VALUING THE FUTURES MARKET CLEARINGHOUSE'S DEFAULT EXPOSURE DURING THE 1987 CRASH

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ABSTRACT

Futures market clearinghouses are intermediaries that make large volume trading between anonymous parties feasible. During the market crash in October 1987 rumors spread that a major clearinghouse might fail. This paper presents estimates of three measures of the default exposure on the popular S&P500 futures contract traded on the Chicago Mercantile Exchange. We estimate the traditional summary statistic for risk exposure: the tail probabilities that the change in the futures price exceeds the margin. And we estimate two economic measures of the risk--the expected value of the payoffs in the tails, and expected value of the payoffs in the tails conditional on landing in the tail. The economic measures of risk reveal exposure from low probability large payoff events--like a crash--that does not show up tail probabilities. The tail probabilities only capture the likelihood of a crash, not the expected loss. The estimated measures of risk follow directly from estimates of the conditional distribution of futures price changes. We infer a jump-diffusion process and a log-normal process from the prices of traded options and we estimate a jump-diffusion process from time-series data on futures prices. After the crash the forward-looking jump-diffusion model inferred from traded options dramatically reflects the fears of another crash voiced by market participants. The model indicates another jump is unlikely, but if it occurred it would be big and negative. The tail probabilities are small, less than 2%. But, the day after the crash the model estimates the expected value of payoffs in the tails conditional on landing in the tail equals of 55% of the S&P500 futures price. According to this estimate roughly $10.5 billion in liquid reserves would be required to weather another crash. On October 20 the Federal Reserve announced it stood ready to supply the necessary liquidity.

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Introduction

Futures and forward contracts are agreements between two parties to buy or sell an asset at a future date at a price set today. Futures contracts, unlike forward contracts, trade on organized exchanges. Futures market clearinghouses are intermediaries that make large volume trading between anonymous parties feasible by guaranteeing performance on all trades between clearing members. Nonmembers execute trades through a clearing member. Clearinghouse intermediation makes futures contracts liquid and isolates traders from individual counter-party default risk.[1]

The margin system is the clearinghouse's first line of defense against default risk. Margin collection and administration are organized in a pyramid structure described in Edwards (1983). The clearinghouse, at the top of the pyramid, collects margins from clearing members. The clearinghouse demands a performance bond (initial margin) when a contract is opened. Thereafter, the clearinghouse "marks" member accounts "to market" to prevent losses from accumulating. It collects funds (variation margin) from clearing members who hold contracts that had a capital loss and distributes funds (also called variation margin) to members who hold contracts that had a capital gain. Clearing member futures commission merchants (FCMs) collect margins from (and distribute gains to) nonclearing FCMs who execute their trades through the clearing member. At the base of the pyramid all FCMs collect margins from and distribute gains to their customers. If the losers don't meet the variation margin call, then the clearinghouse must come up with the funds from its own reserves, or assess the remaining solvent clearing members, or default.

On Monday, October 19, 1987, the S&P 500 futures price declined by 29% -- the largest one day price change since trading began. On that day the Chicago Mercantile Exchange (CME) clearinghouse

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[1] See Hull (1997) p3. Forward contracts are over the counter instruments traded between principals with established credit--usually large banks. Forward contracts are not liquid--most forward contracts terminate in delivery. In contrast, 98% of futures contracts are terminated with an offsetting position, see Fabozzi, et al (1997), p507.

issued variation margin calls for a record \$2.5 billion. The Commodity Futures Trading Commission (the regulatory board for the futures markets) disclosed that during October fourteen FCMs became undersegregated (the FCM had less than the required cash in consumer accounts) and three firms were undercapitalized. In addition eleven firms, including six CME members, had margin calls to a single customer that exceeded their capital. Traders feared that a default by a large customer would trigger a cascade collapsing the pyramid. Rumors spread that a major clearinghouse might fail.[2]

The clearinghouse's default exposure depends on the probability distribution of changes in the futures price. The traditional measure of risk is the probability that a future price change will exceed the margin. Figlewski (1984), Gay, Hunter, and Kolb (1986), Hsieh (1993), and Kupiec (1994) and others estimate the tail probabilities. However, this measure is somewhat limited in ignoring the *consequences* of a futures move that exhausts posted margin. We present two additional methods of assessing clearinghouse exposure, and use them along with tail probability estimates to examine the CME's exposure in late 1987 on the popular S&P 500 futures contract traded on the Chicago Mercantile Exchange.

The first additional measure of default exposure is the expected value of the additional funds required to ensure performance on all futures contracts. If evaluated using asset pricing techniques, this expectation is the premium a clearinghouse would pay for a hypothetical insurance policy that would cover the additional funds. As in some previous examinations of default risk, pricing this insurance draws upon option theory.[3]

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[2]See Bernanke (1990) and the Report of the Presidential Task Force on Market Mechanisms (1988, section VI).

[3]Merton (1974) represented the default risk on risky debt as a put option, while Merton (1977) used put option prices to evaluate the fair price of bank deposit insurance. Previous applications of option pricing to clearinghouse exposure include Craine (1992) and Day and Lewis (1997).

Second, we estimate the expected additional funds requirement conditional on a futures move exhausting the posted margin. Conceptually, this would determine the reserves the insurance provider should hold to cover potential claims. To our knowledge, this measure has not been previously used in examining clearinghouse exposure. Its magnitude is a key determinant of whether the clearinghouse is likely to survive a futures price move that exceeds the margin requirement.

We use two methods to estimate the parameters of the conditional distribution, and to evaluate the three measures of clearinghouse exposure. From time-series data on futures prices we estimate an EGARCH-jump process whose features include volatility persistence, a negative correlation between market returns and volatility shocks, and substantial daily excess kurtosis and/or skewness. We incorporate various informational sources into the conditional variance assessments: lagged shocks, volatilities implied by the price of traded options and intraday high-low price ranges. Since jumps are especially important for default risk, we also condition the current jump probability on the information variables. The model builds upon much previous work in time series econometrics, although some features (e.g., time-varying jump risk) appear to be new.

Second, prices of options on S&P 500 futures contain substantial information regarding traders' assessments of future S&P 500 futures returns. As discussed in Bates (1991), the distributions implicit in S&P 500 futures options exhibited substantial skewness and/or excess kurtosis during the year preceding the stock market crash of October 19, 1987. We use the implicit jump-diffusion parameter estimation approach of Bates (1991) to obtain a second estimate of the conditional distribution of futures price changes from the prices of traded options. We also infer the parameters of a lognormal distribution (no jumps or fat tails) implied by the classic Black-Scholes (1973) model for comparison.

The insurance premium and required reserves measures of risk indicate that the tail probability approach can offer a misleading picture of post-crash CME exposure. Judging only from tail probabilities, the CME's aggressive margin requirement increases in October 1987 combined with shifting conditional distributions reduced clearinghouse exposure to pre-crash levels by or before the end of November. However, the *consequences* of a futures price move in excess of margin were estimated at 1-2 orders of magnitude higher than pre-crash levels, given post-crash time series and option-based estimates of substantial jump risk. Low-probability large-magnitude jumps do not especially show up in tail probabilities, but do show up in the other risk measures. According to S&P 500 futures options prices, clearinghouse exposure peaked on October 20, when reserves of roughly $10.5 billion were required to weather another crash. On October 20 the Federal Reserve announced it stood ready to supply the necessary liquidity.

The paper is organized as follows: Section 1 presents the measures of risk exposure. Section 2 gives the specification of the jump-diffusion model and presents the parameters inferred from traded option prices and estimated from time-series data on the S&P 500 futures contract. Section 3 presents estimates of the measures of risk exposure for October and November of 1987. Section 4 concludes.

Section 1: Measures of the Exposure

This section presents the three measures of exposure associated with a single futures position. For concreteness, we focus on the margin system for clearing members of the CME in 1987. At that time, the CME was one of only two clearinghouses that used a gross margining system, under which margins were required for each contract. Other clearinghouses used a net margining system in which offsetting positions (a long and a short) required no margin. In 1988 the CME moved to a system of margins against a portfolio held by the clearing member; see Kupiec (1994) for details.[4]

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[4]Margins for customer accounts are much more complicated; see Edwards (1983) and Rutz (1989).

It should be emphasized that the measures of exposure presented here do not depend on the particular margining system, nor upon the particular position held. They are general measures that could be used to characterize the risk associated with any position, or portfolio of positions, that is partially secured by a margin requirement. The principles used in assessing the clearinghouse's guarantee of futures positions could equally be used in assessing the clearinghouse's guarantee of written options.

Margins

The exchange clearinghouse demands that clearing members post a performance bond (initial margin) of M, when they enter a futures contract[5] on behalf of customers or on their own account. To prevent losses from accumulating the clearinghouse "marks" members' accounts "to market" at intervals τ[6] and forces them to realize the capital loss, or gain, on their position. The clearinghouse demands "variation margin" equal to the change in the market value of the contract, $\Delta F \equiv F_{t+\tau} - F_t$, where F_t denotes the price of the contract at time t. If the price of the futures contract goes up the short seller must add variation margin equal to the loss in market value of the contract. If the price of the futures contract goes down the clearinghouse credits the variation margin to the short seller's account and he can withdraw the funds. The variation margin for a long position is the negative of the variation margin for a short position.

Exposure

The clearinghouse credits the accounts of positions with a gain and debits the accounts of positions with a loss. If the losers don't come up with the variation margin, the clearinghouse must draw down

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[5] In 1987 clearing members actually posted initial margin on any new positions taken during the day (and still open when the exchange closed) before the next business day. When the clearinghouse raises the initial margin requirement, as it did four times in October, the clearing member must post additional initial margin for all open contracts.

[6] In 1987 the normal interval for the CME was daily. On March 1, 1988, the CME began realizing capital gains or losses on positions twice a day: at noon and at the close. Since June 26, 1992, the noon settlement has been accompanied by a variation margin call.

reserves, or assess solvent clearing members, or default. The clearinghouse holds initial margin against each contract, so the net exposure *ex post* equals the absolute value of the futures price change minus the margin, or zero,

$$V(\Delta F, M) = \max[0, |\Delta F| - M] \tag{1}$$

In this article we focus on the uncollateralized additional funds requirement V that must be raised *in some fashion* to ensure performance on futures positions. The funds may come from customers, or the capital or reserves of FCMs or the clearinghouse. *Who* pays is not addressed here; our focus is on how much *someone* will need to stump up in additional resources.

Figure 1 plots the funds that must be raised as a function of the futures price. For futures price changes smaller (in absolute value) than the margin no additional funds have to be raised. The clearinghouse credits the account of the member with a capital gain. If the member with the capital

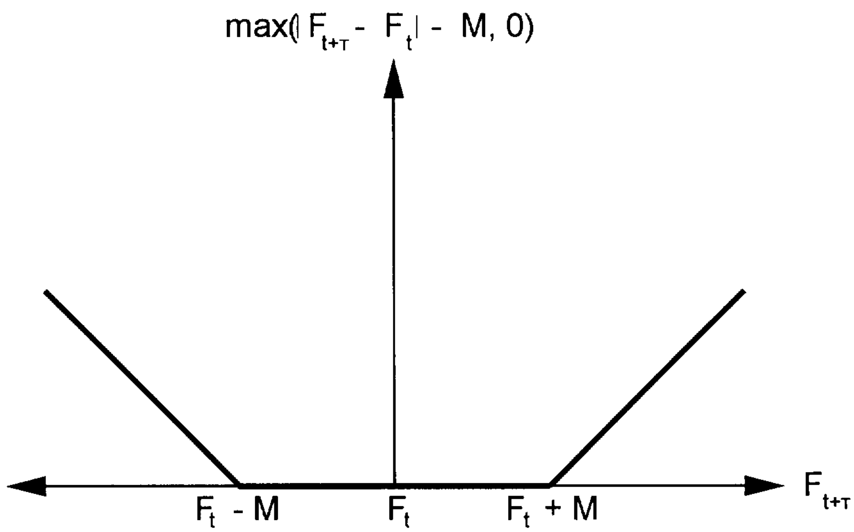

Figure 1. Uncollateralized additional funds requirement per futures contract, as a function of the end-of-period futures price $F_{t+\tau}$.

loss fails to make the variation margin payment the clearinghouse takes the payment out of the initial margin and liquidates the position. For futures price changes greater than the margin additional funds must be raised, or the clearinghouse defaults.

Measures of Exposure

1. $\rho = Prob_t(|\Delta F| > M)$

The tail probability ρ is the conditional probability that additional funds must be raised. This is the typical measure used to evaluate risk exposure in the futures market. See, e.g., the excellent post-crash survey by Warshawsky (1989).

2. $S(F_t, M) = E_t[V(\Delta F, M)]$

The conditional expected value of the additional funds that must be raised. If evaluated using a "risk-neutral" expectation operator E_t^*, this is the daily premium on an insurance policy that would cover the funds if they were needed. To see this, note that equation (1) and Figure 1 are the gross payoff function on a portfolio of options: an "out-of-the-money" put option with strike price $X_c = F_t - M$ *below* the current futures price, and an out-of-the-money call option with strike price $X_c = F_t + M$ *above* the current futures price This option portfolio is known as a *strangle* (Hull, 1997); the market price of the strangle is the insurance premium.

Options have been used to price default risk in finance for some time. Merton (1974) represented the default premium on risky debt as a put option. In 1977 he showed that the fair price of bank deposit insurance was the value of the put option. In this paper we use the option price to assign an economic measure to the funds that must be raised if the change in the futures price exceeds the margin.[7]

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[7]The strangle price S is a summary statistic that measures the economic significance of the exposure just as the tail probability, ρ, is a summary statistic measuring the likelihood of the

3. $\Re = E[V(\Delta F, M) \mid V > 0]$

The expected value of the additional required funds *conditional* on additional funds being needed. $\Re$ is the estimated value of liquid reserves that are required to make the payment of additional funds. In insurance markets, these would be the reserves the insurance company holds to assure the buyers they could pay any claims. Similarly, option writers might be obligated to post a comparably computed amount to ensure against default.

This measure is not commonly used to assess default risk in finance or in the futures market. It is arguably the most appropriate measure. Since futures contracts are not limited liability contracts, if the losers have access to liquid reserves they will meet the margin calls. If liquid assets are not available a large price change triggers a liquidity crisis and clearinghouse default.

Where these liquid reserves come from is irrelevant to the issue of default risk. Obviously, the clearinghouse would prefer that customers meet additional funds requirements. But from a default risk perspective, the relevant reserves for ensuring performance on futures contracts include customers' liquid reserves (lines of credit and assets), the reserves of the FCMs' and clearinghouse, and potentially even the readiness of the Federal Reserve to intervene.

The above three measures of exposure are interrelated. Since V is either positive or zero, the strangle price can be written as

$$\begin{aligned} S &= E_t^*[V \mid V > 0]\ Prob_t^*[V > 0] \\ &= \Re^*\ \wp^*, \end{aligned} \tag{2}$$

exposure. Neither implies default will actually occur when the change in the futures price exceeds the margin.

where $\Re^*$ and $\wp^*$ are variants of $\Re$ and $\wp$ evaluated under the risk-neutral probability measure used in asset pricing, rather than under the conditional probability measure from a time series model. Thus, any two measures suffice to identify the third. As discussed below, it is reasonable to assume that actual and risk-neutral conditional distributions do not deviate substantially at the 1-day horizon considered here.

Section 2: Estimates of the Conditional Distribution

This section presents estimates of the conditional distribution of the price of the S&P 500 futures contract. The S&P 500 futures contract is a high volume contract that was popular with index arbitragers and portfolio insurers. At the beginning of October open interest was roughly 115,000 with a notional value of almost $20 billion.[8] Daily volume ran at approximately 80,000 contracts.

The three measures of exposure depend critically on the conditional distribution of the change in the futures price. Estimating the conditional distribution can be difficult even under normal circumstances. Such estimation is especially difficult during the period following the stock market crash of October 19, 1987, given that the 1987 crash registered by far the largest daily percentage movements in U.S. stock prices since the beginning of accurate record keeping.[9] But even pre-crash data exhibit substantial abnormalities. As shown in Figure 2, the empirical distribution of daily futures returns on the S&P 500 contract (difference of the log of prices) over January 2, 1985 through September 30, 1987 were substantially negatively skewed and leptokurtic, with a Shapiro-Wilks test rejecting

[8] The actual price of an S&P futures contract is $500 times the quote: $\text{Price}_t = \$500 * F_t$.

[9] Shiller (1994) notes that the S&P 500 index fell 20.46% on October 19, 1987 from the preceding Friday's close. The next largest daily movements, of 10-13% in magnitude, occurred in 1929-32; e.g., the -12.34%, -10.16%, and +12.53% moves on October 28-30, 1929 and the 12.36% increase on October 6, 1931.

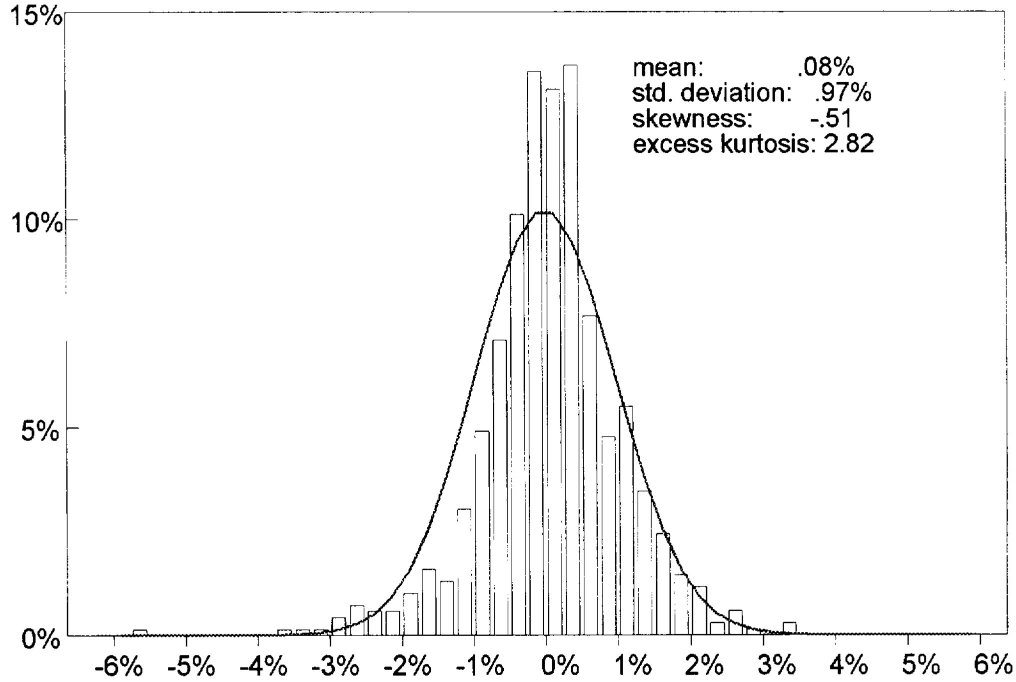

Figure 2. Distribution of daily returns on S&P 500 futures: Jan. 2, 1985 through Sept. 30, 1987. Observed frequencies, and theoretical normal.

normality at a *P*-value of .0001. The 5.7% decline on September 11, 1986 was partially but not fully responsible for the observed negative skewness and excess kurtosis.

We consequently employ two approaches for assessing futures return distributions conditional upon contemporaneous information, each with particular strengths and weaknesses. We infer conditional distributions from options on S&P 500 index futures. Traded options are forward-looking assets that reflect the market information set. The prices are sensitive to salient distributional characteristics: conditional volatility, skewness, and excess kurtosis. Furthermore, observed option prices incorporate the relevant required compensation for assorted untraded risks (jump risk, volatility risk), and therefore in principle could be used to directly price the exposure, ie, the price of the strangle option portfolio. The major difficulty is the mismatch between the monthly/quarterly maturities of traded options[10] and the standard one-day interval for marking accounts to market and collecting variation margin.

We also directly estimate the parameters of the conditional distribution of daily S&P 500 futures returns from the time series of returns. Such estimates are tailored to daily frequencies, and there are many statistical techniques that can be employed in estimating conditional distributions. On the other hand, time series-based estimates are intrinsically backward-looking, are conditioned on an information set which is smaller than the market information set, and have difficulties when an "outlier" of the magnitude of October 19, 1987 is included in the data base.

The time series analysis proceeds in two steps. First, we analyze pre-crash daily futures returns over January 2, 1985 through September 30, 1987, and identify those informational variables most useful in forecasting return distributions. Second, we update conditional distribution estimates on a daily basis over October and November 1987, using a nonlinear "rolling regression" methodology.

Assumed Distribution

We assume the data generating process is well approximated by the jump diffusion process,

$$d\ln F = (\mu_t - \lambda_t \bar{\gamma})dt + \sigma_t dW + \gamma\, dq \tag{3}$$

where

$\mu_t - \lambda_t \bar{\gamma}$ is the drift in the Brownian motion;

W is a Weiner process;

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[10]Options on the S&P 500 futures contract traded on the CME span the succeeding six months and expire on the third Friday of the month. The October 1987 S&P 500 futures options expired on October 16 -- the Friday before the crash.

σ_t is the instantaneous volatility conditional upon no jumps;

q is a Poisson counter with instantaneous intensity λ_t: $Prob(dq = 1) = \lambda_t dt$;

and the jump size γ is normally distributed with mean $\bar{\gamma}$ and variance δ^2.

The jump-diffusion process is a flexible specification that can accommodate most of the features observed in the data. If there are no jumps and the parameters are not time-varying, then the process collapses to the popular geometric Brownian motion specification assumed by Black and Scholes (1973). Jumps produce a distribution with fatter tails, and an asymmetric jump process ($\bar{\gamma} \neq 0$) introduces skewness. Time-varying volatility also generates a fat-tailed unconditional distribution, but has little impact on the higher moments of 1-day conditional distributions.

Options-based Implicit Distributions

Bates (1991, 1996) develops formulas for pricing options on jump-diffusion processes with constant parameters $\theta \equiv \langle\sigma, \lambda^*, \bar{k}^*, \delta\rangle$. Following Bates (1991), we infer daily implicit jump-diffusion parameters from all observed intradaily call and put transaction prices for December 1987 S&P 500 futures options, using nonlinear least squares:

$$\hat{\theta}_t = \underset{\theta}{argmin} \sum_{i=1}^{N_t} \left[\frac{O_i - O(F_i, T_t, X_i; \theta)}{F_i}\right]^2 \tag{4}$$

where O_i is the ith observed call or put price on date t; $O(\bullet)$ is the corresponding value from Bates' option pricing formulas (Section II, equations (13) and (16)) given the underlying futures price F_i, maturity T_t, strike price X_i, and parameters θ; and N_t is the number of options data on date t. Starred parameters indicate parameters of the "risk neutral" process. The divergence of the "risk-neutral" parameters λ^* and $\bar{\gamma}^*$ from the parameters λ and $\bar{\gamma}$ of the true jump-diffusion process of equation (3) reflect required compensation for systematic jump risk. Calibrations in Bates (1991, p. 1034) suggest little divergence between actual and risk-neutral parameters under plausible measures of risk aversion.

Time Series-Based Conditional Distributions: Model Selection

We also estimate the conditional distribution of daily returns using time series data. The discrete time analogue to the jump-diffusion process in equation (3) is a stochastic mixture of normals, randomized over the number of jumps n occurring within a given time interval:

$$\ln(F_{t+1}/F_t) \mid n\ jumps \sim N[(a_1 + a_2\sigma_t^2 - \lambda_t\bar{\gamma})\tau_{t+1}^d + n\bar{\gamma},\ \sigma_t^2\tau_{t+1}^d + n\delta^2]$$

$$Prob(n\ jumps) = \frac{e^{\Lambda_t}\Lambda_t^n}{n!} \qquad \textbf{(5)}$$

$$\Lambda_t \equiv \lambda_t\tau_{t+1}^d = (\lambda_0 + \lambda_1\sigma_t^2)\tau_{t+1}^d; \quad \lambda_0, \lambda_1 \geq 0;$$

where

$N(m, s^2)$ is a Normal distribution with mean m and variance s^2;

σ_t^2 is a conditional variance state variable;

$\bar{\gamma}$ and δ^2 are the mean and variance of the normally distributed jump sizes; and

τ_{t+1} is the time interval between futures observations on dates t and $t+1$, in days.

We assume the conditional variance σ_t^2 affects the drift as in GARCH-in-mean models. Following Engle, Kane, and Noh (1993), τ_{t+1}^d is a variable time scale that parsimoniously captures the impact of weekends and holidays upon the conditional distribution of returns. A d equal to zero implies weekdays and weekends are equivalent, while $d = 1$ implies that 3-day weekends have three times the variance and roughly three times the jump risk of weekdays.

In the spirit of Day and Lewis (1992) and Lamoureux and Lastrapes (1993), the conditional variance state variable σ_t^2 is modeled as an augmented EGARCH process that nests various informational sources:

$$\begin{aligned} \ln \sigma_t^2 &= a_3 + a_4 DUM_t + a_5 \left[|z_t| - \sqrt{\tfrac{2}{\pi}} + a_6 z_t \right] \\ &\quad + a_7 \ln \sigma_{t-1}^2 + a_8 \ln HL_t + a_9 \ln BSIV_t^2 \end{aligned} \tag{6}$$

where

DUM_t is a dummy variable indicating a maturity switch in the S&P 500 futures contract used;

$z_t = \ln(F_t / F_{t-1}) / \sqrt{\sigma_{t-1}^2 \tau_t^d}$ is the previous day's normalized residual;

HL_t is the ratio of the day's high to the day's low; and

BSIV is the per day volatility inferred from pooled intradaily 1-4 month quarterly S&P 500 futures options using a Black-Scholes American option pricing formula.[11]

Day and Lewis (1992) found that the Black-Scholes implicit volatilities inferred from the S&P 100 index options were almost unbiased estimates of subsequent weekly index volatility over 1983-89, but that GARCH and EGARCH volatility estimates provided additional information. We include the Black-Scholes implicit volatility BSIV inferred from S&P 500 futures options as a simple summary measure to incorporate information from the option market. Parkinson (1980) and Garman and Klass (1980) argue that the informational content of an asset's open, high, low, and close considerably exceeds that of the squared daily return. And Chen (1995) shows that the high-low range provides useful *additional* information within an EGARCH framework. We include the log of the high-low ratio, $\ln HL$, to capture that information.

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[11]More precisely, we used the Barone-Adesi and Whaley (1987) formulas for pricing American options on futures. Those formulas maintain the Black-Scholes (1973) and Black (1976) assumption of geometric Brownian motion for the underlying asset price.

The conditional variance state variable σ_t^2 is also allowed to affect conditional distributions through the jump frequency $\lambda_t = \lambda_0 + \lambda_1 \sigma_t^2$ -- a specification Bates (1997) found useful in describing the evolution of distributions implicit in post-'87 S&P 500 futures options. Positive values for λ_1 imply that periods with high conditional volatility are also periods with high jump risk. Because of nonnegativity constraints on jump risk, negative values for λ_0 and λ_1 were precluded through exponential transformations in the estimation procedure.

We tested on various specifications on pre-crash daily log-differenced S&P 500 futures settlement prices over January 2, 1985 through September 30, 1987. We selected the shortest futures maturity available with at least one week to expiration, that being typically the contract with greatest open interest. The model and various submodels were estimated by a maximum likelihood methodology described in the appendix of Jorion (1988).[12]

Tables 1 and 2 summarize the precrash model selection results. Using only the Black-Scholes implicit volatility from traded options to assess conditional volatility and jump risk was unambiguously the best of the models considered in terms of parsimony and informational content. Somewhat unexpectedly, the implicit volatility appears to be a sufficient statistic for pre-crash conditional distributions. Although the high-low range does contribute additional information to an EGARCH specification, as in Chen (1995), neither high-low nor EGARCH provided statistically significant additional information after conditioning on the Black-Scholes implicit volatility from 1-4 month quarterly S&P 500 futures options.

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[12] For models with EGARCH terms, the log of the initial conditional variance, $\ln \sigma_0^2$, was also estimated. Following Ball and Torous (1985) and Jorion (1988), we chose 10 as the maximum feasible number of jumps in any day.

High-frequency low-amplitude jump processes were estimated for all models, with typical pre-crash mean jump size $\bar{\gamma} \approx 0$ and jump standard deviation $\delta \approx 1\%$. While it is not easy to test formally for an absence of jump risk given nonlinear identification issues discussed in Hansen (1992), allowing for conditionally leptokurtic distributions through non-zero jump frequencies strongly increased log likelihoods for all models during the pre-crash period. This, plus the asymmetry of equally out of the money put and call prices presented in Bates (1991) and the estimate of a negatively skewed and leptokurtic unconditional distribution strongly suggest a jump component is present even in pre-crash data. Furthermore, likelihood ratio test comparisons of the last two columns of Table 1 indicate that time-varying jump risk ($\lambda_1 > 0$) was statistically significant for 4 out of 6 models. λ_0 converged to its near-zero constraint whenever $\lambda_1 > 0$ was permitted. The estimates imply that periods of high volatility were historically also periods with higher jump risk -- i.e., with a higher proportion of outliers.

Table 1: Performance (log likelihood) of alternate models

Model	Number of parameters (Gaussian)	log likelihood: conditionally Gaussian ($\lambda_t = 0$)	jumps $\lambda_0 \geq 0$, $\lambda_1 = 0$	jumps $\lambda_0, \lambda_1 \geq 0$
i.i.d.	4	2,232.45	2,263.65	
HL	6	2,236.45	2,264.07	2,265.52
BSIV	**6**	**2,262.30**	**2,275.66**	**2,279.00**
EGARCH	9	2,244.63	2,268.26	2,269.33
HL-EGARCH	10	2,252.14	2,273.93	2,276.69
BSIV-EGARCH	10	2,262.79	2,278.91	2,281.87
HL-BSIV	7	2,262.71	2,275.68	2,279.44

HL: $a_8 \neq 0$; BSIV: $a_9 \neq 0$; EGARCH: a_5, a_6, a_7, $\ln\sigma_0^2 \neq 0$. The first fat-tailed distribution adds three more parameters to the Gaussian specification; the second adds a fourth parameter.

5% significance levels for $\ln L_{UC} - \ln L_C$: 1.92 (1 restriction), 3.00 (2), 3.90 (3), 4.74 (4).

Table 2: Distribution estimation conditional upon the Black-Scholes implicit volatility (BSIV). Log-differenced S&P 500 futures settlement prices, January 2, 1985 through September 30, 1987. See equations (5) and (6) for definitions of parameters. Standard errors are in parentheses.

Model	a_1	a_2	d	a_3	a_4	a_9	λ_1	$\bar{\gamma}$	δ	ln L
BSIV	.001	-.69	.224	-.105	-1.527	.911	0	0	0	2262.30
	(.000)	(5.35)	(.118)	(.430)	(.445)	(.114)				
JD-BSIV	.000	13.85	.218	-1.599	-.931	.705	11434	-.001	.010	2279.00
	(.001)	(22.07)	(.130)	(.624)	(.443)	(.134)	(10862)	(.001)	(.002)	

The initial conditionally Gaussian estimates in Table 2 ($a_3 \approx 0$, $a_9 \approx 1$) show the Black-Scholes implicit volatility was close to an unbiased predictor of future volatility. This was also true for jump-diffusion conditional distributions, with the average pre-crash implicit standard deviation of 16.7% implying an annualized conditional volatility forecast of 13.6%.[13] Typical estimates of $d \approx .20 - .25$ indicate that three-day weekends had roughly 25-30% higher variance than a typical weekday. In no case was the conditional mean of log-differenced futures prices significantly different from zero.

Estimates for October and November of 1987

The specification using only the Black-Scholes implicit volatility from traded options to assess conditional volatility and jump risk was unambiguously the best of the models considered in terms of parsimony and informational content. We selected the JD-BSIV model for assessing conditional distributions over October and November of 1987, with λ_0 set to zero. To fully exploit all available information, the relationship between BSIV and conditional distributions was re-estimated daily via nonlinear "rolling regressions" (JD-RR), using a 692-day (34-month) moving data window.

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[13]The annualized weekday conditional variance is $365\sigma_t^2[1 + \lambda_1(\bar{\gamma}^2 + \delta^2)]$, including jump risk.

In addition, 1-day conditional distributions were inferred from December 1987 S&P 500 futures option prices under two specifications for the underlying driving process: the Black-Scholes option pricing model, which assumes geometric Brownian motion, and the Bates (1991) model, which assumes a jump-diffusion process.

Table 3 shows the daily parameter estimates from the three approaches:

1) Black-Scholes implicit volatility (BSIV's) inferred from option prices assuming lognormality;
2) implicit jump-diffusion parameters (JD-options) inferred assuming a jump-diffusion; and
3) jump-diffusion parameters (JD-RR) estimated from daily futures returns *conditional* upon observed BSIV's, using the model and rolling-regression methodology described above.[14]

[14]For the JD-RR specification, the 1-day assessed jump frequency is $\lambda_t = \hat{\lambda}_1 BSIV_t^2$, where $\hat{\lambda}_1$ is re-estimated daily via rolling regressions.

Table 3: One-day conditional distribution forecasts. All parameters are in daily units.

		Implicit jump-diffusion (JD-options)					Rolling-regression (JD-RR)					
	BSIV	σ	$\lambda*$	$\bar{\gamma}^*$	δ	ν^a	σ	λ	$\bar{\gamma}$	δ	ν^a	*S.e.*[b]
870930	1.06%	1.06%	0.000	-80.7%	4.8%	1.07%	0.76%	0.664	-0.1%	1.0%	1.10%	*0.06%*
871001	1.05%	1.02%	0.000	-20.0%	5.7%	1.06%	0.77%	0.665	-0.1%	1.0%	1.10%	*0.06%*
871002	1.04%	1.00%	0.001	-9.4%	5.8%	1.05%	0.76%	0.650	-0.1%	1.0%	1.09%	*0.05%*
871005	1.04%	0.94%	0.017	-3.0%	1.9%	1.04%	0.75%	0.636	-0.1%	1.0%	1.08%	*0.05%*
871006	1.04%	1.02%	0.000	-34.6%	23.2%	1.09%	0.76%	0.593	-0.1%	1.0%	1.09%	*0.05%*
871007	1.09%	0.25%	0.112	-0.4%	3.2%	1.11%	0.77%	0.552	-0.2%	1.0%	1.09%	*0.05%*
871008	1.07%	1.01%	0.003	-5.7%	3.4%	1.08%	0.80%	0.618	-0.2%	1.0%	1.14%	*0.06%*
871009	1.09%	0.09%	0.231	-0.2%	2.3%	1.10%	0.78%	0.620	-0.1%	1.0%	1.12%	*0.06%*
871012	1.12%	0.04%	0.165	-0.2%	2.8%	1.13%	0.79%	0.636	-0.1%	1.0%	1.14%	*0.06%*
871013	1.11%	0.02%	0.123	-0.2%	3.2%	1.12%	0.82%	0.636	-0.2%	1.0%	1.16%	*0.06%*
871014	1.12%	0.05%	0.177	-0.1%	2.7%	1.13%	0.83%	0.577	-0.2%	1.1%	1.16%	*0.06%*
871015	1.16%	0.16%	0.096	-0.2%	3.8%	1.18%	0.84%	0.618	-0.2%	1.0%	1.18%	*0.06%*
871016	1.34%	1.29%	0.000	401.4%	2.1%	1.30%	0.93%	0.430	-0.3%	1.2%	1.25%	*0.07%*
871019	2.21%	1.30%	0.002	-20.3%	56.4%	3.03%	1.41%	0.016	-9.7%	11.8%	2.40%	*0.58%*
871020	5.58%	1.72%	0.009	-100.4%	0.0%	9.71%	2.47%	0.050	-9.1%	12.0%	4.18%	*1.05%*
871021	3.23%	2.27%	0.004	-52.4%	25.0%	4.48%	7.28%	0.411	-8.4%	12.4%	12.05%	*3.08%*
871022	3.66%	1.56%	0.008	-55.0%	0.0%	5.08%	4.06%	0.129	-7.9%	12.5%	6.68%	*1.70%*
871023	3.29%	1.74%	0.005	-62.0%	0.0%	4.85%	4.56%	0.164	-7.6%	12.6%	7.49%	*1.92%*
871026	4.24%	1.64%	0.010	-52.3%	7.0%	5.54%	4.17%	0.144	-7.1%	12.4%	6.85%	*1.62%*
871027	3.80%	2.05%	0.010	-43.3%	22.2%	5.19%	5.46%	0.240	-7.3%	12.2%	8.86%	*2.14%*
871028	3.37%	2.11%	0.006	-48.6%	0.0%	4.20%	4.75%	0.180	-7.2%	12.2%	7.67%	*1.82%*
871029	2.86%	1.81%	0.004	-49.6%	0.0%	3.61%	4.22%	0.136	-7.7%	12.0%	6.74%	*1.51%*
871030	2.46%	1.60%	0.003	-51.1%	5.5%	3.19%	3.56%	0.094	-8.0%	11.8%	5.64%	*1.21%*
871102	2.52%	1.65%	0.004	-44.1%	21.2%	3.33%	2.99%	0.066	-7.9%	11.9%	4.73%	*0.94%*
871103	2.66%	1.55%	0.007	-29.7%	21.3%	3.35%	3.08%	0.069	-7.7%	12.0%	4.85%	*1.00%*
871104	2.66%	1.54%	0.004	-45.2%	11.5%	3.39%	3.23%	0.077	-7.6%	12.0%	5.09%	*1.09%*
871105	2.49%	1.37%	0.007	-24.3%	23.8%	3.16%	3.20%	0.075	-7.7%	11.9%	5.03%	*1.05%*
871106	2.31%	1.17%	0.008	-20.9%	23.1%	2.99%	2.98%	0.065	-7.5%	12.0%	4.68%	*0.98%*
871109	2.67%	1.56%	0.007	-27.9%	22.9%	3.30%	2.73%	0.054	-7.4%	12.1%	4.28%	*0.92%*
871110	2.65%	1.72%	0.006	-30.5%	19.6%	3.26%	3.17%	0.074	-7.1%	12.2%	4.98%	*1.08%*
871111	2.44%	1.56%	0.008	-19.4%	19.2%	2.95%	3.13%	0.072	-7.1%	12.2%	4.91%	*1.02%*
871112	2.29%	1.51%	0.008	-17.5%	16.8%	2.69%	2.86%	0.059	-7.3%	12.1%	4.48%	*0.89%*
871113	2.21%	1.65%	0.003	-38.0%	3.9%	2.65%	2.65%	0.051	-7.1%	12.2%	4.15%	*0.84%*
871116	2.21%	1.28%	0.016	-11.3%	13.6%	2.55%	2.53%	0.047	-7.1%	12.2%	3.97%	*0.95%*
871117	2.31%	1.32%	0.016	-12.4%	13.2%	2.64%	2.53%	0.047	-6.9%	12.3%	3.96%	*0.84%*
871118	2.19%	1.24%	0.016	-12.2%	12.5%	2.55%	2.63%	0.051	-6.9%	12.3%	4.13%	*0.83%*
871119	2.14%	1.41%	0.008	-21.8%	7.4%	2.51%	2.49%	0.045	-6.7%	12.4%	3.89%	*0.83%*
871120	2.28%	1.32%	0.017	-12.1%	12.0%	2.59%	2.42%	0.042	-6.8%	12.4%	3.77%	*0.88%*
871123	2.09%	1.54%	0.005	-25.0%	3.7%	2.40%	2.57%	0.048	-6.7%	12.4%	4.02%	*0.85%*
871124	1.83%	1.41%	0.004	-25.1%	3.7%	2.09%	2.02%	0.030	-6.7%	12.3%	3.16%	*0.90%*
871125	1.66%	1.26%	0.004	-23.2%	3.0%	1.95%	1.87%	0.026	-6.6%	12.4%	2.93%	*0.57%*
871127	1.78%	1.34%	0.005	-21.7%	0.0%	2.03%	1.80%	0.024	-6.5%	12.5%	2.81%	*0.58%*
871130	2.30%	1.36%	0.025	-10.0%	10.5%	2.64%	1.83%	0.025	-6.3%	12.6%	2.87%	*0.57%*

Comparing the Estimated Distributions

For the measures of default risk exposure the conditional volatility ν and the fraction f of conditional variance due to jump risk are two important statistics of conditional distributions:

$$\nu = \sqrt{\sigma^2 + \lambda(\bar{\gamma}^2 + \delta^2)}$$

$$f = \lambda(\bar{\gamma}^2 + \delta^2)/\nu^2 .$$

For the Black-Scholes model, which has no jump component, the conditional volatility is the implicit volatility $BSIV$, and $f = 0$.

As the conditional volatility increases, ceteris paribus, all risk measures increase: the probability of a futures move in excess of margin, the expected value of additional required funds, the liquid reserves needed to cover potential margin calls. However, this statistic alone is insufficient to summarize the risks facing the clearinghouse. The allocation of risk between "normal" market moves and jumps affects tail probability estimates, and has especially strong implications for the consequences of a margin-exhausting futures price move. Low-probability, large-magnitude jumps generate realizations far out in the tails, which dramatically increases the reserves required to defend against default risk.

As shown in Figure 3, the three different estimates of conditional distributions show extraordinary unanimity prior to the crash regarding conditional variance. All three assess variance in the 1.04% - 1.34% range in the first half of October 1987, with the maximum deviation across forecasts less than .1%. The crash on October 19 and the accompanying sharp increase in implicit volatilities generated substantially higher post-crash assessments of conditional volatilities, with the time series model (JD-RR) generating the highest of the three volatility estimates. Owing to considerable difficulty in estimating jump-related variance over a 692-day sample that includes the crash, the deviation between

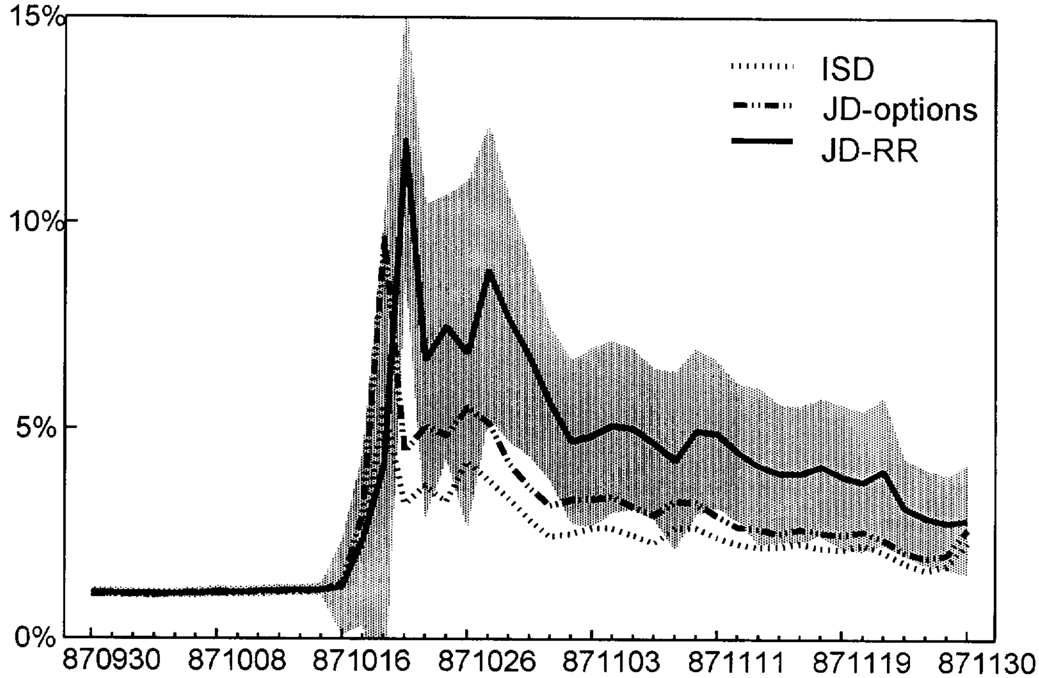

Figure 3. Conditional volatility estimates: alternate models.

the JD-RR and JD-options volatility estimates is not generally statistically significant at the 5% level on any given day. (The shaded area in Figure 3 is the 95% confidence band for the JD-RR model, estimated using the delta method approach described in Lo (1986).) The three volatility estimates roughly re-converge by the end of November.

Figure 4 shows the fraction of the conditional variance due to jump risk. Prior to the crash, the JD-RR estimates in Table 3 attribute roughly 50% of the conditional variance to high-frequency, low-amplitude jumps: .4 - .6 jumps per day, with a slightly negative mean and a standard deviation about 1%. Such low-amplitude jump risk is virtually indistinguishable from Brownian motion at the 2-month horizon of the contemporaneous December 1987 S&P 500 futures options. As noted in Bates (1991, Figure 11), prices of these options did in fact deviate very little from Black-Scholes prices

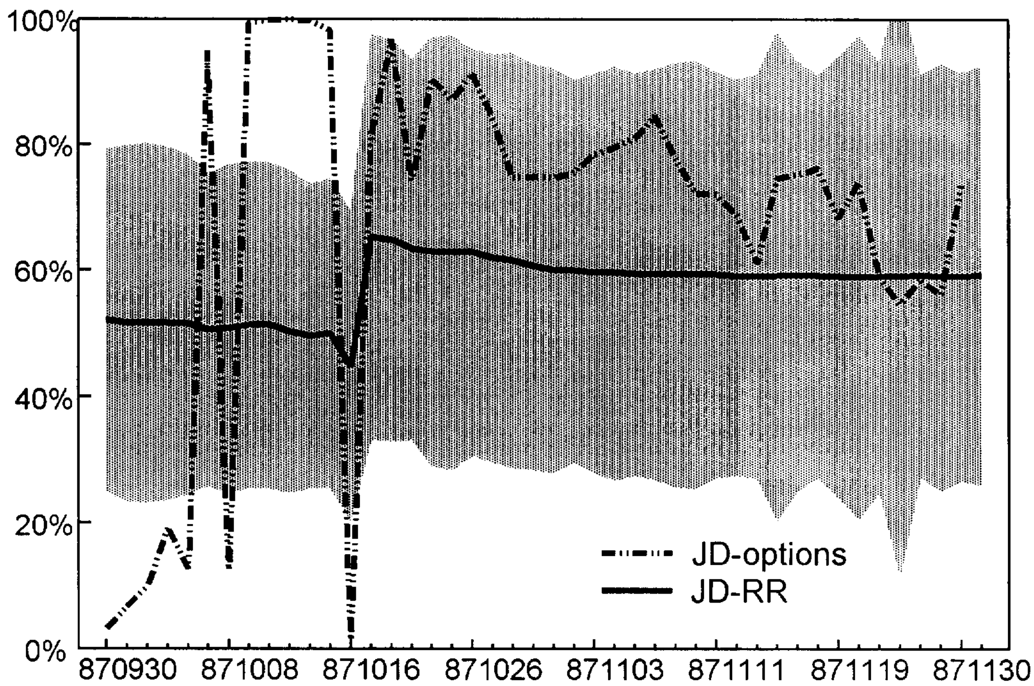

Figure 4. Fraction of conditional variance attributed to jumps.

during the two months immediately preceding the crash -- in contrast to more substantial deviations observed earlier in the year.[15]

Following the crash, time series-based estimates and option prices rapidly incorporated a jump component into the conditional distributions. However, the two approaches fundamentally differ in the form of estimated jump risk. The average post-crash implicit parameter estimates for the JD-options model indicated jumps of mean -26.1% and standard deviation 12.6% occurring with a frequency of .009 jumps per day (3 jumps/year). Implicit parameters changed substantially over late October and November -- especially the jump distribution parameters $\bar{\gamma}^*$ and δ^2.

[15]While implicit jump risk from the JD-options estimates typically accounts for almost all of the implicit variance over October 7-15, excluding October 8, the relatively high implicit jump frequency (.1 - .2 jumps/day) and low implicit jump magnitudes (mean roughly 0; standard deviation less than 5%) imply near-lognormal distributions at the 2-month horizon.

By contrast, the average post-crash time series estimates (JD-RR) were of more frequent (.087 jumps/day, or 32 jumps/year) but smaller jumps: mean size -7.3% and standard deviation 12.2%. The estimated jump distribution parameters were relatively stable post-crash, but the assessed daily jump frequency $\lambda_t = \hat{\lambda}_1 \hat{\sigma}_t^2 = \hat{\lambda}_1 \exp(\hat{a}_3 + \hat{a}_9 \ln BSIV_t^2)$ changed considerably over time. This jump frequency was substantially affected not only by the extreme 28.6% market decline on October 19, 1987, but also by the 7.3% and 19.4% rebounds on October 20 and 21; see Table 4. These moves, which were far larger than the daily moves observed during the preceding 34 months, led to substantial upward revisions in $\hat{\lambda}_1$. By the end of November, however, declining implicit volatilities reduced jump risk assessments, and parameter estimates from both approaches were substantially in agreement.

Table 4: S&P 500 futures settlement prices, margins, volume, and open interest. Settlement prices are for December 1987 futures contracts; volume and open interest are for all S&P 500 future contracts.

Date	F	%change	Margin	(%)	Volume	Open Interest
870930	325.85		$5,000	3.1%	82,444	114,182
871001	331.70	1.8%	$5,000	3.0%	85,128	113,808
871002	331.35	-0.1%	$5,000	3.0%	67,427	113,788
871005	330.80	-0.2%	$5,000	3.0%	69,085	115,312
871006	319.85	-3.3%	$5,000	3.1%	96,869	114,286
871007	320.65	0.3%	$5,000	3.1%	99,673	116,664
871008	315.80	-1.5%	$5,000	3.2%	99,191	119,176
871009	312.20	-1.1%	$5,000	3.2%	76,186	120,728
871012	311.60	-0.2%	$5,000	3.2%	79,907	123,064
871013	315.65	1.3%	$5,000	3.2%	82,040	119,880
871014	305.00	-3.4%	$5,000	3.3%	109,740	127,582
871015	298.25	-2.2%	$5,000	3.4%	124,810	133,696
871016	282.25	-5.4%	$5,000	3.5%	135,344	146,653
871019	201.50	-28.6%	$7,500	7.4%	162,022	172,178
871020	216.25	7.3%	$7,500	6.9%	126,562	174,184
871021	258.25	19.4%	$7,500	5.8%	91,802	169,934
871022	244.50	-5.3%	$10,000	8.2%	57,726	158,774
871023	241.00	-1.4%	$10,000	8.3%	41,945	156,650
871026	220.25	-8.6%	$10,000	9.1%	35,170	158,715
871027	228.60	3.8%	$10,000	8.7%	32,241	157,071
871028	231.25	1.2%	$12,500	10.8%	38,517	156,374
871029	245.70	6.2%	$15,000	12.2%	38,670	153,449
871030	259.35	5.6%	$15,000	11.6%	35,249	152,340
871102	257.75	-0.6%	$15,000	11.6%	33,551	148,164
871103	250.15	-2.9%	$15,000	12.0%	50,335	146,820
871104	250.15	0.0%	$15,000	12.0%	44,268	145,688
871105	255.40	2.1%	$15,000	11.7%	44,978	141,077
871106	249.10	-2.5%	$15,000	12.0%	37,989	140,944
871109	245.60	-1.4%	$15,000	12.2%	43,351	140,388
871110	239.40	-2.5%	$15,000	12.5%	51,590	139,932
871111	242.20	1.2%	$15,000	12.4%	31,745	139,138
871112	249.60	3.1%	$15,000	12.0%	41,769	137,599
871113	247.60	-0.8%	$15,000	12.1%	24,369	138,116
871116	248.20	0.2%	$15,000	12.1%	38,727	139,276
871117	242.70	-2.2%	$15,000	12.4%	48,333	138,508
871118	246.55	1.6%	$15,000	12.2%	56,262	140,462
871119	238.80	-3.1%	$15,000	12.6%	61,291	139,711
871120	241.90	1.3%	$15,000	12.4%	60,725	141,133
871123	244.10	0.9%	$15,000	12.3%	41,218	140,643
871124	246.15	0.8%	$15,000	12.2%	56,396	141,528
871125	244.30	-0.8%	$15,000	12.3%	27,371	141,819
871127	237.00	-3.0%	$15,000	12.7%	17,804	140,352
871130	232.00	-2.1%	$15,000	12.9%	79,552	139,887

Section 3: Estimates of the Clearinghouse Exposure

The Chicago Mercantile Exchange responded aggressively to perceptions of increased default risk on October 19 and in the days that followed. As discussed in Fenn and Kupiec (1993), extraordinary intradaily margin calls occurred three times on October 19, and 10 more times in the remainder of October. Furthermore, margin requirements were rapidly raised. Whereas the margin requirement per futures contract stood at $5000 on October 18, it was raised to $7500 on October 19, to $10,000 on October 22, to $12,500 on October 28, to $15,000 on October 28; see Table 4. Combined with lower futures prices following the crash, the margin requirements effectively went from 3.5% of the futures settlement price on October 16 to 12.2% on October 29. The margin requirements were not lowered again until December 18, to $10,000.

This section applies the three measures of exposure, and the three methods considered above for estimating that exposure, to the S&P 500 futures contract during October and November of 1987. Judging only from the traditional tail probability estimates, the CME's aggressive response was entirely successful in reducing the daily post-crash probability of further margin-exhausting futures price moves to a level comparable to or lower than pre-crash levels. However, conditional distributions that incorporate jump risk indicate substantially higher levels of exposure (price of the options, and the level of reserves required to meet margin calls) following the crash. The difference in risk assessments is attributable to the failure of the tail probability approach to assess the likely consequences of a futures move in excess of posted margin.

1. The Probability that Additional Funds will be Required: $\wp = Prob(|\Delta F| > M)$

The probability that the absolute change in the futures prices exceeds the margin is the probability that additional funds will be required. The jump-diffusion processes postulated in Section 2 model the conditional distribution of 1-day log-differenced futures prices as a probability-weighted mixture of normal distributions, with the weights reflecting assessed probabilities of n jumps occurring within a single day. The upper tail probability is:

$$P_{up} \equiv Prob[F_{t+\tau} > X_c] = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \Phi(d_{2n}) \qquad \textbf{(8)}$$

where

$\Phi(\bullet)$ is the standard normal distribution function,

$b(n) = -\lambda e^{\overline{\gamma} + \frac{1}{2}\delta^2} + n(\overline{\gamma} + \frac{1}{2}\delta^2)/\tau$,

$d_{2n} = [\ln(F/X_c) + b(n)\tau - \frac{1}{2}(\sigma^2\tau + n\delta^2)] / \sqrt{\sigma^2\tau + n\delta^2}$,

$X_c = F + M$, and $\tau = 1$ day.

Similarly, the lower tail probability is $P_{down} \equiv 1 - Prob[F_{t+\tau} > X_p]$, where $X_p = F - M$.

Relevant daily parameter inputs for the time series-based JD-RR model and the options-based JD-options and BSIV are reported above in Table 3. The JD-options model uses the risk-neutral implicit parameter estimates λ^* and $\overline{\gamma}^*$ instead of λ and $\overline{\gamma}$. The lognormal Black-Scholes model has no jump risk ($\lambda = 0$); the infinite sum (8) collapses to the first term $\Phi(d_{20})$ for this model, with n and $b(n)$ equal to zero.

Figure 5 shows the tail probability estimates $\wp = P_{up} + P_{down}$ from the three models, and the 95% confidence interval (shaded area) for JD-RR estimates. Table A in the appendix gives the values. For expositional clarity, we compute all tail probabilities using the standard settlement interval of one day: $\tau = 1$.[16]

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[16]The appropriate computations for weekends and holidays involves replacing τ by τ^d in the above formulas.

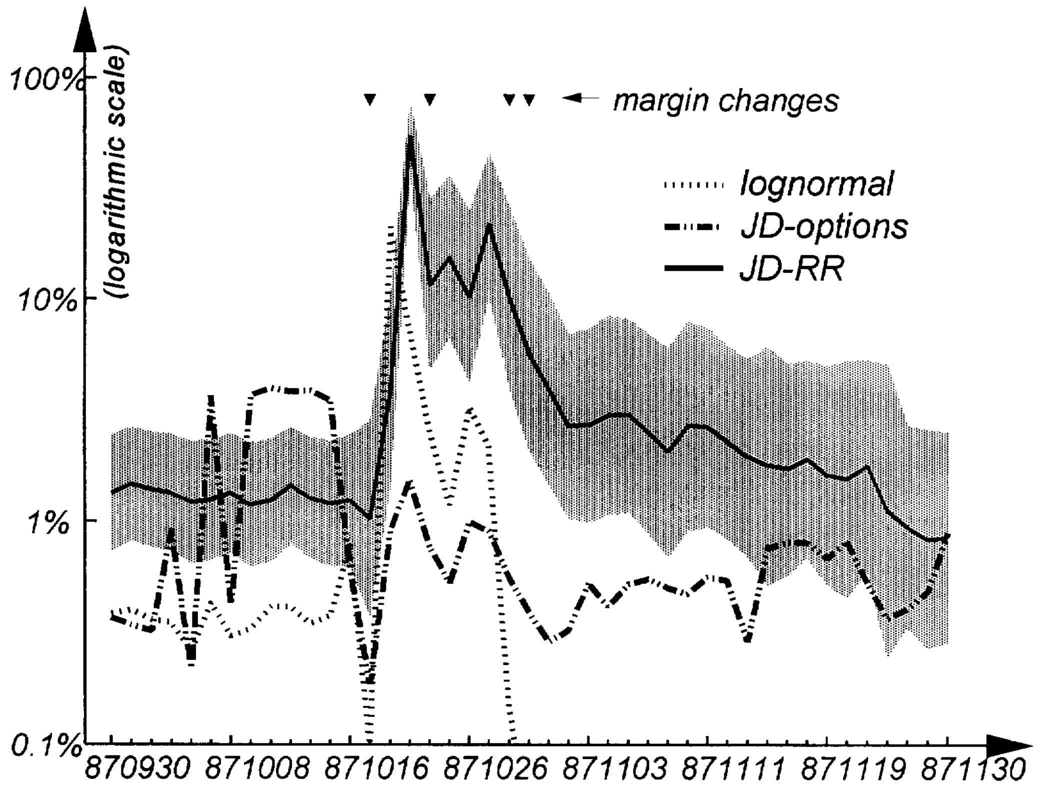

Figure 5. Probability of a margin-exhausting futures move: alternate estimates.

The three tail probability estimates diverge, painting a conflicting picture of the risk. Pre-crash, all models agreed on conditional standard deviation estimates of around 1% per day. However, diverging estimates of the extent of the jump risk generated divergent estimates of the probability of a futures move in excess of the 3 - 3½% margin requirement. JD-RR time series estimates indicated roughly 1% tail probabilities, which may reflect the histogram-based tail probability orientation of the CME margin committee (Kupiec, 1994). JD-options daily tail probability estimates inferred from 3-month December options were more volatile, getting as high as 3-4% in the week preceding the crash. The BSIV lognormal estimates perforce assigned a low probability to observing a 3-3½ standard deviation move.

Post-crash divergences in estimated jump risk created sharp divergences between the JD-RR and JD-options tail probability estimates. Immediately following the crash, S&P 500 futures options prices implicitly attributed most futures price risk to low-frequency large movements; see Figure 4 and Table 3. Consequently, post-crash JD-options tail probability estimates were quite low. By contrast, the rolling-regression estimates were affected by the large moves in futures prices on October 19, 20, and 21, and consequently estimated a higher-frequency but lower-magnitude jump component. The JD-RR estimates of another margin-exhausting futures move peaks at 55% on October 21 and exceeds 10% until October 29. By the end of November, the estimates from the jump-diffusion option model and the jump-diffusion rolling-regression converge at roughly a 1% daily chance of another margin-exhausting futures move.

The Black-Scholes tail probability estimates peaked the day after the crash, when daily implicit volatilities of 5.58% (107% annualized!) implied a 21% probability that the price change will exceed the margin the following day. Subsequent declines in daily implied volatilities to around 2% plus margin increases to a 12% effective level reduced the lognormal tail probability estimates to negligible levels by the end of October.

2. Expected Value of Additional Funds: $S(F_t, M) = E^*[\max(0, |\Delta F| - M)]$

The expected value of price changes that exceed the margin is the price of an option written on the absolute value of the change in the futures price with a strike price equal to the margin. The price of the option gives the current market value of an insurance contract that provides the additional funds if needed.

Option valuations for the jump diffusion processes in Section 2 are a probability-weighted average of Black-Scholes prices, as discussed in Merton (1976) and Bates (1991):

$$c(F,\ \tau;\ X_c) \ = \ \sum_{n=0}^{\infty} Prob(n\ jumps)\ E^*[\ \max(F_{t+\tau} - X_c,\ 0)\ |\ n\ jumps]$$
$$= \ \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \left[F e^{b(n)\tau}\Phi(d_{1n}) - X_c\Phi(d_{2n})\right] \qquad \textbf{(9)}$$
$$p(F,\ \tau;\ X_p) = c(F,\ \tau;\ X_p) + (X_p - F)$$

here $d_{1n} = d_{2n} + \sqrt{\sigma^2\tau + n\delta^2}$, and $\tau = 1$ day. The relevant strangle price is $S = c + p$.

Figure 6 shows the price of the option portfolio as a percentage of the futures price for the three models, and the 95% confidence interval for the JD-RR model. Table B in the Appendix contains the values.

The substantial post-crash increase in hypothetical insurance premia indicate the fundamentally higher

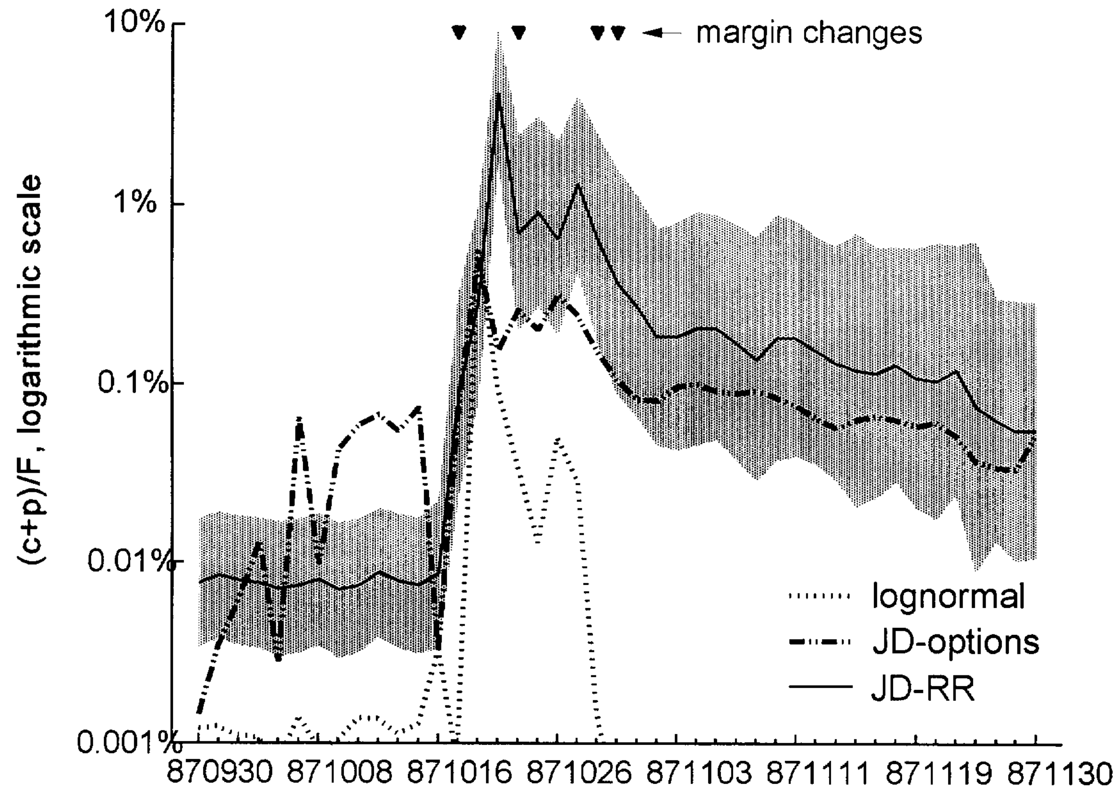

Figure 6. Daily insurance premium, as a percentage of the futures price.

levels of risk faced by the clearinghouse following the crash. While pre-crash estimates put this value at less than 0.1% of the futures price, crash-related revisions in conditional distributions raised these estimates by two to three orders of magnitude. Subsequent declines of JD-RR and JD-options estimates to only an order of magnitude greater than pre-crash values were attributable to two factors: the four CME margin increases in October, and declining assessments of jump risk. Had the CME left the margin requirement at the $5000 level of October 16, JD-RR estimates of insurance premia at the end of November would have been over three times larger.

The degree of unanimity across post-crash jump-based estimates of insurance premia is quite striking. The options-based JD-options model and the time series-based JD-RR model estimate quite different jump processes, with lower frequency jumps of substantially larger magnitudes estimated for the former. Yet the two premia estimates behave quite similarly over the post-crash period, and typically do not diverge significantly in November. This comparable behavior reflects the relative insensitivity of insurance premia to the frequency/magnitude tradeoff in jump risk. As indicated in equation (2), the insurance premia depend upon the *product* of the tail probabilities and the expected consequences of futures price moves in excess of margin. Varying the frequency and jump risk magnitudes while keeping the conditional variance roughly constant across models (see Figure 3) has roughly offsetting effects on these two terms, leaving insurance premia comparable across models.

Black-Scholes estimates of daily insurance premia are negligible following the four margin increases in October. Under the hypothesized (and implausible) lognormal distribution, the combination of a minuscule probability of exceeding the margin and the negligible expected consequences of such a futures move yield small estimated insurance premia.

3. Liquid Assets Required to Cover Additional Funds: $\Re = E^*(|\Delta F| - M \mid |\Delta F| > M)$

The expected value of the additional funds conditional on the fact that additional funds will be needed, $\Re = S/\wp$, is arguably the best measure of the risk exposure. It is the reserves the writer of the strangle option portfolio (or the insurer) would have to hold to pay the option if it expired in the money. If the reserves are adequate, there is no default risk.

A dollar value to these required reserves can be assigned by multiplying $\Re$ by the total open interest on all S&P 500 futures contracts. Under the gross margining system used by the CME in 1987, this is the expected magnitude of additional funds that the losing side of the futures positions will have to post with the clearinghouse if the futures price move exceeds the margin, in order to ensure performance.

Figure 7 shows estimates from the three specifications of the reserves required to insure against default risk, while Table B in the appendix contains the values. The JD-options estimates clearly reflects the crash fears that haunted the options market on and following the crash. The fears of infrequent but large further crashes necessary to match observed transactions prices for S&P 500 futures options on October 20 imply $10.5 billion in reserves (or 55% of the futures price) would be needed to weather another futures price move in excess of margin. The estimates remained in the $5-7 billion range for the remainder of October, and gradually declined to about $1 billion by the end of November.

The time series JD-RR model estimated jumps of somewhat lower magnitudes based essentially upon observed price movements over October 19-21. The resulting estimate of required reserves consequently remained relatively stable at approximately $1.2 billion throughout October and November. In contrast, the lognormal BSIV model predicts that any futures price move in excess of margin is not likely to exceed it by very much.

Whether \$1 billion or \$10 billion represents a substantial risk of clearinghouse default cannot be determined without some knowledge of the remaining liquid assets of the losing customers as well as knowledge of the assets of the FCM's and clearinghouse. However, some perspective comes from considering the margin already posted. The CME's tripling of margin requirements over October implied that the longs and the shorts at end-November had each posted \$15,000/contract × 139,887 open interest = \$2.1 billion in margin. Both the JD-options and JD-RR approaches consequently estimate that at end-November, roughly 50% in additional margin would be required above and beyond what is already posted, conditional upon another margin-exhausting S&P 500 futures move occurring.

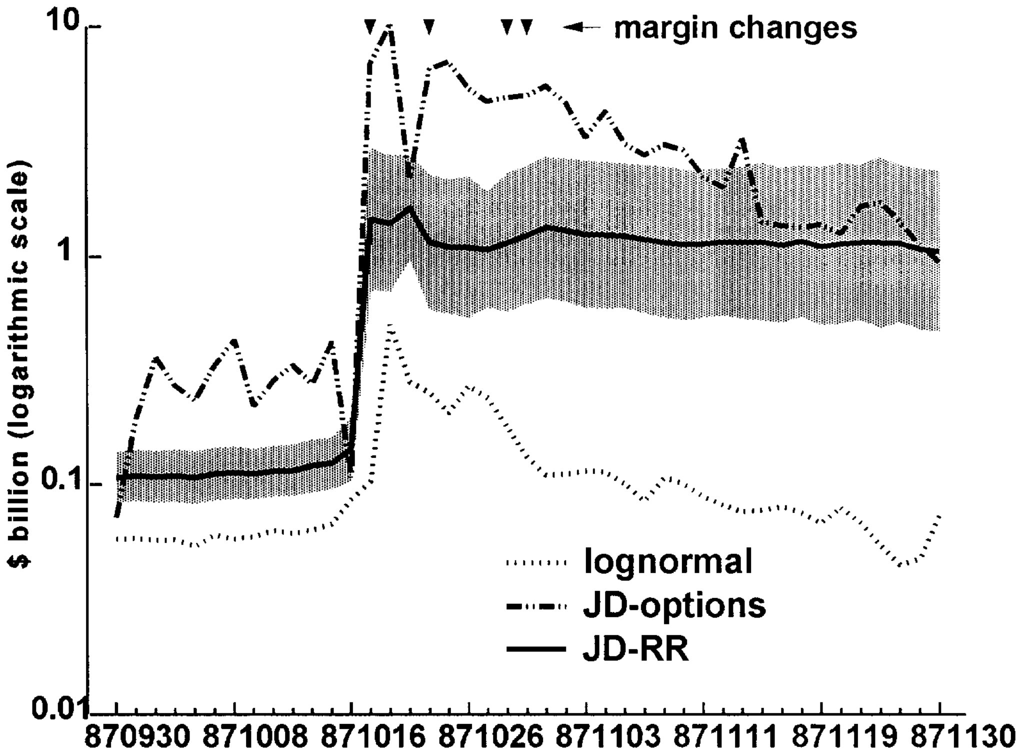

Figure 7 . Expected additional funds requirement *conditional* upon a margin-exhausting futures move: alternate estimates.

Section 4: Summary and Conclusions

This article presented two new measures for assessing the clearinghouse exposure associated with any given margin policy. An application to CME margin policy during October and November 1987 using two alternate estimates of jump-diffusion processes reveals that the earlier focus on the probability of a margin-exhausting futures price move can generate a quite misleading assessment of a clearinghouse's exposure. While post-crash declining risk assessments and the CME's margin requirement increases had reduced tail probabilities to pre-crash levels by the end of November, estimated clearinghouse exposure remained an order of magnitude higher than pre-crash levels. The difference is our approaches also take into account how much in additional funds will be required *conditional* upon a large move occurring; and end-November estimates of conditionally required funds were an order of magnitude higher than pre-crash levels.

The Chicago Mercantile Exchange's margin policy has changed substantially from the system that was in place in 1987. In March 1, 1988 and June 26, 1992, the CME implemented steps to make settlement and variation margin calls at noon as well as at end of day, in contrast to the daily frequency characteristic of pre-1988. Furthermore, the SPAN system introduced in 1988 more explicitly addresses the issue of appropriate margins on potentially offsetting portfolios of positions, such as options positions hedged by futures positions. Kupiec (1994, p. 793) notes that "margins on S&P 500 products have been set more conservatively than on other CME products since the October 1987 stock market crash."

Yet it is striking the degree to which CME margin policy is still based upon tail probability estimates. As described in Kupiec (1994), margins are set at levels sufficient to cover the consequences of the sort of futures price moves observed 95-99% of the time. Those critical futures moves are estimated by the CME margin committee based upon histograms of price changes over the preceding 60-day, 120-day, and 1-year window. Kupiec finds that S&P 500 futures price moves exceeded posted margin less than 0.5% of the time between December 16, 1988 and December 10, 1992.

It may be that current CME margin policy is perfectly adequate. The CME did, after all, survive the crash of 1987 (with some help from the Federal Reserve), as well as substantially smaller drops on January 8, 1988 and October 13, 1989. But focussing on tail probabilities alone is an inadequate criterion for survival, and for clearinghouse regulation. The consequences of a substantial futures price move in excess of margin must also be considered.

Appendix: Values of exposure estimates

Table A. Estimated probabilities of a margin-exhausting price move, and option prices.

Date	margin	2-sided tail probabilities				Option prices: $(c+p)/F$			
		BS	JD-options	JD-RR	s.e.	BS	JD-options	JD-RR	s.e.
870930	3.1%	0.4%	0.4%	1.3%	0.4%	0.001%	0.001%	0.008%	0.003%
871001	3.0%	0.4%	0.3%	1.5%	0.4%	0.001%	0.004%	0.009%	0.004%
871002	3.0%	0.4%	0.3%	1.4%	0.4%	0.001%	0.006%	0.008%	0.003%
871005	3.0%	0.4%	0.9%	1.3%	0.4%	0.001%	0.013%	0.008%	0.003%
871006	3.1%	0.3%	0.2%	1.2%	0.4%	0.001%	0.003%	0.007%	0.003%
871007	3.1%	0.4%	3.6%	1.3%	0.4%	0.001%	0.065%	0.007%	0.003%
871008	3.2%	0.3%	0.4%	1.3%	0.4%	0.001%	0.010%	0.008%	0.003%
871009	3.2%	0.3%	3.7%	1.2%	0.4%	0.001%	0.043%	0.007%	0.003%
871012	3.2%	0.4%	4.0%	1.2%	0.4%	0.001%	0.059%	0.007%	0.003%
871013	3.2%	0.4%	3.8%	1.5%	0.4%	0.001%	0.068%	0.009%	0.004%
871014	3.3%	0.3%	3.8%	1.3%	0.4%	0.001%	0.055%	0.008%	0.003%
871015	3.4%	0.4%	3.5%	1.2%	0.4%	0.001%	0.074%	0.008%	0.003%
871016	3.5%	0.8%	0.6%	1.2%	0.4%	0.003%	0.003%	0.009%	0.004%
871019	7.4%	0.1%	0.2%	1.0%	0.5%	0.000%	0.076%	0.087%	0.058%
871020	6.9%	21.3%	0.9%	3.8%	1.9%	0.573%	0.508%	0.285%	0.182%
871021	5.8%	7.2%	1.6%	55.1%	9.9%	0.093%	0.156%	4.149%	1.786%
871022	8.2%	2.6%	0.8%	11.7%	5.3%	0.033%	0.262%	0.700%	0.447%
871023	8.3%	1.2%	0.5%	15.7%	6.8%	0.013%	0.202%	0.922%	0.576%
871026	9.1%	3.2%	1.0%	10.4%	4.7%	0.050%	0.314%	0.653%	0.414%
871027	8.7%	2.2%	0.9%	22.0%	8.3%	0.029%	0.242%	1.325%	0.758%
871028	10.8%	0.1%	0.6%	10.2%	4.9%	0.001%	0.154%	0.653%	0.445%
871029	12.2%	0.0%	0.4%	5.7%	2.9%	0.000%	0.106%	0.376%	0.278%
871030	11.6%	0.0%	0.3%	4.0%	2.0%	0.000%	0.082%	0.271%	0.198%
871102	11.6%	0.0%	0.3%	2.7%	1.3%	0.000%	0.081%	0.185%	0.131%
871103	12.0%	0.0%	0.5%	2.7%	1.4%	0.000%	0.097%	0.186%	0.139%
871104	12.0%	0.0%	0.4%	3.0%	1.6%	0.000%	0.100%	0.206%	0.158%
871105	11.7%	0.0%	0.5%	3.0%	1.5%	0.000%	0.092%	0.207%	0.153%
871106	12.0%	0.0%	0.6%	2.5%	1.3%	0.000%	0.089%	0.171%	0.132%
871109	12.2%	0.0%	0.5%	2.1%	1.1%	0.000%	0.092%	0.139%	0.112%
871110	12.5%	0.0%	0.5%	2.7%	1.5%	0.000%	0.084%	0.183%	0.149%
871111	12.4%	0.0%	0.6%	2.7%	1.4%	0.000%	0.076%	0.181%	0.140%
871112	12.0%	0.0%	0.5%	2.3%	1.2%	0.000%	0.065%	0.156%	0.116%
871113	12.1%	0.0%	0.3%	1.9%	1.0%	0.000%	0.057%	0.132%	0.102%
871116	12.1%	0.0%	0.8%	1.8%	1.1%	0.000%	0.064%	0.121%	0.109%
871117	12.4%	0.0%	0.8%	1.7%	1.0%	0.000%	0.067%	0.116%	0.096%
871118	12.2%	0.0%	0.8%	1.9%	1.0%	0.000%	0.064%	0.130%	0.100%
871119	12.6%	0.0%	0.7%	1.6%	0.9%	0.000%	0.058%	0.109%	0.093%
871120	12.4%	0.0%	0.8%	1.6%	1.0%	0.000%	0.061%	0.104%	0.095%
871123	12.3%	0.0%	0.5%	1.8%	1.0%	0.000%	0.052%	0.121%	0.099%
871124	12.2%	0.0%	0.4%	1.1%	0.9%	0.000%	0.037%	0.075%	0.082%
871125	12.3%	0.0%	0.4%	1.0%	0.5%	0.000%	0.034%	0.063%	0.051%
871127	12.7%	0.0%	0.5%	0.8%	0.5%	0.000%	0.033%	0.055%	0.047%
871130	12.9%	0.0%	0.9%	0.9%	0.5%	0.000%	0.053%	0.056%	0.047%

Table B. Expected additional funds requirement *conditional* upon a margin-exhausting futures price move. As a percentage of notional futures value, and dollar value given total open interest in all S&P 500 futures contracts.

		Exposure (%)				Exposure ($ billion)			
Date	Margin	BS	JD-options	JD-RR	s.e.	BS	JD-options	JD-RR	s.e.
870930	3.1%	0.3%	0.4%	0.6%	0.1%	$0.06	$0.07	$0.11	$0.01
871001	3.0%	0.3%	1.0%	0.6%	0.1%	$0.06	$0.19	$0.11	$0.01
871002	3.0%	0.3%	1.9%	0.6%	0.1%	$0.06	$0.36	$0.11	$0.01
871005	3.0%	0.3%	1.4%	0.6%	0.1%	$0.06	$0.27	$0.11	$0.01
871006	3.1%	0.3%	1.3%	0.6%	0.1%	$0.05	$0.23	$0.11	$0.01
871007	3.1%	0.3%	1.8%	0.6%	0.1%	$0.06	$0.33	$0.11	$0.01
871008	3.2%	0.3%	2.3%	0.6%	0.1%	$0.06	$0.43	$0.11	$0.01
871009	3.2%	0.3%	1.2%	0.6%	0.1%	$0.06	$0.22	$0.11	$0.01
871012	3.2%	0.3%	1.5%	0.6%	0.1%	$0.06	$0.29	$0.11	$0.01
871013	3.2%	0.3%	1.8%	0.6%	0.1%	$0.06	$0.33	$0.12	$0.01
871014	3.3%	0.3%	1.4%	0.6%	0.1%	$0.06	$0.28	$0.12	$0.02
871015	3.4%	0.3%	2.1%	0.6%	0.1%	$0.07	$0.43	$0.12	$0.02
871016	3.5%	0.4%	0.5%	0.7%	0.1%	$0.09	$0.11	$0.14	$0.02
871019	7.4%	0.6%	40.5%	8.5%	3.1%	$0.10	$7.03	$1.47	$0.54
871020	6.9%	2.7%	55.3%	7.5%	2.6%	$0.51	$10.41	$1.40	$0.49
871021	5.8%	1.3%	10.0%	7.5%	2.0%	$0.28	$2.20	$1.65	$0.44
871022	8.2%	1.3%	34.0%	6.0%	2.1%	$0.25	$6.61	$1.16	$0.40
871023	8.3%	1.1%	37.8%	5.9%	2.0%	$0.21	$7.14	$1.11	$0.38
871026	9.1%	1.6%	31.4%	6.3%	2.3%	$0.27	$5.49	$1.10	$0.40
871027	8.7%	1.3%	26.7%	6.0%	1.8%	$0.24	$4.79	$1.08	$0.32
871028	10.8%	1.0%	27.6%	6.4%	2.3%	$0.18	$4.99	$1.16	$0.41
871029	12.2%	0.7%	26.8%	6.6%	2.4%	$0.13	$5.06	$1.25	$0.45
871030	11.6%	0.6%	28.3%	6.8%	2.5%	$0.11	$5.60	$1.35	$0.49
871102	11.6%	0.6%	24.7%	6.9%	2.5%	$0.11	$4.72	$1.31	$0.48
871103	12.0%	0.6%	18.2%	6.8%	2.6%	$0.12	$3.34	$1.26	$0.47
871104	12.0%	0.6%	24.0%	6.8%	2.6%	$0.11	$4.36	$1.25	$0.47
871105	11.7%	0.6%	17.5%	6.9%	2.5%	$0.10	$3.15	$1.24	$0.46
871106	12.0%	0.5%	16.0%	6.8%	2.6%	$0.08	$2.80	$1.20	$0.46
871109	12.2%	0.6%	18.0%	6.8%	2.6%	$0.11	$3.11	$1.16	$0.45
871110	12.5%	0.6%	17.6%	6.8%	2.6%	$0.10	$2.95	$1.14	$0.44
871111	12.4%	0.5%	13.4%	6.8%	2.6%	$0.09	$2.27	$1.15	$0.44
871112	12.0%	0.5%	11.8%	6.8%	2.6%	$0.08	$2.03	$1.17	$0.44
871113	12.1%	0.4%	19.4%	6.8%	2.6%	$0.08	$3.32	$1.16	$0.45
871116	12.1%	0.4%	8.2%	6.8%	2.7%	$0.08	$1.42	$1.17	$0.47
871117	12.4%	0.5%	8.2%	6.7%	2.7%	$0.08	$1.38	$1.13	$0.45
871118	12.2%	0.4%	7.8%	6.8%	2.6%	$0.08	$1.36	$1.17	$0.46
871119	12.6%	0.4%	8.4%	6.7%	2.7%	$0.07	$1.40	$1.12	$0.45
871120	12.4%	0.5%	7.5%	6.7%	2.8%	$0.08	$1.27	$1.15	$0.47
871123	12.3%	0.4%	9.8%	6.8%	2.7%	$0.07	$1.68	$1.16	$0.46
871124	12.2%	0.3%	10.0%	6.7%	2.9%	$0.05	$1.74	$1.16	$0.51
871125	12.3%	0.3%	8.3%	6.6%	2.7%	$0.04	$1.44	$1.15	$0.46
871127	12.7%	0.3%	6.8%	6.6%	2.7%	$0.05	$1.13	$1.09	$0.45
871130	12.9%	0.5%	5.9%	6.5%	2.7%	$0.07	$0.96	$1.06	$0.44

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