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ABSTRACT

This paper takes a step towards formalizing the theoretical interconnections among four post-Industrial Revolution phenomena - the industrialization and growth take-off of rich 'northern' nations, massive global income divergence, and rapid trade expansion. Specifically, we present a stages-of-growth model in which the four phenomena are jointly endogenous and all are triggered by a gradual fall in the cost of doing business internationally. In the first stage, while trade costs are high, industry is dispersed and growth is low. In the second stage, the north industrializes rapidly, growth takes off and the south diverges. In the third stage, high growth becomes self sustaining. The model shows under which conditions, in a fourth stage, the south can quickly industrialize and converge.

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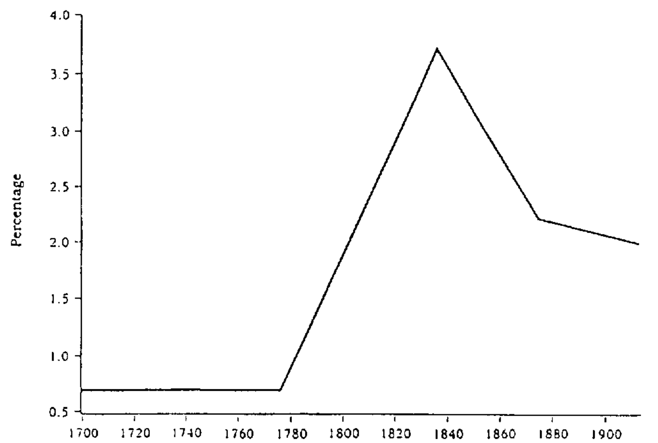
1. Introduction

In 1990 the world's richest nation was 4500% richer than the poorest; in 1870 the figure was 900% and, before the first Industrial Revolution (mid-18th century), West European per capita incomes were only 30% above those of China and India (Maddison 1983; Bairoch 1993). While some scholars disagree with Bairoch's estimate of 18th century income gaps, none disputes the conclusion. Authors as diverse as Braudel, Kuznets, Baumol and Maddison assert that the big north-south income divergence appeared with the first Industrial Revolution. For example, Kuznets (1965 p.20) says: "Before the 19th century and perhaps not much before it, some presently underdeveloped countries, notably China and parts of India, were believed by Europeans to be more highly developed than Europe, and at that earlier time their per capita incomes may have been higher than the then per capita incomes of the presently developed countries". Thus, by the time frame of human history, "the current wide disparities – between rich and poor countries – are recent" (Kuznets 1966, p.393).

The Industrial Revolution caused this rapid income divergence by triggering industrialization and a growth take-off in Europe while incomes stagnated in the now poor nations (see Baumol 1994). At the same time, the world experienced a rapid expansion of international trade. This paper takes a modest step towards formalizing the logical interconnections among these four key phenomena – northern industrialization and growth take-off, income divergence, and trade expansion. Specifically, we present a stages-of-growth model in which the four phenomena are jointly endogenous and all are triggered by a gradual fall in the cost of international transactions. Before turning to the model, we review the phenomena in more detail.

Perhaps the most striking feature of the Industrial Revolution concerns the increase in growth rates. For example, Great Britain's per capita income rose by 14% between 1700 and 1760, by 34% between 1760 and 1820, and by 100% between 1820 and 1870 (Maddison 1983). Population growth also increased sharply during this period (due chiefly to better economic conditions), so Britain's GDP rose even faster than the per capita figures. In their recent re-analysis of growth in this period, Crafts and Harley (1992) and Crafts (1995) have revised downward Maddison's growth figures. Nevertheless, they confirm Maddison's basic message by finding a structural break in the trend growth of industrial production in Great Britain around 1776 (see Figure 1).* Historical data on growth in other countries are scarce, but the available estimates indicate that 19th century per capita GDP in India stagnated (Maddison 1971), or actually regressed (Bairoch 1993; Braudel 1984).

Figure 1: Trend Growth in Industrial Production (Crafts 1994).



During this same period, the sectoral composition of British GDP shifted radically, transforming Britain from a predominantly agrarian country into the world's most industrialized nation (Crafts 1989). In 1700, 18.5% of the labor force was in industry. In 1800, this percentage was 29.5% and in 1840 it was 47.3%. During the same period, Great Britain became a large food importer and a large exporter of industrial goods. Structural changes in what came to be known as the Third World were no less dramatic during this period. India, for instance, switched from a net exporter of manufactures to a net exporter of raw materials (Chaudhuri 1966). To take a specific example, during the 18th century the Indian textile industry was the global leader in terms of quality, production and exports (Braudel 1984). Yet at the end of the 19th century, more than 70% of Indian textile consumption was imported, mainly from Great Britain (Bairoch 1993 and Cohen 1997). A similar, but less dramatic, story can be told for the Indian shipbuilding and steel industries. These shifts in specialization suggest that the massive industrialization in the now rich 'north' may have been accompanied by a de-industrialization in the now poor 'south'.

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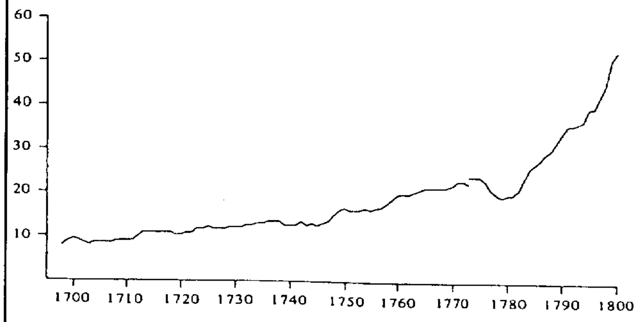
*Additionally, some economic historians (Macer 1993; Crafts 1995) argue that the growth take-off was accompanied by a rapid increase in innovation seen as a profit seeking activity. Sullivan (1989) finds that the growth rate of patenting was 0.5% before 1754 and 3.6% thereafter.

Historians disagree on trade's exact role in the Industrial Revolution (on this debate see Engermann,1996 and O'Brien and Engermann 1991). None, however, disputes its rapid increase during this period (Figure 2, from Deane 1979 illustrates this rapid increase in English foreign trade during the period of the first Industrial Revolution). At least part of this rise was due to the decrease in trade costs during the 18th century (on increased productivity in ocean shipping, see North, 1968).

Figure 2: English Foreign trade (18th century) (net imports plus domestic exports; official values).



On one side of the debate, the fact that exports accounted for a modest fraction of GNP suggests a limited role for trade. On the other side, it is important to note that, during the nineteenth century, Britain (which industrialized first) exported an unusually high proportion of its total output, around 25% compared with 10-12% for France (Crafts 1984, 1989). Also, even before the Industrial Revolution, Britain was a more open economy than any of the continental countries, exporting close to 15% of its GNP. Furthermore, the role of trade was especially important for several leading industries that exported a third of their output product in 1800 (Mokyr 1993). The cotton sector, above all, depended for more than half of its sales on foreign markets and historians (Landes, 1969) stress the sector's importance as a technology driver for the rest of industry. Cotton was also cheap to transport at a time where most goods were not (Crafts 1989). Braudel (1984) takes trade to be a key factor, noting, for instance, that between 1700 and 1800 British industries that focused on domestic sales expanded output by 50% while those that produced for export multiplied production by 500%. Deane (1979) also stresses the role of trade speaking of a Commercial Revolution which transformed London during the eighteen century into "the centre of the wide, intricate, multilateral network of world trade". She also noted that British trade during this period expanded much more rapidly with the West and East Indies and Africa than with Europe, its traditional trade partner. In 1700, 85% of English exports (excluding re-exports) were directed towards Europe, and 9% towards the West and East Indies and Africa. Thus, at the end of the nineteenth century, Europe's share amounted to only 30% while that of the West and East Indies was 38%.

Modelling Strategy In this paper, we posit a model in which these four aspects (northern industrialization and growth take-off, income divergence, and trade expansion) are jointly endogenous. To this end, we combine aspects of the 'economic geography' literature (Krugman 1991) with aspects of the endogenous growth literature (Romer 1986, 1990; Lucas 1988; Grossman and Helpman 1991; Aghion and Howitts 1991). Before presenting our model's logic, we review the relevant lessons of these theoretical literatures.

Consider first the economic geography literature introduced, inter alia, by Krugman (1991a, b), Venables (1996), and Krugman and Venables (1995). This literature (see Fujita, Krugman and Venables 1997 for a synthesis) focuses on the location effects of international integration between identical regions. One remarkable feature of these models is the possibility that a gradual lowering of interregional trade costs can result in the catastrophic agglomeration of industry. In this context, the adjective catastrophic indicates that trade cost reduction has no location effect until a critical level is crossed, and below this level, a discrete jump in agglomeration of industry occurs (often involving total agglomeration). Within each region this sort of 'punctuated equilibrium' time path would appear as a sweeping inter-sector resource shift not unlike the Industrial Revolution.

Consider next the lessons of the trade and endogenous growth literature, such as Romer and Rivera-Batiz (1991), and Grossman and Helpman (1991). Virtually all of these models posit technological externalities – knowledge spillovers or production externalities – as the essential feature that prevents capital's return from falling as the human, physical, and/or knowledge capital stocks rise. A number of empirical studies (Jaffe et al. 1993, Eaton and Kortum 1996; Cabellero and Jaffe 1993) show that these growth-sustaining externalities have an important local component in the sense that international borders seem to dampen the externalities.*

Combining these two sets of lessons can produce a two-region model in which the

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*It is interesting to note that Rosenberg (1994), Macer (1993) and Crafts (1995) explicitly stress the importance of localized cumulative learning processes in their accounts of the Industrial Revolution.

gradual, exogenous lowering of trade costs – driven by lower transportation and communication costs as well as by market opening initiatives – can produce three stages of growth. Due to localized externalities, we have the presumption that agglomerating industry and/or innovation activities is beneficial to growth. From the geography literature, we see that gradual integration may produce a catastrophic agglomeration process marked by three very distinct stages. In the first stage – while trade costs are still quite high – falling trade costs have the usual static effects on prices, trade and welfare, but no location and no growth effects. Growth in this stage may be positive, but it proceeds at a fairly low rate, since the geographical dispersion of industry hinders the externalities that are essential to cease-less innovation and growth. In the middle stage – when trade costs have just entered the 'catastrophic' region – agglomeration occurs very rapidly and, to be specific, say it occurs in the north. This industrialization triggers a take off in northern growth because geographical agglomeration amplifies the exploitation of technical externalities related to innovation.

Agglomeration of industry in the north is accompanied by stagnation in the south, so the agglomeration of industry not only generates industrialization and a growth take-off, it also produces income divergence. Yet despite this, we shall see that the south may still benefit in welfare terms. In the third stage, high growth becomes stable and self-sustaining.

The main focus of this paper is on the four phenomena mentioned above. We also show, however, that the model can generate rapid industrialization in the south and convergence. This emergence of southern industry slows global growth somewhat and forces a relative de-industrialization in the north.

While these stages of growth are highly suggestive, our model is – of course – far too simple to comprehensively track two-centuries of global economic history. We prefer, therefore, to think of it as a first step in examining the internal logic of one trade-and-growth mechanism where international integration spurs industrialization and a growth take-off.

Relation to early literature Our model captures some elements of the informal analyses of the classic growth scholars such as Kuznets and Rostow. Despite their disagreements on important points, both think of the Industrial Revolution as a structural break. Kuznets (1966) divides growth into two types: traditional growth (pre-1750) and modern economic growth (post-1750). The distinctive feature of modern growth, according to Kuznets, is the rapidity of the shifts in industrial structure (he talks of sweeping structural changes) and their magnitude when cumulated over decades. Rostow (1960) goes further, identifying five stages in economic growth: the traditional society, the preconditions for take-off, the take-off, the drive to maturity and the age of high mass-consumption. The take-off can be traced to a sharp stimulus, Rostow asserts, and he lists a number of these, including one that hinges on lower trade costs. The take-off, "may come about through a technological (including transport) innovation which sets in motion a chain of secondary expansion in modern sectors and has powerful potential external economy effects which the society exploits." (Rostow 1960 p.36). Rostow also lists three conditions for a take-off: a rising investment rate, rapid expansion of one or more industrial sectors marked by external economies, and rapid emergence of structures that are necessary for self-sustaining growth. Finally, both Kuznets and Rostow view modern economic growth as a sustained and non-reversible process.

Relation to recent literature The existing formal endogenous growth models deal with modern economic growth, to use Kuznets' phrase. Models such as Romer (1990), Aghion and Howitts (1992), and Grossman and Helpman (1991) do not model the emergence of growth preconditions nor do these models consider the forces that initiate the transition to an endogenous, sustained growth process (see Crafts,1995, however, for an analysis of the British Industrial Revolution in the light of endogenous growth models).

The 'big push' literature of Murphy, Shleifer and Vishny (1989a,b) and others is more closely related. Those models point out that due to pecuniary externalities, an economy may be marked by two growth equilibria: one in which investment and therefore growth is nil, since the economy is too small, and one in which agents invest in anticipation of growth. The jump from one equilibrium to the other generates sudden industrialization. These models do not, however, imply that one of the equilibria must precede the other and as such are not models of growth stages. There is also no clear reason why the economy could not jump back (because of a war for example) to the zero growth equilibrium. Furthermore, in these models, differences in initial conditions (history) or in self-fulfilling expectations explain why an economy experiences a take-off or stays indefinitely in a poverty trap. This point is made clear in Matsuyama (1991) and Krugman (1991). Hence, the difference in the path taken by poor and rich economies, the divergence phenomenon, lies out of the model. Finally, these authors only look at closed economies.

The literature on uneven development, formalized by Krugman (1981), Faini (1984), and Krugman and Venables (1995), analyzes trade and global income divergence. None of these papers endogenises growth, so the long-run growth rate is never affected (in Krugman (1981) the divergence is driven by technological externalities; in Krugman and Venables (1995) it arises due to pecuniary externalities). The interesting recent contribution of Kelly (1997) is the closest to ours as it shows that the expansion of market size through a gradual improvement of transport linkages can lead to a sudden takeoff of the economy. This model does not, however, explain why certain economies have experienced such a take-off and others have not, i.e. the divergence phenomenon that we view as a key issue. Furthermore, Kelly's take-off is only temporary (the long-run growth rate is exogenous) despite the fact that the growth take-off is the most important feature of the transition from traditional to modern growth identified by early growth scholars. Finally, Kind (1997) has independently developed a growth and geography model that shares some elements with our Section 5 model. He does not, however, use his model to study the interconnections among the four phenomena that we focus on.

The remainder of this paper is in five parts. The next section, Section 2, introduces the basic model and equilibrium conditions. The third section studies the stability properties of the model, establishing that the gradual reduction of trade costs eventually produces a catastrophic agglomeration of industry. The fourth section studies the growth and divergence implications. The fifth section discusses an extension and the last section presents our concluding remarks.

2. The Basic Model

The basic logic of our growth take-off – namely, that catastrophic agglomeration speeds growth in the presence of localized learning externalities – would, we conjecture, make sense in a very broad class of models. However, few such models could be solved analytically; Fujita, Krugman and Venables (1997) show that most models with catastrophic agglomeration must be solved numerically. To illustrate the interplay of economic forces as sharply as possible, we want analytic results and this leads us to adopt explicit functional forms and some severe simplifying assumptions. In particular, our model combines Martin and Ottaviano (1996a) and Baldwin (1997) and as such it adopts functional forms and simplifying assumption from the standard product-innovation growth model and from the economic geography literature.

2.1 Basic Assumptions

Consider a world economy with two regions (north and south) each with two factors (labour L and capital K) and three sectors: manufactures M, traditional goods T, and a capital-producing sector I. Regions are symmetric in terms of preferences, technology, trade costs and labour endowments. The Dixit-Stiglitz M-sector (manufactures) consists of differentiated goods where production of each variety entails a fixed cost (one unit of K) and a variable cost (a_M units of labour per unit of output). Its cost function, therefore, is $\pi + wa_M m_i$, where π is K's rental rate, w is the wage rate, and m_i is total output of a typical firm.[1] (Numbered notes refer to the attached 'Supplemental Guide to Calculations'). Traditional goods, which are assumed to be homogenous, are produced by the T-sector under conditions of perfect competition and constant returns. By choice of units, one unit of T is made with one unit of L.

Regional labour stocks are fixed, but each region's K is produced by its I-sector (I is a mnemonic for innovation when interpreting K as knowledge capital, for instruction when interpreting K as human capital, and for investment-goods when interpreting K as physical capital). The I-sector produces one unit of K with a_I units of L. To individual I-firms, a_I is a parameter, however following Romer (1990) and Grossman and Helpman (1991), we assume a sector-wide learning curve. That is, the marginal cost of producing new capital declines (i.e., a_I falls) as the sector's cumulative output rises. Many justifications of this learning are possible. Romer (1990), for instance, rationalizes it by referring to the non-rival nature of knowledge.

The specific production and marginal cost functions assumed are:

$$\dot{K} = Q_K = \frac{L_I}{a_I}, \quad F = wa_I; \qquad a_I \equiv \frac{1}{K^w A}, \quad A \equiv \theta_K + \lambda(1-\theta_K), \quad \theta_K \equiv \frac{K}{K^w} \qquad \textbf{(2-1)}$$

where Q_K and L_I are I-sector output and employment, F is I-sector marginal cost (in equilibrium F is the M-sector's fixed cost), $K^w \equiv K + K^*$ where K and K* are the northern and southern cumulative I-sector production levels, and λ (a mnemonic for learning spillovers) is a parameter governing the internationalization of learning effects. Southern technology is isomorphic with $a_I^*=1/A^*K^w$ and $A^*\equiv\lambda\theta_K+1-\theta_K$. Finally, following Romer (1990) and Grossman and Helpman (1991), depreciation of knowledge capital is ignored, so $\dot{K}=Q_K$. The regional K's therefore represents three quantities: region-specific capital stocks, region-specific cumulative I-sector production (i.e. learning), and region-specific numbers of varieties (recall that there is one unit of K per variety).

The early trade-and-endogenous-growth literature (eg, Grossman and Helpman 1991) considered only the extreme cases of $\lambda=1$ and $\lambda=0$. However recent empirical studies – such as Eaton and Kortum (1996), and Cabellero and Jaffee (1995) – indicate that international learning spillovers are neither perfect nor nonexistent. We therefore assume partially localized learning externalities, i.e. $0<\lambda<1$. When $\lambda<1$, regional I-sector labour productivities, $1/a_I$ and $1/a_I^*$, depend on a common global element K^w and a regional element, namely $A\equiv\theta_K+\lambda(1-\theta_K)$ for the north and $A^*\equiv\lambda\theta_K+1-\theta_K$ for the south.

$$g \equiv \frac{\dot{K}}{K} = \frac{L_I A}{\theta_K} \qquad \textbf{(2-2)}$$

Given (2-1), the growth rate of north's K is related to L_I, λ and θ_K according to: The corresponding expression for K*'s growth is $g^*=L_I{}^*A^*/(1-\theta_K)$.

To keep the analysis tightly focused on key issues, we assume an infinitely-lived representative consumer (in each country) with preferences:

$$U=\int_{t=0}^{\infty} e^{-\rho t}\ln Q\ dt\ , \quad Q \equiv C_T^{1-\alpha}C_M^{\alpha}\ , \quad C_M\equiv\left(\int_{i=0}^{K+K^*} c_i^{1-1/\sigma}di\right)^{\frac{1}{1-1/\sigma}} \quad \sigma>1 \qquad \textbf{(2-3)}$$

where ρ is the time preference parameter, Q is a consumption composite of C_T and C_M (these are, respectively, consumption of T and a CES composite of M-varieties), and c_i is consumption of variety i. Each regional representative consumer acts atomistically even though she owns all her region's L and K. Northern income, Y, is $wL+\pi K$. Southern income is $w^*L+\pi^*K^*$ (recall that $L=L^*$).

While goods (M and T) are traded, factors (L and K) are not.* For goods, we adopt the standard simplifying assumptions (Krugman 1991; Krugman and Venables 1995) that T-trade is costless but trade in M is impeded by frictional (i.e., 'iceberg') import barriers (see Fujita, Krugman and Venables 1997 for detailed discussion of this assumption). Specifically, $\tau \geq 1$ units of M must be exported to sell one unit abroad. τ is viewed as reflecting all costs of doing business abroad. These include everything from the mundane – shipping costs and trade policy barriers (tariffs, etc.) – to more exotic factors such as the difficulty of dealing with cultural and language differences, the cost of providing after-sales services, and the costs of communicating with customers and sales agents.

2.2 Intermediate Results

Utility optimization implies that a constant fraction α of northern consumption expenditure E falls on M-varieties with the rest spent on T. Northern optimization also yields unitary elastic demand for T and the CES demand functions for M varieties:

$$c_j = \frac{s_j \alpha E}{p_j} \quad ; \quad s_j \equiv \frac{p_j^{1-\sigma}}{\int_{i=0}^{K+K^*} p_i^{1-\sigma} di} \tag{2-4}$$

where s_j is variety j's share of expenditure on all M-varieties in the north, E is northern expenditure and the p's are consumer prices. The optimal northern consumption path satisfies the Euler equation $\dot{E}/E = r - \rho$ (r is the north's rate of return on investment) and a transversality condition. Southern optimization conditions are isomorphic.

On the supply side, free trade in T equalizes nominal wage rates as long as both regions produce some T (always true as long as α is not too large). Taking home labour as numeraire and defining p_T as T's price, $p_T = p_T^* = w = w^* = 1$.[2] As for the M-sector, we choose

*We view K as knowledge capital and note that while some aspects of knowledge are easily transfered internationally others are not. Some easily-transfered aspects of knowledge are captured by $\lambda > 0$ in our model, but we also assume that important aspects of variety-specific knowledge are 'tacit' in the sense that they are embodied in the skill and know-how of workers. This component makes K difficult to trade. To keep our results as sharp as possible, we make the simplifying assumption that K is nontraded.

units such that $a_M = 1 - 1/\sigma$. As usual M-sector optimal pricing is given then by $p=1$ and $p^* = \tau$ where p and p* are typical local and export market prices, respectively.[3] Southern M-firms have analogous pricing rules.

With monopolistic competition, equilibrium operating profit is the value of sales divided by σ.[4] Rearranging (2-4), using the optimal pricing rules[5]:

$$\pi = B\left(\frac{\alpha E^w}{\sigma K^w}\right) ; \quad B \equiv \left[\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{\phi(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right], \quad \theta_E \equiv \frac{E}{E^w} \tag{2-5}$$

where E^w is world expenditure, θ_E is north's share of E^w, and $\phi \equiv \tau^{1-\sigma}$. Here ϕ is a mnemonic for the 'free-ness' (phi-ness) of trade since trade gets freer as ϕ rises from $\phi=0$ (prohibitive trade costs) to $\phi=1$ (costless trade). Also, B is a mnemonic for the 'bias' in northern M-sector sales since B measures the extent to which the value of sales of a northern variety (namely, $pc+p^*c^*$) exceeds average sales per variety worldwide (namely, $\alpha E^w/K^w$). The expression for π^* is analogous.[6] Note that the definition of B permits a decomposition of π changes into global developments (measured by $\dot{K}^w$) and local developments (measured by $\dot{\theta}_K$ and $\dot{\theta}_E$).

Finally, differentiating its definition, the law of motion for θ_K is:

$$\dot{\theta}_K = \theta_K(1-\theta_K)(g-g^*) \tag{2-6}$$

Note that the search for steady states is simplified by inspection of (2-6). By definition $\dot{\theta}_K = 0$ in steady state, so the model has only two types of long-run equilibria: those in which $g=g^*$ (nations accumulate capital at equal rates), and those in which θ_K equals either unity or zero. We refer to these two types as, respectively, the interior and core-periphery outcomes. (To avoid repetition, we consider only the core-in-north case, i.e. $\theta_K=1$.)

2.3 Long-Run Equilibria

The simplest way of analysing this model is to take L as numeraire (as assumed above) and L_I, L_I^*, and θ_K as state variables.[7] The L_I's indicate labour devoted to creating new K, so they are the regional levels of real investment. While there may be many ways of determining investment in a general equilibrium model, Tobin's q-approach (Tobin, 1969) is a powerful, intuitive, and well-known method for doing just that. The essence of Tobin's approach is to assert that the equilibrium level of investment is characterised by the equality of the stock market value of a unit of capital – which we denote with the symbol V – and the replacement cost of capital, P_K. Tobin takes the ratio of these, so what trade economists would naturally call the M-sector free-entry condition (namely $V=P_K$) becomes Tobin's famous condition $q \equiv V/P_K=1$.

The denominator of Tobin's q is the price of new capital. Due to I-sector competition, northern and southern prices of K are F and F* (respectively). Calculating the numerator of Tobin's q (the present value of introducing a new variety) requires a discount rate. In steady state, $\dot{E}=0$ in both nations, so the Euler equations imply that $\bar{r}=\bar{r}^*=\rho$, ('bars' indicate steady-state values).[8] Moreover from (2-5), the present value of a new variety also depends upon the rate at which new varieties are created. Since the steady state is marked by time-invariant L_I's, (2-2) implies that the growth rate of K^w is time-invariant in steady state. In particular, the growth rate will either be the common $\bar{g}=\bar{g}^*$ (in the interior case), or north's g (in the core-periphery case). In either case, the steady-state values of investing in new units of K are[9]:

$$\bar{V} = \frac{\bar{\pi}}{\rho+\bar{g}}, \qquad \bar{V}^* = \frac{\bar{\pi}^*}{\rho+\bar{g}} \qquad \textbf{(2-7)}$$

Given this the regional q's depend only on parameters and state variables, i.e.:

$$\bar{q} = \frac{\bar{\pi}\ /\ (\rho+\bar{g})}{\bar{F}}, \qquad \bar{q}^* = \frac{\bar{\pi}^*\ /\ (\rho+\bar{g})}{\bar{F}^*} \qquad \textbf{(2-8)}$$

Optimizing consumers set expenditure at the permanent income hypothesis level in steady state.[10] That is, they consume labor income plus ρ times their steady-state wealth, $\bar{F}K= \bar{\theta}_K/\bar{A}$, and, $\bar{F}^*K^*= (1-\bar{\theta}_K)/\bar{A}^*$ in the north and in the south respectively[11]. Thus:

$$\bar{\theta}_E = \frac{L + \rho\bar{\theta}_K / \bar{A}}{2L + \rho\left[\bar{\theta}_K / \bar{A} + (1-\bar{\theta}_K)/\bar{A}^*\right]} \qquad \textbf{(2-9)}$$

This relation between $\bar{\theta}_E$ and $\bar{\theta}_K$ can be thought as the optimal savings/expenditure function since it is derived from intertemporal utility maximisation.

2.3.1 Interior Steady States

Consider first interior steady states where both nations are investing (innovating), so $\bar{q}=1$ and $\bar{q}^*=1$. Using (2-5) and (2-1) in (2-8), $\bar{q}=\bar{q}^*=1$ gives a second relation between $\bar{\theta}_K$ and $\bar{\theta}_E$ which we can think of as the optimal investment relation. Together with the optimal saving relation of (2-9), it produces three solutions:

$$\bar{\theta}_K = \frac{1}{2}, \quad \bar{\theta}_K = \frac{1}{2}\left[1 \pm \sqrt{\left(\frac{1+\lambda}{1-\lambda}\right)\left(\frac{1+\lambda\Lambda}{1-\lambda\Lambda}\right)}\right]; \quad \Lambda \equiv \left\{1 - \frac{2\rho\phi(1-\lambda\phi)}{[(\lambda(1+\phi^2)-2\phi]L}\right\}^{-1} \qquad \textbf{(2-10)}$$

The first is the symmetric case. The second and third roots – which correspond to interior, non-symmetric steady states – are economically relevant only for a narrow range of ϕ. In particular, the second and third solutions converge to 1/2 as ϕ approachs a particular value which we call ϕ^{cat} (for reasons that become clear below). For levels of ϕ below ϕ^{cat}, the second and third solutions are imaginary and so are irrelevant. For levels of ϕ above another critical value (defined explicitly below), the second solution is negative and the third solution exceeds unity, so both are economically irrelevant.

Given (2-10), the remaining aspects of the interior steady state are easily calculated. In particular, solving $\bar{q}=1$ for $\bar{g}$ and then using (2-2):

$$\bar{L}_I = \frac{\bar{\theta}_K}{\bar{A}}\left\{\frac{\alpha}{\sigma}\left[2L+\rho\left(\frac{\bar{\theta}_K}{\bar{A}}+\frac{1-\bar{\theta}_K}{\bar{A}^*}\right)\right]\bar{A}\bar{B} - \rho\right\} \qquad \textbf{(2-11)}$$

$\bar{L}_I^*$ is found by a similar procedure. Note that for the symmetric case ($\bar{\theta}_K=1/2$):

$$\bar{L}_I = \bar{L}_I^* = \frac{\alpha(1+\lambda)L-\rho(\sigma-\alpha)}{\sigma(1+\lambda)} \qquad \textbf{(2-12)}$$

Using the second and third roots from (2-10) in (2-11) yields analytic solutions for $\bar{L}_I$ in the interior non-symmetric cases, but the expressions are too unwieldy to be revealing.

2.3.2 Core-Periphery Steady States

Expression (2-10) yields values of $\bar{\theta}_K$ that are economically irrelevant when ϕ exceeds a critical level. For such ϕ's, $\bar{\theta}_K$ has two types of solutions: the core-periphery outcome ($\bar{\theta}_K=0$ or 1), or the symmetric outcome (note that $\bar{\theta}_K=1/2$ solves $\bar{q}=\bar{q}^*=1$ for all ϕ). The critical value, call it ϕ^{CP} (a mnemonic for core-periphery), is established by noting that at the core-periphery outcome $\bar{\theta}_K=1$, $\bar{q}=1$ and $\bar{q}^*<1$. That is, continuous innovation is profitable in the north since $\bar{V}=\bar{F}$, but $\bar{V}^*<\bar{F}^*$ so no southern M-firm would choose to setup.* Using (2-1), (2-2), (2-5), (2-8) and $\bar{\theta}_K$, $\bar{q}^*$ with $\bar{\theta}_K=1$ simplifies to[12]:

$$\bar{q}^* = \lambda\frac{(1+\phi^2)L+\phi^2\rho}{(2L+\rho)\phi} \tag{2-13}$$

The ϕ that solves $\bar{q}^*=1$ defines the endpoint of the core-periphery set, namely:

$$\phi^{CP} = \frac{2L+\rho - \sqrt{(2L+\rho)^2-4\lambda^2L(L+\rho)}}{2\lambda(L+\rho)} \tag{2-14}$$

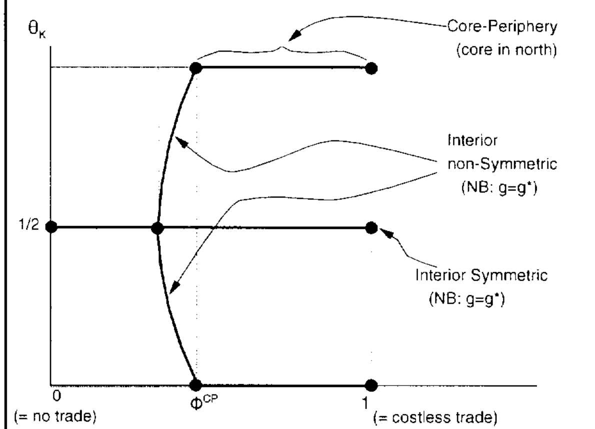
Note that although there are two roots, only one is economically relevant.[13]

Using $\bar{\theta}_K=1$, the remaining aspects of the core-periphery steady state are simple to calculate. In particular, since $\bar{\theta}_K=1$, $\bar{q}=1$ and $\bar{q}^*<1$, we have:

$$\bar{L}_I = \frac{\alpha 2L-\rho(\sigma-\alpha)}{\sigma}, \qquad \bar{L}_I^* = 0 \tag{2-15}$$

2.3.3 A Map of Steady States Figure 3 summarizes the various steady states and their dependence on trade costs. North's share of world K, θ_K, is on the vertical axis and all conceivable levels of trade free-ness are shown with the [0,1] interval on the horizontal axis. As noted above, the symmetric case exists for all ϕ, but the core-periphery outcome (either $\bar{\theta}_K=1$ or 0) is an equilibrium only for $\phi>\phi^{CP}$. The final type of steady state, the interior, non-symmetric case, is shown as the bowed line.

Figure 3



*This may be thought of the dynamic version of Krugman (1991b) thought-experiment of when a firm would wish to 'deviate' from the static core-periphery outcome.

2.4 Steady-State Trade Pattern and Volume

At the symmetric steady state, north and south engage in pure intra-industry trade in differentiated products. When $1/2<\theta_K<1$, the regions have different relative factor stocks and some Heckscher-Ohlin-trade occurs (north is the net exporter of capital-intensive M goods). The last case is when θ_K equals unity and only inter-industry trade occurs; the north exports M-varieties in exchange for T.

The volume of trade is simple to determine. Factors are not traded, so goods trade balances each period. The global volume of exports is thus twice the north's exports. From (2-4), and $\bar{E}^*=L+\rho(1-\bar{\theta}_K)/\bar{A}^*$, the steady-state global volume of exports is:

$$\bar{VT} = \frac{\alpha 2\phi}{1+\phi}\bar{\theta}_K \left[L + \frac{\rho(1-\bar{\theta}_K)}{\lambda\bar{\theta}_K + 1 - \bar{\theta}_K} \right] \qquad \textbf{(2-16)}$$

3. Stability and Catastrophic Agglomeration

Although an equal division of M-varieties is always an equilibrium, it need not be stable, as the economic geography literature has emphasised. Indeed, in our model two cycles of 'circular causality' tend to de-stabilize the symmetric equilibrium.

The first is the well-known demand-linked cycle in which production shifting leads to expenditure shifting and *vice versa*. The particular variant present in our model is based on the mechanism introduced by Baldwin (1997). To see the logic of this linkage, consider a perturbation that exogenously shifts one M-sector firm from the south to the north. Firms are associated with a unit of capital and capital-earnings are spent locally, so 'production shifting' leads to 'expenditure shifting'[*]. Other things equal, this expenditure shifting raises northern operating profits and lowers southern operating profits due to a market-size effect. This tends to raise q and lower q* thereby speeding north's accumulation and retarding south's.[14] The initial exogenous shift thus leads to another round of production shifting and the cycle repeats. As we shall see, if trade costs are sufficiently low, demand-linked circular causality

[*]Production does not literally 'shift' since factors are internationally immobile. Nevertheless θ_K rises since capital accumulation is encouraged in the north and discouraged in the south.

alone can de-stabilize the symmetric equilibrium.

The second link is the growth-linked circular causality introduced by Martin and Ottaviano (1996a). When I-sector technological externalities are transmitted imperfectly across borders, production shifting leads to 'cost shifting' in the I-sector. For instance, suppose an exogenous perturbation increased θ_K slightly. Given localized knowledge spillovers, this shock lowers the northern I-sector's marginal cost and raises that of the south. Other things equal, this raises q and lowers q*, so the initial production shifting raises north's rate of investment and lower south's. Of course, this 'growth shifting' further increases θ_K and the cycle repeats. Again, if trade costs are low enough, growth-linked circular causality alone can yield to total agglomeration.

The sole force opposing agglomeration here is the local competition effect. Namely, raising θ_K (north's share of varieties) tends to raise local competition in the north and lower it in the south. Since competition is bad for profits, raising θ_K tends to lower q and raise q*.[15]

3.1 Stability of the Symmetric Interior Steady State

The appendix shows that in the neighbourhood of the symmetric equilibrium, the linearized system has two positive and one negative real roots when ϕ is less than a critical value. For this range of ϕ's the system is saddle path stable, since only θ_K is a nonjumper. For ϕ beyond the critical value, the linearized system as three positive eigenvalues, so the symmetric equilibrium is unstable. As it turns out, however, an informal approach to stability provides the same answer with greater intuition.

Specifically, to study the symmetric equilibrium's stability, we exogenously increase θ_K by a small amount and check the impact of this perturbation on the regional $\bar{q}$'s, allowing expenditure shares to adjust according to (2-8). In particular, using (2-9), (2-1), (2-2), (2-5) and (2-8), the steady-state q can be expressed as a function θ_K and L_I. Holding L_I constant for the moment, the partial derivative of interest is $\partial\bar{q}/\partial\theta_K$ from $\bar{q}=[\bar{\theta}_K,\bar{L}_K,\bar{\theta}_E[\bar{\theta}_K];\phi]$. The symmetric equilibrium is stable, if and only if $\partial\bar{q}/\partial\theta_K$ is negative for a simple reason. If a unit of capital 'accidentally' disturbed symmetry, the 'accident' lowers Tobin's q in the north and raises it in the south (by symmetry $\partial\bar{q}^*/\partial\theta_K$ and $\partial\bar{q}/\partial\theta_K$ have opposite signs). Moreover, these incipient changes in Tobin's q are sufficient statistics for changes in regional investment levels (see Baldwin and Forslid 1997a Proposition 1 for details). Thus when $\partial\bar{q}/\partial\theta_K<0$, the perturbation generates self-correcting forces in the sense that L_I falls and L_I^* rises. If the derivative is positive, by contrast, the 'accident' boosts L_I and lowers L_I^*, thus amplifying the initial shock to θ_K. Plainly the symmetric equilibrium is unstable in this case. We turn now to signing $\partial\bar{q}/\partial\theta_K$.

Differentiating the definition of q with respect to θ_K, we have:

$$\left(\frac{\partial\bar{q}\,/\bar{q}}{\partial\theta_K}\right)\Bigg|_{\theta_K=1/2} = 2\left(\frac{1-\phi}{1+\phi}\right)\left(\frac{d\bar{\theta}_E}{d\theta_K}\right)\Bigg|_{\theta_K=1/2} + \frac{4}{1+\lambda}\frac{1+\phi^2}{(1+\phi)^2}\left[\frac{-(1-\phi)^2}{1+\phi^2} + 1-\lambda\right] \qquad \textbf{(3-1)}$$

Using (2-8) to find $d\bar{\theta}_E/d\theta_K=2\rho\lambda/[L(1+\lambda)+\rho](1+\lambda)$, we see that the system is unstable for sufficiently low trade costs (i.e. $\phi\approx 1$). And under weak regularity conditions, the system is stable, i.e. (3-1) is negative, for sufficiently high trade costs ($\phi\approx 0$).[16]

(3-1) illustrates the three forces affecting stability. The first and third term are positive, so they represent the destabilizing forces, namely the demand-linked and growth-linked circular causalities (respectively). The negative second term reflects the stabilizing local-competition effect. Clearly, reducing trade costs ($d\phi>0$) erodes the stabilizing force more quickly than it erodes the destabilizing demand-linkage. Moreover, trade free-ness ϕ does not affect the strength of growth-linkage (third term).

To isolate the two distinct cycles of circular causality, suppose, for the sake of argument, that the demand-linkage is cut, so $d\bar{\theta}_E/d\theta_K=0$. In this case, $\partial\bar{q}/\partial\theta_K$ is positive and the system is unstable when $\lambda<2\phi/(1+\phi^2)$. This shows that growth-linked circular causality can by itself produce total agglomeration when trade costs are low enough. (Recall that $0\leq\phi\leq 1$ is a measure of the free-ness of trade, so $\phi=1$ indicates costless trade). To see the dependence of growth-linked circular causality on localized knowledge spillovers, note that with $\lambda=1$ and $d\bar{\theta}_E/d\theta_K=0$, the symmetric equilibrium is always stable. At the other extreme, when spillovers are purely local ($\lambda=0$), the symmetric equilibrium is never stable even without the demand linkage.[*]

Finally the critical level of ϕ at which the symmetric equilibrium becomes unstable is defined by the point where (3-1) switches sign, namely $\partial\bar{q}/\partial\theta_K=0$. This expression is

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[*]This result is reminiscent of the Grossman and Helpman (1991 chap.8) 'hysteresis in growth' result.

quadratic in ϕ, so it has two roots. The economically relevant one is[17]:

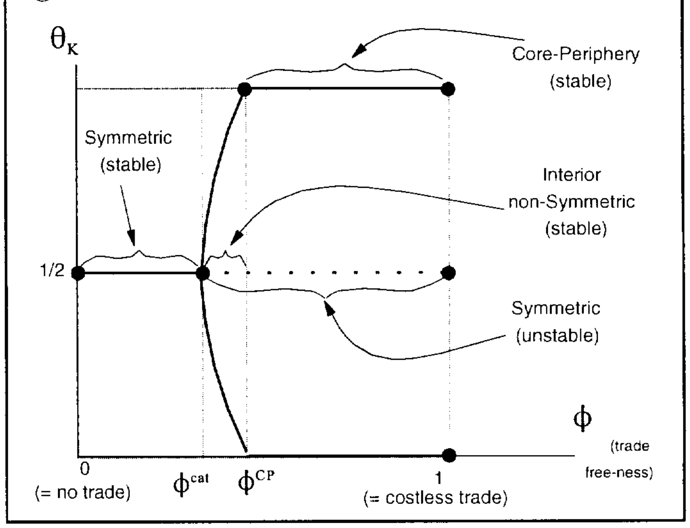
$$\phi^{cat} = \frac{[L(1+\lambda)+\rho] - \sqrt{(1-\lambda^2)[L(1+\lambda)+\rho]^2+\lambda^2\rho^2}}{\lambda[L(1+\lambda)+2\rho]} \tag{3-2}$$

Observe that the range unstable ϕ's, $(\phi^{cat},1]$, gets smaller as the internationalisation of learning effects (as measured by λ) increases. Since the growth-linkage becomes weaker as λ rises, it is easy to understand why the range of trade costs that leads to instability shrinks as λ rises. Additionally the instability set expands as the discount rate ρ rises since this amplifies the demand linkage. That is, the equilibrium return to capital rises with ρ, so a higher ρ amplifies the expenditure shifting that accompanies production shifting.

3.2 Stability of Other Steady States

The stability test for the core-periphery equilibrium case is slightly different since the core-periphery outcome entails $\bar{\theta}_K=1$, $\bar{q}=1$ and $\bar{q}^*<1$. The procedure, therefore, is to find the range of ϕ where $\bar{q}^*<1$, when $\bar{\theta}_K=1$. Since this is exactly the procedure used in Section 2 to determine the range of the core-periphery, we see that the core-periphery steady state is stable wherever it exists.

Figure 4



To examine the stability of the interior-non-symmetric steady state, we adopt the same procedure. Namely, we study $\partial\bar{q}/\partial\theta_K$ evaluated at $\bar{\theta}_K$ given by (2-10). Given the complex nature of $\partial\bar{q}/\partial\theta_K$ and (2-10), we cannot sign the derivative analytically. However, for reasonable values of the parameters the derivative is negative.[18] This finding is robust to sensitivity analysis on parameter values.

Finally, we turn to showing that there is no overlap in the zones of stability. With some difficulty, it is possible to show that $\phi^{CP}>\phi^{cat}$ and that the endpoints of the interior-non-symmetric steady state are exactly equal to ϕ^{cat} and ϕ^{CP}.[19] Using these results, Figure 4 summarizes the model's stability properties in a diagram with ϕ and θ_K on the axes. More precisely, as already argued, 'stable' corresponds to a saddle while .'unstable' correponds to an unstable node/focus. The fact that interior-non-symmetric steady states must correspond to saddles follows from continuity arguments as the dynamical system undergoes a supercritical pitchfork bifurcation when ϕ crosses ϕ^{cat}: the symmetric steady state loses its stability to the two new neighboring steady states.

4. Three Stages of Growth

As history would have it, the cost of doing business internationally has declined sharply since the 18th century. While this trend seems obvious and irreversible with hindsight, it was not obviously predictable in advance, nor was it monotonic. During many periods, and sometimes for decades at a time, the trend was reversed. From 1929 to 1945, for example, international trade became increasingly difficult. Restoration of peace and the founding of GATT allowed the trend to resume, but this was not a foregone conclusion in, say, 1938.

Following Krugman and Venables (1995), this section considers the implications of lowering the cost of trade (as captured by the parameter τ). To keep the analysis as sharp as possible, we take prohibitive trade costs as our initial condition.

4.1 Location and Trade Costs: A Punctuated Equilibrium

When trade costs are high the symmetric equilibrium is stable and gradually reducing trade costs ($d\phi>0$) produces standard, static effects – more trade, lower prices, and higher welfare (more on this below). There is, however, no impact on industrial location, so during an initial phase, the global distribution of industry appears unaffected by ϕ.

As trade free-ness moves beyond ϕ^{cat}, however, the world enters a qualitatively distinct phase. The symmetric distribution of industry becomes unstable, and northern and southern industrial structures begin to diverge; to be concrete, assume industry agglomerates in the north. If θ_K could jump, it would be on the interior-non-symmetric equilibrium (shown as the CC locus in Figure 4). Since CC is vertical at ϕ^{cat}, the impact on location would be catastrophic. That is to say, an infinitesimal change in trade costs would produce a discrete change in the steady-state global distribution of industry.

Since θ_K cannot jump, crossing ϕ^{cat} triggers transitional dynamics in which northern industrial output and investment rise and southern industrial output and investment fall. Moreover, in a very well defined sense, the south would appear to be in the midst of a 'vicious' cycle driven by backward (demand) linkages and forward (cost) linkages. The demand linkages would have southern firms lowering employment and abstaining from investment, because southern wealth is falling, and southern wealth is falling since southern firms are failing to invest. The cost linkages would lead to an incease in the cost of southern investment/innovation relative to the north as θ_K rises (due to localized learning externalities), and to an increase in θ_K since the cost of southern investment/innovation rises relative to the north. By the same logic, the north would appear to be in the midst of a 'virtuous' cycle. Rising θ_K would expand the north's relative market size and reduce its relative cost of investment/innovation.

Although we cannot analytically characterize the transitional dynamics of a system with three non-linear differential equations, we can say that a continuing rise in trade free-ness would raise θ_K until the core-periphery outcome is the only stable long-run equilibrium. Of course, southern knowledge never disappears entirely, so the core-periphery outcome is only reached asymptotically (the number of southern varieties remains fixed, but the value of these drops forever towards zero due to the ceaseless introduction of new northern varieties).

Once the core-periphery outcome is reached – or more precisely, once we can approximate θ_K as unity – the world economy enters a third distinct phase. For trade costs lower than this point, the world economy behaves as it did in the first phase. That is to say, making trade less costly has the usual static effects, but no location effects.

Plainly, the location equilibrium in this world would appear as a punctuated equilibrium. In the first and third phases, lower trade costs have no impact on the distribution of world industry, but in the second phase, the north's share of world industry increases rapidly.

Our paper focuses mainly on the three phases described above, however, the model can also generate a fourth stage in which the south industrializes. The key to this fourth stage is to suppose that $d\phi$ slows as it approaches some natural upper bound, but international integration continues in the form of a rise in the internationalization of knowledge spillovers – namely $d\lambda>0$. Since $d\lambda>0$ weakens agglomeration forces (by facilitating technology transfers), we can identify a critical value of λ beyond which core-periphery outcome becomes unstable and the symmetric outcome is stable. As we shall see, the emergence of southern industry slows global growth somewhat and forces a relative de-industrialization of the north.

4.2 Growth Stages

Long-run growth in this model is driven by the ceaseless accumulation of knowledge capital resulting in an ever greater range of M-varieties. Given preferences, this ceaseless expansion of variety raises real consumption continually. While technology and output in the traditional sector is stagnant, the expansion of M-varieties forces up the price of T relative to that of the composite good C_M. The value of the two sectoral outputs thus grows in tandem.

4.2.1 Stage-One's Growth and Investment Rates

By definition, the initial interior solution entails symmetry, i.e., $\bar{\theta}_E=\bar{\theta}_K=1/2$ and, as long as $\phi<\phi^{cat}$, this outcome is stable. The steady-state rate of K accumulation during this phase is found using the expression for $\bar{L}_I$ from (2-12) in (2-2), to get:

$$\text{Stage } I: \qquad \bar{g}=\bar{g}^{*}=\frac{\alpha(1+\lambda)L-\rho(\sigma-\alpha)}{\sigma} \tag{4-1}$$

This common rate of K-accumulation is unaffected by the level of trade costs, ϕ.

Steady-state growth in real income is nominal Y divided by the perfect consumption price index, P. Given preferences, and $p_z=p_z^*=1$, the perfect price index is P_M^{α}, where P_M is the CES price index[20]. Thus P and P* are:

$$P=K^{w\frac{\alpha}{1-\sigma}}\left[\theta_K+\phi(1-\theta_K)\right]^{\frac{\alpha}{1-\sigma}}, \qquad P^{*}=K^{w\frac{\alpha}{1-\sigma}}\left[\phi\theta_K+1-\theta_K\right]^{\frac{\alpha}{1-\sigma}} \tag{4-2}$$

In steady state, nominal Y and θ_K are time-invariant, yet the P's falls on the steady-state growth path since K^w rises at the common rate of $\bar{g}$. Thus P_M falls at $\bar{g}/(\sigma-1)$ and real income grows at $\alpha\bar{g}/(\sigma-1)$. Using (4-1):[21]:

$$\textit{Stage I}: \quad \bar{g}_{income} = \bar{g}'_{income} = \frac{\alpha^2(1+\lambda)L - \rho\alpha(\sigma-\alpha)}{\sigma(\sigma-1)} \qquad \textbf{(4-3)}$$

where g_{income} is the real income growth rate. By inspection, the growth rate rises with λ and α, but falls with ρ and σ (all of which are standard results in the trade and endogenous growth literature). Again, trade costs do not play a role as long as the economy remains at the symmetric equilibrium.

Consider next, the rate of investment, which plays a central role in Rostow's stages-of-growth approach. With labour as numeraire, the rate of investment in steady state is $\bar{L}_I/\bar{Y}$. Using (2-12) and the definition of $\bar{Y}$, we have[22]:

$$\textit{Stage I}: \quad \frac{\bar{L}_I}{\bar{Y}} = \frac{\alpha(1+\lambda)L - \rho(\sigma-\alpha)}{(\sigma+\alpha)(1+\lambda)L + \alpha\rho} \qquad \textbf{(4-4)}$$

Again, the ratio is rising in λ and α, falling in ρ and σ, and unaffected by ϕ.

These results are simple to establish since stage-one is a steady state. The stage-three growth rate is similarly simple to establish, so we turn to it next.

4.2.2 Stage-Three's Growth and Investment Rates

Once the stage-three steady state is reached (or at least when θ_K is close enough to unity to approximate the steady-state θ_K as equal to unity), (2-2) and (2-15) imply:

$$\textit{Stage III}: \quad \bar{g} = \frac{\alpha 2L - \rho(\sigma-\alpha)}{\sigma} \qquad \textbf{(4-5)}$$

Importantly, this $\bar{g}$ exceeds the stage-one $\bar{g}$ only to the extent that spillovers are localized, i.e. $\lambda<1$. The common real income growth rate, viz. $\bar{g}\alpha/(\sigma-1)$ where $\bar{g}$ is given by (4-5), is also higher than the stage-one growth rates (as long as $\lambda<1$). Observe that the south – which is completely specialized in the traditional sector – engages in no innovation, and indeed makes no investment of any kind. Nevertheless, the south experiences the same rate of growth as the north due to continual terms of trade gains: the price of T (which south exports) is time-invariant but the price of C_M (which south imports) falls.

The stage-three northern investment ratio is:

$$\text{Stage III}: \quad \frac{\bar{L}_I}{\bar{Y}} = \frac{\alpha 2L - \rho(\sigma-\alpha)}{(\sigma+\alpha)L + \alpha\rho/(1+\lambda)} \qquad \textbf{(4-6)}$$

This is greater than that of the first stage even with λ=1 since all investment/innovation occurs in the north (or which ever nation acquires the core).

Finally, notice that while further reduction of ϕ raises both nations' real income trajectories (via one-off drops in the perfect price index), liberalization has no affect on the common slope of their growth paths.

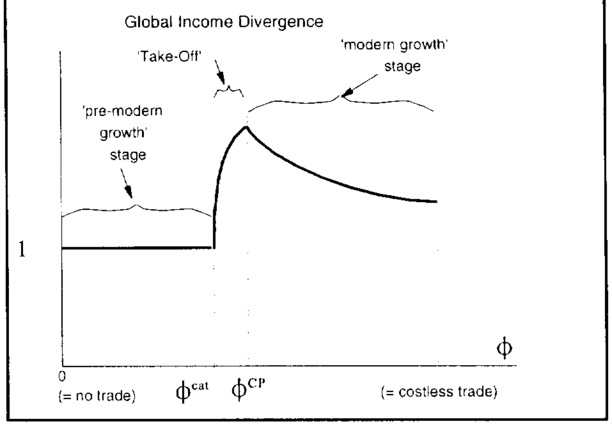
4.2.3 Stage-Two: Growth and Investment During the Take-Off

During the take-off stage, the world economy is in transition between steady states. Characterizing the economies' behaviour during such a phase is a truly difficult problem, since we must work with three nonlinear differential equations.[23] Be that as it may, it is clear that during the course of the takeoff, the rate of investment and economic growth will rise to that of the third stage. Thus two of Rostow's criteria are clearly met in the north: (1) the rate of productive investment (in human, knowledge, and/or physical capital) rises, and (2) one or more manufacturing sectors develop with a high rate of growth. The specialization induced by agglomeration also generates a rapid increase in trade. A generous reading of the model also includes some aspects of his third criteria since the "impulses to expansion in the modern sector and the potential external economy effects of the take-off acquire an on-going character". The third phase of our model, in common with Rostow's, is marked by a high and stable rate of economic growth as well as by a more stable sectoral composition of output.

4.3 Income Divergence

Figure 5 shows how the ratio of steady-state real income levels (north's divided by south's) varies with trade costs. As discussed above, we are unable to analytically characterise the transitional dynamics, so the figure approximates the actual predicted path by assuming that the system is always at the stable steady state

Figure 5



that corresponds to each level of trade free-ness. There are clearly three phases in the figure. In the first phase, trade cost reductions have no impact on this ratio. Per capita income levels are identical since $\bar{\theta}_E=\bar{\theta}_K=1/2$ and, by symmetry, northern and southern price indices are identical. In the second phase, where $\phi>\phi^{cat}$, industry begins to agglomerate in the north. This has two effects, both promoting income divergence. First, as $\bar{\theta}_K$ rises, northern steady-state wealth rises while southern steady-state wealth falls. Second, due the 'home market' effect (Venables 1987), the shift in industry location has a favourable impact on the northern price index and a dilatory impact on the south's price index. That is, as long as trade is not costless, southern consumers face higher consumer prices since all trade costs are passed on to consumers. In the final phase, some of the divergence is reversed. The reason is that although the north's wealth is higher than the south's, lowering trade costs reduces the difference in north and south price indices up to the point $\phi=1$, where the two price indices are identical.

4.4 Welfare

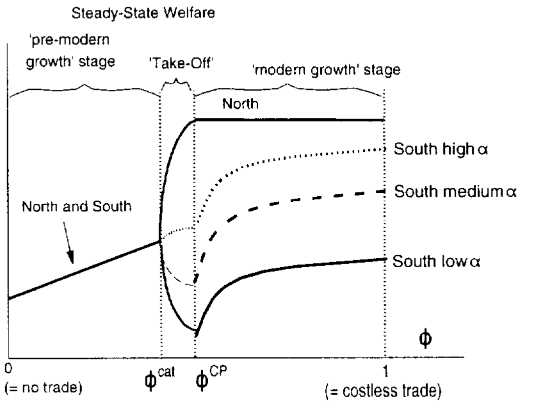
As pointed out above, our model is an example of uneven development in the sense that the agglomeration of industry in the north produces immediate divergence with the south. However, because agglomeration generates a take-off which materializes itself into an acceleration of the world rate of innovation, the take-off also produces benefits for the south. The tension between the negative effect of agglomeration and the positive effect of the increase in the rate of innovation is what makes the welfare effect of the take-off ambiguous for the south. Northern and southern steady-state welfare (i.e., the present value of the utility flows) as functions of ϕ are, respectively:

$$U=\frac{c_0\alpha\bar{g}}{\rho^2(\sigma-1)}\ln\left\{\left(L+\frac{\rho\bar{\theta}_K}{\bar{A}}\right)\left[\bar{\theta}_K+\phi(1-\bar{\theta}_K)\right]^{\frac{\alpha}{\sigma-1}}\right\},\quad U^*=\frac{c_0\alpha\bar{g}}{\rho^2(\sigma-1)}\ln\left\{\left(L+\frac{\rho 1-\bar{\theta}_K}{\bar{A}^*}\right)(\phi\bar{\theta}_K+1-\bar{\theta}_K)^{\frac{\alpha}{\sigma-1}}\right\} \qquad \textbf{(4-7)}$$

where $\bar{g}$ and $\bar{\theta}_K$ depend upon ϕ as described above, and c_o captures terms that do not depend upon ϕ. Notice that while a rise in $\bar{g}$ is welfare enhancing in both regions, raising $\bar{\theta}_K$ raises northern welfare, but lowers that of the south. As discussed above, the impact of raising the steady-state θ_K is twofold: it shifts wealth from south to north and it lowers the northern price index relative to the southern price index, as long as $\phi<1$.

Figure 6 plots the levels of welfare corresponding to the higher arm of the pitchfork represented in Figure 4. We have simulated these levels of welfare as a function of trade costs and found three generic cases (again considering only steady states). Two elements are constant in all cases. First, both regional welfare levels rise as ϕ rises (imports get cheaper) during stage-one (the pre-modern growth phase), and second the north's welfare is insensitive to ϕ in the final stage. Symmetry explains the first element and the fact that $\bar{\theta}_K=1$ in the final phase accounts for the second element. The cases differ only in the south's welfare level in the third phase. In particular, the levels are simulated for three different expenditure shares on manufactured goods, namely $\alpha=0.17$, $\alpha=0.3$, and $\alpha=0.9$; the other parameter values assumed, viz. $\lambda=0.7$, $\sigma=3$, $\rho=0.1$ and $L=1$, are common to the three cases.

Figure 6



Note first that when transaction costs are sufficiently high (ϕ is below the threshold level), a decrease in transaction costs has the usual static effects in both the south and the north. It raises welfare because it lowers the real price of traded manufactured goods. At the point of the take-off, north and south welfare diverge. The north benefits from agglomeration and a higher growth rate. The south benefits only from higher growth; agglomeration actually harms the south. This explains why post-take-off welfare is always lower in the south.

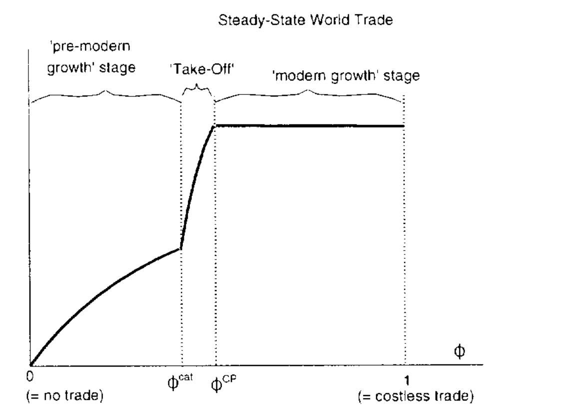
The positive growth effect of the take-off explains why the comparison of welfare before and after the take-off is ambiguous. If the share of manufacturing goods is low enough, the increase in the growth rate of the manufacturing sector does not have a large welfare impact. In this case, the south loses due to agglomeration and its welfare never reaches the level it had before the take-off. In the intermediate α case, the south first loses but eventually attains a welfare level that exceeds its pre-take-off level. Finally, when manufacturing is sufficiently high, the positive growth effect dominates and the take-off benefits both the south and the north. Similar results are obtained when we vary other parameters: the welfare impact is more favourable to the south the higher the growth effect of agglomeration, that is the larger the market size (the higher L), the more local the spillovers (the lower λ), the more the economies of scale (the lower σ) and the lower the subjective discount rate ρ.

Importantly, after the take-off, lowering transaction costs always improves welfare in the south because it decreases the price of goods imported from the north. Thus, even though the south may have been made worse off by agglomeration in the north, resisting further reductions in transaction costs is not welfare improving.

4.5 Trade

Finally, we study the expansion of trade during the three phases. The global trade volume from (2-16) is graphed in Figure 7 (using the same parameters as in the previous figure with α=0.3). Again there are three distinct phases. In the initial phase, the level of trade is fairly low and all trade is intra-industry trade. Furthermore, lowering trade costs promotes trade in a smooth, gradual fashion. Once $\phi>\phi^{cat}$, agglomeration occurs rapidly in the north, so the nature of trade shifts. The north becomes a net exporter of industrial goods and a net importer of traditional goods. Once all industry is in the north, the value of trade is maximized since the south must satisfy all its demand for industrial goods via imports.

Figure 7



4.6 Globalization and Industrialisation of the South

While the radical income disparity between poor and rich countries is still a dominant feature of the global economy, the decades since WWII have also seen some spectacular examples of rapid convergence – what Lucas (1993) calls 'miracles'. Here we show that our model can produce a 'miracle' in the south (with a two region model a miracle in the south produces full income convergence). The key is to take a broader view of international integration.

Up to this point, we have viewed integration as nothing more than the lowering of trade costs. Yet much of the post-war integration, especially that of the past two or three decades, has lowered the cost of 'transporting' ideas more than it has lowered the cost of transporting goods. Due to unprecedented improvement in international communications, the relative cost of virtually all forms of communications – everything from the price of air travel to telephone calls – has fallen substantially in recent years. This trend is strengthened by the development of new communication technologies such as faxes, videoconferencing, overnight courier services, e-mail, etc.

In the context of our model, a decrease in the cost of communications and more generally an increase in the speed of international diffusion of ideas is translated into an increase in λ, which measures the internationalization of knowledge spillovers in the I-sector. To focus sharply on this trend in the relative cost of trading goods and ideas, we make the simplifying assumption that all recent integration consist of rising λ. That is, we start from the stage-three situation of full agglomeration in the north and suppose that trade free-ness ϕ has risen to some natural upper bound, but in a fourth stage λ rises towards unity, i.e. perfect international transmission of learning externalities.

Starting from a situation with full industrial agglomeration in the north ($\bar{\theta}_K=1$), the increase in λ initially has no impact on southern industry or on the global growth rate given by (4-5). However, southern I-sector labour productivity rises with λ, so at some threshold level of λ (call this λ^{mir} for 'miracle'), the steady-state q* exceeds unity. Beyond this, southern M firms find it profitable to invest in new ideas/varieties. The λ that sets $\bar{q}^*=1$ for a given ϕ defines the critical level of λ beyond which the core-periphery outcome is no longer stable. Using (2-13) this level is:

$$\lambda^{mir} = \frac{\phi(2L+\rho)}{L(1+\phi^2)+\rho\phi^2} \qquad \textbf{(4-8)}$$

Clearly λ^{mir} rises with the free-ness of trade.[24]

As in the case of falling trade barriers, there is a second critical value of λ where the symmetric equilibrium becomes stable. This value, denoted as $\lambda^{mir'}$ is the level of λ where $\partial\bar{q}/\partial\theta_K$ evaluated at $\theta_K=1/2$, i.e. (3-2), becomes negative. (3-1) is quadratic in λ and the economically relevant solution defines $\lambda^{mir'}$.

As with the north's take-off, the miracle in the south would appear to be driven by two virtuous circles. When the south invests, its capital stock and therefore permanent income begins to rise, triggering demand-linked circular causality. Rising local expenditures boosts southern profits and this in turn gives a new incentive to innovate/invest. Moreover, as K* rises, the southern I-sector begins to benefit from localized learning externalities, and this triggers cost-linked circular causality. The net effect is a drastic structural change as the south industrializes. During the transitional phase north and south real incomes converge.

The miracle in the south, however, differs from the initial northern take-off in three ways. First, the southern industrialization does not shut down northern innovation. It merely forces a shift of some northern resources from the M-sector to the T-sector (here we think of the T-sector as including services as well as agriculture). Second, the source of the south's take-off is quite different. The miracle occurs due to the south's ability to learn from the north's experience in innovation rather than trade openness *per se*. This is consistent with the account of Rodrick (1995) on the success of the Asians dragons. Finally, the convergence of the south pushes the global steady-state growth rate to a level that is in-between the stage-one (pre-Industrial Revolution) rate and the stage-three (rich-north-poor-south) rate. In particular, defining stage-four as the phase where $\bar{\theta}_K$ has returned to 1/2, the stage-four rate of innovation is:

$$\textit{Stage IV}: \qquad \bar{g} = \bar{g}^{*} = \frac{\alpha(1+\lambda^{mir'})L - \rho(\sigma-\alpha)}{\sigma} \tag{4-9}$$

By inspection this is higher than stage-I rate since $\lambda^{mir'}$ exceeds the λ in (4-1). But it is lower than the stage-three rate in (4-5) since $\lambda^{mir'} < 1$ (assuming that natural barriers prevent λ from reaching unity).

5. Extension: Forward and Backward Linkages

Many of the early growth scholars, including Rostow, had detailed ideas about how growth spreads across sectors. Rostow, for instance, quite explicitly discusses linkages that we would recognize as backward and foward linkages. Including a full set of input-output relationships would render the model analytically intractable, but one particularly important set of backward and forward linkages – those affecting the I-sector – are straightforward to include, as Martin and Ottaviano (1996b) have shown. We turn now to extending the basic model in this direction. The main change is to allow for pecuniary externalities in the I-sector (via traded intermediate inputs into capital production) as the source of localized externalities. As we shall see, this enriches the stage-one growth behaviour of the model, producing a 'soft' take-off.

5.1. Modifications to the Basic Model

Including traded intermediates in the I-sector requires only one significant modification. Instead of (2-1), the modified I-sector production and cost functions assume that new capital is produced directly from consumption goods (as in the Solow model) according to:

$$\dot{K} = \frac{Q - Q_C}{a_I} \quad ; \quad a_I = \frac{1}{(K+K^*)^{\xi}} \quad ; \quad F = p_T^{1-\alpha} P_M^{\alpha} a_I \qquad \textbf{(5-1)}$$

where Q_C is the amount of the composite good Q from (2-3) consumed and F is the marginal cost of capital. A generic condition for steady, endogenous long-run growth is that Tobin's q must be independent of the level of capital stocks. The dual of this is that F must fall at the same rate as V falls. Given Dixit-Stiglitz pricing, V falls at the rate that K grows, so F must fall at the same rate. The implied regularity condition is that $\alpha/(1-\sigma)-\xi=-1$.[25]

Notice that we have set $\lambda=1$ in (5-1). This permits us to isolate the effects of including traded intermediates in the I-sector because, as Section 4 showed, agglomeration had no growth effect when $\lambda=1$.

All L and K are now employed in producing Q, so:

$$\pi = \frac{\alpha Y^w}{\sigma K^w} B \qquad \textbf{(5-2)}$$

where Y^w is world income (equal to $2L+\pi K+\pi^* K^*$); B is as in (2-5) with $\theta_Y \equiv Y/Y^w$ replacing θ_E. Due to markup pricing $Y^w=2L/(1-\alpha/\sigma)$, so the international partition of income, θ_Y, depends upon θ_K according to[26]:

$$\theta_Y = \frac{\sigma-\alpha(1-2B\theta_K)}{2\sigma} \tag{5-3}$$

In steady state, π falls at the rate that K^w grows, so the steady state V's are given by (2-7). The steady state q is therefore[27]:

$$q = \frac{\alpha 2L}{(\rho+g)(\sigma-\alpha)}B\Delta \ ; \qquad \Delta \equiv \left[\theta_K+\phi(1-\theta_K)\right]^{\frac{\alpha}{\sigma-1}} \tag{5-4}$$

and q* is given by a similar expression.

5.2 Stability of Steady States

Stability of the symmetric steady state is investigated as in Section 3. Holding L_I constant, the proportional change in q with respect to θ_K can be written as:

$$\left(\frac{dq/q}{d\theta}\right)\Bigg|_{\theta_K=\frac{1}{2}} = 2\left(\frac{1-\phi}{1+\phi}\right)^2\left[-1+\frac{4\alpha\phi}{\sigma(1-\phi^2)-\alpha(1-\phi)^2}+\left(\frac{1+\phi}{1-\phi}\right)\frac{\alpha}{\sigma-1}\right] \tag{5-5}$$

Again, there is one stabilizing force (the local competition effect shown in the first term in large parantheses) and two destabilizing forces (the last two terms). The first of the destabilizing terms corresponds to the demand link that stems from the expenditure shifting impact of production shifting.[28] The last term reflects the cost-link stemming from the way in which production shifting (i.e. $d\theta_K>0$) lowers F and raises F* via the variety linked cost effect.[29] That is, an increase in the share of firms producing in the north lowers the northern I-sector's marginal cost by lowering the cost of intermediate inputs. This in turn increases the northern accumulation rate, and raises θ_K. Notice that as ϕ approaches unity, the stabilizing force approaches zero faster than the destabilizing forces, so for some ϕ sufficiently close to unity, the symmetric equilibrium is unstable. Notice also that even when trade costs are prohibitive (ϕ=0), the symmetric equilibrium may be unstable when $1<\alpha/(\sigma-1)$. Intuitively, this implies that the cost-linkages in the I-sector must be very strong. In what follows, we assume that this condition does not hold, so there is always a segment of [0,1] for which the symmetric equilibrium is stable.

Setting (5-5) to zero and solving for ϕ, we find that stability of the symmetric equilibrium is assured for ϕ below a critical value ϕ^{cati}:

$$\phi^{cati} = \frac{\sigma-\alpha}{\sigma+\alpha}\left[\frac{1 - \alpha/(\sigma-1)}{1 + \alpha/(\sigma-1)}\right] \tag{5-6}$$

Following the same procedure for checking the stability of the core-periphery equilibrium, we evaluate q* at θ_K=1 to get:

$$(q^*)\big|_{\theta_K=1} = \frac{(\sigma+\alpha)\phi^2 + \sigma-\alpha}{2\sigma}\,\phi^{-\left(\frac{\sigma-1-\alpha}{\sigma-1}\right)} \tag{5-7}$$

and find the range where this q* is less than unity. Since (5-7) involves a non-integer power, we cannot analytically find the critical value of ϕ where this q*=1. Numerically, however, ϕ^{CPi} is simple to find. Importantly, ϕ^{CPi}, is always less than ϕ^{cati}, although the difference disappears as σ gets large. This finding implies that for some range of trade costs, both the symmetric and core-periphery outcomes are stable. The stability properties of this modified model therefore resemble more closely the ones of the standard economic geography models. That is, there is no interior, non-symmetric steady state (as was the case in the basic model).

5.3 Growth Stages

The modified model has three growth stages as does the Section 2 model, however there are two important differences. In the modified model, the catastrophe is much larger in the sense that once the symmetric equilibrium becomes unstable, the only stable equilibrium is the core-periphery outcome. The second difference is that even though there are no local technological spillovers (since we assumed λ=1), we still get a growth take-off because of the pecuniary externalities.

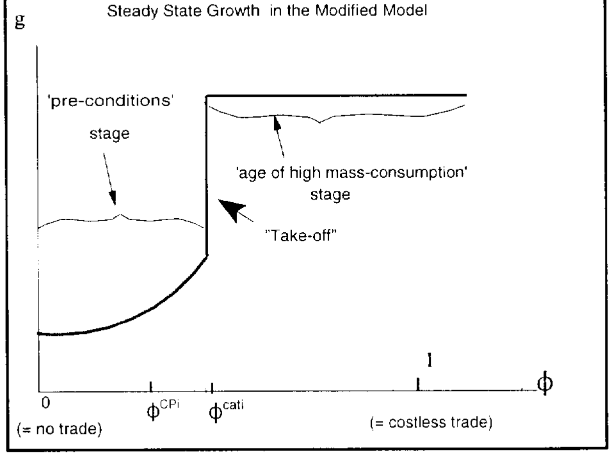
To see this, consider the stage-one growth rate of K. Solving (5-4) for g, using the fact that θ_K=1/2 and B=1, $g^{sym}=[\alpha 2L\Delta/(\sigma-\alpha)]-\rho$. Since Δ is rising in ϕ at θ_K=1/2, we see that g rises as trade costs fall, even in stage-one where θ_K is invariant. This brings out the new elements in this modified model; it contrasts with the Section 2 model, where g was not directly affected by ϕ. Intuitively, the point is that given (5-1), the marginal cost of new K (i.e. F) falls, as transaction costs decrease.[30] This leads to an incipient rise in each nation's q, and thereby draws more resources into the I-sectors. Faster K growth (and therefore real income growth) is the result.

The model enters stage-two, the take-off, once ϕ gets high enough to trigger a collapse to the core-periphery outcome. Since θ_K cannot jump, there must be a phase when θ_K is moving from 1/2 to 1. As before, we cannot formally characterize growth during this transitional phase.

As in the first model, θ_K reaches unity only asymptotically. This means that the third stage of growth when θ_K equals one is only reached asymptotically. Of course at that point $B=\Delta=1$, so the stage-three steady-state growth rate of K is $g^{CP}=[\alpha 2L/(\sigma-\alpha)]-\rho$. This exceeds g^{sym} since $\Delta \leq 1$ at $\theta_K=1/2$.

Figure 8

Steady State Growth in the Modified Model



To summarize, Figure 8 schematically shows the growth rates in the various phases. Figure 8 plots the growth rates corresponding to the symmetric outcome as long as it is a saddle (ϕ below ϕ^{cati}) and to the core-periphery outcome afterwards (ϕ above ϕ^{cati}). When ϕ crosses ϕ^{cati}, g will asymptotically reach the core periphery level. In the modified model the dynamic system undergoes a subcritical (rather than a supercritical as in the previous model) pitchfork bifurcation as ϕ crosses ϕ^{cati}: when the symmetric steady state loses stability, the core-periphery steady state becomes the only stable outcome.

6. Concluding Remarks

From 1750 to 1850, the world experienced dramatic economic changes. Global commerce expanded rapidly and the now-rich nations (the 'north') experienced massive structural shifts from agriculture to industry. The Industrial Revolution is also the starting point of the process of global divergence. Moreover, the nature of economic growth was irreversibly altered in this era. Before the Industrial Revolution, a Malthusian logic seemed to ensure stagnant per-capita-income levels worldwide. After the Industrial Revolution, per capita incomes, especially in the north, came to be governed by a Schumpeterian logic in which a self-sustaining cycle of investment, innovation and higher output permitted ever-rising incomes.

This paper presents a parsimonious model consistent with these facts and which attempts to capture some of the ideas of classic authors such as Braudel, Kuznets and Rostow. In our model, economic geography, trade, global income divergence and stages of growth are jointly endogenous. As is common in economic geography models, the equilibrium location of industry is marked by a punctuated equilibrium. That is, the gradual and exogenous reduction of trade costs results in a three-stage location equilibrium. In the first stage, where trade costs are high, industry is evenly scattered among similar nations. The world economic growth rate stagnates, since dispersed industrial production hinders exploitation of localized knowledge spillovers. In the second stage – the take-off – the economies are in transition. Trade costs have fallen to the point where the symmetric location equilibrium becomes unstable and a core-periphery equilibrium is emerging. During this transition, the global growth rises from the low first stage rate to the higher third stage rate, but we observe massive divergence in real incomes. In the final stage, both nations grow at a common rate that is higher than the stage-one rate. During this third stage, the real income gap narrows, but is not closed. Broadly, these findings are consistent with four key phenomena mentioned in the introduction, namely northern industrialization and growth take-off, rapid trade expansion and income divergence.

The main focus of this paper is on these four key phenomena, however, the model is sufficiently rich to produce eventual southern industrialization. The source of this industrialization is a lowering – in a fourth stage – of the relative costs of transporting ideas versus goods. This enhances the south's ability to learn from the north's experience in innovation, thereby weakening the forces that supported total agglomeration in stage-three. We showed that this emergence of southern industry slows global growth to point between stage-one (pre-Industrial Revolution) and stage-three (core-periphery) levels. It also forces a relative de-industrialization in the north.

Our model also has some interesting and intriguing political economy implications. The most obvious would, at first glance, appear to support notions of 'inequalizing trade'. In our model, the big divergence between rich and poor countries is a necessary implication of

Europe's Industrial Revolution and the expansion of international trade triggered both. Indeed, the creation of a global core-periphery situation is a necessary condition for the growth take-off. Moreover, our paper describes a purely economic mechanism (i.e. other than cultural or political) that explains why the Industrial Revolution did not spread to what we now call the Third World. The traumatic experience of massive de-industrialization in India during the 19th century is consistent with our model and it also suggests one reason why this country, along with most other Third World countries, kept an attitude of distrust towards international trade.

Our model, however, departs sharply from the 'inequalizing trade' paradigm on several key points. First, we showed that the present value of the south's welfare could have been increased the Industrial-Revolution-*cum*-income-divergence. That is, the south could be even poorer than it is today, had the Industrial Revolution not occurred. Second, once a north-south structure has been generated, further trade liberalization narrows the real income gap. Third, our model posits that income divergence was caused by lower trade costs, not by plunder or imperialism. Fourth, in our model, trade liberalization and more generally the reduction in transaction costs first generates massive divergence of real incomes but then is conducive to a process of relative convergence. Finally, we show that to the extent the recent decades of international integration have lowered the cost of trading ideas more than it has lowered the cost of trading goods, integration can be the key to southern industrialization.

Finally, our model may also be taken as providing a long-term perspective on the convergence literature (see Barro and Sala-i-Martin (1992) inter alia). That literature essentially takes the 19th century global income disparity as given and seeks to measure whether this gap has narrowed in the postwar period. Our model attempts to analyse the long term origin of the divergence between North and South by linking it explicitely to the growth take-off of the Industrial Revolution.

Appendix: Formal Stability Analysis of Section 2 (Symmetric Steady State)

This appendix uses standard stability tests involving eigenvalues. As in Baldwin (1997) and Baldwin and Forslid (1997b), we find that the results generated from the more intuitive approach in the text are identical to those of more formal techniques.

It proves convenient to take as state variables, E, E* and θ_K, with the two Euler equations and (2-9) as the system equations. Using $r=\pi/F+\dot{F}/F$, where $\dot{F}/F=-L_I-\lambda A^*L_I^*/A$, and $g=L_IA/\theta_K$, we can express the differential equations in terms of L_I, L_I^* and θ_K. Using the definition of E and E*, namely $E=L-L_I+\alpha BE^w\theta_K/\sigma$ and an analogous expression for E*, as well as $E^w=(2L-L_I-L_I^*)/(1-\alpha/\sigma)$, we can express the L_Is in terms of E's and the two θs. Finally, using $\theta_E=E/(E+E^*)$, we can express the system equations in terms of the state variables: E, E* and θ_K.

Linearizing the system around the symmetric steady state, the eigenvalues of the resulting Jacobian matrix are:

$$e_1 = L(1+\lambda)+\rho; \; , \quad e_{2,3} = \frac{b \pm \sqrt{b^2-4c\sigma(1+\lambda)}}{\sigma(1+\lambda)} \; ; \tag{A-1}$$
$$b \equiv \sigma[e_1(1-\lambda)+2\lambda\rho] + 4\frac{\alpha\phi(1-\lambda\phi)e_1}{(1+\phi)^2} \; , \quad c \equiv -2\alpha e_1\left(e_1[\lambda-2\frac{\phi(1+\lambda)}{(1+\phi)^2}] - \frac{\lambda\rho(1-\phi)}{1+\phi}\right)$$

The first eigenvalue is plainly positive. By inspection, given that $c<0$ at sufficiently low levels of ϕ, we know that at least in this range of trade costs the eigenvalues are all real. In this case, given the positive radicals in e_2 and e_3 are positive, it follows that the eigenvalue that adds the radical – call this e_2 – is always positive. The third eigenvalue changes sign at the point where c=0.[31] Solving this for ϕ, we get exactly (3-2).

Thus in the neighbourhood of the symmetric system, the linearized system has two positive and one negative real roots for $\phi<\phi^{cat}$. This makes it saddle path stable, because only one of the state variables is a nonjumper. For $\phi>\phi^{cat}$ we have three positive eigenvalues and therefore an unstable steady state.

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Supplemental Guide to Calculations:

Baldwin, Martin & Ottaviano "The geography of Growth Take-offs"

1. Note that K's rental rate π is the operating surplus (i.e. the Ricardian surplus). This is due to the fact that each unit of K is variety specific, so K's reward is like that of a specific factor.

2. This follows from perfect competition, constant returns and the choice of units in the T sector.

3. Using the demand functions, the flow of operating profit is:

$$\left(\frac{p_j^{1-\sigma}\alpha E}{\Delta} - wa_M \frac{p_j^{-\sigma}\alpha E}{\Delta} \right) + \left(\frac{p_j^{*\,1-\sigma}\alpha E}{\Delta^*} - (wa_M\tau)\frac{p_i^{*\,-\sigma}\alpha E}{\Delta^*} \right)$$

$$\Delta = \int_{i=0}^{K+K^*} p_i^{1-\sigma} di \ , \quad \Delta^* = \int_{i=0}^{K+K^*} p_i^{*1-\sigma} di$$

The Dixit-Stiglitz monopolistic competitors take other firms' prices as given, so the first-order conditions for a typical variety are:

$$(1-\sigma)c_j - wa_M(-\sigma)\frac{c_j}{p_j} = 0 \ , \quad (1-\sigma)c_j^* - (w\tau a_M)(-\sigma)\frac{c_j^*}{p_j^*} = 0$$

where c_j and c_j* are sales to consumers in the local and export markets (for a typical variety). Using a_M=1-1/σ, simplification of these yields the pricing rules in the text.

4. To see this, consider the first order conditions for a typical variety (dropping the variety subscript for convenience):

$$p(1-\frac{1}{\sigma}) = wa_M \ , \quad p^*(1-\frac{1}{\sigma}) = w\tau a_M$$

Rearranging gives us operating profit per final sale:

$$p - wa_M = \frac{p}{\sigma} \ , \quad p^* - w\tau a_M = \frac{p^*}{\sigma}$$

so total operating profit is

$$(p - wa_M)c + (p^* - w\tau a_M)c^* = \frac{pc}{\sigma} + \frac{p^*c^*}{\sigma}$$

5. Using a cumbersome, but precise notation, northern per-variety sale at consumer prices is $s_N^N E^N + s_N^S E^S$, where subscripts indicate region of origin and superscripts indicate markets. Plugging the optimal pricing rules p=1, p*=τ into the formula for a variety's market share:

$$s_N^N = \frac{1}{K + K^*\phi} , \quad s_N^S = \frac{\phi}{\phi K + K^*}$$

where $\phi = \tau^{1-\sigma}$. Utilizing the share formulas in the expression for operating profit (see previous note), we have (with some rearrangement):

$$\pi_N = \left(\frac{\alpha E^w}{\sigma K^w}\right)\left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{\phi(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right)$$

where $\pi_N \equiv \pi$ is northern operaring profit (for a typical variety), $\theta_E = E/E^w$ and the superscript 'w' indicates world totals. A similar procedure yields an expression for $\pi_S \equiv \pi^*$.

6. Specifically, π^* equals $B^*(\alpha E^w/K^w)$, where

$$B^* \equiv \left(\frac{\phi\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right)$$

7. Since the model has only one primary factor L, expenditure allocation is tantamount to resource allocation, so labour is the natural numeraire. Also the I-sector technology indicates that L_I, L_I^* and θ_K are the determinants of the regional rates of K accumulation, so these are the natural state variables.

8. See Baldwin and Forslid (1997a) for more details on why $\dot{E} = 0$ in steady state.

9. From (2-7) and the facts that $\dot{E} = \dot{\theta}_K = \dot{\theta}_E = 0$ in steady state, B is time invariant in steady state, so π falls at the rate K^w grows. Using the steady-state discount rate ρ, we have:

$$V = \pi_o \int_0^\infty e^{-(\rho+g)t} dt$$

Solution of the integral yields the result in the text.

10. By definition expenditure is income less investment, i.e. $L - L_I + \pi K$ and we wish to show that this equals $L + \rho\theta_K/A$ in steady state. This boils down to showing that $\pi K = L_I + \rho\theta_K/A$. From $\bar{q} = 1$, $\pi = F(\rho + g)$. Multiplying through by K and using growth rate form of the I-sector production function yields the result after some manipulation.

11. To see this, note that at the symmetric steady state, K times V must equal K times F since q=1 implies V=F. Using the definition of F, $KF = K/(K + \lambda K^*) = \theta_K/[\theta_K + \lambda(1-\theta_K)]$. The definition of A yields the result. The south steady-state wealth is derived similarly.

12. Given (2-1), (2-7), and (2-10):

$$q^* = \frac{\alpha E^w B^* A^*}{\sigma(\rho + g)}$$

With $\theta_K = 1$, we have that $E^w = 2L + \rho$, $\theta_E = (L + \rho)/E^w$ and $A^* = \lambda$. Also $B^* = \phi\theta_E + (1-\theta_E)/\phi$, so using $\rho + \bar{g} = \bar{\pi}/F$ with $\bar{q} = 1$ (in the core-periphery steady state), we have:

$$q^* = \lambda \frac{(1+\phi^2)L + \phi^2\rho}{(2L+\rho)\phi}$$

13. Solving for the ϕ where q*=1 yields two solutions:

$$\frac{2L+\rho \pm \sqrt{(2L+\rho)^2 - 4\lambda^2 L(L+\rho)}}{2\lambda(L+\rho)}$$

We wish to show: (1) the roots are both real, (2) the root listed in the text is bounded between zero and unity, (3) the other root always exceeds unity.

To show (1), note that the term under the radical is minimized when λ=1, yet in this case it is $\rho^2>0$. To show (2), we need to demonstrate that:

$$0 < \frac{2L+\rho - \sqrt{(2L+\rho)^2 - 4\lambda^2 L(L+\rho)}}{2\lambda(L+\rho)} < 1$$

the left-hand inequality holds since $4\lambda^2 L(L+\rho)>0$. To see that the right-hand inequality holds, we rearrange it to see that it holds if and only if:

$$2L+\rho - 2\lambda(L+\rho) < \sqrt{(2L+\rho)^2 - 4\lambda^2 L(L+\rho)} \quad <=>$$

$$(2L+\rho)^2 + 4(\lambda(L+\rho))^2 - 4\lambda(L+\rho)(2L+\rho) < (2L+\rho)^2 - 4\lambda^2 L(L+\rho) \quad <=>$$

$$\lambda(2L+\rho) < (2L+\rho)$$

which holds since $\lambda<1$.

The third fact is established by similar algebraic manipulations.

14. Specifically,

$$\pi = B\left(\frac{\alpha E^w}{\sigma K^w}\right), \quad \pi^* = B^*\left(\frac{\alpha E^w}{\sigma K^w}\right);$$

$$B \equiv \left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{\phi(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right), \quad B^* \equiv \left(\frac{\phi\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right);$$

$$\theta_E \equiv \frac{E}{E^w}$$

The home market effect is, by definition, the impact of $d\theta_E$ on π and π^*. By inspection, π is increasing and π^* is decreasing in θ_E, as long as trade is less than perfectly free, i.e. $\phi<1$.

15. Specifically,

$$\pi = B\,(\frac{\alpha E^{w}}{\sigma K^{w}})\,, \qquad \pi^{*} = B^{*}\,(\frac{\alpha E^{w}}{\sigma K^{w}})\;;$$

$$B \equiv \left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{\phi(1-\theta_E)}{\phi\theta_K + 1 - \theta_K} \right), \quad B^{*} \equiv \left(\frac{\phi\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)}{\phi\theta_K + 1 - \theta_K} \right)\;; \theta_E \equiv \frac{E}{E^{w}}$$

The local competition effect is, by definition, the impact of $d\theta_K$ on π and π^*. By inspection, π is decreasing and π^* is increasing in θ_K, as long as trade is less than perfectly free, i.e. $\phi<1$.

16. In particular, we need that $\lambda\rho<(1+\lambda)L$.

17. The two roots are:

$$\phi^{cat} = \frac{[\rho + L(1+\lambda)] \pm \sqrt{\rho^2 + L(1+\lambda)[2\rho + L(1+\lambda)](1-\lambda^2)}}{\lambda[2\rho + L(1+\lambda)]}$$

The root that adds the radical is always greater than unity and so lies outside the economically relevant range. To show this note that the assertion requires

$$\sqrt{\rho^2 + L(1+\lambda)[2\rho + L(1+\lambda)](1-\lambda^2)} \;>\; \lambda[2\rho + L(1+\lambda)] \;-\; [\rho + L(1+\lambda)]$$

Two cases must be considered. If the right side is negative, the assertion holds automatically since the left side is a positive real number. If the right side is positive, we can square both sides without switching the inequality. Performing this and gathering terms we have:

$$2\lambda(1-\lambda)[L(1+\lambda)]^2 \;+\; 6\rho\lambda(1-\lambda)[L(1+\lambda)] \;+\; 4\rho^2\lambda(1-\lambda) \;>\; 0$$

which confirms the result. The same type of reasoning shows that the root that subtracts the radical is always less than unity and never negative.

18. Specifically, we take $L=1, \rho=.1, \lambda=.5$. These may be considered reasonable since from see (2-16) and (2-2), they imply – together with $\alpha=.3$ and $\sigma=3$ – that the symmetric rate of K growth is 6%.

19. These results involve tedious calculations performed in Maple. The worksheet is available upon request.

20. In using the perfect consumption price index, we are guided by microeconomic theory. That is, taking P as the perfect price index, E/P is the indirect utility function for a moment in time. In this way, real income is related to a welfare measure.

In particular, the perfect price and CES price indices are, respectively:

$$P \equiv p_T^{1-\alpha} P_M^{\alpha}\,, \qquad where \qquad P_M \equiv \int_0^{K^w} \left(p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di$$

21. The time-invariant, steady-state Y equals $wL+\pi K$, which reduces to:

$$\bar{Y} = L + \frac{\alpha}{\sigma}[L + \frac{\rho}{1+\lambda}] = \frac{\sigma+\alpha}{\sigma}L + \frac{\alpha\rho}{\sigma(1+\lambda)}$$

P_M reduces to:

$$P_M = K^{\frac{1}{1-\sigma}}(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}$$

where symmetry of regions and varieties, and the optimal pricing rules are used to derive the second expression. Time differentiation of P_M (taking logs first) and the fact that α is the weight on the perfect price index yield the result in the text.

22. The components are:

$$\bar{Y} = \frac{\sigma+\alpha}{\sigma}L + \frac{\alpha\rho}{\sigma(1+\lambda)}, \quad and \quad \bar{L}_I = \frac{\alpha}{\sigma}L - \frac{\rho(\sigma-\alpha)}{\sigma(1+\lambda)}$$

so rearranging

$$\frac{\bar{L}_I}{\bar{Y}} = \frac{\alpha(1+\lambda)L - \rho(\sigma-\alpha)}{(\sigma+\alpha)(1+\lambda)L + \alpha\rho}$$

23. There are two ways forward: linearization and numerical simulation. The first entails approximating transitional behaviour with a system of linear differential equations that corresponds to a linearization of the true equations near the steady state. With three differential equations, this is onerous (it requires, for instance, solution of a system of three third-order polynomials), but it is possible. Unfortunately, the approximation is only reliable in the neighbourhood of the steady state to which the system is heading. The technique, therefore, is of only limited use in characterizing growth and investment in the early part of the take-off when the system is inevitable far from its final, core-periphery, steady state. Despite this limitation future drafts will undertake this approach. The second approach is to attempt to identify the transition path numerically.

24. In particular, $d\lambda^{mir}/d\phi$ equals $(2L+\rho)\,[L(1-\phi^2)-\rho\phi^2]/[L(1+\phi^2)+\rho\phi^2]^2$ which is positive as long as λ^{mir} is less than 1.

25. Note that this may require a negative or positive learning elasticity since $\alpha/(\sigma-1)$ may be above or below unity.

26. Y^w equals 2L plus the sum of operating profits, which equal $\alpha Y^w/\sigma$, since αY^w is total spending on M (M is used in consumption and in investment). Gathering terms yields the expression in the text.

The definition of Y is $L+\pi K$. Using $\pi = B\alpha Y^w/\sigma K^w$ and the expression for Y^w:

$$\theta_Y = \frac{L + \alpha Y^w B\theta_K \frac{1}{\sigma}}{2L + \alpha Y^w/\sigma} = \frac{L(\sigma-\alpha) + \alpha 2LB\theta_K}{2L(\sigma-\alpha) + \alpha 2L}$$

simplification yields the expression in the text. This is not a closed form solution for θ_Y since

B includes θ_Y. However, since θ_Y enters B linearly, a closed form solution is possible (although it is too complex to be revealing).

27. The expression for F is derived using the fact that $p_Z=1$ and :

$$P_X^{\alpha} = \left(K^{w} p^{1-\sigma} [\theta_K + (1-\theta_K)(\frac{p^*}{p})^{1-\sigma}] \right)^{\frac{1}{1-\sigma}} = (K^{w})^{\frac{1}{1-\sigma}} \left((\theta_K + (1+\theta_K)\phi) \right)^{\frac{1}{1-\sigma}}$$

due to the optimal pricing rules, choice of M units and the definition of ϕ. Using the regularity condition on the learning elasticity ξ, K acquires a coefficient of -1 in the expression for F. This cancels out with the K in the expression for π.

28. The term is proportional to $(\partial B/\partial \theta_Y)(\partial \theta_Y/\partial \theta_K)$.

29. Venables (1987) first showed that rising a tariff unilaterally can lower a nation's price index, since the protection leads to production shifting (i.e. the share of varieties produced in the home market rises), which lowers prices by avoiding trade costs. $d\theta_K>0$ has the same impact in our model.

30. To see this, note that protection raises P_M for a given θ_K.

31. The condition is:

$$e_3 = 0 \quad <=> \quad b = \sqrt{b^2 - 4c\sigma(1+\lambda)}$$

$$<=> \quad 0 = -4c\sigma(1+\lambda) \quad <=> \quad c=0$$