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Bennett T. McCallum
Edward Nelson

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ABSTRACT

This paper reports results of simulation exercises that explore several questions relating to the design of rules for monetary policy. Emphasis is given to issues raised by the concept of rule operationality, i.e., reliance on feasible instrument variables and information sets. Many of the results pertain to rules of the Taylor type -- i.e., with an interest rate instrument set in response to inflation and output-gap measures -- but some are reported for rules using a nominal income target and/or a monetary base instrument. The macroeconomic model utilized is small in scale but features a specification designed to represent rational dynamic optimizing choices by the economy's private agents. Saving and portfolio-balance behavior are expressed by optimizing versions of exceptional IS and LM functions, with gradual price adjustments specified differently in two variants of the model. One variant uses the well-known Calvo-Rotemberg price adjustment relation, whereas the second employs a newly-rationalized version of the Mussa-McCallum-Barro-Grossman P-bar model. Parameter values are estimated by instrumental variables on U.S. quarterly data for 1995-1996.

Bennet T. McCallum
GSIA, Carnegie Melon University
Pittsburgh, PA 15213
and NBER
bm05@andrew.cmu.edu

Edward Nelson
GSIA, Carnegie Melon University
Pittsburgh, PA 15213
en27@andrew.cmu.edu

1. Introduction

In a series of studies on monetary policy rules, McCallum (1988, 1990, 1993, 1995) has utilized and promoted a research strategy that emphasizes *operationality* and *robustness*. The first of these properties intentionally limits consideration to policy rules (i) that are expressed in terms of instrument variables that could in fact be controlled on a high-frequency basis by actual central banks and (ii) that require only information that could plausibly be possessed by these central banks. Thus, for example, hypothetical rules that treat (e.g.) M2 as an instrument or that feature instrument responses to current-quarter values of real GDP are ruled out as non-operational. The second property focuses on a candidate rule's tendency to produce at least moderately good performance in a variety of macroeconomic models rather than "optimal" performance in a single model. The idea behind this criterion is that there exists a great deal of professional disagreement over the appropriate specification of crucial features of macroeconomic models, and indeed even over the appropriate objective function to be used by an actual central bank.

Most of the models used in McCallum's own studies have, however, been non-structural vector autoregression or single-equation atheoretic constructs that are quite unlikely to be policy-invariant. Even the so-called "structural" models in McCallum (1988, 1993) are essentially small illustrative systems that are not based on well-motivated theoretical foundations. Thus these studies have not contributed any proposed models of their own to be used in a profession-wide exploration of the robustness of candidate rules' properties.

In the present study, accordingly, we formulate, estimate, and simulate two variants of a

model of the U.S. economy that is intended to have structural properties. The model is quite small - - following in the line of work previously contributed to by Fuhrer and Moore (1995), Yun (1996), Ireland (1997), and Rotemberg and Woodford (1997) among others - - but is based on aggregate demand and supply specifications that are designed to reflect rational optimizing behavior on the part of the economy's private actors. Our formulations pertaining to demand are rather orthodox, but in terms of aggregate supply - - i.e., price adjustment behavior - - we consider two alternatives, one of which is not standard. In particular, we begin with the formulation of Roberts (1995), which is based on the well-known models of Calvo (1983), Rotemberg (1982), and Taylor (1980). In addition, however, we develop a modification of the Mussa-McCallum-Barro-Grossman "P-bar" model, whose theoretical properties are arguably more attractive. Although we consider only two simple variants of our macroeconomic model, we suggest that its design makes it an attractive starting point for a more extensive robustness study. Our estimation is conducted by instrumental variables and utilizes quarterly U.S. data for 1955-1996.

With our estimated model we carry out stochastic and counterfactual historical simulations not only with the class of policy rules promoted in McCallum's previous work, but also rules that are operational versions of the Taylor (1993) type and others with an interest rate instrument. Some of the issues that we explore in these simulations are the following:

(i) Is it true that response coefficients in a rule of the Taylor type should be much larger than recommended by Taylor (1993)?

(ii) Is there any tendency for adoption of a nominal GDP target rule to generate instability

of real GDP and inflation?

(iii) In studying questions such as these, how important is it quantitatively to recognize that actual central banks do not have complete information when setting instrument values for a given period?

(iv) How sensitive to measures of "capacity" output are rules that feature responses to output gaps?

(v) Do interest rates exhibit extreme short-run volatility when base money rules are utilized?

Organizationally, we begin in Section 2 with a discussion of several important background issues. Then Sections 3 and 4 are devoted to specification of the macroeconomic model to be utilized, with the former pertaining to the model's aggregate demand sector and the latter to aggregate supply. Section 5 describes data and estimation, and reports estimates of the model's basic structural parameters. Simulation exercises with various policy rules are then conducted in Sections 6 and 7 for the two variants of the model, and conclusions are summarized in Section 8.

2. Monetary Policy Rules: Alternatives and Issues

We begin by discussing various forms of possible monetary policy rules and some issues raised by the differences among them. In the previous research by McCallum, quarterly data has been utilized and the principal rule specification has been

$$(2.1) \qquad \Delta b_t = \Delta x^* - (\tfrac{1}{16})(x_{t-1} - b_{t-1} - x_{t-17} + b_{t-17}) + \lambda(x^*_{t-1} - x_{t-1}),$$

with $\lambda \geq 0$. Here b_t and x_t denote logarithms of the (adjusted) monetary base and nominal GNP (or GDP), respectively, for period t. The variable x^*_t is the target value of x_t for quarter t, with these targets being specified so as to grow smoothly at the rate Δx^*. This rate is in turn designed to yield an average inflation rate that equals some desired value--e.g., a value such as 0.005, which with quarterly data would represent roughly 2 percent per year.[1] Whereas a growing-level target path $x^{*1}_t = x^{*1}_{t-1} + \Delta x^*$ was used in McCallum's early work (1988), his more recent studies have emphasized growth-rate targets of the form $x^{*2}_t = x_{t-1} + \Delta x^*$ or weighted averages such as $x^{*3}_t = 0.8x^{*2}_t + 0.2x^{*1}_t$. In (2.1), the rule's second term provides a velocity-growth adjustment intended to reflect long-lasting institutional changes, while the third term features feedback adjustment in Δb_t in response to cyclical departures of x_t from the target path x^*_t, with λ chosen to balance the speed of eliminating $x^*_t - x_t$ gaps against the danger of instrument instability.

More prominent in recent years has been the rule form proposed by Taylor (1993), which we write as

$$(2.2) \qquad R_t = r^* + \pi^{av}_{t-1} + \mu_1(\pi^{av}_{t-1} - \pi^*) + \mu_2 \tilde{y}_t.$$

[1] Whatever the desired quarterly inflation rate, Δx^* is set equal to that value plus an estimated long-run average rate of growth of real output, a number assumed to be independent of the policy rule adopted.

Here R_t is the quarter-t value of an interest rate instrument, π_{t-1}^{av} is the average inflation rate over the four quarters prior to t, π^* is the target inflation rate, and $\tilde{y}_t = y_t - \overline{y}_t$ is the difference between the (logs of) real GDP y_t and its capacity or natural-rate value $\overline{y}_t$. The policy feedback parameters μ_1 and μ_2 are positive - - each of them equals 0.5 in Taylor's (1993) example[2] - - so that the interest rate instrument is raised in response to values of inflation and output that are high relative to their targets.

There are two major reasons for the greater prominence of Taylor's rule (2.2) as compared with (2.1). First, it is specified in terms of an interest-rate instrument variable, which is much more realistic.[3] Second, from several studies including Taylor (1993), Stuart (1996), Clarida, Gali, and Gertler (1997a), among others, it appears to be the case that actual policy in recent years - - say, after 1986 - - has been rather well described by a formula such as Taylor's with coefficients quite close to his for some major countries.

As specified by Taylor (1993), however, rule (2.2) is not fully operational since it assumes unrealistically that the central bank knows the value of real GDP for quarter t when setting the instrument value R_t for that quarter. In fact, there is considerable uncertainty regarding the realized value of real GDP even at the end of the quarter in actual economies.[4] In

[2] When annualized values of inflation and the interest rate are used.

[3] Virtually all central banks of industrialized countries use some short-term (nominal) interest rate as their instrument or "operating target" variable. For an extensive recent discussion, see Bank for International Settlements (1997).

[4] In the United States, for example, the recent study of Ingenito and Trehan (1997) indicates that the "forecast" error for real GDP at the end of the quarter is about 1.4 percent, implying that annualized growth rates for the quarter would have a 95 percent confidence interval of about ± 2.8 percent, thereby possibly ranging from boom to deep recession values. This result is based on revised data, so it abstracts from the problem of data revision.

addition it is far from obvious how $\overline{y}_t$ should be measured - - even in principle - - as is emphasized in McCallum (1997), and different measures can imply significantly different instrument settings.[5] The first of these objections can be easily overcome by using the value of y_t expected to prevail at the start of period t. Also, in the same spirit, some more rational representation of expected future inflation could be used in place of π^{av}_{t-1}. Overcoming the second objection, regarding the measurement of $\overline{y}_t$, could be more difficult.

Alterations in rule (2.1) could also be considered, such as using the expectation of x^*_t (or of x^*_{t+1}) rather than actual x^*_{t-1} as the basis for feedback adjustments. More generally, the target values in (2.1) and (2.2) could be exchanged, to provide rules with (i) a base instrument and π^*, $\tilde{y}$ targets and (ii) an interest instrument plus a Δx_t target. In the work that follows, we shall explore several such variants of policy rules.

In this regard, some analysts might suggest that the monetary base instrument be discarded, since actual central banks are not inclined even to consider the use of a b_t instrument.[6] Several academics have hypothesized that policy could be made more effective if a base instrument were utilized, however,[7] and there are clearly some disadvantages of the interest rate scheme. In particular, there is an observable tendency for an interest instrument to become something of a target variable that is thus adjusted too infrequently and too timidly.[8]

[5] These two objections to (2.2) should not be understood as criticisms of Taylor's (1993) paper, which was written mainly to encourage interest in monetary rules on the part of practical policymakers - - and was in that regard extremely successful.

[6] Goodhart (1994) has claimed that tight monetary base control is essentially infeasible.

[7] Among these academics are Brunner and Meltzer (1983), Friedman (1982), McCallum (1988), and Poole (1982).

In any event, the question of the comparative merits of b_t and R_t instruments is one that seems to warrant scientific study - - indeed, more than is provided below.

The foregoing paragraphs have been concerned with policy rules from a normative perspective. In estimating and evaluating a macroeconomic model, however, it is useful to consider what policy rule or rules have in fact been utilized during the sample studied. In that regard, it might be argued that no rule has been in place; that the Federal Reserve has instead behaved in a discretionary manner. But we believe that there has clearly been a major component of Fed behavior that is *systematic*, as opposed to random, and this component can be expressed in terms of a feedback formula.[9] Of course there can be little doubt but that there have been changes during our 1955-1996 sample in the systematic component's specification, with prominent dates for possible changes including October 1979, late summer 1982, August 1987, and a few others.[10] Thus we have experimented with both slope and constant-term dummy variables. After considerable empirical investigation we have ended with an estimated rule of the form

$$R_t = \mu_0 + \mu_1 R_{t-1} + \mu_2 E_{t-1}\Delta x_t + \mu_3 E_{t-1}\tilde{y}_t + \mu_4 d_{1t} + \mu_5 d_{2t} E_{t-1}\Delta x_t + e_{Rt}, \tag{2.3}$$

where $\tilde{y}_t$ is the output gap (the log-deviation of output from its flexible-price level), d_{1t} and d_{2t} are dummy variables that take on the value 1.0 in 1979:4-1982:2 and 1979:4-1996:4 respectively, and e_{Rt} is a serially independent disturbance. Thus our estimated rule for

[8] See Goodhart (1997).

[9] On this topic, see Taylor (1993), McCallum (1997), and Clarida, Gali, and Gertler (1997a).

[10] The study by Clarida, Gali, and Gertler (1997a) considers one possible break - - in October 1979 - - and finds significant differences in estimated policy rule coefficients before and after that date.

1955:1-1996:4 is one that combines the interest rate instrument from (2.2) with a nominal GDP target as in (2.1), as well as an extra countercyclical term. The rule is operational because the monetary authority responds to period $t-1$ forecasts of Δx_t and $\tilde{y}_t$, not their realized values. The inclusion of dummies in equation (2.3) allows for shifts in the policy rule occurring in late 1979, presumably due to the change in operating procedures and anti-inflationary emphasis that was announced on October 6. Of these, the dummy d_{1t} captures a possible intercept shift occurring during the period of nonborrowed reserves targeting, and the interactive dummy $d_{2t}E_{t-1}\Delta x_t$ reflects a permanent shift in the Federal Reserve's objectives after 1979. The empirical results of our investigation are reported below in Section 5.[11]

Returning to the normative topic of effective rule design, there are several prominent issues concerning target variables that will be studied in Sections 6 and 7. One of these involves the claim, expressed by Ball (1997) and Svensson (1997b), that targeting of nominal GDP growth rates (or growing levels) will tend to induce undesirable behavior of inflation and output gap variables. It is not difficult to show that Ball's drastic result of dynamic instability of π_t and $\tilde{y}_t$ holds only under some highly special model specifications, but it is possible that much greater volatility would obtain than with alternative target variables, so a quantitative examination of the issue is needed.

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[11] As the experiments in this paper are concerned with counterfactual policy rules, we do not use rule (2.3) in our simulations in Sections 6 and 7. Our reason for nevertheless estimating and reporting (2.3) is to demonstrate that rule-like behavior is a reasonable characterization of postwar data and to indicate the importance of the regime dummies d_{1t} and d_{2t}, which we include in our instrument set when estimating our structural model in Section 5.

3. Aggregate Demand Specification

This section describes the aggregate demand side of our model; what follows is essentially a condensed presentation of the derivations in McCallum and Nelson (1997). We assume that there is a large number of infinitely-lived households, each of which maximizes

$$(3.1) \qquad E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, (M_{t+j}/P^A_{t+j})),$$

where C_t denotes the household's consumption in period t, and (M_t/P^A_t) denotes its end-of-period real money holdings, M_t being the nominal level of these money balances and P^A_t the general price level. Real money balances generate utility by facilitating household transactions in period t. The instantaneous utility function $U(C_t, (M_t/P^A_t))$ is of the additively separable form:

$$(3.2) \qquad U(C_t, (M_t/P^A_t)) = \sigma(\sigma-1)^{-1} C_t^{\frac{\sigma-1}{\sigma}} \exp(\omega_t) + (1-\gamma)^{-1}(M_t/P^A_t)^{1-\gamma} \exp(\chi_t),$$

with $\sigma > 0$, $\gamma > 0$. Here ω_t and χ_t are both preference shocks, whose properties we specify below.

Each household also acts as a producer of a good, over which it has market power. To this end, it hires N^d_t in labor from the labor market, paying real wage (W_t/P^A_t) for each unit of labor. With this labor and its own capital stock K_t (which depreciates at rate δ) it produces its output Y_t via the technology $Y_t = A_t K^{\alpha}_t (N^d_t)^{1-\alpha}$, where A_t is an exogenous shock which affects all households' production. The household sells its output at price P_t. Each household consumes many goods, consisting of some of the output produced by other households; the C_t that appears in the household's utility function is an index of this consumption, and P^A_t indexes the average price of households' output.

As is standard in the literature, we assume that the demand function for good i is of the Dixit-Stiglitz form, and that also the producer is obliged to set production equal to this demand:

$$A_t K_t^{\alpha}(N_t^d)^{1-\alpha} = (P_t/P_t^A)^{-\theta} Y_t^A, \tag{3.3}$$

with $\theta > 1$, and Y_t^A denoting aggregate output.

The household is also endowed with one unit of labor each period, and supplies N_t^S of this to the labor market. The household's budget constraint each period is then:

$$\begin{aligned}(P_t/P_t^A)^{1-\theta} Y_t^A - C_t - K_{t+1} + (1-\delta)K_t + (W_t/P_t^A)N_t^S \\ -(W_t/P_t^A)N_t^d + TR_t - (M_t/P_t^A) + (M_{t-1}/P_t^A) - B_{t+1}(1+r_t)^{-1} + B_t = 0.\end{aligned} \tag{3.4}$$

In (3.4), B_{t+1} is the quantity of government bonds bought by the household in period t; each of these is purchased for $(1+r_t)^{-1}$ units of output and redeemed for one unit of output in period $t+1$. TR_t denotes lump-sum government transfers paid to the household in period t. Letting ξ_t denote the Lagrange multiplier on constraint (3.3) and λ_t the multiplier on (3.4), the household's first order conditions with respect to C_t, (M_t/P_t^A), K_{t+1}, and B_{t+1} are:

$$C_t^{-1/\sigma} \exp(\omega_t) = \lambda_t. \tag{3.5}$$

$$(M_t/P_t^A)^{-\gamma} \exp(\chi_t) = \lambda_t - \beta E_t \lambda_{t+1}(P_t^A/P_{t+1}^A). \tag{3.6}$$

$$\lambda_t = \beta(1-\delta)E_t\lambda_{t+1} + \alpha\beta E_t \xi_{t+1} A_{t+1} K_{t+1}^{\alpha-1}(N_{t+1}^d)^{1-\alpha}. \tag{3.7}$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1+r_t). \tag{3.8}$$

Because leisure does not enter its utility function, the household's optimal labor supply is $N_t^S = 1$ each period, although, since we assume below that the labor market does not clear, this desired labor supply will not be the realized value of labor utilized.

As an employer of labor, the household's first order condition with respect to N_t^d is

$$\lambda_t(W_t/P_t^A) = (1-\alpha)\xi_t A_t K_t^{\alpha}(N_t^d)^{-\alpha}. \tag{3.9}$$

Equation (3.9) indicates that, as in Ireland (1997), the markup of price over marginal cost is equal to (λ_t/ξ_t). The household has one more first order condition, pertaining to its optimal choice for P_t. We defer the analysis of this decision until Section 4.

We now construct a log-linear model of aggregate demand from the above conditions. While we use (3.7) in our calculations of the implied steady-state level of investment, $\overline{I}$, we do not use an approximation of (3.7) to describe quarter-to-quarter fluctuations in capital or investment. Instead, we treat capital as exogenous and, for tractability, let the movements of log investment around its steady-state value be a random walk. Thus we have

$$i_t = g_k + i_{t-1} + e_{it}, \tag{3.10}$$

where $g_k \succeq 0$ is the average growth rate of capital, $E_{t-1}e_{it} = 0$, and $E(e_{it}^2) = \sigma_{ei}^2$. In (3.10) and below, lower-case letters denote logarithms of variables.

It would be standard practice to complete our specification of technology with the usual log-linear law for capital accumulation,

$$k_{t+1} = \tfrac{(1-\delta)}{(1+g_k)}k_t + \tfrac{(\delta+g_k)}{(1+g_k)}i_t, \tag{3.10a}$$

along with a law of motion for the (log) technology shock a_t. But since we are treating capital movements as exogenous, and since leisure does not appear in the household's utility function, the "flexible-price" or "capacity" level of log output, $\overline{y}_t = a_t + \alpha k_t$, is exogenous in our setup. It makes sense therefore to make assumptions directly about the $\overline{y}_t$ process, instead of its two components. By doing so we lose the connection between investment and

capacity output implied by (3.10a), but this does not seems a serious omission for purposes of business cycle analysis because of the minor contribution that investment makes to the existing capital stock during a typical business cycle. We assume that $\overline{y}_t$ follows an AR(1) process:

$$\overline{y}_t = \varsigma + \rho_{\overline{y}}\overline{y}_{t-1} + e_{yt}, \tag{3.11}$$

where $|\rho_{\overline{y}}| \leq 1$, and $e_{yt} \sim N(0, \sigma^2_{ey})$, $E_{t-1}e_{yt} = 0$.[12]

Define the nominal interest rate as $R_t = r_t + E_t\Delta p_{t+1}$, where $\Delta p_{t+1} \equiv \log(P^A_{t+1}/P^A_t)$. Then (3.5), (3.8), (3.11) and the economy's resource constraint imply (after log-linearization)

$$y_t = E_t y_{t+1} - \sigma(\tfrac{C^{ss}}{Y^{ss}})(R_t - E_t\Delta p_{t+1} - \overline{r}) + \sigma(\tfrac{C^{ss}}{Y^{ss}})(\omega_t - E_t\omega_{t+1}). \tag{3.12}$$

where the superscript ss denotes steady-state value. We assume that the preference shock ω_t is an AR(1) process with AR parameter $|\rho_v| < 1$. Then if we define $v_t \equiv \sigma(1-\rho_v)\omega_t$, it is the case that

$$v_t = \rho_v v_{t-1} + e_{vt}, \tag{3.13}$$

and so (3.12) becomes:

$$y_t = E_t y_{t+1} - \sigma(\tfrac{C^{ss}}{Y^{ss}})(R_t - E_t\Delta p_{t+1} - \overline{r}) + (\tfrac{C^{ss}}{Y^{ss}})v_t, \tag{3.14}$$

which is like the optimizing IS functions of Kerr and King (1996), Woodford (1996), and McCallum and Nelson (1997).

Let $m_t - p_t$ denote the logarithm of (M_t/P^A_t). Then log-linearizing (3.6), we have (up to a constant)

$$m_t - p_t = (\sigma\gamma)^{-1}(\tfrac{Y^{ss}}{C^{ss}})y_t - (\sigma\gamma)^{-1}(\tfrac{I^{ss}}{C^{ss}})i_t - (\gamma R^{ss})^{-1}(R_t - R^{ss}) + \gamma^{-1}(\chi_t - \omega_t), \tag{3.15}$$

[12] In our empirical work we use a measure of $\overline{y}_t$ (described in Section 4) that grows over time, but in stochastic simulations we adopt the standard practice of abstracting from this growth.

where $R^{ss} = r^{ss} + (\Delta p)^{ss}$. This money demand function has scale (consumption) elasticity $(\sigma\gamma)^{-1}$ and (annualized) interest semi-elasticity $-0.25(\gamma R^{ss})^{-1}$. We permit the shocks ω_t and χ_t to be arbitrarily correlated; hence, it is simpler to define the composite disturbance $\eta_t = \gamma^{-1}(\chi_t - \omega_t)$, and make assumptions directly about η_t. Then (3.15) may be written:

$$m_t - p_t = (\sigma\gamma)^{-1}(\tfrac{Y^{ss}}{C^{ss}})y_t - (\sigma\gamma)^{-1}(\tfrac{I^{ss}}{C^{ss}})i_t - (\gamma R^{ss})^{-1}(R_t - R^{ss}) + \eta_t, \tag{3.16}$$

and we assume η_t is AR(1):

$$\eta_t = \rho_\eta \eta_{t-1} + u_t, \tag{3.17}$$

where $|\rho_\eta| < 1$, and $u_t \sim N(0, \sigma_u^2)$, $E_{t-1}u_t = 0$. Since we have allowed u_t and e_{vt} to be correlated, we may write the latter as

$$e_{vt} = \psi_u u_t + \varepsilon_{vt}, \tag{3.18}$$

where $\varepsilon_{vt} \sim N(0, \sigma_{\varepsilon v}^2)$, $E_{t-1}\varepsilon_{vt} = 0$, and $E_t(u_t\varepsilon_{vt}) = 0$. Thus the aggregate demand block of our model consists of the behavioral equations (3.14) and (3.16), together with (3.10) and the laws of motion (3.11), (3.13), (3.17), and (3.18).

4. Price Level Adjustment

In this section we develop the particular model of individual and aggregate price adjustments that will be utilized below. For a typical producer, let $\overline{p}_t$ represent the value of p_t - - its output price in log terms - - that would be optimal in period t if there were no nominal frictions, and let $\overline{y}_t$ be the corresponding level of (log) output y_t, which we will for shorthand refer to as "capacity" output. The producer faces a demand curve of the form

$$y_t = y_t^A - \theta(p_t - p_t^A), \tag{4.1}$$

where y_t^A and p_t^A are indices of aggregate values of y_t and p_t, these being appropriate averages of the values relevant for the individual producers.[13] From (4.1) we note that

$$y_t - \overline{y}_t = \theta(\overline{p}_t - p_t). \tag{4.2}$$

Perhaps the most widely used model of gradual price adjustment at present is the Calvo-Rotemberg model, which is justified by Rotemberg (1987) as follows. Although $\overline{p}_t$ would be charged in t by the typical firm if there were no adjustment costs, in the presence of such costs (assumed quadratic) the producer will instead choose p_t to minimize

$$E_t \textstyle\sum_{j=0}^{\infty} \beta^j [(p_{t+j} - \overline{p}_{t+j})^2 + c_1(p_{t+j} - p_{t+j-1})^2], \tag{4.3}$$

where $c_1 > 0$ reflects the cost of price changes in relation to the opportunity cost of setting a price different from $\overline{p}_t$. From (4.3) one can find the first-order optimality condition and rearrange to obtain the relation

$$\Delta p_t = \beta E_t \Delta p_{t+1} - (1/c_1)(p_t - \overline{p}_t). \tag{4.4}$$

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[13] Thus $p_t^A = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$ and $y_t^A = [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ with $\theta > 1$, where $p_t(i)$ and $y_t(i)$ pertain to producer i, as in Dixit and Stiglitz (1977). In the text, the indices are suppressed for the sake of notational simplicity.

Then using (4.2), we have for the typical producer

$$\Delta p_t = \beta E_t \Delta p_{t+1} + (\theta/c_1)(y_t - \overline{y}_t). \tag{4.5}$$

Assuming symmetry across firms, (4.5) can be used for aggregative analysis. Both Rotemberg (1987) and Roberts (1995) show that an indistinguishable relation is implied by Calvo's (1983) model that emphasizes staggered setting of "contract" prices to prevail until a new price-change opportunity arrives, with probabilities of these arrivals being constant and exogenous. Also, Roberts (1995) shows that the two-period version of Taylor's (1980) well-known model of staggered wage contracts gives a relation that is basically similar.

In what follows, consequently, we shall utilize a quarterly version of Roberts' formulation of the Calvo-Rotemberg model in one variant of our macroeconomic system. There are, however, two theoretical drawbacks to this model. First, the assumed quadratic cost of changing prices is rather unattractive theoretically. One reason is that one might expect the magnitude of price-change costs to be independent of the size of the change, especially if these are to be interpreted as literal resource costs of preparing new price lists, etc. More basically, however, it seems somewhat undesirable to emphasize costs of changing prices, which are rather nebulous, while neglecting the costs of changes in output rates, which are more concrete and arguably quite substantial.[14] Second, as is shown below, the Calvo-Rotemberg model does not satisfy the natural-rate hypothesis.[15]

Accordingly, let us consider a reformulated setup in which the producer chooses p_t to

[14] On this topic see Gordon (1990, p. 1146).

[15] Empirically, it has been suggested that the model does not imply as much persistence of inflation rates as exists in the U.S. data. On this see Ball (1994), Fuhrer and Moore (1995), and Nelson (1998).

minimize (4.6) rather than (4.3):

$$(4.6) \qquad E_{t-1} \sum_{j=0}^{\infty} \beta^j [(p_{t+j} - \overline{p}_{t+j})^2 + c_2(\widetilde{y}_{t+j} - \widetilde{y}_{t+j-1})^2].$$

Here $\widetilde{y}_t = y_t - \overline{y}_t$, so we are assuming that it is costly for a producer to alter his output rate, relative to capacity, from its previous value. The reason for using $(\widetilde{y}_{t+j} - \widetilde{y}_{t+j-1})^2$ rather than $(y_{t+j} - y_{t+j-1})^2$ is that changes in capacity stem primarily from technological improvements or capital installations,[16] neither of which give rise to changes in the labor force needed to produce $\overline{y}_t$ - - but it is labor-force changes that provide the primary rationale for the presumption that output changes are costly.[17] Neither $(\widetilde{y}_{t+j} - \widetilde{y}_{t+j-1})^2$ nor $(y_{t+j} - y_{t+j-1})^2$ is entirely appropriate, perhaps, but the former seems somewhat preferable theoretically - - and it gives rise to a tidy, tractable model, as will be seen shortly. Another feature of (4.6) to be noted is that the presence of E_{t-1} before the summation sign implies that p_t is chosen before the producer knows about demand conditions during t, i.e., p_t is predetermined in each period.[18] Then on the basis of the prevailing p_t, output in t is taken to be demand determined. Labor-leisure trade-offs are assumed relevant for the determination of $\overline{y}_t$, but not for temporary departures of y_t from $\overline{y}_t$. This is in accordance with the "installment payment" nature of current wages, as emphasized by Hall (1980).

Next we can define $\widetilde{p}_t = p_t - \overline{p}_t$ and, in light of relation (4.2), can rewrite (4.6) as

$$(4.7) \qquad E_{t-1} \sum_{j=0}^{\infty} \beta^j [\widetilde{p}_{t+j}^2 + c(\widetilde{p}_{t+j} - \widetilde{p}_{t+j-1})^2],$$

---

[16] There may in actuality be installation costs for new capital goods but if so this can in principle be taken account of in the IS portion of the model, not the price-setting portion.

[17] Models with quadratic costs of changing employment appear frequently in Sargent (1979).

[18] This is our assumption regarding price stickiness *per se*. Implicitly, it embodies the assumption that sellers' costs of changing prices are prohibitive within periods but negligible between periods.

where now $c > 0$ is the cost of output "gap" changes in relation to departures of p_t from $\overline{p}_t$. It might appear that $c\theta^2$ should appear in (4.7) where c does, but θ^2 can be absorbed into c (and indeed this is entirely consistent with a symmetric treatment of the two terms). To minimize (4.7), the relevant first order condition is

$$E_{t-1}[\tilde{p}_t + c(\tilde{p}_t - \tilde{p}_{t-1}) - \beta c(\tilde{p}_{t+1} - \tilde{p}_t)] = 0 \tag{4.8}$$

or

$$E_{t-1}\tilde{p}_t = \alpha\tilde{p}_{t-1} + \alpha\beta E_{t-1}\tilde{p}_{t+1}, \tag{4.9}$$

where $\alpha = c/(1+c+c\beta)$. Then since this relation in effect involves only the single variable $\tilde{p}_t$, we can see that its MSV[19] solution will be of the simple form $E_{t-1}\tilde{p}_t = \phi\tilde{p}_{t-1}$, with $E_{t-1}\tilde{p}_{t+1} = E_{t-1}\phi\tilde{p}_t = \phi^2\tilde{p}_{t-1}$. Substitution into equation (4.9) gives $\phi\tilde{p}_{t-1} = \alpha\tilde{p}_{t-1} + \alpha\beta\phi^2\tilde{p}_{t-1}$, so ϕ must satisfy

$$\alpha\beta\phi^2 - \phi + \alpha = 0. \tag{4.10}$$

Thus the MSV solution for ϕ is

$$\phi = [1 - \sqrt{1 - 4\alpha^2\beta}]/2\alpha\beta. \tag{4.11}$$

From the definition of α, we know that $4\alpha^2\beta < 1$ so ϕ in (4.11) is real. With $0 < \beta < 1$, we have $\phi > \alpha$, so the forward-looking objective increases the inertia of $\tilde{p}_t$. Also, it is the case that ϕ lies in the interval $(0, 1)$.[20]

---

[19] MSV stands for minimal-state-variable. Thus we are adopting the bubble-free solution, in the manner outlined by McCallum (1983).

[20] To show that $4\alpha^2\beta < 1$, it suffices to show that $(1+\beta)^2 > 4\beta$. But that is equivalent to $1+2\beta+\beta^2 > 4\beta$. Then subtracting 4β from each side we have $1 - 2\beta + \beta^2 > 0$ which is certainly true since the left hand side is $(1-\beta)^2$. Next, that $\phi > 0$ is clear from inspection of (4.11), given that $0 < 4\alpha^2\beta < 1$. To see that $\phi < 1$, note that this is the same as $1 - \sqrt{1 - 4\alpha^2\beta} < 2\alpha\beta$ which reduces to $\alpha(1+\beta) < 1$. Since $1/\alpha = (1/c)+1+\beta$, the last inequality holds.

In any event, we have developed a price adjustment rule of the form $p_t - E_{t-1}\overline{p}_t = \phi(p_{t-1} - \overline{p}_{t-1})$. Thus by simple rearrangement we can write

$$p_t - p_{t-1} = (1-\phi)(\overline{p}_{t-1} - p_{t-1}) + E_{t-1}(\overline{p}_t - \overline{p}_{t-1}), \tag{4.12}$$

which can be seen to be equivalent to the price-adjustment formula that was termed the "P-bar model" by McCallum (1994). This model was developed and utilized by Herschel Grossman, Robert Barro, Michael Mussa, and McCallum in the 1970s and early 1980s; for references, see McCallum (1994, pp. 251-252).

An important feature of the model, not noted in previous work, is that (4.2) permits the MSV solution $E_{t-1}\tilde{p}_t = \phi\tilde{p}_{t-1}$ to be alternatively expressible as

$$E_{t-1}\tilde{y}_t = \phi\tilde{y}_{t-1}. \tag{4.13}$$

Thus in analytical or numerical solutions of a macro model that includes the P-bar price adjustment theory, (4.13) can be included as the relation that governs price adjustment behavior. From the perspective of an undetermined-coefficients solution procedure, (4.13) fails to provide conditions relating to the coefficients on current shocks in the solution expression for $\tilde{y}_t$ (or for y_t given $\overline{y}_t$). But these are compensated by the restriction that p_t is predetermined and thus the shock coefficients in its solution equation are zeros. Thus, with this approach, the variable $\overline{p}_t$ need not be included in the analysis at all!

To illustrate the solution approach, suppose only for this paragraph that monetary policy was conducted in a manner that leads nominal income, x_t in log terms, to behave as follows:

$$\Delta x_t = \psi\Delta x_{t-1} + \xi_t, \tag{4.14}$$

where $0 < \psi < 1$, and ξ_t is white noise. Then one could consider the system consisting

of (4.13), (4.14), and the identity $\Delta x_t = \Delta p_t + y_t - y_{t-1}$, where we temporarily adopt the assumption that $\Delta \overline{y}_t = 0$. How does inflation Δp_t behave in this system? By construction, the MSV solution will be of the form

$$\Delta p_t = \phi_{11}\Delta x_{t-1} + \phi_{12} y_{t-1} + \phi_{13}\xi_t \tag{4.15}$$

$$y_t = \phi_{21}\Delta x_{t-1} + \phi_{22} y_{t-1} + \phi_{23}\xi_t, \tag{4.16}$$

in which we know *a priori* that $\phi_{13} = \phi_{21} = 0$ and $\phi_{22} = \phi$. Substitution into (4.14) gives

$$\phi_{11}\Delta x_{t-1} + \phi y_{t-1} + \phi_{23}\xi_t - y_{t-1} = \psi \Delta x_{t-1} + \xi_t. \tag{4.17}$$

Thus $\phi_{11} = \psi$, $\phi_{12} + \phi - 1 = 0$, and $\phi_{23} = 1$ are implied by undetermined-coefficient reasoning, which completes the solution.

It may also be noted that (4.13) provides the basis for an extremely simple proof that the P-bar model satisfies the strict version of the natural rate hypothesis. This version states that $E\tilde{y}_t \equiv 0$, for *any* monetary policy, even one with accelerating inflation. But the application of the unconditional expectation operator to each side of (4.13) yields $E\tilde{y}_t = \phi E\tilde{y}_t$, which with $\phi > 0$ implies that $E\tilde{y}_t \equiv 0$. With the Calvo-Rotemberg model (4.5), by contrast, we have $E(y_t - \overline{y}_t) = (c_1/\theta)E(\Delta p_t - \beta E_t \Delta p_{t+1})$. Using Roberts' (1995) approximation of $\beta \approx 1$, we have $E(y_t - \overline{y}_t) = (c_1/\theta)E(\Delta p_t - E_t \Delta p_{t+1})$, so any policy that yields on average an increasing or decreasing inflation rate will keep $E\tilde{y}_t \neq 0$.[21] Indeed, if $\beta < 1$ is retained, then even a constant $E\Delta p_t \neq 0$ will keep $E\tilde{y}_t \neq 0$.

In implementing our model - - indeed, any model with gradual price adjustment - - a very

[21] It is interesting to note that the Calvo-Rotemberg-Taylor model implies that an increasing inflation rate will reduce $\tilde{y}_t$ whereas a typical NAIRU model implies that an increasing inflation rate will raise $\tilde{y}_t$ - - permanently. Both implications seem theoretically unattractive, although the former is perhaps less implausible (and certainly less dangerous from a policy perspective).

important issue is how to measure $\overline{y}_t$ and therefore $\tilde{y}_t$. Much of the policy-rule literature, including Taylor (1983) and Rotemberg and Woodford (1997), simply uses deviations from a fitted linear time trend for $\tilde{y}_t$, thereby implicitly estimating $\overline{y}_t$ as the fitted trend. This seems unsatisfactory both practically and in principle. Practically, one major difficulty is that the resulting measure can be excessively sensitive to the sample period used in fitting the trend. To illustrate this sensitivity, Figure 1 plots $\tilde{y}_t$ values for the U.S. over 1980-1996 based on trends fitted (i) to a 1980-1996 sample period and (ii) to the 1955-1996 period that we use below. Clearly, they give markedly different pictures of the behavior of $\tilde{y}_t$ over the period 1990-1996. And neither of them reflects the widely-held belief that output has been unusually high relative to capacity in 1995 and 1996.

In principle, the fitted trend method - - even if the detrending is done by a polynomial trend or the Hodrick-Prescott filter - - seems inappropriate because it does not properly reflect the influence of technology shocks. Suppose that the production function is

$$y_t = \alpha_0 + \alpha_1 t + \alpha k_t + (1-\alpha) n_t + a_t, \tag{4.18}$$

where k_t and n_t are logs of capital and labor input, while a_t is a technology shock. Then if $\overline{n}_t$ is the value of n_t under flexible prices, $\overline{y}_t$ equals $\alpha_0 + \alpha_1 t + \alpha k_t + (1-\alpha)\overline{n}_t + a_t$ and so reflects the realization of a_t. But the fitted-trend methods do so either not at all or inadequately.

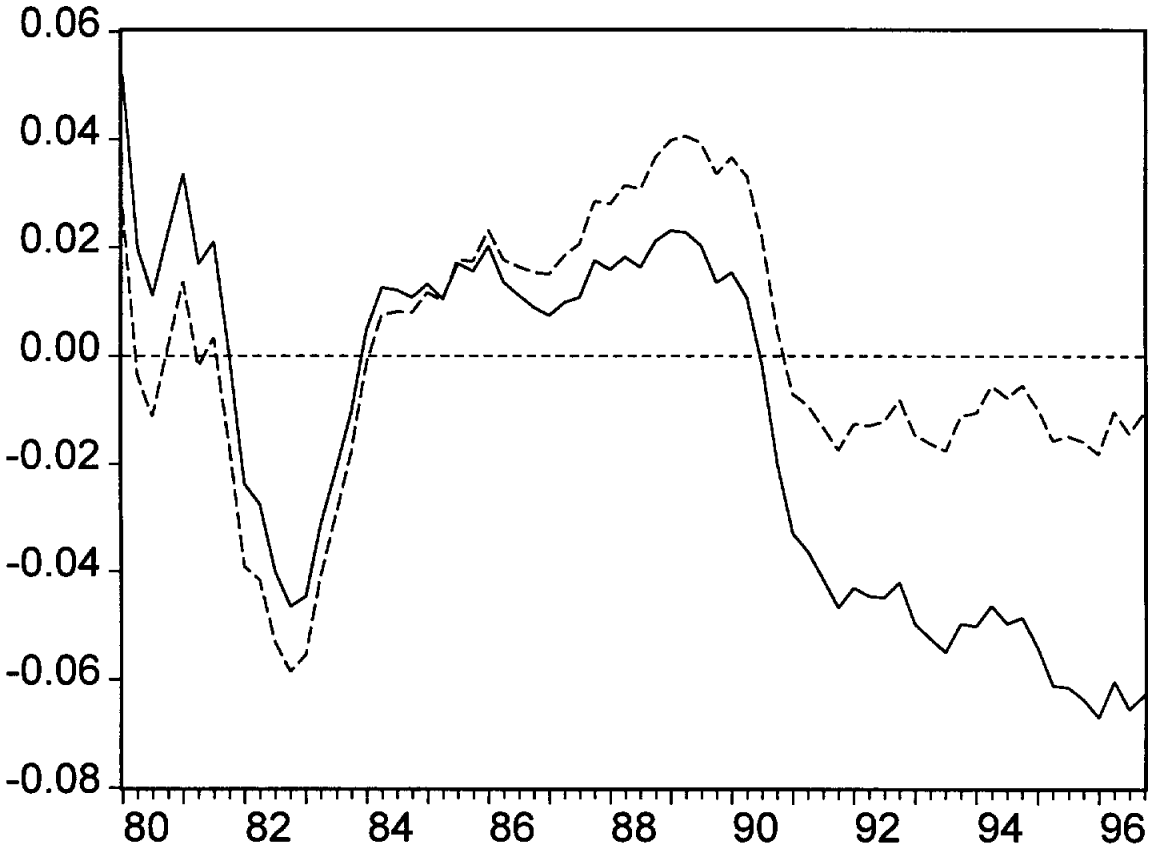
The approach that we use below relies on the observation that (4.18) implies

$$\tilde{y}_t = y_t - \overline{y}_t = (1-\alpha)(n_t - \overline{n}_t). \tag{4.19}$$

Of course this requires that we have some measure of $\overline{n}_t$. In general, it will depend upon households' labor supply behavior as well as producers' demand, but for the present study

Figure 1

Measures of Detrended Output, 1980-1996



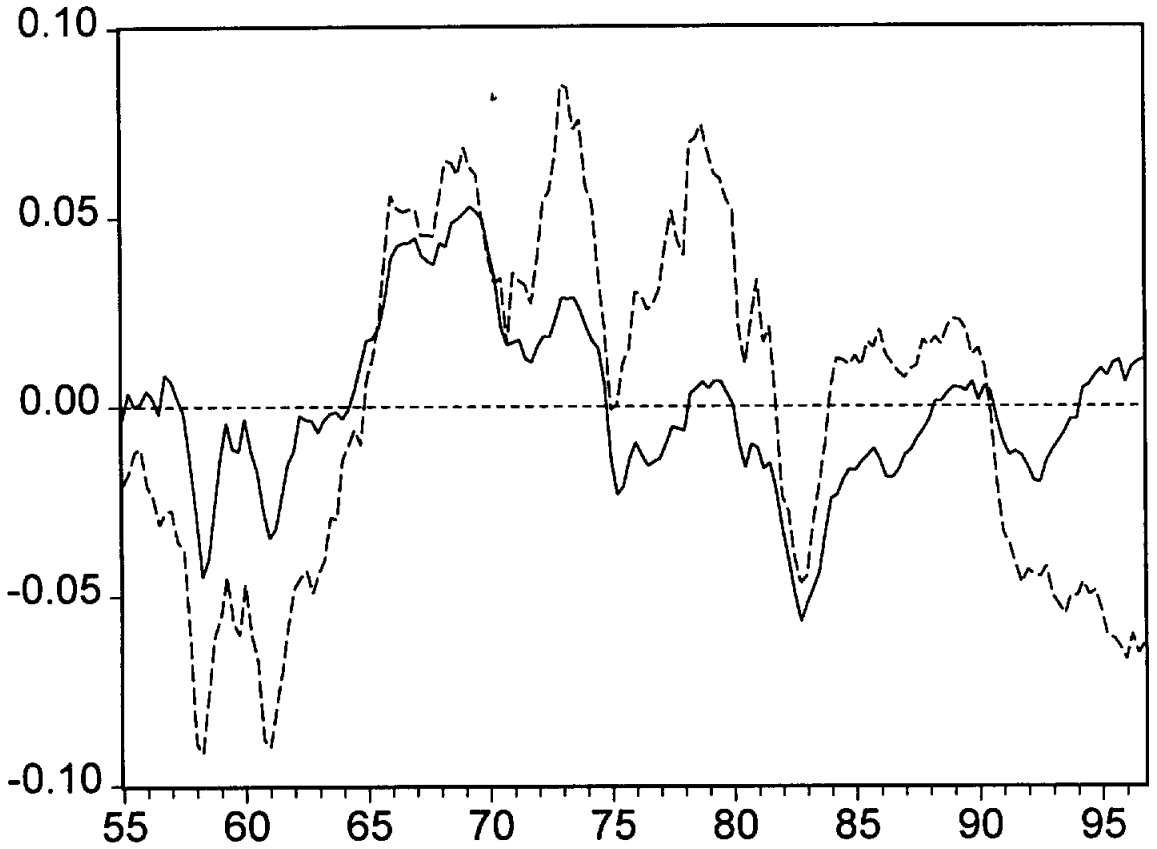
Solid line: 1955-1996 trend removed
Dashed line: 1980-1996 trend removed

we are adopting the simplifying assumption that labor supply is inelastic - - i.e., that $\overline{n}_t$ is a constant. Then variations in $\widetilde{y}_t$ will be proportional to variations in n_t, the hours worked per household under sticky prices. We assume that this actual employment level is demand-determined in each period.[22] The measure that we use for n_t is total manhours employed in nonagricultural private industry divided by the civilian labor force. A plot of the implied $\widetilde{y}_t$, using $\alpha = 0.3$, is shown in Figure 2, together with the fitted trend value based on the 1955-1996 sample period.

[22] Thus, as stated above, we are assuming that current-period wages are irrelevant for determination of current-period employment.

Figure 2

Output Gap Measures



Solid line: Derived from labor input
Dashed line: 1955-1996 trend removed

5. Model Estimation

We estimate our model by instrumental variables. Some of the system's equations are estimated on a single-equation basis, but the two aggregate demand relations are estimated jointly:

$$y_t = b_0 + E_t y_{t+1} - \sigma(\tfrac{C^{ss}}{Y^{ss}})(R_t - E_t \Delta p_{t+1}) + (\tfrac{C^{ss}}{Y^{ss}})v_t \tag{5.1}$$

$$m_t - p_t = c_0 + (\sigma\gamma)^{-1}(\tfrac{Y^{ss}}{C^{ss}})y_t - (\sigma\gamma)^{-1}(\tfrac{I^{ss}}{C^{ss}})i_t - (\gamma R^{ss})^{-1}(R_t) + \eta_t. \tag{5.2}$$

Here (5.1) and (5.2) are the IS and LM equations (3.14) and (3.16), allowing for constant terms. We estimate these equations jointly to take into account the cross-equation restriction (the appearance of the parameter σ in both equations), as well as possible cross-correlation between v_t and η_t via (3.18).

One advantage of the instrumental variables procedure is that, if the orthogonality conditions involving the instruments and the model errors are valid, parameter estimation is consistent under quite general assumptions about the serial correlation of the disturbances, and the precise form of the serial correlation does not have to be specified in estimation. To benefit from this advantage, we do not impose, in our estimation of (5.1) and (5.2), the AR(1) assumptions about the v_t and η_t processes that we make in our general equilibrium model (in (3.13) and (3.17)).

Equations (5.1) and (5.2) contain the expectational variables $E_t y_{t+1}$ and $E_t \Delta p_{t+1}$. We proceed with estimation of the system by replacing these expected values with their corresponding realized values, thereby introducing expectational errors such as $(y_{t+1} - E_t y_{t+1})$ into the equations' composite disturbances. To obtain consistent estimates, we instrument

for all the variables in (5.1) and (5.2). Because of the likely serial correlation in the error terms of the first two equations, lagged endogenous variables are not admissible instruments; only strictly exogenous variables are legitimate candidates. We therefore use as instruments a constant, a time trend, lags one and two of $\Delta gdef_t$ (i.e. the log-change in quarterly defense spending), plus the dummy variables d_{1t} and d_{2t}, which take the value unity in 1979:4-1982:2 and 1979:4-1996:4 respectively.

Money is measured by the St. Louis monetary base, new definition, R_t is the Treasury bill rate (measured in quarterly fractional units) and p_t is the log GDP deflator, defined as $x_t - y_t$. The income variables x_t and y_t are logs of nominal and real GDP, with values of GNP spliced on for observations prior to 1959:1. Also, i_t is gross private fixed investment. All data except interest rates are seasonally adjusted. We fix $\frac{C^{ss}}{Y^{ss}}$ at 0.81, $\frac{I^{ss}}{Y^{ss}}$ at 0.19, and R^{ss} at 0.014. The estimates of equations (5.1) and (5.2) are then:

$$\widehat{y}_t = -0.973 + E_t y_{t+1} - 0.203(\tfrac{C^{ss}}{Y^{ss}})(R_t - E_t \Delta p_{t+1}). \tag{5.3}$$

$$(0.129) \qquad (0.017)$$

$\overline{R}^2 = 0.999$, SEE = 0.0098, DW = 1.35.

$$\widehat{m - p}_t = -0.007 + 0.753(\tfrac{Y^{ss}}{C^{ss}})(y_t - (\tfrac{I^{ss}}{Y^{ss}})i_t) - 0.152(R^{ss})^{-1}(R_t). \tag{5.4}$$

$$(0.001) \qquad (0.015)$$

$\overline{R}^2 = 0.942$, SEE = 0.0617, DW = 0.14.

The estimates imply an intertemporal elasticity of substitution of $\sigma = 0.20$ (standard error 0.018) and an interest elasticity of money demand of $-\gamma^{-1} = -0.15$ (s.e. 0.015). In

turn, these estimates imply a consumption elasticity of money demand of $(\sigma\gamma)^{-1} = 0.75$. The reported standard errors need to be interpreted with caution both because of the residual autocorrelation and because of the trending behavior of the y_t and $(m_t - p_t)$ series.[23]

For the variant of our model that uses the P-bar price-setting specification, aggregate supply behavior is represented compactly by equation (4.13). As in Section 4, we measure $\tilde{y}_t$ by $(1-\alpha)$ times $(n_t - \overline{n})$, where $\overline{n}$ is the mean of log hours, and $\alpha = 0.3$. Equation (4.13) implies that the expectational error $(\tilde{y}_t - \phi\tilde{y}_{t-1})$ should be white noise, but in preliminary estimation of ϕ we found substantial serial correlation in the estimated residuals. We therefore decided to correct for first-order serial correlation in our estimation of ϕ, although such serial correlation is ignored both in our theoretical model and in the stochastic simulations of that model in Section 7 below.[24] Estimation by instrumental variables, with the instruments being those used for (5.1)-(5.2) plus lags two to four of $\tilde{y}_t$, produces:

$$\widehat{E_{t-1}\tilde{y}_t} = \underset{(0.063)}{0.891}\tilde{y}_{t-1} \tag{5.5}$$

$\overline{R}^2 = 0.956$, SEE = 0.0047, DW = 1.95, estimated AR(1) correction parameter = 0.59.[25]

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[23] We assume that $(m_t - p_t - (\sigma\gamma)^{-1}\frac{Y^{ss}}{C^{ss}}(y_t - \frac{I^{ss}}{Y^{ss}}i_t))$ is a stationary process. It is common in the empirical literature instead to estimate money demand functions such as (3.16) using cointegration methods, with $m_t - p_t$, y_t, *and* R_t modelled as I(1) series. We do not do so because treating R_t as I(1) is incompatible with our theoretical model unless Δp_t is I(1). It is also inconsistent with most estimated policy rules, including our own specification (5.10) below, which model nominal interest rates as stationary within each policy regime.

We also experimented with a first-differenced money demand function, finding it produced a poorer fit and less plausible parameter estimates than (5.4).

[24] Our need to correct for serial correlation indicates that the first-order dynamics of the output gap implied by (4.13) are rejected by the data. In future work we hope to generalize the P-bar specification to allow for more realistic dynamics.

Our measure of (log) potential output $\overline{y}_t$ is obtained by adding our estimated $\widetilde{y}_t$ measure to y_t. We found that $\overline{y}_t$ is well described as a random walk ($\rho_{\overline{y}} = 1.0$ in equation (3.11)).[26] Subject to that restriction, the constant (or "drift") term in equation (3.11) becomes interpretable as the long-run growth rate of capacity output.[27] For the investment-output ratio to be a mean-reverting series, the drift terms in (3.10) and (3.11) must be identical, and we therefore estimate those equations jointly subject to that restriction:

(5.6) $\widehat{\Delta i_t} = 0.0073$

(0.0052)

SEE = 0.0250, DW = 0.99,

(5.7) $\widehat{\Delta \widetilde{y}_t} = 0.0073$

(0.0052)

SEE = 0.0070, DW = 2.00,

implying $g_k = \varsigma = 0.0073$, σ^2_{ei}= $(0.0250)^2$, and σ^2_{ey}= $(0.0070)^2$. The Durbin Watson statistic for equation (5.6) indicates strong serial correlation in the estimated residuals, contrary to the assumptions of our model, and suggests some deficiencies in the dynamic specification of the latter.

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[25] Equation (5.5) is based on the assumption that $\widetilde{y}_t = \phi \widetilde{y}_{t-1} + \widetilde{e}_{yt}$, with $\widetilde{e}_{yt}$ following $\widetilde{e}_{yt} = \rho_e \widetilde{e}_{yt-1} + \epsilon_t$, with ϵ_t white noise. By substitution, $\widetilde{y}_t = (\phi + \rho_e)\widetilde{y}_{t-1} - \phi\rho_e \widetilde{y}_{t-2} + \epsilon_t$. The parameters ϕ and ρ_e appear symmetrically in this expression and thus cannot be individually identified without further information; to identify them, we assume that ϕ is the larger of the two parameters.

[26] The behavior of our empirical measure of capacity output therefore supports the analytical model of Clarida, Gali, and Gertler (1997b), in which it is assumed that $\overline{y}_t$ follows a random walk.

[27] And also as the long-run growth rate of actual output, since the output gap is assumed to average zero over our sample period.

To simulate our model, we need to have values for the AR parameters and innovation variances in equations (3.13) and (3.17). Fitting an AR(1) model by least squares to the estimated residuals, $\widehat{\eta}_t$, of equations (5.3) and (5.4) produces

$$\widehat{v}_t = \underset{(0.073)}{0.3233}\,\widehat{v}_{t-1}, \qquad \text{SEE} = 0.0114, \tag{5.8}$$

$$\widehat{\eta}_t = \underset{(0.028)}{0.9346}\widehat{\eta}_{t-1}, \qquad \text{SEE} = 0.0225, \tag{5.9}$$

so that $\rho_v = 0.3233$, $\rho_\eta = 0.9346$, $\sigma^2_{ev} = (0.0114)^2$ and $\sigma^2_u = (0.0225)^2$. The residuals of equation (5.9) are virtually uncorrelated with those of (5.10), leading us to set $\psi_u = 0$ and $\sigma^2_{\varepsilon v} = (0.0114)^2$ in (3.18).

Finally, we turn to the policy rule. To describe actual policy behavior, we use equation (2.3), although our simulations in the next section will consider alternative, counterfactual policy rules. Since we specify the error term in (2.3) as an innovation, lagged endogenous variables are legitimate instruments in the estimation of the equation. Our instrument list for this equation consists of a constant, a time trend, d_{1t}, d_{2t}, Δx_{t-1}, Δx_{t-2}, $d_{2t-1} \cdot \Delta x_{t-1}$, $d_{2t-2} \cdot \Delta x_{t-2}$, Δp_{t-1}, Δp_{t-2}, and n_{t-1}.[28] The resulting estimated rule is:

$$\begin{aligned} \widehat{R}_t = {} & \underset{(0.035)}{0.103} + \underset{(0.049)}{0.866}R_{t-1} + \underset{(0.005)}{0.023}E_{t-1}\widetilde{y}_t + \underset{(0.034)}{0.117}E_{t-1}\Delta x_t \\ & + \underset{(0.001)}{0.002}\,d_{1t} + \underset{(0.031)}{0.064}\,d_{2t} \cdot E_{t-1}\Delta x_t \end{aligned} \tag{5.10}$$

[28] As before, we use $0.7n_t$ to measure (up to a constant) the output gap $\widetilde{y}_t$.

$\overline{R}^2 = 0.939$, SEE = 0.0017, DW = 1.99.

The large coefficient on the lagged dependent variable suggests a high degree of interest rate smoothing. The coefficient on the interactive dummy $d_{2t} \cdot E_{t-1}\Delta x_t$ indicates a substantial permanent increase in the restrictiveness of monetary policy from 1979. After 1979, a 1 percent increase in expected nominal income growth leads to a steady-state increase in the nominal interest rate of 1.35 percentage points, compared to only 0.87 points prior to 1979. This result is similar to the post-1979 increase in the coefficient on expected inflation in Clarida, Gali and Gertler's (1997a) estimates of the Taylor rule. The estimated intercept shift in the 1979-82 period is statistically significant and amounts to an upward shift of 0.8 percentage points when the interest rate is measured in annualized percentage units.

In the variant of our model that includes the Calvo-Rotemberg price-setting specification, the aggregate supply equation (4.5) appears. As is conventional, we set $\beta = 0.99$. The remaining coefficient in the equation is the ratio (θ/c_1). Using annual data, Roberts (1995) estimates this coefficient to be about 0.08. His version of equation (4.5), however, contained an additive disturbance term. Our equation (4.5), by contrast, has no explicit shock term; the randomness in inflation comes only from the stochastic behavior of the right hand side variables $E_t\Delta p_{t+1}$ and $\tilde{y}_t$. As a result, a much higher value of (θ/c_1) than Roberts' estimate, such as 0.30, is required to produce plausible inflation variability, for any of the policy rules that we consider. Thus, 0.30 is the value of (θ/c_1) that we employ. With θ, which is interpretable as the inverse of the aggregate markup under the aggregation scheme that we have used, set to 6, a value of $(\theta/c_1) = 0.30$ implies $c_1 = 20$.

6. Simulation Results: I

In this section we report simulation results for the variant of our macroeconomic model that uses the Calvo-Rotemberg specification of price adjustment behavior. In calculating these results, as well as those in the next section, we have made one change in the aggregate demand portion of our model, replacing $E_t y_{t+1}$ with $E_{t-1} y_{t+1}$ on the right-hand side of the expectational IS function (5.3). This change, which represents a modification of the same basic type as those employed by Rotemberg and Woodford (1997), but less severe, produces more plausible values for the variability of inflation in all our simulations (for both the specifications of aggregate supply that we contemplate).[29]

We begin with simulations involving versions of the Taylor rule, some of them suggested by the conference organizer to facilitate comparison across papers by different researchers. In particular, Table 1 includes results for various values of the policy parameters μ_1, μ_2, and μ_3 in a rule of the form

$$R_t = \mu_0 + \mu_1 \Delta p_t + \mu_2 \tilde{y}_t + \mu_3 R_{t-1}, \tag{6.1}$$

where μ_0 is in principle set so as to deliver the chosen average inflation rate and where policy responses are unrealistically assumed to reflect contemporaneous responses to the state of the economy. In the original Taylor rule $\mu_3 = 0$ but we have also considered cases with $\mu_3 = 1$ (to reflect interest rate smoothing by the Fed) and $\mu_3 = 1.2$ (to investigate a case recommended by Rotemberg and Woodford [1998]). The simulation results reported are

[29] This is particularly important in the context of the P-bar variant, where the two forward-looking components of the model interact in an overly sensitive way. In subsequent work, we plan to explore different modifications of our IS function, as suggested by the results of Campbell and Mankiw (1989) and Fuhrer (1997).

standard deviations (in annualized percentage units) of inflation Δp_t, the output gap $\tilde{y}_t$, and the interest rate R_t.[30] In these simulations constant terms are not included, so the standard deviation of Δp_t is interpretable as the root-mean-square deviation from the inflation target value π^*, as is also the case for $\tilde{y}_t$. The values reported are mean values over 100 replications, with each simulation being for a sample period of 200 quarters.[31] In solving the model, we use the algorithm of Paul Klein (1997), which builds upon that in King and Watson (1995).

Examination of the results in Table 1 shows that they suggest that, for a given value of the smoothing parameter μ_3, stronger responses to Δp_t or $\tilde{y}_t$ - - i.e., higher values of μ_1 or μ_2 - - lead invariably to lower standard deviations of that variable. Indeed, higher values of μ_1 *or* μ_2 lead in most cases to lower standard deviations of both Δp_t and $\tilde{y}_t$ (basically because of the nature of the price adjustment equation). This suggests that if there were no concern for variability of the interest rate, the central bank could achieve extremely good macroeconomic performance merely by responding very strongly to current departures of inflation and output from their target values. In our opinion, however, that would be a highly unrealistic conclusion to draw; the conduct of monetary policy by actual central banks is much more difficult than that. But such a conclusion tends to be obtained from exercises in which the central bank is assumed to possess knowledge of Δp_t and $\tilde{y}_t$ when setting its instrument value (R_t in this case) for period t. In other words, the policy rule (6.1) does not represent an operational specification.

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[30] For the purpose of comparison, the actual historical values over 1955-1996 are 2.41, 2.23, and 2.80.

[31] We ran simulations of 253 periods and ignored the initial 53, so as to abstract from start-up departures from stochastic steady-state conditions.

Table 1
Simulation Results with Calvo-Rotemberg variant
Taylor Rule, Contemporaneous Response
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, R_t respectively (percent per annum)

Values of μ_1, μ_3	Value of μ_2				
	0.0	0.5	1.0	3.0	10.0
1.5, 0.0	2.01	1.96	1.93	1.78	1.40
	1.15	1.12	1.10	1.03	0.82
	3.02	3.94	3.98	5.72	10.31
3.0, 0.0	1.78	1.78	1.72	1.60	1.29
	1.03	1.03	1.00	0.94	0.77
	5.34	5.84	6.13	7.59	11.53
10.0, 0.0	1.24	1.20	1.19	1.14	0.98
	0.75	0.73	0.72	0.69	0.60
	12.35	12.33	12.63	13.49	15.74
1.2, 1.0	1.32	1.25	1.19	1.10	0.97
	1.13	1.11	1.08	1.02	0.85
	2.38	2.94	3.41	5.42	10.63
3.0, 1.0	1.14	1.11	1.09[1]	0.98	0.82
	1.04	1.03	1.03[1]	0.97	0.81
	4.51	4.95	5.14[1]	6.80	10.00
10.0, 1.0	0.85	0.83	0.83	0.78	0.65
	0.86	0.85	0.85	0.82	0.71
	9.32	9.51	9.90	10.71	13.40
1.2, 1.3	1.31[2]	1.32	1.36	1.54	1.64
	1.12[2]	1.11	1.11	1.05	0.94
	2.10[2]	1.64	2.03	5.01	10.51

1. $\mu_2 = 0.8$, not 1.0

2. $\mu_2 = 0.06$, not 0.0

Because of this type of concern, the conference organizer suggested that results also be obtained for a specification like (6.1) but with inflation and the output gap lagged one quarter. Thus we next conduct simulations with

$$R_t = \mu_0 + \mu_1 \Delta p_{t-1} + \mu_2 \tilde{y}_{t-1} + \mu_3 R_{t-1}, \tag{6.2}$$

as the policy rule, and report results in Table 2.

For the cases where $\mu_1 = 1.5$ and there is no interest rate smoothing ($\mu_3 = 0$), the standard deviation of inflation is virtually identical in Table 2 to the corresponding rules in Table 1. As in Table 1, rules with smoothing ($\mu_3 = 1.0$) deliver better results with respect to both inflation and output gap variability than the corresponding rules without smoothing. However, while Table 1 indicated that, with smoothing, the standard deviation of inflation could be reduced to values as low as 0.65, the lowest standard deviation of inflation in Table 2 is 1.00. It is also clear from Table 2 that responding to lagged instead of contemporaneous data reduces the policy-makers' ability to stabilize output: the output gap standard deviation ranges from 0.60 to 1.15 in Table 1, while in Table 2 it ranges from 1.16 to 1.34.

Table 1 suggested that there were benefits in terms of both inflation and output gap variability from high values of μ_1 or μ_2, such as 10.0. In Table 2, on the other hand, these benefits are less clear. Whereas in Table 1, changing the output gap response coefficient μ_2 from 3.0 to 10.0 unambiguously improved performance with respect to both inflation and the output gap, in Table 2 this increase in μ_2 delivers poorer performance on output gap variability and, in most cases with interest rate smoothing, on inflation variability too. Rai-

Table 2
Simulation Results with Calvo-Rotemberg variant
Taylor Rule, Lagged Response
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, R_t respectively (percent per annum)

Values of μ_1, μ_3	Value of μ_2				
	0.0	0.5	1.0	3.0	10.0
1.5, 0.0	1.91	1.88	1.84	1.68	1.37
	1.21	1.22	1.21	1.19	1.19
	2.87	3.43	3.94	6.06	13.86
3.0, 0.0	1.71	1.68	1.66	1.54	1.31
	1.19	1.19	1.19	1.17	1.19
	5.14	5.62	6.17	8.12	15.79
10.0, 0.0	1.33	1.31	1.29	1.26	1.18
	1.19	1.18	1.18	1.20	1.26
	13.26	13.66	14.10	16.20	24.28
1.2, 1.0	1.33	1.27	1.23	1.20	1.25
	1.18	1.18	1.17	1.18	1.26
	2.27	2.78	3.29	5.51	12.76
3.0, 1.0	1.19	1.17	1.15[1]	1.10	1.11
	1.16	1.17	1.17[1]	1.17	1.25
	4.24	4.65	4.96[1]	6.95	13.50
10.0, 1.0	1.03	1.04	1.03	1.01	1.00
	1.17	1.18	1.19	1.20	1.27
	9.50	9.98	10.23	11.78	17.53
1.2, 1.3	1.34[2]	1.34	1.40	1.62	1.88
	1.18[2]	1.16	1.18	1.19	1.34
	2.03[2]	2.49	3.04	5.19	12.61

1. $\mu_2 = 0.8$, not 1.0

2. $\mu_2 = 0.06$, not 0.0

sing μ_1 from 3.0 to 10.0 does improve inflation performance, just as it did in Table 1, but, in contrast to Table 1, it fails to improve output gap performance appreciably.

While the results in Table 2 indicate that there is some deterioration in policy performance with rule (6.2) instead of (6.1), the deterioration is not particularly drastic, and the rules still deliver dynamically stable results with large values of μ_1 and/or μ_2. That finding comes as a surprise to us, but having obtained it we believe that it can be understood as follows. There are two properties of the model at hand that defuse the tendency, mentioned in McCallum (1997, Section 6), for explosive instrument instability to arise when strong feedback responses are based on lagged variables. First, the values of two parameters crucial for the transmission of policy actions to Δp_t are quite small; these are the slope of the "IS function" with respect to the real interest rate ($\sigma \cdot \frac{\bar{c}}{\bar{y}}$ in (3.12))[32] and the slope of the price adjustment relation (θ/c_1 in (4.5)). The smallness of the former implies that aggregate demand responses to changes in R_t are small, and the latter makes aggregate demand changes have small effects on inflation. Second, the Calvo-Rotemberg version of our model is one in which there is no autoregressive structure apart from what is contained in the disturbance terms and the policy rule. The model, that is, is entirely forward looking. We conjecture that models with backward looking IS and price adjustment specifications would possess much more of a tendency to generate dynamic instability for large values of μ_1 and μ_2.[33]

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[32] Our estimated value is less than $\frac{1}{20}$ of the value used by Rotemberg and Woodford (1997, 1998), for example.

[33] Even in the present model we found instability to prevail if μ_1 was raised to 1000 (!) and to prevail at lower values of μ_1 if σ is increased sharply. With contemporaneous feedback, there is no instability even in these cases.

Another operationality concern expressed by McCallum (1997) involves a lack of knowledge about $\overline{y}_t$, the market-clearing value of y_t. Suppose, then, that the central bank believes that a fitted linear trend line represents $\overline{y}_t$, while in fact our measure is correct. Then the central bank would use detrended y_t instead of $\tilde{y}_t$ in its policy rule and would measure output gap fluctuations in relation to this fitted trend. To get an idea of the implications, we redo the Table 1 case with $\mu_1 = 1.2$, $\mu_2 = 1.0$, and $\mu_3 = 1.0$ under this assumption. Then the standard deviation of Δp_t turns out to be 3.41 instead of 1.19, according to our model, and the central bank would believe that the standard deviation of $\tilde{y}_t$ was 3.91 (although it would actually be 1.09 - - almost the same as in Table 1). Also, the standard deviation of R_t would rise from 3.41 to 4.77.[34]

One issue mentioned in our introduction is the stability and desirability of nominal income targeting. To determine whether effects on Δp_t and $\tilde{y}_t$ would be much different if targets were set for $\Delta x_t = \Delta p_t + \Delta y_t$, we have conducted simulations using the rule

$$R_t = \mu_0 + \mu_1 \Delta x_t + \mu_3 R_{t-1}, \tag{6.3}$$

and also with Δx_{t-1} replacing Δx_t, for $\mu_3 = 0$ and $\mu_3 = 1.0$. These results are reported in Table 3. There we see that nominal income targeting with an interest instrument performs reasonably well. It permits considerably more variability of inflation than does the Taylor rule, but tends to stabilize output (in relation to $\overline{y}_t$) almost as well. It should be noted that the good performance in terms of $\tilde{y}_t$ occurs despite the absence of that variable or $\overline{y}_t$ in the policy rule. An advantage of nominal income (growth rate) targeting is that it does not

[34] These results are generated by replacing $\tilde{y}_t$ with y_t in (6.1), re-solving the model, and then looking at simulation results for Δp_t, y_t, and R_t.

Table 3
Simulation Results with Calvo-Rotemberg variant
Nominal Income Target, Interest Rate Instrument
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, Δx_t, R_t respectively (percent per annum)

	Contemporaneous Response Value of μ_3		Lagged Response Value of μ_3	
Value of μ_1	0.0	1.0	0.0	1.0
0.10		1.96 1.22 6.18 0.65		1.96 1.23 6.29 0.63
0.50		1.85 1.13 5.60 2.80		1.84 1.19 6.22 2.78
1.00		1.76 1.05 5.11 5.11		1.79 1.18 6.35 5.17
1.50	2.44 1.11 5.14 7.71	1.71 0.99 4.70 7.06	2.48 1.25 5.23 10.13	1.75 1.18 6.60 7.55
3.00	2.50 1.00 4.17 12.52	1.63 0.87 3.82 10.18	2.53 1.31 7.69 23.07	1.70 1.19 7.17 14.33
10.00	1.92 0.69 2.00 19.96	1.49 0.65 2.05 17.33	all variables explosive	2.00 2.25 18.16 93.84

require the central bank to measure capacity output. More interest rate variability occurs for most parameter values, but such variability is quite low (and the Δp_t and $\tilde{y}_t$ standard deviations are reasonably small) when μ_1 is assigned the small value of 0.1 with $\mu_3 = 1.0$.

As in Table 2, for moderate values of the feedback coefficient there is a deterioration in performance with respect to $\tilde{y}_t$ variability, but little deterioration in Δp_t variability, when feedback is applied with a one-period lag, i.e. to the value of Δx_{t-1} rather than Δx_t. Another similarity with Tables 1 and 2 is that making the feedback coefficient large (in this case, increasing μ_1 in (6.3) from 3.0 to 10.0) delivers an improvement in performance with respect to $\tilde{y}_t$, Δp_t, and Δx_t variability (at the cost of increased R_t volatility) when policy responds to contemporaneous data, but not when policy responds to lagged information. In the latter case, raising μ_1 from 3.0 to 10.0 actually delivers instrument instability when there is no interest rate smoothing. With smoothing, dynamic stability prevails for all variables, but the standard deviations of $\tilde{y}_t$, Δp_t, Δx_t, and R_t are all decidedly increased. Thus, Tables 2 and 3 are both supportive of the notion that assigning very high values to response coefficients is counterproductive when policy can only respond to lagged information.

Next we retain nominal income as the target variable, but consider the use of Δb_t - - the growth rate of the monetary base - - as the instrument. In particular, we consider two versions of McCallum's rule (2.1), one with a "levels" target path $x_t^{*1} = x_{t-1}^{*1} + \Delta x^*$ and the other with a "growth rate" target $x_t^{*2} = x_{t-1} + \Delta x^*$. Stochastic simulation results analogous to those discussed above are presented in Table 4. There it will be seen that performance is quite close to that in Table 3, where nominal income targeting is attempted with R_t as the

Table 4
Simulation Results with Calvo-Rotemberg variant
Nominal Income Target, Monetary Base Instrument
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, Δx_t, R_t respectively (percent per annum)

Value of λ in rule (2.1)	Levels target, x^{*1}	Growth-rate target, x^{*2}
0.25	1.05 1.29 5.25 1.88	1.05 1.30 5.27 1.80
0.50	1.07 1.29 5.25 1.96	1.00 1.27 5.19 1.72
1.00	1.03 1.27 5.25 1.02	1.01 1.28 5.20 1.78
3.00	1.04 1.28 5.39 3.06	1.01 1.26 5.16 2.11
10.00	1.04 1.28 5.89 3.73	0.99 1.25 5.31 4.30

instrument variable. Throughout Table 4, the variability of nominal income growth is about the same as it is with the best of the lagged-response rules in Table 3; moreover, the variability of inflation is lower than it is Table 3, and is comparable to the values obtained in Table 2 with the operational Taylor rule (6.2). In addition, there is no apparent tendency for interest rate variability to increase sharply when the base is used.

A comparison of the levels-target and growth-rate target rule performances in Table 4 shows, somewhat surprisingly, that the results are little different, and, in particular, that $\tilde{y}_t$ variability is not lower with the growth-rate specification. For both rule types, another striking feature is how insensitive the variability of the nominal income growth rate is to changes in the value of the response coefficient λ.[35] Presumably this is the case because the parameter values estimated in Section 5 imply an extremely small response of aggregate demand to real money balances $(b_t - p_t)$.

It should be emphasized that the stochastic simulation exercises underlying Tables 1-4 do not serve to bring out one aspect of operationality claimed by McCallum (1988) for (2.1), namely, its non-dependence on the long-run average growth rate of base velocity. That non-dependence, which is not possessed by most rules with base or reserve aggregate instruments, is basically irrelevant for the stochastic simulations in which constant terms are omitted. Thus the velocity-correction term in (2.1) could be omitted without any appreciable

[35] The levels target results suggest that nominal income growth Δx_t variability is *increasing* in λ; this reflects the fact that, in the simulations, the levels target is a constant, so successful nominal income targeting implies that x_t is I(0). Δx_t is therefore I(−1), and hence will tend to be highly variable, the more so when nominal income targeting is pursued vigorously (i.e., with high values of λ). The standard deviation of the *level* of nominal income in the simulations underlying the first column of Table 1 *is* decreasing in λ, taking the values 1.44, 1.35, 1.26, 1.15, and 1.10 for $\lambda = 0.25, 0.50, 1.00, 3.00$, and 10.00 respectively.

effect on the results of Table 4, which is most definitely not the case for the counterfactual historical simulations reported in (e.g.) McCallum (1988, 1993). Accordingly, we plan to include some simulations of this latter type in subsequent work.

7. Simulation Results: II

In this section we report stochastic simulation results analogous to those of Tables 1-4 but now using the P-bar price adjustment relation. Table 5 gives standard deviations of Δp_t, $\tilde{y}_t$, and R_t for the same values of μ_1, μ_2, and μ_3 as those considered in Table 1, under the assumption of contemporaneous feedback responses to Δp_t and $\tilde{y}_t$. Again it is the case that an increase in μ_1 (μ_2) reduces the variability of Δp_t ($\tilde{y}_t$), but it is not now the case that increasing either μ_1 or μ_2 tends to reduce the variability of both Δp_t and $\tilde{y}_t$. Instead, there is a variability trade-off at work, with increases in μ_2 often increasing the variability of Δp_t. The existence of interest rate smoothing, with $\mu_3 = 1$, is helpful in most cases and is so to a greater extent than in Table 1. Overall, the variability of Δp_t, $\tilde{y}_t$, and R_t is considerably greater than in Table 1. For $\tilde{y}_t$, its magnitude is much more realistic but for Δp_t and/or R_t it is somewhat excessive.

Table 6 is partly but not entirely analogous to Table 2, in which lagged values of Δp_t and $\tilde{y}_t$ are used in rule (6.2). When such values are utilized, dynamically explosive results are obtained for most parameter configurations. Consequently, Table 6 reports values for feedback responses to the lagged value of $\tilde{y}_t$ but to the current value of Δp_t. This modification seems justifiable from an operationality perspective because Δp_t is a predetermined variable in the P-bar variant of our model, so Δp_t is in principle observable at the end of period $t-1$.

Table 5
Simulation Results with P-bar variant
Taylor Rule, Contemporaneous Response
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, R_t respectively (percent per annum)

Values of μ_1, μ_3	Value of μ_2				
	0.0	0.5	1.0	3.0	10.0
1.5, 0.0	8.53	9.67	10.68	14.20	31.38
	2.48	2.39	2.31	1.98	0.55
	12.80	13.70	14.42	16.93	30.24
3.0, 0.0	2.88	3.13	3.37	4.51	12.74
	2.51	2.43	2.34	2.15	0.78
	8.64	8.83	9.01	10.01	19.75
10.0, 0.0	0.76	0.81	1.18	2.00	3.39
	2.49	2.43	2.04	1.57	0.89
	7.63	7.63	8.34	11.78	18.46
1.2, 1.0	3.61	3.71	4.14	6.72	19.00
	2.51	2.42	2.24	1.88	0.44
	5.32	5.68	5.86	7.17	13.86
3.0, 1.0	1.95	1.96	1.98[1]	2.97	6.11
	2.53	2.42	2.36[1]	2.06	1.49
	6.26	6.33	6.27[1]	7.14	10.10
10.0, 1.0	0.70	0.69	0.71	0.95	1.99
	2.42	2.32	2.34	2.12	1.66
	6.88	6.83	6.95	7.52	10.44
1.2, 1.3	3.76[2]	3.74	4.10	6.51	12.61
	2.51[2]	2.35	2.27	1.82	1.15
	4.49[2]	4.65	4.98	6.02	8.72

1. $\mu_2 = 0.8$, not 1.0

2. $\mu_2 = 0.06$, not 0.0

Table 6
Simulation Results with P-bar variant
Taylor Rule, Lagged Response to $\tilde{y}$, Contemporaneous to Δp
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, R_t respectively (percent per annum)

Values of μ_1, μ_3	Value of μ_2				
	0.0	0.5	1.0	3.0	10.0
1.5, 0.0	8.53 2.48 12.80	9.92 2.42 13.92	11.23 2.35 14.88	16.15 2.09 18.52	29.51 1.59 28.78
3.0, 0.0	2.88 2.51 8.64	3.25 2.53 9.04	3.54 2.44 9.13	5.23 2.35 10.18	11.08 2.08 13.78
10.0, 0.0	0.76 2.49 7.63	0.80 2.42 7.57	0.88 2.45 7.55	1.24 2.39 7.67	2.86 2.32 8.33
1.2, 1.0	3.60 2.56 5.32	3.79 2.43 5.74	4.19 2.39 6.12	7.00 2.19 7.99	15.30 1.62 12.75
3.0, 1.0	1.95 2.47 6.16	1.95 2.48 6.31	2.07[1] 2.47[1] 6.43[1]	3.17 2.39 7.17	7.50 2.09 9.51
10.0, 1.0	0.69 2.44 6.83	0.69 2.44 6.80	0.75 2.51 7.01	1.01 2.39 6.95	2.45 2.30 7.50
1.2, 1.3	all variables explosive[2]	3.75 2.50 4.81	all variables explosive	all variables explosive	13.68 1.68 10.79

1. $\mu_2 = 0.8$, not 1.0

2. $\mu_2 = 0.06$, not 0.0

The resulting standard deviations are quite close to those of Table 5 for small and moderate values of μ_1 and μ_2, but are larger for high values of these feedback parameters. There is no evident tendency toward dynamic instability, however, except in the "Rotemberg-Woodford" cases with $\mu_3 = 1.3$.

Next we consider the effect of an incorrect belief by the central bank that a fitted trend line represents $\overline{y}_t$ when in fact our measure is correct. With the P-bar price adjustment relation included, rather than the Calvo-Rotemberg version, this effect is considerably smaller. Thus in the particular case mentioned in Section 6 - - i.e., with $\mu_1 = 1.2$, $\mu_2 = 1.0$ and $\mu_3 = 1.0$ - - the Δp_t and $\tilde{y}_t$ standard deviations increase only from 4.14 and 2.24 (respectively) to 4.80 and 2.35. The reduction in this effect obtains, clearly, because the P-bar specification makes $\tilde{y}_t$ very strongly related to $\tilde{y}_{t-1}$. If the central bank responds more vigorously to its (incorrect) beliefs about $\tilde{y}_t$, however, the deleterious effect will be somewhat larger. With $\mu_2 = 3.0$, for example,[36] the standard deviations increase from 6.72 and 1.88 to 10.50 and 2.15.

With nominal income targeting and an interest instrument, the results with the P-bar variant of our model are given in Table 7. There the results are much more favorable with μ_3 equal to 1.0 rather than zero, i.e., with interest smoothing. The ability of rule (6.3) to keep Δx_t close to its target value is about the same as with the Calvo-Rotemberg variant, but results in terms of the variability of Δp_t (and to a lesser extent $\tilde{y}_t$) are much less desirable. Clearly, the dynamic relationship between Δp_t and $\tilde{y}_t$ is very different with these two price-

[36] With μ_1 and μ_3 as before.

Table 7
Simulation Results with P-bar variant
Nominal Income Target, Interest Rate Instrument
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, Δx_t, R_t respectively (percent per annum)

	Contemporaneous Response Value of μ_3		Lagged Response Value of μ_3	
Value of μ_1	0.0	1.0	0.0	1.0
0.10		7.46 2.07 8.50 1.77		7.51 2.10 8.56 1.72
0.50		4.36 2.27 5.85 4.07		4.67 2.19 6.13 3.72
1.00	27.65 1.77 28.62 28.62	2.97 2.24 4.58 5.67	38.87 1.73 39.42 39.28	3.56 2.21 5.16 5.12
1.50	6.82 2.31 8.22 12.33	2.25 2.09 3.90 6.78	all variables explosive	3.41 2.17 4.96 6.26
3.00	2.70 2.06 3.91 11.75	1.47 1.97 2.95 9.54	all variables explosive	all variables explosive
10.00	0.98 1.67 1.67 16.71	0.96 1.64 1.67 15.77	all variables explosive	all variables explosive

Table 8
Simulation Results with P-bar variant
Nominal Income Target, Monetary Base Instrument
Reported figures are standard deviations of
Δp_t, $\tilde{y}_t$, Δx_t, R_t respectively (percent per annum)

Value of λ in rule (2.1)	Levels target, x^{*1}		Growth-rate target, x^{*2}
0.25	6.67 2.24 7.70 2.47		7.17 2.20 8.26 2.29
0.50	6.38 2.27 7.51 2.61		6.88 2.27 7.95 2.40
1.00	6.08 2.28 7.20 2.83		6.38 2.27 7.49 2.56
3.00	5.48 2.18 6.61 3.55		5.28 2.30 6.62 3.23
10.00	5.00 2.19 6.09 5.50		3.62 2.26 5.14 5.02

adjustment specifications.

Finally, in the Table 8 case with rule (2.1), in which Δb_t is the instrument variable (and x_t or Δx_t the target variable), the performance is about the same as in Table 7. For a given level of R_t variability, that is, the standard deviations of Δx_t, Δp_t, and $\tilde{y}_t$ are about the same. Furthermore, the figures indicate a low degree of responsiveness of nominal income variability to the feedback parameter λ, although the responsiveness is considerably greater than it was with the Calvo-Rotemberg variant of our model (in Table 4). Again, this low responsiveness is largely a result of the optimizing IS specification that we employ, which implies that aggregate demand is quite insensitive to the quantity of real money balances.

8. Conclusions

There are some conclusions from the simulation results that hold for both variants of our model - - i.e., with both price adjustment relations. The first of these is that the inclusion of the R_{t-1} interest-smoothing term in the Taylor rule is helpful in reducing the variability of Δp_t and $\tilde{y}_t$ for given values of the policy-response parameters μ_1 and μ_2, while also reducing R_t variability. Second, for moderate values of response coefficients, the use of lagged rather than contemporaneous values of $\tilde{y}_t$ does not bring about any major deterioration in results and does not generate any severe danger of instrument instability.[37] Third, nominal income targeting with an R_t instrument is only mildly effective but shows no noticeable tendency to generate dynamic instability, provided that interest rate smoothing is employed.[38] Fourth,

[37] This is not true, as mentioned, for lagged Δp_t values in the P-bar variant, in which case Δp_t is itself a predetermined variable.

[38] With strong feedback or with $\mu_3 = 0$ in the lagged response cases, dynamic instability obtains. It is not, however, of the type mentioned by Ball (1997), which involves instability of Δp_t and $\tilde{y}_t$ even though

nominal income targeting with a monetary base instrument does not imply drastically greater R_t variability than with an interest instrument. It is, however, only weakly effective - - the standard deviation of Δx_t is not very responsive to the feedback parameter λ.[39]

Other conclusions are more sensitive to the model variant. For example, pure inflation targeting ($\mu_1 > 0$, $\mu_2 = 0$) is quite effective in the Calvo-Rotemberg specification but significantly less so with the P-bar relation. More generally, increasing μ_1 *or* μ_2 tends (for moderate ranges of those parameters) to reduce *both* inflation and output gap variability with the Calvo-Rotemberg variant; by contrast, the P-bar specification generates a trade-off between inflation and output gap variability, so that raising μ_2 for a given μ_1 yields improved output gap performance at the expense of more variable inflation. Furthermore, performance deteriorates sharply if the central bank responds to an incorrect measure of capacity output ($\overline{y}_t$) when the Calvo-Rotemberg relation is used, but does so only moderately with the P-bar specification. And nominal income targeting holds down inflation variability much better with the Calvo-Rotemberg version of the model. Finally, when policy responds to lagged rather than contemporaneous output gap data, increasing the value of the Taylor rule response coefficient on the output gap to a very high level (say, 10) tends to be counterproductive - - in the sense of increasing rather than decreasing output gap variability - - when the Calvo-Rotemberg specification of aggregate supply is used. This result does not carry over when the P-bar specification is employed.

Δx_t is stabilized.

[39] This conclusion might be changed by alternative specifications of relations analogous to our (3.14) and (3.16).

These last-mentioned conclusions illustrate the importance, mentioned in our introduction, of the robustness of proposed rules to model specification. In future work, we hope to conduct a small robustness study of our own while also investigating several issues that we have not yet been able to explore.

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