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HIGH WATER MARKS

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ABSTRACT

Incentive fees for money managers are frequently accompanied by high-water mark provisions that condition the payment of the performance fee upon exceeding the previously achieved maximum share value. In this paper, we show that hedge fund performance fees are valuable to money managers, and conversely, represent a claim on a significant proportion of investor wealth. The high-water mark provisions in these contracts limit the value of the performance fees. We provide a closed-form solution to the cost of the high-water mark contract under certain conditions. Our results provide a framework for valuation of a hedge fund management company.

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High-Water Marks and
Hedge Fund Management Contracts

The growth of the hedge fund industry over the past decade has brought an unusual form of performance contract to the attention of the investment community.[1] Hedge fund managers typically receive a fraction of the fund's return each year in excess of the high-water mark. The high-water mark for each investor is the maximum share value since his or her investment in the fund.[2] These performance fees generally range from 15 percent to 25 percent of the new profits earned each year. In addition, managers also charge a regular annual fee of one percent to two percent of portfolio assets. For example, George Soros' Quantum Fund charges investors an annual fee of one percent of net asset value with a high-water mark based performance fee of 20 percent of net new profits earned annually. As a result, the Quantum Fund returned 49 percent (pre-fee) in 1995 on net assets of $3.7 billion resulting in an estimated total compensation of $393 million for that year, most of which was due to the incentive terms.[3] Of course, when the high-water mark is not reached, manager returns are substantially reduced. In 1996, the Quantum fund lost 1.5 percent, and thus, earned only their regular annual fee $54 million (1 percent of the $5.4 billion of assets).

While the Quantum Fund stands out as an unusually good performer over the past decade, its compensation terms are typical of the hedge fund industry. High-water mark contracts have the appealing feature of paying the manager a bonus only when the investors make a profit, and in addition, requiring that the manager make up any earlier losses before becoming eligible for the bonus payment. On the other hand, their option-like (non-linear) characteristics clearly could induce the manager to alter his investment strategy, and the large bonus of 20 percent above the benchmark clearly reduces long-term asset growth.

In this paper, we examine the costs and benefits of high-water mark compensation to investors. To do so, we develop a valuation equation that allows us to estimate the division of wealth that the investor implicitly makes with the portfolio manager upon entering into such a contract. We find for reasonable

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[1] The term hedge fund is used to characterize a broad class of skill-based asset management firms that, for a variety of reasons, do not qualify as mutual funds or money managers regulated by the Investment Company Act of 1940. For recent academic research on the hedge fund industry, performance incentives and performance, see Fung and Hsieh (1997, 1999, 2000), Brown, Goetzmann and Ibbotson (1999), Brown, Goetzmann, and Park (1997) and Ackerman, MacEnally, and Ravenscraft (1999).

[2] The various partners' funds are all pooled so they earn the same rate of return, but different partners may have a different high-water mark depending on the maximum share value reached since their investment in the fund.

[3] Figures are from the *U.S. Offshore Funds Directory*, 1995 and 1996 editions for the Quantum Fund N.V. Returns assume re-investment of income. Manager fees are calculated from reported changes in net asset value.

parameters of the valuation equation that the present value of fees and other costs could be as high as 33 percent of the amount invested. A more representative number, though, is probably 10 percent to 20 percent. A significant proportion of this compensation is due to the incentive feature of the contract; however, the tradeoff between regular fees and high-water mark fees depends upon the volatility of the portfolio and the investor's withdrawal policy. We find that this proportion is high when money is "hot," *i.e.*, when the probability of investors leaving the fund is high, and when the volatility of the assets is high. In contrast, when investors are likely to remain for the long term, and when volatility is low, the regular-fee portion of the contract provides the greatest value to the manager.

This apparently significant transfer of wealth to the manager may, however, be economically justified. We show that excess performance as small as an alpha of three percent could compensate the investors for such charges.

We also consider why high-water mark contracts exist, and in particular, why they are used by hedge funds as opposed to mutual funds. While their prevalence in the hedge fund industry might be an accident of history, the high-water mark compensation contract may have features particularly suited to the types of investment strategies employed by hedge funds. The role of volatility and investor withdrawal, for example, may account for why we find high-water mark incentives used in asset classes such as hedge funds, commodity funds, and venture capital funds. In these asset classes, investor payoff is presumably based more upon expectations of superior manager skill and less upon the expected returns to an undifferentiated or passively managed portfolio of assets. Given that hedge fund investment is, in a sense, a pure bet on manager skill, our analysis provides a framework for considering how much skill a hedge fund manager must have to justify earning such high fees.

In addition to the valuation of the high-water mark contract, we explore the question of whether the high-water mark compensation is due to the fact that hedge fund technology may have diminishing returns to scale. Most hedge fund managers are engaged in some form of "arbitrage in expectations," in the domestic and global debt, equity, currency, and commodities markets. By their very nature, scaling these arbitrage returns may not be possible as investors purchase more fund shares. Most mutual funds can compensate their managers for past performance with a fixed percentage fee on assets, since good performance will attract new money. Hedge funds, however, may not be able to take or even want new funds.

To test whether the high-water mark contract may be a substitute for increasing compensation through fund growth, we examine the empirical relationship between hedge fund investor cash-flows and performance. In contrast to similar studies in the mutual fund industry, we find that neither large funds nor funds with superior performance sell new shares — indeed we find evidence that they experience net share repurchases. This is consistent with the hypothesis that the hedge fund industry itself has important limits to growth. This also has implications for investors seeking alternative investments to equities and debt. While hedge fund performance over the past ten years has been strong on a risk-adjusted basis, this performance may be in part due to the relatively small size of the hedge fund sector. The unwillingness of successful funds to accept new money may be indicative of diminishing returns in the industry as a whole as investment dollars flow in. We conjecture that the option-like fees commanded by hedge funds exist because managers cannot expect to trade on past superior performance to increase compensation through growth.

The paper is structured as follows. Section I develops a valuation model for determining the cost of the manager's contract. Section II estimates parameters for the model, using data on hedge funds. Section III provides some comparative statics and discusses the implications of our results. Section IV illustrates some extensions to the model. Section V presents evidence on hedge fund performance, size, and fund flows. Section VI concludes.

I. The Management Contract Cost Model

The hedge fund management contract has interesting option-like characteristics. It is a potentially perpetual contract with a path-dependent payoff. The payoff at any time depends on the high-water mark that is related to the maximum asset value achieved. As such, the contract can be valued using option-pricing methods. The major difference in our approach is to use an equilibrium derivation. Typically, option pricing is based on replicating a claim. Replicating a hedge fund is not possible since the information used in finding the arbitrages, which give it value, are, by necessity, private. Furthermore, since our approach is an equilibrium one, the valuation is not necessarily valid for an agent like the fund manager who likely holds a undiversified portfolio with his wealth concentrated in the fund's assets.

We work in a continuous-time framework and assume that, in the absence of payouts, the assets of the fund follow a lognormal diffusion process with expected rate of return μ and logarithmic variance σ^2. Although the investors' assets are pooled for management, we model a single investor's position in the fund because each investor may have different withdrawal expectations and a different high-water mark if investment occurred at different times. The variable S represents the market value (or net asset value) of the investor's position, and H is the current high-water mark; it is the highest level that the net asset value has reached subject to certain adjustments. The net asset value is reduced when the client makes withdrawals or receives distributions from the fund. There may be regular or periodic withdrawals or distributions, which we approximate as continuous occurring at the rate $W(S, H, t)$:

$$dS = [\mu S - W(S,H,t) - cS]dt + \sigma S d\omega\,, \quad S < H\,. \tag{1}$$

The fund may experience a complete withdrawal of an investor's assets. We model this as occurring when the asset value drops to some low level, $\underline{S}(H, t)$, representing a loss of confidence by this investor in the hedge fund's managers. In addition, an investor may withdraw his funds before this level is reached. Such a withdrawal might represent a liquidity need or more profitable investment opportunities elsewhere. The probability per unit time of such a liquidation is $\Lambda(S, H, t)$.[4]

The fund has operating expenses, including a regular management fee that must be paid from the assets. We assume these expenses are proportional to the value of the fund, cS per unit time. When the asset price moves above the high-water mark, the manager also collects an extra or performance fee equal to the fraction k of this return. In the stylized setting of the model, the performance fee is earned continuously. In practice, the performance fee is usually accrued on a monthly basis with H being reset annually or quarterly.

In the simplest case, the high-water mark is the highest level the asset value has reached in the past. For some incentive contracts, the high-water mark grows at the rate of interest or other contractually stated rate, g. It is also adjusted by withdrawals and often certain expenses are allocated to its reduction. When the asset value is below the high-water mark, it is not affected by the random variation in S. It is only adjusted due to withdrawals, allocated expenses, and the contractual growth rate. Since these are not locally random in our model, the evolution of H is locally deterministic. The common practice is to adjust H by the same proportion that withdrawals and allocated expenses have to the asset value. So for $S < H$, the evolution of H is

$$dH = \left(g - \frac{W(S,H,t) + c'S}{S} \right) H\, dt \tag{2}$$

where g is the contractual growth rate of the high water mark (usually zero or r) and $c'S$ is the costs and fees allocated to reducing the high-water mark. When the asset value reaches a new high, the high-water mark is reset to this higher level.

We are interested in determining the values of the performance fees, $P(S, H, t)$; the regular annual fees, $A(S, H, t)$; the sum of the two, $F(S, H, t)$; and the investor's claim, $I(S, H, t)$. The variable S represents the market value of the assets held and does not recognize that the managers have superior information that makes the assets more valuable (at least in this combination) than the market believes. On the other hand, the values, $P(\cdot)$, $A(\cdot)$, $F(\cdot)$, and $I(\cdot)$, are not market values; indeed, the claims are not marketable. Rather, they are the values of the claims as computed by a representative investor who knows the premium performance the managers of the fund are providing. In particular, we shall see that $I + F$ exceeds the market value of the

[4] We refer to the complete withdrawal of an investor's money as a liquidation to distinguish it from the regular or periodic withdrawals. This does not imply that the fund as a whole is liquidated. Other investors may have different "rules" or different high-water marks due to investing at different times.

assets held.

Let f represent the value of a generic claim, then, for $S < H$, the evolution of this value is

$$df = \left[\frac{\partial f}{\partial S}[\mu S - W(S,H,t) - cS] + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \left(g - \frac{W(S,H,t) + c'S}{S} \right) H \frac{\partial f}{\partial H} + \frac{\partial f}{\partial t} \right] dt \\ + \frac{\partial f}{\partial S}\sigma S d\omega + [\, f_\Lambda(S,H,t) - f\,] d\phi \quad . \tag{3}$$

There are no $\partial^2 f / \partial H^2$ or $\partial^2 f / \partial H \, \partial S$ terms because H is locally deterministic when $H < S$. The different values are distinguished by different payouts from the fund to the claim; different boundary conditions; and different post-liquidation values, $f_\Lambda(S, H, t)$. This is the *known* value of the claim after the investor's funds are withdrawn. For any of the fee claims, this is 0; for the investor's claim, it is S. The first two terms in (3) are the usual Itô description of the evolution; $d\phi$ is the increment to a point process; $\phi = 0$ and $\phi = 1$ before and after the investor's liquidation, respectively with $\mathbb{E}[d\phi] = \Lambda(S,H,t)dt$.

If the assets of the fund could be purchased, we could now determine the values through the usual no-arbitrage arguments of the Black-Scholes model. However, it is unrealistic to assume that investors can replicate the hedge fund without the special knowledge of the managers. Furthermore, even if they could replicate the claim, doing so would require a short position in the very assets the managers believe provide the superior returns to the hedge fund.

In place of the absence of arbitrage, we will use an equilibrium derivation as in Merton (1976) or Ingersoll (2002). Quite generally, the value of any asset can be determined with a martingale pricing operator, Θ, and the valuation relation $\mathbb{E}[d(\Theta f) + D(S,H,t)dt] = 0$, where $D(\cdot)$ is any payment made to the claim being valued. If we select as the martingale process the marginal utility of any investor, then the value determined is this investor's subjective value. If the investor selected is the representative investor, then we derive the market value of the asset. For the representative investor, the evolution of the martingale pricing operator is $d\Theta/\Theta = -r\,dt + [(\mu_m - r)/\sigma_m]d\omega_m$, where μ_m and σ_m are the expected rate of return and logarithmic standard deviation on the market, and $d\omega_m$ is the Weiner process for the market.

The pricing relation for the claim is then[5]

[5] The derivation assumes there is no relation between changes in Θ and a liquidation. There can be no relation between the changes in Θ and the value of f when liquidation occurs, since the liquidation change in the magnitude of f is deterministic. The only relation could be between changes in Θ and the event of a liquidation. In a more general model, the liquidation probability Λ could depend on changes in Θ or its level.

$$
\begin{aligned}
0 &= \mathbb{E}\left[d(\Theta f) + D(\cdot)\,dt \right] \\
&= \mathbb{E}\left[\Theta[f_S\, dS + \tfrac{1}{2} f_{SS} dS^2 + f_t\, dt + f_H\, dH + (f_\Lambda - f)d\phi] + f\, d\Theta + d\Theta\, f_S\, dS \right] + D\,dt \\
&= \Theta\left[f_S[\mu S - W(S,H,t) - cS] + \frac{1}{2}\sigma^2 S^2 f_{SS} + f_t + \Lambda(S,H,t)[f_\Lambda(S,H,t) - f] \right. \\
&\quad \left. + \left(g - \frac{W(S,H,t) + c'S}{S} \right) H f_H - rf + D \right] dt - f_S \sigma S \frac{\mu_m - r}{\sigma_m} \Theta \mathbb{E}[d\omega\, d\omega_m] \quad .
\end{aligned} \tag{4}
$$

The expression $(\sigma/\sigma_m)\mathbb{E}[d\omega\, d\omega_m]$ in the final term is the standard CAPM beta for the fund's asset value. We define the premium return on the fund's assets as $\alpha = \mu - r - \beta(\mu_m - r)$.[6] Then, while the fund's assets are below the high-water mark, the present value functions satisfy the option-like partial differential equation:

$$
\begin{aligned}
0 = \tfrac{1}{2}\sigma^2 S^2 f_{SS} + [(r+\alpha)S - W(S,H,t) - cS] f_S + \left(g - \frac{W(S,H,t) + c'S}{S} \right) H f_H \\
+ f_t + \Lambda(S,H,t)[f_\Lambda(S,H,t) - f] - rf + D(S,H,t) \qquad \text{for } H < S\,.
\end{aligned} \tag{5}
$$

Although a different derivation was used, this equation has the standard Black-Scholes interpretation. The first four terms are the expected risk-neutral change in the value of the fees due to the changes in S, H, and t. The expected rate of return on S has been "risk-neutralized" to $r + \alpha$. There are no f_{HH} or f_{HS} terms because H is locally deterministic when $H < S$. For the same reason, the expected change in H requires no risk-neutral adjustment.

The function $\Lambda(S, H, t)$ is the probability per unit time of the investor liquidating his position. The term $\Lambda(S,H,t)[f_\Lambda(S,H,t) - f]$ is the risk-neutral expected change in the claim due to liquidation. If the investor liquidates, which happens with probability Λdt, the value of the claim changes from f to f_Λ; this is the *known* value of the claim after the investor's funds are withdrawn. For any of the fee claims, this is 0; for the investor's claim, it is S.

The term $D(S, H, t)$ is the flow rate of costs whose present value (along with that of the performance fees) we are determining. It is like a dividend paid to the derivative asset in the Black-Scholes model. For the performance fees, the regular annual costs and fees, and the investor's claim, the payouts are $D(S, H, t) = 0$, $D(S, H, t) = cS$, and $D(S, H, t) = W(S, H, t)$, respectively.

Note that we are doing this valuation from the point of view of an investor in a competitive market. To the extent that the manager cannot hedge away the risk inherent in the funds and the fees, he may assign a

[6] This definition of α is completely general. There is no need that the CAPM holds in this economy to derive our formulas. The CAPM is consistent with our assertion that managers who can provide a positive α have ability, but this is simply a matter of interpretation. Our model would still be valid for any other zero-ability value for α. What we *have* assumed is that any good-buy opportunities that the active manager finds in the market are limited. If the hedge fund could find true arbitrages, then α could be made as large as desired by taking zero-cost arbitrages at unlimited scale.

personal utility-adjusted value to the fees that is less than the market value.[7]

Four boundary conditions are required to solve this equation. Three of the boundary conditions for the problem are

$$
\begin{aligned}
&A\left(\underline{S}(H,t),\ H,\ t\right)=0\ , \quad A_H(S,\infty,\ t)=0\ , \ \text{and}\ A(S,H,T)=\Xi_A(S,H) \\
&P\left(\underline{S}(H,t),\ H,\ t\right)=0\ , \quad P_H(S,\infty,\ t)=0\ , \ \text{and}\ P(S,H,T)=\Xi_P(S,H) \\
&I\left(\underline{S}(H,t),\ H,\ t\right)=\underline{S}(H,t)\ , \quad I_H(S,\infty,\ t)=0\ , \ \text{and}\ I(S,H,T)=\Xi_I(S,H)\ .
\end{aligned}
\tag{6}
$$

The first conditions indicate that when the asset value falls to the liquidation level, $\underline{S}(H, t)$, then the investor will withdraw all his money and there are no further costs or fees. The second conditions say if the high-water mark is very high relative to the asset value, then there is little chance of ever receiving an incentive payment, so a change in the high-water mark will not affect the value of the fees or the investor's claim. The third conditions apply the contractual sharing rules at the maturity of the management contract, T. Invariably, these hedge funds have no contractual termination, so such a boundary condition would not apply.

A fourth condition applies along the boundary $S = H$. When the asset value rises above the high-water mark to $H + \varepsilon$, the high-water mark is reset to $H + \varepsilon$, and a performance fee of $k\varepsilon$ is paid, reducing the asset value to $H + \varepsilon(1 - k)$. Therefore, $P(H + \varepsilon, H, t) = k\varepsilon + P(H + \varepsilon - k\varepsilon, H + \varepsilon, t)$ or

$$
k\epsilon = P(H+\epsilon,H,t)-P(H+\epsilon-k\epsilon,H+\epsilon,t)\approx\left[k\epsilon\frac{\partial P}{\partial S}-\epsilon\frac{\partial P}{\partial H}\right]\Bigg|_{S=H+\epsilon}\quad . \tag{7}
$$

In the limit, as $\varepsilon \to 0$, this is exact, giving the fourth boundary condition

$$
\left[k\frac{\partial P}{\partial S}-\frac{\partial P}{\partial H}\right]\Bigg|_{S=H}=k\quad . \tag{8}
$$

This condition applies to both the total value of the fees and the performance fees alone. The boundary conditions for the regular annual fees and the investor's claim are $[k\cdot\partial A/\partial S - \partial A/\partial H]\big|_{S=H} = [k\cdot\partial I/\partial S - \partial I/\partial H]\big|_{S=H} = 0$, because these claims do not receive the performance payments.

To obtain a closed-form solution for the present values, we make several simplifying assumptions:

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[7] See Ingersoll (2002) for a discussion of subjective pricing and an analytical model in the context of incentive options. Applying those results to this model, the valuation equation for a fund manager with a relative risk aversion of ξ who holds the fraction δ of wealth in the hedge fund with a residual variance (relative to the market) of v^2 would be

$$
\begin{aligned}
0 = \tfrac{1}{2}\sigma^2 S^2 f_{SS} &+ \left[[r+\alpha-\xi\delta v^2]S - W(S,H,t) - cS\right] f_S + \left(g - \frac{W(S,H,t)+c'S}{S}\right) H f_H \\
&+ f_t + \Lambda(S,H,t)[f_\Lambda(S,H,t) - f] - [r-\xi\delta^2 v^2] f + D(S,H,t).
\end{aligned}
$$

The analysis and numerical results of this paper are still valid with the modified parameters $r \to r - \xi\delta^2 v^2$ and $c \to c + \xi\delta(1-\delta)v^2$.

(i) The liquidation level is a constant fraction of the high-water mark, $\underline{S}(H, t) = bH$.

(ii) Withdrawals are proportional to asset value $W(S, H, t) = wS$.

(iii) The premium return, α, and probability of liquidation are constant, $\Lambda(S, H, t) = \lambda$.

Since withdrawals, liquidation, and the premium were the only time-dependent features of the problem, the present value function f no longer depends explicitly on time under these assumptions, and $f_t = 0$. Furthermore, it is clear by the economics of the problem that f is now homogeneous of degree one in S and H, so the solution has the form $f(S, H, t) = HG(x)$, where $x \equiv S/H$. Substituting this and the derivatives $f_H = G - xG_x$, $f_S = G_x$ and $f_{SS} = G_{xx}/H$ into (5) gives an ordinary differential equation[8]

$$\tfrac{1}{2}\sigma^2 x^2 G_{xx} + (r + \alpha + c' - g - c)xG_x - (r + c' - g + w + \lambda)G + \theta x = 0 , \tag{9}$$

where $\theta = 0$ for $f = P$, $\theta = c$ for $f = A$ or $f = F$, and $\theta = w + \lambda$ for $f = I$. As well as simplifying the solution, this differential equation provides insight into the effects of different parameters.

There are ten parameters in the valuation equation: r, α, and σ are environmental; k, c, c', and g are contractual; and w, λ, and b are actually endogenous choices, which we have assumed here to be constant. The first two "choice" variables enter the solution in a symmetric fashion; an increase in the withdrawal rate, w, is exactly the same as an increase in the probability intensity of liquidation, λ. In other words, for our model, withdrawing funds at the constant rate w has exactly the same effect on the present value as a probability w per unit time of withdrawing all the funds. Therefore, it will be simplest to just think of $w + \lambda$ as the effective withdrawal rate.

Similarly, the parameters r, c', and $-g$ have symmetric effects. An increase in the interest rate is equivalent to a contractual decrease in the growth rate of the high-water mark. It is also equivalent to an increase in the fees that are allocated to reducing the high-water mark. Note that an increase in c' is an accounting change that does not directly increase costs. It does, however, indirectly increase the present value of future costs by lowering the high-water mark and thereby shortening the time until a performance fee is charged. An increase in c' or a decrease in g are therefore like an increase in r, which increases the risk-neutral expected rate of growth in the assets, which also shortens the expected time until a performance fee is charged.

The solution to this equation is $G(x) = \theta x/(c + w + \lambda - \alpha) + Ax^{\gamma} + Bx^{\eta}$, where A and B are constants of integration and γ and η are the larger and smaller roots of the characteristic quadratic equation; *i.e.*,

[8] When the contractual growth rate in the high-water mark, g, is not zero, it is usually set equal to the interest rate. For these contracts, equation (9) does not depend on r. Therefore, in such cases, the formulas below would hold even with a stochastic rate of interest, provided that the other variables like α, w, and λ do not depend on r.

$$\binom{\gamma}{\eta} \equiv \frac{\frac{1}{2}\sigma^2 + c - r - \alpha - c' + g \pm \sqrt{\left(\frac{1}{2}\sigma^2 + c - r - \alpha - c' + g\right)^2 + 2\sigma^2(r + c' - g + w + \lambda)}}{\sigma^2} . \tag{10}$$

Use of the positive and negative signs gives γ and η, respectively. Note that $\eta < 1 < \gamma$.

The values of the total fees, performance fees, and the investor's claim are

$$\begin{aligned} F(S,H) &= \frac{c}{c+w+\lambda-\alpha}S + \frac{(w+\lambda-\alpha)k + [\eta(1+k)-1]cb^{1-\eta}}{(c+w+\lambda-\alpha)\left\{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]\right\}}H^{1-\gamma}S^{\gamma} \\ &\quad - \frac{b^{\gamma-\eta}(w+\lambda-\alpha)k + [\gamma(1+k)-1]cb^{1-\eta}}{(c+w+\lambda-\alpha)\left\{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]\right\}}H^{1-\eta}S^{\eta} \end{aligned} \tag{11a}$$

$$P(S,H) = k\frac{H^{1-\gamma}S^{\gamma} - b^{\gamma-\eta}H^{1-\eta}S^{\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]} \tag{11b}$$

$$\begin{aligned} I(S,H) &= \frac{w+\lambda}{c+w+\lambda-\alpha}S - \frac{(w+\lambda)k + [\eta(1+k)-1](c-\alpha)b^{1-\eta}}{(c+w+\lambda-\alpha)\left\{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]\right\}}H^{1-\gamma}S^{\gamma} \\ &\quad + \frac{b^{\gamma-\eta}(w+\lambda)k + [\gamma(1+k)-1](c-\alpha)b^{1-\eta}}{(c+w+\lambda-\alpha)\left\{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]\right\}}H^{1-\eta}S^{\eta} . \end{aligned} \tag{11c}$$

The present value of the annual fees is then $A(S, H) = F(S, H) - P(S, H)$.

To gain a better understanding of the solution, it is helpful to look at the simpler problem when there is no liquidation due to a drop in the asset value; *i.e.*, $b = 0$. In the absence of a lower liquidation barrier, the present value of the performance fees alone and all the fees are

$$\begin{aligned} P(S,H) &= \frac{kH}{\gamma(1+k)-1}(S/H)^{\gamma} \\ F(S,H) &= \frac{c}{c+w+\lambda-\alpha}S + \frac{w+\lambda-\alpha}{c+w+\lambda-\alpha}\frac{kH}{\gamma(1+k)-1}(S/H)^{\gamma} \quad . \end{aligned} \tag{12}$$

The first factor in $P(\cdot)$, $kH/[\gamma(1+k)-1]$ measures the present value of the performance fee at the inception of the fund (or whenever $S = H$). The factor $(S/H)^{\gamma}$ is the reduction in the present value of the future performance fees due to the time required before the asset value reaches the high-water mark and they can be earned; that is, $(S/H)^{\gamma}$ is the present value of \$1 paid the first time the stock price rises from S to H.[9] The product of these two factors gives the present value of the performance fees at any level of asset value.

As discussed earlier, the effective withdrawal rate of the investor is $w + \lambda$, so in the absence of a performance fee, the present value of the perpetuity of the regular fees would be proportional to the fraction

[9] See Ingersoll (2000) for a discussion of the valuation of these contracts known as *first-touch digital*s. The first-touch digital in (13) is a combination of the simple first-touch digitals for hitting H and bH, whose values are $(S/H)^{\gamma}$ and $(S/bH)^{\eta}$.

that the fees are of all outflows adjusted for the premium return, $c/(c + w + \lambda - \alpha)$. With a performance fee, the present value of the perpetuity of regular fees is this same fraction of the asset value net of the performance fees. So the value of the regular annual fees is $A(S, H) = [c/(c + w + \lambda - \alpha)][S - P(S, H)]$. The total value of the fees is the sum of these two quantities.

Now we return to the case when the fund is liquidated when the asset value falls to bH. A similar interpretation can be given. The present value of \$1 paid the first time the stock price rises from S to H without first hitting bH is

$$\mathcal{T}(S;H \mid S_{\min} > bH) = \frac{(S/H)^{\gamma} - b^{\gamma-\eta}(S/H)^{\eta}}{1 - b^{\gamma-\eta}}. \tag{13}$$

The value of the performance fees at the inception of the contract (when $S = H$) is given in (11b) as

$$P(H,H) = kH \frac{1 - b^{\gamma-\eta}}{[\gamma(1+k) - 1] - b^{\gamma-\eta}[\eta(1+k) - 1]}. \tag{14}$$

The value of the performance fees at any other stock price S is this initial value of the performance fees multiplied by the present value function in (13); that is, $P(S, H) = \mathcal{T}(S, H; \mid S_{\min} > bH) \times P(H, H)$. The value of the regular annual fees is the fraction $c/(c + w + \lambda - \alpha)$ of $S - P(S, H)$, less an additional first-touch correction to account for the cancellation of the regular fees when the stock price hits bH.

The total value of the assets under management; *i.e.*, the value of the fees and the investor's claim together is

$$\begin{aligned} F(S,H) + I(S,H) = {} & \frac{c + w + \lambda}{c + w + \lambda - \alpha} S \\ & -\alpha \frac{k - [\eta(1+k) - 1]b^{1-\eta}}{(c + w + \lambda - \alpha)\left\{\gamma(1+k) - 1 - b^{\gamma-\eta}[\eta(1+k) - 1]\right\}} H^{1-\gamma} S^{\gamma} \\ & +\alpha \frac{b^{\gamma-\eta}k - [\gamma(1+k) - 1]b^{1-\eta}}{(c + w + \lambda - \alpha)\left\{\gamma(1+k) - 1 - b^{\gamma-\eta}[\eta(1+k) - 1]\right\}} H^{1-\eta} S^{\eta} \quad . \end{aligned} \tag{15}$$

This is value based upon knowing the managers' superior performance and, therefore, is more than the value of assets under management, S.

In the absence of performance fees, the expected rate of growth of the funds under management would be $\mu - c - w - \lambda$. The total expected payout at time t is $(c + w + \lambda)\mathbb{E}[S_t]$ so the superior performance would give an effective value to the managed assets of

$$\begin{aligned} \mathbb{E}\left[\int_0^{\infty} e^{-\mu t}(c + w + \lambda)S_t \,\mathrm{d}t\right] &= (c + w + \lambda)\int_0^{\infty} e^{-\mu t} e^{(\mu + \alpha - c - w - \lambda)t} S \,\mathrm{d}t \\ &= \frac{c + w + \lambda}{c + w + \lambda - \alpha} S \quad . \end{aligned} \tag{16}$$

This value exceeds the market value of the assets without management, S,[10] and the value of the fund computed in (15). The presence of performance fees affects the value, because they are removed from the funds under management and no longer earn the premium, α.

In Section III below, we provide some typical numerical values for the contract, after determining relevant value for the parameters in the next section.

II. Model Parameters

To address the question of what are reasonable parameter values for the valuation model, we turn to the database of hedge fund returns used in Brown, Goetzmann, and Ibbotson (1999) (hereafter BGI). The data are annual returns and fund characteristics gathered from the 1990 through 1996 volumes of the *U.S. Offshore Funds Directory*. Offshore funds in the directory represent a substantial portion of the hedge funds in operation and include most of the major managers.[11]

A. Fund Volatility

To estimate the fund volatility, we calculate the sample standard deviation for all funds. Of 610 hedge funds in the sample, 229 have return histories exceeding two years. Of this group, the median and mean sample standard deviation are 18.7 percent and 23.0 percent per year, respectively. There are two reasons why such a small percentage of funds have enough data to calculate volatility. First, many funds have started recently, so a large number of the extant funds have only a short track record. Second, the attrition rate for funds is relatively high — about 20 percent of these funds fail each year. Since we are effectively conditioning upon fund survival, we are presumably losing the funds that had such poor returns that they failed in their second year. This may bias our volatility estimate downward.

B. Withdrawal Rate, w, and the liquidation parameters, λ and b

In our model, the payout policy w is a flow; however, it is unlikely that all hedge fund investors conceive of it that way. A constant payout ratio is a reasonable assumption for certain institutional investors such as university endowments and charitable foundations, which choose payout ratios as a matter of policy; however, it may not be a reasonable assumption for the most common type of hedge fund investor —

[10] Note that the total withdrawals from the assets, $c + w + \lambda$, must exceed the superior performance, α; otherwise, the fund will have a residual value at infinity whose present value is infinite. A similar result is true with performance fees, although the exact value of α for which this transversality violation occurs is higher and depends on the other parameters as well, since the performance fees also limit the growth of the asset value.

[11] See Brown, Goetzmann, and Ibbotson (1999) for a complete discussion of the coverage of the database.

traditionally, a high-net worth individuals. Life-cycle issues are potentially important, and in addition, modeling the conditional probability of withdrawal may be useful in determining a realistic value for *w*.

BGI estimate that the annual attrition rate for funds is 20 percent per year. This estimate is conditioned upon the fund appearing originally in the annual database. Thus, it neglects funds that started and disappeared before year end. Consequently, a fund must survive through the end of the year of its inception to be counted. This would suggest that new funds have a probability of liquidation greater than 20 percent. By the same token, some of the largest funds, such as the Quantum fund, are long-lived. The 20 percent is not a dollar-weighted estimate of fund disappearance. Consequently, the probability of liquidation might be lower than 20 percent on a dollar-weighted basis. In any case, individual investors may have completely different "rules" for liquidation, which are not captured by the aggregate disappearance rate.

In addition, it is difficult to separately estimate λ and b. Without information concerning the reason for the liquidation from any given hedge fund, we cannot tell if the intention was to liquidate when the assets fell to this level or if the liquidation was a chance occurrence (captured by λ). And, of course, it is the intentions and expectations of the participants that determine the pricing. It is our belief that most investors expect to liquidate if the assets do not perform well. To cover as wide a range as possible, we look at liquidations policies of $b = 0, 0.5, 0.8$. The value $b = 0$ corresponds to no asset-based liquidations. We present these numbers mostly for the purpose of comparison.

We suspect that many investors (apart from the managing partners) would liquidate if the asset value fell by 15 percent to 25 percent from their personal high-water mark. This corresponds to $b = 0.85$ to 0.75. We report present values for $b = 0.8$. These numbers should be representative of the present value of the management fee contract of a single hedge fund. They may not, however, be appropriate to measure the total hedge fund cost for an investor who merely transfers his money to a different hedge fund when liquidating. The middle value, $b = 0.5$, can be interpreted as giving the total present value to the investor of the management fees from investing in a series of hedge funds. The investor withdraws his money completely from hedge funds only when the value of the assets falls to 50 percent of the original investment's high-water mark.[12]

C. Performance Fee, k*, and Regular Annual Fee,* c

The vast majority of the funds have performance fees of 20 percent. In 1996, this was true of 213 of

[12] In fact, the numbers reported for $b = 0.5$ understate the present value of the costs of management fees under this scenario. When the investor moves his money to a new hedge fund, he also "agrees" to lowering the high-water mark to the current asset value. The effect of this adjustment is discussed in Section IV.

the 301 funds. The fees ranged from zero to 42.5 percent. Regular annual fees are usually around one to two percent. In 1996, 254 of the 301 funds had fees in this range, with the former being slightly more common. The annual fee rates ranged from 0.5 percent to 6 percent. A natural question is what factors differentiate funds on the basis of fees. We tried volatility, past performance, and fund size as predictors, and found none to explain differences in performance fees.

III. Interpretation of Model

Table I shows the present value of the management fees and costs as a fraction of the asset value for typical parameter values $r + c' - g = 5$ percent, $k = 20$ percent, $c = 1.5$ percent, $w + \lambda = 5$ percent and 10 percent, $\sigma = 15$ percent and 25 percent, and $b = 0$, 0.5, and 0.8. The values of both fees types are increasing in S since the fees paid are always in proportion to the asset value. The performance fee increases more than proportionally to S since the higher is S/H, the shorter will it be until the high-water mark is hit again and the performance fee can be collected. The regular annual fees are affected differently by the ratio S/H. When $b = 0$, the proportional value of the regular annual fees is decreasing in this ratio, since any payment from the fund's assets (like the performance fee) reduces the value of the assets and hence the value of future regular annual fees. But, the former effect is stronger, and the value of the two fee components together is increasing in the ratio S/H. When there is a liquidation barrier ($b > 0$), then the value of the regular fees is increasing in the ratio for small values of S/H because hitting the barrier cancels all future fee payments so a nearby barrier reduces the value of future fees. For the same reason, the total value of the fees and each component separately is decreasing in the liquidation barrier, b.

Place Table I here

The value of the fees and of each component is decreasing in the withdrawal rate $w + \lambda$, since any reduction in the asset value decreases the base on which these fees are paid. The value of the performance fee is generally increasing in σ, since they have option-like characteristics. The exception is when the asset value is close to the liquidation barrier. An increase in σ decreases the average time until the liquidation barrier is hit and if the barrier is close, this effect dominates the other and the performance fees value is decreasing in σ. The value of the regular annual fee is decreasing in σ. As σ increases, the average time before the liquidation barrier or the high-water mark is hit decreases. Both of these events decrease future annual fees, the former because the contract is canceled, the latter because the performance fee payment decreases the asset value.

Due to the perpetual nature of the investment problem, the present values of the fees are very sensitive to the withdrawal policy, $w + \lambda$. As seen in Table I, an increase in the withdrawal rate from 5 to 10 percent decreases the value of the regular annual fees by about 40 percent and the performance fees by 30 to 50 percent (except with the highest liquidation barrier $b = 0.8$). The effect is stronger in percentage terms at lower volatilities, though the decrease in the dollar value of the fees is greater at larger volatilities. Notice that for low asset volatility, the regular annual fee portion of the compensation is the dominant source of value, particularly when the withdrawal rate, $w + \lambda$, is small. This is not surprising, since the "option" value is increasing in σ and the present value in perpetuity of the regular fees is decreasing in w. This suggests that manager compensation contracts may separate according to the volatility of the strategies and investment outflows.

The present value of the fees is a large fraction of the value of the assets under management. Even with a sizable withdrawal rate, the fees can be expected to absorb one-fifth to one-third of the funds assets. Whether the manager provides investment advice commensurate with these fees is addressed below. Of course, even without a performance fee, the fraction of wealth paid as fees is very high. For instance, in the simple case with no performance fee or liquidation barrier, costs and fees come to the fraction $c/(w + \lambda + c)$ of the asset value. With a five percent payout and a 1.5 percent regular annual fee, this is 23 percent of the asset value.[13] With a 20 percent performance fee, the cost of the regular annual fee drops to 20.1 percent, but the performance fees are worth 13.1 percent bringing the total present value of the fees to 33.1 percent of the assets under management. Thus, even low regular annual fees claim a non-trivial proportion of investment assets.

Table II gives the value of the regular annual fees, the performance fees, and the investor's claim when the manager provides a premium return of α = 300 basis points. The cases are the same as given in Table I. The fees are worth more with a premium return because the assets will grow at a faster rate and both provide a higher base on which the fees are paid and exceed the high-water mark more often. The total value of the fees and the investor's stake sum to more than 100 percent because the manager's ability to earn a premium return means the managed assets are worth more than their market value. Note that the values of the fees are less with a higher withdrawal rate since this reduces the assets on which the fees are based. Conversely, a higher withdrawal rate increases the value of the investor's stake under most circumstances for the same reason. However, when the investor's portion by itself is worth somewhat more than the market value of the assets, then *reducing* the withdrawal rate may increase the investor's value since withdrawing assets prevents the premium return, α, from being earned on them.

Place Table II here

[13] Of course, the regular fee is not all profit to the manager. It must cover management expenses. For active managers, these costs may be high. Even a low-cost equity index fund may have expenses of 40 basis points. With the payout rule of 5 percent, index fund expenses translate into an eight percent fraction of the investor wealth. With a payout ratio equal to current dividend yields, this fraction increases to about 13 percent.

Similarly, unlike the case $\alpha = 0$, a higher liquidation barrier is not always beneficial to the investor when a premium is being earned. Withdrawing assets from the fund recovers the full asset value, but also prevents any sharing of the future premium earnings. As shown in the first and second panels of Table II, both the investor and the managers would prefer no liquidation ($b = 0$) to liquidating when the assets drop to half the high-water mark. Obviously, the managers would prefer to close the entire fund and start a new one (with $H = S$) assuming, less obviously, they could convince these or other investors to provide them with the money. In addition, the decision to liquidate is actually endogenous and clearly depends on perceptions about the excess performance to be provided. Once the assets drop to half of the high-water mark, the investor may no longer believe that the managers can provide excess performance.

This also means that the high-water mark contract may create a type of lock-in for underperforming hedge funds. Consider two hedge funds with the same parameter values as give in Table II ($w+\lambda = 10$ percent, $\sigma = 15$ percent, $b = 0.5$), but assume one hedge fund has an α of 2.5 percent and the other has an α of three percent. Suppose an investor holds \$70 million in the lower performing fund with a high-water mark of \$100 million. His position is currently worth \$70.40 million. If he moved it to the better-performing fund and could keep the \$100 million high-water mark, it would be worth 1.0467×\$70 million = \$73.3 million. However, moving the money to a new fund would establish a new high-water mark of \$70 million so, in the new fund, the assets would be worth only 1.0016×\$70 million = \$70.11 million. This decrease in the value is due to the write-down of the high-water mark and is what creates the lock-in.

How does a high-water mark contract compare to a simple, regular annual fee contract? Absent any incentive differences induced by the contracts, it is possible to characterize the trade-off between a higher regular annual fee and the performance fee that gives a fixed total fee value of F. This comparison should be made at the inception of the contract when $S = H$. Solving (11a) for the performance fee, k, gives

$$k(F) = \frac{(1-b^{\gamma-\eta})(w+\lambda+c-\alpha)(1-F/S)}{(w+\lambda-\alpha)(1-b^{\gamma-\eta}) - cb^{1-\eta}(\gamma-\eta) - \left[\frac{F}{S}(c+w+\lambda-\alpha)-c\right]\left(\gamma - b^{\gamma-\eta}\eta\right)} - 1, \qquad (17)$$

which, along with the regular annual fee, c, is the compensation required to make the total of the fees worth F. Assuming that investors are indifferent among contracts that cost the same, this fixed point provides a measure of the trade-off between the two fee types.

Table III shows the tradeoffs for a representative set of parameters for a benchmark case of a contract with a 20 percent performance fee, a 1.5 percent regular annual fee, and no superior performance. The table shows the fee structures that would have the same present value for various withdrawal rates and volatilities. For example, with a withdrawal rate of five percent, a volatility of 15 percent, and no liquidation, the benchmark incentive contract has the same cost as a one percent regular annual fee contract with a performance fee of 31.24 percent or a 2 percent regular fee and a 9.44 percent performance fee. This trade-off is valid at the inception of the contract (or whenever $S = H$); when the asset value is below the high-water mark, then the incentive contract has a smaller value so the performance fee, k, would have to be larger. However, the comparison is properly made at the inception of the contract when S is equal to H.

Place Table III here

As shown in the table, this trade-off is dramatically affected by the volatility of the assets and the possibility of liquidation, but not so much by the withdrawal policy $w + \lambda$. With asset volatility at 25 percent, the investor would be willing to pay a 27.62 percent performance fee to reduce the regular annual fee to one percent. If the contract were to be liquidated if the asset value fell to half the high-water mark, the investor would be willing to pay a 29.02 percent performance fee to reduce the regular annual fee to one percent.

An important question for both the investor and the manager is how large a performance fee is justified by a given level of performance. The active manager's contribution will just merit its cost to the investor if the value of the investor's claim equals S when the contract commences. Therefore, the excess return required to make the investor indifferent about entering into the compensation contract at the start of the fund is the solution to $I(S, S) = S$. Using (11c) to solve $I(S, S) = S$ for k gives the maximum high-water performance fee justified by a particular α. This is

$$k^*(\alpha) = \frac{1-\gamma+(\gamma-\eta)b^{1-\eta}+(\eta-1)b^{\gamma-\eta}}{\gamma-(\gamma-\eta)b^{1-\eta}-\eta b^{\gamma-\eta}-\dfrac{w+\lambda}{\alpha-c}\left(1-b^{\gamma-\eta}\right)} \quad . \tag{18}$$

Equation (18) gives the maximum performance fee rate. Hedge funds may well charge less than this. If different hedge funds compete for the same capital, they may charge substantially less than the rates indicated here. In particular, if investment capital is a scarce resource relative to potential hedge fund managers, virtually all benefits of the hedge funds may go to the investors. Then, managers might be earning fees only just sufficient to draw them into the business.

Table IV shows the maximum performance fee justified by a given α for different levels of asset volatility and withdrawal policies. The justified fee rate is decreasing in σ, since for a given rate, the value of the fee is larger for a larger volatility due to the option characteristic of the valuation equation. When the withdrawal rate increases, the performance required to compensate the investor at a given fee also increases. It may seem strange that better performance is required with higher withdrawal rates, but recall that α measures the extra return per unit time. With a higher withdrawal rate, the funds are managed for a shorter period of time on average, so a high per-period premium must be earned to offset the fee discount in value.

Place Table IV here

For an asset volatility of 15 percent, the required excess return is 300 to 400 basis points to justify a performance fee of 15 to 20 percent. For an asset volatility of 25 percent the excess return required to justify a performance fee of 20 percent ranges from 350 to 750 basis points.[14] This is certainly within the range of the performance provided by many hedge funds, at least in the early 1990s. Whether it is consistent with investors' beliefs *ex ante* is more difficult to determine.

Although it seems natural to identify the manager's contribution in terms of a positive additional rate of return (*i.e.*, an alpha), this might not be the appropriate way of considering the benefits to investing in a hedge fund. The benefits expressed by alpha are linear in the capitalization of the fund, but hedge funds might in fact provide decreasing returns to scale. An alternative way of thinking of hedge funds is that they are firms that can capture a fixed amount of "arbitrage" profits in the economy. In other words, they have a limited net present value. The choice of how to finance this venture is a capital structure decision. From this perspective, the issuance of additional shares has a diluting effect on the outstanding claims — investors simply divide a fixed pie of arbitrage gains. In this framework, new money — *i.e.*, a positive flow of funds into the account from new investors — has only limited attraction to the hedge fund manager. It benefits him only to the extent that he is unable to borrow the funds his activities require or to the extent that he fears bankruptcy through a margin call.

IV. Extensions to the Model

The model developed here can be extended in many ways to capture additional features of interest. Many people have claimed that the convex payoff structure in hedge funds fees creates an incentive for the managers to take on excess risk and, in particular, to take on more risk when the asset value is substantially below the high-water mark. Carpenter (2000) has proven this is optimal behavior when the compensation is with an option-like payoff based on the portfolio's terminal value.

If the fee structure induces the manager to alter the portfolio, the volatility of the managed assets may not be constant but vary systematically with asset value. Rather than assume a particular functional form for the managed volatility, we adopt a simple but general approach that allows the managed volatility to have a wide variety of forms. We divide the range $S \in (bH, H)$ into N regions $\zeta_{n-1}H < S < \zeta_n H$ with $\zeta_0 = b$ and $\zeta_N = 1$. The volatility can be different in each region

[14] If the hedge fund is compared to a passive investment with fees or other costs like an index fund, then a more appropriate measurement of required superior performance might be α less these fees. Typically, costs for index funds are very low. For example, Vanguard Index Trust has costs of less than 20 basis points.

$$\sigma(S,H)=\sigma_n \quad \text{for} \quad \zeta_{n-1}<S/H<\zeta_n \quad . \tag{19}$$

If Carpenter's (2000) analysis applies, then we should find σ large for the very low regions of asset value; however, we permit any relations amongst the various values of the volatility.

The value of the investor's claim and the various components of the fees still satisfies the same pricing equation with the same general solution in each range. The parameter σ is of course different in each solution. The lower boundary condition for the first region and the upper boundary condition for the highest region are as before. The extra boundary conditions are that the functions and their first derivatives must match across each change of region. For example, the boundary conditions for the total fees are

$$\begin{aligned} &F_1(bH,H)=0 && \left[k\frac{\partial F_N(S,H)}{\partial S}-\frac{\partial F_N(S,H)}{\partial H}\right]\Bigg|_{S=H}=k \\ &F_n(\zeta_n H,H)=F_{n+1}(\zeta_n H,H) && \frac{\partial F_n(S,H)}{\partial S}\bigg|_{S=\zeta_n H}=\frac{\partial F_{n+1}(S,H)}{\partial S}\bigg|_{S=\zeta_n H} \quad . \end{aligned} \tag{20}$$

The boundary conditions for the performance fees alone are the same. The region-matching boundary conditions for the investor's claim are also the same. The first and second boundary conditions for the investor's claim are $I_1(bH, H) = bH$ and $[\partial I_N/\partial S - \partial I_N/\partial H]|_{S=H} = 0$.

The value of the annual fees, performance fees, total fees, and investor's claims in the nth region are

$$\begin{aligned} A_n(S,H)&=\frac{c}{c+w+\lambda-\alpha}S+K_n^A H^{1-\gamma_n}S^{\gamma_n}+B_n^A H^{1-\eta_n}S^{\eta_n} \\ P_n(S,H)&=K_n^P H^{1-\gamma_n}S^{\gamma_n}+B_n^P H^{1-\eta_n}S^{\eta_n} \\ F_n(S,H)&=A_n(S,H)+P_n(S,H) \\ I_n(S,H)&=\frac{w+\lambda}{c+w+\lambda-\alpha}S+K_n^I H^{1-\gamma_n}S^{\gamma_n}+B_n^I H^{1-\eta_n}S^{\eta_n} \quad . \end{aligned} \tag{21}$$

The parameters γ_n and η_n are given in equation (10) for the various values of σ_n. The constants of integration are

$$(K_1^J, B_1^J, K_2^J, B_2^J, \dots, K_N^J, B_N^J)' = \mathbf{M}^{-1}\mathbf{m}^J \ , \tag{22}$$

where
$$\mathbf{m}^A \equiv \frac{-c}{c+w+\lambda-\alpha}\left(b,0,\dots,k\right)' \qquad \mathbf{m}^P \equiv \left(0,\dots,0,k\right)'$$
$$\mathbf{m}^I \equiv \frac{1}{c+w+\lambda-\alpha}\left(b(c-\alpha),0,\dots,-k(w+\lambda)\right)'$$

and

$$
\mathbf{M} \equiv \begin{pmatrix}
b^{\gamma_1} & b^{\eta_1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\zeta_1^{\gamma_1} & \zeta_1^{\eta_1} & -\zeta_1^{\gamma_2} & -\zeta_1^{\eta_2} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\gamma_1\zeta_1^{\gamma_1-1} & \eta_1\zeta_1^{\eta_1-1} & -\gamma_2\zeta_1^{\gamma_2-1} & -\eta_2\zeta_1^{\eta_2-1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \zeta_2^{\gamma_2} & \zeta_2^{\eta_2} & -\zeta_2^{\gamma_3} & -\zeta_2^{\eta_3} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_2\zeta_2^{\gamma_2-1} & \eta_2\zeta_2^{\eta_2-1} & -\gamma_3\zeta_2^{\gamma_3-1} & -\eta_3\zeta_2^{\eta_3-1} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & \zeta_{N-1}^{\gamma_{N-1}} & \zeta_{N-1}^{\eta_{N-1}} & -\zeta_{N-1}^{\gamma_N} & -\zeta_{N-1}^{\eta_N} \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & \gamma_{N-1}\zeta_{N-1}^{\gamma_{N-1}-1} & \eta_{N-1}\zeta_{N-1}^{\eta_{N-1}-1} & -\gamma_N\zeta_{N-1}^{\gamma_N-1} & -\eta_N\zeta_{N-1}^{\eta_N-1} \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & (1+k)\gamma_N - 1 & (1+k)\eta_N - 1
\end{pmatrix}.
$$

This valuation of the fees permits an analysis of the incentives to manage volatility.

When there is no lower boundary liquidation ($\zeta_0 = b = 0$), then the solution to (21) shows that the value of fees is higher in all regions for larger volatilities. However, in this case, there is also no incentive to micro-manage the volatilities. The volatility in each region should be set as high as possible if the goal is to maximize the present value of future fees.

When there is liquidation at low asset values ($\zeta_0 = b > 0$), then volatility should be managed. In particular, since the fee value is zero at liquidation, the volatility should be reduced as the asset value drops near the liquidation level to ensure that liquidation does not occur. At higher asset values, a larger volatility should be adopted to increase the value of the performance fee based on the high-water mark (again assuming a goal of fee value maximization). This conclusion is inconsistent with that of Carpenter (2000) in which volatility goes to infinity as asset value goes to zero.

For example, consider a hedge fund with two regimes; the volatility differs when the asset value is below or above 75 percent of the high-water mark ($\zeta = 0.75$). If the withdrawal rate is $w + \lambda$ = five percent, $b = 0.5$, and the other parameters are as given in Table III, then the volatility should optimally be decreased to 8.6 percent from 15 percent when the asset value drops into the lower regime. If the volatility regime changes at $\zeta = 0.8$ or 0.7 instead, then the lower regime volatility should be set to 9.2 percent or 7.8 percent, respectively.

A similar procedure can be applied if the withdrawal rate or liquidation probability, w or λ, changes with asset value or in response to performance. Of course, further analysis should be done to determine how investors might choose these parameters endogenously.

We can also use this region-based method to determine the total cost of being invested in a series of hedge funds. As mentioned in footnote 12, the implicit cost of investing in a first hedge fund exceeds the present value of the fees paid to just that one fund if the money is to be withdrawn after poor performance and invested into a second hedge fund. When this transfer is made, the high-water mark for the new fund is equal to the amount transferred, but the high-water mark of the fund from which the money has been withdrawn is higher, and probably substantially higher since the reason for withdrawal is poor performance. This step-down in the high-water mark is an additional cost facing the investor, because performance fees will be owed when the asset value rises above the high-water mark — something that occurs sooner with the lowered high-water mark. The investor is willing to “pay” this additional cost since he presumably no longer believes the second fund has an alpha that is sufficiently higher to justify it.

Suppose money is withdrawn from the first fund and invested into a second if the asset value drops to b_1H. Assume, for the moment, that the money will be withdrawn from this hedge fund and not reinvested if the asset value drops further to the fraction b_2 of the new fund’s high-water mark. Then the present values generated by this second fund are just the solutions as given before. We will refer to the investor’s claim as $I_2(S, H; b_2)$. The present value of the investor’s claim while in the first fund, $I_1(S, H)$, includes the step-down cost of resetting the high-water mark. The factor $I_1(\cdot)$ is the solution to the standard pricing equation[15] as well, with the boundary condition

$$I_1(b_1H, H; b_1, b_2) = I_2(b_1H, b_1H; b_2)\,, \tag{23}$$

which is used in place of $I(bH, H) = bH$. The right-hand side of (23) is the present value of the claim on the second fund when the investment is transferred and high-water mark is reset equal to the value of the money transferred.

The present value of the investor’s claim when invested in the first of two funds with the same parameters is

$$I_1(S,H) = \frac{w+\lambda}{c+w+\lambda-\alpha}S + K_1 H^{1-\gamma}S^{\gamma} + B_1 H^{1-\eta}S^{\eta}$$

$$\text{where}\quad K_1 \equiv -\frac{k(w+\lambda)b_1^{\eta-1} + [\eta(1+k)-1]\Psi}{(c+w+\lambda-\alpha)\left([\gamma(1+k)-1]b_1^{\eta-1} - [\eta(1+k)-1]b_1^{\gamma-1}\right)} \tag{24}$$

$$B_1 \equiv \frac{[\gamma(1+k)-1]\Psi + k(w+\lambda)b_1^{\gamma-1}}{(c+w+\lambda-\alpha)\left([\gamma(1+k)-1]b_1^{\eta-1} - [\eta(1+k)-1]b_1^{\gamma-1}\right)}$$

$$\Psi \equiv \frac{(b_2^{\gamma-\eta}-1)(w+\lambda)k + (c-\alpha)b_2^{1-\eta}(1+k)(\gamma-\eta)}{\gamma(1+k)-1-b_2^{\gamma-\eta}[\eta(1+k)-1]}\quad .$$

[15] The withdrawal rate, costs, alpha, or other parameters can be different for the two funds. We use the parameters of the second fund to determine $I_2(\cdot)$. We use the parameters of the first fund in the partial differential equation to determine the total present value from both funds, $I_1(\cdot)$. The parameters of the second fund are captured in the total value through the boundary matching condition.

If the investor uses a series of more than two funds, then the same method is applied sequentially. The present value of the benefits of the final fund are determined in the usual way. The present value of the benefits of the last two funds are determined as just described with the boundary condition $I_{N-1}(b_{N-1}H, H; b_{N-1}) = I_N(b_{N-1}H, b_{N-1}H; b_N)$. This procedure is repeated for all earlier hedge funds. In this case, $I_n(\cdot)$ represents the present value of the benefits of the n^{th} and all subsequent funds.

V. Incentives and New Money

Over the long term, the real compensation function of the manager depends on both the explicit contract and implicit relation between performance and capital inflows. Because the technology of hedge funds is different from that of mutual funds, the performance-flow relationship may potentially be different from that observed in the mutual fund industry. In a stylized framework in which the hedge fund manager identifies and exploits limited arbitrage-in-expectations opportunities, capital can be put to profitable use without incurring systematic risk only up to a point. Beyond that point, presumably the manager has no comparative advantage. A natural question is whether a performance fee structure might induce the manager to accept investment beyond the point at which the capital can be used efficiently. While a high regular annual fee and performance fee might tempt such behavior, it may be difficult for a manager to conceal the risk characteristics of the portfolio for long. Increasing systematic risk exposure by a hedge fund would presumably indicate that the limits to skill at pure arbitrage in expectations have been reached. Performance fees may exist to offset a possible negative relationship between performance and capital inflow.

Do hedge funds take new money when they do well? If the manager’s technology were linear, then on balance, more money would be welcome at any time. If, instead, there is a declining schedule of profitable arbitrage opportunities, then new money would be accepted when the fund decreased in scale, rather than when it grew, at least for large funds. To test the hypothesis that hedge fund managers do not accept new money when they do well, we examine the relationship between flow of funds and past performance for hedge funds by regressing net fund growth on lagged return in cross section. If managers accept new money after a good year, and/or investors pull out of poorly performing funds, we would expect to find a positive regression coefficient. On the other hand, if managers refuse new money after a good year, and seek additional funding after a bad year, then we would expect to find a negative regression coefficient on past returns. We define net fund growth as the increase in net asset value of the fund due to the purchase of new shares, as opposed to the investment return of the fund. This requires us to make the simplifying assumption that new shares are purchased only at the beginning of the year — purchases during the year will be interpreted as investment return.[16] Another problematic issue is survivorship. Although we have defunct fund data, we must make some assumption regarding the fund outflow in the year of its disappearance. We address survival issues by assuming a 100 percent outflow in the year a fund closes. We control for year effects by performing the regression separately for each year, and also by including year dummies for the stacked regression.

A. New Money Regression Results

Besides estimating a single linear response, we also consider how the response differs depending upon past fund performance. Following Sirri and Tufano (1992) and Goetzmann and Peles (1997) we examine the differential response of new money to past returns via a piecewise linear regression. We separate fund return in cross section into quintiles each year, and allow the coefficients to differ across quintiles. We test for the equality of the coefficients across quintiles via a Chow test. The results for the single response regression are reported in the first panel of Table V, and the results for the piecewise regression are reported in second panel of Table V. The year-by year results for the piecewise regression are reported in Table VI.

Place Tables V and VI here

The results from panel 1 indicate that new money responds negatively to past positive performance. The response differs across quintiles of lagged returns, however. The best and worst performers have quite different coefficients. Panel 2 shows that new money responds by flowing out of the poorest performers as one might expect. The positive sign of the regression coefficient for the first quintile indicates that flows in the subsequent period have the same sign as the returns in the preceding period. However, money also flows out of the good performers. Funds in the top quintile show a negative response to positive performance. These results are quite different from the pattern observed in mutual funds. Sirri and Tufano (1992), Chevalier and Ellison (1995) and Goetzmann and Peles (1997), for example, all find money flows into top performers and flows into poor performers as well. This is exactly the opposite of what we find for our hedge fund sample. The negative response to top performance we find in the hedge fund universe provides some support for the hypothesis that good performers may not readily accept new money.

Although a Chow test rejects equality of the coefficients across the quintiles, the t-statistics on the smallest quintile in our sample are marginal at the 5 percent level, meaning we should be cautious about interpreting the positive coefficient as strong evidence of a negative response to poor performance. In fact, in

[16] When we assumed that money flowed in at the end of the period, the results were essentially the same.

Table VI, the year-by-year regression results indicate that the pattern differs considerably over time.

B. Sorting on Size

Another approach to the issue of whether the technology of hedge funds is linear is to test whether larger funds continue to take new money. We can address this question simply by sorting on size, and then averaging a measure of new money. Table VII reports the results of this exercise. We break funds into size quintiles in the first period, and then we average the net growth of the fund in the following period for each quintile. We define growth slightly differently, under the assumption that money flows in at the end of the period. As in the previous test, we find this change makes no difference in our results. Table VII shows that the largest size funds have net cash outflows, while the smallest performers have net cash inflows. Unlike the flow of funds regressions above, this pattern is relatively consistent throughout the period, with negative flows for large funds and positive flows for small funds each year. The second panel of the table shows the results of t-tests for each group, annually as well as in the aggregate – the extreme quintiles have means different from 0. As in the previous test, this pattern is consistent with the story that well-capitalized funds avoid taking new money. It differs in that it is also consistent with the hypothesis that smaller funds raise capital. Since we did not sort on performance, many of the funds in the first quintile may be good performers and thus able to raise new money, or stop funds from flowing out.

Place Table VII here

Taken together, the empirical tests suggest that hedge fund managers may behave differently from mutual fund managers with respect to accepting new money. While mutual funds demonstrate dramatic positive inflows into superior performers, this appears not to be the case with hedge funds. In addition, large funds do not seem to grow at a rate as high as smaller funds — even when growth is measured in dollar terms rather than percentage terms. We conjecture that this may be due to the limits of the investment strategies employed by hedge fund managers. To the extent that they engage in "arbitrage in expectations," success creates its own limitations.

VI. Conclusion

Hedge funds are an interesting investment class with an unusual form of manager compensation. In this paper, we provide a closed-form expression for the cost of a hedge fund manager contract and examine its implications to both the manager and the investors. We also provide estimates of the typical parameter values for the equation. The high-water mark provision creates a distinct option-like feature to the contract. Our computed cost of the contract increases in the variance of the portfolio when the asset value of the fund is not far below the high-water mark. As a result, the manager has an incentive to increase risk, provided other non-modeled considerations are not overriding.[17]

Depending on the variance, the performance fee effectively "costs" the investors 10 to 20 percent of the portfolio. Including the regular fees, the total percentage of wealth claimed by the hedge fund manager can be between 30 and 40 percent. Investing with a hedge fund manager would only appear to be rational if he or she provided a large, positive risk-adjusted return in compensation. In particular, we find that rational investors would expect 200 to 500 basis points in additional risk-adjusted return (alpha) when they enter into a hedge fund contract. Interestingly, BGI report that alphas for hedge funds over the 1989 through 1995 period are positive, and range from four to 8 percent annually. Consequently, hedge fund contracts may be priced about right.

How exactly are we to interpret these valuation formulas? They were derived by standard continuous-time pricing methodology. From the investors' viewpoint, this can be interpreted as a Black-Scholes hedging derivation, but generally the manager cannot do any form of delta hedging. Indeed, the very form of the contract is no doubt designed to solve some complex agency problem. We have argued, though, that it is probably best to think of the contract between the manager and the investor more as a way of financing the firm than as an incentive contract for the manager. In that light, then, the exact contract form will be a function of all of the different forces that impact corporate form, in addition to the usual culprits of moral hazard and observability.

Despite these caveats, though, the valuation formulas do determine the cost to the market of investing in the hedge fund, given any particular withdrawal policy. This permits us to derive the alpha required to offset this cost. Alpha, in turn, is a function both of the manager's abilities and of the incentives implicit in the contract, so ours is a partial equilibrium analysis that takes a given contract form — the one prevalent in the market — and examines the required alpha generated. Any exercise that changes the parameters of the contract will generate a new required alpha. Candidate equilibrium contracts will be fixed points in which this unanalyzed mapping from incentive contracts to alphas generates the same alpha as the market cost of the contract, which we have derived. Furthermore, given the withdrawal policy, the valuations we obtain describe the market (or risk-neutral) value to the manager. This is appropriate for a firm that offers such a fund, but since the contract cannot be monetized without negating its incentive value, the actual value to a risk averse manager would be lower.

In considering why high-water mark contracts exist in the hedge fund industry, we considered how

[17] An obvious non-modeled feature is the effect of this increased risk on a risk-averse manager who cannot hedge or diversify away the increased risk.

hedge funds differ in terms of the product they offer. An analysis of the relative benefits of the regular annual fee vs. the performance fee to the manager suggests that high variance strategies and strategies for which the investors may pull out soon, lend themselves to high-water mark contracting. The relative value of the regular annual fee portion on the contract decreases as the time until the investor's withdrawal decreases. Empirical evidence on the short half-life of hedge funds may explain why hedge fund managers choose to use high-water mark contracts.

It has become nearly axiomatic in studies of the investment management industry that managers seek to increase the size of assets under management. This presumes, however, that the benefits to investment in the fund can be scaled up with the growth in net asset value. While our sample focuses on the off-shore hedge fund industry, there are important regulatory limits to growth in the on-shore hedge fund industry. Until 1996, the number of investors in a hedge fund was limited to 99. Recent regulatory changes have increased this limit to 499 under certain conditions, but even this limit is likely to effectively cap the growth in assets that a fund manager might expect. Hedge fund strategies are fundamentally different from "long" asset portfolio strategies, however. Large sectors of the hedge fund industry have nearly zero "beta" exposure. Many hedge funds use the invested money as margin for maintaining offsetting long and short positions. Hedge fund managers are made up of event arbitrageurs, global macro market and debt market speculators, pairs traders and opportunistic managers exploiting undervalued securities. They use leverage of all types to exploit these opportunities – from short-selling equities to sophisticated debt repurchase agreements. In this context, the dollar investment benefits the manager only to the extent that he is credit constrained in his strategy. By their very nature, arbitrages in expectations are not infinitely exploitable.

Since it is not possible to directly investigate the relationship between scale and strategy payoff, we use flow of funds, return, and size data from the hedge fund industry over the period 1989 through 1995 to explore the issue of linear vs. non-linear returns to scale. Regression of net growth in fund assets on lagged returns indicates that, unlike the mutual fund industry, the hedge funds show a net decrease in investment, conditional upon past performance. We conjecture that this is due to the manager's unwillingness to increase the fund size. A sort on fund size, however, shows that small funds tend to grow (net of returns), while large funds tend to shrink.

This pattern may help explain the usefulness of the high-water mark compensation to the hedge fund manager. While mutual fund managers and pension fund managers can increase their compensation by growing assets under management, hedge fund managers cannot. Thus, they must explicitly build in benefits conditional upon positive returns, since they appear to resist net growth.

The implications of these results extend beyond the issue of the cost of compensation within an important sector of the investment industry. The existence of high-water mark contracts may in fact be a signal to investors that the returns in the industry are diminishing in scale. Option-like incentive contracts are scarce in the mutual fund industry and pension fund management industry, but are prevalent in the real estate sector, the venture capital sector, and the hedge fund sector. Perhaps the compensation structure itself is telling us that future returns in these asset classes depend crucially upon how much money is chasing a limited set of unique opportunities.

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Table I
Values of Fees and Investor's Claim

Value of fees and investor's claim as a percentage of the asset value of the fund. Values are computed using the formulas in (11). Parameter values are $r + c' - g$ = 5 percent, k = 20 percent, c = 1.5 percent, $\alpha = 0$, σ = 15 or 25 percent in left and right panels respectively, $w + \lambda$ = five or 10 percent in top and bottom panels, respectively. The liquidation point is b = 0, 0.5, 0.8 in the three sections.

$$f(S,H) \equiv \frac{F(S,H)}{S} = \frac{1}{c+w+\lambda-\alpha}\left[c + \frac{(w+\lambda-\alpha)k + [\eta(1+k)-1]cb^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\gamma-1} - \frac{b^{\gamma-\eta}(w+\lambda-\alpha)k + [\gamma(1+k)-1]cb^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\eta-1}\right]$$

$$p(S,H) \equiv \frac{P(S,H)}{S} = k\frac{(S/H)^{\gamma-1} - b^{\gamma-\eta}(S/H)^{\eta-1}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]} \qquad a(S,H) \equiv \frac{A(S,H)}{S} = f(S,H) - p(S,H)$$

For $\alpha = 0$: $i(S, H) = 1 - F(S, H)/S = 1 - f(S, H)$

$b = 0$

σ = 15% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $P(S, H)$	Total $f(S, H)$
1.0	20.1%	13.1%	33.1%
0.9	20.4%	11.6%	32.0%
0.8	20.7%	10.2%	30.9%
0.7	21.0%	8.8%	29.9%
0.6	21.4%	7.4%	28.8%
0.5	21.7%	6.1%	27.7%

σ = 25% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	18.8%	18.6%	37.4%
0.9	19.1%	17.2%	36.3%
0.8	19.4%	15.8%	35.2%
0.7	19.8%	14.3%	34.1%
0.6	20.1%	12.8%	32.9%
0.5	20.5%	11.2%	31.7%

σ = 15% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $P(S, H)$	Total $f(S, H)$
1.0	11.9%	8.7%	20.6%
0.9	12.1%	7.3%	19.3%
0.8	12.3%	5.9%	18.2%
0.7	12.4%	4.7%	17.1%
0.6	12.6%	3.6%	16.2%
0.5	12.7%	2.6%	15.3%

σ = 25% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	11.4%	12.8%	24.2%
0.9	11.6%	11.4%	23.0%
0.8	11.7%	10.0%	21.7%
0.7	11.9%	8.6%	20.5%
0.6	12.1%	7.2%	19.3%
0.5	12.3%	5.9%	18.1%

Table I
Values of Regular Annual Fees and Incentive Fees (cont)

b = 0.5

σ = 15% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	17.30%	12.31%	29.61%
0.9	17.47%	10.82%	28.30%
0.8	17.22%	9.24%	26.46%
0.7	15.99%	7.40%	23.39%
0.6	12.14%	4.85%	16.98%
0.5	0.00%	0.00%	0.00%

σ = 25% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	9.71%	13.62%	23.34%
0.9	9.74%	12.09%	21.82%
0.8	9.36%	10.29%	19.64%
0.7	8.24%	8.02%	16.27%
0.6	5.69%	4.91%	10.60%
0.5	0.00%	0.00%	0.00%

σ = 15% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	11.05%	8.54%	19.59%
0.9	11.16%	7.06%	18.22%
0.8	11.04%	5.65%	16.69%
0.7	10.34%	4.25%	14.59%
0.6	8.01%	2.64%	10.65%
0.5	0.00%	0.00%	0.00%

σ = 25% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	7.46%	10.97%	18.43%
0.9	7.49%	9.45%	16.94%
0.8	7.22%	7.81%	15.03%
0.7	6.40%	5.94%	12.33%
0.6	4.47%	3.56%	8.03%
0.5	0.00%	0.00%	0.00%

b = 0.8

σ = 15% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	3.73%	5.03%	8.76%
0.9	3.09%	3.11%	6.20%
0.8	0.00%	0.00%	0.00%

σ = 25% $w + \lambda$ = 5%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	1.28%	4.36%	5.64%
0.9	1.03%	2.52%	3.56%
0.8	0.00%	0.00%	0.00%

σ = 15% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	3.36%	4.61%	7.97%
0.9	2.79%	2.78%	5.57%
0.8	0.00%	0.00%	0.00%

σ = 25% $w + \lambda$ = 10%

S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$
1.0	1.23%	4.23%	5.46%
0.9	1.00%	2.43%	3.42%
0.8	0.00%	0.00%	0.00%

Table II
Value of Regular Annual Fees, Incentive Fees and Investor's Claim When Fund Has Superior Performance

Value of fees and investor's claim as a percentage of the asset value of the fund. Values are computed using the formulas in (11). Parameter values are: $r + c' - g$ = 5 percent, k = 20 percent, c = 1.5%, $\alpha = 0$, σ = 15 or 25 percent in left and right panels respectively, and $w + \lambda$ = 5 or 10 percent in top and bottom panels, respectively. The liquidation point is b = 0, 0.5, 0.8 in the three sections.

$$f(S,H) \equiv \frac{F(S,H)}{S} = \frac{1}{c+w+\lambda-\alpha}\left[c + \frac{(w+\lambda-\alpha)k + [\eta(1+k)-1]cb^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\gamma-1} - \frac{b^{\gamma-\eta}(w+\lambda-\alpha)k + [\gamma(1+k)-1]cb^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\eta-1} \right]$$

$$p(S,H) \equiv \frac{P(S,H)}{S} = k\frac{(S/H)^{\gamma-1} - b^{\gamma-\eta}(S/H)^{\eta-1}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]} \qquad a(S,H) \equiv \frac{A(S,H)}{S} = f(S,H) - p(S,H)$$

$$i(S,H) \equiv \frac{I(S,H)}{S} = \frac{1}{c+w+\lambda-\alpha}\left[w+\lambda - \frac{(w+\lambda)k + [\eta(1+k)-1](c-\alpha)b^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\gamma-1} + \frac{b^{\gamma-\eta}(w+\lambda)k + [\gamma(1+k)-1](c-\alpha)b^{1-\eta}}{\gamma(1+k)-1-b^{\gamma-\eta}[\eta(1+k)-1]}(S/H)^{\eta-1} \right]$$

$b = 0$

	σ = 15% $w + \lambda$ = 5%					σ = 25% $w + \lambda$ = 5%			
S/H	Regular *a*(*S*, *H*)	Perform *p*(*S*, *H*)	Total *f*(*S*, *H*)	Investor *i*(*S*, *H*)	*S/H*	Regular *a*(*S*, *H*)	Perform *p*(*S*, *H*)	Total *f*(*S*, *H*)	Investor *i*(*S*, *H*)
1.0	30.9%	27.9%	58.8%	103.1%	1.0	28.4%	33.7%	62.1%	94.8%
0.9	31.4%	26.6%	58.1%	104.8%	0.9	28.9%	32.5%	61.4%	96.4%
0.8	32.0%	25.3%	57.3%	106.7%	0.8	29.5%	31.3%	60.7%	98.2%
0.7	32.6%	23.9%	56.5%	108.7%	0.7	30.0%	29.9%	60.0%	100.1%
0.6	33.3%	22.3%	55.6%	110.9%	0.6	30.7%	28.5%	59.1%	102.2%
0.5	34.0%	20.7%	54.7%	113.3%	0.5	31.4%	26.8%	58.2%	104.6%

	σ = 15% $w + \lambda$ = 10%					σ = 25% $w + \lambda$ = 10%			
S/H	Regular *a*(*S*, *H*)	Perform *p*(*S*, *H*)	Total *f*(*S*, *H*)	Investor *i*(*S*, *H*)	*S/H*	Regular *a*(*S*, *H*)	Perform *p*(*S*, *H*)	Total *f*(*S*, *H*)	Investor *i*(*S*, *H*)
1.0	15.1%	14.6%	29.7%	100.5%	1.0	14.3%	18.9%	33.2%	95.4%
0.9	15.3%	13.2%	28.5%	102.1%	0.9	14.6%	17.5%	32.1%	97.1%
0.8	15.6%	11.7%	27.3%	103.8%	0.8	14.8%	16.1%	30.9%	98.7%
0.7	15.8%	10.3%	26.1%	105.5%	0.7	15.1%	14.6%	29.7%	100.5%
0.6	16.1%	8.9%	25.0%	107.2%	0.6	15.3%	13.1%	28.4%	102.2%
0.5	16.3%	7.4%	23.8%	108.9%	0.5	15.6%	11.5%	27.1%	104.1%

Table II
Value of Regular and Incentive Fees and Shareholder's Claim
When Fund Has Superior Performance (continued)

b = 0.5

	σ = 15% $w + \lambda$ = 5%					σ = 25% $w + \lambda$ = 5%			
S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$	*S/H*	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$
1.0	28.15%	26.18%	54.32%	101.97%	1.0	13.48%	20.88%	34.36%	92.60%
0.9	28.54%	24.86%	53.40%	103.68%	0.9	13.56%	19.38%	32.94%	94.18%
0.8	28.61%	23.24%	51.84%	105.37%	0.8	13.16%	17.36%	30.51%	95.80%
0.7	27.59%	20.83%	48.42%	106.76%	0.7	11.81%	14.37%	26.18%	97.44%
0.6	22.60%	15.86%	38.47%	106.74%	0.6	8.39%	9.43%	17.82%	98.96%
0.5	0.00%	0.00%	0.00%	100.00%	0.5	0.00%	0.00%	0.00%	100.00%

	σ = 15% $w + \lambda$ = 10%					σ = 25% $w + \lambda$ = 10%			
S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$	*S/H*	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$
1.0	14.50%	14.34%	28.84%	100.16%	1.0	9.45%	15.29%	24.74%	94.16%
0.9	14.71%	12.90%	27.60%	101.81%	0.9	9.51%	13.79%	23.29%	95.72%
0.8	14.75%	11.38%	26.13%	103.38%	0.8	9.25%	12.00%	21.24%	97.25%
0.7	14.29%	9.62%	23.91%	104.67%	0.7	8.34%	9.67%	18.02%	98.67%
0.6	11.87%	6.96%	18.83%	104.91%	0.6	6.00%	6.22%	12.22%	99.78%
0.5	0.00%	0.00%	0.00%	100.00%	0.5	0.00%	0.00%	0.00%	100.00%

b = 0.8

	σ = 15% $w + \lambda$ = 5%					σ = 25% $w + \lambda$ = 5%			
S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$	*S/H*	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$
1.0	4.85%	7.26%	12.11%	97.59%	1.0	1.40%	4.95%	6.36%	96.45%
0.9	4.13%	5.03%	9.16%	99.10%	0.9	1.15%	3.01%	4.16%	98.13%
0.8	0.00%	0.00%	0.00%	100.00%	0.8	0.00%	0.00%	0.00%	100.00%

	σ = 15% $w + \lambda$ = 10%					σ = 25% $w + \lambda$ = 10%			
S/H	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$	*S/H*	Regular $a(S, H)$	Perform $p(S, H)$	Total $f(S, H)$	Investor $i(S, H)$
1.0	4.23%	6.43%	10.66%	97.80%	1.0	1.35%	4.79%	6.14%	96.56%
0.9	3.61%	4.34%	7.96%	99.27%	0.9	1.10%	2.88%	3.99%	98.22%
0.8	0.00%	0.00%	0.00%	100.00%	0.8	0.00%	0.00%	0.00%	100.00%

Table III
Regular Annual and Incentive Fee Trade-offs

Incentive and regular annual fee combinations that have same value when the asset value is at the high-water mark, as given in equation (17). Parameter values are: $r + c' - g$ = 5 percent, $\alpha = 0$, σ = 15 or 25 percent and $w + \lambda$ = 5 or 10 percent, b = 0, 0.5, and 0.8

$$k(F) = \frac{(1-b^{\gamma-\eta})(w+\lambda+c-\alpha)(1-F/S)}{(w+\lambda-\alpha)(1-b^{\gamma-\eta}) - cb^{1-\eta}(\gamma-\eta) - \left[\frac{F}{S}(c+w+\lambda-\alpha)-c\right]\left(\gamma - b^{\gamma-\eta}\eta\right)} - 1 \quad .$$

	$b = 0$				$b = 0.5$				$b = 0.8$			
	$w + \lambda$ = 5%		$w + \lambda$ = 10%		$w + \lambda$ = 5%		$w + \lambda$ = 10%		$w + \lambda$ = 5%		$w + \lambda$ = 10%	
c	σ =15%	σ = 25%	σ = 15%	σ = 25%	σ = 15%	σ = 25%	σ = 15%	σ = 25%	σ =15%	σ = 25%	σ = 15%	σ = 25%
4.00%	–25.62%	–11.58%	–21.33%	–8.23%	–16.53%	0.42%	–16.64%	0.14%	0.96%	11.19%	0.65%	11.18%
3.75%	–21.87%	–8.88%	–17.86%	–5.76%	–13.49%	2.19%	–13.58%	1.93%	2.69%	12.03%	2.41%	12.03%
3.50%	–17.94%	–6.08%	–14.26%	–3.22%	–10.33%	3.99%	–10.39%	3.77%	4.46%	12.88%	4.20%	12.88%
3.25%	–13.83%	–3.19%	–10.50%	–0.60%	–7.03%	5.84%	–7.07%	5.64%	6.27%	13.74%	6.03%	13.74%
3.00%	–9.54%	–0.19%	–6.59%	2.10%	–3.59%	7.73%	–3.61%	7.56%	8.11%	14.61%	7.91%	14.61%
2.75%	–5.07%	2.91%	–2.54%	4.87%	–0.02%	9.67%	–0.03%	9.52%	9.99%	15.49%	9.82%	15.48%
2.50%	–0.41%	6.11%	1.67%	7.73%	3.70%	11.64%	3.70%	11.52%	11.91%	16.37%	11.77%	16.37%
2.25%	4.43%	9.42%	6.03%	10.67%	7.56%	13.66%	7.56%	13.57%	13.87%	17.27%	13.76%	17.26%
2.00%	9.44%	12.84%	10.53%	13.69%	11.56%	15.73%	11.57%	15.67%	15.87%	18.17%	15.80%	18.17%
1.75%	14.63%	16.36%	15.19%	16.80%	15.71%	17.84%	15.71%	17.81%	17.92%	19.08%	17.88%	19.08%
1.50%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
1.25%	25.53%	23.75%	24.96%	23.28%	24.44%	22.21%	24.43%	22.24%	22.13%	20.93%	22.17%	20.93%
1.00%	31.24%	27.62%	30.06%	26.66%	29.02%	24.46%	29.01%	24.53%	24.30%	21.87%	24.38%	21.87%
0.75%	37.10%	31.60%	35.31%	30.12%	33.76%	26.76%	33.73%	26.87%	26.51%	22.82%	26.63%	22.82%
0.50%	43.11%	35.69%	40.70%	33.67%	38.63%	29.12%	38.59%	29.26%	28.77%	23.77%	28.94%	23.78%
0.25%	49.28%	39.90%	46.23%	37.32%	43.65%	31.52%	43.59%	31.70%	31.07%	24.74%	31.29%	24.75%
0.00%	55.60%	44.24%	51.90%	41.05%	48.81%	33.98%	48.74%	34.20%	33.42%	25.72%	33.68%	25.72%

Table IV
Maximum Incentive Fee Consistent with a Given Level of Superior Performance

The maximum incentive fee justified by a given level of superior performance as given in equation (18). Parameter values are: $r + c' - g$ = five percent, c = 1.5 percent, σ = 15 or 25 percent, $w + \lambda$ = five or 10 percent, and b = 0, 0.5, and 08..

$$k^*(\alpha) = \frac{1-\gamma+(\gamma-\eta)b^{1-\eta}+(\eta-1)b^{\gamma-\eta}}{\gamma-(\gamma-\eta)b^{1-\eta}-\eta b^{\gamma-\eta}-\dfrac{w+\lambda}{\alpha-c}\left(1-b^{\gamma-\eta}\right)} \quad .$$

	$b = 0$				$b = 0.5$				$b = 0.8$			
	$w + \lambda$ = 5%		$w + \lambda$ = 10%		$w + \lambda$ = 5%		$w + \lambda$ = 10%		$w + \lambda$ = 5%		$w + \lambda$ = 10%	
α	σ = 15%	σ = 25%	σ = 15%	σ = 25%	σ = 15%	σ = 25%	σ = 15%	σ = 25%	σ = 15%	σ =25%	σ = 15%	σ = 25%
2.0%	7.33%	5.25%	6.70%	4.65%	6.98%	3.85%	6.49%	3.68%	4.00%	1.62%	3.94%	1.61%
2.5%	14.91%	10.71%	13.63%	9.47%	14.27%	7.87%	13.25%	7.50%	8.18%	3.27%	8.04%	3.25%
3.0%	22.69%	16.39%	20.77%	14.47%	21.83%	12.06%	20.26%	11.48%	12.54%	4.96%	12.33%	4.93%
3.5%	30.67%	22.27%	28.12%	19.65%	29.65%	16.42%	27.51%	15.61%	17.08%	6.68%	16.79%	6.64%
4.0%	38.82%	28.36%	35.65%	25.00%	37.70%	20.96%	34.98%	19.90%	21.82%	8.44%	21.43%	8.38%
4.5%	47.13%	34.64%	43.36%	30.52%	45.95%	25.68%	42.64%	24.36%	26.75%	10.23%	26.27%	10.17%
5.0%	55.58%	41.10%	51.22%	36.20%	54.38%	30.59%	50.49%	28.97%	31.89%	12.07%	31.30%	11.98%
5.5%	64.15%	47.75%	59.24%	42.05%	62.98%	35.68%	58.51%	33.76%	37.23%	13.94%	36.52%	13.84%
6.0%	72.84%	54.56%	67.38%	48.05%	71.71%	40.96%	66.68%	38.71%	42.78%	15.84%	41.94%	15.73%
6.5%	81.63%	61.54%	75.65%	54.21%	80.57%	46.44%	74.98%	43.83%	48.54%	17.79%	47.57%	17.66%
7.0%	90.51%	68.67%	84.03%	60.51%	89.52%	52.11%	83.40%	49.13%	54.52%	19.78%	53.40%	19.64%
7.5%	99.48%	75.95%	92.52%	66.96%	98.57%	57.99%	91.94%	54.60%	60.71%	21.81%	59.44%	21.65%
8.0%	108.52%	83.36%	101.10%	73.55%	107.70%	64.05%	100.57%	60.24%	67.13%	23.88%	65.70%	23.70%
8.5%	117.62%	90.91%	109.76%	80.26%	116.90%	70.32%	109.29%	66.06%	73.76%	25.99%	72.16%	25.79%
9.0%	†	98.58%	118.51%	87.11%	†	76.78%	118.09%	72.05%	80.62%	28.15%	78.84%	27.93%
9.5%	†	†	127.34%	94.08%	†	83.44%	126.97%	78.22%	87.70%	30.35%	85.72%	30.11%
10.0%	†	†	136.23%	101.16%	†	90.29%	135.91%	84.55%	95.01%	32.59%	92.83%	32.34%

† transversality condition violated.

Table V

Net Fund Growth and Lagged Returns, 1990 - 1995

The table reports the results of two linear regressions of net fund growth on previous year returns. The growth in net asset value of fund *i* in year *t*, N_{it} , is defined as the new dollar cash flow into the fund (in millions) in the year following the return observation. It is calculated as $N_{it} = \mathrm{NAV}_{i,t-1}[(1+G_{it})/(1+R_{it}) - 1]$, where NAV_{it} is the fund net asset value in year *t*, R_{it} is the total return for fund *i* in year *t*, and G_{it} is the percent change in net asset value for fund in the year. This assumes that money is only invested at the beginning of the year, and that reinvested dividends are defined as growth. The form of the regressions are:

$$N_{i,t+1} = \beta_0 + \sum_{j=1}^{5} \beta_j I_j + \beta_6 R_{i,t} + e_{i,t} \qquad (a)$$

$$N_{i,t+1} = \beta_0 + \sum_{j=1}^{5} \beta_j I_j + \sum_{q=6}^{10} \beta_q R_{i,t,q} + e_{i,t} \; . \qquad (b)$$

Year effects (defined as differences from 1995) are captured by dummies I_j. Coefficients on returns are allowed to differ according to quintiles each year: $R_{i,t\text{-}1,q}$ where coefficients 6 through 10 capture quintiles 1

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Year effects (defined as differences from 1995) are captured by dummies I_j. Coefficients on returns are allowed to differ according to quintiles each year: $R_{i,t\text{-}1,q}$ where coefficients 6 through 10 capture quintiles 1

Regression 1 Results

	coef	**std.err**	**t.stat**	**p.value**
Intercept	-1.71	19.4	-0.08	0.92
1990	0.01	24.7	0.00	0.99
1991	10.56	23.4	0.45	0.65
1992	39.75	21.9	1.81	0.06
1993	-18.37	21.0	-0.87	0.38
1994	-16.83	21.1	-0.79	0.42
Return	-62.28	24.0	-2.60	0.00

Multiple R-Square = 0.0306 *N* = 934

Regression 2 Results

Intercept	-7.104	20.4	-0.3484	0.7276
1990	14.357	25.1	0.5709	0.5682
1991	18.254	23.4	0.7815	0.4347
1992	45.338	21.9	2.0695	0.0388
1993	-19.063	20.9	-0.9116	0.3622
1994	-0.585	22.4	-0.0261	0.9792
Smallest 1	127.621	65.6	1.9450	0.0521
2	42.556	130.1	0.3272	0.7436
3	60.703	92.0	0.6601	0.5094
4	9.453	61.4	0.1541	0.8776
Largest 5	-112.260	27.9	-4.0292	0.0001

Multiple R-Square = 0.0491

Chow test of coefficient equality: F = 5.23, 3,872 p-value = .998