, real income and consumption growth rates are identical in both regions in either the CP or symmetric configuration. This statement is obvious for the symmetric case. It holds for the CP case since once the CP outcome is obtained, the perfect price index in both nations will fall (thereby yield rising real incomes) at the same rate. The rate of rise will be $\mu/(\sigma\text{-}1)$ times the growth rate. Importantly, however, even though the slopes of the core's and the periphery's price-index paths are identical, the periphery's path is permanently higher, as long as $f < 1$, due to the usual variety effect first explored by Venables (1987).

Plainly many welfare exercises suggest themselves, for example tracing out the welfare of a particular integration path. However to run a very clean comparison and keep the analytics simple, we consider a very stark policy experiment. That is, we consider a policy that forces the symmetric equilibrium to switch to a CP equilibrium without changing ϕ or λ. For this to be possible, the world must initially find itself in the overlapping stability region where there are two perfectly valid long-run equilibria, one in which industry is equally divided and one in which industry is concentrated in the core. It is well known that in the static core-periphery model moving from the symmetric to CP outcomes is always good for the mobile factor, always good for agriculturists in the core, but always bad for the agriculturists in the periphery (since they must pay higher prices for their X-sector consumption). When the agglomeration also changes the growth rate, however, the outcome is ambiguous. In particular, it would seem possible that the dynamic gain more than compensates periphery-based agriculturists for their static losses. In this case the catastrophic agglomeration would be a Pareto improving transformation.

We turn now to evaluating this possibility in our model by comparing steady-state utility levels

5.1 Welfare Impact of Agglomeration: Static and Dynamic Effects

Whenever, the economy is forever in a single long-run equilibrium, the present value of an agriculturist's utility flow is[†]:

[*] Of course the first-best policy would be free trade in X and a production subsidy to the I-sector in the core country. In this case, agglomeration would be unambiguously welfare improving for both regions.
[†] This is simply the present value of E_A/P, where E_A is the expenditure of a typical agriculturist. The fact

$$U_A = \ln(\frac{A}{P_0}) - \ln\left(1 - b\,(1+g)^{m/(s-1)}\right) \qquad \textbf{(17)}$$

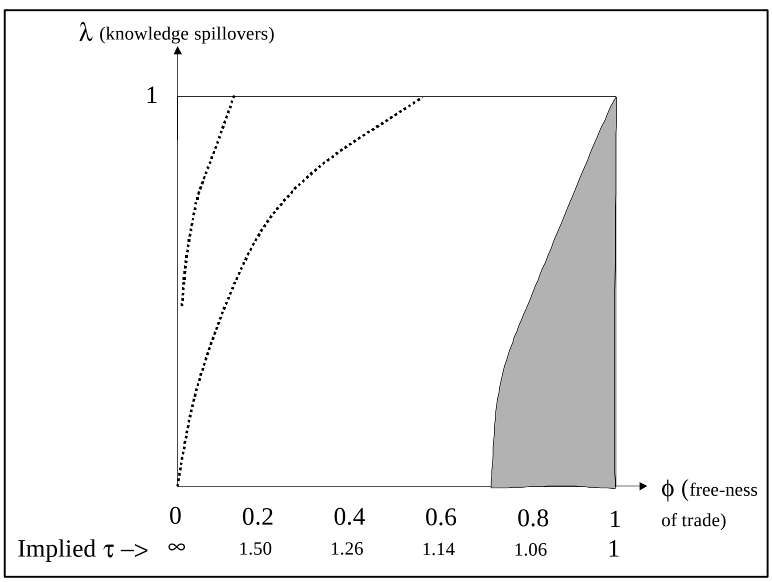
The first term in this expression captures static effects, i.e. one-time changes in income or prices. The second term captures dynamic effects, i.e. changes in the rate at which the price index falls.

To focus tightly on key issues, we consider the U_A for an agriculturist based in the south, i.e. the region that has no industry in the CP equilibrium. The symmetric equilibrium price index, call it P_o^S equals $(K_o^w(1+\phi)/2)^{\mu/1-\sigma}$, where K_o^w is the period zero level of the world capital stock, i.e. the number of varieties produced world wide. In the CP equilibrium, the corresponding index, P_o^C, equals $(K_o^w\phi)^{\mu/1-\sigma}$. The growth factors, i.e. 1+g, in the symmetric and CP equilibria are $L^w(1+\lambda)/2\sigma$ and L^w/σ, respectively. Using these facts in (17), we have:

$$U_A^C - U_A^S = \frac{-m}{s-1}\ln(\frac{1+f}{2f}) + \ln\left(\frac{1 - b\,(1+L^w(1+l)/2s)^{m/(s-1)}}{1 - b\,(1+L^w/s)^{m/(s-1)}}\right) \qquad \textbf{(18)}$$

The first term shows the static welfare effect (this is unambiguously negative) and the second term shows the dynamic welfare effect (this is strictly positive when $\lambda<1$). The lower is λ, the larger is the dynamic gain and so the more likely it is that the (18) is positive. Similarly, raising ϕ lowers the static loss to periphery agriculturist and so makes a gain more likely.

Figure 3: Dynamic Gains from Agglomeration



From (18) it is clear that there is some combination of ϕ and λ where periphery agriculturists are just indifferent to agglomeration. A convenient way to characterise this

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that A= E_A is not obvious. It follows from the fact that in this model, investment spending is exactly equal to capital's income.

line is to simply plot the zero level curve of (18) in ϕ, λ space, using the same underlying parameter values employed in Figures 1 and 2. The result is shown in Figure 3.

Interestingly, the range of ϕ and λ where the dynamic effect is large enough to outweigh the static loss—shown by the shaded area--does not overlap with the stability region for the symmetric equilibrium. This finding, which is robust to sensitivity analysis on μ, σ and β, is not unexpected. For the dynamic effect to be large, λ must be low, however when λ is low, symmetry is unstable since we approach the Grossman-Helpman hysteresis-in-growth world. For (18) to be positive for high values of λ, the level of ϕ must be very close to unity (to keep the static loss low). Again in this case, the symmetric equilibrium is unstable since the model approaches the standard static core-periphery model.

This lack of overlap suggests the compensating growth effects can only mitigate the periphery's loss. That is, any small change in λ and ϕ that switches the economy from a symmetric to asymmetric outcome will unambiguously harm residents of the periphery. Moreover, once the total agglomeration has occurred, further changes in λ have no impact on the welfare of any consumer.

6. Summary and Concluding Remarks

Using the q-approach to trade-and-endogenous growth models developed in Baldwin and Forslid (1999), this paper shows that it is relatively simple to introduce endogenous growth into the standard Krugman (1991) core-periphery model with footloose labour. The resulting model can be thought of as a marriage between the Krugman economic geography model and a Romer-Grossman-Helpman endogenous growth model. The paper shows that growth can be a powerful destabilising force. We also show that inter-regional learning spillovers are a stabilising force.

Including endogenous growth explicitly also allows us to take a more subtle view of integration. Traditionally, integration is viewed simply lowering the cost of trading goods. Many aspects of integration, however, might more naturally viewed as lowering the cost of trading information, i.e. as reducing the extent to which learning externalities in growth sectors are localised. We show that this form of integration tends to be stabilising. This contrasts sharply with the traditional analysis that views integration as inevitably destabilising. This expansion of the research agenda is welcomed since many real-world regional policies—e.g. encouragement of regional universities, subsidisation of high-technology industrial parks in disadvantaged regions—seem aimed at combating the localisation of learning externalities in the core.

Finally we show that agglomeration of industry is favourable to growth. Since laissez-faire growth is socially sub-optimal in the symmetric equilibrium, agglomeration produces a welfare-enhancing pro-growth effect. We show that this can mitigate loses to residents of the periphery, but it cannot turn catastrophic agglomeration into a Pareto improvement.

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Supplemental Guide to Calculations

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[1] We can justify the migration equation as follows (we work in continuous time to allow for more compact expressions; the conversion to discrete time is obvious). Consider a southern household that is considering sending some labour to the north to maximise its real earnings. The problem is faces is to the migration rate in order to solve the following problem:

$$\max \int_0^\infty e^{-rt} \ln\left(Lw + (1-L)w* - gm^2 / 2(1-L)\right) dt$$

where we have assumed migration costs are quadratic in the rate of migration (as a proportion of the sending population) and $m = \dot{L}$.

The current valued Hamiltonian is $e^{-rt} \ln\left(s_L w + (1-s_L)w* - gm^2 / 2\right) + e^{-rt} Wm$ and the necessary conditions are:

$$m = \dot{L} = W(1-L)/g \,,$$

$$\dot{W} = rW - \ln(w - w*)$$

Aggregating over household yields a law of motion similar to the one in the text.

[2] With (1) and (7), q=1 can be written as:

$$\frac{w(L - L_I)/K(s\ -1)}{w/(K_{-1} + lK^*_{-1})} = 1$$

Realising that 1+g=K/K_{-1}, using the growth form of the I-sector production function, namely, (1+g)=$L_I(1+\lambda(1-\theta_K)/\theta_K)$, and simplifying, we get the expression in the text.