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ABSTRACT

A search-theoretic general equilibrium model of frictional unemployment is shown to be consistent with some of the key regularities of unemployment over the business cycle. In the model the return to a job moves stochastically. Agents can choose either to quit and search for a better job, or continue working. Search generates job offers that agents can accept or reject. Two distinguishing features of current work relative to the existing business cycle literature on labor market fluctuations are: (i) the decision to accept or reject jobs is modeled explicitly, and (ii) there is imperfect insurance against unemployment.

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1 Introduction

What determines the rate of unemployment and its movement over the business cycle? In the U.S. economy the unemployment rate moves countercyclically. So too does the average duration of unemployment, implying that it is easier to find a job in booms than in busts. Furthermore, the flows into and out of unemployment are positively correlated and move countercyclically. Last, people experience a drop in consumption upon entering unemployment. This paper discusses whether these key facts about unemployment behavior are consistent with a basic general equilibrium search model in which individual job opportunities are affected by both aggregate and idiosyncratic shocks.

During the last decade there has been a resurgence of interest in unemployment models.[1] Most research on the cyclical behavior of unemployment conducted in general equilibrium has postulated the presence of perfect unemployment insurance to allow the analysis to proceed within a representative agent framework. The search-theoretic model of frictional unemployment studied in this paper, which is constructed along the lines of Lucas and Prescott (1974) and Jovanovic (1987), abstracts from perfect unemployment insurance. Employed agents decide whether to keep their current job opportunities or search for better ones. Unemployed agents have to choose between accepting employment or continuing search. Markets are incomplete, so agents cannot insure themselves against the idiosyncratic risk they face in their job opportunities; the best they can do is to smooth the effects of these shocks by borrowing and lending in the economy-wide capital market, subject to a borrowing constraint. Characterizing the competitive equilibrium of this model is much more complex than studying

[1] One widely used class of models stresses nonconvexities in labor demand or supply, a la Rogerson (1988) and Hansen (1985). A second class stresses the job matching process following the influential work of Mortenson and Pissarides (1994). Examples that incorporate this paradigm into real business cycle models include Andolfatto (1996) and Merz (1995). This framework has recently been evaluated by Cole and Rogerson (1996). The model in this paper belongs to a third class which features search with incomplete markets. Examples of prior work along these lines includes Andolfatto and Gomme (1996), Hansen and Imrohoroglu (1992) and Zhang (1995). These papers study the effects of unemployment insurance. Andolfatto and Gomme (1996) don't allow for personal asset holdings and Hansen and Imrohoroglu (1992) abstract from physical capital, features that are not essential to these analyses. None of this work incorporates aggregate uncertainty, a significant complexity that is necessary for undertaking business cycle analysis.

a representative agent environment. This complexity pays off, however, in the sense that the model can address some important empirical regularities of the labor market. At the same time, it is consistent with the key cyclical properties of consumption, investment and output.

There are some interesting interactions between the nature of search and the absence of complete markets. In the environment studied here the reservation job productivity of each individual agent depends on his wealth. Wealthier agents have higher reservation job productivities than poor ones. This means that unemployed agents become, over time, less choosy about the jobs they are willing to take. While unemployed, agents consume out of their past savings, thus reducing their wealth, which leads to a reduction in their reservation productivities. The dependency of the reservation productivity on the agent's wealth means that there are two possible sources of inefficiency in terms of labor allocation. First, the rate of unemployment will generally be different from that of a complete markets economy. Second, wealthy agents tend to engage in too much search, while poor agents search too little, in comparison with what would be optimal. Thus the allocation of search across agents is also inefficient.

The paper is organized as follows. Section 2 presents the model and provides some theoretical results. Section 3 discusses the main features of a calibrated version of the model that abstracts from aggregate shocks. The competitive equilibrium of this economy is compared with the Pareto Optimum. Aggregate shocks are then incorporated in the model and the resulting cyclical properties are compared with those of U.S. data. A final section summarizes the main findings and discusses possible extensions of the model.

2 The Model

Consider an economy with a continuum of workers distributed over the unit interval. An agent's goal in life is to maximize the expected value of his lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\tilde{c}_t - D(l_t)), \quad 0 < \beta < 1.$$

His momentary utility function is given by

$$U(\widetilde{c} - D(l)) = (\widetilde{c} - \frac{l^{1+\theta}}{1+\theta})^{1-\sigma}/(1-\sigma), \qquad \theta, \sigma > 0,$$

where $\widetilde{c}$ and l represent his consumption and labor effort in the current period. The agent derives income from either working or past savings in the form of physical capital. He uses this income either for consumption or saving for the future. Capital depreciates at the rate δ and yields a rate of return (net of depreciation) of r. He can borrow and lend freely in the economy-wide capital market at this real rate of interest r but is subject to a borrowing constraint: the level of assets, a, has to be greater than a minimum level $\bar{a}$. The agent can devote his time to working in his current job, searching for a new one, or enjoying leisure.

At the beginning of each period every agent has a job opportunity, represented by the production function

$$O(k, l; \varepsilon, \lambda) = \exp(\lambda + \varepsilon) k^{\alpha} l^{1-\alpha},$$

where l and k are inputs of labor and capital, and λ and ε represent aggregate and idiosyncratic technology shocks. An agent uses his own labor effort to operate the project. He rents capital from a competitive capital market at a rental rate of $(r + \delta)$. If the agent chooses to work this job opportunity he will earn income in the amount

$$\max_{k} \, [O(k, l; \lambda, \varepsilon) - (r + \delta)k].$$

The aggregate technology shock is drawn from the distribution function $F(\lambda'|\lambda) \equiv \Pr[\lambda_{t+1} \leq \lambda' | \lambda_t = \lambda]$; this is common to all production technologies in the economy. The idiosyncratic shock for this job opportunity evolves according to the distribution function $G(\varepsilon'|\varepsilon) \equiv \Pr[\varepsilon_{t+1} \leq \varepsilon' | \varepsilon_t = \varepsilon]$.

The timing of events is as follows. At the beginning of each period an agent has a job opportunity described by the pair (λ, ε). Depending on the values of λ and ε he will decide to work this opportunity or to abandon it. If the agent works he will earn labor income in the amount $O(k, l; \lambda, \varepsilon) - (r + \delta)k$. Suppose that last period the agent had saved the amount a in physical capital. Then, he will also receive the amount ra in rental income.

The individual uses these two sources of income for consumption, c, and saving for the future, a'. Now in standard fashion, denote the value of the agent's idiosyncratic shock for next period by ε'. If the agent accepts the current job opportunity, then ε' will be drawn from the distribution $G(\varepsilon'|\varepsilon)$. If the individual instead rejects the job opportunity, then he searches for a new production technology. The simplest job sampling rule is to allow a searching agent to sample one new job prospect per period. In line with simplicity, let the agent draw a new technology for operation next period from the distribution function $H(\varepsilon')$. When the agent rejects his job opportunity he must live solely off of the income from his past savings or $(1+r)a$.

Thus, at the beginning of each period an individual must decide whether to work or search. Clearly, the values for the technology shocks, ε and λ, are relevant for this decision as well as the individual's wealth, a. So too is the economy distribution of wealth since this determines the rental rate on capital, R. Let $Z(a, \varepsilon, \lambda)$ represent the cumulative distribution of agents over the state $(a, \varepsilon, \lambda)$, and $z(a, \varepsilon, \lambda)$ denote the associated density. Suppose that this distribution function evolves according to transition operator $\mathbf{T}$ so that $Z' = \mathbf{T}Z$. The equilibrium interest rate will be a function of the aggregate technology shock and the distribution of wealth across agents so that $r = R(\lambda; Z)$. Next, let the expected lifetime utility of a worker and a searcher in state $(a, \varepsilon, \lambda; Z)$ be represented by $W(a, \varepsilon, \lambda; Z)$ and $S(a, \varepsilon, \lambda; Z)$. Finally, define $Y(\varepsilon, \lambda; Z)$ to be the income earned by a worker net of the disutility of working so that

$$Y(\varepsilon, \lambda; Z) = \max_{k,l} [O(k, l; \lambda, \varepsilon) - (r+\delta)k - D(l)].$$

Represent the decision rules for k and l by $L(\varepsilon, \lambda; Z)$, and $K(\varepsilon, \lambda; Z)$.

The choice problem for a worker can now be expressed as:

$$W(a, \varepsilon, \lambda; Z) = \max_{c,a'} \{U(c) + \beta \int \max[W(a', \varepsilon', \lambda'; Z'), S(a', \lambda'; Z')] G_1(\varepsilon'|\varepsilon) F_1(\lambda'|\lambda) d\varepsilon' d\lambda'\}, \quad \text{P(1)}$$

subject to

$$\begin{aligned} c + a' &= Y(\lambda, \varepsilon; Z) + [1 + R(\lambda; Z)]\, a, \\ a' &\geq \bar{a}, \end{aligned} \tag{1}$$

and $Z' = \mathbf{T}Z$. Denote the worker's decision rules for c and a', by $C^w(a, \varepsilon, \lambda; Z)$, and $A^w(a, \varepsilon, \lambda; Z)$.

The programming problem for a searcher is defined by:

$$S(a, \lambda; Z) = \max_{c, a'} \{U(c) + \beta \int \max[W(a', \varepsilon', \lambda'; Z'), S(a', \lambda'; Z')] H_1(\varepsilon') F_1(\lambda'|\lambda) d\varepsilon' d\lambda'\} \quad \text{P(2)}$$

subject to

$$\begin{aligned} c + a' &= [1 + R(\lambda; Z)]\, a, \\ a' &\geq \bar{a}, \end{aligned}$$

and $Z' = \mathbf{T}Z$. Note that, for simplicity that search is assumed to be effortless. The searcher's decision rules for consumption and asset accumulation read $c = C^s(a, \lambda; Z)$ and $a' = A^s(a, \lambda; Z)$.

Clearly, an agent will choose to work in the current period if $W(a, \varepsilon, \lambda; Z) \geq S(a, \lambda; Z)$; otherwise he will search. Let $\Omega(a, \varepsilon, \lambda; Z)$ be the decision rule governing whether an individual works or not. This decision rule is specified by

$$\Omega(a, \varepsilon, \lambda; Z) = \begin{cases} 1, \text{ if } W(a, \varepsilon, \lambda; Z) \geq S(a, \lambda; Z) \\ 0, \text{ otherwise.} \end{cases} \tag{2}$$

It it then easy to see that an agent who finds himself in state $(a, \varepsilon, \lambda; Z)$ will save the amount

$$a' = A(a, \varepsilon, \lambda; Z) \equiv \Omega(a, \varepsilon, \lambda; Z) A^W(a, \varepsilon, \lambda; Z) + (1 - \Omega(a, \varepsilon, \lambda; Z)) A^S(a, \lambda; Z).$$

Last, in a competitive equilibrium the demand and supply of capital should always be equal. In an equilibrium the market clearing condition for the capital market will read

$$\int K(\varepsilon, \lambda) \Omega(a, \varepsilon, \lambda) z(a, \varepsilon, \lambda) da d\varepsilon d\lambda = \int a z(a, \varepsilon, \lambda) da d\varepsilon d\lambda. \tag{3}$$

The lefthand side of above expression represents the total demand for capital by working agents while the righthand side gives the total supply from all agents.

The following definition of a competitive equilibrium takes stock of the discussion so far.

Definition A competitive equilibrium is a set of decisions rules, A^w, L, K, A^s, Ω, a set of value functions, W, S, a pricing function, R, and a law of motion for the aggregate wealth distribution, $Z' = \mathbf{T}Z$, such that

1. The decision rules A^w, L, and K, and value function W, solve problem P(1), given the functions S, R, and $\mathbf{T}$.

2. The decision rule A^s, and value function S, solve problem P(2), given the functions W, r, and $\mathbf{T}$.

3. The work/search decision rule, Ω, is determined in line with (2), given W and S.

4. The capital market clears so that (3) holds.

5. The law of motion for the economy-wide distribution of wealth, or $Z' = \mathbf{T}Z$, is described by

$$\begin{aligned} Z'(a', \varepsilon', \lambda') &= \int I(A(a, \varepsilon, \lambda) - a')[\Omega(a, \varepsilon, \lambda)G(\varepsilon'|\varepsilon) \\ &\quad + (1 - \Omega(a, \varepsilon, \lambda))H(\varepsilon')]F(\lambda'|\lambda)z(a, \varepsilon, \lambda)da d\varepsilon d\lambda, \end{aligned} \tag{4}$$

   where $I(x) = 1$ if $x \leq 0$ and $I(x) = 0$ otherwise.

2.1 Discussion

Is unemployment voluntary or involuntary in the model? It will be argued that this distinction isn't useful, a point made by Lucas (1978). On the one hand, all unemployed agents choose not to work. In this sense unemployment is voluntary. On the other hand, they all left their job due to bad luck or misfortune. Nobody chose to get a poor (ε, λ) combination, and an agent is made worse off by having one. For example suppose that an agent drew a value of zero for his idiosyncratic productivity $\exp(\varepsilon + \lambda)$. Formally this is the same as not having a job opportunity, or as being involuntary employed. Now, assume instead that the agent drew a value for his productivity $\exp(\varepsilon + \lambda)$ that is arbitrarily close to zero and that he rationally turns down this fruitless job opportunity. Surely, one should classify this as involuntary unemployment too. Thus, the question turns on whether the agent will work for a reasonable income. In the model, as in the real world, this reasonable wage is agent specific.

2.2 Deterministic Steady-State Results

The presence of $\max[W, S]$ operation on the righthand side of P(1) and P(2) greatly complicates the analysis of the model. Still with a few assumptions (that are satisfied in the computational results) some intuition about how the model is likely to work can be developed. Now, let $T(a)$ give the value of the shocks for which an agent is indifferent between working and searching. This threshold rule is defined by the equation

$$W(a, T(a)) = S(a), \tag{5}$$

where λ and Z have been dropped from value functions given the focus on a deterministic steady state. Since W is monotonically increasing and continuous in ε, T will be a function and

$$W(a,\varepsilon) \mathrel{\substack{\geq \\ <}} S(a) \text{ as } \varepsilon \mathrel{\substack{\geq \\ <}} T(a).$$

To develop the situation further (in a heuristic way) make the following assumption:

Assumption: W and S are C^1 functions.

By the implicit function theorem it then follows that $T(a)$ is a C^1 function too. It now transpires that P(1) and P(2) will have the form

$$W(a,\varepsilon) = \max_{a'} \{U(Y(\varepsilon)+(1+r)a-a')+\beta[\int^{T(a')} S(a')G_1(\varepsilon'|\varepsilon)d\varepsilon' + \int_{T(a')} W(a',\varepsilon')G_1(\varepsilon'|\varepsilon)d\varepsilon']\},$$

and

$$S(a) = \max_{a'} \{U((1+r)a-a') + \beta[\int^{T(a')} S(a')H_1(\varepsilon')d\varepsilon' + \int_{T(a')} W(a',\varepsilon')H_1(\varepsilon')d\varepsilon']\}.$$

Using the envelope theorem and the definition of a threshold it then follows that

$$W_1(a,\varepsilon) = U_1(Y(\varepsilon) + (1+r)a - a')(1+r) = U_1(C^w(a,\varepsilon))(1+r)$$

and

$$S_1(a) = U_1((1+r)a - a')(1+r) = U_1(C^s(a))(1+r).$$

Clearly, C^w and C^s are increasing in a if and only if W and S are strictly concave. The decreasing marginal utility of wealth associated with concavity is a natural property for this type of environment, which holds in all the model calibrations that we considered.[2]

Assuming that this property holds, what is required if an agent is to experience a drop in consumption upon crossing the threshold from work to search? It is easy to see that $C^w(a,T(a)) > C^s(a)$ if and only if $S_1(a) > W_1(a,T(a))$. Furthermore, when $S_1(a) > W_1(a,T(a))$ then by the implicit function theorem $T(a)$ is increasing in a. Thus, wealthy agents will be choosier about accepting job opportunities. Last, let W_1 be strictly decreasing in ε. For a worker a higher value for the shock implies higher current income, and a greater likelihood of higher future income, so that an extra unit of savings should be worth less. When $W_{12} < 0$ a worker's consumption is increasing in the shock. Therefore a worker's consumption must always exceed a searcher's for the same level of wealth. This transpires since if a worker's consumption is greater than a searcher's at the threshold level of the shock then it must be greater still at higher levels of the shock. The situation just discussed is summarized in Figure 1, which plots data obtained from the model.

Agents in the model are unable to insure perfectly against the possibility of becoming unemployed. Upon entering a spell of unemployment they experience a drop in consumption. This clearly may affect an agent's saving behavior. Aiyagari (1994) and Huggett (1995) have illustrated how the presence of borrowing constraints in a model with heterogeneous agents leads to over savings in the sense that equilibrium interest rate lies below the rate of time

[2] Note that concavity can be preserved since the uncertainty due to the idiosyncratic shock smooths out the kinks in the value functions that are due to the $\max[W,S]$ operation.

preference. Their argument would appear to apply here too.[3] Thus, one cannot assume that in a steady state $r = 1/\beta$.

3 Findings

The quantitative implications of the model are developed via simulation. Simulating a model such as this is not an easy task. It is made difficult by (i) the form of programming problems P(1) and P(2) that aren't readily amenable to linearization or L.Q. approximation techniques, and (ii) the necessity to carry around some measure of the distribution of wealth as a state variable. The details of algorithm employed to simulate this model are provided in Appendix A.

3.1 Calibration

In order to compute the equilibrium for the deterministic steady state a time series process must be specified for the idiosyncratic shock. Specifically, assume that for a worker ε evolves according to

$$\varepsilon' = \rho_\varepsilon \varepsilon + \xi,$$

where $\xi \sim N(0, \sigma_\xi)$. A searcher draws a value of ε according to

$$\varepsilon = \nu,$$

where $\nu \sim N(\mu_v, \sigma_v)$.

[3] A worker's asset accumulation in determined in line with the efficiency condition $U_1(C^w(a,\varepsilon)) \geq \beta(1+r)[\int_{T(a')} U_1(C^w(a',\varepsilon'))G_1(\varepsilon'|\varepsilon)d\varepsilon' + \int^{T(a')} U_1(C^s(a'))G_1(\varepsilon'|\varepsilon)d\varepsilon']$. Likewise, the searcher's asset accumulation is governed by $U_1(C^s(a)) \geq \beta(1+r)[\int_{T(a')} U_1(C^w(a',\varepsilon'))H_1(\varepsilon')d\varepsilon' + \int^{T(a')} U_1(C^s(a'))H_1(\varepsilon')d\varepsilon']$. These equations hold with equality whenever the borrowing constraint does not bind. Next, integrate both sides of the worker's Euler equation with respect to the stationary density z over the part of the state space applying to him. Perform the analogous operation on the searcher's Euler equation. Sum the resulting equations. Use the definition of a stationary distribution on the right hand side of the resulting expression to get $\int\{\int_{T(a)} U_1(C^w(a,\varepsilon))z(a,\varepsilon)d\varepsilon + \int^{T(a)} U_1(C^s(a))z(a,\varepsilon)d\varepsilon\}da \geq \beta(1+r)\int\{\int_{T(a')} U_1(C^w(a',\varepsilon'))z(a',\varepsilon')d\varepsilon' + \int^{T(a')} U_1(C^s(a'))z(a',\varepsilon')d\varepsilon'\}da'$. But this can only hold if $\beta(1+r) \leq 1$ (assuming that the integrals are bounded). If the set of liquidity constrained agents has strictly positive measure then the inequality is strict, implying that $(1+r) < 1/\beta$. Thus, the possibility of over-accumulation continues to hold for this economy despite the presence of search. The line of argument employed above was developed in Huggett (1995).

The time period chosen for decision making was taken to be six weeks. This short time horizon seems appropriate given that the average duration of unemployment is about a quarter. Since most macro-data is only available at the quarterly frequency, the output of the model was aggregated up to this periodicity. For the benchmark model the following parameters values are assumed.

Parameter Values	
Preferences	$\sigma = 2$, $\beta = 0.993$, $\theta = 2.0$
Technology	$\alpha = 0.36$, $\delta = 0.006$,
Idiosyncratic Shock, Worker	$\mu_\xi = 0$, $\rho_\varepsilon = 0.92$, $\sigma_\varepsilon = 0.0372$
Idiosyncratic Shock, Searcher	$\mu_v = -0.025$, $\sigma_v = 0.06$

The model's parameters of preferences and technology were set equal to values that are standard in the real business cycle literature. This configuration of parameter values implies an annual interest rate of 6% and a depreciation rate of 4.8%. In order to prevent the borrowing constraint from playing a large role, the value of $\bar{a}$ was chosen so that this constraint is binding only for a very small fraction of agents. As a result, the model's equilibrium real interest rate is extremely close to the rate of time preference.

The stochastic process for the idiosyncratic shocks were chosen so that the version of this economy with aggregate shocks will mimic the average rate of unemployment (5.7%) and the average median duration (6.4 weeks) in the U.S. economy during the post war period.

3.2 Properties of the Stationary Distribution

With this configuration of parameter values and driving processes for the shocks the mean level of unemployment in the model is 4.6%, while the average duration of unemployment is about 8 weeks. Both of these numbers are a little on the low side. Figure 2 shows the exit rates from unemployment. As can be seen 81% of agents exit the pool of unemployed one period after entry with another 16% leaving after two periods. This means that the model is consistent with the observation by Clark and Summers (1979) that a large fraction of unemployment spells (79% in 1969, 60% in 1974 and 55% in 1975) end within one month. However, the model generates a much smaller number of workers who search for more than two periods than that suggested by the Clark-Summers data. Introducing the possibility of part time work in the model would lower the cost of search and induce a higher number of agents to endure long unemployment spells.

The coefficient of variation for annual income in the model is about 13%, which is somewhat shy of the 20-40% range that Aiyagari (1994) states is reasonable vis a vis the U.S. data. This is not surprising since in the model all differences in skill levels are temporary. It is clear that much of the income inequality in the data results from permanent differences in human capital.

Dynarski and Sheffrin (1987) report a drop in consumption at the time an individual becomes unemployed. Grueber (1994) estimates the drop in food consumption to be 6.8% under the current unemployment insurance system and 22% in its absence. These findings are usually interpreted as evidence of lack of full insurance against the possibility of unemployment.[4] The model is consistent with this property of consumption behavior: in the model agents reduce their consumption by 34% upon becoming unemployed.

One common implication of search models shared by this economy is that labor income in the first quarter of a new job is, on average, greater that the income earned in the last quarter on the previous job. After all, if search did not result in an improvement in job prospects, workers would not be willing to endure voluntary spells of unemployment. The model implies that labor income is 11.5% higher in the first quarter on the new job, when compared to the last quarter in the previous job. This is similar to the 12% increase reported by Topel and Ward (1992) in their study of mobility among young men.

A key implication of the model is that the threshold job productivity above which agents will choose to work is increasing in the agent's financial wealth, a. Richer agents are more

[4] This consumption drop is consistent with complete markets, however, if the marginal utility of consumption is declining in leisure. This is not a property of standard utility functions such as the Cobb-Douglas form.

selective in the jobs that they are willing to take. In a recent study Rendón (1996) has found empirical evidence for this effect in a sample of white male high school graduates who did not attend college. In this group an increase of \$5,000 in initial assets produces after 20 quarters an increase of \$50 in quarterly wages.

Finally, the model is consistent with the positive correlation between tenure and labor income found in the data [Abraham and Farber (1987)]. This results from the persistent character of the worker's idiosyncratic shock. Workers who found a high productivity job are less likely to see their productivity fall below the search threshold in the near future.

3.3 Comparative Statics Results

To gain some intuition about how the model works some comparative statics experiments will be undertaken. The qualitative results of these experiments are summarized in the Table below. The different lines reports the new values of the unemployment rate and average duration that result from changing each individual parameter to the value indicated. In all experiments except the last β was adjusted so that the real interest rate remained the same as in the baseline calibration. The different parameters were reduced by 20%, relative to their benchmark values, with four exceptions. The value of μ_ξ was increased to 0.01 from its benchmark value of zero. The value of σ was raised so as to lower the elasticity of intertemporal substitution, $1/\sigma$, by 20%. Similarly, θ was increased so as to raise the elasticity of labor supply, $1/\theta$, by 20%. Finally, β was increased to 0.9938. This corresponds to a reduction in the annual real interest rate from 6% to 5%.

First, consider an increase in the standard deviation of the searcher's idiosyncratic shock, σ_ν. This increases the rate of unemployment because the value of search has now risen. The odds of getting a good job have improved. Note that while the odds of getting a bad job offer have worsened, the agent does not have to accept bad realizations of the idiosyncratic shock. Thus, an increase in the variance of job seeker's shock raises the option value of search.

What happens if the variance of the worker's shock is increased? Now there is a greater likelihood that a worker will experience bad luck with his job. Presumably, this should operate to increase the number of agents engaging in job search in order to improve their life. In fact, increasing σ_ξ results in a rise in unemployment.

Parameters	Unemployment Rate	Duration (weeks)
Baseline	4.6	8.0
$\sigma = 2.5$	4.5	8.0
$\theta = 2.5$	4.2	7.9
$\alpha = 0.3$	4.1	7.8
$\delta = 0.005$	4.6	8.0
$\mu_\xi = 0.01$	4.1	8.1
$\mu_\nu = -0.02$	4.7	8.0
$\sigma_\xi = 0.03$	3.9	8.1
$\sigma_\nu = 0.048$	4.1	7.6
$\rho = 0.75$	2.5	6.6
$\beta = 0.9938$	4.6	8.0

Changing the mean values of ξ and ν has obvious consequences. Increasing the mean of ξ from its benchmark value of zero makes it less likely that a worker will transit into a low enough productivity state that he will choose to search. This leads to a decline in the rate of unemployment. Raising the mean value of ν increases the rewards to searching and the rate of unemployment.

Increasing the persistence of the worker's shock, ρ, raises both the duration and the rate of unemployment. When shocks are more persistent a worker who receives a bad shock is more likely to remain in a low productivity state. This lowers the opportunity cost of search. When ρ increases a worker who receives a high ε is likely to remain highly productive for a longer period of time. This raises the rewards to searching. Both of these effects conspire to make search more attractive, raising the unemployment rate and the unemployment duration.

Increases in θ (which lower the elasticity of labor supply $1/\theta$) and decreases in α have similar effects: they raise the duration and the rate of unemployment. To see the economic mechanism that produces this effect it is useful to calculate how these parameters change the current value of a job opportunity. The surplus earned by the worker, in terms of labor income net of the disutility of labor, is

$$\max_{l,k} [\exp(\varepsilon)k^{\alpha}l^{1-\alpha} - (r+\delta)k - \frac{l^{1+\theta}}{1+\theta}].$$

Optimizing out k and l it is easy to see that the value of this expression is proportional to $[\exp(\varepsilon)]^{\frac{1+1/\theta}{1-\alpha}}$, and hence is convex in $\exp(\varepsilon)$. Notice that both increases in θ and reductions in α increase the degree of convexity in this expression, raising the desirability of high values of ε. This provides more incentive to search, raising the rate of unemployment.

Finally, both the duration and unemployment rate seem reasonably insensitive to changes in σ, β, and δ.

3.4 The Social Versus the Private Rate of Unemployment

Imagine that all allocations in the model are determined by a benevolent social planner. Clearly, the planner would like to move workers out of unproductive jobs and have them search for better opportunities. This will result in unemployment. The question is how much?

The planner will enter each period with a given quantity of capital, $\mathbf{k}$, and some distribution of idiosyncratic shocks represented in density function form by, $z(\varepsilon)$. He'll need to pick a capital stock for next period, $\mathbf{k}'$, and choose an optimal threshold value for the shock, ε^*, such that an agent works if $\varepsilon \geq \varepsilon^*$ and searches otherwise. Furthermore, he will also have to assign a certain quantity of capital to each agent who works. Let $k(\varepsilon)$ denote the amount of capital that is assigned to a worker with shock level ε. The surplus earned by the worker, net of the disutility of working, will be

$$\Pi(k(\varepsilon),\varepsilon) = \max_{l} [O(k(\varepsilon),l,\varepsilon) - \frac{l^{1+\theta}}{1+\theta}].$$

The total amount of surplus produced in the period is therefore given by $\int_{\varepsilon^*} \Pi(k(\varepsilon), \varepsilon) z(\varepsilon) d\varepsilon$. The planner will distribute this surplus [net of gross investment] over individuals so as to equalize the marginal utility of consumption across agents. The momentary utility function has the form $U(c, l) = U(c - \frac{l^{1+\theta}}{1+\theta})$ so that this implies that $c - \frac{l^{1+\theta}}{1+\theta}$ will be equalized across agents. Employed agents will consume more than unemployed ones, but just by an amount that exactly compensates them for their work effort. Everybody will realize the same level of utility.

Formally the planning problem is described by

$$J(\mathbf{k}, z) = \max_{\mathbf{k}', \varepsilon^*, k(\varepsilon)} \{U(\int_{\varepsilon^*} \Pi(k(\varepsilon), \varepsilon) z(\varepsilon) d\varepsilon + (1-\delta)\mathbf{k} - \mathbf{k}') + \beta J(\mathbf{k}', z')\},$$

subject to the law of motion for the density z

$$z'(\varepsilon') = \int_{\varepsilon^*} G_1(\varepsilon'|\varepsilon) z(\varepsilon) d\varepsilon + H_1(\varepsilon') \int^{\varepsilon^*} z(\varepsilon) d\varepsilon,$$

and the capital constraint

$$\mathbf{k} - \int_{\varepsilon^*} k(\varepsilon) z(\varepsilon) d\varepsilon = 0. \tag{6}$$

The first order conditions are:

$$-U_1(c) + \beta J_k(k', z') = 0, \tag{7}$$

$$\begin{aligned} & U_1(c)\Pi(k(\varepsilon^*), \varepsilon^*) + \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') G_1(\varepsilon'|\varepsilon^*) d\varepsilon' \\ = \; & \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') H_1(\varepsilon') d\varepsilon' + \lambda k(\varepsilon^*), \end{aligned} \tag{8}$$

and

$$U_1(c)\Pi_k(k(\varepsilon), \varepsilon) = \lambda, \text{ for } \varepsilon \geq \varepsilon^*, \tag{9}$$

where λ is the multiplier attached to (6). The notation $J_{z(\varepsilon)}(\mathbf{k}, z)$ signifies the Volterra derivative of the function J with respect to z. It measures the impact that a small perturbation in the function z at the point ε will have on J.

Next, observe that

$$J_k(\mathbf{k}, z) = U_1(c)(1-\delta) + \lambda, \tag{10}$$

and that

$$J_{z(\varepsilon)}(\mathbf{k}, z) = \begin{cases} \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') H_1(\varepsilon') d\varepsilon', & \text{if } \varepsilon \leq \varepsilon^*, \\ U_1(c)\Pi(k(\varepsilon), \varepsilon) - \lambda k(\varepsilon) + \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') G_1(\varepsilon'|\varepsilon) d\varepsilon', & \text{if } \varepsilon \geq \varepsilon^*. \end{cases} \tag{11}$$

Substituting (9) and (10) into (7) gives a standard looking condition for capital accumulation.

$$U_1(c) = \beta[\Pi_k(k(\varepsilon), \varepsilon) + (1-\delta)]U_1(c').$$

Next substituting (9) into (8) gives

$$\begin{aligned} & U_1(c)[\Pi(k(\varepsilon^*), \varepsilon^*) - \Pi_k(k(\varepsilon^*), \varepsilon^*)k(\varepsilon^*)] + \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') G_1(\varepsilon'|\varepsilon^*) d\varepsilon' \\ = & \ \beta \int J_{z(\varepsilon')}(\mathbf{k}', z') H_1(\varepsilon') d\varepsilon'. \end{aligned} \tag{12}$$

Clearly, the Euler equation for capital accumulation implies that in a steady state

$$\Pi_k(k(\varepsilon), \varepsilon) = 1/\beta + \delta - 1.$$

The value of $k(\varepsilon)$ that solves this single equation will be represented by $k^*(\varepsilon)$. Now, let $V(\varepsilon) = J_{z(\varepsilon)}(\mathbf{k}, z)/U_1(c)$, evaluated at the steady state $\mathbf{k}$ and z. The function $V(\varepsilon)$ is defined by

$$V(\varepsilon) = \begin{cases} \beta \int V(\varepsilon') H_1(\varepsilon') d\varepsilon', & \text{if } \varepsilon \leq \varepsilon^*, \\ \Pi(k^*(\varepsilon), \varepsilon) - \Pi_k(k^*(\varepsilon^*), \varepsilon^*)k^*(\varepsilon^*) + \beta \int V(\varepsilon') G_1(\varepsilon'|\varepsilon) d\varepsilon', & \text{if } \varepsilon \geq \varepsilon^*. \end{cases}$$

The function $V(\varepsilon)$ gives the expected present-value of the surplus that will be realized from a worker who currently has productivity level, ε, assuming that the threshold rule ε^* is followed. The steady-state optimality condition that determines ε^* is then easily seen from (12) to be

$$[\Pi(k^*(\varepsilon^*), \varepsilon^*) - \Pi_k(k^*(\varepsilon^*), \varepsilon^*)k^*(\varepsilon^*)] + \beta \int V(\varepsilon') G_1(\varepsilon'|\varepsilon^*) d\varepsilon' = \beta \int V(\varepsilon') H_1(\varepsilon') d\varepsilon'.$$

This expression has a simple interpretation. A marginal increase in the threshold ε^* implies the loss of the surplus that a worker with productivity ε^* would generate this period,

$\Pi(k^*(\varepsilon^*),\varepsilon^*) - \Pi_k(k^*(\varepsilon^*),\varepsilon^*)k^*(\varepsilon^*)$, and also the loss in continuation value associated with this worker's productivity, $\beta \int V(\varepsilon')G_1(\varepsilon'|\varepsilon^*)d\varepsilon'$. The benefit from changing the threshold rule is in the fact that ε will be sampled from the distribution $H(\varepsilon)$, which generates the expected gain $\beta \int V(\varepsilon')H_1(\varepsilon')d\varepsilon'$. The optimality condition for ε^* can be rewritten as

$$V(\varepsilon^*) = \beta \int V(\varepsilon')H_1(\varepsilon')d\varepsilon'.$$

Figure 3 provides a (qualitative) comparison between the threshold rules arising in the decentralized competitive equilibrium and the one obtained in planning problem. The threshold rule for the competitive equilibrium is shown by the solid line. As an agent's asset holding increase he becomes choosier about the job opportunities that he will work. Relative to the planning problem (the dashed line), rich agents will search too much while poor ones will search too little. In the model over a reasonable range for asset holdings private agents always have a higher threshold value for the shock than does the planner. Thus, in the decentralized competitive equilibrium there is excessive search and unemployment. The unemployment rate for the planning problem is 4.2% (which corresponds to ε^*=-0.0824), as compared with 4.6% for the benchmark competitive equilibrium. The welfare gain from moving from the competitive equilibrium to the Pareto Optimum, measured in units of c, is on the order of 3.7%. This welfare gain is in large part due to the reduction in the volatility of c associated with the presence of complete unemployment insurance.

3.5 Business Cycle Properties

In order to simulate the model with aggregate shocks a time series process for λ must be specified. The method of Tauchen and Hussey (1991) was used to compute a three-point Markov chain for λ that approximates an AR(1) process with serial correlation ρ_λ and standard deviation σ_λ.

To solve this model with aggregate shocks it is necessary to keep track of the evolution of the wealth distribution, Z. In order to make their decisions individual agents have to forecast the future values of the real interest rate. These values are influenced by the wealth distribution, Z. The algorithm employed, detailed in the Appendix, makes appeal to the results of Krusell and Smith (1995) that suggest in this type of environment it may be sufficient to approximate the wealth distribution Z by its mean, which will be denoted by $\mathbf{k} = \int az(a, \varepsilon, \lambda)da d\varepsilon d\lambda$.

The behavior of unemployment in the model depends critically on the response of the threshold rule to an aggregate productivity shock. This rule now takes the form $\varepsilon = T(a, \mathbf{k}, \lambda)$, where $\mathbf{k}$ represents the aggregate capital stock in the economy. It is difficult to predict theoretically the response of this threshold rule to a change in λ because there are two contradictory effects at play. When λ rises, the opportunity cost of search increases, which should lead to less search. On the other hand, since λ is serially correlated, an increase in λ raises the conditional dispersion of future productivity $\exp(\varepsilon' + \lambda')$. As the comparative statics experiment of Section 3.3 have shown, this dispersion increase raises the option value of search, which should lead to more search. Figure 4 depicts the threshold rules for the three possible values of λ and for a value of $\mathbf{k}$ equal to the mean value obtained in our simulations. This figure shows that the opportunity cost effect dominates: when λ increases the threshold line is displaced downward, leading to less search.

One interesting aspect of Figure 4 is that it shows that recessions have a "cleansing" effect in the model. In expansions, jobs with low idiosyncratic productivity are not abandoned because agents want to take advantage of the temporarily high aggregate productivity. It is in recessions that these low productivity jobs are eliminated as workers search for better opportunities.

A related point is that λ does not coincide with the logarithm of the Solow residual, measured as the logarithm of aggregate output minus a weighted average (with weight α) of the logarithms of aggregate capital and total hours worked. The source of the discrepancy is the cyclical movement in the threshold rule depicted in Figure 4. In an expansion many low ε production opportunities are retained, only to later be discarded in a recession. This means that the volatility of the measured Solow residual underestimates the volatility of the true productivity shock. The values of ρ_λ and σ_λ^2 were chosen so that the Solow residual, conventionally measured using our simulated data, exhibits roughly the same serial correlation and variance reported by Cooley and Prescott (1995) for their estimate of the Solow residual.

Figure 5 depicts the threshold rules for the mean value of the aggregate shock λ and two values of $\mathbf{k}$, the mean plus and minus four standard deviations. An increase in the aggregate stock of capital lowers the real interest rate. This increases the probability that an individual agent will choose to work for two reasons: (i) the rental price of capital drops, raising the value of labor income associated with a given idiosyncratic productivity; and (ii) it reduces ra is reduced, thus lowering end of period wealth. However, the figure shows that quantitatively these effects are very small.

Figure 6 depicts the average response of the system across all instances in our 10,000 period simulation in which the productivity shock transited from its mean value to its high value. Figure 7 reports the same information for transitions from the mean value of the shock to its lowest value. These figures are analogous to impulse response functions. They show clearly that the model is capable of retaining the successful features of real business cycle models, in terms of the behavior of consumption, output and investment, while, at the same time, being consistent with some key regularities of unemployment behavior.[5] Table 1 shows that the volatility persistence and comovement of consumption, output and investment are similar to those of U.S. data. In both Table 1 and Table 2, discussed below, the model and U.S. data series were detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600.

Figures 6 and 7 show that unemployment is clearly countercyclical. This is what the threshold rules in Figure 4 had suggested: workers are willing to accept a low ε job when aggregate productivity is high. The flow into unemployment declines in an expansion as workers become less willing to quit their jobs. Average duration declines in the first period as searchers become more inclined to accept low ε offers. A further decline takes place

[5] The volatility of output is higher in our model than in the data (2.61 versus 1.72). This accords with Diaz-Gimenez's (1996) finding that an economy in which agents smooth idiosyncratic shocks through borrowing and lending is more volatile than an analogous economy with complete contracts.

in the second period, reflecting the fact that most of the agents who become unemployed in period 1 accept jobs in period 2. The flow out of unemployment increases initially, as agents become employed to take advantage of the high aggregate productivity. Then this flow declines, simply because the number of unemployed workers is sharply reduced.

Table 2 confirms that the model reproduces the comovement patterns of labor market variables that characterize U.S. data: average hours and employment are procyclical, while the unemployment rate, the duration of unemployment are countercyclical. Note, however, that the correlation between these variables and output is weaker in the model than in the data, with the exception of average weekly hours. The volatility of most labor series is also generally lower in the model than in the data. This may possibly reflect the fact that the model abstracts from flows in and out of the labor force.

One salient feature of labor market data is the countercyclical character of flows into and out of unemployment. This feature has been documented for the U.S. [Davis, Haltiwanger and Schuh (1996), Merz (1996)] and for several European countries [Burda and Wyplosz (1994)]. The model is consistent the negative correlation between job flows and output, but it exhibits a weaker correlation than that found in the US data. At the same time, the model produces realistic volatilities for flows in and out of unemployment and it implies a similar volatility for both of these flows. Blanchard and Diamond (1990) stressed that in the U.S. flows into unemployment are more volatile than flows out of it. Merz (1996) has, however, recently disputed these findings, arguing that the difference between the volatility in these two flows is not statistically significant.

3.6 Decomposing Aggregate Unemployment

How much of aggregate unemployment is due to aggregate versus idiosyncratic shocks? Note that without idiosyncratic shocks there would be no unemployment in the model. Nobody would expect that they could improve their lot by quitting their current job and searching for a better one because all jobs would be the same. An estimate of how much unemployment is caused by business cycle factors can be obtained by shutting off the aggregate shock.

When this is done the aggregate unemployment rate falls from 5.2% to 4.6%. Clearly, the model predicts that idiosyncratic shocks are much more important in determining the unemployment rate than are aggregate ones. In particular, by this accounting aggregate shocks contribute no more than 0.6 percentage points to the unemployment rate. These results seem in agreement with Greenwood, MacDonald and Zhang (1996) who estimate, using a two-sector model with indivisible labor, that approximately one percentage point of the unemployment rate can be accounted for by a combination of aggregate and sectoral shocks. It seems that idiosyncratic shocks are important for modelling the unemployment process. Therefore, the emphasis on stabilization policy as a remedy for unemployment may be misplaced.

4 Conclusions

It is time to take stock of the properties of the prototypical search model presented here. The model was successful in accounting for some key labor market regularities. Furthermore, there are several extensions of the model that can potentially rationalize some additional empirical observations. Some of these extensions would be challenging to implement at this time, but then computing a fully specified general equilibrium search model of the sort presented here would have been unimaginable ten years ago. So, what can be explained by the basic search model studied here, and what extensions look potentially promising?

First, search is productive in the model, in the sense that an agent who engages in search ends up, on average, with a new job that is better than the job he just left. The limited evidence that is available for the U.S. economy [Topel and Ward (1992)] agrees with this feature of the model. The model predicts that low wages agents will those who will be more likely to search.

Second, the model is consistent with the observed positive relation between tenure and wages. As Abraham and Farber (1987) note, this relation can result either from high wage workers being less likely to quit their jobs — the story told here — or because there is on-the-job human capital formation. This later aspect could be accommodated in the model by introducing learning-by-doing.

Third, Clark and Summers (1979) document that most spells of unemployment last less than a month. The model is consistent with these rapid exit rates from unemployment. However, the model cannot generate as much long term unemployment as observed in the U.S. data. This is not surprising given the model's emphasis on frictional unemployment. This shortcoming could be overcome in several ways. Perhaps an agent's mean skill level could depreciate with time spent in unemployment. In particular, the more time an agent spends in unemployment the lower the odds are of drawing a good employment opportunity. By itself this could increase the exit rates from unemployment. But combined either with possibilities to do part time or home work, or policies such as unemployment insurance, this might go a long way toward increasing the number of agents willing to endure long spells of unemployment. In a similar vein heterogeneity in mean skill levels could be introduced. A related observation is that in the real world a high of fraction of workers change jobs without experiencing unemployment. This could be captured by allowing for on-the-job search.

Fourth, in the U.S. data consumption drops significantly as agents become unemployed [Dynarski and Sheffrin (1987), Grueber (1994)]. This same drop in consumption, accompanied by lower welfare, takes place in the model when agents become unemployed.

Fifth, in terms of its cyclical properties, the model is capable of mimicking the countercyclical nature of unemployment and its duration. The model is also consistent with the volatility and countercyclical character of the flows in and out of unemployment. These countercyclical movements are, in the model, the result of a simple intuitive mechanism: when aggregate productivity is high the opportunity cost of search rises, leading fewer agents to search and more searchers to accept jobs. The puzzling reduction in the flows out of unemployment during expansions simply reflects an immediate reduction in the pool of unemployed workers when a boom takes place. The fact that in expansions the opportunity cost of search rises, leading to less search, leads to a "cleansing effect" associated with downturns: low productivity jobs tend to persist during expansions but are eliminated in recessions. The volatility of employment is a little anemic in the model. This could be rectified by allowing for a home state representing withdrawal from the labor market. Flows in and out of the labor force are important empirically

There are clearly some important features of the labor market that the model is not equipped to address. The model abstracts from vacancies; the number of new job openings always coincide with the number of unemployed workers. This prevents the model from confronting regularities such as the negative relation between vacancies and unemployment (the Beveridge curve). Vacancies can be introduced by allowing unemployed agents to sample more than once at a cost for each draw. Sample size would now be an endogenous variable. Presumably, in booms agents will sample more so that vacancies (unfilled job opportunities) would be high.

Sixth, in the model's eyes idiosyncratic shocks are much more important for determining the aggregate rate of unemployment than are aggregate ones. When realistic shocks to the economy were incorporated, the rate of unemployment increased by less than 1%. This implies that, from the model's perspective, countercyclical policy is unlikely to be an effective remedy against unemployment.

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A Appendix: Computation

A variant of the Den Haan (1993) and Krusell and Smith (1995) algorithm will be used to solve the model with aggregate shocks. The idea is to approximate the wealth distribution, Z, by a limited set of statistics, such as a set of points characterizing a frequency distribution or a set of moments. A law of motion is also specified for the statistics characterizing Z. In line with the results contained in Krusell and Smith (1995), it will be assumed that approximating the wealth distribution Z by its mean is adequate for the analysis. Denote the mean level of the capital stock by $\mathbf{k}$ so that $\mathbf{k} = \int aZ(da, d\varepsilon, d\lambda)$. In order to solve the model parametric forms must be specified for the law of motion for the aggregate capital stock and the equilibrium interest rate. Assume that the aggregate capital stock has a law of motion of the following form

$$\mathbf{k}' = \kappa_0 + \kappa_1 \mathbf{k} + \kappa_2 \lambda \tag{A.1}$$

and that the equilibrium interest rate can be written as

$$\ln r = \iota_0 + \iota_1 \ln \mathbf{k} + \iota_2 \lambda. \tag{A.2}$$

A.1 Computing the Model's General Equilibrium

The algorithm for computing the solution to the model with aggregate shocks proceeds as follows.

1. *Initialization.* Call up $n(m+1)$ normally distributed random variables. Here n represents the number of periods in the simulation and m is the number of agents. Initialize each agent i's asset holdings at some level, say $a_{i,0}$. This could be done in accordance with the stationary distribution obtained from the deterministic version of the model. Next, an initial guess for the laws of motion for the aggregate capital stock and interest rate is made:

$$\mathbf{k}' = \kappa_0^0 + \kappa_1^0 \mathbf{k} + \kappa_2^0 \lambda$$

and

$$\ln r = \iota_0^0 + \iota_1^0 \ln \mathbf{k} + \iota_2^0 \lambda.$$

A good guess for the ρ's comes from (A.5) below. A good guess for κ_0^0 and κ_1^1 may come from the law of motion for the standard neoclassical growth model.

2. *Computing the Sample Path (Iteration $j+1$)* . The first step is to solve the dynamic programming problems for workers and searchers, taking as given the law of motions for the aggregate capital stock and the equilibrium interest rate:

$$\mathbf{k}_{t+1} = \kappa_0^j + \kappa_1^j \mathbf{k}_t + \kappa_2^j \lambda_t, \tag{A.3}$$

and

$$\ln r_t = \iota_0^j + \iota_1^j \ln \mathbf{k}_t + \iota_2^j \lambda_t. \tag{A.4}$$

This gives a solution for the value functions W^{j+1} and S^{j+1}. The procedure for obtaining these solutions is discussed in detail in section below. Now, suppose that agent i's state in period t is characterized by $(a_{i,t}, \varepsilon_{i,t}, \lambda_t, \mathbf{k}_t)$. To compute his state for $t+1$:

(a) Check whether $W^{j+1}(a_{i,t}; \varepsilon_{i,t}, \lambda_t, \mathbf{k}_t) > S^{j+1}(a_{i,t}, \lambda_t, \mathbf{k}_t)$ to determine whether agent i will work or not in the current period.

(b) Compute the agent's asset holding for period $t+1$, or $a_{i,t+1}$. If the agent is a worker compute his asset holdings for period $t+1$, or $a_{i,t+1}$, using the decision rule $a_{i,t+1} = A^{w,j+1}(a_{i,t}, \varepsilon_{i,t}, \lambda_t, \mathbf{k}_t)$. Alternatively, one could solve the worker's decision problem at the point $(a_t, \varepsilon_{i,t}, \lambda_t, \mathbf{k}_t)$. Note that worker i will hire capital in the amount $k_{i,t} = K(\varepsilon_{i,t}, \lambda_t, r_t)$.

(c) If agent i is a searcher compute his asset holdings using decision rule $a_{i,t+1} = A^{s,j+1}(a_{i,t}, \lambda_t, \mathbf{k}_t)$. Again, one could instead solve the searcher's decision problem at the point $(a_{i,t}, \lambda_t, \mathbf{k}_t)$. A searcher hires no capital so that $k_{i,t} = 0$.

(d) The aggregate supply of capital stock can be computed by calculating $\sum_{i=0}^{m} a_{i,t} = \mathbf{k}_t$. The demand for capital is calculated by computed $\sum_{i=0}^{m} k_{i,t}$. Now, if i is a worker $k_{i,t} = Const \exp(\varepsilon_{i,t} + \lambda_t)^{(1+\theta)/[\theta(1-\alpha)]} \times (1/r_t)^{(\alpha+\theta)/(\theta(1-\alpha))}$, otherwise $k_{i,t} = 0$. Therefore, the equilibrium interest rate can be computed from the formula

$$r_t = [\frac{Const}{\mathbf{k}_t} \sum_{i \in \mathcal{W}} \exp(\varepsilon_{i,t} + \lambda_t)^{(1+\theta)/[\theta(1-\alpha)]}]^{\theta(1-\alpha)/(\alpha+\theta)}, \tag{A.5}$$

where $\mathcal{W}$ is the set of worker's indices.

3. *Updating the Aggregate Law of Motion.* By collecting the time series $\{\mathbf{k}_t\}_{t=0}^n$ and $\{\lambda_t, r_t\}_{t=0}^n$ revised aggregate laws of motion can be computed by running the following regressions

$$\mathbf{k}_{t+1} = \kappa_0^{j+1} + \kappa_1^{j+1}\mathbf{k}_t + \kappa_2^{j+1}\lambda_t = \mathbf{K}(\mathbf{k},\lambda),$$

and

$$\ln r_t = \iota_0^{j+1} + \iota_1^{j+1} \ln \mathbf{k}_t + \iota_2^{j+1}\lambda_t.$$

The procedure should be repeated until $\text{dist}([\kappa^{j+1}, \iota^{j+1}], [\kappa^j, \iota^j]) < \text{tol}$.

A.2 Computing the Value Functions

In Step 1 of the algorithm the value functions W^{j+1} and S^{j+1} needed to be computed. A loop is nested within the main algorithm to do this. Suppose one had a guess for the functions (A.1) and (A.2) as given by (A.3) and (A.4). Given this guess the dynamic programming problems P(1) and P(2) can be solved. In particular the worker's problem would have the general form

$$\begin{aligned} W^{j+1}(a,\varepsilon,\lambda,\mathbf{k}) &= \max_{c,a'k,l} \{U(c) + \\ &\beta \int \max[W^{j+1}(a',\varepsilon',\lambda',\mathbf{K}^j(\mathbf{k},\lambda)), S^{j+1}(a',\lambda',\mathbf{K}^j(\mathbf{k},\lambda))] \\ &\times G_1(\varepsilon'|\varepsilon)F_1(\lambda'|\lambda)d\varepsilon' d\lambda'\} \end{aligned}$$

subject to

$$c + a' = Y(\lambda, \varepsilon; R^j(\mathbf{k},\lambda)) + [1 + R^j(\mathbf{k},\lambda)]a,$$

where the functions R^j and $\mathbf{K}^j$ are defined by (A.3) and (A.4). The searcher's problem would appear as

$$S^{j+1}(a,\lambda,\mathbf{k}) = \max_{c,a'k,l} \{U(c) + \beta \int \max[W^{j+1}(a',\varepsilon',\lambda',\mathbf{K}^j(\mathbf{k},\lambda)), S^{j+1}(a',\lambda',\mathbf{K}^j(\mathbf{k},\lambda))] H_1(\varepsilon')F_1(\lambda'|\lambda)\} d\varepsilon' d\lambda'$$

subject to

$$c + a' = [1 + R^j(\mathbf{k},\lambda)]a.$$

In order to compute these problems the functions W^{j+1} and S^{j+1} are approximated by low order polynomials, specifically quadratics. First, a grid was specified over the model's state space for the continuous variables a, ε, and $\mathbf{k}$ — recall that λ only has three values contained in the $\mathcal{L}$. Denote these sets of grid points by $\mathcal{A}, \mathcal{E}$, and $\mathcal{K}$. Second, an initial guess for the 2nd degree polynomials used to approximate W^{j+1} and S^{j+1} is made. Denote this guess by $W^{j+1,0}$ and $S^{j+1,0}$. A good initial guess may be the solution for the value functions obtained on the previous iteration of the main algorithm, or W^j and S^j. Third, given a guess for W^{j+1} and S^{j+1} at the i-th iteration of this inner loop, or $W^{j+1,i}$ and $S^{j+1,i}$, problems P(1) and P(2) are solved at each point in the set $\mathcal{A} \times \mathcal{E} \times \mathcal{L} \times \mathcal{K}$ by using this guess on the righthand side of P(1) and P(2). This results in lefthand values for W^{j+1} and S^{j+1} at each of these points. Fourth, two new second degree polynomials are then fitted to these points via least squares. The new functions are represented by $W^{j+1,i+1}$ and $S^{j+1,i+1}$. Fifth, the procedure is repeated until convergence is obtained.

Remark Note that the deterministic version of the model can easily be computed by just solving this inner loop for computing W and S for a given value of r, which is adjusted iteratively until the demand and supply of capital are equated. Here W and S are just functions of a and ε so there is no need to find the laws of motion (A.1) and (A.2).

Table 1: Standard Business Cycle Facts

U.S. Quarterly Data — 1954:1 - 1991:2		
Variable	**Rel. Std. Dev.**[1] (%)	**Corr. with Output**
Output	1.72	1.00
Consumption	0.73	0.83
Investment	2.97	0.90
Hours	0.92	0.92
Productivity	0.42	0.34

Source: Cooley and Prescott (1995)

[1] All standard deviations (except output) are reported relative to output.

Model		
Variable	**Rel. Std. Dev.** (%)	**Corr. with Output**
Output	1.00	1.00
Consumption	0.81	.98
Investment	2.06	.94
Total Hours	0.53	1.00
Productivity	0.48	1.00

Table 2: Labor Market Facts

U.S. Quarterly Data		
Variable	**Rel. Std. Dev. (%)**	**Corr. with Output**
Employment[1]	0.81	0.89
Average Weekly Hours[1]	0.27	0.62
Unemployment[2]	0.48	-0.89
Duration[2]	6.87	-0.37
Unemployment — Flow In	3.11[3]	-0.78[4]
Unemployment — Flow Out	2.50[3]	-0.51[4]
Corr(Flow In, Flow Out) = 0.64[4]		

[1] Cooley and Prescott (1995), Table 1.1; period 1954:1-1991:2.

[2]Computed for period 1954:1-1991:2.

[3] Merz (1996), Table 2; period 1959:1-1988:2.

[4] Merz (1996), Table 2; period 1959:1-1981:4.

Model		
Variable	**Rel. Std. Dev. (%)**	**Corr. with Output**
Employment	0.12	0.71
Average Weekly Hours	0.44	0.99
Unemployment	2.06	-0.71
Duration	0.30	-0.83
Unemployment — Flow In	2.04	-0.66
Unemployment — Flow Out	2.06	-0.56
Corr(Flow In, Flow Out) = 0.11		

Figure 1: Determination of Consumption

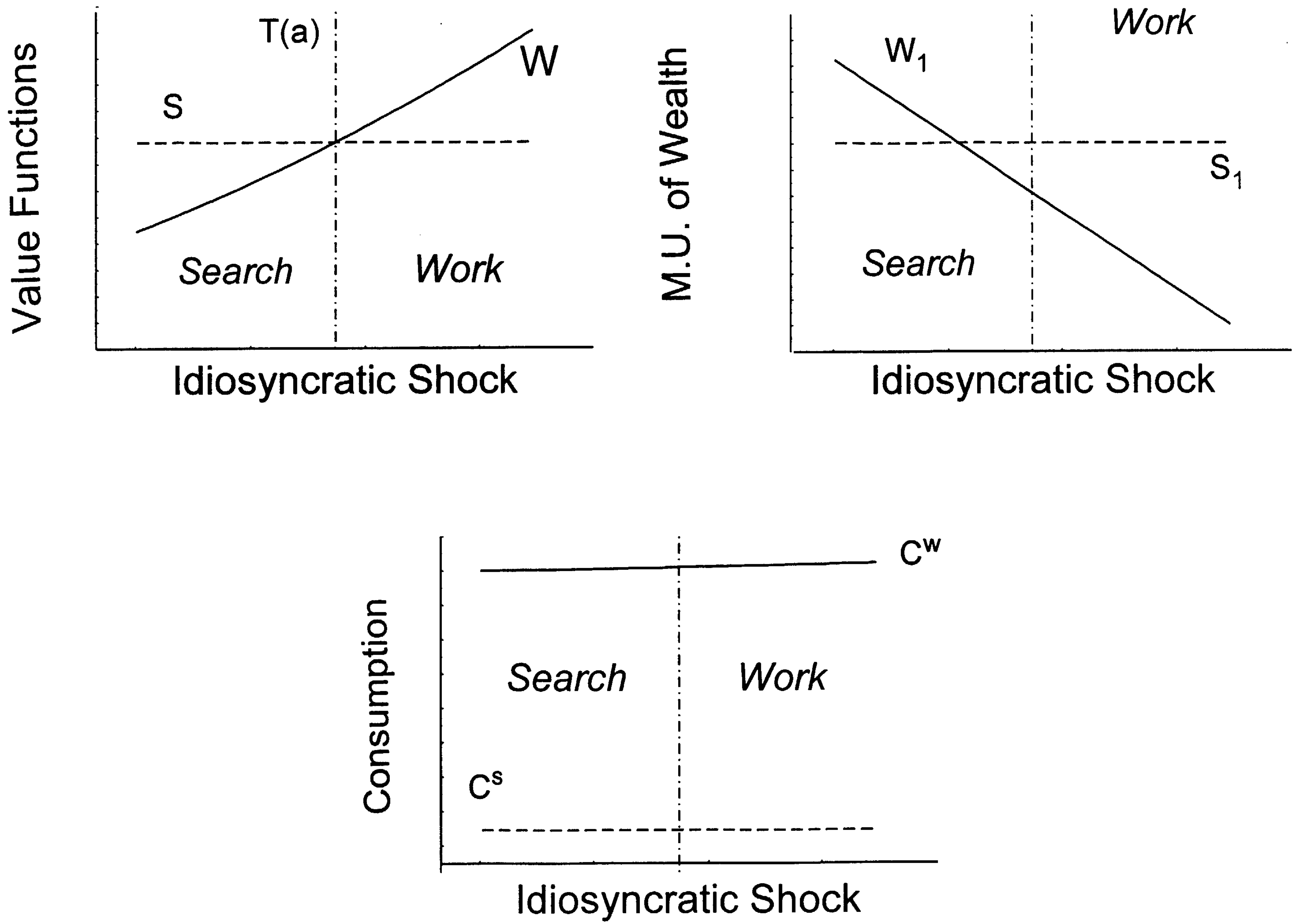


Figure 2: Exit Rates from Unemployment

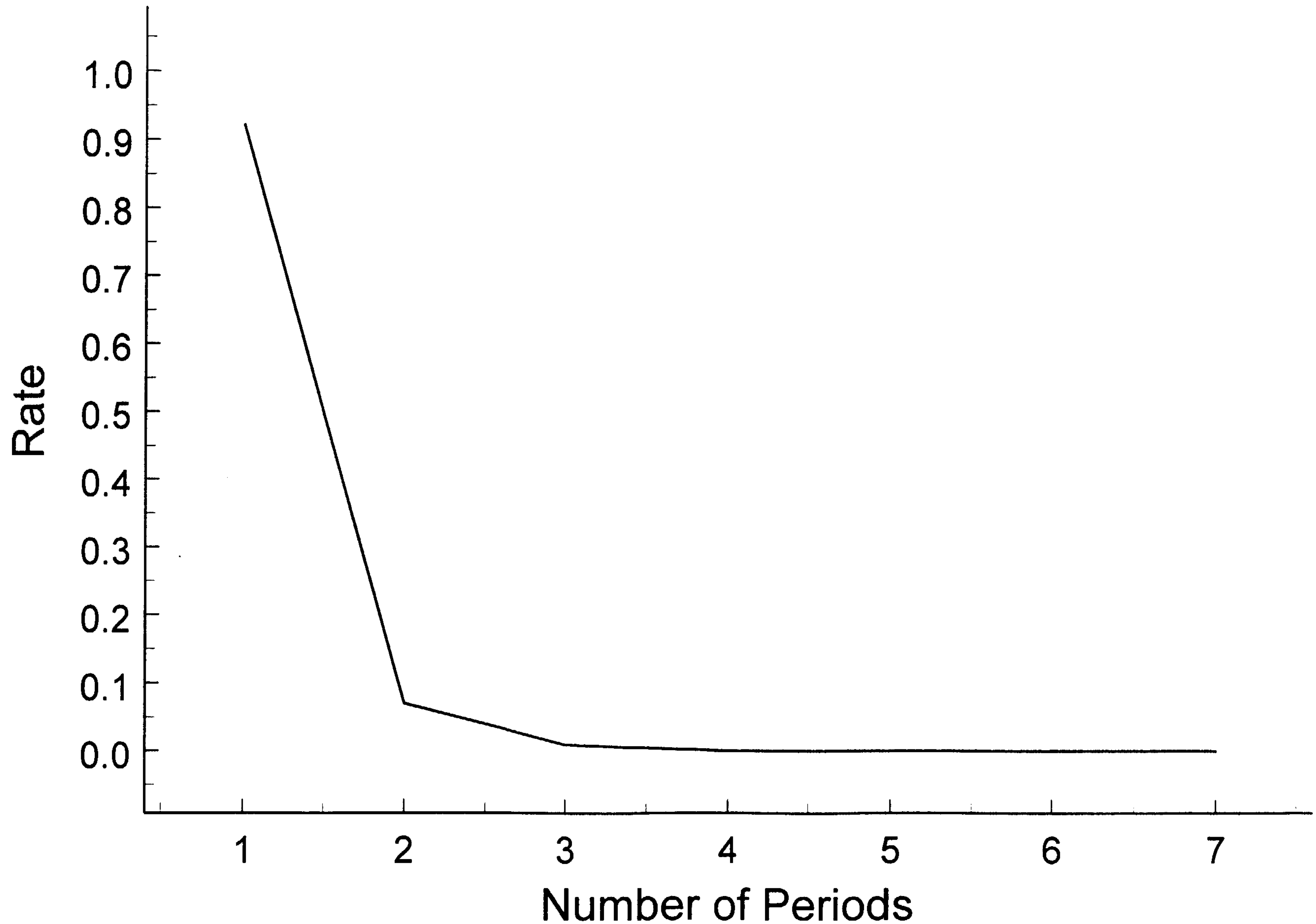


Figure 3: Work vs. Search

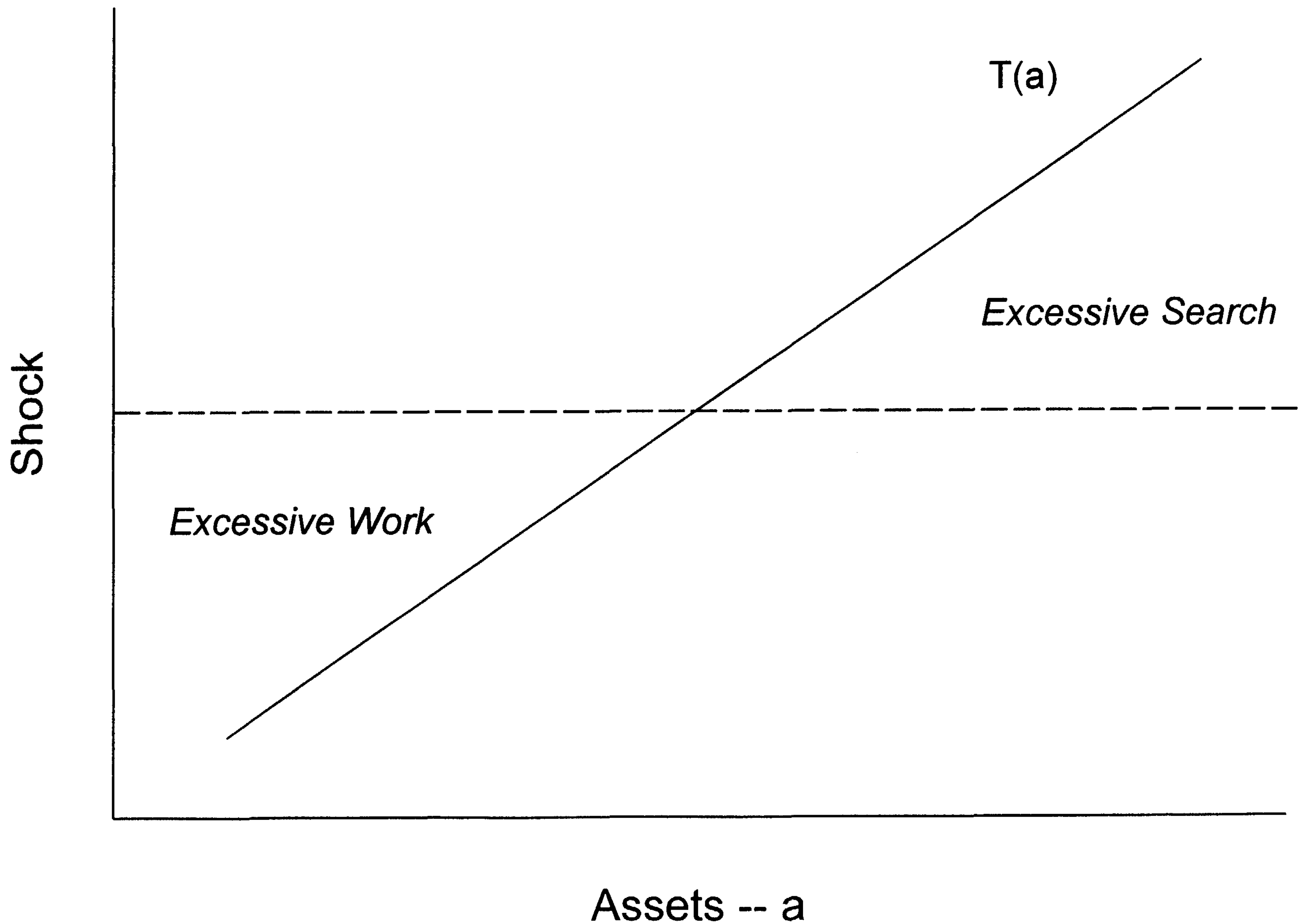


Figure 4: Threshold Rules

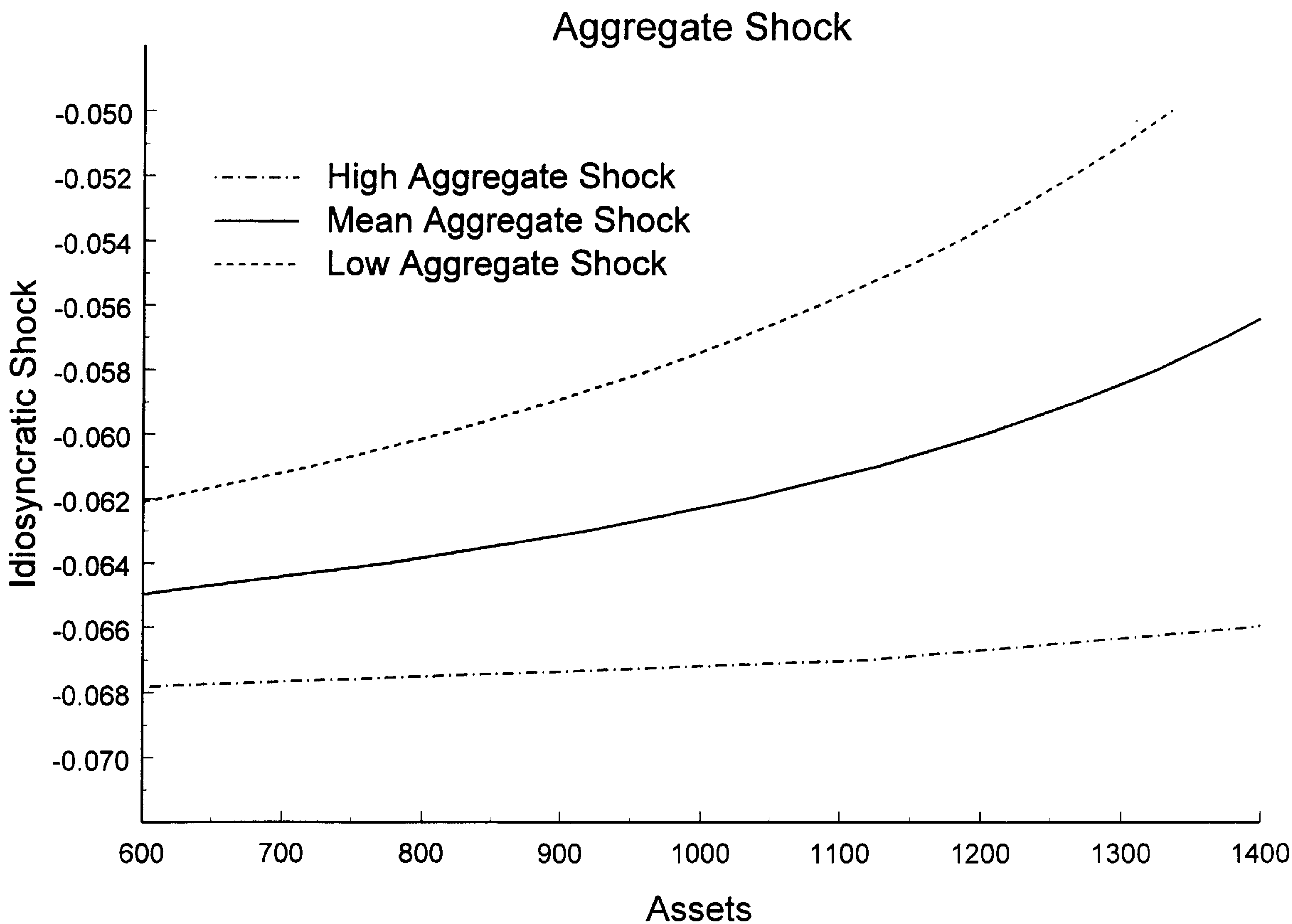


Figure 5: Threshold Rules

Aggregate Capital Stock

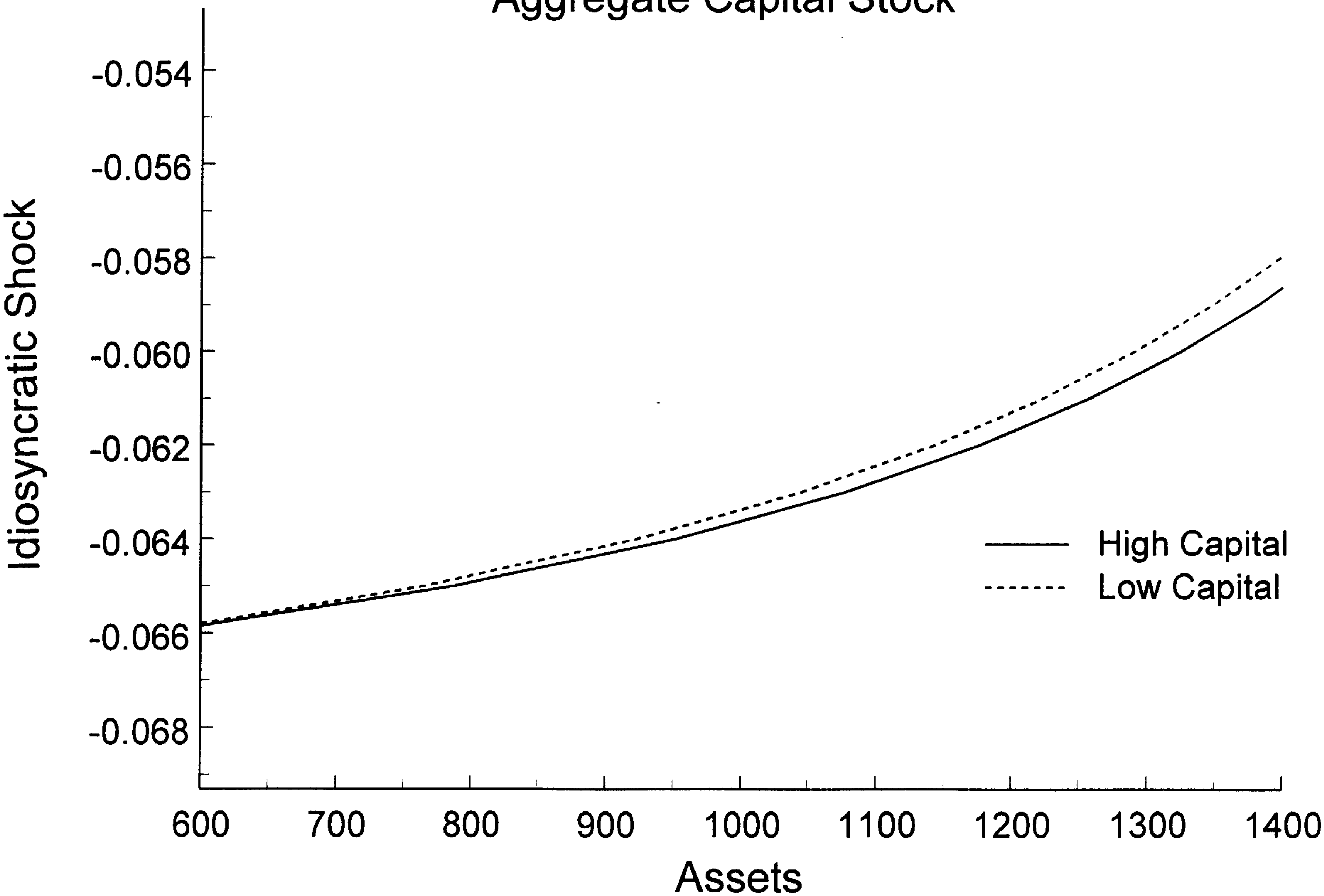


Figure 6a: Impulse Response

Positive Shock

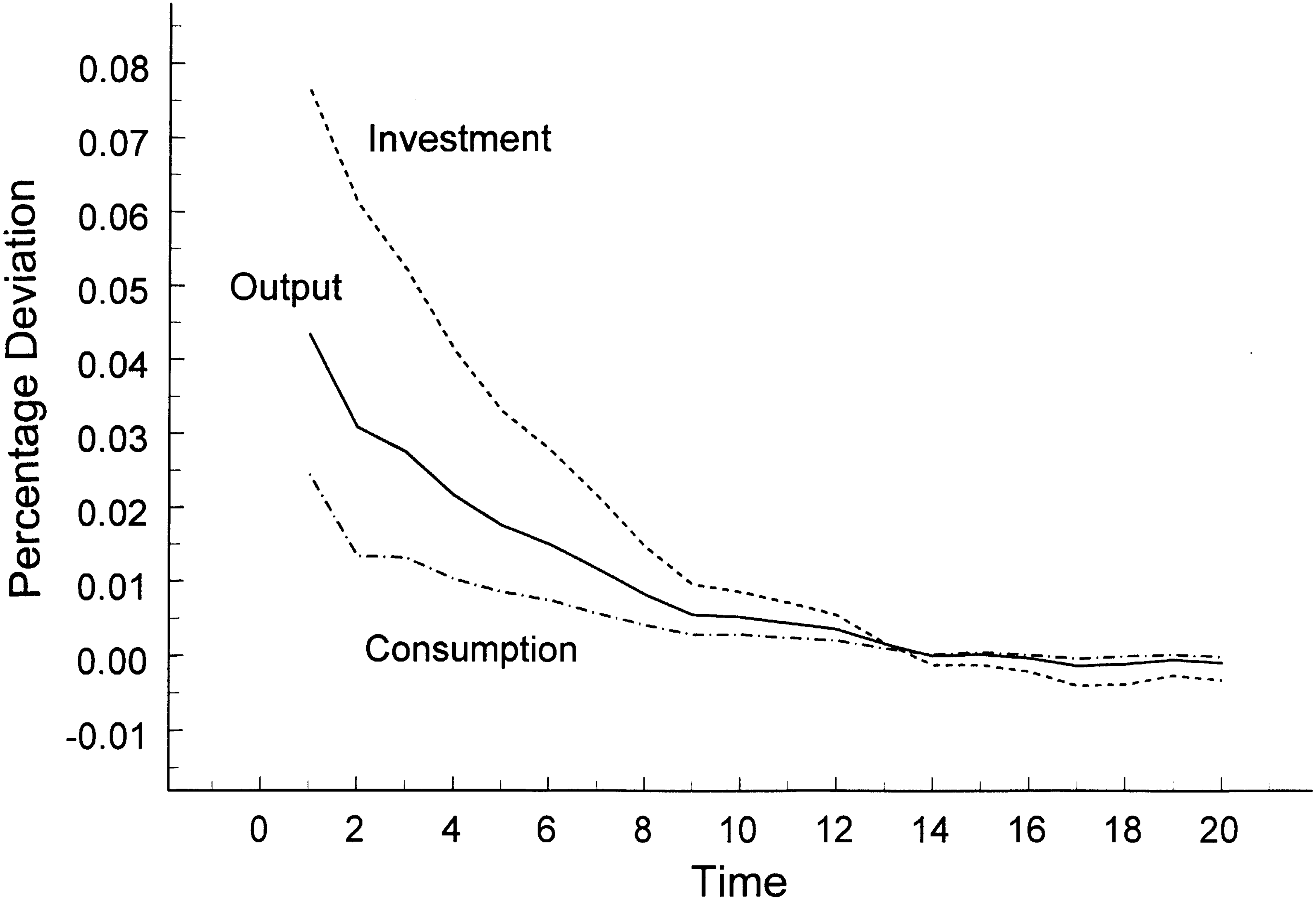
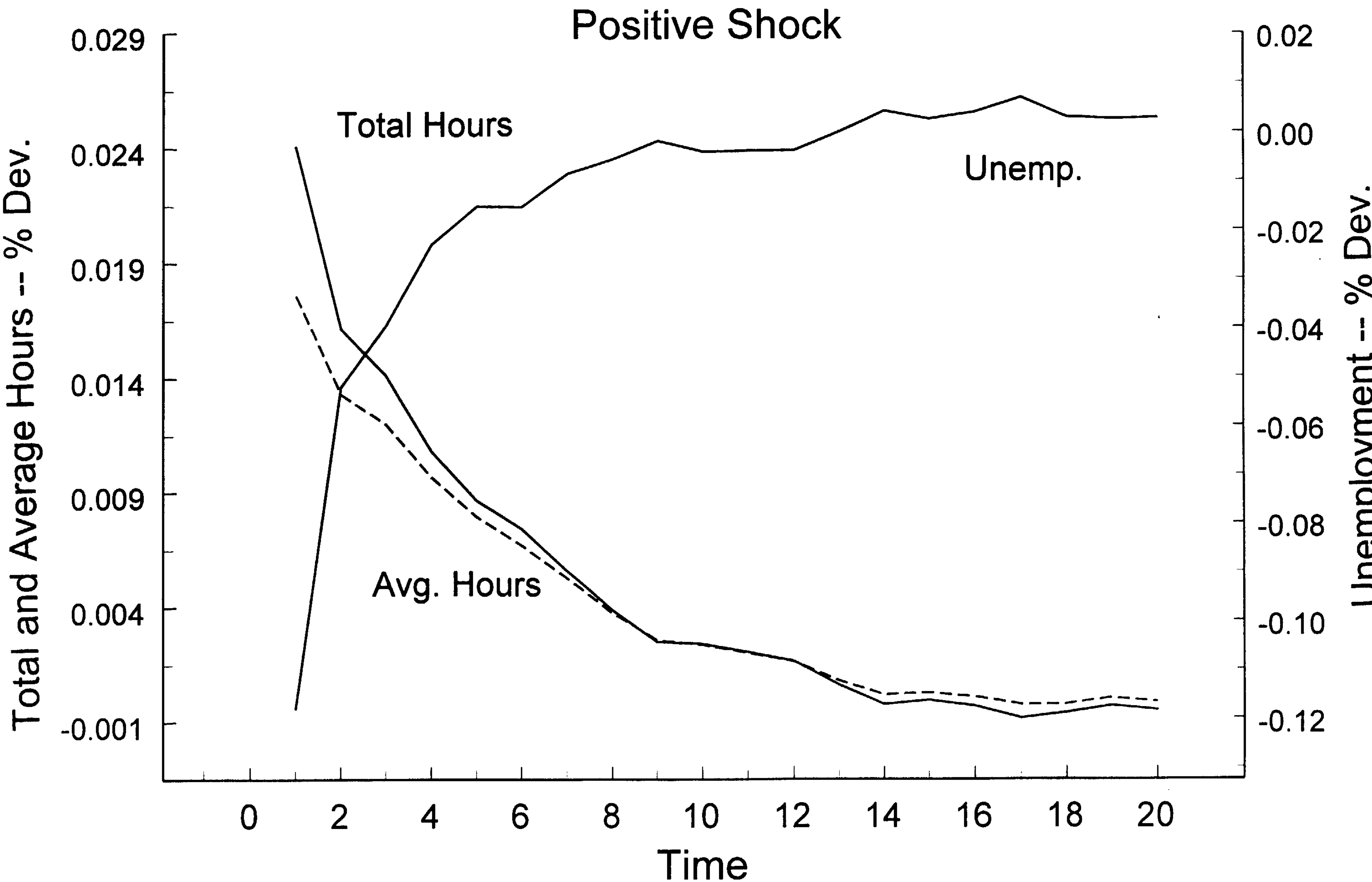


Figure 6b: Impulse Response



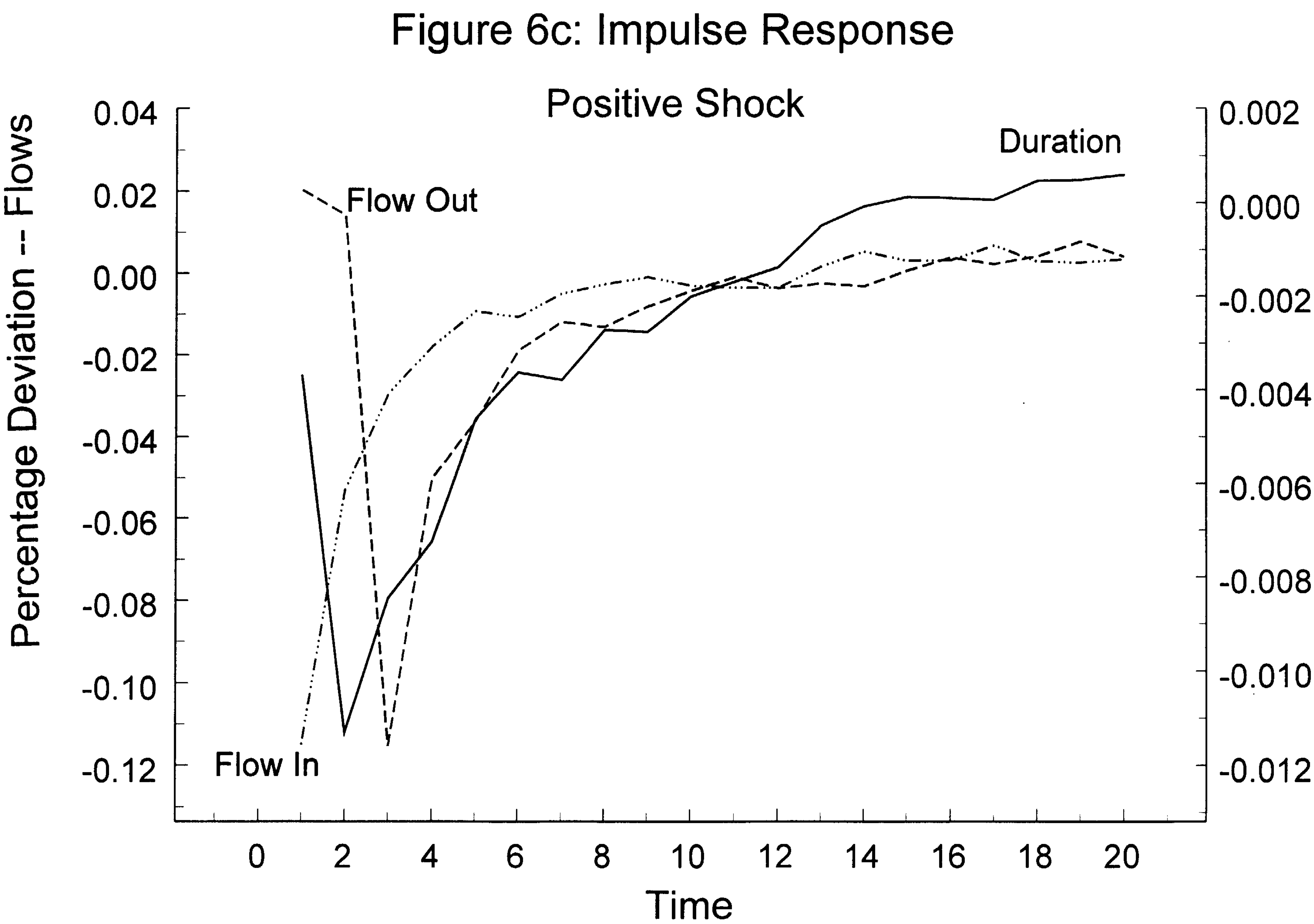


Figure 6c: Impulse Response
Positive Shock
Percentage Deviation -- Flows
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
0.002
0.000
-0.002
-0.004
-0.006
-0.008
-0.010
-0.012
Duration
Flow Out
Flow In
0
2
4
6
8
10
12
14
16
18
20
Time

Figure 7a: Impulse Response

Negative Shock

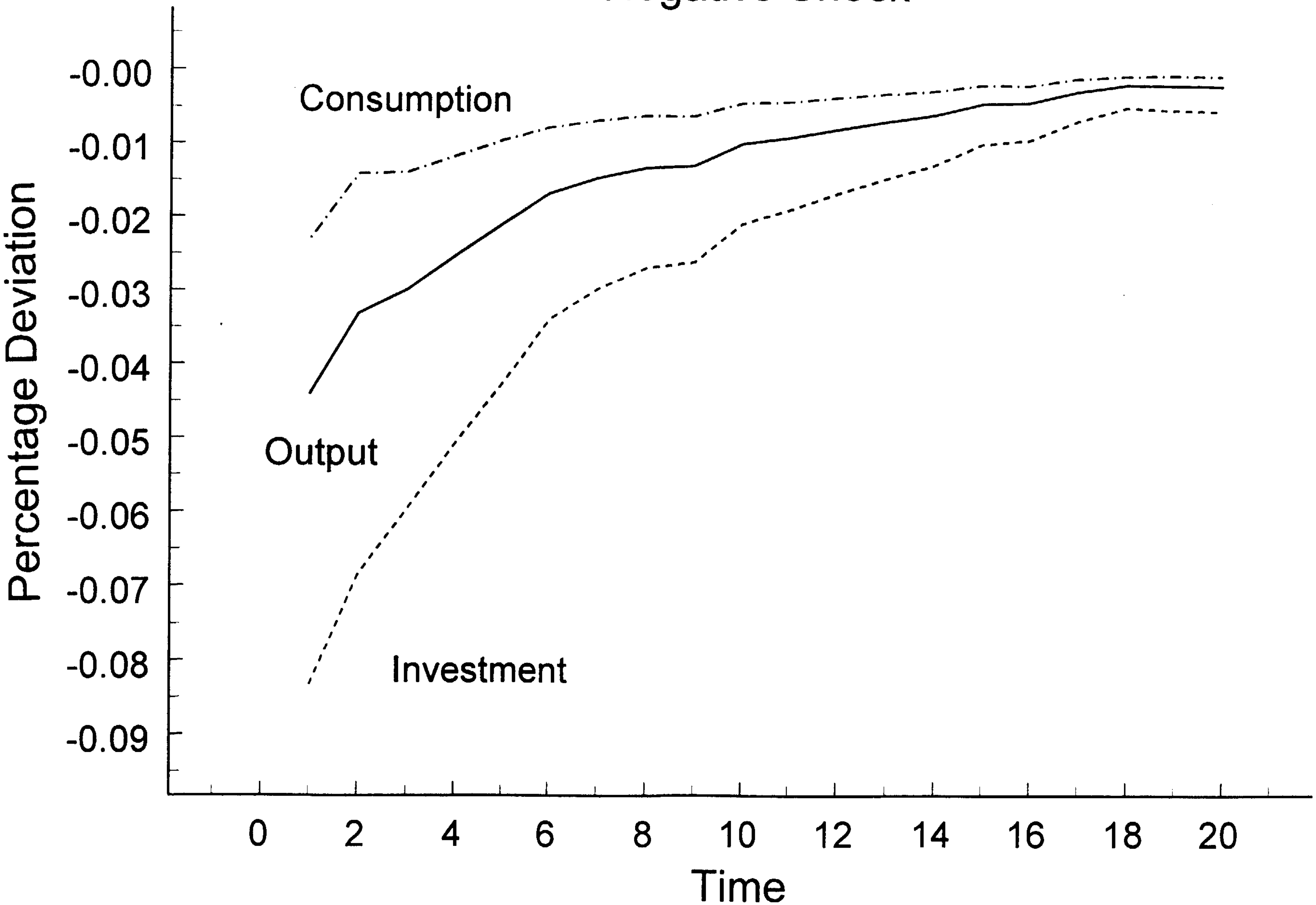


Figure 7b: Impulse Response

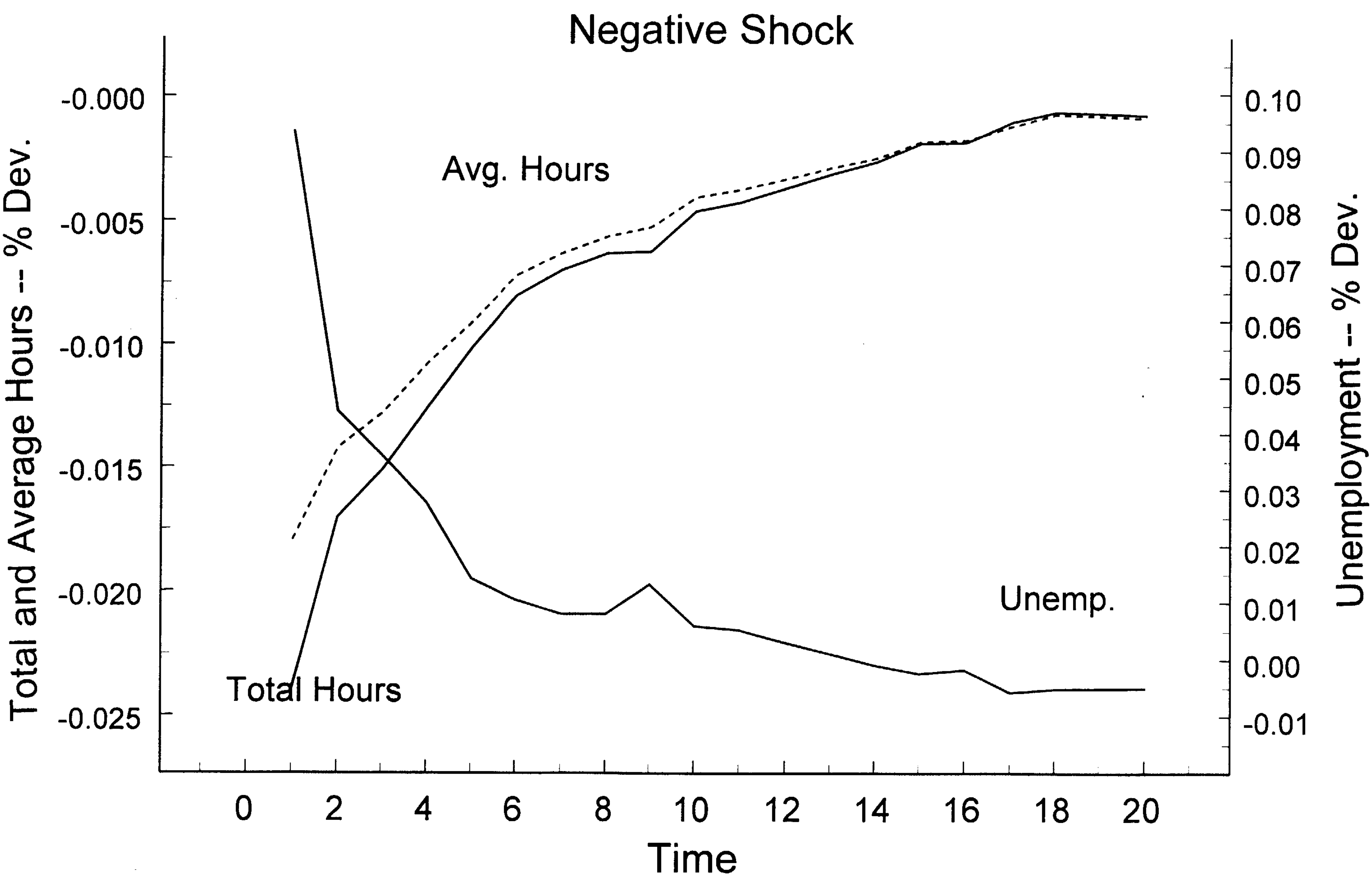


Figure 7c: Impulse Response