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ELASTICITIES OF SUBSTITUTION IN REAL BUSINESS CYCLE MODELS WITH HOME PRODUCTION

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ABSTRACT

This paper constructs a simple model of home production that demonstrates the connection between the intertemporal elasticity of substitution in market consumption (IES) and the static elasticity of substitution between home and market consumption (SES), when the utility function is additively separable over home and market consumption. Understanding this connection is important because there is a large body of empirical evidence suggesting that the IES is small, but little evidence on the size of the SES. We use our framework to shed light on the properties of a home production model with a low IES. We find that such a model must have two fundamental properties in order to match key aspects of the U.S. aggregate data. First, the steady-state growth rate of technology must be the same across sectors. Second, shocks to technology must be sufficiently positively correlated across sectors.

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1. Introduction

Recently there has been considerable interest in modifying the standard real business cycle model to include home production. Authors such as Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), Greenwood, Rogerson and Wright (1993), and Rupert, Rogerson and Wright (1997) have documented the importance of the home sector in the U.S. economy, and have shown that home production can improve the quantitative performance of the standard model.

In almost all these studies, households derive utility from three "goods": market consumption, home consumption, and leisure. Home consumption is considered to be a substitute for market consumption, as for example a home-cooked meal is a substitute for a meal in a restaurant. Leisure is distinct from both these forms of consumption and is modelled in a traditional manner, as time not occupied by home or market production.

In this paper we adopt a different perspective. We argue that households value their leisure time because of what they can do with it. Valued leisure is not the residual time unoccupied by production; after all, time spent in prison is unproductive, but does not generate utility in the same way as time spent at home. From this point of view, it is natural to think that valued leisure is the output from a home production function in which home or leisure time, home capital, and home technology appear just as market time, market capital, and market technology do in the market production function. Accordingly we follow Greenwood and Hercowitz (1991) and use a model in which households derive utility from two "goods": market consumption and home consumption, where the latter replaces the traditional leisure variable.

Furthermore, we assume that home consumption enters the utility function separably from market consumption. We do this for two reasons. First, the traditional real business cycle literature commonly assumes that utility is additively separable over consumption and leisure. Hansen (1985), for example, writes utility as the sum of the logarithms of consumption and leisure. Since we are treating home production as a generalization of the traditional concept of leisure, it is logical for us to modify the traditional model by simply replacing leisure with home consumption in the utility function. By preserving the additively separable specification when adding a home production sector, we can easily fix the value of the intertemporal elasticity of substitution in consumption at a common level across the traditional and home production models, thereby making the two frameworks more directly comparable.

Second, aggregate data offer no evidence of any important nonseparability between market consumption and labor hours (see Eichenbaum, Hansen and Singleton [1988]; Campbell and Mankiw [1990]; Beaudry and van Wincoop [1996]). For example, Campbell and Mankiw find that although there is substantial predictable variation in hours, it is not significantly related to predictable consumption growth as it should be if utility over leisure and consumption were additively nonseparable. This evidence suggests that consumption and nonmarket hours can be well characterized by an additively separable utility function over consumption and nonmarket time.

An important advantage of our framework is that it allows us to investigate more general preferences than those typically specified in the real business cycle literature. A long-standing difficulty with real business cycle models first pointed out by King, Plosser, and Rebelo (1988) is that log utility for consumption is required to obtain a constant steady state labor supply when utility is additively separable over consumption and leisure. This restriction is undesirable because it confines the intertemporal elasticity of substitution in consumption (IES) to unity, a value that is inconsistent with a large and growing number of empirical estimates that are much lower, indeed close to zero.[1] By introducing steady-state

[1]E.g., see Attanasio and Weber, 1993; Campbell, Lo, and MacKinlay, 1997; Campbell and Mankiw, 1989; Hall, 1988; Ludvigson (forthcoming). For a dissenting analysis see Beaudry and van Wincoop (1996).

technological progress in the home sector, we can free up the curvature of the utility function and study the effects of introducing an empirically plausible IES into the standard model.

In our model there is a tight link between the intertemporal elasticity of substitution in market consumption and the static elasticity of substitution between home and market consumption (SES). Our assumption of additive separability implies that these two elasticities are equal. While equality follows only from additive separability, a positive relationship between the IES and the SES is not unique to our framework; using parameter values typically assumed, it is also a feature of the most commonly employed model in the existing home production literature. We find this implication intuitively appealing: the more willing agents are to substitute consumption over time in response to technology-driven shocks to the ex-ante real interest rate, the more willing they are to substitute between home and market consumption in response to technology-driven productivity differentials between the home and market sectors.

This positive relationship between the IES and the SES is important because there is little direct empirical evidence on the value of the SES. We argue that existing evidence for a low IES suggests that the SES is also low. By contrast, the existing home production literature assumes a high SES, and this assumption is critical for the improvements in the quantitative performance of real business cycle models documented in the literature.

To explore the theoretical properties of a model with time-separable preferences across home and market consumption, we use a standard representative-agent framework with isoelastic utility. Leisure time interacts with a home technology process, and possibly with home capital, to produce home goods and services, and affects utility only through its role as an input to home production. We solve the model using the analytical approach of Campbell (1994). To facilitate comparison with the existing literature, we use the solution to simulate the model's endogenous variables, comparing their relative variability and comovements with those found in aggregate U.S. data. Four special cases are studied, including a benchmark model which assigns a minimal role to the home sector, and a more general model which allows for home technological change and the use of home capital.

Our results suggest two important insights about the underlying structure of a home production model with a low IES. First, in steady state, balanced growth requires the home and market sectors to display the same long-run growth rate of technology. Second, procyclical variation in both market hours and market consumption around the steady state requires a sufficiently positive correlation between the technology shocks to the home and market sectors. Equivalently, intersectoral productivity shocks must be small; and as the IES approaches zero, they must disappear altogether. This result contrasts with the existing literature which typically requires large intersectoral productivity shocks to improve the quantitative performance of the traditional RBC model.

The rest of this paper is organized as follows. Section 2 presents the model and assumptions. Section 2.1 discusses the steady state, while section 2.2 outlines the solution procedure for studying the economy's response to technology shocks out of steady state. Section 3 presents the approximate analytical solutions, focusing on how technology shocks influence the model economy. Section 4 presents time series simulations of the model, and compares its dynamic properties with those of the standard RBC model, and with the U.S. data. Section 5 concludes.

2. The Model

Consider an individual who receives utility from consumption of market goods, C, and home goods, H. This representative agent maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \rho^t U(C_t, H_t), \tag{2.1}$$

where ρ is the discount factor restricted to be between zero and one, and preferences are specified as

$$U(C_t, H_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \theta \frac{H_t^{1-\lambda}}{1-\lambda}. \tag{2.2}$$

The intertemporal elasticity of substitution in market consumption is given by $1/\gamma \equiv \sigma$. Output is produced in both the home and market sectors according to the following Cobb-Douglas production technologies:

$$Y_t = K_t^{1-\alpha}(A_t N_t)^{\alpha}, \tag{2.3}$$

and

$$H_t = D_t^{1-\beta}(Z_t(1-N_t))^{\beta}, \tag{2.4}$$

where Y_t is market output, K_t is market capital, D_t is household capital, and N_t is the portion of labor's endowed time allocated to market activities. A_t and Z_t are labor augmenting technological shocks to the market and home production sectors, respectively.

Two points about the preferences and technologies specified above deserve mention. First, all nonmarket time is assumed to be devoted to home production rather than dividing it between leisure and home production hours. This captures the idea that leisure is not valued for its own sake, but for what can be done with it. Second, equations (2.4) and (2.2) taken together indicate that home production equals home consumption period by period; hence investment can take place only in the market sector. More specifically, if the evolution of each capital stock is denoted by

$$K_{t+1} = (1 - \delta)K_t + I_{kt} \tag{2.5}$$

and

$$D_{t+1} = (1 - \delta)D_t + I_{dt}, \tag{2.6}$$

where δ is the common rate of depreciation, I_{kt} is gross business investment and I_{dt} is gross household investment, the resource constraint for market output is given by

$$Y_t = C_t + I_{kt} + I_{dt}. \tag{2.7}$$

The first order conditions are as follows. For market capital accumulation, the standard Euler equation holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\alpha)K_{t+1}^{-\alpha}(A_{t+1}N_{t+1})^{\alpha} + (1-\delta)], \tag{2.8}$$

where the quantity in brackets is the gross marginal product of market capital. For household capital accumulation a similar intertemporal condition holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\beta)D_{t+1}^{-\beta}(Z_{t+1}N_{t+1})^{\beta}\theta\frac{H_{t+1}^{-\lambda}}{C_{t+1}^{-\gamma}} + (1-\delta)], \tag{2.9}$$

where we define the quantity in brackets analogously as the gross marginal product of home capital. Finally, for the allocation of labor between market and home activities there is a static first order condition:

$$\alpha K_t^{1-\alpha}A_t^{\alpha}N_t^{\alpha-1}C_t^{-\gamma} = \theta H_t^{-\lambda}\beta D_t^{1-\beta}Z_t^{\beta}(1-N_t)^{\beta-1}. \tag{2.10}$$

2.1. The balanced growth path

The driving force of steady-state growth is technological progress, and we assume that A_t, Z_t, Y_t, K_t, C_t, D_t, and H_t grow at the common gross rate, G, along a balanced growth path with constant hours, N.

Equation (2.10) illustrates how hours can be constant along the balanced growth path in this model, even if utility over market consumption is not logarithmic. Taking logs and first differences of both sides, the equation implies that the following relationship holds along the balanced growth path:

$$(1-\alpha)g + \alpha g - \gamma g = -\lambda g + (1-\beta)g + \beta g, \tag{2.11}$$

where lowercase letters denote logs of variables, and the approximation $\log(G) \approx 1 + g$ has been used. If $\beta = 1$ and there are no shocks to Z_t, the model is essentially the standard one, except that there is now steady state growth in Z. Without steady-state technological progress in the home sector, the right hand side of (2.10) would be constant. This would imply that the right hand side of (2.11) is zero, requiring $\gamma = 1$. This restriction in traditional RBC models pins down the curvature of the utility function over consumption. By contrast, steady-state technological progress in the home production sector permits a continuum of values for γ without violating balanced growth.

Two other aspects of equation (2.11) are worth noting. First, no matter what the values of α, β, γ, and λ, balanced growth requires the steady-state growth rates of A_t and Z_t to be the same. Second, no matter what the values of α and β, a restriction necessary for balanced growth is $\lambda = \gamma$. We impose this from now on.

Along the balanced growth path, (2.8) becomes

$$G^{\gamma} = \rho R, \tag{2.12}$$

where,

$$R_{t+1} \equiv (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta). \tag{2.13}$$

R_{t+1} is the gross marginal product of market capital, equal to a constant, R, along the balanced growth path. Note that equation (2.12) pins down the value of ρ, given G, R, and γ. By combining (2.12), (2.8), and (2.3), the steady-state output to capital ratio can be obtained:

$$\frac{Y}{K} = \left(\frac{A_t N}{K_t}\right)^{\alpha} \approx \frac{r+\delta}{(1-\alpha)}, \tag{2.14}$$

where the approximate equality arises from setting $R \approx 1 + r$. Because the first order condition for market capital accumulation is the same as in the standard RBC model, (2.14) is the standard result for the output to capital ratio.

Equations (2.12) and (2.9) can be combined to yield the steady-state ratio of home production to home capital:

$$\frac{H}{D} = \left(\frac{Z_t(1-N)}{D_t}\right)^{\beta} \approx \frac{(r+\delta)H_t^{\gamma}}{(1-\beta)\theta C_t^{\gamma}}. \tag{2.15}$$

From (2.10), the steady-state ratio of home to market capital is

$$\frac{D}{K} = \frac{(1-N)\alpha(1-\beta)}{N\beta(1-\alpha)}. \tag{2.16}$$

Equation (2.16) implies that the steady-state ratio of home to market capital is equal to the steady-state ratio of home to market hours, if the share of market capital in market output is the same as the share of home capital in home output.

Finally, equation (2.16), along with (2.5), (2.6), and (2.7) together imply that the steady-state market consumption to market capital ratio is:

$$\frac{C}{Y} = 1 - \frac{\alpha}{\beta}(1-\beta)\left(\frac{g+\delta}{r+\delta}\right)\frac{(1-N)}{N} - \frac{(g+\delta)(1-\alpha)}{r+\delta}. \tag{2.17}$$

By combining (2.17), (2.14), and (2.16), an expression for steady-state C/D can be obtained. This can be equated with the ratio $C/D = \frac{H/D}{H/C}$, implied from (2.15), which explicitly links A/Z and $(1-N)/N$ given θ. It is difficult to know how to calibrate θ. Fortunately, it is much easier to calibrate N, and by considering a number of special cases for A/Z, we can leave the constant θ undefined in modeling fluctuations. We discuss these cases along with calibration assumptions next.

2.2. Fluctuations around steady state

Away from steady state, the model consists of a system of nonlinear expectational equations. To solve this model we use the analytical technique of Campbell (1994), which seeks an approximate solution by transforming nonlinear equations into loglinear difference equations. Each equation is loglinearized around steady state ratios of variables given above, so that variables in logs represent deviations from steady state. Below, we review the procedure only briefly, and refer the reader to Campbell (1994) for details.

Before solving the model, a number of parameter values must be chosen. Two difficult parameters to set are β and the ratio A/Z. Given that there is little evidence available to assess what values for these parameters would be reasonable, we limit our analysis to the following four cases: Case 1: $\beta = 1$, $Z_t = 1$, $\alpha < 1$, A_t varies; Case 2: $\beta = 1$, $\alpha < 1$, $A_t^{\alpha} = Z_t$; Case 3: $\alpha = \beta$, $Z_t = 1$, A_t varies; Case 4: $\alpha = \beta$, $A_t = Z_t$. To close the model, we assume that log deviations from steady-state technological progress follow a first-order autoregressive process, $a_{t+1} = \phi a_t + \varepsilon_{t+1}, 0 \leq \phi \leq 1$.

These four cases cover a range of possibilities. Case 1 minimizes the role of the home production sector by eliminating both home capital and innovations in home technology; thus, it is most similar to the standard RBC setup. The only difference from the standard RBC model is that home technology grows nonstochastically in steady state. With the further assumption that $\sigma \equiv 1/\gamma = 1$, this case is observationally equivalent to Hansen's (1985) divisible labor model with log utility over consumption and leisure. We will refer to Case 1 with $\sigma \equiv 1/\gamma = 1$ as the *standard model.*

Case 2 allows fluctuations in home technology (scaled by the same factor as fluctuations in market technology), but assumes that home capital does not enter the household production function. Case 3 adds home capital, but assumes only nonstochastic growth in home technology. Cases 1 and 3 deliver the maximum degree of *relative* productivity variation across sectors in response to technology shocks. Finally, Case 4 restricts both the technology shock and the share of capital to be the same across production functions. We discuss the model's solution in each of these cases below.

Other parameters in the model are calibrated at quarterly rates as follows. The steady state growth rate g is set to 0.005 (2 percent at an annual rate), the steady state real interest rate, r, is set equal to 0.015 (6 percent at an annual rate); α, labor's share in the market production process, is set to 0.667; the discount rate, δ, is set equal to 0.025 (10 percent at an annual rate), and N, the steady state allocation of hours to market activities is taken to be 1/3. We allow for σ and ϕ to take on a range of values, discussed below.

In cases 3 and 4 we further assume that capital can be re-allocated between the home and market sectors within the period. This assumption implies that the gross marginal products of home and market capital (defined implicitly by the two intertemporal first order conditions (2.8) and (2.9)) are equated within the period, and allows us to define a single summary capital stock state variable, $F_t \equiv K_t + D_t$, rather than having each capital stock enter the model separately. Defining a single capital stock state variable greatly simplifies the analytical solution procedure.[2]

An analytical solution to the system of nonlinear equations is sought by trans-

[2]When each capital stock enters the problem separately, the analytical solution procedure requires solving a pair of quadratic equations for the elasticity of market consumption with respect to each capital stock. This makes the problem intractable since the solution to this highly nonlinear system has at least four roots.

forming the model into a system of approximate loglinear expectational difference equations. As before, lower case letters denote logs of variables. In cases 1 and 2, this procedure yields a loglinear solution for the log deviation from steady state as a function of the two state variables, k_t and a_t, equal to:

$$v_t = \eta_{vk} k_t + \eta_{va} a_t \tag{2.18}$$

for $v_t = c_t, k_{t+1}, n_t, y_t, h_t$,and where η_{yx} denotes the partial elasticity of y with respect to x, assumed constant. Similarly, for Cases 3 and 4, the procedure yields a solution for the log deviation from steady state as a function of the two state variables, f_t and a_t :

$$v_t = \eta_{vf} f_t + \eta_{va} a_t, \tag{2.19}$$

for $v_t = c_t, f_{t+1}, k_t, d_{t,} n_t, y_t,$ and h_t. The elasticities are complex functions of the parameters in the model and the steady-state ratios of variables discussed above. Appendix A gives the complete analytical solutions for each case.

A simplifying feature that the model shares with the standard model is that elasticities with respect to the current period capital stock ($\eta_{.k}$) depend on the IES (and therefore on the elasticity of substitution between home and market consumption), but not on the persistence parameter in the technology process (see Campbell, 1994). This is because elasticities with respect to the capital stock measure the effect on current variables of an increase in capital, holding fixed the level of technology.

3. Elasticities and their Interpretation

3.1. General properties of the model

The model we have presented has two important properties. First, the static elasticity of substitution between home and market consumption, which we will denote σ_{ch}, is equal to σ, the intertemporal elasticity of substitution in market consumption.[3] This model yields a one-to-one correspondence between willingness to substitute market consumption over time, and willingness to substitute between market and home produced goods. Intuitively, if individuals are relatively unresponsive to technology-induced shifts in the expected real interest rate, they will also be relatively unresponsive to technology-induced changes in relative productivity differentials across sectors.

Though the separable specification we consider makes this intuition straight-

[3]The static elasticity of substitution between home and market consumption is defined as $\frac{\partial \ln(H_t/C_t)}{\partial \ln(P_h/P_c)}$, where P_h/P_c is the shadow price of home goods, equal to $\frac{U_2(C_t,H_t)}{U_1(C_t,H_t)}$.

forward, a positive relationship between σ and σ_{ch} is not unique to our framework, and applies more generally to popular nonseparable specifications. For example, Benhabib et al. (1993) and McGrattan et al. (1993) use a nonseparable specification with leisure, home consumption, and market consumption in which $U = u(\widetilde{C})v(L) \equiv \widetilde{C}^{b(1-r)}L^{(1-b)(1-r)}/(1-r)$, where L is leisure and $\widetilde{C}$ is a composite consumption good consisting of market and home consumption equal to $[aC^e + (1-a)H^e]^{1/e}$. In this model the static elasticity $\sigma_{ch} = 1/(1-e)$.

Defining the IES in this case as $\sigma \equiv -u_c/(u_{cc}C)$, it is straightforward to show that

$$\sigma = \frac{-\partial\widetilde{C}/\partial C}{[(b(1-r)-1)\widetilde{C}^{-1}(\partial\widetilde{C}/\partial C)^2 + \partial^2\widetilde{C}/\partial C^2]C}.$$

The relationship between σ and σ_{ch} depends on the value of r; the studies which use this specification set $r = 1$.[4] Although σ depends on the amount of home consumption relative to market consumption, as well as on the parameters a and b, over a grid covering reasonable ranges of these parameters, σ is increasing in e, and therefore increasing in σ_{ch}. Moreover when $r = 1$, the value of σ is at least as

[4] McGrattan, Rogerson and Wright (1991) estimate that r is about 1. McGrattan, Rogerson and Wright (1993) estimate a higher r (about 5), but because the standard error is large, they cannot reject the hypothesis that $r = 1$ and so they too keep r at unity. Greenwood, Rogerson and Wright (1993) and Benhabib, Rogerson and Wright (1991) also set $r = 1$.

large as one, and only approaches one when e is close to zero. Values of e typically assumed in the home production literature are as high as 0.8, implying that σ is considerably above unity, at odds with a large number of empirical estimates cited above which suggest that the IES is close to zero.[5]

A second property of the general model concerns the behavior of market hours as market and home consumption become highly complementary. As σ approaches zero, even though the agent is very averse to shifting the ratio of C to H, N can shift, and adjusts passively to insure a fixed ratio of home to market consumption.

3.2. The effects of home and market technology shocks

In this section, we consider how innovations to market and home technology influence consumption, labor supply, output, and the capital stock. These effects are given by the partial elasticities with respect to a_t. We focus our discussion on these elasticities, though the capital elasticities are also provided in the tables for reference.

Table 1 gives consumption, capital, employment, and output elasticities for

[5]We are aware of only two studies which attempt to estimate the value of e in the home production model specified above. McGrattan et al. (1993) use aggregate data, and Rupert et al. (1995) use household level data from the Panel Study of Income Dynamics (PSID). It should be noted that neither of these studies estimate values for e that are nearly as large as 0.8; the former study estimates $e = 0.385$, while the latter study finds a very small (and imprecisely estimated) value of e for single men, and a statistically significant but small value of e for single women.

Case 1. The table shows the numerical values of the elasticities, for the benchmark values of the parameters discussed above, and for various values of σ and ϕ. σ is set equal to $0, 0.2, 1, 5$, and ∞. ϕ is set equal to $0, 0.5, 0.95$, and 1.

Case 1 generalizes the standard RBC model by freeing up the curvature in the utility function over consumption. As a result, it is worthwhile elaborating on several features of this case. First, as already noted, when $\sigma = 1$, this case collapses to the standard Hansen RBC model with divisible labor and log utility over consumption and leisure. Hence the elasticities in the middle column of Table 1 are the same as those given in Campbell (1994), which uses the analytical technique employed here to solve the standard model.

Second, the elasticity of consumption with respect to a positive technology shock, η_{ca}, is increasing in persistence for low σ, but decreasing for high σ. When σ is low, substitution effects are weak and the agent responds primarily to income effects which increase with the persistence of a technology shock. When σ is high, substitution effects are important, and a more persistent technology shock increases the interest rate today and in the future, motivating a large substitution into consumption tomorrow; hence consumption elasticities can be very small, or even negative.

Third, in the extreme case when $\sigma = \infty$, a positive technology shock leads to a very large decrease in consumption and a very large increase in next period's capital stock for $\phi > 0$. In this case, the representative agent is risk neutral and consumers are infinitely willing to substitute consumption over time in response to fluctuations in the marginal product of capital. Since risk neutrality fixes the *ex-ante* real interest rate, a positive technology shock produces a very large substitution out of today's consumption, and into tomorrow's consumption.[6] This effect is absent when $\phi = 0$ because a purely transitory technology shock does not directly affect the *ex-ante* real interest rate.

Fourth, when the IES is less than one, the response of labor supply to a positive technology shock (η_{na}) is negative. This implies that consumption and hours are negatively correlated since η_{ca} is positive for small values of σ. This counterfactual prediction can be understood by referring back to (2.10), setting $\beta = Z_t = 1$, and $H_t = (1 - N_t)$: if the real wage is constant, the marginal utility of consumption will be perfectly correlated with the marginal disutility of labor

[6]To see why the consumption response in this case must be so large, recall that tomorrow's consumption effects the expected real interest rate through it's influence on tomorrow's labor supply. An increase in consumption tomorrow lowers the marginal utility of consumption, motivating a decrease in labor supply. As labor supply declines, the marginal product of capital is driven down. Since linear utility fixes the *ex-ante* real interest rate, this decline in the marginal product of capital is needed to offset the increase brought about by a positive shock to technology. However, when $\sigma = \infty(\gamma = 0)$, labor supply is very unresponsive to movements in consumption, because it responds to the *marginal utility* of consumption. Consequently, infinite changes in consumption are required to induce a change in labor supply. In the standard RBC model, the response of consumption to a technology shock is never infinite because the curvature of the utility function over consumption is fixed at $\gamma = 1$.

hours. If consumption rises, the marginal utility of consumption falls, requiring a decrease in the marginal disutility of work, or an increase in leisure, producing a negative correlation between market hours and consumption. This problem can be resolved if the real wage is procyclical, but only if marginal utility does not decline too rapidly, a condition that does not hold when $\sigma < 1$. This explains why hours and consumption are negatively correlated when the intertemporal elasticity of substitution is less than one.

Hence, Case 1, where technology shocks only affect the market sector, illustrates a fundamental difficulty with the standard RBC model with low IES: it predicts that market hours will be countercyclical. Table 2 demonstrates how adding a home production sector to the standard model can remedy this problem. The table shows the elasticities for Case 2, when $\beta = 1$, but $Z_t = A_t^{\alpha}$. In this case, shocks to technology affect the home and market sectors symmetrically. First, note that a shock to technology now leads to an increase in home production ($\eta_{ha} > 0$) since it increases productivity in both sectors. More importantly, the labor supply elasticities are now *decreasing* in σ for all values of ϕ except $\phi = 1$. Thus when σ is low, market hours respond positively to an increase in technology, and labor supply is both procyclical and positively correlated with market consumption.

To understand this difference from Case 1, it is easiest to think about how the elasticities vary with ϕ, for a given σ. As previously discussed, the elasticity of consumption with respect to a positive technology shock, η_{ca}, is increasing in persistence for low σ, but decreasing for high σ. Since higher consumption reduces the marginal utility of (market) income, this leads to the opposite pattern for labor supply: the elasticity of market hours with respect to a positive technology shock, η_{na}, is *decreasing* in persistence for low σ, but *increasing* for high σ. This pattern also holds in Case 1 for the same reason. The difference in Case 2 is that higher market consumption (resulting from a positive technology shock) does not reduce the marginal utility of market consumption *relative* to home consumption as much as in Case 1, so that the labor supply elasticities are much larger when σ is low (and income effects are strong) than they are in Case 1.

Intersectoral shocks—which are important in Case 1 but not in Case 2—are unhelpful because they exacerbate the inverse relation between market hours and market consumption that already exists due to the rapidly declining marginal utility that preferences with low σ imply. This suggests that home production models with low IES require a sufficient degree of positive correlation in technology shocks across sectors. Put another way, they require a minimal degree of *relative* productivity variation. This finding contrasts with the results of previous home production studies which emphasize the importance of a relatively high degree of productivity variation across sectors in order to improve the quantitative performance of the standard model. A quick inspection of Table 1 (as well as Table 3—discussed in more detail below) displays the reason for the difference: we permit a much lower value for the IES (and therefore a much lower value for σ_{ch}) than has been implicitly assumed elsewhere. As the tables show, when σ is sufficiently high, market hours and market consumption are both procyclical even in cases where there is a large amount of relative productivity variation across sectors.

Table 3 gives the results for Case 3, where there is home capital, but no technology shocks to the household sector. As in Case 1, a shock to a_t creates a large differential in productivity across the home and market sectors.

The value of σ has several notable affects on the elasticities in Case 3. First, elasticities with respect to market consumption, market output, and market hours are generally increasing in σ; the more willing individuals are to substitute both intertemporally and intratemporally, the larger are the effects on the economy of a technology shock to the market sector. Second, when σ is very large, a positive technology shock induces a very large substitution into market consumption and market output, and out of home consumption (home output). Third, when σ

is sufficiently large, a positive shock to technology generates a large increase in market capital and market time ($\eta_{ka} = \eta_{na} = \infty$) which must be offset by a large decrease in home capital and home time ($\eta_{da} = \eta_{1-na} = -\infty$).

Finally, Table 4 shows elasticities for Case 4. In this case, both sectors utilize capital and technology in the same proportion, and shocks to technology across sectors are perfectly correlated. This implies that $\eta_{ca} = \eta_{ha}$ for all σ and ϕ. To understand this, it is easiest to consider parameter values for which substitution effects are strong. Combinations of high σ and high ϕ produce strong *intertemporal* substitution effects, but *intratemporal* substitution effects are washed out because a technology shock produces no relative productivity differential between the two sectors. The only way to substitute intertemporally is to increase market output over market consumption; hence market hours rise ($\eta_{na} > 0$) and market consumption falls ($\eta_{ca} < 0$). The relative scarcity of market consumption reduces the benefit of additional home capital, motivating a reallocation to the market sector ($\eta_{da} < 0, \eta_{ka} > 0$), and a decline in home consumption ($\eta_{ha} < 0$). Note too that, like Case 2, none of the labor supply elasticities are negative regardless of how small σ is. Thus, as σ approaches zero, only those cases which assume that shocks across sectors are the same predict that both market consumption and market hours will be procyclical.

Elasticities with respect to capital are the same in Cases 3 and 4, and are shown in Table 5.

4. Simulation Results

The elasticities discussed above summarize how the model's properties change with key parameter values. The results indicate that the solution is very sensitive to the assumed values of σ and ϕ. To gain further understanding into the model's predictions at empirically plausible values of σ and ϕ, and to compare them with those of other RBC and home production models, it is useful to undertake simple time-series simulations of the model. We can then carry out the exercise typically performed in the RBC literature of asking how well moments from the simulated data match those from the U.S. data.

We focus on the model's properties when the IES is set equal to empirically plausible levels. A survey of the many studies cited above which estimate this parameter suggests that it is well below one, and in many cases close to zero. Therefore, we consider 0.20 to be a conservative value for this parameter, and we use it in the simulations reported below.

We choose parameters for the technology process that are fairly standard.[7]

[7] Our parameter choice for the variance of technology shocks coincides with those in Benhabib,

In particular, we assume that the AR(1) process for the log technology is given by $a_t = 0.95a_{t-1} + \epsilon_t$, where ϵ_t is normally distributed with a standard deviation equal to 0.007. Using allocation rules implied by the elasticities reported above, 100 simulations of 150 periods each are computed.[8] The simulated data are Hodrick-Prescott filtered before computing any statistics, again following the home production literature.

Panel A of Table 6 gives selected moments from U.S. quarterly data over the period 1959:1-1996:4[9] for the following log real variables: output, y, consumption, c, investment, i, average productivity, w, market capital, k, and market hours, n. For each of these variables, the table gives the percent standard deviation in the variable relative to the percent standard deviation of y, and the cross correlation of the variable with y. Data details are given in Appendix B.

Panel B uses simulated data to summarize the cyclical properties of the standard model (Case 1 with $\sigma = 1$). The panel reveals several well-known discrepancies between the model's predictions and key aspects of the U.S. data. These

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Rogerson, Wright, 1993, Greenwood and Hercowitz, 1993 and Greenwood, Rogerson and Wright, 1995, while our choice of ϕ is consistent with Benhabib et al. and Greenwood, Rogerson, and Wright.

[8]Each simulation consists of a random sample of 150 realiztions of ϵ_t, which is then used to compute the values of each of the other variables in the model using the decision rules reported above.

[9]This sample period applies for all series except the capital stock, for which the most recent data runs from 1959:1-1994:1.

discrepancies can be summarized as follows: compared to the data, output is not volatile enough; relative to output, consumption and hours are not volatile enough; relative to output, investment is too volatile, and productivity (w) is too highly correlated with output. Existing home production studies have documented significant improvements in the standard model's performance, along all of these dimensions, as the result of explicitly incorporating a household sector into the standard model (e.g. see Benhabib, et al., 1991). Next, we ask whether those improvements are maintained in our model with low intertemporal elasticity of substitution in consumption.

Panels C-F of Table 6 show statistics computed from the simulated data for Cases 1–4. Note that Case 1 is simply the standard model with $\sigma = 0.2$ instead of $\sigma = 1$.

Focusing on the problems discussed above, Table 6 reveals that none of the cases produce results that represent a clear improvement over the standard model's performance. Instead, generalizing the standard model to include home technological progress and a low intertemporal substitution elasticity appears to significantly deteriorate its quantitative performance along several dimensions. For example, in every case, the volatility of investment relative to output is larger, the volatility of consumption relative to output is smaller, and the correlation of productivity with output is higher than in the standard model. Furthermore, Cases 1 and 3 have output less volatile than the standard model, and only Case 4 yields output that is more volatile.

The existing home production literature documents that models with a household sector perform significantly better than the RBC benchmark along all of these dimensions. As previously noted, however, the most popular of these specifications implies that the IES is greater than one by imposing a value for the SES that is greater than one. Yet the RBC benchmark to which these models are compared is the standard model which fixes the IES at unity, making it difficult to determine how much of the documented improvement is due to the inclusion of a home production sector, and how much is due to the higher IES assumed in the home production framework. Our specification allows us to both fix the value of the IES at a common level across the RBC benchmark and home production model, and to give the IES an empirically plausible value. Thus, although the four home production models considered in Table 6 perform worse than the standard model with log utility over consumption and hours, we do not believe the latter is an appropriate benchmark to which models of home production with low IES should be compared. Instead, we argue that Case 1, which minimizes the role of the home production sector but permits a more empirically plausible IES than the standard model, is the relevant benchmark.

Table 6 shows that, relative to Case 1, Cases 2 and 4, which permit an explicit role for home production with symmetric technology shocks, generally represent an improvement over the low σ benchmark. For example, Case 1 predicts a counter-factual negative correlation between market hours and output. This is consistent with the negative labor supply elasticities (η_{na}) found in Table 1 when $\sigma = 0.2$. And while Case 3 does not produce a negative value for this correlation when $\sigma = 0.2$, a quick inspection of Table 3 indicates that it will produce a negative value for smaller σ (i.e., $\eta_{na} < 0$ when $\sigma = 0$). By contrast, both cases which impose the same technology shocks across sectors (Case 2 and Case 4) yield procyclical market hours. Moreover, unlike Case 3, Cases 2 and 4 also represent a clear improvement in the relative volatility of both hours and wages over the low σ benchmark.

Table 6 nevertheless demonstrates that some problems remain with the low σ models, even when technology shocks across sectors are the same. The most notable difficulty is that both Cases 2 and 4 continue to predict investment that is too volatile, and consumption that is too smooth, relative to output. While this difficulty may at first seem rather glaring, others (e.g., Baxter and Crucini, 1993) have shown that the problem can be resolved by allowing for adjustment costs in market capital.

In summary, simulation results presented in this section demonstrate that the simple model of home production studied here, with a relatively low value of the IES, does not yield the quantitative improvements over the standard RBC model which has a higher IES. However, if one accepts that the standard model imposes an implausibly high value for the IES, the introduction of a home production sector that shares a common technology shock with the market sector, does improve the quantitative performance of the RBC model relative to a more appropriate *low* σ benchmark. And, while the home production models we consider have difficulty matching the relative volatility of investment and consumption found in the U.S. data, it seems likely that these problems can be addressed in a richer model with adjustment costs in market capital.

5. Conclusions

Little evidence is available to assess the empirical validity of several key parameters in models with home production. One such parameter is the static elasticity of substitution between home and market consumption (SES). Yet theoretical models in the existing household production literature typically assume that home and market consumption are highly substitutable. Our strategy for calibrating the SES is to calibrate the intertemporal elasticity of substitution (IES) instead, making use of the positive relation that exists between the two parameters. In doing so, we rely on a large body of empirical evidence which suggests that the value of the IES is substantially below unity.

The framework studied in this paper allows us to explore several possible generalizations of the standard real business cycle model. A minimal generalization de-emphasizes the role of the home production sector, but relaxes the restriction of the standard model that the IES must be one to permit balanced growth. The most general specification incorporates a complete home production function with household capital and stochastic shifts in household technological progress. The value of the IES has a critical effect on the time-series properties of all these models.

While previous studies have concentrated on home production models with high values for the IES, we have explored the properties of a home production model with a low IES. We develop a low IES benchmark by introducing steady-state technological growth into the home sector of an otherwise standard, time-separable RBC model.

Our results provide two key insights about the underlying structure of a home production model with a low IES. First, freeing up the curvature of the utility function while maintaining balanced growth requires that the home and market sectors display the same long-run rate of technological progress. Second, the cyclical behavior of market hours is not well captured in a home production model with a high degree of intersectoral productivity variation. In contrast to models which impose higher values for the IES, intersectoral technology shocks are not helpful in our model because they tend to make market hours and market consumption move inversely.

Appendix A

This appendix provides complete solutions to the loglinearized model for each of the four cases we consider. We use the method of Campbell (1994). Here we provide only the solutions for the elasticities, and refer the reader to Campbell (1994) for details about the procedure. In each case, the model's equations are made linear in logs by approximating them with first order Taylor expansions around steady state values. We start with the most general case and proceed backwards.

Case 4

Combining (2.6) and (2.5) we get an accumulation equation for $F_t \equiv K_t + D_t$:

$$F_{t+1} = (1 - \delta)F_t + Y_t - C_t. \tag{A.1}$$

Taking logs of both sides and linearizing the right hand side yields an equation for f_{t+1} :

$$f_{t+1} = \lambda_1 k_t + \lambda_2(a_t + n_t) + \lambda_3 c_t \lambda_4 f_t, \tag{A.2}$$

where,

$$\lambda_1 \equiv \frac{(r+\delta)N}{1+g}, \quad \lambda_2 \equiv \frac{(r+\delta)N\alpha}{(1+g)(1-\alpha)}, \quad \lambda_3 \equiv \frac{(\delta+g)}{1+g} - \frac{(r+\delta)N}{(1+g)(1-\alpha)}, \quad \lambda_4 \equiv 1 - \frac{\delta+g}{1+g}.$$

Loglinearizing the work-wage first-order condition (2.10) yields an equation for log hours:

$$n_t = \nu_1 k_t + \nu_2 d_t + \nu_3 a_t + \nu_4 c_t, \tag{A.3}$$

where,

$$\nu_1 \equiv \nu^*(1-\alpha), \quad \nu_2 \equiv \nu^*(1/\sigma - 1)(1-\alpha), \quad \nu_3 \equiv \nu^*\alpha/\sigma, \quad \nu_4 \equiv -\nu^*/\sigma,$$

and where,

$$\nu^* \equiv (\nu(1-N)\sigma)/((1-N)\sigma + \nu\alpha N), \quad \nu \equiv (1-N)/(1-\alpha).$$

Equation (2.8) is loglinearized assuming that R_{t+1} and C_{t+1} are jointly log-normal and homoskedastic to obtain:

$$E_t \Delta c_t = E_t[\xi_1 k_{t+1} + \xi_2 d_{t+1} + \xi_3 a_{t+1} + \xi_4 c_{t+1}], \tag{A.4}$$

where,

$$\xi_1 \equiv (\sigma\alpha(r+\delta)(\nu_1 - 1))/(1+r), \quad \xi_2 \equiv (\sigma\alpha(r+\delta)\nu_2)/(1+r)$$

$$\xi_3 \equiv (\sigma\alpha(r+\delta)(\nu_3 + 1))/(1+r), \quad \xi_4 \equiv (\sigma\alpha(r+\delta)\nu_4)/(1+r).$$

We assume that individuals can reallocate capital between the home and market sectors within the period. This allows us to equate the gross marginal products of each type of capital in (2.8) and (2.9) yielding an equation for k_t and d_t in terms of $f_{t,}a_t$, and c_t:

$$k_t = \pi_1 f_t + \pi_2 a_t + \pi_3 c_t \tag{A.5}$$

$$d_t = \chi_1 f_t + \chi_2 a_t + \chi_3 c_t, \tag{A.6}$$

where,

$$\chi_1 \equiv \omega_1^* \pi_1, \quad \chi_2 \equiv \omega_1^* \pi_2 + \omega_2^*, \quad \chi_3 \equiv \omega_1^* \pi_3 + \omega_3^*,$$

$$\omega_1^* \equiv (\omega_1 \nu_1 + \omega_3)/(1 - \omega_1 \nu_2), \quad \omega_2^* \equiv (\omega_1 \nu_3 + \omega_2)/(1 - \omega_1 \nu_2)$$

$$\omega_3^* \equiv (\omega_1 \nu_4 + \omega_4)/(1 - \omega_1 \nu_2),$$

and where,

$$\omega_1 \equiv -(N/(1-N)\alpha(1-1/\sigma)\sigma/((1-\alpha)+\alpha\sigma) - \alpha\sigma/((1-\alpha)+\alpha\sigma),$$

$$\omega_2 \equiv -\alpha/((1-\alpha)+\alpha\sigma), \quad \omega_3 \equiv \alpha\sigma/((1-\alpha)+\alpha\sigma), \quad \omega_4 \equiv 1/((1-\alpha)+\alpha\sigma),$$

$$\pi_1 \equiv (1/N)/(1+(1-N)\omega_1^*/N), \quad \pi_2 \equiv -(1-N)\omega_2^*/(N+(1-N)\omega_1^*),$$

$$\pi_3 \equiv -\omega_3^*(1-N)/(N+(1-N)\omega_1^*).$$

The solution proceeds by the method of undetermined coefficients, by making an initial guess that the loglinear solution will be of the form specified in (2.19).

η_{cf} solves the quadratic equation:

$$Q_2\eta_{cf}^2 + Q_1\eta_{cf} + Q_0 = 0, \tag{A.7}$$

where,

$$Q_2 \equiv (\psi_3\mu_3 - \mu_3), \quad Q_1 \equiv (1 + \psi_1\mu_3 + \psi_3\mu_1 - \mu_1), \quad Q_0 \equiv \psi_1\mu_1.$$

where,

$\psi_1 \equiv \xi_1\pi_1 + \xi_2\chi_1, \quad \psi_2 \equiv \xi_1\pi_2 + \xi_2\chi_2 + \xi_3, \quad \psi_3 \equiv \xi_1\pi_3 + \xi_2\chi_3 + \xi_4,$

and where,

$$\mu_1 \equiv \lambda_1^*\pi_1 + \lambda_3^*\chi_1 + \lambda_5^*, \quad \mu_2 \equiv \lambda_1^*\pi_2 + \lambda_3^*\chi_2 + \lambda_2^*, \quad \mu_3 \equiv \lambda_1^*\pi_3 + \lambda_3^*\chi_3 + \lambda_4^*,$$

where,

$$\lambda_1^* \equiv \lambda_1 + \lambda_2\nu_1, \quad \lambda_2^* \equiv \lambda_2 + \lambda_2\nu_3, \quad \lambda_3^* \equiv \lambda_2\nu_2, \quad \lambda_4^* \equiv \lambda_3 + \lambda_2\nu_4, \quad \lambda_5^* \equiv \lambda_4.$$

η_{ca} is given by

$$\eta_{ca} = \frac{-(\psi_1\mu_2 + \psi_3\mu_2\eta_{cf} - \mu_2\eta_{cf} + \psi_2\phi)}{\psi_1\mu_3 + \psi_3\mu_3\eta_{cf} - \mu_3\eta_{cf} + \psi_3\phi + 1 - \phi} \tag{A.8}$$

Elasticities of total capital with respect to last period's total capital and the log technology shock are then found as

$$\eta_{ff} = \mu_1 + \mu_3\eta_{cf}, \quad \eta_{fa} = \mu_2 + \mu_3\eta_{ca}.$$

All other elasticities are defined in terms of the elasticities above:

$$\eta_{kf} = \pi_1 + \pi_3\eta_{cf}, \quad \eta_{ka} = \pi_2 + \pi_3\eta_{ca}, \quad \eta_{df} = \chi_1 + \chi_3\eta_{cf}, \quad \eta_{da} = \chi_2 + \chi_3\eta_{ca},$$

$$\eta_{nf} = \nu_1\eta_{kf} + \nu_2\eta_{df} + \nu_4\eta_{cf}, \quad \eta_{na} = \nu_1\eta_{ka} + \nu_2\eta_{da} + \nu_3 + \nu_4\eta_{ca},$$

$$\eta_{yf} = \alpha\eta_{nf} + (1-\alpha)\eta_{kf}, \quad \eta_{ya} = \alpha(1+\eta_{na}) + (1-\alpha)\eta_{ka},$$

$$\eta_{hf} = (1-\alpha)\eta_{df} - \alpha N\eta_{nf}/(1-N), \quad \eta_{ha} = (1-\alpha)\eta_{da} + \alpha - \alpha N\eta_{na}/(1-N).$$

Case 3

Parameter definitions for Case 3 are the same as in Case 4, with the following exceptions:

$$\omega_2 \equiv -\alpha\sigma/((1-\alpha)+\alpha\sigma), \quad \nu_3 \equiv \nu^*\alpha, \quad \eta_{ha} = (1-\alpha)\eta_{da} - \alpha N\eta_{na}/(1-N).$$

Case 2

The solutions for the elasticities given in (2.18) for Case 2 yield a quadratic equation in k for η_{ck},

$$Q_2\eta_{ck}^2 + Q_1\eta_{ck} + Q_0 = 0, \tag{A.9}$$

where,

$$Q_2 \equiv (\psi_3\mu_3 - \mu_3), \quad Q_1 \equiv (1 + \psi_1\mu_3 + \psi_3\mu_1 - \mu_1), \quad Q_0 \equiv \psi_1\mu_1.$$

where,

$$\psi_1 \equiv (\nu_1 - 1)\lambda_3\sigma, \quad \psi_2 \equiv (\nu_2 - 1)\lambda_3\sigma, \quad \psi_3 \equiv \nu_3\lambda_3\sigma,$$

and where,

$$\mu_1 \equiv \lambda_2\nu_1 + \lambda_1, \quad \mu_2 \equiv \lambda_2\nu_1 + \lambda_2, \quad \mu_3 \equiv 1 - \lambda_1 - \lambda_2 + \lambda_2\nu_3,$$

$$\nu_1 \equiv \nu^*(1-\alpha), \quad \nu_2 \equiv \nu^*\alpha(1/\sigma), \quad \nu_3 \equiv -\nu^*\sigma,$$

$$\nu^* \equiv \tfrac{(1-N)\sigma}{(1-\alpha)(1-N)\sigma+N},$$

$$\lambda_1 \equiv \tfrac{1+r}{1+g}, \quad \lambda_2 \equiv \tfrac{(r+\delta)\alpha}{(1+g)(1-\alpha)}, \quad \lambda_3 \equiv \tfrac{(r+\delta)\alpha}{1+r},$$

$$\psi_1 \equiv (\nu_1 - 1)\lambda_3\sigma, \quad \psi_2 \equiv (\nu_2 + 1)\lambda_3\sigma, \quad \psi_3 \equiv \nu_3\lambda_3\sigma.$$

The elasticity of consumption with respect to technology is a function of η_{ck}:

$$\eta_{ca} = \frac{-(\psi_1\mu_2 + \psi_3\mu_2\eta_{ck} - \mu_2\eta_{ck} + \psi_2\phi)}{\psi_1\mu_3 + \psi_3\mu_3\eta_{ck} - \mu_3\eta_{ck} + \psi_3\phi + 1 - \phi}, \tag{A.10}$$

and the rest of the elasticities are defined in terms of the consumption elasticities:

$$\eta_{kk} = \mu_1 + \mu_3\eta_{ck}, \quad \eta_{ka} = \mu_2 + \mu_3\eta_{ca},$$

$$\eta_{nk} = \nu_1 + \nu_3\eta_{ck}, \quad \eta_{na} = \nu_2 + \nu_3\eta_{ca},$$

$$\eta_{yk} = 1 - \alpha + \alpha\eta_{nk}, \qquad \eta_{ya} = \alpha + \alpha\eta_{na},$$

$$\eta_{hk} = -N\eta_{nk}/(1-N), \qquad \eta_{ha} = \alpha - N\eta_{na}/(1-N).$$

Case 1

Parameter definitions for Case 1 are the same as in Case 2, with the following exceptions:

$$\nu_2 \equiv \nu^*\alpha, \qquad \eta_{ha} = -N\eta_{na}/(1-N).$$

Appendix B

This appendix reviews the data used to compute the summary statistics in the first panel of Table 6. All series are per capita, measured at quarterly frequency, seasonally adjusted, and chain weighted in 1992 dollars, except where otherwise noted.

Consumption

Consumption is the sum of personal consumption expenditures (PCE) on non-durables and services, excluding expenditure on housing services, 1959:3-1996:4. Source: Bureau of Economic Analysis (BEA).

Investment

Total investment series is defined as residential and non-residential investment plus personal expenditure on consumer durables. Source: Bureau of Economic Analysis (BEA).

Hours

This series is aggregate hours of all wage and salary workers in non-agricultural industries, in millions. These data are monthly and converted to quarterly averages over the period 1959:1-1997:2. Source: Bureau of Labor Statistics.

Capital Stock

This series is the constant-cost net stock of fixed nonresidential structures and equipment, in billions of 1987 dollars from 1959-1994 at annual frequency. The data are linearly interpolated to quarterly frequency. Source: Bureau of Economic Analysis.

Output

The output series is constructed as consumption plus investment, following Benhabib et al.

Productivity

Average productivity (proportional to the real wage with Cobb-Douglas technology) is output divided by hours, defined above.

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Table 1

Elasticities for Case 1: $\alpha < 1, \beta = 1, Z_t = 1, A_t$ varies

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.04	0.21	0.54	1.19	2.00
	1.00	0.97	0.94	0.92	0.91
All ϕ	-0.08	-0.26	-0.24	0.22	1.00
	0.28	0.16	0.17	0.48	1.00
	0.04	0.13	0.12	-0.11	-0.50
	0.00	0.02	.07	0.23	0.44
	0.08	0.09	0.13	0.17	0.20
0.00	-0.01	0.20	0.71	1.43	2.00
	0.66	0.80	1.14	1.62	2.00
	0.00	-0.10	-0.36	-0.72	-1.0
	0.01	0.03	0.10	0.15	$-\infty$
	0.08	0.09	0.13	0.18	∞
0.5	-0.01	0.18	0.68	1.47	2.18
	0.66	0.78	1.12	1.65	2.12
	0.01	-0.09	-0.34	-0.74	-1.09
	0.05	0.17	0.29	-0.36	$-\infty$
	0.07	0.06	0.09	0.25	∞
0.95	-0.11	-0.06	0.45	1.70	3.31
	0.60	0.63	0.97	1.80	2.87
	0.05	0.03	-0.22	-0.85	-1.65
	0.32	0.37	0.46	-7.0	$-\infty$
	0.00	0.01	0.06	0.29	∞
1.0	-0.64	-0.41	0.24	1.86	4.00
	0.24	0.39	0.83	1.91	3.34
	0.32	0.21	-0.12	-0.93	-2.0

Notes: α is the share of labor in market production; β is the share of labor in home production; ϕ is the persistence parameter on the market technology process; σ is the intertemporal elasticity of substitution in consumption. Each column of numbers in each panel gives the elasticities of market consumption, next period's market capital, market labor supply, market output, and home output, respectively. In the first panel are η_{ck}, η_{kk}, η_{nk}, η_{yk}, and η_{hk}, the elasticities with respect to this period's market capital which do not vary with ϕ. The last four panels give η_{ca}, η_{ka}, η_{na}, η_{ya}, and η_{ha}, the elasticities with respect to market technology for selected values of ϕ.

Table 2

Elasticities for Case 2: $\alpha < 1, \beta = 1, Z_t = A_t^\alpha$

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.04	0.21	0.54	1.19	2.00
	1.00	0.97	0.94	0.92	0.91
All ϕ	-0.08	-0.26	-0.24	0.22	1.00
	0.28	0.16	0.17	0.48	1.00
	0.04	0.13	0.12	-0.11	-0.50
	0.01	0.04	0.07	0.12	0.15
	0.18	0.17	0.13	0.09	0.07
0.00	1.32	1.11	0.71	0.25	0.00
	1.55	1.41	1.14	0.84	0.67
	0.01	0.11	0.31	0.54	0.67
	0.01	0.06	0.10	0.08	$-\infty$
	0.18	0.16	0.13	0.09	∞
0.5	1.30	1.07	0.68	0.27	0.06
	1.54	1.38	1.12	0.85	0.71
	0.01	0.13	0.33	0.53	0.64
	0.12	0.30	0.29	-0.18	$-\infty$
	0.16	0.11	0.09	0.13	∞
0.95	1.09	0.65	0.45	0.39	0.44
	1.39	1.10	0.97	0.93	0.96
	0.12	0.34	0.76	0.47	0.45
	0.75	0.65	0.46	-0.36	$-\infty$
	0.00	0.02	0.06	0.15	∞
1.0	-0.16	0.04	0.24	0.47	0.67
	0.56	0.69	0.83	0.98	1.11
	0.75	0.65	0.55	0.43	0.33

Notes: See Table 1.

Table 3

Elasticities for Case 3: $\alpha = \beta, Z_t = 1, A_t$ varies

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.00	0.01	.05	0.35	∞
	0.03	0.04	0.08	0.26	22.6
	-0.01	0.24	1.24	5.97	∞
0.00	0.00	-0.12	-0.62	-2.98	$-\infty$
	-0.01	0.24	1.24	5.97	∞
	0.66	0.91	1.91	6.64	∞
	0.00	-0.12	-0.62	-2.98	$-\infty$
	0.01	0.02	0.06	0.52	∞
	0.03	0.04	0.07	0.25	11.8
	-0.01	0.23	1.20	5.62	∞
0.5	0.01	-0.12	-0.60	-2.81	$-\infty$
	-0.01	0.23	1.20	5.62	∞
	0.66	0.90	1.87	6.28	∞
	0.01	-0.12	-0.60	-2.81	$-\infty$
	0.05	0.08	0.23	1.57	∞
	0.02	0.03	0.06	0.15	2.08
	-0.10	0.10	0.88	3.54	∞
0.95	0.05	-0.05	-0.44	-1.77	$-\infty$
	-0.10	0.10	0.88	3.54	∞
	0.57	0.77	1.54	4.20	∞
	0.05	-0.05	-0.44	-1.77	$-\infty$
	0.30	0.22	0.41	2.16	∞
	0.00	0.02	0.04	0.10	1.00
	-0.59	-0.18	0.51	2.34	∞
1.0	0.30	0.09	-0.26	-1.17	$-\infty$
	-0.59	-0.18	0.51	2.34	∞
	0.07	0.49	1.18	3.01	∞
	0.30	0.09	-0.26	-1.17	$-\infty$

Notes: α is the share of labor in market production; β is the share of labor in home production; ϕ is the persistence parameter on the market technology process; σ is the intertemporal elasticity of substitution in consumption. Each column of numbers in each panel gives the elasticities of market consumption, next period's total capital, this period's market capital, this period's home capital, market labor supply, market output, and home output, respectively. The four panels report η_{ca}, η_{fa}, η_{ka}, η_{da}, η_{na}, η_{ya}, and η_{ha}, the elasticities with respect to market technology, for selected values of ϕ.

Table 4

Elasticities for Case 4: $\alpha = \beta, Z_t = A_t$

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.01	0.02	0.05	0.10	0.89
	0.08	0.08	0.08	0.07	0.00
	1.32	1.29	1.24	1.14	-0.44
0.00	-0.66	-0.64	-0.62	-0.57	0.22
	1.32	1.29	1.24	1.14	-0.44
	1.98	1.95	1.91	1.81	0.22
	0.01	0.02	0.05	0.10	0.89
	0.02	0.04	0.06	0.07	-4.69
	0.08	0.08	0.07	0.07	0.50
	1.30	1.25	1.20	1.20	10.7
0.5	-0.65	-0.63	-0.60	-0.60	-5.36
	1.30	1.25	1.20	1.20	10.7
	1.97	1.92	1.87	1.87	11.4
	0.02	0.04	0.06	0.07	-4.69
	0.15	0.25	0.23	-0.11	-9.72
	0.07	0.06	0.06	0.09	0.95
	1.04	0.83	0.88	1.55	20.8
0.95	-0.52	-0.42	-0.44	-0.77	-10.4
	1.04	0.83	0.88	1.55	20.8
	1.70	1.50	1.54	2.21	21.4
	0.15	0.25	0.23	-0.18	-9.72
	0.89	0.70	0.41	-0.21	-10.3
	0.00	0.02	0.04	0.10	1.00
	-0.44	-0.06	0.51	1.75	21.9
1.0	-0.22	0.03	-0.26	-0.87	-10.9
	-0.44	-0.06	0.51	1.75	21.9
	0.22	0.60	1.18	2.41	22.6
	0.89	0.70	0.41	-0.21	-10.3

Notes: See Table 3.

Table 5

Elasticities with respect to capital for Cases 3 and 4

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.11	0.31	0.59	1.21	11.3
	1.00	0.98	0.96	0.90	0.00
	1.44	1.06	0.49	-0.75	-20.9
All ϕ	0.78	0.97	1.26	1.87	11.9
	0.44	0.06	-0.51	-1.75	-21.9
	0.78	0.40	-0.18	-1.14	-21.9
	0.11	0.31	0.59	1.21	11.3

Notes: See Table 4. Each column of numbers gives the elasticities of market consumption, next period's total capital, this period's market capital, this period's home capital, market labor supply, market output, and home output, respectively. The elasticities reported are η_{cf}, η_{ff}, η_{kf}, η_{df}, η_{nf}, η_{df}, η_{nf}, η_{yf}, and η_{hf}, the elasticities with respect to total capital.

Table 6

$x =$

	c	i	w	k	n
A. U.S. Data: std(y) = 2.0					
$\frac{std(x)}{std(y)}$	.49	2.44	.65	.25	.76
$corr(x,y)$	.90	.97	.65	.38	.76
B. Standard Model, $\sigma = 1$: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.35	3.10	.56	.33	.47
$corr(X,Y)$	.90	.99	.98	.04	.98
C. Case 1, $\sigma = 0.2$: std(y) = 0.7					
$\frac{std(x)}{std(y)}$	.28	3.20	1.10	.33	.13
$corr(x,y)$	.98	.99	.99	.01	-.77
D. Case 2, $\sigma = 0.2$: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.28	3.21	.43	.35	.60
$corr(x,y)$	.98	.99	.96	.02	.98
E. Case 3, $\sigma = 0.2$: std(Y) = .7					
$\frac{std(x)}{std(y)}$	.11	3.71	.88	.19	.15
$corr(x,y)$	.94	.99	.99	.68	.85
F. Case 4, $\sigma = 0.2$: std(Y) = 1.4					
$\frac{std(x)}{std(y)}$	.17	3.58	.45	.56	.55
$corr(x,y)$	.98	.99	.99	.98	.99

Notes: All series are filtered using the Hodrick-Prescott technique. The following variables are in logs: y is output, c is market consumption, i is investment, k is market capital, n is market hours, and w is average productivity. The top of each panel is the percentage standard deviation of output; std(x)/std(y) gives the standard deviation of the series x relative to that of Y, and corr(x,y) gives the correlation of x with y. The numbers are averages over 100 simulations of 150 periods each.