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THE STRATEGIC USE OF TYING TO PRESERVE AND
CREATE MARKET POWER IN EVOLVING INDUSTRIES

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Working Paper 6831
<http://www.nber.org/papers/w6831>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1998

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ABSTRACT

This paper investigates how the tying of complementary products can be used to preserve and extend monopoly positions. We first show how a firm that is a monopolist of a product in the current period can use tying to preserve its monopoly position in future periods. We then show using related arguments how a monopolist in one market can employ tying to extend its monopoly position into a newly emerging market. The analysis focuses on the importance of entry costs and network externalities. The paper includes a discussion of antitrust implications.

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I. INTRODUCTION

Tying is a common practice in many markets, i.e., the seller of product A refuses to sell product A to a consumer unless the consumer also purchases B (in this scenario product A is referred to as the tying product and B as the tied product). Examples are numerous such as IBM's famous practice of requiring purchasers of IBM's tabulating machines to also purchase tabulating cards from IBM, and Microsoft's more recent attempts to bundle Internet Explorer with Windows. Due to the ongoing battles between Microsoft and the U.S. Justice Department concerning Microsoft's practices, the motivations and rationales behind tying arrangements have become the subject of both public policy and academic debates. In this paper we use dynamic models to show that a firm that is currently a monopolist in its primary market can use tying to preserve and extend its monopoly position by deterring future entry into both the primary market and related markets. We call this dynamic motive for tying "strategic tying of complementary products."¹

Most previous analyses of tying have not focused on the ability of tying to enhance a monopolist's market power in its primary market and related markets, but instead have focused either on the ability of tying to achieve price discrimination or its ability to foreclose competition in the tied market.² A classic analysis in the price discrimination vein is that of metered sales. In this argument consumers vary in terms of the quantity of the tied good demanded, where high valuation consumers are assumed to have a high demand for the tied good while low valuation consumers have a low demand. The argument is that by tying and charging a high price for the tied good the monopolist is able to extract more of the surplus from the high valuation/high demand consumers. This is the standard interpretation for why IBM required consumers of its machines to also purchase cards from IBM.

The foreclosure argument is quite different. One variant of this argument is that the monopolist of one product increases its profits by earning monopoly profits in the now monopolized tied market. This argument was for a long time quite controversial because many believed that the monopolist need not monopolize the tied market to earn all the potential monopoly profits (see, e.g., Director and Levi (1956), Bowman (1957), Posner (1976), and Bork (1978)). In an important recent paper, however, Whinston

¹One of the authors (Carlton) has worked for Sun Microsystems in its lawsuit against Microsoft which alleges breach of contract and antitrust violations. The opinions expressed here are those of the authors alone.

²Other justifications for tying include increasing efficiency in the presence of variable proportions, avoiding price regulations, giving secret price discounts, and quality assurance. See Carlton and Perloff (1994) for a discussion of these other rationales.

(1990) has shown that criticisms of the foreclosure argument depend on the tied market being characterized by perfect competition and constant returns to scale and, given economies of scale and imperfect competition, tying can increase monopoly profitability.

In most of Whinston's analyses tying is used to induce exit in the tied market, and the subsequent lack of substitute producers in the tied market enables the firm to increase its current profits in that market. For example, suppose that a hotel on a resort island requires its guests to eat their meals at the hotel restaurant. Local residents may then have fewer choices and be forced to also frequent the hotel restaurant. In this case tying is profitable because it eliminates competition in the other market.³ Whinston does consider one setting, however, in which tying is used to increase the firm's profits in the initially monopolized market. In this analysis there is a competitively supplied inferior substitute for the firm's initial monopoly product, and products in this market and the potentially tied market are complementary. The result is that tying and inducing exit in the tied market can be profitable because it eliminates the competitively supplied inferior product as a potential substitute.⁴

In this paper we build on and extend Whinston's important work. Our analysis is related to Whinston's in that we also focus on tying and foreclosure. However, we do not concentrate on the monopolist's ability to use tying and foreclosure to increase current profitability in the tied market, or to eliminate inferior substitutes in the initially monopolized market. Rather, we use dynamic models to concentrate on the monopolist's ability to use tying and foreclosure to increase future profits by deterring entry of efficient firms into the monopolist's primary market and related markets. It is the strategic use of tying to deter the entry of efficient firms that raises the most interesting and difficult public policy issues. In our argument there is a complementary good characterized by intertemporal economies of scope, which makes it more profitable for the alternative producer to be active in the complementary market in the second period if it was also active in that market in the first period (as we discuss, intertemporal economies of scope result from many simple and standard assumptions in the literature). We show that, because of such scope economies, tying a complementary product can deter entry into the monopolist's primary market and related markets. Indeed, our work suggests that intertemporal economies of scope has not received the

³We thank R. Gertner for this example.

⁴See Ordover and Willig (1981) and Ordover, Sykes, and Willig (1985) for analyses related to Whinston's paper.

attention it deserves in the entry-deterrence literature.⁵

Our first argument is that tying can be used to preserve a monopoly position. We begin with a two-period setting in which a firm operates in both its primary market and a market for a complementary good. In the first period the firm is a monopolist in the primary market, say due to patenting, but there is the potential for entry in the second period. The complementary good, on the other hand, can be produced both by the monopolist and another producer, where the alternative producer faces a cost of entering the complementary market. We consider a specification in which the monopolist has no incentive to tie if there is no threat of entry into the primary market, but does have such an incentive when entry into the primary market is possible. The logic is that the entry cost for the complementary good creates an intertemporal economy of scope, and as a result it is less profitable for a rival to enter the primary market in the second period when entry requires entering both markets rather than one.

In addition to the two-period entry-cost analysis just described, we also consider the monopoly preservation role for tying in a network externalities analysis and a T-period analysis. In the first analysis we show that network externalities are similar to intertemporal economies of scope, so that we obtain similar results when the alternative producer faces no entry costs for the complementary good but the demand side of the market is characterized by network externalities. In the T-period analysis we again assume entry costs for the complementary good rather than network externalities, and show that the results of the two-period analysis described above generalize to a T-period setting when the monopolist can introduce new versions of the primary and complementary products every other period.

In addition to Whinston (1990), the above argument is related to some other earlier papers which show that tying and related activities can sometimes allow a firm to preserve and/or enhance monopoly positions. For example, Williamson (1979) argues that tying can reduce the probability of entry if the potential entrant only has experience relevant for producing one of the goods. His logic is that, if the potential entrant lacks experience in one of the products, then tying can inhibit entry because it forces the firm to enter both markets which given its inexperience in one of the markets results in a higher cost of capital. Another related analysis appears in Comanor (1967) which considers vertical mergers used for

⁵Papers in this literature include Spence (1977), Dixit (1979, 1980), Milgrom and Roberts (1982), and Fudenberg and Tirole (1984).

foreclosure rather than tying.⁶ In Comanor's argument merging deters entry by causing entry to occur in two markets simultaneously, where two-stage entry is difficult because higher capital requirements serve as a barrier to entry.

Our second argument is that tying sometimes allows a monopolist to acquire a monopoly position in a newly emerging market. We use two different approaches to show this result. The key is that there are complementary links between the primary and newly emerging markets. In our first approach the newly emerging market is associated with the same complementary product as the primary market. In that case, because of the entry cost associated with the complementary product, tying primary and complementary goods lowers the profitability of a rival entering the newly emerging market in much the same way that it lowered the profitability of a rival entering the primary market in the above discussion. The conclusion is that tying primary and complementary goods can result in the firm monopolizing the newly emerging market by lowering the other producer's return to entering that market.

In our second approach the primary and newly emerging markets are initially complementary but the newly emerging market eventually makes the primary market obsolete. To develop the argument, we assume a monopolized primary market and a newly emerging market characterized by network externalities, where the two products are complements in the first period but new versions of the newly emerging market product make the monopolist's primary product obsolete in the second period. We show that by initially tying the two goods, the firm can establish a monopoly position in the newly emerging market and thus retain its monopoly profits even after the primary product becomes obsolete. At the end of this analysis, we relate this argument to arguments put forth in the recent Justice Department case against Microsoft.

Throughout the paper, we frequently refer to a firm tying two products without distinguishing whether the tie is achieved through contract or through product design. In real world settings there are examples of both types of ties. For example, in the original 1936 IBM case, purchasers of IBM's tabulating machines were required to also purchase their tabulating cards from IBM -- a tie achieved through contracting. In contrast, in the later 1970s case, there was an allegation that IBM's new central processing unit was interface incompatible with the plug-in components of rivals -- a tie achieved through product design. Similarly, in a 1983 case, Kodak was accused of designing its camera and film so as to

⁶Other papers concerning vertical mergers and foreclosure include Salinger (1988), Hart and Tirole (1990), and Ordovery, Saloner, and Salop (1990).

achieve incompatibility with rivals' products. We believe our arguments apply similarly to both types of ties, so we do not distinguish between the two cases as we proceed through the various theoretical models. We do distinguish between the two cases in our discussion of antitrust implications, however, because the importance of alternative efficiency rationales for tying may vary across the two types of ties.

In addition to ties achieved through contracting and product design, we also consider the possibility that a monopolist can achieve a virtual tie through pricing. For example, suppose the primary and complementary products are used in fixed proportions. Then a monopolist of the primary product can achieve a virtual tie by setting a high price on the primary product and a very low price (say zero) on the complementary product. This achieves a virtual tie since alternative producers of the complementary product cannot operate profitably given the very low price charged for this product by the monopolist. We discuss the situations in which a monopolist may employ virtual as opposed to real ties, and show in particular why a virtual tie may be used in settings characterized by network externalities, but not in settings characterized by entry costs for the complementary good.

The outline for the paper is as follows. Section II looks at a two-period setting, and shows that a monopolist in a primary market can sometimes use a tie with a complementary product to preserve its monopoly in the primary market. Section III uses related arguments to show that tying can sometimes be used to extend a monopoly position into a newly emerging market. Section IV demonstrates the monopoly preservation role for tying in a T-period setting. Section V discusses antitrust implications of our analysis. Section VI presents concluding remarks.

II. PRESERVING MONOPOLY THROUGH STRATEGIC TYING OF COMPLEMENTARY PRODUCTS

In this section we show how a monopolist can strategically use the tying of complementary products to preserve an initial monopoly position. We first show how this works in a two-period setting characterized by entry costs for the complementary good, and then derive similar results when there are no entry costs for the complementary good but the demand side of the market is characterized by network externalities. Our focus is on the ability of tying to make an alternative producer's future production of the complementary good less profitable, which because of complementarity deters future entry into the primary market.

A. A Two-Period Model

The monopolist is the sole producer in the primary market in period 1, say due to patenting, while there is the potential for entry into the primary market in period 2 by a single alternative producer.⁷ The monopolist and the primary market's alternative producer have the same constant marginal cost for producing the primary good, denoted c_p . There is a complementary good that can be produced by the monopolist and a single other firm, where the monopolist and the complementary market's alternative producer have the same constant marginal cost for producing the complementary good, denoted c_c . Also, as described in more detail below, there is a sunk cost associated with entry into each market, and firms engage in Bertrand competition when more than one firm is active.

We assume that the primary market's single alternative producer and the complementary market's single alternative producer are the same firm. Without this assumption, an entrant into the primary market would lose money under the model's assumptions of constant marginal costs in the primary market, positive entry costs, and Bertrand competition. Alternatively, we could assume two different potential entrants and allow for payments between the firms. Our choice is to assume a single potential entrant that can produce both products since this avoids the need to describe the bargaining process that would determine the size of such payments. The results that follow would be qualitatively unchanged if we assumed two different potential entrants each having the ability to produce only one product and Coasian-type bargaining.

The alternative producer has an entry or R&D cost associated with producing the first unit of the primary product, denoted E_{ap} , while its entry cost for producing the first unit of the complementary product is denoted E_{ac} . The monopolist has entry costs for the primary and complementary products, although we assume these costs are sufficiently small that the monopolist always incurs both costs in the first period. This allows us to focus on the entry decisions of the alternative producer. We denote the sum of the monopolist's entry costs as E_m . Note, for both the primary and complementary markets we could add to the analysis fixed costs in addition to entry costs, but this would not change the qualitative nature of the results.

Consumers purchase at most one unit of each good which eliminates any variable proportions rationale for tying. A primary unit can be used either by itself or in combination with a complementary unit, while a complementary unit cannot be used by itself (e.g., a computer and a printer). We refer to a

⁷The analysis that follows is also consistent with there being a pool of alternative producers, where the focus is on pure-strategy equilibria.

primary unit and complementary unit consumed together as a system. Consumers are indifferent between a unit of the primary good produced by the monopolist and a unit produced by the alternative producer. In contrast, in order to address the role that tying can play in deterring entry, we assume that consumers prefer the alternative producer's version of the complementary good (if consumers exhibited indifference concerning both the primary and complementary products there would never be entry in this model).⁸

To be precise, a consumer derives a gross benefit from a primary unit by itself equal to V' , he derives a gross benefit from a system in which the complementary good is produced by the monopolist equal to V , while his gross benefit from a system in which the complementary good is produced by the alternative producer is $V+\Delta$, where $V-V'>c_c$ and $V'-c_p>\Delta/2$.⁹ The restriction $V-V'>c_c$ ensures that the monopolist would sell complementary units if there was no alternative producer. The restriction $V'-c_p>\Delta/2$ ensures that the primary market monopoly is more valuable to the monopolist than the potential benefits associated with having the alternative producer offer its higher quality complementary product. We assume there are two cohorts of N identical consumers. Consumers in cohort 1 are in the market in period 1 while consumers in cohort 2 are in the market in period 2 (to simplify the analysis, consumers in cohort 1 are assumed not to be in the market in period 2 even if they do not purchase anything in period 1). We assume there is no discounting by both the firms and consumers, although incorporating discounting would not materially change the results.

In the beginning of the first period the monopolist decides whether to offer a tied product consisting of one unit of its primary and complementary goods or whether to offer the products individually.¹⁰ This decision is binding for both periods 1 and 2 (footnote 14 discusses how the results change if the monopolist's product choices for period 2 are decided at the beginning of period 2). Following Whinston, we assume that if the two goods are tied, a consumer cannot undo the tie (if $\Delta\leq c_c$, then the results are

⁸As long as consumers do not prefer one version of the primary good much more than the other, the qualitative nature of the results would be unchanged by allowing consumers to either prefer the alternative producer's version of the primary good or the monopolist's primary good (see the end of this section for a related discussion). The qualitative nature of the results would also be unchanged if we assumed that consumers were indifferent between the monopolist's and alternative producer's versions of the complementary good, but the alternative producer had a lower marginal cost of production for the complementary good.

⁹Under the alternative specification that there is a pool of alternative producers (see footnote 7), all that is required is that a subset of the alternative producers can produce a superior complementary product.

¹⁰There is no reason for a firm to offer both tied and individual products in this model because consumers are identical rather than heterogeneous. See Adams and Yellen (1976) for an analysis in which consumers are heterogeneous and firms sometimes offer both tied and individual products to better price discriminate.

unchanged under the assumption that consumers can undo ties). That is, if a consumer purchases a tied good consisting of one unit of the monopolist's primary good and one unit of its complementary good, then the consumer cannot purchase a unit of the complementary good from the alternative producer and create a system consisting of the monopolist's primary good and the alternative producer's complementary good. This means that if the monopolist offers only a tied product, then in the first period the alternative producer will not be able to sell any units of the complementary good. In contrast, in the second period the alternative producer would not be locked out of the market because it can produce both products.

In the first period, if the monopolist decides to offer its primary and complementary goods as individual products, then the alternative producer must decide whether or not to enter the complementary market. If the monopolist offers its primary and complementary goods as individual products and the alternative producer enters, then prices are determined by Bertrand competition (footnote 21 discusses how the results change if in the first period the monopolist's product choice and pricing decisions are simultaneous). In the second period, the alternative producer decides whether or not to enter the primary market and, if it did not enter the complementary market previously, whether or not to enter the complementary market. Since in this model there is no incentive for the alternative producer to tie, to simplify the exposition we assume that when the alternative producer is in both markets in the second period it offers individual products. As in the first period, if both firms are active in the second period then prices are determined by Bertrand competition. Finally, we restrict attention to pure-strategy subgame-perfect Nash equilibria.

In this model, Bertrand competition will not typically result in a unique set of prices. To see this, suppose the alternative producer has entered the complementary market in period 1. One equilibrium set of prices in period 1 is that the monopolist charges $V - c_c$ for its primary product and the alternative producer charges $\Delta + c_c$ for its complementary product (here and in the following set of equilibrium prices, consumers purchase the complementary good from the alternative producer as long as the monopolist charges more than c_c for its complementary product). In this equilibrium the alternative producer receives all the surplus associated with consumers preferring its version of the complementary product. However, another set of equilibrium prices is the monopolist charges $V + \Delta - c_c$ for its primary product and the alternative producer charges c_c for its complementary product. In this equilibrium the monopolist receives all the surplus associated with consumers preferring the alternative producer's version of the complementary good. In fact, any division across the two sellers is consistent with equilibrium.

In our analysis, we assume the prices that emerge divide evenly across the two sellers the surplus associated with consumers preferring the alternative producer's version of the complementary good.¹¹ The same qualitative results would follow from any division that gave each firm a strictly positive proportion of the surplus, but the results would not follow if the surplus was either all received by the monopolist or all received by the alternative producer. In the former case entry into the complementary market in the first period would be associated with negative profits, and thus the monopolist would not need to tie to deter first-period entry. In the latter case, if the alternative producer were to enter the primary market in the second period after entering the complementary market in the first, there would be no increase in the proportion of the surplus received by the alternative producer. The result is that the alternative producer would never enter the primary market.

B. Analysis

As a result of our assumption that it faces low entry costs, the monopolist enters both markets in the first period. Our focus is on the entry decisions of the alternative producer, and the extent to which the monopolist tries to affect these decisions by offering a tied product. Our main result is that the monopolist in the first period will sometimes use tying to deter entry by the alternative producer into both the primary and complementary markets. This strategy increases the monopolist's profitability by preserving its monopoly in the primary market in the second period.

We begin with a benchmark analysis in which the alternative producer cannot enter the primary market in either the first or second periods (a simple interpretation is that $E_{ap} = \infty$). In this benchmark analysis, however, there is still the possibility that the monopolist will use tying to deter entry into the complementary market, although we will show that the monopolist does not have an incentive to deter entry in this case. By comparing the results in this case with what happens when the alternative producer can also enter the primary market, we are able to more clearly show the role that preserving its monopoly position in the primary market plays in the monopolist's decision concerning whether or not to tie.

Suppose the alternative producer can never enter the primary market. In that case the monopolist can deter the alternative producer from ever entering the complementary market by offering a tied product. Proposition 1 answers three questions. First, under which circumstances does the alternative producer

¹¹One interpretation of our assumption that the surplus is divided equally between the firms is that prices are determined by the Nash bargaining solution (see Nash (1950)).

enter and under which does it stay out? Second, when entry does occur, in which period does it take place? Third, is the monopolist better off or worse off when the alternative producer stays out of the complementary market? All proofs are in the Appendix.

Proposition 1: Suppose $E_{ap} = \infty$. Then there exists a value E_{ac}^* , $E_{ac}^* > 0$, such that i) and ii) describe equilibrium behavior if $E_{ac} > E_{ac}^*$, while iii) and iv) describe equilibrium behavior if $E_{ac} < E_{ac}^*$. Also, overall monopoly profitability is higher when $E_{ac} < E_{ac}^*$.

- i) The monopolist offers individual or tied products.
- ii) The alternative producer never enters the complementary market.
- iii) The monopolist offers individual products.
- iv) The alternative producer enters the complementary market in the first period.

There are three results of interest in Proposition 1. First, the alternative producer enters the complementary market if its entry cost is small, and does not enter if its entry cost is large. Second, when entry occurs the alternative producer enters in the first period. This result is not surprising, since there is a bigger return to entering when entry allows the firm to operate in the market for two periods rather than one. Third, the monopolist earns higher profits when entry occurs than when it does not. The logic here is that, since the monopolist is able to capture some of the surplus associated with consumers preferring the alternative producer's complementary product, the monopolist's profitability rises upon entry.

The last result above tells us that in this model, if there is no threat of entry into the primary market, then the monopolist has no incentive to deter entry into the complementary market. Thus, although in this case offering a tied product is an optimal strategy for the monopolist when $E_{ac} > E_{ac}^*$ (but not the only optimal strategy), this is not because the monopolist wants to deter entry. Rather, the cause and effect are in the other direction. Because the alternative producer's entry cost is sufficiently high that no entry will take place, there is no cost (and no return) to the monopolist from offering a tied product.

We now consider what happens when the alternative producer has the option of entering the primary market in the second period. This case works quite differently than the benchmark case analyzed above. The reason is that, as opposed to what is true when the alternative producer only enters the complementary market, when it enters both the primary and complementary markets overall monopoly

profitability is hurt rather than helped. As a result, the monopolist sometimes has an incentive to deter entry into both markets by offering a tied product. We begin with a preliminary result concerning when the alternative producer has an incentive to enter the primary market in the second period. Below, π_{a2}^{pc} denotes the alternative producer's second-period profitability when the alternative producer entered the complementary market in period 1 and the primary market in period 2, and π_{a2}^c denotes the alternative producer's second-period profitability when the alternative producer entered the complementary market in period 1 and does not enter the primary market in period 2 (and the monopolist offers independent products).

Lemma 1: There exists a value E_{ap}^* , $E_{ap}^* > 0$, such that $\pi_{a2}^{pc} > \pi_{a2}^c$ if $E_{ap} < E_{ap}^*$, while $\pi_{a2}^{pc} < \pi_{a2}^c$ if $E_{ap} > E_{ap}^*$.

Lemma 1 is straightforward. It simply says that, if the alternative producer entered the complementary market in the first period and has the option of entering the primary market in the second period, it will stay out of the primary market if the cost of entering that market is sufficiently high. But it will enter if the cost of entry is sufficiently low. The next step is to consider in more detail what happens if the alternative producer has entered both markets by the beginning of the second period. Below, π_m denotes overall monopoly profitability, π_{mj} denotes monopoly profitability in period j , π_a denotes the overall profitability of the alternative producer, and π_{aj} denotes the alternative producer's profitability in period j .

Suppose the alternative producer has entered both markets by the beginning of the second period. Bertrand competition yields that purchasing a system from the monopolist will cost $c_p + c_c$, purchasing a system from the alternative producer will cost $c_p + c_c + \Delta$, and consumers purchase the complementary product from the alternative producer (sales of the primary product may be split across the two firms).¹² In turn, second-period monopoly profitability is given by $\pi_{m2} = 0$, while $\pi_{a2} = N\Delta - E_{ap}$ if the alternative producer had entered the complementary market in period 1 but $\pi_{a2} = N\Delta - E_{ap} - E_{ac}$ if it had not. There are two results of interest here. First, as opposed to what was true when the alternative producer only entered the complementary market, second-period monopoly profitability is now below rather than above second-

¹²If we allowed the alternative producer to tie, then by tying it could force all consumers to purchase primary units from itself. However, this would not increase the alternative producer's profitability.

period profitability in the absence of any entry. Second, the alternative producer's second-period profitability depends on whether it had entered the complementary market in the first period. If it had not, then the alternative producer bears that entry cost in the second period with a resulting decrease in second-period profitability.

The above analysis suggests that the monopolist sometimes has an incentive to deter entry into the complementary market in the first period, where this arises not from the effect on first-period profitability but rather because of the effect on second-period and overall profitability. That is, the benchmark analysis told us that deterring entry into the complementary market in the first period reduces the monopolist's first-period profitability. However, this action raises the alternative producer's cost of operating in the complementary market in the second period, with the possible result that entry into both markets is deterred in which case second-period and overall monopoly profitability are increased. Proposition 2 shows that this argument sometimes results in the monopolist offering a tied product.

Proposition 2: If $E_{ap} < E_{ap}^*$, then there exist values E_{ac}^* and E_{ac}^{**} , $0 < E_{ac}^* < E_{ac}^{**}$, such that for all $E_{ac}^* < E_{ac} < E_{ac}^{**}$ the unique equilibrium is described by i) and ii).

- i) The monopolist offers a tied product.
- ii) The alternative producer never enters either market.

Proposition 2 says that, if the alternative producer would enter the primary market in the second period if it had previously entered the complementary market ($E_{ap} < E_{ap}^*$), then the monopolist sometimes offers a tied product and in this way deters entry into both markets. In particular, the monopolist does this when the alternative producer's cost of entering the complementary market falls in an intermediate range. The logic is that if this entry cost is low ($E_{ac} < E_{ac}^*$) the monopolist has no incentive to tie because the alternative producer would respond by entering both markets in the second period, while if this cost is high ($E_{ac} > E_{ac}^{**}$) there is no incentive (or disincentive) for the monopolist to tie because the alternative producer would never enter either market even if the monopolist offered individual products. However, for intermediate values the alternative producer would enter both markets if the monopolist offered individual

products, but never enters either market if the monopolist ties.^{13,14}

To clarify the argument, consider the following simple example. Let $V=20$, $V'=19$, $\Delta=10$, $c_p=c_c=0$, $N=10$, $E_m=0$, and $E_{ap}=25$. Given this parameterization, suppose the monopolist offers individual products, and the alternative producer enters the complementary market in the first period and the primary market in the second period. In the first period the firms will evenly split the per consumer surplus of 10 created by the fact consumers prefer the alternative producer's complementary good, which means $\pi_{m1}=10(25)=250$ and $\pi_{a1}=10(5)-E_{ac}$. In the second period Bertrand competition yields that consumers will get the primary product for free, but each consumer will pay a price of 10 to the alternative producer for its complementary product. This means $\pi_{m2}=0$ and $\pi_{a2}=10(10)-25=75$. These calculations yield that if the monopolist offers individual products, as long as $E_{ac}<125$, the alternative producer enters the complementary market in the first period, it enters the primary market in the second period, and $\pi_m=250$.

Now suppose the monopolist offers a tied product and the alternative producer enters both markets in the second period. Bertrand competition now yields that the monopolist will offer its system at a price of zero, and that each consumer will pay a price of 10 to the alternative producer for its system. This means $\pi_{m2}=0$ and $\pi_{a2}=10(10)-25-E_{ac}=75-E_{ac}$. These calculations yield that if the monopolist offers a tied product, as long as $E_{ac}>75$, the alternative producer will never enter either market which in turn means overall monopoly profitability is given by $\pi_m=2[10(20)]=400>250$. In other words, if $75<E_{ac}<125$, the monopolist offers a tied product because this deters entry into both markets which increases overall monopoly profitability by preserving the firm's primary market monopoly in the second period.

The essential feature of our argument is that the alternative producer's entry cost for the complementary market creates an intertemporal economy of scope, which makes it more profitable for the

¹³The statements concerning what happens when $E_{ac}<E_{ac}^*$ and $E_{ac}>E_{ac}^{**}$ are proved in the proof of Proposition 2 in the Appendix. Also, consistent with these statements, in this model every equilibrium characterized by entry into the complementary market has that entry take place in the first period. This occurs for two reasons. First, as was true in the benchmark analysis, if the monopolist does not tie then entry into the complementary market is more profitable when the alternative producer operates in the market for two periods rather than one. Second, since entry into the complementary market helps the monopolist's first-period profitability (see the benchmark analysis), if the monopolist anticipates that even with a tie the alternative producer will enter the complementary market in the second period, then the monopolist has an incentive to offer individual products and have that entry occur in the first period.

¹⁴Suppose the monopolist's product choices for period 2 are decided at the beginning of period 2. Then for every $E_{ac}^*<E_{ac}<E_{ac}^{**}$ the monopolist would still have an incentive to deter entry into both markets. However, this would now mean tying in the first period and offering either a tied product or individual products in the second period.

alternative producer to be active in the complementary market in the second period if it was also active in that market in the first period. Because of this economy of scope, tying can deter entry into both markets because it reduces the alternative producer's potential profitability for being active in the complementary market in the second period. This perspective suggests it is not the entry cost per se that is critical for tying to be used for monopoly preservation, but rather any factor that creates a similar economy of scope should yield this result. For example, learning by doing also creates an intertemporal economy of scope, and we conjecture that a model without entry costs but with learning by doing for the complementary good would also yield a monopoly preservation role for tying.

As a final point, one could use the model analyzed in this subsection to illustrate a different but related argument for why a monopolist might tie. In this subsection we assumed that consumers are indifferent between the monopolist's and alternative producer's primary products, but that consumers prefer the alternative producer's complementary product. Suppose everything in the model is the same except now consumers prefer the alternative producer's primary product and the monopolist's complementary product. Under this alternative specification, just as in Proposition 2, the monopolist may tie its primary and complementary goods in period 1 in order to stop the alternative producer from entering the primary market in period 2. The logic underlying the result, however, is somewhat different.

In Proposition 2 the monopolist ties in order to stop the alternative producer from entering the complementary market, and, since consumers are indifferent between the two primary products, the alternative producer does not enter the primary market given no entry into the complementary market. Under the alternative specification just described, tying is not used to stop the alternative producer from entering the complementary market. That is, since consumers prefer the monopolist's complementary product, the alternative producer would not enter the complementary market if the monopolist did not tie. Rather, under this alternative specification tying can deter the alternative producer from entering the primary market because of a direct reduction in the alternative producer's return from entering that market. The logic is that tying reduces this return because consumers are unable to combine the alternative producer's superior primary product with the monopolist's superior complementary product.

C. Network Externalities

The previous subsection demonstrated a monopoly preservation role for tying when the alternative producer faces entry costs for the complementary good. In this subsection we assume away entry costs for the complementary good, and show that the presence of network externalities for the complementary good can similarly result in the strategic use of tying to deter entry into the primary market. As we will show, one interesting aspect of the network externalities case is that the tie can take the form of a virtual tie achieved through pricing as was discussed in the Introduction.¹⁵

In addition to assuming no entry costs for the complementary good, we make the following changes to the model analyzed in the previous subsection. Let N_{mj} be the number of consumers in cohort j who own a system consisting of one unit of the primary good and one unit of the monopolist's complementary good, while N_{aj} is the number of consumers in cohort j who own a system consisting of one unit of the primary good and one unit of the alternative producer's complementary good. A consumer derives a gross benefit from a system in which the complementary good is produced by the monopolist equal to $V+v(N_{m1}+N_{m2})$, $v'>0$, while a consumer derives a gross benefit from a system in which the complementary good is produced by the alternative producer equal to $V+\Delta+v(N_{a1}+N_{a2})$. In this specification, $v(\cdot)$ embodies the network externalities, i.e., the gross benefit a consumer derives from a system is positively related to the number of other consumers who own a similar system.¹⁶ One example of a complementary product with network externalities is an applications program, such as Word (a word processing program), where files can be traded among users.

In each of periods 1 and 2, because of network externalities, a given set of prices will frequently not result in a unique set of purchase decisions by the consumers. Similar to the approach taken in Katz and Shapiro (1986), we assume that purchase decisions are made as if consumers could coordinate behavior. That is, when there are multiple equilibria for a subgame that starts with consumer purchase decisions, we rule out the equilibria that are Pareto dominated for the consumers purchasing that period. Another way to put this is that we restrict attention to Perfectly Coalition-Proof Nash equilibria (see Bernheim, Peleg, and Whinston (1987) for a discussion of this refinement). Additionally, in this model

¹⁵Papers on the network externalities issue include Katz and Shapiro (1986, 1994), Farrell and Saloner (1986, 1992), and Matutes and Regibeau (1988, 1992).

¹⁶In this specification every consumer derives equal benefit from consumers in the same cohort who purchase a similar system as from consumers in the other cohort. Allowing for differential benefits would complicate the analysis without changing the qualitative nature of the results.

Bertrand competition will sometimes not result in a unique set of prices. Similar to the previous subsection, if this occurs in period 2 we assume that the prices that emerge evenly split across the two sellers the surplus associated with consumers preferring the alternative producer's version of the complementary good, although as before any split that gave each firm a strictly positive proportion of the surplus would yield the same qualitative results.¹⁷ Finally, we now assume $V'-c_p > 2\Delta$ rather than the assumption of the previous subsection that $V'-c_p > \Delta/2$.¹⁸

Consider first the alternative producer's incentive to enter the primary market in the second period. This issue yields a result similar to Lemma 1 of the previous subsection. That is, there exists a value E_{ap}^* such that, if $E_{ap} < E_{ap}^*$, then the alternative producer will enter the primary market in period 2 if it had previously sold complementary units to all consumers in period 1. The logic is that entering the primary market allows the alternative producer to capture more of the surplus associated with the superiority of its complementary product, and thus it enters if the entry cost is sufficiently low.

We now consider in more detail what happens if the alternative producer enters the primary market in period 2, where our focus is on parameterizations characterized by strong network externalities, i.e., $v(2N)-v(N) > \Delta$ (see footnote 19). There are two cases. The first case is that consumers purchased complementary units from this firm in period 1. In this case Bertrand competition yields that purchasing a system from the monopolist will cost c_p+c_c , purchasing a system from the alternative producer will cost $c_p+c_c+\Delta+v(2N)-v(N)$, and consumers purchase the complementary product from the alternative producer (sales of the primary product may be split across the two firms -- see footnote 12). In turn, second-period

¹⁷In the previous subsection, if the alternative producer entered the complementary market in the first period, the outcome of the second-period pricing game is independent of the outcome of the first-period pricing game. As a result, in that subsection the assumption that surplus is divided equally across the two sellers has a well defined meaning for both the first-period pricing game and the second-period pricing game. In contrast, in this subsection the outcome of the second-period pricing game depends on the outcome of the first-period pricing game. As a result, in this subsection the assumption that the surplus is divided equally across the two sellers has a well defined meaning for the second-period pricing game but is open to interpretation for the first-period pricing game. Given this problem and that all the results in this subsection hold for any way of resolving the first-period pricing game (although see footnote 18), we only impose the assumption for the second-period pricing game.

¹⁸The reason the condition is more restrictive in this subsection than in the previous subsection is because, when Bertrand competition does not result in a unique set of prices in the first period, we do not impose any assumption concerning how this multiple equilibria problem is resolved (see footnote 17). As a result, it is possible the monopolist gets all the surplus associated with cohort 1 consumers and cohort 2 consumers preferring the alternative producer's version of the complementary good, and this means $V'-c_p > 2\Delta$ is needed to ensure that the primary-market monopoly is more valuable to the monopolist than the potential benefits associated with having the alternative producer sell its higher quality complementary product.

monopoly profitability is given by $\pi_{m2}=0$, while $\pi_{a2}=N(\Delta+v(2N)-v(N))-E_{ap}$. The second case is that consumers purchased complementary units from the monopolist in period 1. In this case, given $v(2N)-v(N)>\Delta$, Bertrand competition yields that purchasing a system from the monopolist will cost $c_p+c_c+v(2N)-v(N)-\Delta$, purchasing a system from the alternative producer will cost c_p+c_c , and consumers purchase the complementary product from the monopolist. In turn, $\pi_{m2}=N(v(2N)-v(N)-\Delta)$ and $\pi_{a2}=-E_{ap}$.

There are two results of interest in the above analysis. First, if cohort 1 consumers purchase complementary units from the alternative producer, then entry into the primary market in the second period hurts the monopolist's second-period and overall profitability. Second, if cohort 1 consumers purchase complementary units from the monopolist, then the alternative producer would not enter the primary market in the second period because such entry results in negative profits. Together, these two results suggest that the monopolist will sometimes deter entry into the primary market in period 2 by behaving in a manner that causes cohort 1 consumers to purchase complementary units from the monopolist. We formalize this argument in Proposition 3.

Proposition 3: If $E_{ap} < E_{ap}^*$ and $v(2N)-v(N) > \Delta$, then every equilibrium is characterized by i) and ii).

- i) There is no entry into the primary market in the second period, and consumers in both cohorts purchase both primary and complementary goods from the monopolist.
- ii) The monopolist offers a tied product, or offers individual products but in the first period charges a "high" price for the primary product and a "low" price for the complementary product (see the proof for the exact definitions of "high" and "low").¹⁹

Proposition 3 tells us that the monopolist will sometimes use either a real tie or a virtual tie achieved through pricing to both stop the alternative producer from selling complementary units and deter its entry into the primary market. The logic was discussed above. When the monopolist sells its products in a manner that causes cohort 1 consumers to purchase complementary units from the monopolist, it makes entry into the primary market unprofitable and this in turn increases overall monopoly profitability.

¹⁹If $v(2N)-v(N) < \Delta$, then there are still parameterizations in which the monopolist either ties or uses a virtual tie through pricing to deter entry into the primary market. However, analysis of that case is significantly more complicated. The reason is that when $v(2N)-v(N) < \Delta$, if the monopolist ties its products and then an alternative producer enters the primary market in the second period, cohort 2 consumers would purchase from the alternative producer rather than the monopolist.

An interesting aspect of this result is that the monopolist need not actually tie its products to achieve its goal, but can rather employ a virtual tie achieved through first-period prices. The logic is that, if the complementary good in the first period is priced sufficiently low, then an alternative producer cannot profitably sell complementary units.²⁰

A natural question that arises is, why is a virtual tie achieved through pricing an entry deterring strategy in the network-externalities case but not in the entry-cost case. The reason is the different goals the monopolist is trying to achieve through tying in the two cases. In the network-externalities case, the goal of the monopolist in tying is to force cohort 1 consumers to purchase the complementary good from the monopolist because this is what stops entry into the primary market in the second period. This can be achieved either by using a real tie in which case cohort 1 consumers are directly forced to purchase the complementary good from the monopolist, or by a virtual tie where cohort 1 consumers purchase the complementary good from the monopolist because its price is set so low.

Now consider the entry-cost analysis. In that analysis, the goal of the monopolist in tying is to stop the alternative producer from entering the complementary market in the first period because in that case this is what stops entry into the primary market in the second period. This can be achieved by a real tie because the alternative producer will not enter the complementary market in the first period if it knows it cannot sell any complementary units in the first period. However, a virtual tie achieved through pricing will not work. The reason is that, once the alternative producer has entered the complementary market in the first period, the monopolist's incentive is not to employ a virtual tie but rather price in such a fashion that the alternative producer sells complementary units. In other words, attempting to deter entry using a virtual tie is not a credible strategy in the entry-cost case (another way to put this is that in the entry-cost case, deterring entry using a virtual tie is time inconsistent).²¹

Related to the underlying argument of the previous subsection, the essential feature of the argument here is that the presence of network externalities is similar to an intertemporal economy of scope. That is, because of network externalities, it is more profitable for an alternative producer to be active in the

²⁰Because of a non-negativity constraint on the price for the monopolist's complementary good, a virtual tie achieved through pricing is not always feasible. See the proof of Proposition 3 for details.

²¹Suppose that in the first period the monopolist's product choice and pricing decisions are simultaneous (and that in the entry-cost case this occurred after the alternative producer's entry decision for the complementary market). Consistent with the above discussion, this would have no effect on the analysis in the network-externalities case but would eliminate the ability of the monopolist to use tying for entry deterrence in the entry-cost case.

complementary market in the second period if it was also active in the complementary market in the first period. Tying thus deters entry into the primary market in the second period by reducing the alternative producer's profitability of being active in the complementary market in the second period.

One interesting aspect of both arguments in this section is that in our model tying can be used to exclude efficient competitors. Although we focus on the case in which consumers are indifferent between the monopolist's primary product and the alternative producer's primary product, the results in both subsections easily generalize to the case in which consumers prefer the alternative producer's primary good by a small amount (depending on the parameterization, tying may even be used to deter entry when consumers prefer the alternative producer's primary product by a large amount). Hence, in contrast to Whinston's analysis in which tying was used to eliminate an inferior producer of the monopolist's primary product, in our arguments tying can be used to deter entry of a firm that produces superior versions of both the primary and complementary goods.

III. EXTENDING MONOPOLY THROUGH STRATEGIC TYING OF COMPLEMENTARY PRODUCTS

In Section II we showed how a monopolist can strategically use the tying of complementary products to preserve an initial monopoly position. In this section we show how a monopolist can strategically use the tying of complementary products to extend a monopoly position into a newly emerging market. We first consider a model closely related to that analyzed in Subsection II.B, and show how tying can be used by the monopolist to "swing" or transfer his monopoly to the newly emerging market in a setting in which the newly emerging market is associated with the same complementary good as the monopolist's primary market. We then consider a model closely related to that analyzed in Subsection II.C, and show how tying allows the monopolist to monopolize the newly emerging market in a setting in which the primary market and the newly emerging market are initially complementary, but the newly emerging market eventually makes the primary market obsolete.

A. The Newly Emerging Market Uses the Same Complementary Product

Starting from the model analyzed in Subsection II.B (where there was a monopolist of a primary product in period 1, and a single alternative producer that could enter the complementary market in period

1 and the primary market in period 2), we make the following changes. First, the monopolist now faces no threat of entry into its primary market, and thus tying is not needed to deter entry into that market. Second, there is now a newly emerging market that is associated with the same complementary good as the primary market (although see footnote 22). The newly emerging market does not exist in the first period, but both the monopolist and the alternative producer can enter this market at the beginning of the second period at a cost E_n . Both firms have the same constant marginal cost for producing the newly emerging market good, denoted c_n .

Similar to what is true for primary units, a newly-emerging-market or simply new-market unit can either be used by itself or in combination with a complementary unit. Also, consumers are indifferent between a new-market unit produced by the monopolist and a new-market unit produced by the alternative producer, but prefer the alternative producer's complementary good for their new-market systems. To be precise, a consumer derives a gross benefit from a new-market unit by itself equal to V_n' , he derives a gross benefit from a new-market system in which the complementary unit is produced by the monopolist equal to V_n , while his gross benefit from a new-market system in which the complementary unit is produced by the alternative producer is equal to $V_n + \Delta_n$, where $V_n - V_n' > c_c$ and $N(V_n' - c_n) - E_n > N\Delta$. The restriction $V_n - V_n' > c_c$ ensures that the monopolist would sell complementary units for use in new-market systems if there was no alternative producer. The restriction $N(V_n' - c_n) - E_n > N\Delta$ ensures that being the sole producer of the new-market good in the second period is more valuable to the monopolist than the potential benefits associated with consumers being able to purchase primary-market systems containing the alternative producer's superior complementary product.

The timing of moves in the game is as follows. The timing for period 1 is the same as for the model analyzed in Subsection II.B. Period 2 has the following stages. First, the monopolist and the alternative producer simultaneously decide whether or not to enter the newly emerging market, and if the alternative producer has not previously entered the complementary market, it also decides whether to enter the complementary market at this time. Note, since there are no later entry decisions there is no return for a firm to offer a tied product at this date, and thus to simplify the exposition we do not allow tying in the newly emerging market. Second, prices are determined, where this means Bertrand competition if the alternative producer has entered one or more markets. Similar to before, if in either the primary market or the newly emerging market the alternative producer only offers a complementary product while the

monopolist offers both products as individual products, then the prices that emerge divide evenly across the two sellers the surplus associated with consumers preferring the alternative producer's version of the complementary good.^{22,23}

Our focus is on how the monopolist can use tying of primary and complementary goods in period 1 to deter the alternative producer from entering the newly emerging market in period 2, and in this way establish a monopoly position in the newly emerging market in period 2. We begin the analysis by considering who enters the newly emerging market in the second period as a function of the alternative producer's behavior in the complementary market.

Lemma 2: There exists a value E_n^* , $E_n^* > 0$, such that, if $E_n < E_n^*$, then i) and ii) hold.

- i) If the alternative producer enters the complementary market in the first period, then only the alternative producer enters the newly emerging market in period 2.
- ii) If the alternative producer never enters the complementary market, then only the monopolist enters the newly emerging market in period 2.

The logic behind Lemma 2.i) is as follows. Because the alternative producer has a superior complementary product, if the alternative producer enters the complementary market in the first period and the entry cost for the newly emerging market is sufficiently low, the alternative producer will enter the newly emerging market in period 2 whether or not it expects the monopolist to enter (if the alternative producer expects the monopolist to enter, then the alternative producer also enters because entry allows the alternative producer to capture more of the surplus associated with its superior complementary product). In contrast, because of Bertrand competition and that the monopolist has an inferior complementary product, the monopolist will not enter the newly emerging market in period 2 if it expects the alternative

²²We assume that a firm that has entered the complementary market can charge different prices for complementary units used in primary-market systems and complementary units used in new-market systems. One way to think about this assumption is that the two markets are not associated with the exact same complementary good, but rather there is a single entry cost that allows a firm to produce both complementary units used in primary-market systems and complementary units used in new-market systems. An example would be a software program that can be used on both personal computers and mainframes, where each type of computer requires a different version of the program.

²³We now also assume that, if the alternative producer is the only firm that enters the newly emerging market but the alternative producer does not enter the complementary market, then the prices that emerge divide evenly across the two sellers the surplus associated with complementary units used in new-market systems.

producer to enter. The result is that, if the alternative producer entered the complementary market in the first period and the entry cost for the newly emerging market is sufficiently small, then only the alternative producer enters the newly emerging market in the second period. A similar logic explains why, if the alternative producer never enters the complementary market and the entry cost for the newly emerging market is sufficiently low, then only the monopolist enters the newly emerging market in the second period (Lemma 2.ii)).

Lemma 2 tells us that, if E_n is sufficiently small, then monopolization of the newly emerging market by one firm or the other is quite possible. If the alternative producer enters the complementary market in the first period, then the alternative producer monopolizes the newly emerging market in the second period. In contrast, if the alternative producer is deterred from ever entering the complementary market, then the monopolist monopolizes the newly emerging market in the second period. This suggests that the monopolist will be able to increase its profits if it can deter the alternative producer from ever entering the complementary market. We now turn our attention to this issue.

Suppose the alternative producer has entered both the newly emerging and complementary markets by the beginning of the second period and the monopolist has not entered the newly emerging market. Then the alternative producer will price its products such that a new-market system consisting of a unit of the alternative producer's new-market good and a unit of its complementary good will cost $V_n + \Delta_n$. There are two relevant cases. Suppose the monopolist sells its primary and complementary goods as independent products, and as a result the alternative producer entered the complementary market in the first period. In this case, cohort 2 consumers purchase complementary units from the alternative producer for their primary-market systems, and the alternative producer's second-period profitability is given by $\pi_{a2} = N[V_n + \Delta_n + (\Delta/2) - c_n - c_c] - E_n$.

The other case is that the monopolist sells its primary and complementary goods as a tied product, and as a result the alternative producer does not enter the complementary market in the first period. In this case, cohort 2 consumers do not purchase complementary units from the alternative producer for their primary-market systems, and the alternative producer's second-period profitability is given by $\pi_{a2} = N[V_n + \Delta_n - c_n - c_c] - E_n - E_{ac}$. In other words, if the alternative producer does not enter the complementary market in the first period because the monopolist ties its primary and complementary goods, the result is a decrease in the alternative producer's second-period profitability due to the alternative producer bearing the

complementary market entry cost in the second period and to cohort 2 consumers not purchasing the alternative producer's complementary units for use in primary-market systems.

Since by tying its primary and complementary goods the monopolist lowers the alternative producer's profitability for being active in the complementary market in the second period, the above analysis suggests that the monopolist may be able to use tying of its primary and complementary goods to deter the alternative producer from ever entering the complementary market. In turn, combining this result with Lemma 2 suggests that tying may sometimes enable the monopolist to establish a monopoly position in the newly emerging market in the second period. We formalize this argument in Proposition 4.

Proposition 4: If $E_n < E_n^*$, then there exist values E_{ac}^* and E_{ac}^{**} , $0 < E_{ac}^* < E_{ac}^{**}$, such that for all $E_{ac}^* < E_{ac} < E_{ac}^{**}$ every equilibrium is characterized by i)-iii).²⁴

- i) The monopolist offers its primary and complementary goods as a tied product.
- ii) The monopolist enters the newly emerging market at the beginning of the second period.
- iii) The alternative producer never enters either the newly emerging or complementary markets.

Proposition 4 says that, if the entry cost for the newly emerging market is small, then the monopolist sometimes ties its primary and complementary goods and in this way extends its monopoly position into the newly emerging market. Similar to what was true in Proposition 2 in Subsection II.B, the monopolist does this when the alternative producer's cost of entering the complementary market falls in an intermediate range. As before, the logic is that if this entry cost is low ($E_{ac} < E_{ac}^*$) the monopolist does not tie because the alternative producer would respond by entering the newly emerging and complementary markets in the second period, while if the cost is high ($E_{ac} > E_{ac}^{**}$) the alternative producer would never enter either market even if the monopolist did not tie. However, if the alternative producer's entry cost for the complementary market is in an intermediate range, then the alternative producer enters neither market if the monopolist ties but enters both markets if the monopolist does not tie.²⁵

²⁴If $E_n > E_n^*$, there are still equilibria in which the monopolist ties its products in the first period in order to monopolize the newly emerging market in the second period, but those parameterizations are more difficult to analyze because of a multiple equilibria problem concerning who enters the newly emerging market in the second period.

²⁵The statements concerning what happens when $E_{ac} < E_{ac}^*$ and $E_{ac} > E_{ac}^{**}$ are proved in the proof of Proposition 4 in the Appendix.

It should be clear that the argument of this subsection is closely related to that of Subsection II.B. In Subsection II.B the monopolist tied its primary and complementary goods and in this way reduced the alternative producer's second-period return from being active in the primary and complementary markets. The result was that the alternative producer never entered these markets and the monopolist preserved its monopoly position in the primary market in the second period. Here, the monopolist ties its primary and complementary goods and in this way reduces the alternative producer's second-period return from being active in the newly emerging and complementary markets. The result is that the alternative producer never enters these markets and the monopolist establishes a monopoly position in the newly emerging market in the second period.

There is one interesting difference between the argument presented here and that of Subsection II.B. The essential feature of the argument in Subsection II.B was that the alternative producer's entry cost for the complementary market created an intertemporal economy of scope, where this made the alternative producer's profitability for being active in the complementary market in the second period higher if the firm entered the complementary market in the first period. That is, because of this scope economy, tying could deter entry into both the primary and complementary markets by reducing the alternative producer's potential profitability for being active in the complementary market in the second period.

The interesting difference between the two arguments is that the argument in this subsection relies on both an intertemporal economy of scope and the alternative producer's profitability for selling complementary units for use in primary-market systems. As in Subsection II.B, there is an intertemporal economy of scope which makes the alternative producer's profitability for being active in the complementary market in the second period higher if the firm entered the complementary market in the first period. There is now another important factor, however, in that, if the monopolist ties its primary and complementary goods, the alternative producer's profitability for being active in the complementary market in the second period is lower because the alternative producer cannot sell complementary units for use in primary-market systems. The result is that in this model tying can deter entry into both the newly emerging and complementary markets because of both the intertemporal economy of scope and the lost profits associated with selling complementary units for use in primary-market systems.

One implication of this difference concerns the importance of the two-period structure. Because the analysis of Subsection II.B depends solely on the intertemporal economy of scope, in that analysis the two-period structure is crucial. That is, if in that analysis there was a single period and the alternative

producer could enter both markets in period 1, the monopolist could not deter entry by tying its products. In contrast, because a consequence of tying in this subsection is the lost profits associated with the alternative producer selling complementary units for use in primary-market systems, in this subsection the two-period structure is not crucial. In this analysis, if there was a single period and the monopolist's tying decision in the primary market occurred before the alternative producer's entry decisions, then by tying its primary and complementary goods the monopolist could sometimes deter the alternative producer from entering both the newly emerging and complementary markets.

Another implication of this perspective is that the sizes of the primary and newly emerging markets should be important in determining whether the monopolist can use tying to establish a monopoly in the newly emerging market. If the primary market is large, then by tying its primary and complementary goods the monopolist significantly decreases the alternative producer's profitability for being active in the complementary market in the second period. Hence, when the primary market is large tying is likely to be useful for deterring entry into the newly emerging and complementary markets. In contrast, a large newly emerging market makes it unlikely tying will be useful for deterring entry. If the newly emerging market is large (or is growing quickly), then the alternative producer should be able to profitably operate in the complementary market even if it does not sell any complementary units for use in primary-market systems.

B. The Newly Emerging Market Eventually Makes the Primary Market Obsolete

In this subsection we explore a second avenue through which complementary links between a primary market and a newly emerging market allow a monopolist to extend its monopoly position to the newly emerging market. In particular, we consider a variant of the model analyzed in Subsection II.C, and show how this can work if the monopolist's primary market and the newly emerging market are initially complementary but the newly emerging market eventually makes the primary market obsolete.²⁶ At the end of the subsection we relate our analysis to arguments put forth in the recent Justice Department case against Microsoft.

We make the following three changes to the model analyzed in Subsection II.C. First, as in the previous subsection, the monopolist now faces no threat of entry into its primary market, and thus tying is not needed to deter entry into that market. Second, in the second period the primary good and the other

²⁶Previous papers that have studied the obsolescence issue include Levinthal and Purohit (1989), Waldman (1993, 1996), and Choi (1994).

good can be substitutes rather than complements. In Subsection II.C this other good was referred to as the complementary good, but to avoid confusion in this subsection we refer to the other good as the new-market good. Third, to make clear what it means to say that primary goods and new-market goods are complements in the first period but can be substitutes in the second period, we now introduce the idea of consumption activities. That is, consumers now derive a gross benefit from the performance of each of two activities -- a primary-market activity and a new-market activity.

Saying that primary and new-market goods are complements in the first period but can be substitutes in the second period means the following. Consider first cohort 1 consumers. For these consumers a primary unit can only be used to perform the primary-market activity, while a new-market unit can only be used to perform the new-market activity. Further, cohort 1 consumers can only use a new-market unit in combination with a primary unit, where the specification for the gross benefits that cohort 1 consumers derive from the various goods is exactly the same as in Subsection II.C. One should now think of V' as the gross benefit that a cohort 1 consumer derives from using the monopolist's primary good to perform the primary-market activity. Additionally, $V-V'+v(N_{m1}+N_{m2})$ is the gross benefit a cohort 1 consumer derives if he uses the monopolist's new-market good to perform the new-market activity, while $V-V'+\Delta+v(N_{a1}+N_{a2})$ is the gross benefit he derives if he uses the alternative producer's new-market good to perform the new-market activity. In this description the two goods are complements in the first period because a new-market unit can only be used in combination with a primary unit, and there is no element of substitutability in that each good is used to perform a different activity.

For cohort 2 consumers the situation is different because at the beginning of period 2 each firm can invest R_n and acquire the ability to produce a new version of the new-market product. If a firm does not invest, then in the second period it can produce the same new-market product it produced in the first period. Thus, a firm that does not invest produces new-market units in the second period that can only be used in combination with a primary unit, and that can only be used to perform the new-market activity. For a firm that does invest there are two changes. First, new-market units produced by this firm in the second period can be used without an accompanying primary unit. Second, new-market units produced by this firm in the second period can be used to perform both the new-market activity and the primary-market activity. In this description the monopolist's primary good and a new version of the new-market good are substitutes because they can both be used to perform the primary-market activity, and there is no element of

complementarity because a new version of the new-market good can be used without an accompanying primary unit.

To be precise, cohort 2 consumers derive gross benefits for the various goods as follows. A cohort 2 consumer who purchases the monopolist's primary good and either no new-market unit or an old-version new-market unit derives gross benefits exactly the same as cohort 1 consumers. A cohort 2 consumer who purchases only a new-version new-market unit produced by the monopolist derives a gross benefit equal to $V+v(N_{m1}+N_{m2})$, while a cohort 2 consumer who purchases only a new-version new-market unit produced by the alternative producer derives a gross benefit equal to $V+\Delta+v(N_{a1}+N_{a2})$.²⁷ Notice that a cohort 2 consumer who purchases either firm's new-version new-market good never has an incentive to purchase the monopolist's primary good. Related to this last point and as is discussed in more detail below, our focus is on values for R_n sufficiently small that at least one firm always invests in equilibrium. Since new-version new-market products can be used to perform the primary-market activity and do not require an accompanying primary unit, that at least one firm always invests means the monopolist's primary product becomes obsolete in the second period.

As in Subsection II.C, due to network externalities a given set of prices will frequently not result in a unique set of purchase decisions by the consumers, and we resolve this potential multiplicity by assuming that purchase decisions are made as if consumers could coordinate behavior. That is, when there are multiple equilibria for a subgame that starts with consumer purchase decisions, we rule out the equilibria that are Pareto dominated for the consumers purchasing that period. Also as in Subsection II.C, Bertrand competition will sometimes not result in a unique set of prices, and when this occurs in period 2 we assume that the prices that emerge evenly split across the two sellers the surplus associated with consumers preferring the alternative producer's versions of the new-market good.

Our focus is on how the monopolist can use tying of its primary and new-market goods in period 1 to monopolize the newly emerging market, and in this way retain its monopoly profits even after technological progress in the newly emerging market makes the monopolist's primary good obsolete. We begin by considering the manner in which second-period investment decisions depend on first-period

²⁷To be complete, a cohort 2 consumer who purchases a new version of the monopolist's new-market good and a new or old version of the alternative producer's new-market good derives a gross benefit equal to $\max\{V+v(N_{m1}+N_{m2}), V+\Delta+v(N_{a1}+N_{a2})\}$, while a cohort 2 consumer who purchases a new version of the alternative producer's new-market good and a new or old version of the monopolist's new-market good also derives a gross benefit equal to $\max\{V+v(N_{m1}+N_{m2}), V+\Delta+v(N_{a1}+N_{a2})\}$. A cohort 2 consumer who purchases a primary good in addition to a new-version new-market good gets the same gross benefit as from the new-version new-market good by itself.

consumption decisions. Note that as in Subsection II.C, our focus is on the strong network externalities case, i.e., $v(2N)-v(N)>\Delta$.

Lemma 3: Suppose $v(2N)-v(N)>\Delta$. There exists a value R_n^* , $R_n^*>0$, such that, if $R_n < R_n^*$, then period 2 is characterized by i) and ii).

- i) If all cohort 1 consumers purchased the monopolist's new-market product, then only the monopolist invests and all cohort 2 consumers purchase only the monopolist's new-version new-market product.
- ii) If all cohort 1 consumers purchased the alternative producer's new-market product, then only the alternative producer invests and all cohort 2 consumers purchase only the alternative producer's new-version new-market product.

The logic behind Lemma 3 is similar to that for Lemma 2 in the previous subsection. Because cohort 2 consumers prefer new-market products produced by the first-period seller of new-market units (this follows from $v(2N)-v(N)>\Delta$), if R_n is sufficiently low, then this first-period seller invests in period 2 whether or not it expects the other firm to invest. Further, because of Bertrand competition and that a new-version new-market product produced by the first-period seller of new-market units will be more attractive to cohort 2 consumers than a new version of the other firm's new-market product, the other firm will not invest if it expects the first-period seller of new-market units to invest. The result is that, if R_n is sufficiently small, then only the first-period seller of new-market units invests in the second period.²⁸

Lemma 3 has two results of interest. First, if cohort 1 consumers purchase new-market units from the alternative producer, then the monopolist sells nothing in the second period which hurts the monopolist's second-period and overall profitability. Second, if cohort 1 consumers purchase new-market units from the monopolist, then in the second period only the monopolist invests and it sells new-version new-market units to all cohort 2 consumers. Together, these two results suggest that the monopolist can sometimes increase its overall profitability by behaving in a manner that causes cohort 1 consumers to purchase new-market units from the monopolist. We formalize this argument in Proposition 5.

²⁸If R_n is sufficiently large, then neither firm invests. We have not considered the case where $R_n > R_n^*$ but is not so large that neither firm invests.

Proposition 5: If $R_n < R_n^*$ and $v(2N) - v(N) > \Delta$, then every equilibrium is characterized by i)-iii).

- i) Cohort 1 consumers purchase both primary units and new-market units from the monopolist.
- ii) In period 2 only the monopolist invests, and cohort 2 consumers purchase only new-version new-market units from the monopolist.
- iii) The monopolist offers a tied product in the first period, or offers individual products but charges a "high" price for the primary product and a "low" price for the new-market product (see the proof for the exact definitions of "high" and "low").

Proposition 5 tells us that the monopolist will sometimes use either a real tie or a virtual tie achieved through pricing to stop the alternative producer from selling new-market units in the first period, and in this way establish a monopoly position in the newly emerging market in the second period. The logic was discussed above. If the alternative producer sells new-market units in the first period, then the alternative producer monopolizes the newly emerging market in the second period and there is a resulting decrease in overall monopoly profitability. To stop this from occurring, in the first period the monopolist uses either a real tie or a virtual tie achieved through pricing to ensure that cohort 1 consumers purchase new-market units from the monopolist, and due to network externalities the result is that the firm establishes a monopoly position in the newly emerging market in the second period.²⁹

The results captured in Proposition 5 are related to the current antitrust case against Microsoft. One of the Justice Department's main allegations is that Microsoft has attempted to monopolize the Internet browser market through tying and a variety of other practices, and that its goal is to preserve its monopoly position in the operating systems market.³⁰ The Department argues that a successful rival Internet browser could potentially evolve into a substitute for Windows. It then argues that tying will allow Microsoft to monopolize the Internet browser market, and monopolizing that market will allow Microsoft to preserve its monopoly position in the operating systems market. Our analysis is closely related. In our analysis technological progress makes new-market products substitutes in the second period for the monopolist's

²⁹As was true for Proposition 3 (see footnote 20), because of a non-negativity constraint on the price for the monopolist's new-market good, a virtual tie achieved through pricing is not always feasible. See the proof of Proposition 5 for details.

³⁰In addition to tying, the Justice Department alleges, for example, that Microsoft has attempted to monopolize the Internet browser market through the use of exclusionary contracts between Microsoft and personal computer manufacturers.

primary product (in our analysis superior substitutes). In turn, tying allows the monopolist to extend its monopoly position to the newly emerging market, which means that even after its primary product is obsolete the monopolist retains the monopoly profits associated with the primary-market activity.³¹

As was true for the models in Section II, an interesting aspect of the models investigated in this section is that tying can be used to deter an efficient competitor. This is most easily seen in the analysis of this subsection. Since consumers prefer the alternative producer's new-version new-market product for performing the new-market activity, in this model the alternative producer's new-version new-market product is superior to the monopolist's new-version new-market product. Yet, the monopolist sometimes ties in the first period, and in this way establishes a monopoly position in the second period for its inferior product.

IV. A T-PERIOD ANALYSIS

One drawback of the analyses in Sections II and III is that they all focus on two-period settings. Here we show that the results of Subsection II.B generalize to a T-period setting when the monopolist can introduce new versions of the primary and complementary goods every other period. In our analysis, after the alternative producer enters a market, it has the same cost as the monopolist for producing future versions of the product, and in the absence of tying the alternative producer enters the complementary market in the first period and the primary market in the second. Yet, despite the fact that after entry there is no cost advantage to being the initial monopolist, in our analysis the monopolist sometimes ties and deters entry into both markets for all T periods. The analysis thus shows that even if the monopolist only has a temporary one-period advantage over the alternative producer, tying can sometimes be used to preserve its monopoly position in the primary market into the indefinite future.

³¹Regarding Microsoft's behavior concerning Windows and Internet Explorer, another interesting aspect of the analysis in this subsection is that the tie can take the form of a virtual tie achieved through pricing. Microsoft's initial behavior concerning Windows95 and Explorer was to form a contractual tie between the two products, but the firm was required by the Justice Department to discontinue the practice. Microsoft's response was to make Explorer free for purchasers of Windows95, and the result was that Microsoft's share of the Internet browser market continued to grow. This sequence of events is consistent with the results here and in Subsection II.C that, in industries characterized by network externalities, real ties and virtual ties achieved through pricing are close substitutes in forcing consumers to purchase the complementary or new-market goods from the primary-market monopolist.

A. The Model

We again consider a setting characterized by a monopolist and a single alternative producer except now there are T periods rather than two, where T is an even number (we also discuss the infinite-period case). The monopolist enters both the primary market and the complementary market in the first period, and then in every subsequent odd-numbered period an investment of R_p gives the monopolist the ability to produce a new and superior version of the primary product, while an investment of R_c gives the monopolist the ability to produce a new and superior version of the complementary product. A generation t primary product will refer to the version of the product that the monopolist acquires when it invests R_p in period t , while a generation t complementary product is defined similarly.³²

The alternative producer can enter the complementary market in any period starting with the first, while it can enter the primary market in any period starting with the second. Once the alternative producer has entered the primary market, then in every subsequent odd-numbered period t it can invest R_p and acquire the ability to produce a generation t primary product. Similarly, once the alternative producer has entered the complementary market, then in every subsequent odd-numbered period t it can invest R_c and acquire the ability to produce a generation t complementary product.

As already indicated, the monopolist's entry costs are sufficiently small that it enters both markets in the first period. The alternative producer's entry cost for the primary good is denoted E_{ap} , while its entry cost for the complementary good is denoted E_{ac} . When the alternative producer enters the complementary market it acquires the ability to produce a complementary product of the current generation. When the alternative producer enters the primary market in period t it acquires the ability to produce a primary product of the generation the monopolist offered for sale in period $t-1$. In other words, entry into the primary market is only possible by imitating or copying the monopolist's product (or equivalently, entry into the primary market without copying is sufficiently costly that it is never cost effective). However, once entry has occurred, the alternative producer is on an equal footing with the monopolist for all subsequent periods. As in the earlier analysis, each firm has a constant marginal cost of c_p for producing units of the primary good and a constant marginal cost c_c for producing units of the complementary good. Further, the firms are risk neutral and there is no discounting (although see footnote 34).

³²A more general specification would allow new versions of the primary and complementary products in periods $t'+1, 2t'+1, \dots, T-t'+1$. The results in this subsection do generalize to this alternative specification, but for ease of exposition we focus solely on the case $t'=2$.

Consumers are indifferent between a primary unit produced by the monopolist and a unit produced by the alternative producer if the products are of the same generation, but consumers prefer later generations to earlier generations. In contrast, complementary units of the same generation can be either high or low quality. Entry into the complementary market by the alternative producer gives it a probability p of acquiring a high-quality complementary product of the current generation and a probability $1-p$ of acquiring a low-quality complementary product of the current generation. For either firm, an investment of R_c in odd-numbered period t gives the firm a probability p of acquiring a high-quality generation t complementary product and a probability $1-p$ of acquiring a low-quality generation t complementary product. It follows that the alternative producer would never enter either market in a period in which the monopolist can produce high-quality complementary units. Hence, we assume that the monopolist produces low-quality complementary units in the very first period so that we are starting the analysis with the first period in which entry might occur.

A consumer derives a gross benefit equal to V_t' from a generation t primary unit by itself, he derives a gross benefit V_t from a system consisting of a generation t primary unit and a low-quality generation t complementary unit, while he derives a gross benefit equal to $V_t+\Delta$ from a system consisting of a generation t primary unit and a high-quality generation t complementary unit, where similar to Subsection II.B we assume $V_t-V_t'>c_c$ for all odd values for t and $V_1-c_p>\Delta/2$. We also assume that $N[p(1-p)\Delta]>R_p+R_c$ and $V_{t+2}'>V_t+\Delta$ for all odd values for t . These two restrictions ensure that, once the alternative producer has entered both markets, both firms will invest in new versions of the primary and complementary goods in every subsequent odd-numbered period. Suppose a system consists of one generation t product and one generation t' product, where $t'>t$. From the standpoint of the gross benefit a consumer derives from the system, it is as if both products were of generation t . In other words, a system with a new generation product is only an improvement over an old system if both products are of the new generation. Finally, there are T cohorts of N identical consumers, where consumers in cohort t are in the market in period t .

In the beginning of the first period the monopolist decides whether to offer a tied product consisting of one unit of its first generation primary and complementary goods or whether to offer the products individually. This decision is binding for all periods in which it sells generation 1 products. In the first period in which the alternative producer has entered both markets, it must decide whether to offer a tied product consisting of one unit of its current primary and complementary goods or whether to offer the products individually. This decision is binding for all periods in which it sells the products. If either the

monopolist or the alternative producer invests R_p and/or R_c in period t , then in the beginning of period t it decides whether to offer a tied product consisting of one unit of its current primary and complementary goods or whether to offer the products individually.³³ This decision is binding for all periods in which it sells these products. As earlier, we assume that if two goods are tied, a consumer cannot undo the tie.

The timing of moves in the game is as follows. The timing for periods 1 and 2 is the same as for the entry-cost model analyzed in Subsection II.B. Starting with period 3 each period has the following stages. First, in every odd-numbered period the monopolist and the alternative producer simultaneously decide whether to invest R_p and/or R_c (the alternative producer can only invest and acquire a new version of a product if it previously entered that market), and if a firm invests then it subsequently decides whether to offer tied or individual products. After these decisions, the quality of any new complementary product becomes publicly known. Second, if the alternative producer has not previously entered a market, it decides whether to enter. If the alternative producer does enter a market and has now entered both markets, it also decides whether to offer tied or individual products. Third, prices are determined. As before, if the alternative producer has previously entered one or both markets, then prices are determined by Bertrand competition. Further, also as before, if the alternative producer has only entered the complementary market and has a superior complementary product, then the prices that emerge divide evenly across the two sellers the surplus associated with consumers preferring the alternative producer's version of the complementary product.

B. Analysis

Consider first the benchmark analysis in which the alternative producer cannot enter the primary market in any period (a simple interpretation is that $E_{ap} = \infty$). Just as in Subsection II.B, the monopolist would have no incentive to tie in this benchmark case. The logic is as in the earlier analysis. When there is no threat of entry into the primary market, the monopolist earns higher expected profits when there is entry into the complementary market because it captures some of the surplus when the alternative producer offers a higher quality complementary product. Hence, in the absence of an entry threat into the primary market, the monopolist has no incentive to tie because it has no incentive to deter entry into the complementary market.

³³Although we allow the alternative producer to tie in this model, as was true in Subsection II.B, in this model the alternative producer never has an incentive to tie.

We now consider what happens when the alternative producer has the option of entering the primary market. Similar to what was true in Subsection II.B, we show that in this case the monopolist sometimes ties and deters entry into both markets. The reason is that entry into both markets hurts rather than helps monopoly profitability, and thus the monopolist sometimes finds it optimal to tie and in this way deter entry. A preliminary result in this case is similar to Lemma 1 of Subsection II.B. That is, there exists a value E_{ap}^* such that, if $E_{ap} < E_{ap}^*$, then entering the primary market in the second period raises the alternative producer's second-period profitability if in period 1 the monopolist offered individual products and the alternative producer had entered the complementary market. The logic is that the second-period profitability associated with entering the primary market in the second period is negatively related to that market's entry cost, and thus the alternative producer's second-period profitability rises with entry as long as that entry cost is sufficiently low.

Now consider what happens if the monopolist offers individual products in the first period and $E_{ap} < E_{ap}^*$. If E_{ac} is also low, then the alternative producer enters the complementary market in the first period and the primary market in the second period, where this increases the monopolist's first-period profitability but decreases its second-period profitability. In turn, given our assumption $N[p(1-p)\Delta] > R_p + R_c$, in every subsequent odd-numbered period both firms invest R_p and R_c and offer new generation primary and complementary products. The result is that the monopolist's expected overall profitability is significantly below what it achieves when there is no threat of entry into either market. We now consider how the analysis changes when the monopolist can tie.

Proposition 6: If $E_{ap} < E_{ap}^*$, then there exist values E_{ac}^* and E_{ac}^{**} , $0 < E_{ac}^* < E_{ac}^{**}$, such that for all $E_{ac}^* < E_{ac} < E_{ac}^{**}$ equilibrium behavior is described by i)-iii).

- i) There exists a value t^* , $1 < t^* \leq T$, such that the monopolist offers a tied product in every period t , $1 \leq t \leq t^*$, and offers individual or tied products in every period t , $t^* < t \leq T$.
- ii) The monopolist invests R_p and R_c in every odd-numbered period starting with period 3.
- iii) The alternative producer never enters either market.

Proposition 6 tells us that the monopolist uses tying to deter entry in similar circumstances to when tying was used in Proposition 2. That is, if the alternative producer's entry cost for the primary market is

low and its entry cost for the complementary market falls in an intermediate range, then the monopolist uses tying to deter entry into both markets. The reason this result is restricted to an intermediate range of values for the complementary market's entry cost is the same as in Proposition 2. When this entry cost is very low there is no return to tying because tying does not deter entry, while when it is very high there is no return to tying because there would be no entry even in the absence of tying. For intermediate values, however, the alternative producer enters both markets in the absence of tying but stays out of both markets when the monopolist ties.

The basic logic here is closely related to that underlying Proposition 2. When the monopolist ties in period 1, the alternative producer has no incentive to enter the complementary market in the first period because it cannot sell complementary units. The result is a decrease in the profitability associated with the alternative producer being active in the complementary market in the second period, which, in turn, results in the alternative producer not entering either market in the second period. In period 3 the monopolist invests R_p and R_c and, because there is a one-period lag before the alternative producer can enter the primary market with a new-generation primary product, the monopolist can similarly deter entry in periods 3 and 4 by tying. That is, tying stops the alternative producer from entering the complementary market in period 3, and similar to before the resulting decrease in the profitability associated with the alternative producer being active in the complementary market in period 4 causes the alternative producer not to enter either market in period 4. In turn, repeating this argument for periods 5, 7, ..., T-1 tells us that tying can be used by the monopolist to stop entry for all periods.

An interesting aspect of Proposition 6 is that the monopolist ties in early periods but may not need to tie in later periods. The logic for this result is as follows. The alternative producer's expected return to entering the two markets is positively related to the total number of periods the firm can be active in the two markets. This means that the alternative producer's return to entry falls over time because entry at a later date means a smaller number of periods in which the firm can be active. Hence, as t approaches T the monopolist may not need to tie to deter entry because the return to entry has fallen, and thus there would be no entry even in the absence of tying.

One interesting question is how would the analysis change if there were an infinite number of periods rather than T periods? The answer is not very much. That is, given that the alternative producer's entry cost for the primary market is low, there would still be an intermediate range of values for the complementary market's entry cost such that the monopolist uses tying to deter entry into both markets.

One difference, however, is that the infinite-period case would not exhibit the property just discussed that the monopolist ties in early periods but may not need to tie in later periods. Remember, in the T-period case this occurred because entry at a later date meant the alternative producer was active in the two markets for fewer periods. In the infinite-period case the number of periods the alternative producer is active in the two markets is independent of the date of entry, and thus if tying is required to deter entry in period 1 it will also be required to deter entry at all later dates.³⁴

As in the model analyzed in Subsection II.B, the underlying factor driving the results in this section is the presence of intertemporal economies of scope. By tying in period t , the monopolist stops the alternative producer from entering the complementary market in t . In turn, because there are intertemporal economies of scope due to the entry cost for the complementary good, the alternative producer's return to being active in the complementary market in $t+1$ is decreased and the result is that the alternative producer does not enter either market in $t+1$. In other words, even though entry into a market gives the alternative producer the same cost as the monopolist for producing future versions of the product, due to the economies of scope tying can deter entry by reducing its return.

In summary, in this section we have considered a T-period setting in which the monopolist's advantage over the alternative producer is not very large. In our analysis, the alternative producer can enter the monopolist's primary market in period 2, and once it has entered the alternative producer has the same cost as the monopolist for producing future versions of the product. Yet, we show that the monopolist sometimes ties and deters entry into both markets for all T periods. In other words, the analysis of this section demonstrates that the initial monopoly advantage need not be very large for tying to be effective in preserving monopoly profits into the indefinite future.

³⁴In our discussion of the infinite-period case, we are assuming discounting and that after the alternative producer has entered one or both markets prices are still determined by the static Bertrand solution. In the finite-period case tying may be used to deter entry whether or not there is discounting (assuming no discounting simplifies the proof). In contrast, in the infinite-period case no discounting means tying would never be used to deter entry. The reason is that, if the alternative producer enters both markets in period t and the net present value of the firm's profit stream starting with period $t+1$ is strictly positive, then infinite periods and no discounting means the alternative producer will enter both markets whether or not the monopolist ties. As for our assumption that prices are still determined by the static Bertrand solution, this is only one among many possible outcomes in the infinite-period case (see Friedman (1971, 1977) and Fudenberg and Tirole (1986) for related analyses), and the ability of the monopolist to use tying to deter entry would be reduced if pricing were close to the collusive levels.

V. ANTITRUST IMPLICATIONS

This paper has shown how a monopolist can engage in strategic tying over time in order to both preserve and extend its monopoly. Such strategic behavior is akin to various types of pre-positioning and pre-commitment behavior, well examined in the economic literature. It would be a grievous mistake to condemn such strategic behavior and attempt to use the antitrust laws to prevent it without an analysis of the welfare consequences of such behavior and without an analysis of the likelihood of being able to correctly identify such behavior without simultaneously condemning welfare enhancing behavior. Too often in the past, antitrust advocates have confused the theoretical possibility of harm with an empirical demonstration of such a harm. In this section, we attempt to evaluate the antitrust implications of our theory and contrast those implications with the standard antitrust doctrine regarding tie-ins.

The primary focus in the antitrust laws on tying has been the foreclosure of competition in the tied product. It was this focus that the Chicago School labeled misguided because in the litigated cases (e.g., International Salt, IBM), the tie facilitated price discrimination and had no effect on competition in the tied product. It is well known that the welfare effects of price discrimination are ambiguous -- in general, to the extent that tying allows a firm to come closer to practicing perfect price discrimination, the more likely is welfare enhanced. Hence, there is no intellectual basis to justify traditional antitrust hostility toward tie-ins when such tie-ins are used for reasons of price discrimination.

Whinston's (1990) work shows that, absent free entry and constant returns to scale in the tied product, tying can adversely affect competition in the tied product. This provides a justification -- though quite a limited one -- for antitrust doctrine. We simply observe that we are aware of no evidence or study examining whether scale economies in the tied product were important in the litigated cases and whether the tying led to the existence of market power in the pricing of the tied product.

In contrast to the focus of the antitrust laws on foreclosure of competition in the tied product, this paper has described a theory in which tying is used to both preserve and extend the monopoly in the tying product. This approach addresses and explains what appears to be an important phenomenon in industries where technology is constantly being improved, such as in computing. We now address some difficulties that arise in trying to turn our positive theory of market behavior into a prescriptive theory for antitrust enforcement.

The central legal issue in a tie-in case is whether the two products are really separate. The mere fact that product A and product B could be separately defined, produced, and consumed does not answer

the question. Since the production of A and B into a combination product C (a package with the characteristics of A and B) can have properties that A and B separately do not have (e.g., convenience of use, added functionality), a difficult issue is evaluating the motivation for product C. Specifically, do the consumer benefits of C justify its introduction or is its introduction solely to allow the firm to engage in strategic tying? This, to us, is, in general, a horrendously complex trade-off to evaluate. Fear of antitrust scrutiny could easily prevent an innovator from introducing new desirable products. The flip side, of course, is that failure for antitrust enforcers to act can turn an industry like the ones we have studied -- those with either intertemporal economies of scope or network externalities -- from competition to monopoly.

Our views on evaluating this complex trade-off are as follows. First, great weight should be given to any plausible efficiency from the tie. Efficiencies may be hard to quantify, but foregoing an efficiency can generate substantial deadweight loss. Second, evidence on motivation can assist in exceptional cases in figuring out the reason for the tie and could provide a justification for intervention. For example, evidence that the sole purpose of a design change was to stymie competitors by creating an effective tie could be the type of evidence that allows one to avoid analyzing the technological benefits of the design change -- a task which we predict will fail to lead to consensus. This type of evidence (such as memos) is of the kind usually examined by lawyers not economists. Third, efficiencies achieved through physical integration (as when A and B are produced together in a package C) should receive greater weight than efficiencies achieved through contract (in which the combined use of A and B are mandated by contract). The antitrust laws have always shown greater deference to activities within the firm compared to interfering in activities outside of the firm. For example, an antitrust court is much more apt to negate an exclusive dealing contract with distributors than it is to order divestiture of an internal division engaged in marketing. The logic, and in general it sounds correct, is that the cost of interfering inside a firm (where many unspecified relationships and transactions are not mediated by the price system) is likely to be higher than interfering in the contractual relations between two firms.

Our analysis showed that, in the presence of network externalities, a virtual tie can sometimes be effectively created by setting the price of the complementary good sufficiently low. Since a virtual tie is similar to a contractual tie in that the firm has shown by its actions that there is no efficiency reason to physically produce the components as one, the antitrust analysis should proceed under a rule of reason, as is standard in exclusionary contractual disputes such as those involving exclusive dealing. However, the

virtual tie raises the issue of whether a claim of predation makes sense. Creating a virtual tie through pricing may require a price below marginal production cost, but especially in a market characterized by network effects, there could be an efficiency justification for such pricing. What is interesting and novel about this pricing is that, if it is used strategically to deter entry, the recoupment does not occur in the complementary market but in the primary market. This means that ease of entry into the primary market, not the complementary market, is key and that the failure to observe an elevation of price in the complementary market cannot be regarded as definitive proof against predation or strategic behavior.

A particularly vexsome issue -- and one wholly ignored by antitrust courts -- is whether raising the rate of return is desirable in industries undergoing rapid technological change. The argument would be that strategic behavior that entrenches monopoly raises the return to being the first in the industry. By raising this return, more innovation is encouraged. If, as empirical studies appear to show (e.g., Mansfield et al. (1977), Bernstein and Nadiri (1988), and Mansfield (1991)) the social rate of return from innovation exceeds the private rate of return, such an action would be desirable. However, despite its logic, we have never seen an antitrust court use the importance of innovation as a decision criteria for whether to allow monopolization.

In the dynamic models of this paper, the welfare consequences of encouraging more innovation are even harder to analyze than in a simple model of a single patent race. The reason is that although at early stages of industry evolution strategic behavior could raise the rate of return and thereby encourage more innovation, the consequence of strategic behavior could be to dampen the incentives for subsequent innovations. Especially in a growing market, the value of the subsequent innovations could easily swamp the value of the initial ones.

In summary, the difficulty of using cost-benefit analyses to identify harmful tie-ins leads us to the conclusion that, other than in exceptional cases, plausible efficiency justifications for a physical tie should defeat an antitrust attack on tying. For contractual ties and virtual ties achieved through pricing, the standard can be lower and a balancing of competitive costs versus benefits can be done much as is now done in exclusive-dealing cases. The key insight of this paper is to cease to view tying as a special category of offence that forecloses competition in the tied product (see Hyde) and more as strategic behavior akin to exclusive dealing that permits the preservation and extension of monopoly.³⁵

³⁵Jefferson Parish Hospital District No. 2 v. Hyde, 466 U.S. 2 (1984). Lynk (1994) analyzes this case.

VI. CONCLUSION

Most previous analyses of tying have focused either on the ability of tying to achieve price discrimination or its ability to foreclose competition in the tied market. In contrast, our focus is on the use of tying to preserve and extend a monopoly position in the tying market. In particular, we present a series of analyses in which a firm is currently a monopolist in its primary market, where the firm uses tying of a complementary product to deter future entry into both the primary market and related markets. In each of our analyses when the monopolist ties there is a direct reduction in the alternative producer's return to selling the complementary product. In turn, because of the complementary links between the products, if the alternative producer is deterred from selling the complementary product there is also a corresponding reduction in the alternative producer's return to entering the primary market and related markets. We show how this can work both in models characterized by intertemporal economies of scope and in models characterized by network externalities.

Our analysis suggests that the use of tying to preserve and extend a monopoly position will be most important in industries characterized by substantial innovation where product lifetimes are short. Consider, for example, the analyses of Subsections II.B and III.B. In those analyses the direct result of tying is that the alternative producer cannot sell any complementary units in the first period which lowers the alternative producer's return to entering the complementary market in either period. In each analysis the lifetime of the complementary product is two periods, so eliminating one period of sales has a significant effect on this return. Suppose instead product lifetimes were very long and tying only eliminated the alternative producer's sales of the complementary product for one or at most a few periods. In that case tying still reduces the alternative producer's return to entering the complementary market, but because only a small fraction of potential sales are eliminated, tying has only a minimal effect on this return. In turn, since there is little effect on the alternative producer's return to entering the complementary market, there is also likely to be little effect on the alternative producer's return to entering the primary market and related markets. In other words, it is only in markets with substantial innovation where product lifetimes are short that tying is likely to be an effective tool for preserving and extending an initial monopoly position.

There are many directions in which the analysis in this paper could be extended. Two specific directions come to mind. First, it might be of interest to analyze a setting in which the monopolist can control the speed of innovation. Our conjecture is that, in such a setting, a primary-market monopolist would sometimes preserve and extend its monopoly by both introducing new products quickly and tying

each new generation of its primary and complementary products. The logic is that, consistent with the above discussion, tying is likely to be a more effective tool in markets in which product lifetimes are short.³⁶ Second, all our analyses focus on the case of Bertrand competition and no heterogeneity among consumers. The result is that, at least in the complementary market, within a cohort of consumers it is never the case that some consumers purchase the monopolist's complementary good while others purchase the alternative producer's complementary good. We do not believe that our results depend on this property of our models, but it might be worthwhile formally demonstrating that our results are robust by either moving away from Bertrand competition or introducing consumer heterogeneity.

³⁶See Cadot and Lippman (1996) for a related analysis that does not incorporate tying.

APPENDIX

Due to space considerations, proofs are somewhat abbreviated.

Proof of Proposition 1: Given $E_{ap} = \infty$, the alternative producer will never enter the primary market. Given this, suppose for the moment that the monopolist offers individual products. There are three possibilities. First, if the alternative producer does not enter the complementary market in either period, then $\pi_m = 2N[V - c_p - c_c] - E_m$ and $\pi_a = 0$. Second, if the alternative producer enters the complementary market in period 2, then the prices that emerge evenly split the surplus associated with consumers preferring the alternative producer's version of the complementary good. This means the alternative producer charges $c_c + (\Delta/2)$ for its complementary product, the monopolist charges $V - c_c + (\Delta/2)$ for its primary product, the monopolist charges any price greater than or equal to c_c for its complementary product, and consumers purchase the primary product from the monopolist and the complementary product from the alternative producer. This yields $\pi_m = 2N[V - c_p - c_c] + N(\Delta/2) - E_m$ and $\pi_a = N(\Delta/2) - E_{ac}$. Third, if the alternative producer enters the complementary market in period 1, then pricing in each period is as in the second period in case 2 which means $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$ and $\pi_a = N\Delta - E_{ac}$.

Let $E_{ac}' = N\Delta$. If $E_{ac} > E_{ac}'$, then the above analysis indicates the alternative producer does not enter the complementary market in either period and $\pi_m = 2N[V - c_p - c_c] - E_m$. If $E_{ac} < E_{ac}'$, then the above analysis indicates the alternative producer enters the complementary market in period 1 and $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$.

Now suppose the monopolist offers a tied product. This means the alternative producer will not enter the complementary market in either period, and this in turn means $\pi_m = 2N[V - c_p - c_c] - E_m$ and $\pi_a = 0$. We now have that, if $E_{ac} < E_{ac}'$, then the monopolist offers individual products, earns $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$, and the alternative producer enters the complementary market in the first period. On the other hand, if $E_{ac} > E_{ac}'$, then offering individual and tied products are both equilibrium choices for the monopolist, $\pi_m = 2N[V - c_p - c_c] - E_m$ for each choice, and for each choice the alternative producer never enters the complementary market. Notice, since $N\Delta > 0$, we further see that the monopolist earns higher profits when $E_{ac} < E_{ac}'$.

Proof of Lemma 1: From the proof of Proposition 1 we know that $\pi_{a2}^c = N\Delta/2$. Now suppose the alternative producer entered the complementary market in period 1 and the primary market in period 2. We consider the case in which the monopolist offers independent products, but the profitabilities for the two firms are the same if the monopolist offers a tied product. For the primary product, Bertrand competition yields that each firm charges a price equal to c_p , and sales are split between the two firms. For the complementary product, Bertrand competition yields that the monopolist charges c_c , the alternative producer charges $c_c + \Delta$,

and cohort 2 consumers purchase complementary units from the alternative producer.³⁷ This yields $\pi_{a2}^{pc} = N\Delta - E_{ap}$. Let $E_{ap}^* = N\Delta/2$. We now have $\pi_{a2}^{pc} > \pi_{a2}^c$ if $E_{ap} < E_{ap}^*$, while $\pi_{a2}^{pc} < \pi_{a2}^c$ if $E_{ap} > E_{ap}^*$.

Proof of Proposition 2: Suppose the monopolist offers a tied product. There are four possibilities. First, the alternative producer never enters either market in which case $\pi_m = 2N[V - c_p - c_c] - E_m$ and $\pi_a = 0$. Second, the alternative producer enters the complementary market in either period 1 or period 2 and never enters the primary market. Because the monopolist is only offering a tied product, the alternative producer would be unable to sell any complementary units if it only entered the complementary market and thus in this case $\pi_a = -E_{ac}$. Third, the alternative producer enters the primary market in period 2 and never enters the complementary market. Because the monopolist is offering a tied product, the alternative producer would be unable to sell any primary units in period 2 if it only entered the primary market and thus in this case $\pi_a = -E_{ap}$. Fourth, the alternative producer enters the primary market in period 2 and enters the complementary market in either period 1 or period 2. Because the monopolist is offering a tied product, the alternative producer would be unable to sell any complementary units in the first period whether or not it enters the complementary market in the first period. In the second period, Bertrand competition yields that the monopolist charges $c_p + c_c$ for its system, the alternative producer charges an aggregate price of $c_p + c_c$ for its two products, and consumers purchase both primary and complementary products from the alternative producer. Thus, in this case $\pi_m = N[V - c_p - c_c] - E_m$ and $\pi_a = N\Delta - E_{ap} - E_{ac}$.

Suppose the monopolist offers individual products. There are six possibilities. First, the alternative producer never enters either market in which case $\pi_m = 2N[V - c_p - c_c] - E_m$ and $\pi_a = 0$. Second, the alternative producer enters the complementary market in period 1 and never enters the primary market. From the proof of Proposition 1 we know that in this case $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$ and $\pi_a = N\Delta - E_{ac}$. Third, the alternative producer enters the complementary market in period 2 and never enters the primary market. From the proof of Proposition 1 we know that in this case $\pi_m = 2N[V - c_p - c_c] + N(\Delta/2) - E_m$ and $\pi_a = N(\Delta/2) - E_{ac}$. Fourth, the alternative producer enters the primary market in period 2 and never enters the complementary market. Bertrand competition means that in period 2 both firms charge c_p for the primary product which in turn means $\pi_a = -E_{ap}$. Fifth, the alternative producer enters the primary and complementary markets in period 2. In the second period, Bertrand competition yields that both firms charge c_p for the primary product, the monopolist charges c_c for its complementary product while the alternative producer charges $c_c + \Delta$ for its complementary product, and consumers purchase complementary units from the alternative producer while purchases of primary units are split across the two firms. Thus, in this case $\pi_m = N[V - c_p - c_c] - E_m$ and $\pi_a = N\Delta - E_{ap} - E_{ac}$. Sixth, the alternative producer enters the

³⁷There are other price pairs consistent with Bertrand competition in which the monopolist charges a price for the complementary good that is below its marginal cost. Here and in later analyses we rule out pricing of this sort. Formally, any trembling-hand-type refinement concerning consumer purchase decisions would rule out such pricing.

complementary market in period 1 and the primary market in period 2. Pricing in the second period is as in the fifth case above. In the first period, the prices that emerge evenly split the surplus associated with consumers preferring the alternative producer's version of the complementary product. This means the alternative producer charges $c_c+(\Delta/2)$ for its complementary product, the monopolist charges $V-c_c+(\Delta/2)$ for its primary product, the monopolist charges any price greater than or equal to c_c for its complementary product, and consumers purchase the primary product from the monopolist and the complementary product from the alternative producer. Thus, in this case $\pi_m=N[V-c_p-c_c]+N(\Delta/2)-E_m$ and $\pi_a=N(3\Delta/2)-E_{ap}-E_{ac}$.

Let $E_{ac}^*=N\Delta-E_{ap}$ and $E_{ac}^{**}=N(3\Delta/2)-E_{ap}$. Suppose $0<E_{ac}<E_{ac}^*$. If the monopolist ties, then from above we know the alternative producer enters the primary market in period 2 and enters the complementary market in either period 1 or period 2. Further, $\pi_m=N[V-c_p-c_c]-E_m$ and $\pi_a=N\Delta-E_{ap}-E_{ac}$. If the monopolist offers individual products and given $E_{ap}<E_{ap}^*=N\Delta/2$, then from above we know the alternative producer enters the primary market in period 2 and the complementary market in period 1. Further, $\pi_m=N[V-c_p-c_c]+N(\Delta/2)-E_m$ and $\pi_a=N(3\Delta/2)-E_{ap}-E_{ac}$. Since $N\Delta/2>0$, the monopolist offers individual products and the alternative producer enters the primary market in period 2 and the complementary market in period 1.

Suppose $E_{ac}>E_{ac}^{**}$. If the monopolist ties, then from above we know the alternative producer never enters either market. Further, $\pi_m=2N[V-c_p-c_c]-E_m$ and $\pi_a=0$. If the monopolist offers individual products and given $E_{ap}<E_{ap}^*=N\Delta/2$, then from above we know the alternative producer never enters either market. Further, $\pi_m=2N[V-c_p-c_c]-E_m$ and $\pi_a=0$. Thus, in this case there are two equilibria. In one equilibrium the monopolist ties and the alternative producer never enters either market, while in the other equilibrium the monopolist offers individual products and the alternative producer never enters either market.

Suppose $E_{ac}^*<E_{ac}<E_{ac}^{**}$. If the monopolist ties, then from above we know the alternative producer never enters either market. Further, $\pi_m=2N[V-c_p-c_c]-E_m$ and $\pi_a=0$. If the monopolist offers individual products and given $E_{ap}<E_{ap}^*=N\Delta/2$, then from above we know the alternative producer enters the complementary market in period 1 and the primary market in period 2. Further, $\pi_m=N[V-c_p-c_c]+N(\Delta/2)-E_m$ and $\pi_a=N(3\Delta/2)-E_{ap}-E_{ac}$. Given $V-V'>c_c$ and $V'-c_p>\Delta/2$, we know $2N[V-c_p-c_c]-E_m>N[V-c_p-c_c]+N(\Delta/2)-E_m$. Thus, in this case the unique equilibrium is the monopolist offers a tied product and the alternative producer never enters either market.

Proof of Proposition 3: Throughout the proof we assume $v(2N)-v(N)>\Delta$. Suppose all cohort 1 consumers purchase complementary units from the monopolist. There are two possibilities for what happens in period 2. First, the alternative producer enters the primary market in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, Bertrand competition in this case yields that both firms will charge c_p for a primary unit, the alternative producer will charge c_c for a

complementary unit, the monopolist will charge $c_c + v(2N) - v(N) - \Delta$ for a complementary unit, and consumers purchase complementary units from the monopolist while purchases of primary units may be split between the firms (if the monopolist is offering a tied product, then all primary units are purchased from the monopolist). This yields $\pi_{m2} = N[v(2N) - v(N) - \Delta]$ and $\pi_{a2} = -E_{ap}$.

Second, the alternative producer does not enter the primary market in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, consumers purchase both primary and complementary units from the monopolist. In particular, the alternative producer charges c_c for a complementary unit, while the monopolist charges an aggregate price of $V + v(2N)$ for a system (if the monopolist sells individual products, then the price for the primary product is in the interval $(V + \Delta + v(N) - c_c, V + v(2N))$ while the complementary-good price equals $V + v(2N)$ minus the primary-good price). This yields $\pi_{m2} = N[V + v(2N) - c_p - c_c]$ and $\pi_{a2} = 0$. Comparing this expression for the alternative producer's second-period profit with the expression above yields that, if all cohort 1 consumers purchase complementary units from the monopolist, then the alternative producer does not enter the primary market in period 2, cohort 2 consumers purchase both primary and complementary goods from the monopolist, $\pi_{m2} = N[V + v(2N) - c_p - c_c]$, and $\pi_{a2} = 0$.

Suppose all cohort 1 consumers purchase complementary units from the alternative producer (this will only be the case if the monopolist offers individual products). There are again two possibilities for what happens in period 2. First, the alternative producer enters the primary market in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, Bertrand competition in this case yields that both firms will charge c_p for a primary unit, the monopolist will charge c_c for a complementary unit, the alternative producer will charge $c_c + \Delta + v(2N) - v(N)$ for a complementary unit, and consumers purchase complementary units from the alternative producer while purchases of primary units may be split between the firms. This yields $\pi_{m2} = 0$ and $\pi_{a2} = N[\Delta + v(2N) - v(N)] - E_{ap}$.

Second, the alternative producer does not enter the primary market in period 2. By assumption the prices that emerge evenly split the surplus associated with consumers preferring the alternative producer's superior complementary product. This means the alternative producer charges $c_c + [(\Delta + v(2N) - v(N))/2]$ for a complementary unit, the monopolist charges c_c or more for a complementary unit, the monopolist charges $V + \Delta + v(2N) - c_c - [(\Delta + v(2N) - v(N))/2]$ for a primary unit, and consumers purchase primary units from the monopolist and complementary units from the alternative producer. This yields $\pi_{m2} = N[V + \Delta + v(2N) - c_p - c_c - [(\Delta + v(2N) - v(N))/2]]$ and $\pi_{a2} = N[(\Delta + v(2N) - v(N))/2]$. Let $E_{ap}^* = N[(\Delta + v(2N) - v(N))/2]$. Comparing this expression for the alternative producer's second-period profit with the expression above yields that, if $E_{ap} < E_{ap}^*$ and all cohort 1 consumers purchase complementary units from the alternative producer, then the alternative producer enters the primary market in period 2, cohort 2 consumers purchase complementary units from the alternative producer while purchases of primary units may be split between the firms, $\pi_{m2} = 0$, and $\pi_{a2} = N[\Delta + v(2N) - v(N)] - E_{ap}$.

Now consider period 1. Suppose the monopolist ties in period 1. If all cohort 1 consumers purchase complementary units from the monopolist, then from above we know that in period 2 all cohort 2 consumers will also purchase complementary units from the monopolist. Let P_s denote the price the monopolist charges for a system. Given that a cohort 1 consumer's net benefit from purchasing a system from the monopolist equals $V+v(N_{m1}+N_{m2})-P_s$, for any P_s that satisfies $P_s \leq V+v(2N)$, each cohort 1 consumer's utility is maximized by having every cohort 1 consumer purchase a system (if $P_s = V+v(2N)$, then each cohort 1 consumer is indifferent between having all purchase and not purchasing). Similarly, for any P_s that satisfies $P_s > V+v(2N)$, each cohort 1 consumer's utility is maximized by not purchasing. Hence, since purchase decisions are made as if consumers could coordinate behavior and since if all cohort 1 consumers purchase the resulting second-period profit is independent of the first-period price, the monopolist sets $P_s = V+v(2N)$.³⁸ This, in turn, yields $\pi_{m1} = N[V+v(2N)-c_p-c_c]-E_m$, $\pi_{m2} = N[V+v(2N)-c_p-c_c]$, and $\pi_m = 2N[V+v(2N)-c_p-c_c]-E_m$.

Suppose the monopolist offers individual products in period 1. Let P_p denote the monopolist's price for its primary product, P_c denote the monopolist's price for its complementary product, and P_{ac} denote the alternative producer's price for its complementary product. A cohort 1 consumer's net benefit from purchasing a system consisting of one unit of the monopolist's primary product and one unit of the alternative producer's complementary product is $V+\Delta+v(N_{a1}+N_{a2})-P_p-P_{ac}$. This means that, if the alternative producer is to sell any complementary units in period 1, the alternative producer's price for complementary units must satisfy $P_{ac} \leq V+\Delta+v(2N)-P_p$. Given this, if the alternative producer sells complementary units in the first period, its overall profit cannot exceed what it earns if it sets $P_{ac} = V+\Delta+v(2N)-P_p$ and sells complementary units to all cohort 1 consumers, i.e., $\pi_a \leq N[V+\Delta+v(2N)-P_p-c_c] + N[\Delta+v(2N)-v(N)]-E_{ap}$. Further, let P^* satisfy $N[V+\Delta+v(2N)-P^*-c_c] + N[\Delta+v(2N)-v(N)]-E_{ap} = 0$. We now have that the alternative producer can only sell complementary units in the first period if $P_p \leq P^*$.

If the monopolist offers individual products in period 1, there are three possibilities. First, the alternative producer sells complementary units to some but not all cohort 1 consumers in the first period. There are two cases. First, the alternative producer does not enter the primary market in the second period. Consider a cohort 1 consumer who purchased the alternative producer's complementary product. This consumer's first-period utility is given by $V+\Delta+v(N_{a1}+N_{a2})-P_p-P_{ac} \geq V+v(N_{m1}+N_{m2})-P_p-P_c$ (if this inequality did not hold, the consumer could have increased his utility by purchasing the monopolist's complementary product).³⁹ But if this condition holds, then all cohort 1 consumers purchasing the

³⁸Because price is a continuous variable, for any P_s strictly below $V+v(2N)$, there is a higher value for P_s that increases the monopolist's overall profit. This means that, if the monopolist ties, the unique equilibrium to the subsequent subgame is characterized by $P_s = V+v(2N)$.

³⁹For this step of the proof we are assuming that each cohort of consumers consists of a continuum of individuals of mass N . By assuming this, we ensure that no single cohort 1 consumer's first-period purchase decisions determine the equilibrium to the second-period subgame.

alternative producer's complementary product is also equilibrium behavior, and under this alternative set of purchase strategies this consumer's first-period utility is given by $V+\Delta+v(2N)-P_p-P_{ac}>V+\Delta+v(N_{a1}+N_{a2})-P_p-P_{ac}$. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, we have just shown that it cannot be the case that some but not all cohort 1 consumers purchase complementary units from the alternative producer in the first period and the alternative producer does not enter the primary market in the second period. Second, the alternative producer enters the primary market in the second period. This case is ruled out using an argument similar to that used to rule out the first case.

Second, the alternative producer sells complementary units to all cohort 1 consumers in the first period. In this case, $\pi_{m1}=N[P_p-c_p]-E_m$ and from above we know $\pi_{m2}=0$. This means $\pi_m=N[P_p-c_p]-E_m$ and since we also know from earlier that $P_p \leq P^*$, we have $\pi_m \leq N[P^*-c_p]$. Given P^* satisfies $N[V+\Delta+v(2N)-P^*-c_c]+N[\Delta+v(2N)-v(N)]-E_{ap}=0$, we know $P^* < [V+\Delta+v(2N)-c_c]+[\Delta+v(2N)-v(N)]$. Thus, $\pi_m < N[V+\Delta+v(2N)-c_p-c_c]+N[\Delta+v(2N)-v(N)]-E_m$. Given $V-V' > c_c$ and $V'-c_p > 2\Delta$, this expression for overall monopoly profit is strictly less than overall monopoly profit when the monopolist ties. Thus, the monopolist's best tying strategy dominates any strategy in which the monopolist offers individual products and the alternative producer sells complementary units to all cohort 1 consumers in the first period.

Third, the alternative producer sells no complementary units in the first period. Clearly the best the monopolist can do in this case is sell primary and complementary units to all consumers in both periods and extract all the surplus. When it can do this $\pi_m=2N[V+v(2N)-c_p-c_c]-E_m$, i.e., overall monopoly profit is the same as with tying. Achieving this result requires $P_c=V+v(2N)-P_p$. Assuming that strictly negative prices are not feasible, there are two cases. First, suppose $P^* > V+v(2N)$. If the monopolist sets $P_p \geq P^*$, then $P_c=V+v(2N)-P_p$ yields $P_c < 0$ which is not a feasible strategy. If the monopolist sets $P_p < P^*$ and $P_c=V+v(2N)-P_p$, then the alternative producer's best response is to set P_{ac} in such a fashion that cohort 1 consumers purchase complementary units from the alternative producer. Hence, if $P^* > V+v(2N)$, then the monopolist cannot achieve $\pi_m=2N[V+v(2N)-c_p-c_c]-E_m$. Second, suppose $P^* \leq V+v(2N)$. If the monopolist sets $P_p > V+v(2N)$, then $P_c=V+v(2N)-P_p$ yields $P_c < 0$ which is not a feasible strategy. If the monopolist sets $P_p < P^*$ and $P_c=V+v(2N)-P_p$, then the alternative producer's best response is again to set P_{ac} in such a fashion that cohort 1 consumers purchase complementary units from the alternative producer. However, suppose the monopolist sets $P^* \leq P_p \leq V+v(2N)$ and $P_c=V+v(2N)-P_p$. Then $P_c \geq 0$ and the alternative producer's best response is to price in such a way that the alternative producer sells no complementary units in the first period (if $P_p=P^*$, then the alternative producer is indifferent between pricing in this fashion and pricing such that it sells complementary units to all cohort 1 consumers in the first period). In other words, if $P^* \leq V+v(2N)$, then there are multiple price pairs for which overall monopoly profit is the same as with tying.

In summary, there are two parameter regimes. If $P^* > V+v(2N)$, then there is a unique equilibrium in which the monopolist offers a tied product, the alternative producer does not enter the primary market in the second period, and consumers in both cohorts purchase both primary and complementary goods from

the monopolist. If $P^* \leq V + v(2N)$, then there are multiple equilibria. One equilibrium is the same as when $P^* > V + v(2N)$. The other equilibria are identical to the tying equilibrium, except that in the first period the monopolist sells individual products and charges a high price for the primary product and a low price for the complementary product.

Proof of Lemma 2: Suppose that in period 1 the alternative producer entered the complementary market and the monopolist offered a tied product. There are two possibilities. First, suppose the alternative producer enters the newly emerging market in the second period. In this case the alternative producer's second-period profit will be minimized if the monopolist also enters the newly emerging market. Bertrand competition in this case means $\pi_{m2} = N[V - c_p - c_c] - E_n$ and $\pi_{a2} = N\Delta_n - E_n$. Second, suppose the alternative producer does not enter the newly emerging market in the second period. In this case the alternative producer's second-period profit will be maximized if the monopolist enters the newly emerging market. The prices that emerge in this case evenly split the surplus associated with consumers preferring the alternative producer's complementary units used in new-market systems. This and Bertrand competition means $\pi_{m2} = N[V - c_p - c_c] + N[V_n - c_n - c_c] + N(\Delta_n/2) - E_n$ and $\pi_{a2} = N\Delta_n/2$. Let $E_n^* = \min\{N\Delta_n/2, N(V_n - V_n' - c_c)/2\}$. If $E_n < E_n^*$, then $N\Delta_n - E_n > N\Delta_n/2$. Hence, if $E_n < E_n^*$ and in period 1 the alternative producer entered the complementary market and the monopolist offered a tied product, then the alternative producer enters the newly emerging market in period 2.

Suppose that in period 1 the alternative producer entered the complementary market and the monopolist offered individual products. There are two possibilities. First, suppose the alternative producer enters the newly emerging market in the second period. In this case the alternative producer's second-period profit will be minimized if the monopolist also enters the newly emerging market. The prices that emerge in this case evenly split the surplus associated with consumers preferring the alternative producer's complementary units used in primary-market systems. Given this and Bertrand competition, second-period profits in this case are given by $\pi_{m2} = N[V - c_p - c_c] + N(\Delta/2) - E_n$ and $\pi_{a2} = N(\Delta/2) + N\Delta_n - E_n$. Second, suppose the alternative producer does not enter the newly emerging market in the second period. In this case the alternative producer's second-period profit will be maximized if the monopolist enters the newly emerging market. The prices that emerge in this case evenly split the surplus associated with consumers preferring the alternative producer's complementary units used in both primary-market and new-market systems. This and Bertrand competition means $\pi_{m2} = N[V - c_p - c_c] + N[V_n - c_n - c_c] + N(\Delta/2) + N(\Delta_n/2) - E_n$ and $\pi_{a2} = N(\Delta/2) + N(\Delta_n/2)$. If $E_n < E_n^*$, then $N(\Delta/2) + N\Delta_n - E_n > N(\Delta/2) + N(\Delta_n/2)$. Hence, if $E_n < E_n^*$ and in period 1 the alternative producer entered the complementary market and the monopolist offered individual products, then the alternative producer enters the newly emerging market in period 2. Combining this result with the result of the above paragraph we have that, if $E_n < E_n^*$ and the alternative producer entered the

complementary market in period 1, then the alternative producer enters the newly emerging market in period 2.

Suppose the alternative producer entered the complementary market in period 1, the monopolist offered a tied product in period 1, and the alternative producer enters the newly emerging market in period 2. From above we know that if the monopolist also enters the newly emerging market in period 2, then $\pi_{m2}=N[V-c_p-c_c]-E_n$. If the monopolist does not enter, then $\pi_{m2}=N[V-c_p-c_c]$. Since $E_n>0$, we have that in this case the monopolist will not enter the newly emerging market in period 2. Suppose the alternative producer entered the complementary market in period 1, the monopolist offered individual products in period 1, and the alternative producer enters the newly emerging market in period 2. From above we know that if the monopolist also enters the newly emerging market in period 2, then $\pi_{m2}=N[V-c_p-c_c]+N(\Delta/2)-E_n$. If the monopolist does not enter, then $\pi_{m2}=N[V-c_p-c_c]+N(\Delta/2)$. Since $E_n>0$, we have that in this case the monopolist will not enter the newly emerging market in period 2. Hence, if the alternative producer enters the complementary market in the first period and is sure to enter the newly emerging market in the second period, then whether or not the monopolist ties in the first period it does not enter the newly emerging market in the second period. Combining this result with the result of the above paragraph we have that, if $E_n<E_n^*$ and the alternative producer entered the complementary market in period 1, then only the alternative producer enters the newly emerging market in period 2. This proves i).

Suppose the alternative producer never enters the complementary market (since the alternative producer never enters the complementary market, second-period profit levels are independent of whether the monopolist offered tied or individual products in period 1). There are two possibilities. First, suppose the monopolist enters the newly emerging market in the second period. In this case the monopolist's second-period profit will be minimized if the alternative producer also enters the newly emerging market. Bertrand competition in this case means $\pi_{m2}=N[V-c_p-c_c]+N[V_n-V_n'-c_c]-E_n$ and $\pi_{a2}=-E_n$. Second, suppose the monopolist does not enter the newly emerging market in the second period. In this case the monopolist's second-period profit will be maximized if the alternative producer enters the newly emerging market. The prices that emerge in this case evenly split the surplus associated with complementary units used in new-market systems. This and Bertrand competition means $\pi_{m2}=N[V-c_p-c_c]+N[(V_n-V_n'-c_c)/2]$ and $\pi_{a2}=N[(V_n'-c_n)+((V_n-V_n'-c_c)/2)]-E_n$. If $E_n<E_n^*$, then $N[V-c_p-c_c]+N[V_n-V_n'-c_c]-E_n>N[V-c_p-c_c]+N[(V_n-V_n'-c_c)/2]$. Hence, if $E_n<E_n^*$ and the alternative producer never enters the complementary market, then the monopolist enters the newly emerging market in period 2.

Suppose the alternative producer never enters the complementary market and the monopolist enters the newly emerging market in period 2 (again, since the alternative producer never enters the complementary market, second-period profit levels are independent of whether the monopolist offered tied or individual products in period 1). From above we know that if the alternative producer also enters the newly emerging market in period 2, then $\pi_{a2}=-E_n$. If the alternative producer does not enter, then $\pi_{a2}=0$. Given $E_n>0$, we now have that if the alternative producer never enters the complementary market and the

monopolist is sure to enter the newly emerging market in the second period, then the alternative producer does not enter the newly emerging market in the second period. Combining this result with the result of the above paragraph we have that, if $E_n < E_n^*$ and the alternative producer never enters the complementary market, then only the monopolist enters the newly emerging market in period 2. This proves ii).

Proof of Proposition 4: Suppose the alternative producer enters the complementary market in the first period. There are two possibilities. First, the monopolist offered a tied product in the first period. From Lemma 2 we know that in this case the alternative producer would be the sole entrant into the newly emerging market in period 2. Since in this case the alternative producer monopolizes the newly emerging market in period 2 but sells no complementary units for use in primary-market systems, the alternative producer's and monopolist's overall profits are given by $\pi_a = N[V_n + \Delta_n - c_n - c_c] - E_{ac} - E_n$ and $\pi_m = 2N[V - c_p - c_c] - E_m$. Second, the monopolist offered individual products in the first period. From Lemma 2 we know that in this case the alternative producer would again be the sole entrant into the newly emerging market in period 2. Since in this case the alternative producer both monopolizes the newly emerging market in period 2 and sells complementary units for use in primary-market systems, the alternative producer's and monopolist's overall profits are given by $\pi_a = N[V_n + \Delta_n - c_n - c_c] + N\Delta - E_{ac} - E_n$ and $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$.

Suppose the monopolist ties in the first period and the alternative producer does not enter the complementary market in the first period. In this case the alternative producer's second-period and overall profits are maximized either if the alternative producer enters neither market in period 2 which given Lemma 2 means the monopolist is the sole entrant into the newly emerging market in period 2, or if the alternative producer enters both the complementary and newly emerging markets in period 2 and the monopolist does not enter the newly emerging market (given Lemma 2, it cannot be the case that the alternative producer does not enter the complementary market in period 2 but is the sole entrant into the newly emerging market). In the former case the alternative producer's second-period and overall profits are given by $\pi_{a2} = \pi_a = 0$, while the monopolist's overall profit is given by $\pi_m = 2N[V - c_p - c_c] + N[V_n - c_n - c_c] - E_m - E_n$. In the latter case the alternative producer's second-period and overall profits are given by $\pi_{a2} = \pi_a = N[V_s + \Delta_n - c_n - c_c] - E_{ac} - E_n$, while the monopolist's overall profit is given by $\pi_m = 2N[V - c_p - c_c] - E_m$. Notice, if the monopolist ties in the first period and the alternative producer does not enter the complementary market in the first period, then the alternative producer's second-period and overall profits must satisfy $\pi_{a2} = \pi_a \leq \max\{0, N[V_n + \Delta_n - c_n - c_c] - E_{ac} - E_n\}$.

Suppose the monopolist offers individual products in the first period and the alternative producer does not enter the complementary market in the first period. In this case the alternative producer's second-period and overall profits are maximized either if the alternative producer enters neither market in period 2 which given Lemma 2 means the monopolist is the sole entrant into the newly emerging market in period 2, or if the alternative producer enters both the complementary and newly emerging markets in period 2 and

the monopolist does not enter the newly emerging market (given Lemma 2, it cannot be the case that the alternative producer does not enter the complementary market in period 2 but is the sole entrant into the newly emerging market). In the former case the alternative producer's second-period and overall profits are given by $\pi_{a2}=\pi_a=0$, while the monopolist's overall profit is given by $\pi_m=2N[V-c_p-c_c]+N[V_n-c_n-c_c]-E_m-E_n$. In the latter case the alternative producer's second-period and overall profits are given by $\pi_{a2}=\pi_a=N[V_n+\Delta_n-c_n-c_c]+N(\Delta/2)-E_{ac}-E_n$, while the monopolist's overall profit is given by $\pi_m=2N[V-c_p-c_c]+N(\Delta/2)-E_m$. Notice, if the monopolist offers individual products in the first period and the alternative producer does not enter the complementary market in the first period, then the alternative producer's second-period and overall profits are equal and must satisfy $\pi_{a2}=\pi_a\leq\max\{0, N[V_n+\Delta_n-c_n-c_c]+N(\Delta/2)-E_{ac}-E_n\}$.

Let $E_{ac}^*=N[V_n+\Delta_n-c_n-c_c]-E_n$ and $E_{ac}^{**}=N[V_n+\Delta_n-c_n-c_c]+N\Delta-E_n$. Suppose $0<E_{ac}<E_{ac}^*$. If in period 1 the monopolist ties and the alternative producer enters the complementary market, from above we know the alternative producer is the sole entrant into the newly emerging market in period 2 and $\pi_a=N[V_n+\Delta_n-c_n-c_c]-E_{ac}-E_n$. If in period 1 the monopolist ties and the alternative producer does not enter the complementary market, from above we know the alternative producer's overall profit must satisfy $\pi_a\leq N[V_n+\Delta_n-c_n-c_c]-E_{ac}-E_n$ (since $E_{ac}<E_{ac}^*$ we know $N[V_n+\Delta_n-c_n-c_c]-E_{ac}-E_n>0$). Further, $\pi_a=N[V_n+\Delta_n-c_n-c_c]-E_{ac}-E_n$ is only achieved if the alternative producer enters the complementary market in the second period and is the sole entrant into the newly emerging market in period 2. Hence, if the monopolist ties in the first period, then either the alternative producer enters the complementary market in the first period and is the sole entrant into the newly emerging market in period 2, or the alternative producer enters the complementary market in the second period and is the sole entrant into the newly emerging market in period 2 (for the alternative producer not to enter the complementary market in period 1, the subsequent subgame must have the alternative producer enter the complementary market in the second period and be the sole entrant into the newly emerging market in period 2). In either case, $\pi_a=N[V_n+\Delta_n-c_n-c_c]-E_{ac}-E_n$ and $\pi_m=2N[V-c_p-c_c]-E_m$.

If in period 1 the monopolist offers individual products and the alternative producer enters the complementary market, from above we know the alternative producer is the sole entrant into the newly emerging market in period 2 and $\pi_a=N[V_n+\Delta_n-c_n-c_c]+N\Delta-E_{ac}-E_n$. If in period 1 the monopolist offers individual products and the alternative producer does not enter the complementary market, from above we know the alternative producer's overall profit must satisfy $\pi_a\leq N[V_n+\Delta_n-c_n-c_c]+N(\Delta/2)-E_{ac}-E_n$ (since $E_{ac}<E_{ac}^*$ we know $N[V_n+\Delta_n-c_n-c_c]+N(\Delta/2)-E_{ac}-E_n>0$). Hence, if the monopolist offers individual products in the first period, then the alternative producer enters the complementary market in the first period and is the sole entrant into the newly emerging market in period 2. In turn, the alternative producer's and monopolist's overall profits are given by $\pi_a=N[V_n+\Delta_n-c_n-c_c]+N\Delta-E_{ac}-E_n$ and $\pi_m=2N[V-c_p-c_c]+N\Delta-E_m$. Comparing this expression for overall monopoly profit with the expression in the above paragraph yields that, if $0<E_{ac}<E_{ac}^*$, the monopolist offers individual products in the first period.

Suppose $E_{ac} > E_{ac}^{**}$. If in period 1 the monopolist ties and the alternative producer enters the complementary market, from above we know the alternative producer is the sole entrant into the newly emerging market in period 2 and $\pi_a = N[V_n + \Delta_n - c_n - c_c] - E_{ac} - E_n$. This means $\pi_a < 0$ since $E_{ac} > E_{ac}^{**}$. If in period 1 the monopolist ties and the alternative producer does not enter the complementary market, from above we know the alternative producer's second-period and overall profits must satisfy $\pi_{a2} = \pi_a \leq 0$ (since $E_{ac} > E_{ac}^{**}$ we know $0 > N[V_n + \Delta_n - c_n - c_c] - E_{ac} - E_n$). Further, $\pi_{a2} = \pi_a = 0$ is only achieved if the alternative producer enters neither market in the second period, which given Lemma 2 means that the monopolist is the sole entrant into the newly emerging market in period 2. Hence, if the monopolist ties in the first period, then the alternative producer never enters either market and the monopolist is the sole entrant into the newly emerging market in period 2 (the alternative producer does not enter the complementary market in period 1 knowing that the equilibrium to the subsequent subgame is that it won't enter either market in period 2). In turn, the alternative producer's and monopolist's overall profits are given by $\pi_a = 0$ and $\pi_m = 2N[V - c_p - c_c] + N[V_n - c_n - c_c] - E_m - E_n$.

If in period 1 the monopolist offers individual products and the alternative producer enters the complementary market, from above we know the alternative producer is the sole entrant into the newly emerging market in period 2 and $\pi_a = N[V_n + \Delta_n - c_n - c_c] + N\Delta - E_{ac} - E_n$. This means $\pi_a < 0$ since $E_{ac} > E_{ac}^{**}$. If in period 1 the monopolist ties and the alternative producer does not enter the complementary market, from above we know the alternative producer's second-period and overall profits must satisfy $\pi_{a2} = \pi_a \leq 0$ (since $E_{ac} > E_{ac}^{**}$ we know $0 > N[V_n + \Delta_n - c_n - c_c] + N(\Delta/2) - E_{ac} - E_n$). Further, $\pi_{a2} = \pi_a = 0$ is only achieved if the alternative producer enters neither market in the second period, which given Lemma 2 means the monopolist is the sole entrant into the newly emerging market in period 2. Hence, if the monopolist offers individual products in the first period, then the alternative producer never enters either market and the monopolist is the sole entrant into the newly emerging market in period 2 (the alternative producer does not enter the complementary market in period 1 knowing that the equilibrium to the subsequent subgame is that it won't enter either market in period 2). In turn, the alternative producer's and monopolist's overall profits are given by $\pi_a = 0$ and $\pi_m = 2N[V - c_p - c_c] + N[V_n - c_n - c_c] - E_m - E_n$. Comparing this expression for overall monopoly profit with the expression in the above paragraph yields that, if $E_{ac} > E_{ac}^{**}$, the monopolist offers either tied or individual products in the first period.

Suppose $E_{ac}^* < E_{ac} < E_{ac}^{**}$. Using an argument similar to one above, if the monopolist ties in the first period, then the alternative producer never enters either market and the monopolist is the sole entrant into the newly emerging market in period 2. In turn, the monopolist's overall profit is given by $\pi_m = 2N[V - c_p - c_c] + N[V_n - c_n - c_c] - E_m - E_n$. Also using an argument similar to one above, if the monopolist offers individual products in the first period, then the alternative producer enters the complementary market in the first period and is the sole entrant into the newly emerging market in period 2. In turn, the monopolist's overall profit is given by $\pi_m = 2N[V - c_p - c_c] + N\Delta - E_m$. Given $V_n - V_n' > c_c$ and $N(V_n' - c_n) - E_n > N\Delta$, a comparison of this expression for overall monopoly profit with the one above yields that, if $E_{ac}^* < E_{ac} < E_{ac}^{**}$, then the

monopolist ties in the first period, the alternative producer never enters either market, and the monopolist is the sole entrant into the newly emerging market in period 2.

Proof of Lemma 3: Throughout the proof we assume $v(2N)-v(N)>\Delta$. Also, to keep the notation clear the marginal cost of the new-market product will be denoted c_n . Suppose all cohort 1 consumers purchase new-market units from the monopolist. There are five possibilities for what happens in period 2. First, neither firm invests R_n in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, consumers purchase both primary and new-market units from the monopolist. In particular, the alternative producer charges c_n for a new-market unit, while the monopolist charges an aggregate price of $V+v(2N)$ for a system composed of its primary and new-market units (if the monopolist sells individual products, then the price for the primary product is in the interval $(V+\Delta+v(N)-c_n, V+v(2N))$ while the new-market-good price equals $V+v(2N)$ minus the primary-good price). This yields $\pi_{m2}=N[V+v(2N)-c_p-c_n]$ and $\pi_{a2}=0$.

Second, the monopolist invests R_n in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, consumers purchase the monopolist's new-version new-market product. In particular, the alternative producer charges c_n for a new-market unit, while the monopolist charges $V+v(2N)$ for a new-version new-market unit. This yields $\pi_{m2}=N[V+v(2N)-c_n]-R_n$ and $\pi_{a2}=0$. Let $R_n^*=N\min\{\min\{c_p, c_n\}, \min\{[v(2N)-v(N)-\Delta], [(\Delta+v(2N)-v(N))/2]\}\}$. A comparison of this expression for second-period monopoly profit with the expression above yields that, if $R_n < R_n^*$ and the alternative producer does not invest, then the monopolist invests and all cohort 2 consumers purchase the monopolist's new-version new-market product.

Third, both firms invest R_n in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, Bertrand competition in this case yields that the alternative producer charges c_n for a new-version new-market unit, the monopolist charges $c_n+v(2N)-v(N)-\Delta$ for a new-version new-market unit, and consumers purchase new-version new-market units from the monopolist. This yields $\pi_{m2}=N[v(2N)-v(N)-\Delta]-R_n$ and $\pi_{a2}=-R_n$.

Fourth, the alternative producer invests R_n in period 2 and the monopolist offers a tied product. There are three cases. First, suppose $c_p < v(2N)-v(N)-\Delta$. Bertrand competition in this case yields the alternative producer charges c_n for its new-version new-market product, the monopolist charges $c_n+v(2N)-v(N)-\Delta$ for a system composed of its primary and new-market products, and consumers purchase a system from the monopolist. This yields $\pi_{m2}=N[v(2N)-v(N)-\Delta-c_p]$ and $\pi_{a2}=-R_n$. Second, suppose $c_p > v(2N)-v(N)-\Delta$. Bertrand competition in this case yields that the monopolist charges c_p+c_n for a system composed of its primary and new-market products, the alternative producer charges $c_p+c_n+v(N)+\Delta-v(2N)$ for its new-version new-market product, and consumers purchase new-version new-market units from the alternative producer. This yields $\pi_{m2}=0$ and $\pi_{a2}=N[v(N)+\Delta-v(2N)+c_p]-R_n$. Third, suppose $c_p = v(2N)-v(N)-\Delta$. This

case is consistent with both the equilibrium in the first case in which the alternative producer sells no new-market units, and the equilibrium in the second case in which the alternative producer sells new-version new-market units to all cohort 2 consumers. In both of these equilibria, however, $\pi_{m2}=0$ and $\pi_{a2}=-R_n$.

Fifth, the alternative producer invests R_n and the monopolist offers individual products. There are four cases. First, suppose $\min\{c_p, c_n\} > v(2N) - v(N) - \Delta$. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, consumers purchase the alternative producer's new-version new-market product. In particular, the monopolist charges c_p for its primary product and c_n for its new-market product, the alternative producer charges $c_p + c_n + v(N) + \Delta - v(2N)$ for its new-version new-market product, and consumers purchase new-version new-market units from the alternative producer. This yields $\pi_{m2}=0$ and $\pi_{a2}=N[v(N) + \Delta - v(2N) + c_p] - R_n$. Second, suppose $\min\{c_p, c_n\} < v(2N) - v(N) - \Delta$ and $c_p < c_n$. This case is similar to the first case under the fourth possibility above with the result that $\pi_{m2}=N[v(2N) - v(N) - \Delta - c_p]$ and $\pi_{a2}=-R_n$. Third, suppose $\min\{c_p, c_n\} < v(2N) - v(N) - \Delta$ and $c_n < c_p$. Bertrand competition in this case yields that the alternative producer charges c_p for its new-version new-market product, the monopolist charges c_p for its primary product and $v(2N) - v(N) - \Delta$ for its new-market product, and consumers purchase new-version new-market units from the alternative producer and old-version new-market units from the monopolist. This yields $\pi_{m2}=N[v(2N) - v(N) - \Delta - c_n]$ and $\pi_{a2}=N[c_p - c_n] - R_n$. Fourth, suppose $\min\{c_p, c_n\} = v(2N) - v(N) - \Delta$. This case is consistent with both the equilibrium in the first case in which the monopolist sells nothing in the second period, and equilibria in the second and third cases in which the monopolist sells primary and/or new-market units in the second period. In all of these equilibria, however, $\pi_{m2}=0$ and $\pi_{a2}=N[v(N) + \Delta - v(2N) + c_p] - R_n$.

If $R_n < R_n^*$, then $\max\{N[v(2N) - v(N) - \Delta - \min\{c_p, c_n\}], 0\} < N[v(2N) - v(N) - \Delta] - R_n$. Hence, if $R_n < R_n^*$ and the alternative producer invests, then the monopolist also invests and all cohort 2 consumers purchase the monopolist's new-version new-market product. Combining this conclusion with one above yields that, if $R_n < R_n^*$ and all cohort 1 consumers purchase the monopolist's new-market product, then the monopolist invests and all cohort 2 consumers purchase only the monopolist's new-version new-market product.

Now consider the alternative producer. From above we know that if the monopolist invests and the alternative producer does not, then $\pi_{a2}=0$. We also know that if both firms invest, then $\pi_{a2}=-R_n$. Hence, if the monopolist is sure to invest, then the alternative producer does not. Combining this conclusion with one above yields that, if $R_n < R_n^*$ and all cohort 1 consumers purchase the monopolist's new-market product, then only the monopolist invests and all cohort 2 consumers purchase only the monopolist's new-version new-market product.

Now suppose all cohort 1 consumers purchase new-market units from the alternative producer (this will only be the case if the monopolist offers individual products). There are again four possibilities. First, neither firm invests R_n in period 2. By assumption the prices that emerge evenly split the surplus associated with consumers preferring the alternative producer's superior new-market products. This means the alternative producer charges $c_n + [(\Delta + v(2N) - v(N))/2]$ for a new-market unit, the monopolist charges c_n

or more for a new-market unit, the monopolist charges $V+\Delta+v(2N)-c_n-[(\Delta+v(2N)-v(N))/2]$ for a primary unit, and consumers purchase primary units from the monopolist and new-market units from the alternative producer. This yields $\pi_{m2}=N[V+\Delta+v(2N)-c_p-c_n-[(\Delta+v(2N)-v(N))/2]]$ and $\pi_{a2}=N[(\Delta+v(2N)-v(N))/2]$.

Second, the alternative producer invests R_n in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, Bertrand competition in this case yields that the monopolist charges c_p for its primary product and c_n for its new-market product, the alternative producer charges $c_p+c_n+\Delta+v(2N)-v(N)$ for its new-version new-market product, and consumers purchase new-version new-market units from the alternative producer. This yields $\pi_{m2}=0$ and $\pi_{a2}=N[\Delta+v(2N)-v(N)+c_p]-R_n$. A comparison of this expression for second-period alternative producer profit with the expression above yields that, if $R_n < R_n^*$ and the monopolist does not invest, then the alternative producer invests and all cohort 2 consumers purchase the alternative producer's new-version new-market product.

Third, both firms invest R_n in period 2. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, Bertrand competition in this case yields that the monopolist charges c_n for a new-version new-market unit, the alternative producer charges $c_n+\Delta+v(2N)-v(N)$ for a new-version new-market unit, and consumers purchase new-version new-market units from the alternative producer. This yields $\pi_{m2}=-R_n$ and $\pi_{a2}=N[\Delta+v(2N)-v(N)]-R_n$.

Fourth, the monopolist invests R_n in period 2. There are three cases. First, suppose $\min\{c_p, c_n\} > \Delta+v(2N)-v(N)$. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, consumers purchase the monopolist's new-version new-market product. In particular, the alternative producer charges c_n for a new-market unit, while the monopolist charges $V+v(N)$ for a new-version new-market unit. This yields $\pi_{m2}=N[V+v(N)-c_n]-R_n$ and $\pi_{a2}=0$. Second, suppose $\min\{c_p, c_n\} < \Delta+v(2N)-v(N)$. By assumption the prices that emerge evenly split the surplus associated with consumers preferring the alternative producer's superior new-market products. This means the alternative producer charges $c_n+[(\Delta+v(2N)-v(N)-\min\{c_p, c_n\})/2]$ for a new-market unit, the monopolist charges $V+\Delta+v(2N)-c_n-[(\Delta+v(2N)-v(N)-\min\{c_p, c_n\})/2]$ for either primary units (if $c_p \leq c_n$) or new-version new-market units (if $c_n \leq c_p$), and each consumer purchases a new-market unit from the alternative producer and either a primary unit (if $c_p \leq c_n$) or a new-version new-market unit (if $c_n \leq c_p$) from the monopolist. This yields $\pi_{m2}=N[V+\Delta+v(2N)-\min\{c_p, c_n\}-c_n-[(\Delta+v(2N)-v(N)-\min\{c_p, c_n\})/2]]-R_n$ and $\pi_{a2}=N[(\Delta+v(2N)-v(N)-\min\{c_p, c_n\})/2]$. Third, suppose $\min\{c_p, c_n\} = \Delta+v(2N)-v(N)$. This case is consistent with both the equilibrium in the first case in which the alternative producer sells no new-market units, and the equilibrium in the second case in which the alternative producer sells new-market units to all cohort 2 consumers. In both of these equilibria, however, $\pi_{m2}=N[V+v(N)-c_n]$ and $\pi_{a2}=0$.

If $R_n < R_n^*$, then $\max\{N[(\Delta+v(2N)-v(N)-\min\{c_p, c_n\})/2], 0\} < N[\Delta+v(2N)-v(N)]-R_n$. Hence, if $R_n < R_n^*$ and the monopolist invests, then the alternative producer also invests and all cohort 2 consumers purchase the alternative producer's new-version new-market product. Combining this conclusion with one above yields that, if $R_n < R_n^*$ and all cohort 1 consumers purchase the alternative producer's new-market

product, then the alternative producer invests and all cohort 2 consumers purchase only the alternative producer's new-version new-market product.

Now consider the monopolist. From above we know that if the alternative producer invests and the monopolist does not, then $\pi_{m2}=0$. We also know that if both firms invest, then $\pi_{m2}=-R_n$. Hence, if the alternative producer is sure to invest, then the monopolist does not. Combining this conclusion with one above yields that, if $R_n < R_n^*$ and all cohort 1 consumers purchase the alternative producer's new-market product, then only the alternative producer invests and all cohort 2 consumers purchase only the alternative producer's new-version new-market product.

Proof of Proposition 5: Throughout the proof we assume $R_n < R_n^*$ and $v(2N)-v(N) > \Delta$. Suppose the monopolist ties in period 1. If all cohort 1 consumers purchase new-market units from the monopolist, then from Lemma 3 we know that in period 2 all cohort 2 consumers will purchase new-version new-market units from the monopolist. Let P_s denote the price the monopolist charges for a system. Given that a cohort 1 consumer's net benefit from purchasing a system from the monopolist equals $V+v(N_{m1}+N_{m2})-P_s$, for any P_s that satisfies $P_s \leq V+v(2N)$, each cohort 1 consumer's utility is maximized by having every cohort 1 consumer purchase a system (if $P_s = V+v(2N)$, then each cohort 1 consumer is indifferent between having all purchase and not purchasing). Similarly, for any P_s that satisfies $P_s > V+v(2N)$, each cohort 1 consumer's utility is maximized by not purchasing. Hence, since purchase decisions are made as if consumers could coordinate behavior and since if all cohort 1 consumers purchase the resulting second-period profit is independent of the first-period price, the monopolist sets $P_s = V+v(2N)$.⁴⁰ Given results in the proof of Lemma 3, this means $\pi_{m1} = N[V+v(2N)-c_p-c_n]-E_m$, $\pi_{m2} = N[V+v(2N)-c_n]-R_n$, and $\pi_m = N[V+v(2N)-c_p-c_n]+N[V+v(2N)-c_n]-E_m-R_n$.

Suppose the monopolist offers individual products in period 1. Let P_p denote the monopolist's price for its primary product, P_n denote the monopolist's price for its new-market product, and P_{an} denote the alternative producer's price for its new-market product. A cohort 1 consumer's net benefit from purchasing a system consisting of one unit of the monopolist's primary product and one unit of the alternative producer's new-market product is $V+\Delta+v(N_{a1}+N_{a2})-P_p-P_{an}$. This means that, if the alternative producer is to sell any new-market units in period 1, the alternative producer's price for new-market units must satisfy $P_{an} \leq V+\Delta+v(2N)-P_p$. Given this, if the alternative producer sells new-market units in the first period, its overall profit cannot exceed what it earns if it sets $P_{an} = V+\Delta+v(2N)-P_p$ and sells new-market units to all cohort 1 consumers. Given results in the proof of Lemma 3, this means $\pi_a \leq N[V+\Delta+v(2N)-P_p-$

⁴⁰Because price is a continuous variable, for any P_s strictly below $V+v(2N)$, there is a higher value for P_s that increases the monopolist's overall profit. This means that, if the monopolist ties, the unique equilibrium to the subsequent subgame is characterized by $P_s = V+v(2N)$.

$c_n] + N[\Delta + v(2N) - v(N) + c_p] - R_n$. Further, let P^* satisfy $N[V + \Delta + v(2N) - P^* - c_n] + N[\Delta + v(2N) - v(N) + c_p] - R_n = 0$. We now have that the alternative producer can only sell new-market units in the first period if $P_p \leq P^*$.

If the monopolist offers individual products in period 1, there are three possibilities. First, the alternative producer sells new-market units to some but not all cohort 1 consumers in the first period. There are two cases. First, the alternative producer does not invest in the second period. Consider a cohort 1 consumer who purchased the alternative producer's new-market product. This consumer's first-period utility is given by $V + \Delta + v(N_{a1} + N_{a2}) - P_p - P_{an} \geq V + v(N_{m1} + N_{m2}) - P_p - P_n$ (if this inequality did not hold, the consumer could have increased his utility by purchasing the monopolist's new-market product).⁴¹ But if this condition holds, then all cohort 1 consumers purchasing the alternative producer's new-market product is also equilibrium behavior, and, given Lemma 3, under this alternative set of purchase strategies this consumer's first-period utility is given by $V + \Delta + v(2N) - P_p - P_{an} > V + \Delta + v(N_{a1} + N_{a2}) - P_p - P_{an}$. Given our assumption that purchase decisions are made as if consumers could coordinate behavior, we have just shown that it cannot be the case that some but not all cohort 1 consumers purchase new-market units from the alternative producer in the first period and the alternative producer does not invest in the second period. Second, the alternative producer invests in the second period. This case is ruled out using an argument similar to that used to rule out the first case.

Second, the alternative producer sells new-market units to all cohort 1 consumers in the first period. In this case, $\pi_{m1} = N[P_p - c_p] - E_m$ and from the proof of Lemma 3 we know $\pi_{m2} = 0$. This means $\pi_m = N[P_p - c_p] - E_m$ and since we also know from earlier that $P_p \leq P^*$, we have $\pi_m \leq N[P^* - c_p]$. Since P^* satisfies $N[V + \Delta + v(2N) - P^* - c_n] + N[\Delta + v(2N) - v(N) + c_p] - R_n = 0$, we have $P^* = [V + \Delta + v(2N) - c_n] + [\Delta + v(2N) - v(N) + c_p] - (R_n/N)$. Thus, $\pi_m \leq N[V + \Delta + v(2N) - c_p - c_n] + N[\Delta + v(2N) - v(N) + c_p] - E_m - R_n$. Given $V - V' > c_n$ and $V' - c_p > 2\Delta$, this expression for overall monopoly profit is strictly less than overall monopoly profit when the monopolist ties. Thus, the monopolist's best tying strategy dominates any strategy in which the monopolist offers individual products and the alternative producer sells new-market units to all cohort 1 consumers in the first period.

Third, the alternative producer sells no new-market units in the first period. Clearly the best the monopolist can do in this case is sell primary and new-market units to all cohort 1 consumers in the first period and extract all the surplus from these consumers, and, given $R_n < Nc_p$ (see the definition of R_n^* in the proof of Lemma 3), sell new-version new-market units to all cohort 2 consumers in the second period and extract all the surplus from these consumers. When it can do this $\pi_m = N[V + v(2N) - c_p - c_n] + N[V + v(2N) - c_n] - E_m - R_n$, i.e., overall monopoly profit is the same as with tying. Achieving this result requires $P_n = V + v(2N) - P_p$. Assuming that strictly negative prices are not feasible, there are two cases. First, suppose $P^* > V + v(2N)$. If the monopolist sets $P_p \geq P^*$, then $P_n = V + v(2N) - P_p$ yields $P_n < 0$ which is not a feasible

⁴¹For this step of the proof we are assuming that each cohort of consumers consists of a continuum of individuals of mass N . By assuming this, we ensure that no single cohort 1 consumer's first-period purchase decisions determine the equilibrium to the second-period subgame.

strategy. If the monopolist sets $P_p < P^*$ and $P_n = V + v(2N) - P_p$, then the alternative producer's best response is to set P_{an} in such a fashion that cohort 1 consumers purchase new-market units from the alternative producer. Hence, if $P^* > V + v(2N)$, then the monopolist cannot achieve $\pi_m = N[V + v(2N) - c_p - c_n] + N[V + v(2N) - c_n] - E_m - R_n$. Second, suppose $P^* \leq V + v(2N)$. If the monopolist sets $P_p > V + v(2N)$, then $P_n = V + v(2N) - P_p$ yields $P_n < 0$ which is not a feasible strategy. If the monopolist sets $P_p < P^*$ and $P_n = V + v(2N) - P_p$, then the alternative producer's best response is again to set P_{an} in such a fashion that cohort 1 consumers purchase new-market units from the alternative producer. However, suppose the monopolist sets $P^* \leq P_p \leq V + v(2N)$ and $P_n = V + v(2N) - P_p$. Then $P_n \geq 0$ and the alternative producer's best response is to price in such a way that the alternative producer sells no new-market units in the first period (if $P_p = P^*$, then the alternative producer is indifferent between pricing in this fashion and pricing such that it sells new-market units to all cohort 1 consumers in the first period). In other words, if $P^* \leq V + v(2N)$, then there are multiple price pairs for which overall monopoly profit is the same as with tying.

In summary, there are two parameter regimes. If $P^* > V + v(2N)$, then there is a unique equilibrium in which the monopolist offers a tied product in the first period, in period 2 only the monopolist invests, cohort 1 consumers purchase both primary and new-market units from the monopolist, and cohort 2 consumers purchase new-version new-market units from the monopolist. If $P^* \leq V + v(2N)$, then there are multiple equilibria. One equilibrium is the same as when $P^* > V + v(2N)$. The other equilibria are identical to the tying equilibrium, except that in the first period the monopolist sells individual products and charges a high price for the primary product and a low price for the new-market product.

Proof of Proposition 6: Suppose the alternative producer has entered both markets prior to period $T-1$. Consider the monopolist's decision concerning whether to invest R_p and R_c in period $T-1$. Suppose the monopolist has decided to invest R_p in period $T-1$ and is now deciding whether to invest R_c . Holding fixed the alternative producer's investment choices in period $T-1$, the incremental expected net return for investing R_c in period $T-1$ is greater than or equal to $N[2p(1-p)\Delta] - R_c$. Given $N[p(1-p)\Delta] > R_p + R_c$, we have that, if the monopolist invests R_p in period $T-1$, then it also invests R_c in period $T-1$.

Suppose the monopolist has decided to invest R_c in period $T-1$ and is now deciding whether to invest R_p . Holding fixed the alternative producer's investment choices in period $T-1$, the incremental expected net return for investing R_p in period $T-1$ is greater than or equal to $N\min\{p(1-p)\Delta, 2(V_{T-1} - V_{T-3}) - p(1-p)\Delta\} - R_p$. Given $N[p(1-p)\Delta] > R_p + R_c$ and $V_{t+2} > V_t + \Delta$ for all odd values for t , we have that, if the monopolist invests R_c in period $T-1$, then it also invests R_p in period $T-1$.

Suppose the monopolist is deciding between investing R_p and R_c in period $T-1$ and investing nothing in period $T-1$. Holding fixed the alternative producer's investment choices in period $T-1$, the incremental expected net return for investing R_p and R_c in period $T-1$ is greater than or equal to $N\min\{2p(1-p)\Delta, 2(V_{T-1} - V_{T-3}) - p(1-p)\Delta\} - (R_p + R_c)$. Given $N[p(1-p)\Delta] > R_p + R_c$ and $V_{t+2} > V_t + \Delta$ for all odd

values for t , we have that the monopolist prefers to invest R_p and R_c in period $T-1$ rather than investing nothing in period $T-1$.

The three arguments together yield that, if the alternative producer has entered both markets prior to period $T-1$, the monopolist invests R_p and R_c in period $T-1$. Further, repeating the argument for the alternative producer yields that, if the alternative producer has entered both markets prior to period $T-1$, both the monopolist and the alternative producer invest R_p and R_c in period $T-1$. Further, repeating the argument for periods $T-3, T-5, \dots, 3$ yields that, if the alternative producer has entered both markets prior to odd-numbered period t' , then both the monopolist and the alternative producer invest R_p and R_c in period t' and in every subsequent odd-numbered period.

Suppose the alternative producer has entered neither market prior to period $T-1$ and consider the monopolist's choice of whether or not to tie in period $T-1$. Also, assume for the moment that the monopolist has decided to invest R_p and R_c in period $T-1$. There are two possibilities. First, the monopolist offers a tied product in period $T-1$. If the alternative producer decides to enter either market in period $T-1$ or period T , its best option is to enter the primary market in period T only when the monopolist offers a low-quality complementary product and to enter the complementary market in either period $T-1$ or period T only when the monopolist offers a low-quality complementary product. This yields $E(\pi_{mT-1} + \pi_{mT}) = N[V_{T-1} + p\Delta - c_p - c_c] + N[p(1-p)\Delta] - (R_p + R_c)$ and $E(\pi_{aT-1} + \pi_{aT}) = N[p(1-p)\Delta] - (1-p)(E_{ap} + E_{ac})$. Note, if the monopolist ties and the alternative producer stays out of both markets in both periods, then $E(\pi_{mT-1} + \pi_{mT}) = 2N[V_{T-1} + p\Delta - c_p - c_c] - (R_p + R_c)$.

Second, the monopolist offers individual products in period $T-1$. If the alternative producer decides to enter either market in period $T-1$ or period T , there are two possibilities for its best strategy. If it enters the complementary market in period $T-1$ only when the monopolist offers a low-quality complementary product and the primary market in period T only when the monopolist offers a low-quality complementary product, then $E(\pi_{mT-1} + \pi_{mT}) = N[V_{T-1} + p\Delta + p(1-p)(\Delta/2) - c_p - c_c] + N[p(1-p)\Delta] - (R_p + R_c)$ and $E(\pi_{aT-1} + \pi_{aT}) = N[p(1-p)(\Delta/2)] + N[p(1-p)\Delta] - (1-p)(E_{ap} + E_{ac})$. If it enters the complementary market in period $T-1$ only when the monopolist offers a low-quality complementary product, then $E(\pi_{aT-1} + \pi_{aT}) = N[p(1-p)\Delta] - (1-p)E_{ac}$. Let $E_{ap}^* = N[p(\Delta/2)]$. Given $E_{ap} < E_{ap}^*$, the first option dominates the second. Note, if the monopolist offers individual products and the alternative producer stays out of both markets in both periods, then as before $E(\pi_{mT-1} + \pi_{mT}) = 2N[V_{T-1} + p\Delta - c_p - c_c] - (R_p + R_c)$.

Let $E_{T-1}^* = Np\Delta - E_{ap}$ and $E_{T-1}^{**} = Np(3\Delta/2) - E_{ap}$. There are three cases. First, if $E_{ac} \geq E_{T-1}^{**}$, then the alternative producer stays out of both markets in both periods whether or not the monopolist ties (if $E_{ac} = E_{T-1}^{**}$ this is one of two possible outcomes when the monopolist offers individual products). Second, if $E_{ac} \leq E_{T-1}^*$, then the alternative producer enters both markets whether or not the monopolist ties (if $E_{ac} = E_{T-1}^*$ this is one of two possible outcomes when the monopolist ties).

Third, if $E_{T-1}^* \leq E_{ac} \leq E_{T-1}^{**}$, then the alternative producer stays out of both markets in both periods when the monopolist ties (if $E_{ac} = E_{T-1}^*$ this is one of two possible outcomes). However, if the monopolist

offers individual products, then the alternative producer enters the complementary market in period T-1 when the monopolist offers a low-quality complementary product and the primary market in period T when the monopolist offers a low-quality complementary product (if $E_{ac}=E_{T-1}^{**}$ this is one of two possible outcomes. This means $E(\pi_{mT-1}+\pi_{mT})=2N[V_{T-1}+p\Delta-c_p-c_c]-(R_p+R_c)$ if the monopolist ties, while $E(\pi_{mT-1}+\pi_{mT})=p[2N[V_{T-1}+\Delta-c_p-c_c]-(R_p+R_c)]+(1-p)[N[V_{T-1}+p(\Delta/2)-c_p-c_c]-(R_p+R_c)]$ if the monopolist offers individual products. Given $V_1'-c_p>\Delta/2$ and $V_t-V_t'>c_c$ for all odd values for t, we know $2N[V_{T-1}+p\Delta-c_p-c_c]-(R_p+R_c)>p[2N[V_{T-1}+\Delta-c_p-c_c]-(R_p+R_c)]+(1-p)[N[V_{T-1}+p(\Delta/2)-c_p-c_c]-(R_p+R_c)]$. Hence, if $E_{T-1}^*<E_{ac}<E_{T-1}^{**}$, the monopolist ties and the alternative producer stays out of both markets in both periods.

We now know that, if $E_{ac}>E_{T-1}^*$, an investment of R_p and R_c by the monopolist in period T-1 means the alternative producer stays out of both markets in both periods. Given this and using arguments similar to those presented earlier yields that, if $E_{ac}>E_{T-1}^*$, the monopolist invests R_p and R_c in period T-1. In turn, this means the above analysis is valid if $E_{ac}>E_{T-1}^*$.

Suppose the alternative producer has entered neither market prior to period T-3 and consider the monopolist's choice of whether or not to tie in period T-3. Also, assume $E_{ac}>E_{T-1}^*$ and for the moment that the monopolist has decided to invest R_p and R_c in period T-3. We know from above that, if the alternative producer enters both markets prior to period T-1, then $E(\pi_{mT-1}+\pi_{mT})=E(\pi_{aT-1}+\pi_{aT})=2N[p(1-p)\Delta]-(R_p+R_c)>0$. Let $Z=2N[p(1-p)\Delta]-(R_p+R_c)$, $E_{T-3}^*=Np\Delta-E_{ap}+Z$, and $E_{T-3}^{**}=Np(3\Delta/2)-E_{ap}+Z$. Using the same arguments as above yields there are three cases. First, if $E_{ac}\geq E_{T-3}^{**}$, then the alternative producer stays out of both markets in period T-3 and period T-2 whether or not the monopolist ties (if $E_{ac}=E_{T-3}^{**}$ this is one of two possible outcomes when the monopolist offers individual products). Second, if $E_{ac}\leq E_{T-3}^*$, then the alternative producer enters both markets in period T-3 and period T-2 whether or not the monopolist ties (if $E_{ac}=E_{T-3}^*$ this is one of two possible outcomes when the monopolist ties).

Third, if $E_{T-3}^*\leq E_{ac}\leq E_{T-3}^{**}$, then the alternative producer stays out of both markets in period T-3 and period T-2 when the monopolist ties (if $E_{ac}=E_{T-3}^*$ this is one of two possible outcomes). However, if the monopolist offers individual products, then the alternative producer enters the complementary market in period T-3 when the monopolist offers a low-quality complementary product and enters the primary market in period T-2 when the monopolist offers a low-quality complementary product (if $E_{ac}=E_{T-3}^{**}$ this is one of two possible outcomes). This means $E(\pi_{mT-3}+\pi_{mT-2}+\pi_{mT-1}+\pi_{mT})=2N[V_{T-3}+p\Delta-c_p-c_c]+2N[V_{T-1}+p\Delta-c_p-c_c]-2(R_p+R_c)$ if the monopolist ties, while $E(\pi_{mT-3}+\pi_{mT-2}+\pi_{mT-1}+\pi_{mT})=p[2N[V_{T-3}+\Delta-c_p-c_c]+2N[V_{T-1}+p\Delta-c_p-c_c]-2(R_p+R_c)]+(1-p)[N[V_{T-3}+p(\Delta/2)-c_p-c_c]+2N[p(1-p)\Delta]-2(R_p+R_c)]$ if the monopolist offers individual products. Given $V_1'-c_p>\Delta/2$ and $V_t-V_t'>c_c$ for all odd values for t, we know $2N[V_{T-3}+p\Delta-c_p-c_c]+2N[V_{T-1}+p\Delta-c_p-c_c]-2(R_p+R_c)>p[2N[V_{T-3}+\Delta-c_p-c_c]+2N[V_{T-1}+p\Delta-c_p-c_c]-2(R_p+R_c)]+(1-p)[N[V_{T-3}+p(\Delta/2)-c_p-c_c]+2N[p(1-p)\Delta]-2(R_p+R_c)]$. Hence, if $E_{T-3}^*<E_{ac}<E_{T-3}^{**}$, then the monopolist ties and the alternative producer stays out of both markets in period T-3 and period T-2. Note, there are two subcases. If $E_{T-3}^*<E_{ac}<E_{T-3}^{**}$ and $E_{ac}<E_{T-1}^{**}$, then the monopolist ties in both period T-3 and period T-1 and the alternative producer never enters either market. If $E_{T-3}^*<E_{ac}<E_{T-3}^{**}$ and $E_{ac}\geq E_{T-1}^{**}$, then the monopolist

ties in period $T-3$, the monopolist offers either tied or individual products in period $T-1$, and the alternative producer never enters either market.

We now know that, if $E_{ac} > E_{T-3}^*$, an investment of R_p and R_c by the monopolist in period $T-3$ means the alternative producer never enters either market. Given this and using arguments similar to those presented earlier yields that, if $E_{ac} > E_{T-3}^*$, the monopolist invests R_p and R_c in period $T-3$. In turn, this means the above analysis is valid if $E_{ac} > E_{T-3}^*$.

Repeating the above argument for periods $T-5, T-7, \dots, 1$ yields the following. For each odd-numbered period t , let $E_t^* = Np\Delta - E_{ap} + [(T-t-1)/2]Z$ and $E_t^{**} = Np(3\Delta/2) - E_{ap} + [(T-t-1)/2]Z$. Denote E_1^* as E_{ac}^* and E_1^{**} as E_{ac}^{**} . If $E_{ac}^* < E_{ac} < E_{ac}^{**}$, then the monopolist invests R_p and R_c in every odd-numbered period starting with period 3 and the alternative producer never enters either market. Further, the monopolist offers a tied product in every odd-numbered period t such that $E_{ac} < E_t^{**}$, and offers either tied or individual products in every odd-numbered period t such that $E_{ac} \geq E_t^{**}$. Let t^{**} be the largest odd value for t such that $E_{ac} < E_t^{**}$ and let $t^* = t^{**} + 1$.

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