INDETERMINACY, BUBBLES, AND THE FISCAL THEORY OF PRICE LEVEL DETERMINATION

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ABSTRACT

The recently-developed fiscal theory of price level determination contends that there is an important class of policy rules in which there exists a unique rational expectations solution that shows the price level to be dependent upon fiscal policy and independent of monetary variables. The present paper argues, however, that there is an alternative solution to these models that has entirely traditional (or "monetarist") properties. This latter solution is perhaps the more plausible since it is the solution that is typically regarded as the bubble-free "fundamentals" solution. The argument involves a respecification of feasible instrument variables.

Bennett T. McCallum
Graduate School of Industrial Administration
Carnegie Mellon University
Pittsburgh, PA 15213
and NBER
bm05@andrew.cmu.edu

Introduction

During the past few years a number of writings have appeared in which fiscal policy is viewed as the predominant determinant of macroeconomic phenomena that have traditionally been regarded as primarily of a monetary nature. The most striking of the developments in this fiscalist literature is a "fiscal theory of price level determination," which has been spelled out by Woodford (1994, 1995) and Sims (1994, 1996).[1] Specifically, these papers contend that there is an important class of policy rules for which, in models of the Sidrauski-Brock type,[2] there is a unique solution that shows the price level to be dependent upon fiscal policy and independent of monetary variables.

The main purpose of the present paper is to argue that the just-stated fiscalist contention is partially incorrect, i.e., that there is an alternative solution to these models that has traditional or monetarist properties. Furthermore, this traditional solution is arguably the more plausible since it represents the model's fundamentals or bubble-free solution, whereas the fiscalist price level solution involves a bubble component. As a related matter, other fiscalist results promoting a policy of interest rate pegging are here given an interpretation that reduces the claimed attractiveness of such an approach to policymaking.

II. Fiscalist Proposition

To make the argument as transparent as possible, let us adopt an extremely simple analytical framework that excludes capital accumulation and stochastic shocks.[3] Specifically, the economy is populated with a large number of identical infinite-lived households, each of which seeks at time t to maximize $u(c_t, m_t) + \beta\, u(c_{t+1}, m_{t+1}) + \beta^2 u(c_{t+2}, m_{t+2}) + \ldots$, where c_t is consumption during t and $m_t = M_t/P_t$,[4] with P_t the money price of a consumption unit and M_t

denoting a household's nominal money holding at the start of period t.[5] Also $\beta = 1/(1+\rho)$, with $\rho>0$. In order to have a convenient parametric example, let us suppose that

$$(1) \quad u(c_t,m_t) = (1-\sigma)^{-1}A_1\, c_t^{1-\sigma} + (1-\eta)^{-1}A_2\, m_t^{1-\eta}$$

where $\sigma, \eta > 0$. Each household uses its inelastically-supplied labor to produce y units of fully perishable output in each period. Also it pays lump-sum taxes in the amount v_t so if B_{t+1} is the number of \$1 government bonds purchased in t at the price $(1+R_t)^{-1}$, and $b_t = B_t/P_t$, the household's budget constraints is[6]

$$(2) \quad y - v_t = c_t + (P_{t-1}/P_t)m_{t+1} - m_t + (P_{t-1}/P_t)(1+R_t)^{-1}b_{t+1} - b_t.$$

With the foregoing specification, the household first-order optimality conditions are

$$(3) \quad \frac{u_2(c_{t+1},m_{t+1})}{u_1(c_{t+1},m_{t+1})} = \frac{A_2 m_{t+1}^{-\eta}}{A_1 c_{t+1}^{-\sigma}} = R_t$$

and

$$(4) \quad \frac{u_1(c_t,m_t)}{u_1(c_{t+1},m_{t+1})} = \frac{c_t^{-\sigma}}{c_{t+1}^{-\sigma}} = \frac{\beta(1+R_t)}{P_{t+1}/P_t}$$

In addition, we assume that the government does not lend to households so that $B_t \geq 0$ for all t.

This implies that the household's optimal choices must also satisfy the two following transversality conditions (TCs):

$$(5) \quad \lim_{t\to\infty} \beta^t m_{t+1} = 0$$

$$(6) \quad \lim_{t\to\infty} \beta^t b_{t+1} = 0.$$

For competitive equilibrium, the government budget constraint must be satisfied. In per-household terms, this can be written as

$$(7) \quad g_t - v_t = (P_{t+1}/P_t)m_{t+1} - m_t + (P_{t+1}/P_t)(1+R_t)^{-1}b_{t+1} - b_t$$

where g_t is government purchases. For simplicity, let us set $g_t = g \geq 0$, all t, in what follows. Then (2) and (7) together imply that

$$(8) \quad c_t = y - g.$$

In the exercise at hand, monetary policy settings are taken to be exogenous—non-responsive to the state of the economy. For simplicity (again), let us suppose that $M_t = M$ and also that $v_t = v$ with $v - g > 0$ for all $t = 1,2,\ldots$. Then with $c_t = y - g$, the equilibrium time paths for P_t, R_t, and b_{t+1} are given for $t = 1,2,\ldots$ by

$$(9) \quad M/P_{t+1} = AR_t^{-1/\eta} \qquad\qquad A \equiv (y^{\sigma}A_1/A_2)^{1/\eta}$$

$$(10) \quad 1 + \rho = (1 + R_t)P_t/P_{t+1} \qquad\qquad \rho = /\beta$$

$$(11) \quad b_{t+1} = (1+\rho)b_t + (1+\rho)(g - v)$$

provided that the TCs hold.

According to the fiscalist argument, the time path for the price level is determined in this economy as follows. From (11) it is apparent that b_t will explode, violating the TC (6), unless it is the case that $b_1 = (v - g)(1+ \rho)/\rho$ which would induce b_t to remain constant at that value. Meanwhile, conditions (9) and (10) imply a difference equation that relates P_{t+1} to P_t in an explosive manner. For $0<\eta<1$, the situation is as illustrated in Figure 1. There we see that if $P_1 > P^* \equiv M\rho^{1/\eta}/A$, then $P_t \rightarrow \infty$ as $t \rightarrow \infty$ whereas $P_t \rightarrow 0$ if $P_1 < P^*$.[7] Supposing then that $P_1 > P^*$,[8] it will be true that $M/P_t \rightarrow 0$ so the TC (5) is satisfied. In this case, then, all the requirements for equilibrium will be satisfied even though P_t is exploding. Furthermore, the position of the P_t

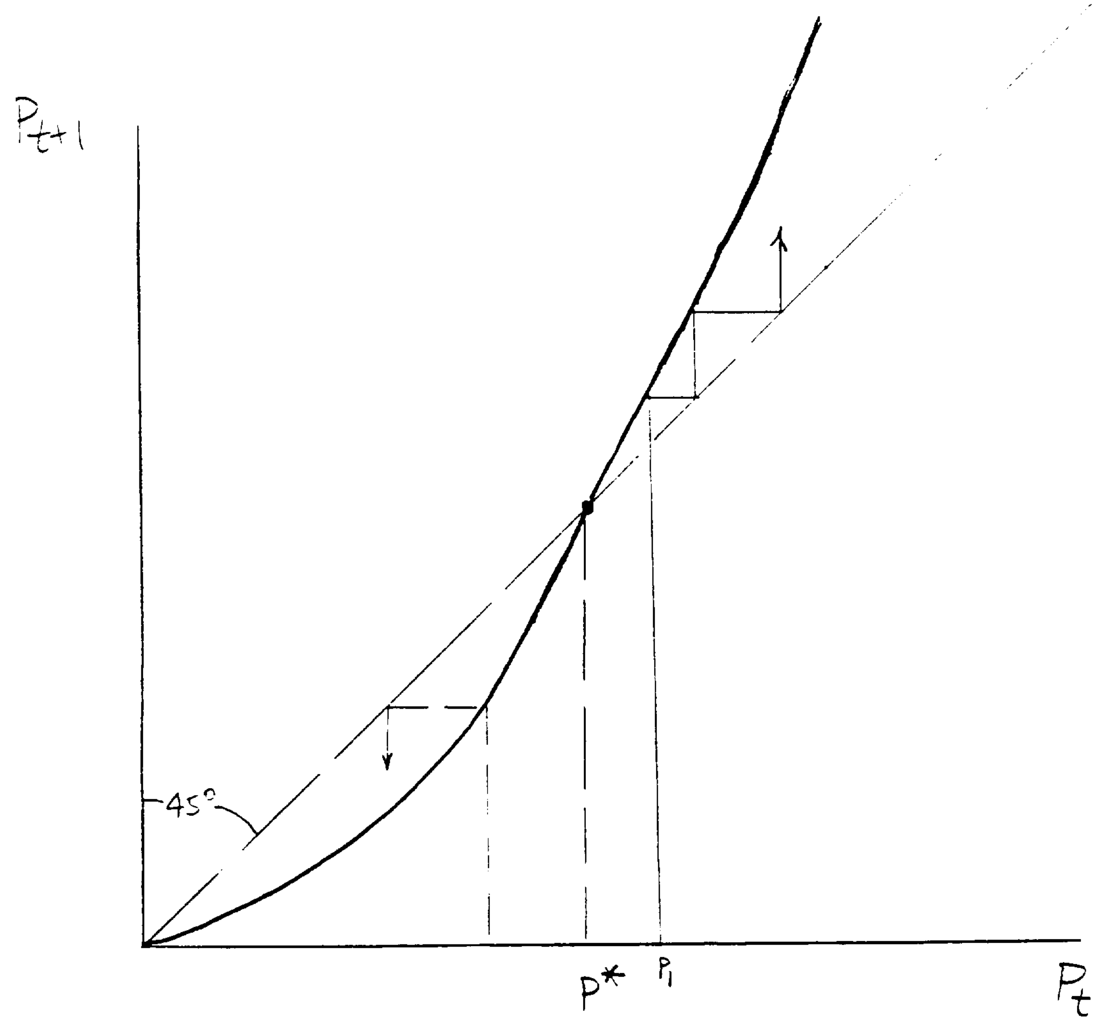

Figure 1

path will depend upon the initial bond stock B_1 since we must have $B_1/P_1 = (v - g)(1+\rho)/\rho$ for the TC (6) to be satisfied. And the P_t path depends in no way upon the value of $M_t = M$, so we have an outcome that is about as strongly "fiscalist" as anyone could imagine. This is our example of the fiscalist result.

At this point we pause to emphasize that the foregoing result holds not just for the particular preference specification (1), but for any that gives rise to a P_{t+1} vs. P_t mapping of the form shown in Figure 1.[9] That there is a tendency toward such a shape can be argued loosely as follows. Suppose that u(c,m) is additively separable. Then with $u_{22}(c,m)<0$, condition (3) implies that R_t is an increasing function of P_{t+1}. Next, with $c_t = c_{t+1} = c$ and $u_{12}(c,m) = 0$, equation (4) makes P_{t+1}/P_t increasing in R_t, so the ratio of P_t/P_{t+1} decreases with P_{t+1} as in Figure 1. Also, $P_{t+1}/P_t<1$ for $R_t = 0$ is implied by (4) and $R_t \to 0$ as $P_{t+1} \to 0$ is implied by (3). The main obstacle is that the P_{t+1} vs. P_t plot can be backward bending, and indeed will be if we have (1) with $\eta > 1$. On the other hand, separability of u(c,m) is not required for a mapping like that of Figure 1.

Is such a mapping required for the argument of the present paper? The answer is no. Such a mapping is required for the tidy exposition of the fiscalist position given above, but a clean-cut case of this type is evidently not necessary for the fiscalist position—a rather bewildering set of cases is examined by Sims (1994). But in any case, the purpose of this paper is to question the fiscalist position, and a mapping like that of Figure 1 is certainly not necessary for our argument. We emphasize Figure 1 merely because it does provide a tidy fiscalist argument and because our reasons for questioning the fiscalist theory do not include any contention that the shape of Figure 1 is unrealistic.

There is a problem with the analysis presented above that is recognized but barely mentioned by fiscalist writers, as follows.[10] What is the solution for P_t if it happens that the initial bond stock is such that $B_1/P_1 = (v - g)(1+\rho)/\rho$ implies that $P_1 < P^*$? The formal answer, apparently, is that in this case equilibrium does not exist (because $P_t \rightarrow 0$ violates the TC (5)). Less formally, the model does not explain how P_t will evolve if the policy with constant M and $v - g$ begins at t=1 with $B_1 < M\rho^{1/\eta}(v - g)(1+\rho)/\rho A$. One might ask, then, what is the defect of the model that leaves it unable to explain how P_t would behave in this case? As it happens, an answer is generated as a by-product of the following discussion, which puts forth a basic objection to the fiscalist analysis.

III. Alternative Solution

It will now be argued that the fiscalist analysis summarized above has overlooked an alternative equilibrium path for the variables P_t, R_t, and b_t. This more traditional - - perhaps the term "monetarist" would be appropriate - - equilibrium path has $P_t = P^* = M\rho^{1/\eta}/A$ together with $R_t = \rho$ and $B_{t+1} = 0$ for all t = 1,2,.... Clearly that path satisfies (9), (10), and both TCs. Furthermore, it also satisfies (11) if we let $v_1 - g = B_1/P_1$ and $v_t - g = 0$ for t = 2,3,.... Now, it might be quickly objected that the latter does not conform to the assumption that $v_t - g > 0$ for all t = 1,2,... but that objection can be answered, I contend, as follows. From a realistic perspective, it is not legitimate to specify fiscal policy in terms of the primary surplus $v_t - g_t$ (or the deficit g_t

Therefore the present argument is applicable to other cases, which have various existence and uniqueness outcomes according to the fiscalist solution.

IV. Bubble and Fundamental Solutions

The two different solutions for P_t presented in the two preceding sections are, of course, familiar ones. In particular, the monetarist solution is the bubble-free or "fundamentals" solution suggested by the minimal-state-variable (MSV) approach to solving rational expectations models. It is, in other words, the solution that does not include any arbitrary though self-justifying bubble or bootstrap components. To see this, it is useful to recall that P_{t+1} in equations (9) and (10) is actually the expected future price level; these equations determine P_t and R_t in response to M_t and expected P_{t+1}, not P_t in response to M_{t-1} and P_{t-1}. Thus with M_t constant we can solve out R_t and express the system (9)-(10) as

(12) $P_t = \psi(P_{t+1}{}^e)$,

where the superscript "e" is purely for heuristic purposes. Since with M_t constant there are no relevant state variables, we then conjecture a MSV solution of the form $P_t = \phi$. Then it is implied that $P_{t+1}{}^e = \phi$ so substitution into (12) yields $\phi = \psi(\phi)$, which we can solve, in an undetermined-coefficients manner, for ϕ. The latter turns out, of course, to be P^*, so we have $P_t = P^*$ as the MSV solution.

$- v_t$), because that is not a variable that the fiscal authority can directly control. For a given path of the base money stock, directly controlled by the central bank, the fiscal authority's primary surplus or deficit is determined (with constant g) by the amount of revenue that is raised by the sale of bonds to the public. So formally a model such as the one at hand should be specified with B_{t+j}^{S}, the bonds <u>supplied</u> by the fiscal authority, as its policy variable. Then with B_{t+j}^{D} representing bond demand by the public, we would have $B_t^D \leq B^S$ for all t = 2,3... as the equilibrium condition. In the monetarist equilibrium, the result is $B_t^D = 0$ and thus $b_t = 0$ for all t = 2,3,.... But how about v_t? The interpretation is as follows. Although the fiscal authority is collecting taxes in the amount vP_t each period, by not using these to purchase goods from the public (recall that $g_t \leq v$), the fiscal authority is also in effect providing a transfer payment to households, one that just offsets the apparent surplus and leaves the net magnitude of v – g as zero. With zero revenue raised by money issue and by bond sales—the latter being required by (6) and (11)—the fiscal authority's primary surplus (deficit) magnitude is constrained to equal zero.

An attractive feature of this solution is that it is not subject to the problem described above for the case in which $B_1 < M\rho^{1/\eta}(v - g)(1+\rho)/\rho A$. Instead, $P_t = P^*$ regardless of the size of the initial stock of bonds. More fundamentally, it is apparent that nothing in the argument of this section requires that the P_{t+1} vs. P_t mapping of (9) and (10) be of the form shown in Figure 1.

But although there are no relevant state variables, one can conjecture a solution to (12) of the form $P_t = \Phi\ (P_{t-1})$, thereby introducing an "extraneous" state variable, in the language of McCallum (1983). Then $P_{t+1}{}^e = \Phi(P_t) = \Phi[\Phi(P_{t-1})]$ so substitution into (12) yields $\Phi(P_{t-1}) = \psi\{\Phi[\Phi(P_{t-1})]\}$ which can typically be solved for the function Φ. This bubble solution, $P_t = \Phi(P_{t-1})$, is precisely the one provided for the problem at hand by the fiscalist approach given in Section II.[11]

In this case, as in many others, economic theory does not determine whether the fundamentals or bubble solution will prevail. Someone who believes, however, that bubble solutions rarely if ever are empirically relevant in a macroeconomic context would tend to hypothesize that the MSV monetarist solution would obtain empirically. What we have is two competing predictions about price level behavior in a setting in which money stock growth is kept close to zero by an independent central bank while the fiscal authority attempts to maintain a budgetary surplus. The bubble-free monetarist prediction is that the price level will remain basically constant over time, whereas the fiscalist prediction is that the price level will grow explosively. It is unfortunate that a controlled experiment cannot be conducted in an actual economy: the present author would certainly welcome the opportunity of wagering a few thousand dollars on the outcome.

V. Interest Rate Instrument

A second theme in the fiscalist literature is the suggestion that price-level indeterminacy problems can be solved by having the central bank peg the nominal interest rate at a level consistent with the central bank's desired inflation rate, rather than by controlling the growth rate of the (base) money supply.[12] In evaluating this suggestion it is important to distinguish clearly between two different types of price level behavior that have been referred to in the literature as constituting "indeterminacy." Both involve aberrational price level behavior, but they are nevertheless quite different both analytically and economically. Accordingly, McCallum (1986, p. 137) proposed that they be referred to by terms that would recognize this distinction and thereby add precision to the discussion. The proposed terms are nominal indeterminacy and solution multiplicity (or nonuniqueness).[13] The former, nominal indeterminacy, refers to cases in which the model at hand fails to pin down the value of any nominal variable (i.e., any variable measured in monetary units). Thus values of all money-stock aggregates and (e.g.) nominal income, as well as the price level, are left undetermined by the model's restrictions. Paths of all real variables, however, are typically well determined. In terms of real-world behavior, such a situation could in principle obtain if the central bank failed entirely to provide a nominal anchor. This is the type of phenomenon that has been discussed in classic works by Patinkin (1949, 1961) and Gurley and Shaw (1960), as well as in more recent contributions by Sargent and Wallace (1975), Sargent (1979, pp. 360-363), McCallum (1981, 1986), and Canzoneri, Henderson, and Rogoff (1983).

Solution multiplicity by contrast, refers to aberrational behavior involving "bubbles" or "sunspots" that affect the price level. In these cases, the path of the money stock—or possibly some other nominal variable controlled by the central bank—is perfectly well specified. Nevertheless, more than one path for the price level—often an infinity of such paths—will satisfy all the conditions of the model. In terms of real-world behavior, the source of this phenomenon is arbitrary yet self-justifying expectations. It has been discussed by a vast number of writers including Sargent and Wallace (1973), Black (1974), Brock (1975), Flood and Garber (1980), and Taylor (1977).

In light of this distinction, let us consider the results of Woodford (1990, pp. 1119-1122; 1995, pp. 32-35), Brock (1975, pp. 144-147), and Sims (1994, pp. 388, 391-393). In all of these discussions, the cases under study are ones in which the (base) money stock is directly controlled by the central bank. Thus the non-uniqueness of rational expectations equilibria that are shown to obtain for some given money growth rate is clearly not of the nominal indeterminacy type, but instead reflects solution multiplicity involving price level bubbles or sunspots. Whether such multiplicities are of any empirical relevance is a controversial topic that is much too large to be discussed here. But in any event the cases under consideration are not examples of nominal indeterminacy in the sense of Patinkin (1949, 1961), Gurley and Shaw (1960), Sargent (1979, pp. 360-363), or Sargent and Wallace (1975). Consequently, the interest-rate pegging result of the fiscalist papers mentioned above—a result that claims that pegging will eliminate multiple solutions—does not provide a remedy for nominal indeterminacy problems.

VI. Conclusion

In the foregoing sections, I have argued that the fiscal theory of price level determination is of dubious validity. It should be emphasized that this argument is not meant to deny that fiscal policy will in fact often have an enormous influence on the price level because the central bank chooses to accommodate fiscal tendencies, perhaps because of political pressures. But in that case there is no basic dispute between fiscalists and monetarists. What is at issue is the fiscalist claim that P_t is fiscally determined in cases in which the central bank refuses to accommodate and keeps the monetary base on its predetermined path. Finally, it should be mentioned that the argument does not depend upon the assumption that M_t is constant. With M_t growing, the two competing equilibria would pertain to the question of whether P_t explodes relative to the M_t path--as the fiscalist solution implies--or conforms to the M_t path as in traditional analysis.

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Endnotes

[1] The same argument appears in Sargent (1997, Ch. 8) while other notable fiscalist writings include Leeper (1991), and Leeper and Sims (1994).

[2] That is, dynamic general equilibrium models with infinite-lived households, money in the utility function, and rational expectations. Such models are ones in which it is common for Ricardian equivalence results to be obtained, which makes the fiscalist theory's claim more remarkable than if developed in a model of (e.g.) the overlapping generations type.

[3] In this regard, the setup is similar to Woodford (1994, 1995). Sims (1994, 1996) includes shocks but no capital.

[4] The relation $m_t = M_t/P_t$ might be viewed not as a identity, but rather as a market-clearing condition with m_t being real balances demanded and M_t nominal money supplied (per household).

[5] Woodford (1995) and Sims (1994) specify that it is end-of-period money balances that facilitate transactions rather than start-of-period balances. That difference in specification is not crucial for the arguments presented below. The alternative is adopted here to conform more closely to the cash-in-advance approach and to yield the case depicted in Figure 1.

[6] With preferences as in (1), positive quantities of c_t and m_t will be chosen so the constraint can be written as an equality.

[7] That the relationship between P_{t+1} and P_t is of the form depicted in Figure 1 when $0<\eta<1$ will be shown momentarily.

[8] We will consider $P_1 \leq P^*$ below.

[9] The crucial features of the mapping are that $P_{t+1} = f(P_t)$ with f( ) a twice differentiable function such that $f(0) = 0$, $f'(0) < 1$, $f'(P^*) > 0$ where $P^* = f(P^*)$, $f''(P) > 0$ for $0 < P < P^*$, and $f(P) > P$ for $P > P^*$.

[10] I am not here suggesting that fiscalist writers have barely mentioned the possibility of non-existence of a rational expectations solution, but rather that in a prominent case the nature of their proposed solution is drastically different for different initial values of B_1 (with non-existence prevailing for a range of values). In the alternative solution proposed below, the behavior of P_t does not depend on any initial condition.

[11] To see that P_{t-1} is extraneous, note that the relevant system for determining P_t, R_t, c_t, m_t, b_t, and v_t for exogenous paths of M_t, g_t, and B_t is (2), (3), (4), (7), $m_t = M_t/P_t$, and $b_t = B_t/P_t$ (plus the transversality conditions). But (2) and (7) imply (8), as an instance of Walras's Law. Then the subsystem (3), (4), (8), and $m_t = M_t/P_t$ determines P_t, R_t, c_t, and m_t without reference to (7) or to $b_t = B_t/P_t$. In that subsystem there are no lagged variables.

[12] See, e.g, Woodford (1990, pp.1128-1134; 1995), Sims (1994).

[13] Actually, McCallum (1986) proposed "indeterminacy" for the former, but the inclusion of the adjective now seems clearly desirable.