NBER WORKING PAPER SERIES

AN OPTIMIZING IS-LM SPECIFICATION FOR MONETARY POLICY AND BUSINESS CYCLE ANALYSIS

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Working Paper 5875

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 1997

We are indebted to Olivier Blanchard, Jinill Kim, Bob King, Allan Meltzer, Lars Svensson, and Alex Wolman for helpful comments. This paper is part of NBER's research programs in Economic Fluctuations and Growth and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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JEL Nos. E10, E30, E40
Economic Fluctuations and Growth
and Monetary Economics

ABSTRACT

This paper asks whether relations of the IS-LM type can sensibly be used for the aggregate demand portion of a dynamic optimizing general equilibrium model intended for analysis of issues regarding monetary policy and cyclical fluctuations. The main result is that only one change -- the addition of a term regarding expected future income -- is needed to make the IS function match a fully optimizing model, whereas no changes are needed for the LM function. This modification imparts a dynamic, forward-looking aspect to saving behavior and leads to a model of aggregate demand that is tractable and usable with a wide variety of aggregate supply specifications. Theoretical applications concerning price level determinacy and inflation persistence are included.

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I. Introduction

Although a few scattered words of defense can be found in the literature,[1] the once-dominant IS-LM framework for macroeconomic analysis has been sharply criticized by many leading researchers over the last 20 years or more. Among the critics it is possible to list authors as prominent and diverse as Barro (1984), Brunner and Meltzer (1974, 1993), Friedman (1976), King (1993), Leijonhufvud (1968, 1983), Lucas (1994), Sims (1992), Tobin (1969), and Wallace (1980). There is also a considerable amount of diversity in the reasons or logical bases for the criticisms, with at least six distinct failings being mentioned. Nevertheless, most undergraduate macroeconomics textbooks continue to feature IS-LM models,[2] and variants are frequently utilized in both theoretical and empirical analyses by a substantial number of workers.

From a historical perspective it might be regarded as unlikely that the IS-LM construct would be truly incoherent, since its creator, J.R. Hicks, was unquestionably competent in general equilibrium analysis.[3] But Hicks's (1937) famous article was intended as an exposition of Keynes's *General Theory* views, and it has been firmly established that these involve numerous logical inconsistencies,[4] so it is conceivable that a wholesale criticism might be warranted. In any event, it has long been the belief of the first-named author of the present

[1] See for example Mankiw (1990), Ball and Mankiw (1994), Gali (1992), Hall (1980), Hoover (1995), Taylor (1993), and McCallum (1989, 1995). See also footnote 7 below.

[2] See, for example, recent texts by Abel and Bernanke (1992) or Mankiw (1994). A notable exception is Barro (1993), whose IS-LM section comes at the very end of the book.

[3] This is of course an understatement, as students of his *Value and Capital* (1939) will know.

[4] See Patinkin (1976). Patinkin himself paid tribute to Keynes's work as a great contribution, but his cited book (and other writings) detail an enormous number of significant theoretical errors in the *General Theory*.

paper that sensible macro and monetary analysis of many issues[5] can be conducted using IS-LM relations, provided that they are accompanied by "aggregate supply" or "price adjustment" sectors that may reflect temporary price stickiness, but have reasonable - - i.e., classical - - long-run properties. To put the matter bluntly, it is suggested in McCallum (1989, pp. 102-107) that useful insights into monetary policy and business cycle behavior may be provided by a macroeconomic structure such as

$$(IS) \qquad \log y_t = b_0 + b_1[R_t - E_t(\log P_{t+1} - \log P_t)] + v_t$$

$$(LM) \qquad \log M_t - \log P_t = c_0 + c_1 \log y_t + c_2 R_t + \eta_t$$

$$(AS) \qquad \log y_t = a_0 + a_1(\log P_t - E_{t-1} \log P_t) + a_2 \log y_{t-1} + u_t$$

plus a policy rule for M_t (or R_t). Here y_t, P_t, and M_t are measures of real output, the price level, and nominal money balances, respectively, while R_t is a nominal interest rate and $E_t(\cdot) = E(\cdot|\Omega_t)$, with Ω_t representing the set of information available in period t. In such a system, the IS and LM relations pertain to the "demand side" of the macroeconomic system, while AS represents aggregate supply behavior.[6] But the defense of IS-LM demand-side specifications offered in those pages can be regarded as successful only for the LM half of the combination (at best); in the case of the IS function, a much weaker justification is provided. Similarly, Blanchard and Fischer (1989, p. 532) find the LM function to be "quite

[5] Not including those related to economic growth.

[6] We do not mean to suggest that the particular specification given by (AS) is an appropriate one. There is an enormous amount of professional disagreement over the proper specification of this sector of the model. We wish merely to indicate that the IS and LM relations do not themselves constitute a complete macroeconomic structure.

consistent with the demand for money that emerges" from their more detailed optimizing treatment, but conclude that the IS function "is only a pale reflection of our analysis of optimal [saving and investment] behavior under uncertainty."

In the present paper, we seek to provide a reconsidered and reasonably wide-ranging analysis of the issue. In particular, we will in Section II review the main criticisms of the IS-LM approach, concluding that most of them are not crippling in the context of monetary policy and business cycle (as opposed to growth) analysis. For consistency with optimizing behavior, however, one simple but crucial modification to the usual IS specification is needed.[7] This central point is developed in Section III for a basic setup, after which Section IV provides extensions for fiscal and foreign influences. Two illustrative applications of the resulting specification to issues including price level determinacy and inflation persistence are then included in Sections V and VI. The paper concludes with a recapitulation in Section VII.

II. Weaknesses of IS-LM Models

Let us begin by cataloging some of the principal objections to IS-LM models that have been expressed over the years. Among these are the following:

(i) IS-LM analysis presumes a fixed, rigid price level

[7] This statement does not refer to the absence of fiscal variables from equation (IS) above. Instead, the modification involves the inclusion of an additional variable reflecting expected future income. A similar modification has recently been suggested by Kerr and King (1996). Fane (1985) and Koenig (1989, 1993a, 1993b) represent previous efforts with objectives similiar to those of the present paper, but they only show that some comparative-static properties of their models are like those of an IS-LM setup. In particular, they do not develop dynamic equations analogous to IS and LM functions, as is done below. Auerbach and Kotlikoff (1995, pp. 312-315) derive IS and LM equations from an overlapping generations framework under the highly restrictive assumption of rigid prices.

(ii) It does not distinguish between real and nominal interest rates

(iii) It does not recognize enough distinct assets

(iv) It permits only short-run analysis

(v) It treats the capital stock as fixed

(vi) It is not derivable from explicit maximizing analysis of rational economic agents.[8]

We now discuss each objection in turn.

It is certainly true that textbook-style IS-LM analysis of the 1950s and 1960s was often guilty of charges (i) and (ii), and we share the opinion that such analysis is fundamentally misguided - - in part because it creates the impression that real output movements and levels are readily manipulatable by the monetary authorities. But models with such weaknesses are not the type under discussion. Instead, as indicated above, we are concerned with the use of IS and LM functions to represent aggregate demand behavior, as opposed to aggregate supply, in macroeconomic models that do recognize price level variability and the real vs. nominal interest rate distinction. Such usage was emphasized by Bailey (1962), and is utilized in the famous paper of Sargent and Wallace (1975), as well as textbooks by Sargent (1979) and McCallum (1989).

Criticism (iii), expressed by Brunner and Meltzer (1974, 1993) and Tobin (1969), is clearly correct for some purposes. Since the usual IS-LM model recognizes only one (nominal) interest rate, it implicitly lumps all assets into two categories, termed "money" and "bonds".

[8] The point stressed by King (1993), that the standard IS-LM model omits important expectational influences, is a significant special case of this objection.

Thus there is no distinction between treasury bills, commercial paper, long-term private and government bonds, or physical capital – all are simply treated as perfectly substitutable components of a single interest-bearing nonmonetary asset. Accordingly, many interesting macro or monetary issues cannot be addressed by such IS-LM models. But for a considerable range of problems in these areas, it would appear that the two-asset restriction is not critical. Many critics of IS-LM analysis are evidently willing to use models of other types with two (or fewer!) assets for a variety of issues – see, e.g., Wallace (1980), Lucas (1972), or King (1993). Or, to put the point in another way, disaggregation provides benefits but also costs, so two-asset models will often prove convenient and satisfactory.

Criticisms (iv) and (v) are closely related, since the traditional justification for the fixed capital stock assumption is that short-run analysis was originally the model's reason for existence. For the purpose of business cycle analysis, however, it is clearly unsatisfactory to maintain the short-run limitation that was common in the literature of the 1940s and 1950s. The problem is not only that the duration of a typical cycle seems too long to justify a "short-run" assumption; in addition it is the case that macroeconomics in the era of rational expectations is inherently dynamic. The emphasis of recent IS-LM supporters is therefore directed toward expectational phenomena, gradual adjustment to various shocks, and the consequences of alternative maintained policy rules.[9] Thus the short-run assumption will not be a feature of the framework to be constructed below. Instead, it will be presumed

[9] Taylor's work (e.g., 1993) in this area is outstanding. Although he tends not to use the term IS-LM, his models amount to IS-LM structures with some disaggregation in the IS portion and with open-economy influences recognized.

that the model is designed for quarterly time series data over sample periods of many years' duration (e.g., 10 to 50 years).

What, then, becomes of the original fixed-capital assumption? In principle it might be possible to incorporate an endogenously determined capital stock,[10] but for the present discussion we will utilize a simpler approach by treating capital's time path as exogenous. Thus in a theoretical analysis one might assume a constant or steadily-growing capital stock, or in an empirical study either steady growth or the actual historical values might be used, but in either case movements in capital would not be explained endogenously. Clearly, this is a treatment that could be challenged, so we will spend the remainder of this section on the presentation of our rationale.

Basically, our argument is that the assumption of an exogenously given time path for the capital stock is not a crippling flaw in the context of business cycle analysis because there is very little connection at cyclical frequencies between capital stock movements and those in aggregate output and consumption variables. In large part this is because a typical year's investment is very small in relation to the existing stock of capital. That empirically there is a very small correlation between capital and output measures, k_t and y_t, say, was briefly mentioned by Kydland and Prescott (1990, p. 12), and will be documented more fully below. Before turning to that documentation, however, let us briefly describe a line of reasoning recently put forth by Wen (1996). This reasoning was developed to explain why typical real business cycle (RBC) models do not yield impulse response patterns, and other

[10] As in Sargent and Wallace (1975).

dynamic features, similar to those found in actual U.S. time series.

Wen's explanation can be approximated as follows. Suppose that labor employment is fixed, so that the usual production function implies

(1) $$\log \hat{y}_t = \log \hat{A}_t + \alpha \log \hat{k}_t ,$$

where A_t is a technology shock, and where the hats indicate values measured relative to their steady-state magnitudes. Now let us log-linearize the income identity $y_t = c_t + i_t$ and also the capital accumulation equation $k_{t+1} = i_t + (1-\delta)\, k_t$ around their steady-state paths, obtaining

(2) $$\log \hat{y}_t = (1-s) \log \hat{c}_t + s \log \hat{i}_t$$

(3) $$\log \hat{k}_{t+1} = d \log \hat{i}_t + (1-d) \log \hat{k}_t , \qquad d = (g+\delta)/(1+g),$$

where c_t denotes consumption i_t investment, and g the average growth rate. In this context, a 1.0 percent departure of A_t from its expected value will yield directly a 1.0 percent change in y_t. Then with s equal to about 1/3, i_t will experience a 3 percent change, so with d equal to about 0.025 (on a quarterly basis), the effect on k_{t+1} will be about 0.075 percent. With α equal to (say) 1/3, the effect on y_{t+1} will then be about 0.025 percent, which is virtually negligible in relation to the direct effect of the shock. Thus the serial correlation pattern of y_t will tend to mimic that of A_t, with capital accumulation playing a very small role. Furthermore, Wen observed that "incorporating endogenous labor choice into the model can help only to amplify shocks at the initial period, but not to propagate shocks for the following periods" (1996, p. 5).

Wen's result was obtained using the basic stochastic growth model, which imposes several restrictive conditions, including fully flexible prices and only one source of shocks. In practice, these conditions are likely to be violated, so the empirical relevance of Wen's conclusions may be questioned. Let us therefore consider a bit of empirical evidence for the U.S. economy in support of the proposition that capital and output movements are in fact not strongly related over business cycles.

We will use two capital stock measures, one fairly narrow and the other comparatively broad. The narrow measure pertains to net private non-residential fixed capital while the broader series includes private residential capital as well, plus government capital and household stocks of consumer durables. Our end-of-quarter measures have been calculated from Musgrave's (1992) annual capital stock series by distributing the annual changes over quarters in proportion to quarterly shares of each year's values for a gross investment variable that corresponds to the capital component.[11] The resulting series are reported in Appendix A.

For these two capital measures, time series plots of their logs are shown, along with logs of real GNP, in panels A and B of Figure 1. That there is very little, if any, relation at cyclical frequencies is apparent. Figure 2 shows that somewhat more of a relationship is induced, although not at cyclical frequencies, if per capita values are considered. (Here civilian population in the second month of each quarter is used as the population measure.) In addition, we have plotted in Figures 3 and 4 first differences and Hodrick-Prescott filtered (with $\lambda = 1600$) values of the logs of the per capita series. In Figure 4 there is a clearly observable

[11] These investment series are gross private nonresidential investment, gross private residential investment, government purchases of goods and services, and purchases of consumer durables (all in 1987 prices).

tendency for capital per person to fall during recessions and to rise during episodes of high output, but even in this case there is no apparent effect of capital on output. Furthermore, it is demonstrated by Cogley and Nason (1995) that the Hodrick-Prescott filter tends to produce spurious cyclical patterns when applied to cycle-free but persistent data. For the most part, then, these figures provide support for our contention that for cyclical purposes it is satisfactory to treat capital stock movements as smooth and/or exogenous.

Figure 1: Log Levels

Figure 1A

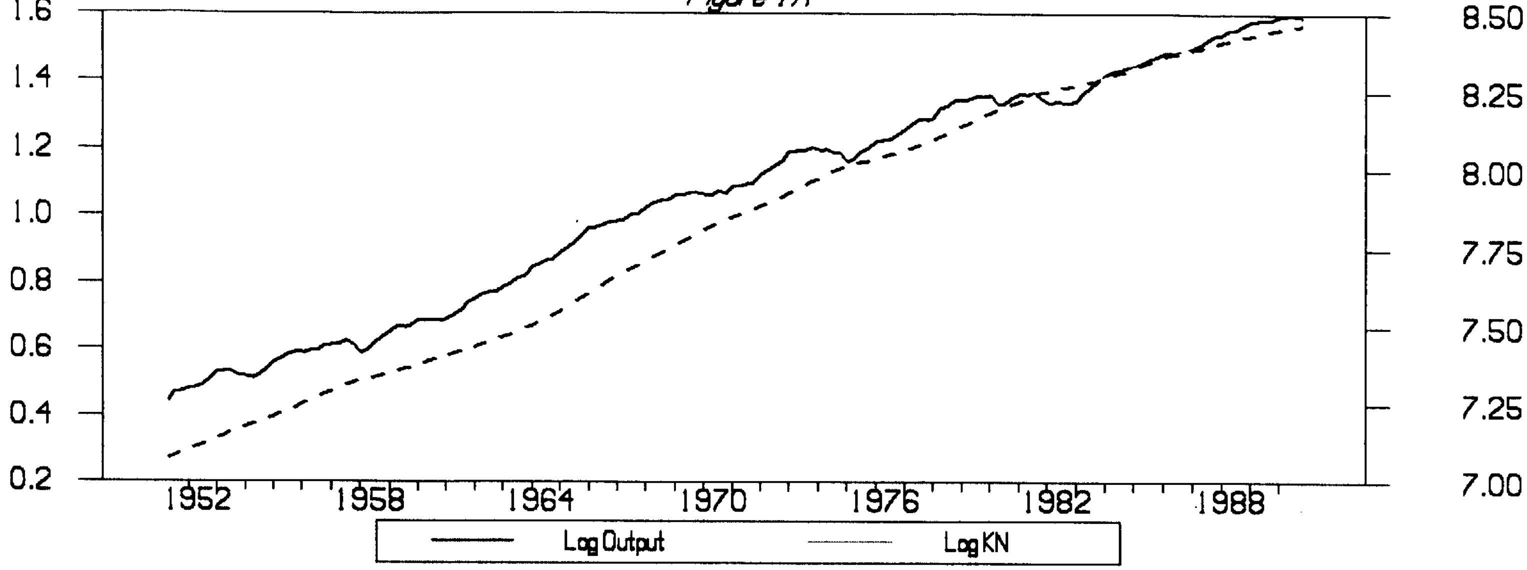


Figure 1B

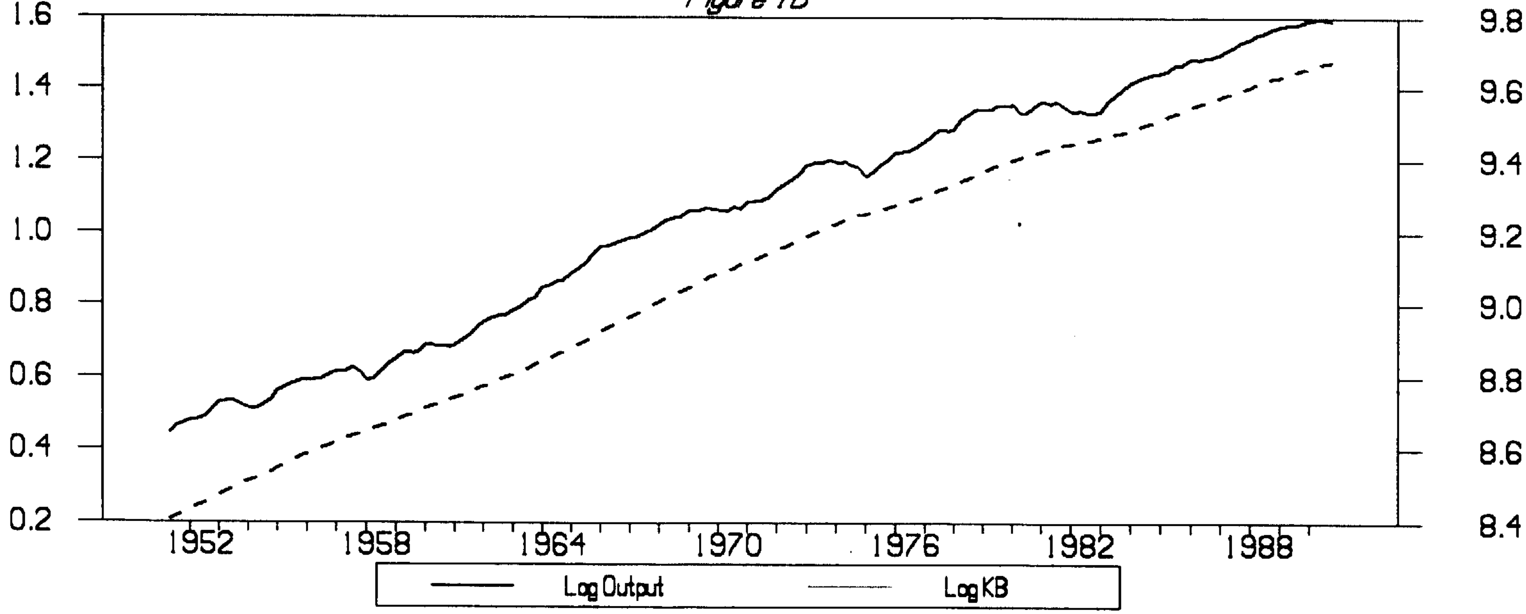


Figure 2: Per Capita Logs

Figure 2A

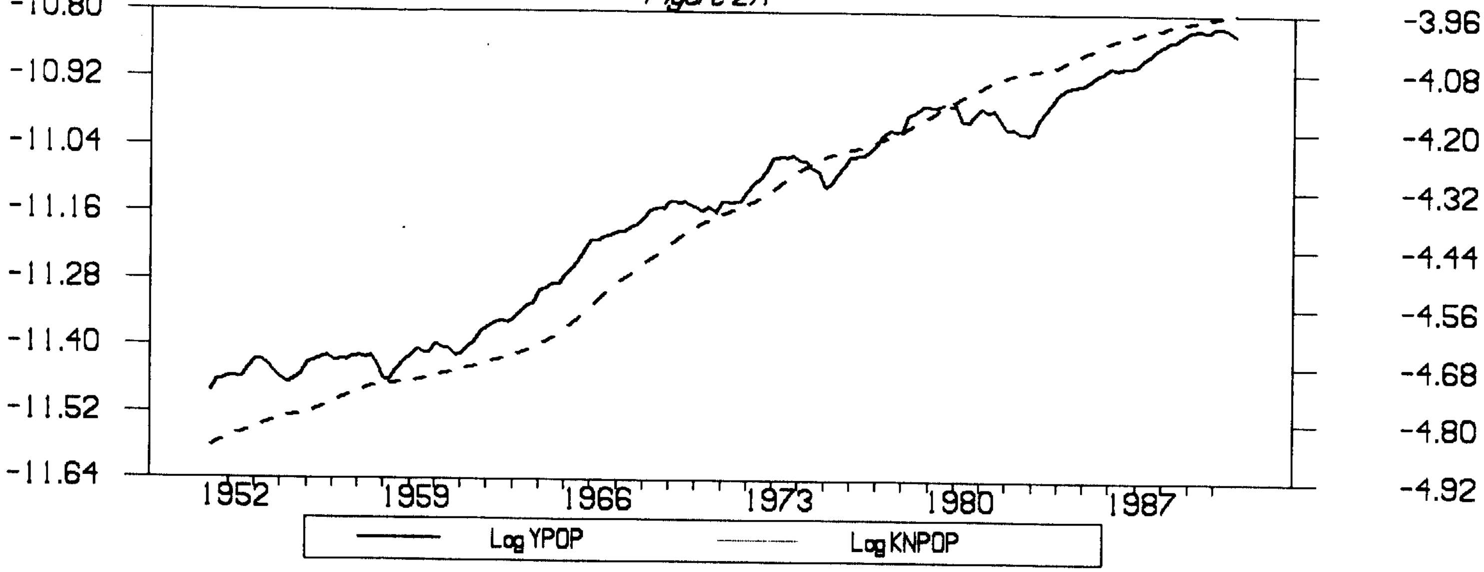


Figure 2B

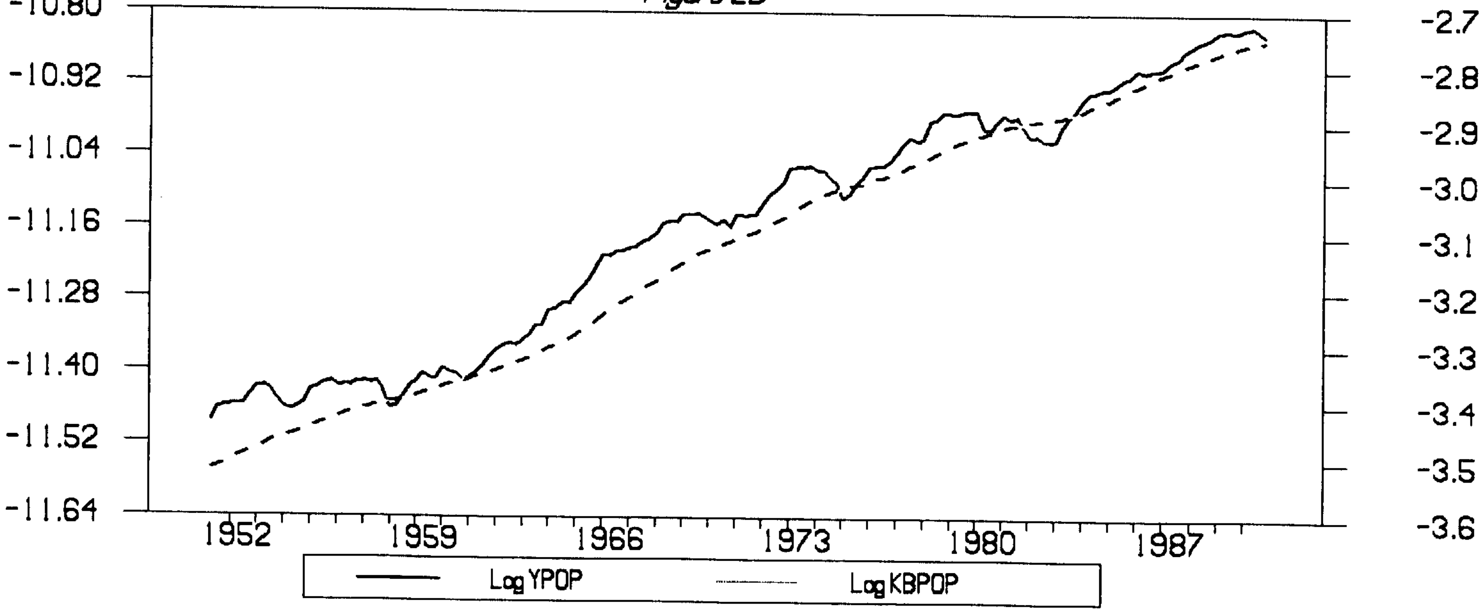


Figure 3: Log Differences

Figure 3A

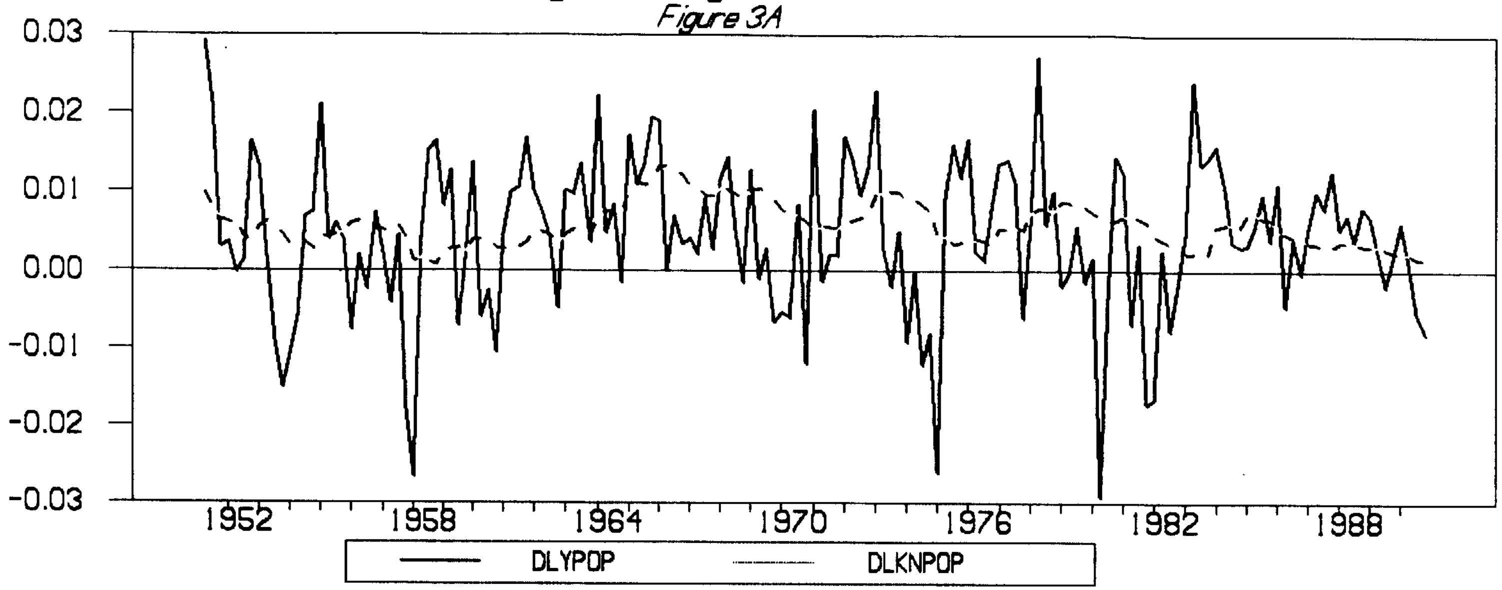


Figure 3B

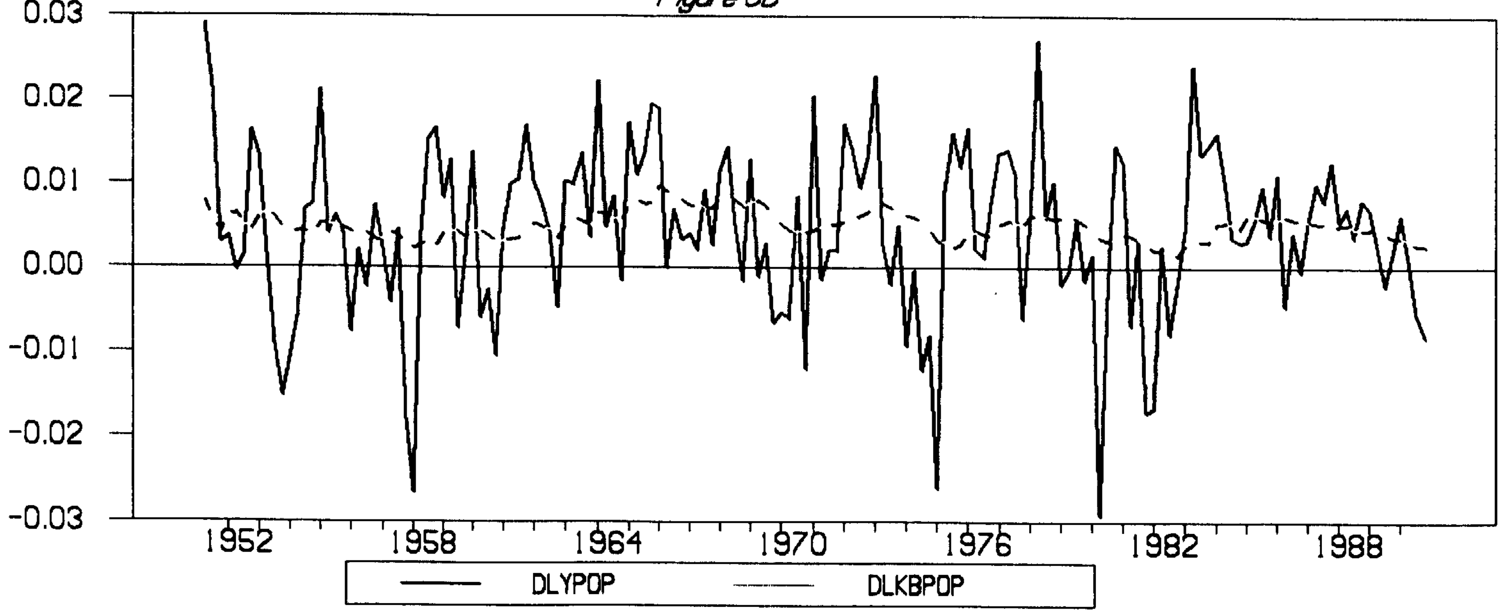


Figure 4: HP-FILTERED DATA

Figure 4A

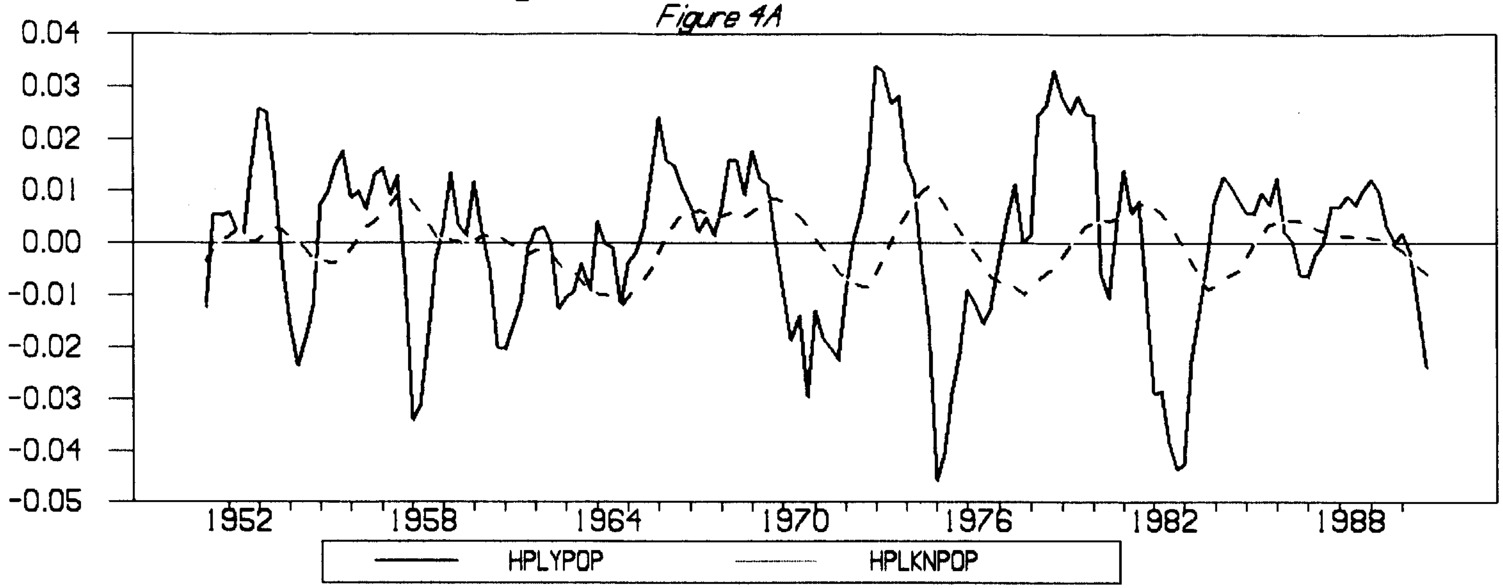
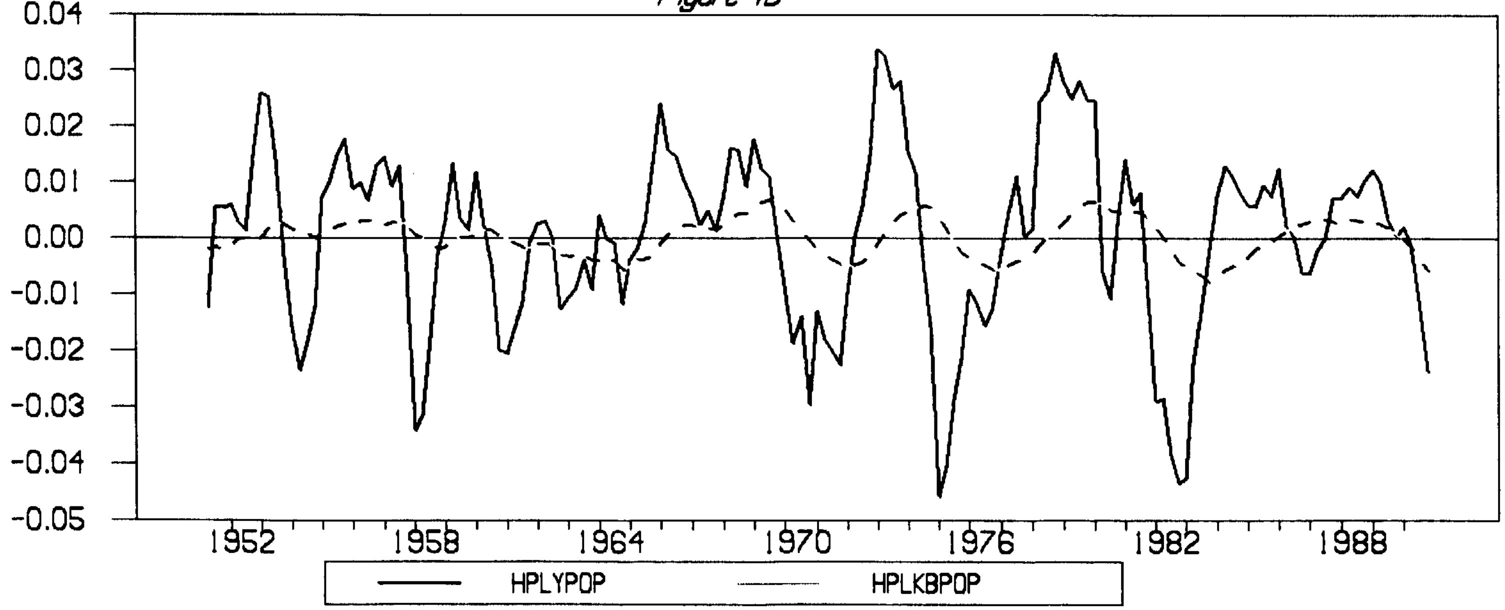


Figure 4B



III. Basic Result

Let us now turn our attention to the final criticism mentioned above, concerning the compatibility of IS and LM relations with explicit analysis of the maximizing behavior of rational economic agents. For this purpose, we shall consider an economy consisting of numerous individual households, beginning with a deterministic setting and then extending the analysis so as to accommodate stochastic shocks. Each household seeks at time t to maximize the time-separable utility function $\sum_{\tau=0}^{\infty}\beta^{\tau}u(c_{t+\tau}, m_{t+\tau})$, where $\beta \in (0, 1)$ is the household's discount factor, c_t denotes the household's consumption during t, and m_t is the stock of real money balances held at the start of the period. The rationale for the inclusion of m_t is of course that holdings of the economy's medium of exchange provide transaction services that reduce the time or other resources needed in "shopping" for the numerous distinct consumption goods whose aggregate is represented by c_t.[12] This method of introducing money is less restrictive, and more convenient for present purposes, than adoption of a cash-in-advance constraint.

Although households consume many goods, they specialize in production. Each one produces a single good as restricted by the production function $y_t = f(n_t, k_t)$, where y_t is output, n_t is labor input, and k_t is the stock of capital held by the household at the start of t. The functions $f(\cdot)$ and $u(\cdot)$ are assumed to be well-behaved, i.e., to satisfy the Inada conditions.

[12] This venerable and familiar assumption can be rationalized by drawing on the argument developed in Lucas (1980), which supposes that technology, preferences, and markets are such that relative prices of goods are constant, so that aggregation is possible. Alternative aggregation setups are utilized by Blanchard and Kiyotaki (1987) and Kim (1996), among others.

Each household inelastically supplies one unit of labor per period to a labor market, from which the households as producers purchase labor inputs at the real wage rate w_t.[13] In addition, there is a market for one-period government bonds on which the real rate of interest is r_t, where $(1+r_t)^{-1}$ is the real purchase price of a bond that is redeemed for one unit of output in the next period. Letting $\pi_t = (P_{t+1} - P_t)/P_t$ denote the inflation rate, where P_t is the money price of goods, a typical household's budget constraint is

$$f(n_t, k_t) - v_t = c_t + k_{t+1} - (1-\delta)k_t + w_t(n_t - 1) + (1+\pi_t)m_{t+1} - m_t + b_{t+1}(1+r_t)^{-1} - b_t \quad (4)$$

for period t, with similar constraints for all future periods. In (4), b_{t+1} is the number of real bonds purchased in t and v_t is the magnitude of lump-sum taxes levied on the household, while δ is the capital stock depreciation rate.

In this setup, the household's optimality conditions include (4) and[14]

$$u_1(c_t, m_t) - \lambda_t = 0. \quad (5)$$

$$\beta u_2(c_{t+1}, m_{t+1}) - \lambda_t(1+\pi_t) + \beta\lambda_{t+1} = 0. \quad (6)$$

$$-\lambda_t + \beta\lambda_{t+1}[f_2(n_{t+1}, k_{t+1}) + (1-\delta)] = 0. \quad (7)$$

$$f_1(n_t, k_t) - w_t = 0. \quad (8)$$

$$-\lambda_t(1+r_t)^{-1} + \beta\lambda_{t+1} = 0. \quad (9)$$

These difference equations determine the household's choices of sequences for c_t, b_{t+1},

[13] A rental market for capital goods could also be recognized but, with households treated (for simplicity) as alike, this would not alter the workings of the model. We abstract from population growth to avoid unnecessary notational clutter.

[14] Here and below, for any function $g(\cdot)$ that possesses multiple arguments, the notation $g_i(\cdot)$ denotes the partial derivative of g with respect to its ith argument.

m_{t+1}, n_t, k_{t+1} and λ_t in response to paths for w_t, r_t, π_t, and v_t that it faces.[15]

For competitive equilibrium, we also have the government's budget constraint, written in per-household terms as

$$(10) \qquad -v_t = (1+\pi_t)m_{t+1} - m_t + (1+r_t)^{-1}b_{t+1} - b_t,$$

and the market-clearing conditions

$$(11) \qquad n_t = 1$$

and

$$(12) \qquad m_t = M_t/P_t.$$

In the latter, M_t is the per-household nominal money supply. Thus the ten equations (4) – (12) plus the identity

$$(13) \qquad \pi_t = (P_{t+1} - P_t)/P_t$$

determine paths for w_t, r_t, π_t, and P_t plus the six variables mentioned above, in response to government-chosen sequences for M_t and v_t. For the moment, we assume no government consumption.

The foregoing is clearly a flexible-price model. But our objective is to obtain from it a pair of relations that are analogous to IS and LM functions, which could then be used sensibly in a setting with slow price adjustment. To that end, we first use (7) in (6) to obtain

$$(14) \qquad \beta u_2(c_{t+1}, m_{t+1}) - (1+\pi_t)\beta\lambda_{t+1}[f_2(n_{t+1}, k_{t+1}) + 1 - \delta)] + \beta\lambda_{t+1} = 0,$$

which with $\lambda_{t+1} = u_1(c_{t+1}, m_{t+1})$ implies

[15] We assume that three transversality conditions - - pertaining to the household's accumulation of capital, money and bonds - - are satisfied.

(15) $$\beta u_2(c_{t+1}, m_{t+1}) = \beta u_1(c_{t+1}, m_{t+1})[(1+\pi_t)(f_2(n_{t+1}, k_{t+1}) + 1 - \delta) - 1].$$

But (7) and (9) imply $r_t = f_2(n_{t+1}, k_{t+1}) - \delta$, so with real and nominal (R_t) interest rates related as in $1 + r_t = (1 + R_t)/(1 + \pi_t)$, equation (15) collapses to

(16) $$u_2(c_{t+1}, m_{t+1})/u_1(c_{t+1}, m_{t+1}) = R_t.$$

And under reasonably standard assumptions, the latter can be solved for m_{t+1}, as in

(17) $$m_{t+1} = L(c_{t+1}, R_t).$$

Thus we have a relation expressing end-of-period real money balances as a function of the upcoming period's consumption spending and the current nominal interest rate. Under reasonable specifications for $u(\cdot)$ and $f(\cdot)$, $L(\cdot)$ will be increasing in c_{t+1} and decreasing in R_t. Thus, since we are taking c_t to provide a satisfactory index of fluctuations in total output, equation (17) describes essentially the same type of behavior as that of the standard LM equation - - real money balances are positively related to a transactions variable and negatively related to an opportunity-cost variable.[16]

The timing in (17) is not quite the same as in (LM) of Section I, but could be made to coincide exactly by specifying that it is end-of-period real money balances that facilitate transactions.[17] To some, that might seem a questionable proposition, but we would suggest that neither the end-of-period nor the start-of-period specification is fully "correct"; indeed, some average over the period might arguably be more appropriate (as in the Baumol-Tobin

[16] It is emphasized by McCallum and Goodfriend (1987) that relations such as (17) are not properly termed "demand functions" since the spending variable is not exogenous to the individual economic unit in question. But such relations are typically referred to as money demand functions in the literature.

[17] The analysis is conducted in that fashion in McCallum and Goodfriend (1987). If we interpret m_t as end-of-period real money balances, then the budget constraint (4) includes $m_t - m_{t-1}(1+\pi_t)^{-1}$on the right-hand side and, following the same steps as above, we obtain $u_2(c_t, m_t)/u_1(c_t, m_t) = R_t(1+R_t)^{-1}$ in place of (16).

model).[18] But each of these specifications is actually just an approximation or a metaphor designed to represent the transaction-facilitating services of the medium of exchange. Accordingly, we contend that the model of equations (4)-(17) provides adequate justification for the use of a money-demand relation taking a form such as (LM).

Arguments similar to the foregoing have, as stated above, been developed previously by several writers including McCallum and Goodfriend (1987) and Blanchard and Fischer (1989). The more novel portion of our current task is to attempt an analogous derivation of a relation implied by the model at hand that represents behavior of the sort described by IS functions. Proceeding toward that goal, we find that substitution of (5) into (7) gives

$$u_1(c_t, m_t) = u_1(c_{t+1}, m_{t+1})\beta(1 + r_t), \tag{18}$$

where we have used $r_t = f_2(n_{t+1}, k_{t+1}) - \delta$. Now we add one new assumption, namely, that the functional form of $u(c_t, m_t)$ is separable[19] and of the form

$$u(c_t, m_t) = \theta\sigma(\sigma - 1)^{-1}c_t^{(\sigma-1)/\sigma} + (1 - \theta)\Psi(m_t), \tag{19}$$

where $\theta \in (0,1)$, $\sigma > 0$ with $\Psi'(\cdot) > 0$ and $\Psi''(\cdot) < 0$ over the empirically relevant range.[20] For the case $\sigma = 1$ in (20), we take preferences to be logarithmic in consumption. With that convention, it is the case that for all $\sigma > 0$, $u_1(c_t, m_t) = \theta c_t^{-1/\sigma}$ and we have

$$\theta c_t^{-1/\sigma} = \theta c_{t+1}^{-1/\sigma}\beta(1 + r_t) \tag{20}$$

[18] In addition, we would point out that in several recent papers, it is assumed that end-of-period money balances are relevant for facilitating transactions. See, for example, Obstfeld and Rogoff (1995), Kim (1996), and Ireland (1996).

[19] We do not claim that separability is theoretically an appropriate assumption. But we believe that for many purposes an approximation that neglects interaction effects will be satisfactory. Such approximations are certainly quite common in the literature.

[20] For some purposes, such as optimal inflation rate analysis, it would be appropriate to assume that there exists a satiation level of real money balances.

or

$$c_t = c_{t+1}[\beta(1 + r_t)]^{-\sigma} \tag{21}$$

so that, upon taking natural logarithms,

$$\log\ c_t = \log\ c_{t+1} - \sigma \log(1 + r_t) - \sigma \log\ \beta. \tag{22}$$

The crucial thing to realize about this relation is that, behaviorally, it represents the typical household's choice in period t of c_t in response to r_t and expectations concerning c_{t+1} - - *not* the choice of c_{t+1} in response to the lagged values c_t and r_t. Thus the analysis provides justification for a consumption equation such as

$$\log\ c_t = b_0' + b_1' r_t + E_t \log c_{t+1}, \qquad b_1' < 0, \tag{23}$$

where we have used the common approximation $\log(1 + x) = x$ (for x small relative to 1.0). But furthermore, as explained above, for business cycle purposes we are able to approximate fluctuations in y_t with those in c_t. Thus our basic conclusion is that a relation of the form

$$\log\ y_t = b_0 + b_1 r_t + E_t \log\ y_{t+1}, \qquad b_1 < 0, \tag{24}$$

is justifiable by the foregoing analysis of a maximizing model.[21] Equation (24) is, however, of the common IS form that we set out to consider - - but with one significant difference. Specifically, the expected value of next period's output is an important determinant of the quantity of output demanded in the current period. This extra term gives a forward-looking aspect to (24) that is not present in typical IS-LM analysis, and which could possibly have a major effect on the dynamic properties of a macroeconomic system. Some examples of models with expectational IS functions of this type will be investigated below, in Sections

[21] A more explicit and rigorous treatment of this step is presented below in Section IV. It indicates that the coefficient on $E_t \log y_{t+1}$ should be 1.0.

V and VI.

The preceding maximizing analysis took place in a deterministic setting. Analogous results can be obtained in a stochastic version of the model, provided that we employ some commonly-made approximations. To be specific, suppose first that the production function is stochastic, given by $y_t = Z_t f(n_t, k_t)$, where Z_t is a random variable such that log Z_t is a covariance stationary process. Accordingly, $Z_t f(n_t, k_t)$ replaces $f(n_t, k_t)$ in the household's constraint (4). The household's problem is now to maximize $E_t \sum_{\tau=0}^{\infty} \beta^\tau u(c_{t+\tau}, m_{t+\tau})$ and its optimality conditions include (4), (5), and the stochastic analogues to (6) – (9), i.e.,

(25) $$\beta E_t u_2(c_{t+1}, m_{t+1}) - \lambda_t(1 + E_t \pi_t) + \beta E_t \lambda_{t+1} = 0.$$

(26) $$- \lambda_t + \beta E_t \lambda_{t+1}[Z_{t+1} f_2(n_{t+1}, k_{t+1}) + 1 - \delta] = 0.$$

(27) $$z_t f_1(n_t, k_t) - w_t = 0.$$

(28) $$- \lambda_t (1 + r_t)^{-1} + \beta E_t \lambda_{t+1} = 0.$$

The competitive equilibrium conditions again include (10) – (13). From this stochastic system, we can obtain the following counterpart to (15):

(29) $$\beta E_t u_2(c_{t+1}, m_{t+1}) = \beta E_t u_1(c_{t+1}, m_{t+1})[(1 + E_t \pi_t)(E_t Z_{t+1} f_2(n_{t+1}, k_{t+1}) + 1 - \delta) - 1]$$

where (29) follows Sargent (1987, pp. 94-95) in approximating the conditional covariance term in the general formula for the expectation of a product of two random variables d and x, namely $E_t dx = E_t d E_t x + cov_t(d, x)$, by zero.[22] With this additional assumption, furthermore, (26) and (28) imply $r_t = E_t[Z_{t+1} f_2(n_{t+1}, z_{t+1})] - \delta$, and equation (29) becomes

(30) $$[E_t u_2(c_{t+1}, m_{t+1})]/[E_t u_1(c_{t+1}, m_{t+1})] = R_t,$$

[22] Specifically, (29) requires that $E_t \lambda_{t+1} z_{t+1} f_2(n_{t+1}, k_{t+1}) = E_t \lambda_{t+1}[E_t z_{t+1} f_2(n_{t+1}, k_{t+1})]$.

which differs only randomly from the LM relation (16).

Next, we assume preferences are given by

$$u(c_t, m_t) = \theta\sigma(\sigma - 1)^{-1} c_t^{(\sigma-1)/\sigma} \exp(\varepsilon_t) + (1 - \theta)\Psi(m_t). \tag{31}$$

Here, $\{\varepsilon_t\}_{t=1}^{\infty}$ is a normally, independently distributed sequence of preference shocks that is assumed to possess the property $E_t\varepsilon_{t+1} = 0$.We then have the stochastic analogue to (20):

$$\theta(c_t)^{-(1/\sigma)} \exp(\varepsilon_t) = E_t\theta(c_{t+1})^{-(1/\sigma)} \exp(\varepsilon_{t+1})\beta(1 + r_t) \tag{32}$$

or, approximately,

$$\varepsilon_t - \sigma^{-1} \log c_t \simeq -\sigma^{-1} E_t \log c_{t+1} + \log \beta + \log(1 + r_t) \tag{33}$$

Rearranging (33), we obtain the counterpart to result (22) :

$$\log c_t \simeq E_t \log c_{t+1} - \sigma r_t - \sigma \log \beta + v_t, \tag{34}$$

where $v_t \equiv \sigma\varepsilon_t$ is distributed $N(0, \sigma^2 V_\varepsilon)$, and (33) involves approximations of the form $\log E_t d_{t+1} \simeq E_t \log d_{t+1}$. As shown in equations (21)-(22) above, (33) and (34) hold exactly in a deterministic environment. In the stochastic model at hand, however, both equations are approximations, arising from passing nonlinear functions through the linear expectations operator. Approximations of this nature have been used frequently in stochastic optimizing models for the sake of tractability, recent examples being the studies by Cooley and Hansen (1995, p. 216) and Kim (1996).

The derivation of (34) shows that by introducing uncertainty, we may include an additive disturbance in (22). By including a shock term in the specification of $\Psi(m_t)$ in (31), it is also possible to derive log-linearized versions of (30) where a money demand shock enters explicitly, just as in (LM) in Section I.[23]

It might be noted that the optimizing model that we have utilized is one in which there are no adjustment costs associated with the process of investing in physical capital. Since standard IS-LM frameworks are frequently regarded as implying the existence of capital adjustment costs - - see, e.g., Sargent (1979) - - this feature of our setup might be questioned. It is our intention, however, to demonstrate that relations such as (17) and (24) can be obtained via maximizing analysis, so use of the simplest and most standard version of the Sidrauski-Brock model seems appropriate. And our treatment of the capital stock, as smoothly changing at an exogenously given rate for a typical household, will keep the model from possessing non-standard implications that might otherwise obtain.

Recently, capital adjustment costs have become a popular modification of sticky-price quantitative general equilibrium models (see Kimball [1995], King and Watson [1996], and Kim [1996]). Evidently, models without this modification exhibit very strong contemporaneous responses of investment to monetary shocks. These responses tend to produce some counterfactual model properties; for example, real and nominal interest rates increase in reaction to monetary expansion. The presence of capital adjustment costs dampens the investment response, and thereby tends to produce smooth behavior of the capital stock. In their practical effect, therefore, capital adjustment costs appear to have much the same effect on an optimizing model as our constant-growth assumption.[24]

[23] Nelson (1996) gives a parameterization of $\Psi(m_t)$ from which (LM) may be derived. Kim (1996) and Ireland (1996) show that a standard log-linear stochastic money demand function may be derived from a nonseparable specification of $u(c_t, m_t)$ that includes a preference shock.

[24] Indeed, Leeper and Sims (1994, p. 83) give the desire for a model with smooth capital growth as one

IV. Extensions

Before turning to applications, we wish to explore the possibility of extending the framework just developed so as to incorporate fiscal policy variables - - in particular, government spending - - and to provide an open-economy version. For the sake of brevity we shall work with non-stochastic versions of the model.

The Sidrauski-Brock model that we used as a starting point in Section III is, of course, one in which Ricardian equivalence prevails, provided that taxes are of a lump-sum nature. But even in such a setting it will be the case that government purchases of goods and services ("spending") will affect aggregate demand. To reflect such purchases we rewrite the per-household government budget constraint as

$$g_t - v_t = (1+\pi_t)m_{t+1} - m_t + (1+r_t)^{-1}b_{t+1} - b_t, \quad (35)$$

where g_t denotes government spending per household. With that variable's values being chosen by the government, the only change to the model is that (35) replaces (10). But of course the government and household budget constraints together with (11) imply the economy's overall resource constraint, which in per-household terms is:

$$f(n_t, k_t) = c_t + k_{t+1} - (1-\delta)k_t + g_t = c_t + i_t + g_t, \quad (36)$$

where i_t is gross investment. Consequently, as an instance of Walras' Law, we can use (36) in place of (35) as one of the model's equations that serve to determine time paths of the endogenous variables.

Now, let us adopt a log-linear approximation to (36), namely,

$$\log y_t = d_1 \log c_t + d_2 \log i_t + d_3 \log g_t, \quad (37)$$

reason for their inclusion of adjustment costs and other modifications of the standard optimizing model.

where the d_j are average shares of consumption, investment and government spending. Using (23), we then obtain

$$\text{(38)} \quad \log y_t = d_1(b_0' + b_1' r_t + E_t \log c_{t+1}) + d_2 \log i_t + d_3 \log g_t$$
$$= d_1 b_0' + d_1 b_1' r_t + E_t(\log y_{t+1} - d_2 \log i_{t+1} - d_3 \log g_{t+1}) + d_2 \log i_t + d_3 \log g_t.$$

But under our assumption that capital grows at a constant rate ξ, $E_t(\log i_{t+1} - \log i_t) = \log(1+\xi) \simeq \xi$. Thus we end up with

$$\text{(39)} \quad \log y_t = b_0 + b_1 r_t + E_t \log y_{t+1} + b_3 \log g_t - b_3 E_t \log g_{t+1}.$$

The latter provides an IS function, analogous to (24) or (25), usable when government spending is a non-negligible contributor to cyclical fluctuations. Some implications will be discussed below. Note that this derivation justifies the passage from (23) to (24) above.

Incorporation of open-economy influences is somewhat more complex. For the derivation, let c_t^F denote per-household consumption of a (composite) good - - different from the home economy's composite good - - that is produced abroad, and suppose that preferences are given by

$$\text{(40)} \qquad u(c_t, c_t^F, m_t) + \beta u(c_{t+1}, c_{t+1}^F, m_{t+1}) + \ldots$$

where $u_2 > 0$, $u_{22} < 0$. Domestic households do not derive utility from holding foreign real money balances, i.e., such balances are not useful in facilitating transactions. Continuing to let P_t denote the nominal price of one unit of the domestic consumption good, a unit of the foreign consumption good costs the domestic household $S_t P_t^*$ in domestic currency. Here P_t^* is the foreign-country price of its own composite good, while S_t is the nominal exchange rate.

The household's exports to the rest of the world, expressed in domestic prices, are $P_t x_t$. In order to allow, in a tractable manner, for discrepancies between x_t and c_t^F - - that is, nonzero current account balances - - it is useful to assume that the home government does not engage in fiscal policy, so that $b_t = 0$ in (10), and $g_t = 0 \; \forall \; t$. Foreigners may, however, issue bonds, which both domestic and foreign households are permitted to purchase. Consequently, in real terms, the typical household's budget constraint is:

$$(41) \qquad f(n_t, k_t) - v_t + x_t = c_t + k_{t+1} - (1-\delta)k_t + w_t(n_t - 1)$$
$$+(1+\pi_t)m_{t+1} - m_t + b_{t+1}^F(1+r_t)^{-1} - b_t^F + S_t(P_t^*/P_t)c_t^F.$$

where b_t^F is the household's net holding of foreign bonds in period t. In this open economy, the household has one additional choice variable, c_t^F. Its optimality conditions are (41) and:

$$(42) \qquad u_1(c_t, c_t^F, m_t) = \lambda_t.$$

$$(43) \qquad \beta\, u_3(c_{t+1}, c_{t+1}^F, m_{t+1}) - \lambda_t(1 + \pi_t) + \beta\, \lambda_{t+1} = 0.$$

$$(44) \qquad -\lambda_t + \beta\, \lambda_{t+1}[f_2(n_{t+1}, k_{t+1}) + (1-\delta)] = 0.$$

$$(45) \qquad f_1(n_t, k_t) - w_t = 0.$$

$$(46) \qquad -\lambda_t(1+r_t)^{-1} + \beta\, \lambda_{t+1} = 0.$$

$$(47) \qquad u_2(c_t, c_t^F, m_t) = S_t(P_t^*/P_t)\lambda_t.$$

Our principal interest is in the form of the equation for aggregate demand implied by (41) – (47).[25] To this end, we adopt the following specification of utility:

$$(48) \qquad u(c_t, c_t^F, m_t) = \alpha_1 \sigma(\sigma - 1)^{-1} c_t^{(\sigma-1)/\sigma} + \alpha_2 \zeta(c_t^F) + \alpha_3 \Psi(m_t),$$

with $\sum_1^3 \alpha_i = 1, \alpha_i \in (0,1), \zeta'(\cdot) > 0$,and $\zeta''(\cdot) < 0$. Inspection of conditions (42), (43), (44)

[25] It is the case that the money demand function will also be affected if money balances help to facilitate purchases of foreign-produced goods.

and (46) indicates that they are identical[26] to (5)-(7) and (9) in the closed economy model of Section III. The latter set of equations, as well as the assumption that utility took the form of (19), led to our consumption equation (22). In the open economy, with preferences taking the separable form (48), we deduce that (22) continues to hold:

$$\log\ c_t = b_0' + b_1' r_t + E_t \log c_{t+1}. \tag{49}$$

where $b_0' = -\sigma \log \beta$, $b_1' = -\sigma$.

Relative to the closed-economy model of Section III, the open-economy version contains an additional optimality condition, namely (47). This equation, combined with the form of preferences given by (48), produces (letting $Q_t = S_t(P_t^*/P_t)$ denote the real exchange rate),

$$(\alpha_2/\alpha_1)\zeta'(c_t^F)\, c_t^\sigma = Q_t \tag{50}$$

Equation (50) implies the existence of a demand function for imports, whose arguments are the real exchange rate and consumption of the domestic good:

$$c_t^F = h(Q_t, c_t), \qquad h_1(\cdot) < 0, h_2(\cdot) > 0. \tag{51}$$

We assume that in the rest of the world a parallel maximization exercise has taken place, with a typical foreign household's preferences described by the utility function

$$u(c_t^*, c_t^{F*}, m_t^*) = \omega_1 \sigma(\sigma - 1)^{-1}[c_t^*]^{(\sigma-1)/\sigma} + \omega_2 \varphi(c_t^{F*}) + \omega_3 \Psi(m_t^*). \tag{52}$$

where $\sum_1^3 \omega_i = 1, \omega_i \in (0,1), \varphi'(\cdot) > 0,$and $\varphi''(\cdot) < 0$. Since $c_t^{F*} = x_t$, the foreign sector's maximizing behavior will have produced a decision rule analogous to (51), which gives the rest of the world's demand function for the domestic household's exports:

$$x_t = \gamma(Q_t, c_t^*), \qquad \gamma_1(\cdot) > 0, \gamma_2(\cdot) > 0. \tag{53}$$

[26] Except for the number of arguments of $u(\cdot)$.

The domestic resource constraint per household is the sum of domestic consumption of domestic output (c_t), investment (i_t), and exports (x_t, foreign consumption of domestic output):

(54) $$y_t = c_t + i_t + x_t.$$

Letting μ_y, μ_c, μ_i, and μ_x respectively denote steady-state values of y_t, c_t, i_t and x_t, the log-linearization of (54) becomes:

(55) $$\log y_t = d_1 \log c_t + d_2 \log i_t + d_3 \log \hat{x}_t,$$

where $d_1 = \mu_c/\mu_y$, $d_2 = \mu_i/\mu_y$, and $d_3 = \mu_x/\mu_y$. Using (49) and (53), (55) becomes

(56) $$\log y_t = d_1 b_0' + d_1 b_1' r_t + d_1 E_t \log c_{t+1} + d_2 \log i_t + d_3 \log x_t(Q_t, c_t^*).$$

By proceeding on the same argument that led from (38) to (39), (56) can be used to justify the following equation:

(57) $$\log y_t = b_0 + d_1 b_1' r_t + E_t \log y_{t+1} + d_3 \log x_t(Q_t, c_t^*) - d_3 E_t \log x_{t+1}(Q_{t+1}, c_{t+1}^*).$$

Equation (57) indicates that short-run output demand is principally determined by the expected value of next period's consumption, the real interest rate, and exports. Exports, in turn, are affected both by the real exchange rate (Q_t) and foreign households' consumption of their own output (c_t^*). Moreover, if c_t^* moves largely in unison with a measure of foreign economic activity, such as rest-of-the-world output y_t^*, then (57) indicates that foreign output and real exchange rate fluctuations should both be useful in explaining domestic output behavior. Empirical findings of this sort have, in fact, been reported by Ghosh and Masson (1991) and Gruen and Shuetrim (1994).[27]

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[27] These papers found positive, significant effects of current and lagged $\log Q_t$ and $\log y_t^*$ on output. This can be rationalized by (57), as follows. Log-linearizing (53), we have $\log x_t = \pi_0 \log Q_t + \pi_1 \log y_t^*$, with

Having extended the analysis as intended, we now turn to two applications in each of which the modified and legitimized version of an IS-LM model is used for a substantive purpose.

V. Price Level Determinacy

Our first application involves an issue of fundamental importance in monetary theory, namely the possible analytical indeterminacy of prices and other nominal variables in a setting in which all private agents are free of money illusion and form their expectations rationally. In a famous article, Sargent and Wallace (1975) put forth the claim that in a model with those properties, nominal magnitudes would be formally indeterminate if the central bank used an interest rate as its instrument variable, i.e., if it set the interest rate R_t each period by means of a policy feedback rule that specifies R_t as a linear function of (any) data from previous periods.[28] Sargent (1979, p. 362) summarized the conclusion as follows: "There is no interest rate rule that is associated with a determinate price level." The specific model used by Sargent and Wallace (1975) was of the IS-LM-AS type, similar to the one exhibited above in Section I.[29] Subsequently, however, McCallum (1981, 1986) showed that the Sargent-Wallace claim was actually incorrect in such a model; instead, all

both π_i's positive, and $E_t \log x_{t+1} = \pi_0 E_t \log Q_{t+1} + \pi_1 E_t \log y^*_{t+1}$. Suppose that we approximate these conditional expectations by univariate AR forecasts. Assume that both $\log Q_t$ and $\log y^*_t$ are stationary processes whose univariate AR representations satisfy:

$\log Q_t = \tau_0 + \sum_{j=1}^{p} \tau_j \log Q_{t-i} + \varepsilon_{Qt}$

$\log y^*_t = \phi_0 + \sum_{j=1}^{k} \phi_j \log y^*_{t-i} + \varepsilon_{yt}$

Then writing $d_3 \log x_t - d_3 E_t \log x_{t+1}$ of (57), in terms of $\{\log Q_{t-i}\}_{i=0}^{p-1}$ and $\{\log y^*_{t-i}\}_{i=0}^{k-1}$, we obtain a coefficient sum of $d_3\pi_0(1-\sum_{j=1}^{p} \tau_j) > 0$ on the distributed lag of $\log Q_t$, and a sum of $d_3\pi_1(1-\sum_{j=1}^{k} \phi_j) > 0$ on the distributed lag of $\log y^*_t$.

[28] Linearity of the feedback rule is not of central importance, but was assumed because the analysis was being conducted in a linear model.

[29] Capacity output was treated as endogenous, but that is of no relevance for the issue at hand.

nominal variables will be fully determinate provided that the policy rule utilized for the interest rate instrument involves some nominal variable, just as suggested previously by Parkin (1978) and in the classic discussion of Patinkin (1965, pp. 295-310). Our objective here, consequently, will be to reconsider that result in the context of our modified IS-LM framework, that is, using the expectational version of the IS function.[30]

At this point it will be useful to adopt a *change in notation* relative to preceding sections. Specifically, from this point onward we will let y_t, p_t and m_t denote the logarithms of real output, the price level, and the nominal money stock with R_t representing the (nominal) rate of interest. Thus the modified IS-LM portion of the model to be used in the present section can be written as

$$y_t = b_0 + b_1(R_t - E_t p_{t+1} + p_t) + b_2 E_t y_{t+1} + v_t \tag{58}$$

$$m_t - p_t = c_0 + c_1 y_t + c_2 R_t + \eta_t, \tag{59}$$

where v_t and η_t are stochastic disturbances that are for simplicity assumed to be white noise, and where $b_2 = 1.0$ is a prominent possibility. For the aggregate supply portion of this model it would be possible to use some sticky-price specification, but in order to maximize the possibility of nominal indeterminacy we shall assume that prices are fully flexible, with y_t being determined exogenously according to

[30] It should be mentioned that Woodford (1995) has recently suggested that the determinacy issue should be considered in a model with explicitly optimizing agents. His own analysis is generally supportive of the results in McCallum (1981, 1986), but involves some issues involving transversality conditions that are too complex to be taken up here. Earlier, Sargent and Wallace (1982) put forth arguments quite different from those of their 1975 paper, and attributed this difference to the use of an optimizing model. In fact, however, the main relevant difference is that their 1982 analysis is based on a model in which monetary and nonmonetary assets cannot be distinguished — and indeterminacy does not actually prevail in any case. On this, see McCallum (1986, pp. 144-154).

(60) $$y_t = \rho_0 + \rho_1 y_{t-1} + u_t,$$

where $|\rho_1| < 1.0$ and u_t is white noise.[31]

To illustrate the notion of nominal indeterminacy, initially suppose that in an economy represented by (58)-(60), the monetary authority conducts policy by manipulating R_t according to the following rule:

(61) $$R_t = \mu_0 + \mu_1 y_{t-1}.$$

Then we can substitute from (60) and (61) into (58) to obtain

(62) $$\rho_0 + \rho_1 y_{t-1} + u_t = b_0 + b_1(\mu_0 + \mu_1 y_{t-1}) - b_1 E_t p_{t+1} + b_1 p_t$$
$$+ b_2[\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1} + u_t)] + v_t.$$

Since y_{t-1}, u_t, and v_t are apparently the only relevant state variables, we conjecture that the minimal-state-variable (MSV) solution for p_t will be of the form

(63) $$p_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 u_t + \phi_3 v_t$$

and thus that $E_t p_{t+1} = \phi_0 + \phi_1(\rho_0 + \rho_1 y_{t-1} + u_t)$. Substitution of these into (62) gives an expression that will hold for all values of y_{t-1}, u_t, and v_t (thereby making (63) a solution) only if the following "undetermined coefficient" restrictions hold:

(64) $$\rho_0 = b_0 + b_1\mu_0 - b_1\phi_1\rho_0 + b_2\rho_0(1 + \rho_1)$$
$$\rho_1 = b_1\mu_1 - b_1\phi_1\rho_1 + b_1\phi_1 + b_2\rho_1^2$$
$$1 = b_1\phi_2 - b_1\phi_1 + b_2\rho_1$$
$$0 = b_1\phi_3 + 1.$$

[31] Here the process generating y_t is more general than that adopted in McCallum (1981), where output is treated as a constant. It is that property that has led us to use a different example here, because with a constant output the modified expectational IS function (58) cannot be distinguished from the unmodified special case in which $b_2 = 0$. Logically, then, it follows that the analysis in McCallum (1981) is valid even under the assumption that the modified IS function is relevant, but only because output does not vary.

From the latter three conditions we see that $\phi_1 = (\rho_1 - b_1\mu_1 - b_2\rho_1^2)/b_1(1-\rho_1)$, $\phi_2 = (1 - b_1\mu_1 - b_2\rho_1)/b_1(1-\rho_1)$, $\phi_3 = -1/b_1$. But the coefficient ϕ_0 does not appear in any of the conditions, so its value is not determined. In this sense, p_t is indeterminate in the model at hand. Then from (59) it follows that m_t, the system's other nominal variable, is indeterminate as well.

But the hypothetical policy rule (61) does not meet the proviso mentioned in the first paragraph of this section, namely, that of involving some nominal variable in an essential way.[32] Accordingly, let us now consider a policy rule that does involve a nominal variable. Suppose then that R_t is set each period so as to make the expected value of p_t equal to an exogenously chosen target value p^*. Thus R_t is set according to

$$(65)\qquad \begin{aligned} R_t &= E_{t-1}[E_t p_{t+1} - p^* + (1/b_1)(y_t - b_0 - b_2 E_t y_{t+1}) - (1/b_1)v_t)] \\ &= E_{t-1}p_{t+1} - p^* + (1/b_1)[\rho_0 + \rho_1 y_{t-1} - b_0 - b_2[\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1})]]. \end{aligned}$$

Since $E_{t-1}p_{t+1} = \phi_0 + \phi_1(\rho_0 + \rho_1 y_{t-1})$ under the conjecture that (63) is the solution form, rule (65) is a feedback rule for R_t of the class considered by Sargent and Wallace (1975).

Putting (65) instead of (61) into (58) yields

$$(66)\qquad \begin{aligned} \rho_0 + \rho_1 y_{t-1} + u_t &= b_0 - b_1 p^* + [\rho_0 + \rho_1 y_{t-1} - b_0 - b_2\rho_0 \\ &- b_2\rho_1(\rho_0 + \rho_1 y_{t-1})] - b_1\phi_1 u_t + b_1[\phi_0 + \phi_1 y_{t-1} + \phi_2 u_t + \phi_3 v_t] \\ &+ b_2[\rho_0 + \rho_1(\rho_0 + \rho_1 y_{t-1} + u_t)] + v_t. \end{aligned}$$

Consequently, the conditions analogous to (64) are:

[32] To illustrate the meaning of the last phrase, consider a specification that includes $m_{t-1} - p_{t-1}$. This represents the inclusion of the log of the previous period's *real* money balances, so would not actually involve any nominal variable.

$$
\begin{aligned}
(67) \qquad 0 &= -b_1 p^* + b_1 \phi_0 \\
\rho_1 &= \rho_1 + b_1 \phi_1 \\
1 &= -b_1 \phi_1 + b_1 \phi_2 + b_2 \rho_1 \\
0 &= b_1 \phi_3 + 1.
\end{aligned}
$$

Here the solutions for ϕ_2 and ϕ_3 are as before whereas that for ϕ_1 is now equal to zero. But the important difference is that the coefficient ϕ_0 appears precisely once in the first of equations (67) so its value is clearly and uniquely determined. Thus the conjectured solution form (63) is shown to be justified, and to yield a fully determinate solution for p_t. This remains true, moreover, if the value 1.0 is assigned to b_2, as the discussion of Section III would suggest, or if $b_2 = 0$ (as in the unmodified IS function). Furthermore, if the price level target was a *path*, $\{p_t^*\}$, instead of a constant value, then ϕ_0 would equal zero and p_t^* would appear in its place in the solution; thus determinacy would again result.

The foregoing example provides an illustration of price level determinacy with an interest rate rule that is rather different than the one used in McCallum (1981), $E_{t-1}p_t$ rather than $E_{t-1}m_t$ being the targeted nominal "anchor" and interest-rate smoothing being absent. Because of the latter property, there is here no need to decide which value of ϕ_1 represents the bubble-free or "fundamentals" MSV solution, as is necessary for π_1 in the 1981 paper. The important aspect of the comparison, however, is the similarity of the two examples with regard to the issue of nominal determinacy with a R_t policy instrument. In both cases, a nominal magnitude enters the policy rule and determinacy prevails. The basic reason is that indeterminacy results only when the monetary authority totally neglects to provide a nominal anchor, in which case there is no economic actor that is in any way concerned with nominal magnitudes - - rational private actors being concerned only with real variables such as y_t or $m_t - p_t$.

If interest rate smoothing - - i.e., a tendency by the monetary authority to keep R_t close to R_{t-1} - - were present in our current example, there would be two values of some coefficients that would satisfy the undetermined coefficient conditions analogous to (67). Thus there might be a multiplicity of rational expectation solutions to the model, if one of the values was not ruled out by an assumed transversality condition. But even if a multiplicity were to occur, there would be no implication of nominal indeterminacy. In this regard it is important to distinguish between solution multiplicities and nominal indeterminacies, i.e., between bubbles and the absence of any nominal anchor. The former phenomenon typically involves multiple solution paths for *real* variables (e.g., multiple solution paths for p_t even with m_t paths given), whereas the latter refers to situations in which the model provides a single solution for all real variables but simply fails to determine a solution path for *any* nominal variable.[33] These two types of "aberrant" behavior are conceptually quite distinct, a fact that is sometimes masked by an unsatisfactory terminology that refers to both merely as "indeterminacies".[34]

From the standpoint of the present paper's main issue, our conclusion is that use of

[33] This conceptual distinction was emphasized by McCallum (1986, p. 137).

[34] In a recent paper that uses a modified IS function similar to ours, Kerr and King (1996) develop several interesting results, one of which may appear to conflict with our finding since it suggests that, for some parameter values, an interest rate rule that involves a nominal variable will not result in a "unique equilibrium". But there is in fact no conflict, since a non-unique equilibrium is not the same form of aberrant behavior as nominal inderminacy.

the modified IS function does not alter results pertaining to potential nominal indeterminacy. Just as with conventional IS-LM specifications, nominal prices and money-stock values are determinate even with interest rate policy rules provided that these rules involve some nominal magnitude in an essential way.

VI. Inflation Persistence

In our second application we shall be concerned with an interesting recent contribution by Fuhrer and Moore (1995). The centerpiece of that study is a model of dynamic aggregate supply behavior - - in other words, a short-run Phillips relationship - - that involves overlapping nominal contracts but posits a different adjustment mechanism than the one familiar from the work of Taylor (1979, 1993). Fuhrer and Moore hint that their nominal contracting specification may be preferable theoretically, but its main attraction is that it evidently yields the implication that inflation rates will have considerable persistence, nicely matching that property (and others) of the actual U.S. quarterly data, whereas Taylor's basic scheme implies persistence of price level movements but not inflation rates.

This finding of inflation persistence is obtained by Fuhrer and Moore (1995) by means of simulations of a small estimated macro model that incorporates their aggregate supply specification, a traditional IS function, and a policy rule for setting short-term nominal interest rates. There are theoretical objections that could be raised regarding details of the latter two relationships, however, including the point that the IS specification is not of the expectational variety developed above.[35] Consequently, the following paragraphs will develop

[35] Another problem is that the real interest rate - - a long rate, incidentally - - enters the IS equation with a one quarter lag. In addition, the policy rule specification permits the Fed to respond to current-period

an analytical result concerning inflation persistence in a model that includes a Fuhrer-Moore supply sector, an expectational IS relation, and an interest rate policy rule with responses to lagged but not current values of output and inflation. The precise specification of the policy rule will be based on the formula proposed by Taylor (1994), which matches actual U.S. data very well for the period 1988-1992 and which has recently attracted considerable interest within the Federal Reserve System.

In specifying the relations of this model, we now let y_t represent the log of output measured relative to some (exogenous) capacity or natural-rate value, with m_t and p_t denoting logs of the money stock and price level, respectively. Also, R_t is the one-period (nominal) interest rate and x_t is the log of a contract price (or wage) that is set in period t so as to prevail in periods t and $t+1$ for half of the economy's sellers. Then the model is:

(68) $$y_t = b_0 + b_1(R_t - E_t\Delta p_{t+1}) + b_2 E_t y_{t+1} + v_t$$

(69) $$m_t - p_t = c_0 + c_1 y_t + c_2 R_t + \eta_t$$

(70) $$p_t = 0.5(x_t + x_{t-1})$$

(71) $$x_t - p_t = 0.5[x_{t-1} - p_{t-1} + E_t(x_{t+1} - p_{t+1})] + \theta(y_t + E_t y_{t+1})$$

(72) $$R_t = E_{t-1}\Delta p_{t+1} + \mu_0 + \mu_1 E_{t-1} y_t + \mu_1 E_{t-1}\Delta p_t + e_t.$$

Given exogenous processes for v_t, η_t and e_t, these five equations determine time paths for the endogenous variables y_t, p_t, m_t, R_t, and x_t.

Equation (68) is, clearly, an expectational IS function as derived above. Equation (69), the LM or money demand function, is actually superfluous in this model since R_t is used as the central bank's instrument. Its only function, that is, is to determine the behavior of

values of GNP, which seems rather implausible.

m_t that is required to support (72). Next, a two-period version of the Fuhrer-Moore supply specification is given by equations (70) and (71).[36] The basic premise here, as mentioned above, is that the (log) contract price is set in t by half of the economy's producers and then prevails for their sales over the next two periods. Thus the average price level in t is given by (70).[37] Let us refer to $x_t - p_t$ as the "relative price" for producers setting contract prices in t. Equation (71) specifies that x_t is chosen so as to equate the relative price $x_t - p_t$ to the average value (over the periods during which x_t will prevail) of the relative price for the other producers in the economy, with an adjustment (as in Taylor [1979]) for current and expected values of output relative to capacity. Thus relative prices are set, under the Fuhrer-Moore specification, in the same manner as are nominal prices in Taylor's (1979, 1993) formulation.[38]

Taylor's (1994) monetary policy rule, finally, sets the (nominal) interest rate R_t so as to adjust the expected[39] real interest rate $E_{t-1}(R_t - E_t\Delta p_{t+1})$ upward or downward to tighten or loosen monetary conditions, with those adjustments made in response to predicted values

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[36] Fuhrer and Moore (1995) use a four-period version with quarterly data, and also let the weights on lagged prices in their relation analogous to (70) be determined empirically.

[37] Henceforth we shall often say "price" where "log price" would be, strictly speaking, more correct.

[38] With prices set as in (70) and (71), it of course becomes the case that some relation used to derive (68) and (69) no longer holds - in particular, (11). This is one standard way of proceeding in sticky-price models, i.e., to assume that production equals the quantity demanded, with labor departing from its supply curve temporarily.

A further modification of the framework of Section III is required for the pricing setup of (70) – (71) to be applicable. The typical household of that section is hereafter assumed to produce *two* goods, which are distinct from one another but which provide the same utility level. One of these goods has its period t price contracted in period $t-1$ while the other has its period t price contracted in t. The first good is aggregated across households to produce a composite good, while a second composite good is constructed by aggregating the second good across households. Under this interpretation, p_t in (70) is the period t price of a *further* composite good, obtained by aggregation of the two composite goods.

[39] We use $E_{t-1}\Delta p_{t+1}$ in (72), rather than $E_t\Delta p_{t+1}$, to keep the policy rule informationally operational.

of output relative to capacity and inflation relative to a target value such as 0.5 percent per quarter.[40]

To develop a solution, first note that (70) implies that $x_t - p_t = 0.5\Delta x_t$. Therefore, equation (71) can be rewritten as

$$\Delta x_t = (0.5\Delta x_{t-1} + 0.5E_t\Delta x_{t+1}) + 2\theta(y_t + E_t y_{t+1}). \tag{73}$$

Then (72) can be substituted into (68), with Δp_t expressed as $0.5(\Delta x_t + \Delta x_{t-1})$ from (70), yielding

$$y_t = b_0 + b_1\{\mu_0 + \mu_1 E_{t-1}y_t - 0.5[E_t(\Delta x_{t+1} + \Delta x_t) - E_{t-1}(\Delta x_{t+1} - \Delta x_t)] + 0.5\mu_1 E_{t-1}(\Delta x_{t-1} + \Delta x_t) + e_t\} + b_2 E_t y_{t+1} + v_t. \tag{74}$$

Since relations (73) and (74) include only y_t and Δx_t as endogenous variables, we can in principle solve for those two in isolation and then use the results to find solutions for p_t, R_t, and m_t.

Let us assume that the policy-equation disturbance e_t - - i.e., the unsystematic component of policy - - is white noise and that v_t is an AR(1) process:

$$v_t = \rho v_{t-1} + \xi_t, \tag{75}$$

where $0 \leq \rho < 1$ and ξ_t is white noise. Then we can look for a MSV solution of the form

$$y_t = \phi_{10} + \phi_{11}\Delta x_{t-1} + \phi_{12}v_{t-1} + \phi_{13}\xi_t + \phi_{14}e_t \tag{76}$$

$$\Delta x_t = \phi_{20} + \phi_{21}\Delta x_{t-1} + \phi_{22}v_{t-1} + \phi_{23}\xi_t + \phi_{24}e_t. \tag{77}$$

Then $E_t y_{t+1} = \phi_{10} + \phi_{11}(\phi_{20} + \phi_{21}\Delta x_{t-1} + \phi_{22}v_{t-1} + \phi_{23}\xi_t + \phi_{24}e_t) + \phi_{12}(\rho v_{t-1} + \xi_t)$ and $E_t\Delta x_{t+1} = \phi_{20} + \phi_{21}(\phi_{20} + \phi_{21}\Delta x_{t-1} + \phi_{22}v_{t-1} + \phi_{23}\xi_t + \phi_{24}e_t) + \phi_{22}(\rho v_{t-1} + \xi_t)$.

[40] In (72), this target value is absorbed into the constant term, μ_0.

Substituting these into (73) and (74) yields ten "undetermined coefficient" conditions, which determine the ϕ_{ij} values necessary for (76) and (77) to be solutions. Among these conditions are the following pair:

$$\phi_{21} = 0.5 + 0.5\phi_{21}^2 + 2\theta\phi_{11} + 2\theta\phi_{11}\phi_{21} \tag{78}$$

$$\phi_{11} = b_1\mu_1\phi_{11} + b_1\mu_1 0.5\phi_{21} + b_1\mu_1 0.5 + b_2\phi_{11}\phi_{21}. \tag{79}$$

Eliminating ϕ_{11} from these, we obtain the cubic equation

$$\begin{aligned} &0.25b_2\phi_{21}^3 - 0.5[b_2 + 0.5(1 - b_1\mu_1) + \theta b_1\mu_1]\phi_{21}^2 \\ &+[0.5(1 - b_1\mu_1) + 0.25b_2 - \theta b_1\mu_1]\phi_{21} \\ &-[0.25(1 - b_1\mu_1) + 0.5\theta b_1\mu_1] = 0. \end{aligned} \tag{80}$$

Since $b_2 = 1.0$, while $b_1 < 0$ and $0 < \mu_1 < 1$ with θ positive but very small, the three terms in square brackets are all almost certainly positive.[41] Therefore, the coefficients in this polynomial for ϕ_{21} alternate in sign, as follows: $+, -, +, -$. There are then three changes in sign, so Descartes' "rule of signs" implies that there are either three or one positive real roots to (80). Viewing (80) as a polynomial in $-\phi_{21}$ leads to the sign pattern $-, -, -, -$, which has zero sign changes and therefore implies that there are no negative real roots. Since the minimal state variable procedure requires real values for all ϕ_{ij} coefficients, we can then conclude that ϕ_{21} is positive. But this implies that the model's solution for Δx_t has a positive coefficient on Δx_{t-1}, and in that sense features persistence in contract inflation rates.[42] Since $\Delta p_t = 0.5(\Delta x_t + \Delta x_{t-1})$, persistence also characterizes the inflation rate in

[41] Formally, this conclusion can be assured by assuming that $\theta < 0.5$. The estimate of θ obtained by Fuhrer and Moore (1995) is less than $\frac{1}{50}$ of that magnitude. And even $\theta < 0.5$ is far from being necessary.

[42] We have not shown that $\phi_{21} < 1$, but Nelson (1996) finds that to be the case for all reasonably calibrated numerical versions of the model.

terms of the average price level.

Thus we conclude that the Fuhrer-Moore finding of inflation persistence obtains in a two-period version of their model that includes an expectational IS function and a monetary policy rule of the type discussed by Taylor (1994). The importance of this persistence property is illustrated in a recent paper by Nelson (1996) concerning the "liquidity effect" - - i.e. the negative relationship between money growth surprises and nominal interest rate levels (both contemporaneous and subsequent). Most existing models consistent with maximizing behavior either imply the absence of a liquidity effect or else include some specificational features that are highly improbable and/or analytically awkward.[43] Nelson has shown, however, that a realistically calibrated version of a model like the one of the present section does give rise to a reasonably strong and lasting liquidity effect.[44] The basic reason is that persistence of inflation leads to persistence in rationally expected inflation rates, thereby permitting nominal as well as real interest rates to fall in response to a money growth surprise.

VII. Conclusions

In the preceding sections we have shown that a dynamic, optimizing general equilibrium model of the Sidrauski-Brock type gives rise, when rather orthodox preference and production

[43] We consider the models of Christiano and Eichenbaum (1995), and Chari, Christiano and Eichenbaum (1995) to fall into the first category, as they prohibit households from adjusting their portfolios to a contemporaneous monetary shock. Dotsey and Ireland (1995) show that partial relaxations of this restriction can remove the liquidity effects in the Christiano-Eichenbaum model. Other studies, such as Kimball (1995) and Kim (1996), have found that liquidity effects obtain if both price stickiness and capital adjustment costs are introduced, while Wolman (1995) demonstrates that liquidity effects occur if money growth consists of both permanent and transitory components, with agents unable to distinguish between either component.

[44] As conjectured by Dotsey and Ireland (1995, p. 1456).

functions are specified, to a pair of linear equations that are analogous to traditional IS and LM functions. One of these equations is similar - - except for timing - - to a typical money demand specification, and can be made identical by a minor (and frequently utilized) timing modification. The other equation differs from a basic IS specification in one respect: an additional variable reflecting expected next-period income is present.[45] This modification gives a dynamic, forward-looking aspect to saving behavior. Together, the two equations provide a model of aggregate demand behavior that is reasonably tractable and yet usable with a wide variety of aggregate supply specifications - - from full price flexibility to ones with overlapping nominal contracts.[46]

Our specification treats capital (and therefore capacity output) as exogenous and steadily growing at its trend rate, so the model is not usable for issues concerning capital accumulation. Subject to that proviso, we in effect argue that traditional IS and LM functions need to be modified, for monetary policy or business cycle issues, only by the addition to the former of one forward-looking term. Consequently, the question naturally arises: Does this one modification result in a framework that implies traditional or non-traditional answers to substantive problems? To this question there can evidently be no single answer, except that "it depends" upon the problem at hand. But some tentative conclusions are possible, at least as conjectures. Thus it seems clear that issues involving details of dynamic behavior will have answers that differ from the traditional ones unless income (per-capita) has a constant

[45] In Section III we also presented stochastic versions, which require a few rather common approximations.

[46] Additional variables will also be present, in our equation and in standard IS functions, when government purchases and/or foreign trade is included in the model.

expected future value. On the other hand, issues that hinge on the distinction between real and nominal variables would appear to yield familiar and traditional answers.

Appendix A: Measures of Capital Stock ($ billion, 1987 prices)

	Narrow Measure				Broad Measure			
1948	1027.5	1041.5	1055.4	1069.8	4121.8	4140.4	4157.8	4173.4
1949	1080.4	1090.3	1099.8	1109.0	4194.0	4214.0	4234.6	4257.0
1950	1119.4	1130.8	1143.1	1155.5	4290.9	4327.7	4368.5	4407.0
1951	1167.7	1180.3	1193.1	1205.7	4448.5	4487.2	4525.1	4563.2
1952	1217.0	1228.4	1238.9	1250.1	4607.0	4651.7	4695.2	4740.9
1953	1262.5	1274.8	1287.4	1300.0	4789.9	4839.1	4887.9	4936.8
1954	1310.6	1321.2	1331.9	1342.5	4979.9	5023.5	5068.3	5114.1
1955	1354.5	1367.2	1380.5	1394.3	5164.8	5216.7	5268.9	5320.6
1956	1409.3	1424.5	1439.8	1454.9	5367.6	5414.1	5460.5	5506.6
1957	1469.0	1483.0	1497.3	1511.3	5551.4	5595.6	5639.8	5683.8
1958	1520.0	1528.3	1536.4	1544.7	5721.9	5759.8	5798.9	5840.5
1959	1555.2	1566.0	1577.1	1588.2	5889.1	5939.0	5988.7	6037.6
1960	1600.6	1612.9	1625.0	1637.1	6086.8	6134.4	6181.2	6227.9
1961	1648.3	1659.6	1670.9	1682.6	6273.3	6319.0	6366.0	6414.5
1962	1696.1	1710.1	1724.2	1738.1	6466.8	6520.4	6574.3	6628.0
1963	1752.2	1766.8	1781.8	1797.2	6685.0	6744.4	6905.1	6867.0
1964	1815.7	1834.8	1854.4	1874.5	6932.7	6997.9	7063.0	7128.0
1965	1899.7	1926.0	1953.2	1981.4	7203.2	7280.1	7357.7	7436.8
1966	2011.4	2041.7	2072.3	2102.5	7521.4	7604.4	7687.6	7767.9

Narrow Measure					Broad Measure			
1967	2129.3	2156.1	2182.7	2209.9	7841.2	7917.9	7995.8	8076.1
1968	2237.6	2264.7	2292.1	2320.3	8160.2	8245.0	8330.7	8417.9
1969	2349.6	2379.2	2409.4	2439.3	8502.6	8587.2	8671.9	8753.8
1970	2465.6	2491.7	2518.0	2543.6	8825.7	8895.9	8967.7	9041.0
1971	2566.5	2589.5	2612.4	2635.5	9113.3	9189.2	9267.3	9347.1
1972	2659.7	2684.3	2709.3	2735.7	9429.1	9512.0	9595.7	9683.3
1973	2768.9	2803.3	2838.5	2873.9	9781.7	9878.4	9973.7	10068
1974	2906.3	2938.6	2970.2	3000.7	10151	10232	10312	10386
1975	3020.3	3039.3	3058.5	3077.8	10440	10495	10552	10610
1976	3096.0	3114.3	3133.1	3152.1	10677	10744	10811	10882
1977	3176.1	3200.8	3226.0	3251.9	10963	11049	11137	11224
1978	3283.7	3317.7	3352.3	3387.5	11318	11418	11519	11620
1979	3425.8	3464.2	3503.6	3543.0	11721	11821	11922	12022
1980	3578.2	3611.3	3644.1	3677.4	12104	12177	12250	12327
1981	3709.9	3743.2	3777.1	3810.6	12400	12472	12543	12611
1982	3834.3	3857.0	3879.0	3900.6	12661	12711	12760	12811
1983	3917.1	3933.9	3951.4	3970.2	12871	12935	13004	13074
1984	4000.1	4031.5	4063.7	4096.8	13167	13263	13360	13458
1985	4134.1	4172.2	4209.4	4247.8	13565	13673	13784	13895

Narrow Measure					Broad Measure			
1986	4276.9	4305.3	4333.3	4361.5	14004	14116	14229	14344
1987	4384.4	4408.0	4432.5	4457.1	14447	14552	14659	14765
1988	4482.6	4509.1	4535.7	4561.9	14870	14977	15083	15191
1989	4585.4	4609.2	4633.5	4657.5	15291	15390	15490	15588
1990	4678.7	4699.6	4721.0	4742.4	15680	15770	15859	15946

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