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PRIVATE AND PUBLIC SUPPLY OF LIQUIDITY

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ABSTRACT

This paper addresses a basic yet unresolved question: Do claims on private assets provide sufficient liquidity for an efficient functioning of the productive sector? Or does the State have a role in creating liquidity and regulating it either through adjustments in the stock of government securities or by other means?

In our model, firms can meet future liquidity needs in three ways: by issuing new claims and diluting old ones, by obtaining a credit line from a financial intermediary, and by holding claims on other firms. When there is no aggregate uncertainty, we show that these instruments are sufficient for attaining the socially optimal (second-best) contract between investors and firms. Such a contract imposes both a maximum leverage ratio and a liquidity constraint on firms. Intermediaries coordinate the use of liquidity. Without intermediation, scarce liquidity may be wasted and the social optimum may not be attainable.

When there is only aggregate uncertainty, the private sector is no longer self-sufficient with regard to liquidity. The government can improve liquidity by issuing bonds that commit future consumer income. Government bonds command a liquidity premium over private claims. The supply of liquidity can be managed by loosening liquidity (boosting the value of its securities) when the aggregate liquidity shock is high and tightening liquidity when the shock is low. The paper thus provides a microeconomic example of government supplied liquidity as well as of the possibility of active government policy.

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1. Introduction

This paper addresses a basic, yet unresolved question: Do private assets provide sufficient liquidity for the efficient functioning of the productive sector? Or does the State have a role in supplying additional liquidity and regulating it through adjustments in the stock of government securities or by other means?

These questions presume a demand for liquidity; without demand there is no need to worry about supply. So why do firms demand liquidity? In the standard theory of finance (and in the Arrow-Debreu world of general equilibrium), firms can at any time issue claims up to the full value of their expected returns. They have no reason to hold liquid reserves in order to meet future financing needs, because they can just as well issue new claims as funding needs arise. As long as they use their funds to finance positive net present value projects, such firms will never encounter any liquidity problems.

The first step in our analysis then is to build a model in which there is a demand for liquidity, that is, advance financing is essential. For this purpose we use an entrepreneurial model of moral hazard in which the value of external claims are strictly less than the full value of the firm for the standard reason that the entrepreneur must be given a minimum share of the firm in order to be motivated. The wedge between the full value and the external value of the firm limits the firm's ability to raise funds to finance all projects that have positive net present value. In a dynamic setting, this feature implies that liquidity shocks could force the firm to terminate a project mid-stream even though the project has a positive continuation value. To protect itself against such risks, the firm wants to hold liquid reserves either in the form of marketed assets that can be readily sold or by arranging for credit in advance.

We study the determinants of the firm's liquidity demand in a simple, dynamic moral hazard model. There are three periods. At date 0 the firm raises funds to invest in a variable-sized project that pays off at date 2. At date 1, the firm experiences a liquidity shock. The shock is a random fraction of the date-0 investment and represents the amount of additional investment that must be made to continue the project.[1] If the necessary funds can be raised, the project

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[1] The liquidity shock admits several interpretations. It can be a cost overrun on the initial investment. Alternatively, it can represent a shortfall in the date-1 revenue, which could have

proceeds, delivering a stochastic date-2 return that depends on the entrepreneur's effort.

We show that the optimal date-0 contract between the firm and the outside investors limits both the initial investment level and the amount that the firm is allowed to spend on the liquidity shock, both constraints being proportional to the firm's initial assets. Because the firm is credit constrained, the second-best solution trades off the benefits of a higher initial investment against the increased likelihood of having to terminate the project early and see it all go to waste. This solution can be implemented in several ways. One is to give the firm all the necessary funds in advance, but add a *liquidity covenant* in which the firm promises to set aside a certain amount of funds to cover future liquidity needs. Alternatively, intermediaries could fund future liquidity needs via a *credit line*.[2]

Having established a demand for liquidity, we move on to study its supply by embedding the model of an individual firm in a general equilibrium model with a large number of such firms. Each firm (or intermediary) can issue arbitrary claims contingent on its date-2 financial position and these claims can be freely traded in the market.[3] The government can also supply liquidity by selling securities such as Treasury bonds. There are no other assets that firms and intermediaries can use to transfer wealth from one period to the next.[4] Thus, a firm can meet liquidity needs in four ways: by issuing claims on its own productive assets, by holding claims on other firms, by holding government issued claims, or by using a credit line.

Our main interest is in understanding the government's role in supplying and managing liquidity as a function of the severity and nature of the liquidity shocks. We consider two polar

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been used to finance date-1 operating expenses. Or the shock might simply be information about the firm's date-2 returns. The central feature in all these cases is that the shock affects the firm's net worth, which determines its financing capacity.

[2] Our theory is too sparse to distinguish between different financial institutions. When we use language (intermediary, bank, etc) that suggests an institutional interpretation, this should be interpreted as illustrative of many similar arrangements.

[3] In contrast to much of the literature, we do not restrict the set of claims that firms can issue, in order to avoid a spurious demand for publicly supplied liquidity.

[4] We can allow other assets, such as real estate, as long as their value does not fulfill the private sector's liquidity needs. See the conclusion.

cases, one in which the firms' liquidity shocks are independent, so that there is no aggregate uncertainty, and the opposite case in which the firms' liquidity shocks are identical.

In the absence of aggregate uncertainty, we show that government securities do not add useful liquidity beyond that provided by private claims, nor does the government have any role in managing aggregate liquidity.[5] There is a simple intuition behind the self-sufficiency of the private sector: The date-1 continuation value of the private sector, which is deterministic, must exceed its date-0 value, else it could not have invested at date 0. Therefore the private sector can raise sufficient funds at date 1 by diluting its shareholdings, or equivalently, by leveraging itself up with new debt.

However, even though the private sector is self-sufficient in the aggregate, we show that individual firms will in general be unable to satisfy their liquidity needs by holding only private market instruments. One might have thought that it would suffice for each firm to hold a share of the market portfolio, which it could sell when the liquidity shock hits. However, this market solution is in general not efficient, because lucky firms with low liquidity shocks end up holding excess liquidity. Thus, scarce liquidity is wasted.

We show that an optimal allocation can always be achieved by intermediaries. Intermediaries hold corporate securities and simultaneously grant credit lines to firms. They take into account the fact that some firms (namely, those with low liquidity needs) will not exhaust their credit lines at date 1. Intermediaries can therefore redistribute excess liquidity to firms that do need extra funds. They act as *liquidity pools* or insurers in preventing a wasteful accumulation of liquidity by firms that end up being lucky at date 1. Thus, *intermediation may strictly dominate financial markets.*

Note that it is the anonymity of market claims, and not an informational handicap, that explains why financial markets can be inferior to intermediation. The critical feature of the credit line arrangement that yields a productive optimum is that it allows negative net present value investments at date 1. By contrast, claimholders can dispose of negative value claims in an anonymous financial market.

In the presence of pure aggregate uncertainty, the earlier argument for why the private

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[5] If issued, government securities will carry no premium.

sector is self-sufficient breaks down. Now there can be no cross-subsidization, because each firm faces trouble exactly when the other firms do. As the private sector is unable to provide insurance there will be a liquidity shortage whenever the reinvestment need exceeds the date-1 value of corporate claims. This creates a potential demand for government-provided liquidity. We show that the government can achieve a Pareto improvement by issuing Treasury bonds, and that these bonds can be sold at a *liquidity premium*. The private sector is willing to purchase low-yielding securities because they serve as an input into the production process. In our risk-neutral model securities with different rates of return co-exist in equilibrium.

It is important to point out what the government can achieve that the private sector cannot. In our model, individual investors can refuse to bring in new funds to the corporate sector at date 1, if this is unprofitable. More precisely, we assume that individual investors cannot at date 0 promise to supply new funds at date 1: For example they may pretend at date 1 that they have no new endowment. This assumption is both theoretically and empirically reasonable. In practice individuals can borrow very limited amounts without pledging assets as collateral (as in the case of a mortgage). Because the private sector has no power to audit income and levy taxes, it is unable to check who is lying about endowments at date 1. By contrast, the government has the power to audit incomes and impose non-financial penalties (such as jail) to enforce tax payments. It can therefore, on behalf of the consumers, commit new funds at date 1.[6] This is what Treasury bonds issued at date 0 achieve.

When the government issues securities at a premium and when firms are constrained to issue a single security (equity), a firm has an incentive to free-ride on the liquidity provided by those firms that purchase government securities. The firm can buy their shares instead of government securities. We show that firms respond to the possibility of free riding by issuing multiple securities so as to price discriminate between investors who value liquidity services much (corporations) and those who don't (consumers). Corporate debt is then sold at a liquidity premium relative to equity.

However, these outcomes are not socially efficient in that they lead to distorted

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[6] If our model were one with overlapping generations, the government could commit future consumers as well as current ones.

reinvestments in an effort to economize on the use of government provided liquidity. We show that it is more efficient to have the government issue state-contingent bonds, which supply liquidity only as needed by the private sector. Such bonds have a high value when liquidity needs are high and a low value when liquidity needs are low. We interpret state-contingent bonds as a metaphor for an active government policy in which liquidity is made tight when the aggregate liquidity shock is favorable (low) and loose when the aggregate shock is unfavorable (high). Thus, the model offers an example, not only of government supplied liquidity, but also for the possibility of an active management of liquidity by the government.

The paper is organized as follows. Section 2 sets up a partial equilibrium model of liquidity demanded by a single firm. Section 3 embeds this model in a general equilibrium model without aggregate uncertainty and shows that the private sector is self-sufficient and that intermediaries are used as liquidity pools. Section 4 studies aggregate uncertainty and analyzes the demand for and supply of government securities, including the optimal use of state-contingent bonds. Section 5 relates our results to the literatures on credit rationing, intermediation and government debt. Section 6 concludes with remarks on the key features of the model and suggestions for future research.

2. Exogenous Liquidity Supply

2.1 *The model.*

This section studies the investment decision of an individual firm.

There are three periods, $t = 0,1,2$ and two types of agents, firms (entrepreneurs) and investors (consumers). There is one (universal) good used both for consumption and investment. All agents are risk neutral with an additively separable utility function over undiscounted consumption streams. For the moment we assume that the good can be stored at a zero rate of return and simply refer to it as "cash". This guarantees that there is no shortage of liquidity. In later sections we will drop this assumption as we are interested in the endogenous supply of liquidity, that is, in how well financial assets can service future funding needs of firms.

The firm has access to a stochastic constant-returns-to-scale technology, which for an initial investment I returns RI if the project succeeds and 0 if it fails. The scale of the investment

I can be varied freely, subject only to financial constraints. The investment is made at date 0. At date 1, an additional, uncertain amount $\rho I > 0$ of cash is needed to finance cost overrruns or date-1 operating expenditures and thus to carry on with the project. The *liquidity shock* ρ is distributed according to the cumulative distribution function F, with a density function f. If ρI is not paid, the project terminates and yields nothing. If ρI is paid, the project continues and its payoff is realized at date 2.

Investment is subject to moral hazard in that the firm (entrepreneur) privately chooses the probability p that the project succeeds. The firm can either "behave" or "shirk". If the firm behaves the probability of success is p_H (high), if it shirks the probability of success is p_L (low), where $p_H - p_L \equiv \Delta p > 0$. If the firm shirks, it enjoys a private benefit, $BI > 0$, proportional to the level of its investment I. The firm makes the decision on p after the continuation decision. If the project is abandoned, no decision on p needs to be made.

The timing of events is described in Figure 1.

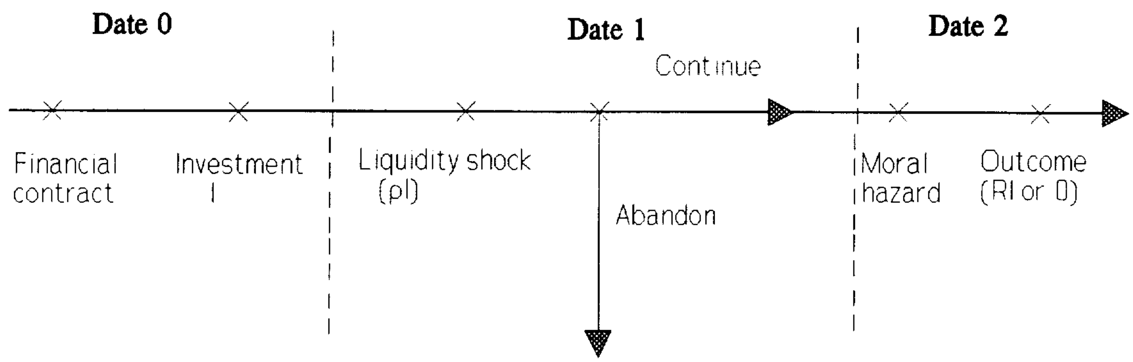

Figure 1

The firm has a date-0 endowment of cash, A, and no endowments at dates 1 and 2.[7] If the firm wants to invest $I > A$, it will need to raise $I - A$ from outside investors. For the moment, we assume that the initial investment level, the project outcome and the liquidity shock are all verifiable (as we will see, nothing changes if only the entrepreneur observes the liquidity

[7] There is no loss of generality in assuming this. In the presence of a random date-1 revenue stream, the liquidity shock must be thought of as being *net* of the date-1 revenue.

shock). A contract with the investors specifies the amount that the investors will contribute at date 0, the scale I of the project, the contingencies in which the project is continued at date-1, who pays for the liquidity shocks, and finally how the proceeds from the investment are distributed. A key constraint on the contract is that the firm cannot pay out more funds than it has; there is *limited liability*.

It is easy to show that an optimal allocation has the following simple structure. The firm invests all its funds initially with the balance of the investment, I - A, paid by the investors. The project is continued if and only if the liquidity shock falls below a threshold $\hat{\rho}$, $\rho \leq \hat{\rho}$. Neither the investors nor the firm receive anything if the project fails or is abandoned. If the project succeeds, investors are paid $R_i I$ and the firm is paid $R_f I$, where $R_i + R_f = R$.[8] This allocation has three free parameters to be determined: the scale of the project I, the cut-off $\hat{\rho}$, and the investors' share R_i.

To make the analysis interesting, we will assume that the project has a positive net present value if the firm is diligent, but not if it shirks:

$$\max_{\tilde{\rho}} \left\{ F(\tilde{\rho}) p_H R - 1 - \int_0^{\tilde{\rho}} \rho f(\rho) d\rho \right\} > 0 > \max_{\tilde{\rho}} \left\{ F(\tilde{\rho}) p_L R + B - 1 - \int_0^{\tilde{\rho}} \rho f(\rho) d\rho \right\}. \tag{1}$$

Note that the first maximum is achieved by setting $\tilde{\rho} = \rho_1 \equiv p_H R$. That is, net present value is maximized by continuing the project whenever the expected return exceeds the additional need for funding. We will refer to ρ_1 as the *first-best threshold.*

We turn to the second-best solution. If the firm continues, it must be given an incentive to be diligent. This requires that

$$(\Delta p) R_f I \geq BI. \tag{2}$$

As is standard in models with limited liability, the firm must be bribed to be diligent. According

[8] In this two-outcome world the investors' claim can either be interpreted as debt (the firm must pay back $R_i I$ and defaults if it cannot) or as equity (there is no debt and the investors hold an equity share R_i/R).

to (2), the minimum bribe per unit of investment is $R_f = R_b \equiv B/\Delta p$.[9] We assume that this minimum is so high that the residual share that can be pledged to investors, $R - R_b$, is insufficient to cover the expected cost of the investors' investment. If the return required by investors is zero, this assumption holds if

$$F(\hat{\rho})\rho_0 < 1 + \int_0^{\hat{\rho}} \rho f(\rho) d\rho \quad \text{for all } \hat{\rho}, \tag{3}$$

where

$$\rho_0 \equiv p_H(R-R_b)$$

is the date-1 *pledgeable per-unit return*. Note that condition (3) is equivalent to the condition that some equity is needed to implement the threshold that is most favorable to outside investors :

$$F(\rho_0)\rho_0 < 1 + \int_0^{\rho_0} \rho f(\rho) d\rho.$$

Given that the investors' time preference is zero, the return they will require will also be zero as long as they collectively have more cash than demanded by the firms. We will maintain this assumption throughout the paper. With a zero rate of return, and for a given cutoff $\hat{\rho}$, the investors' break-even constraint is

$$F(\hat{\rho})\, p_H(R-R_f)I \geq I - A + I \int_0^{\hat{\rho}} \rho\, f(\rho) d\rho. \tag{4}$$

The left-hand side represents the ex ante "pledgable expected income", which must exceed the investors' expected outlays found on the right-hand side. Given (3), we see that the break-even constraint limits the firm's investment to a finite level, which we might term its *investment capacity*. Because investors break even, the firm's net utility equals the social surplus of the project, namely

$$U(\hat{\rho}, I) \equiv [F(\hat{\rho})\rho_1 - 1 - \int_0^{\hat{\rho}} \rho f(\rho) d\rho]\; I \equiv m(\hat{\rho}) I, \tag{5}$$

where $m(\hat{\rho})$, the term in brackets in (5), represents the marginal return on a unit of initial investment. For a given $\hat{\rho}$, the second-best solution therefore maximizes I, implying, by (4),

9 The minimum payment to the entrepreneur can more broadly be interpreted as a rent shared by all employees of the firm. Without such rents, employees will not work as efficiently.

that it is optimal to set $R_f = R_b$. The firm's investment capacity is then

$$I = k(\hat{\rho})A, \tag{6a}$$

where

$$k(\hat{\rho}) \equiv \frac{1}{1+\int_0^{\hat{\rho}} \rho f(\rho)d\rho - F(\hat{\rho})\rho_0} \tag{6b}$$

is the *equity multiplier*. Maximal leverage is achieved by setting $\hat{\rho} = \rho_0$. The equity multiplier can be less than 1 if the expected liquidity demand at date 1 is sufficiently high. In that case I-A < 0. (The total investment, that is, the initial investment plus the expected liquidity payments, will of course exceed the firm's initial cash A.) Unless otherwise specified, we will assume that I-A > 0. That is, that the firm is a net borrower at date 0 (this will be the case if the denominator in (6b) is less than 1 for $\hat{\rho} = \rho_1$).

It remains to determine the optimal continuation threshold. Intuition might suggest that it would be optimal to let the firm continue whenever continuation is ex post efficient ($\rho \leq \rho_1$). Indeed, the first-best cut-off $\rho_1 = p_H R$ maximizes the per-unit profit of investing, $m(\hat{\rho})$. However, at ρ_1, the equity multiplier $k(\hat{\rho})$ is decreasing in $\hat{\rho}$, indicating that one ought to choose a lower treshold than the ex post efficient one. Substituting (6) into (5) and dividing through by $F(\hat{\rho})$, one finds that the optimal (second-best) threshold, ρ^*, minimizes the expected unit cost of total expected investment:

$$\rho^* \text{ minimizes } \frac{1+\int_0^{\hat{\rho}} \rho f(\rho)d\rho}{F(\hat{\rho})}. \tag{7}$$

It is not difficult to show that the optimal threshold satisfies [10]

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[10] Note that ρ^* does not depend on either ρ_0 or ρ_1, so how can (8) hold? The answer is that program (7) gives the optimal cut-off only if (1) and (3) hold. We must have $\rho^* > \rho_0$, else the firm would not be credit constrained at all and condition (3) would be violated. If $\rho^* > \rho_1$, investment would not be profitable and condition (1) would be violated.

Note also that the first-order condition for (7) can be written

$$\int_0^{\rho^*} F(\rho)d\rho = 1.$$

$$p_H(R-R_b) = \rho_0 < \rho^* < \rho_1 = p_H R. \tag{8}$$

These bounds can be given a simple economic interpretation. The lower bound is explained by the fact that if $\rho < \rho_0$, both the firm and the investors prefer to continue with the project ex post. The upper bound is explained by the fact that if $\rho > \rho_1$, the net present value of continuing is negative, in which case an ex post Pareto improvement could be achieved by having investors compensate the firm (which always likes to continue ex post) for abandoning the project. Since transfers from investors to the firm are not constrained by incentive compatibility, both parties would agree to abandon the project given such a liquidity shock.

Since $k(\rho_0)$ represents the largest investment scale that the firm can achieve, while ρ_1 maximizes the ex post surplus, we see that the second-best solution trades off a larger investment scale at date 0 against fewer liquidations (more investment) at date 1. The firm cannot freely choose both variables, because by (2) it is limited in its ability to transfer social surplus to investors. Consequently, the firm distributes available funds so that it is credit rationed both ex ante and ex post.

For future reference, we record the following result:

Lemma 1: *It is optimal for the firm to adopt a cut-off policy in which liquidity needs are met if and only if* $\rho \leq \hat{\rho}$.

(i) *The firm's investment capacity* $I = k(\hat{\rho})A$ *is quasi-concave in* $\hat{\rho}$, *with a maximum at* $\hat{\rho} = \rho_0$.

(ii) *The marginal return on investment* $m(\hat{\rho})$ *is quasi-concave in* $\hat{\rho}$, *with a maximum at* $\hat{\rho} = \rho_1$.

(iii) *The firm's expected output* $(F(\hat{\rho})\rho_1 k(\hat{\rho})A)$ *and the value of external claims on this output* $(F(\hat{\rho})\rho_0 k(\hat{\rho})A)$ *are quasi-concave in* $\hat{\rho}$, *with a maximum at the second-best optimum* $\hat{\rho} = \rho^*$.

A mean-preserving spread in the distribution of the liquidity shock therefore decreases ρ^*.

2.2 *Implementation of the optimal policy.*

Suppose the decision to continue the project is left to be negotiated at date 1. In that case, investors would be willing to inject more cash into the project if $\rho < \rho_0$, but not if liquidity needs exceed ρ_0. In the latter case, even if the firm were allowed to dilute the claims of the initial investors by issuing senior securities at date 1, it could not raise enough cash, because the maximal value of the firm's external claims is $\rho_0 I$. Thus, to cover the firm's liquidity needs at date 1 when $\rho \in (\rho_0, \rho^*)$ one must find a way for investors to commit sufficient funds at date 0.

One solution is to give the firm I - A at date 0 and extend it an *irrevocable line of credit* in the amount $\rho^* I$ to be used for liquidity needs at date 1. Given that the firm cannot consume funds, and always prefers to continue, it will do so as long as the credit line permits.[11] This implements the optimal solution. [Equivalently, the investors can extend an irrevocable credit line in the amount $(\rho^* - \rho_0) I$ and grant the firm the right to dilute outside claims as needed to cover the liquidity shock.]

In practice, individuals do not grant credit lines, because they cannot commit future income or endowments. Ex post, credit line commitments would be either unprofitable, in which case investors would claim that they have no funds, or be profitable, in which case a commitment was unnecessary. Instead, credit lines are granted by intermediaries such as banks, which can make commitments (both in practice and in our model), assuming of course that they have enough funds to meet later credit needs. This raises the question: How do intermediaries guarantee that their own liquidity needs can be met? In the next section, we show that they can do so by issuing shares that pay off only at date 2.

An alternative solution is to give the firm $I(1+\rho^*)$ - A at date 1, and require that $\rho^* I$ of this amount is kept in liquid assets. In other words, investors demand that the firm maintains a *liquidity ratio* equal to $\rho^*/(1+\rho^*)$, at least up to date 1. Indeed, a minimum liquidity ratio is a

[11] Could the line of credit be renegotiated for mutual advantage once ρ is realized? The answer is no. If $\rho \leq \rho^*$, then $\rho < p_H R$ and it is ex post efficient to continue so there is no scope for renegotiation. Nor can both parties benefit from an increase in the line of credit when $\rho^* < \rho \leq p_H R$. Even though this increase raises total surplus, there is no way to distribute the surplus in a manner that satisfies the investors.

commonly observed debt covenant. As in the case of credit lines, intermediaries are the natural party for overseeing the compliance of such a covenant. It is worth stressing that the partial equilibrium model does not distinguish between a credit line and the hoarding of liquid assets by the firm. But as the next section shows these two methods of providing reserves are no longer equivalent once general equilibrium aspects (namely, that the supply of liquid assets is not unlimited) are brought into consideration.

Remark 1: We assumed that the liquidity shock is verifiable. Nothing is altered if only the firm observes the liquidity shock as long as the firm cannot use part of the credit line or its financial assets for consumption at date 1. Given the firm's reserves, it can continue whenever $\rho \leq \rho^*$. If $\rho > \rho^*$, it cannot continue, since no outsider can be induced to invest in the firm. A rational investor would infer that a firm that tries to raise new funds does so because it has exceeded its reserves and therefore has experienced a shock $\rho > \rho^* > \rho_0$.

Remark 2: If investors cannot control the firm's use of funds, one can show that the firm does not want to distribute the funds between date-0 investment and date-1 investment in accordance with the second-best solution. Either the firm wants to invest less initially in order to be able to continue in more states at date 1. Or the firm wants to invest excessively at date 0, relying partly on being bailed out by the investors when $\rho < \rho_0$ and there is a cash shortfall (a version of the soft-budget constraint problem). To illustrate the latter possibility, suppose that the firm invests the full amount $I(1+\rho^*) \equiv I^*$ in illiquid assets. Despite the lack of cash left for reinvestment, the project will often be continued, as the investors have an incentive to rescue the firm as long as $\rho \leq \rho_0$. Of course, the investors might want to claim initially that they will not put any more money into the venture, but this is not a credible commitment. Anticipating a ''soft budget constraint'', the firm may overinvest. Indeed the firm prefers investing I^* rather than I if

$$F(\rho^*)p_H R_b I < F(\rho_0) p_H R_b I^*, \text{ or}$$
$$F(\rho^*) < F(\rho_0)(1+\rho^*).$$

This condition can be satisfied by adjusting B so that ρ_0 falls just below ρ^*.

We summarize this section in:

Proposition 1. *In the second-best allocation:*

(i) *A firm with initial capital A invests $I = k(\rho^*)A$ where $k(\cdot)$ is given by (6b).*

(ii) The project is continued if and only if the liquidity shock falls below the cutoff ρ^*. The cutoff ρ^* lies strictly between the (per-unit-of-investment) pledgeable expected return $\rho_0 = p_H(R-B/\Delta p)$ and the expected return $\rho_1 = p_H R$.

(iii) Neither party is paid anything if the project is terminated or fails. If the project succeeds, the firm is paid $(B/\Delta p)I$ and the investors $(R-B/\Delta p)I$.

(iv) This second-best allocation can be implemented by raising (I-A) in external funds for the initial investment and extending the firm a line of credit in the amount $\rho^* I$ to be used for reinvestment. Alternatively, the firm gets $(1 + \rho^*)I$ at date 0 with the convenant that it keeps $\rho^* I$ in reserve for reinvestments.

3. Endogenous Liquidity Supply without Aggregate Uncertainty.

3.1 *The economy.*

Next we embed our model in a general equilibrium framework. Our objective is to analyze the endogenous supply of liquidity. To focus on the problem of creating assets that can meet the demand for liquidity at date 1, we drop the assumption that there is an exogenously given storage technology. The single consumption good can no longer be stored (there is no longer cash), nor are there any other private assets (like real estate) that can be used to transfer wealth from one period to the next. The only private way to transfer wealth across periods is by buying claims issued by firms. Later on, we will introduce government bonds.

There is a continuum of firms with unit mass. Each firm is endowed with the stochastic technology described in section 2. Because of constant returns to scale, there is no loss in assuming that firms have identical endowments; the representative firm is endowed with A units of the good at date 0 and none at dates 1 and 2.

As before, consumers and entrepreneurs are risk neutral and do not care about the timing of consumption. Their expected utility for consumption is

$$E[c_0 + c_1 + c_2].$$

Consumers receive per-period endowments that collectively are large enough to finance all required investments and taxes that may be levied. Consumers cannot sell claims on their future endowments, because they can default with impunity.

Any assets purchased by the consumers must yield a zero expected return given that their collective endowments exceed investment demand. By contrast, firms may be willing to purchase assets at a premium (a negative return) as long as those assets help them to meet their liquidity needs. Thus, rates of return, to the extent they deviate from zero, will be entirely determined by the productive sector. This is the convenience afforded by a linear preference structure.

In this section we will consider the case where liquidity shocks are independent. Because there is a continuum of firms, there is no aggregate uncertainty and $F(\rho)$ denotes both the ex ante probability that a given firm faces a liquidity shock below ρ and the realized fraction of firms with liquidity shock below ρ. The independence assumption and the analysis in section 2 imply that the date-1 funds needed by the productive sector to achieve the productive optimum is the deterministic amount

$$D \equiv [\int_0^{\rho^*} \rho f(\rho) d\rho] I, \tag{9}$$

where I is the representative firm's investment.

The two ways of implementing the productive optimum discussed in section 2, an irrevocable credit line or a liquidity requirement, both relied on an exogenously supplied liquid asset, cash. With cash no longer available, we are led to investigate whether firms can finance their liquidity needs D by using available market instruments, that is, by issuing additional claims at date 1 or by holding stakes in other firms.

3.2 *Financial markets.*

For the time being, assume that there are no intermediaries. There is only a financial market for claims issued by firms. The financial market is anonymous in that the return on a given claim is independent of who holds it.

We first compute the maximum amount that a firm can raise at date 1 by selling new (senior) securities in the financial market. Assuming that the firm is able to continue with the amount thus raised, its market value (the maximum amount it can raise) is

$$p_H(R-R_b)I = \rho_0 I.$$

The market value $\rho_0 I$ reflects the fact that the firm must retain the stake $R_b I$ as an inside claim in order to behave. Since date-0 investors, those who bought the firm's initial securities, get nothing if the firm cannot meet its liquidity needs, they are willing (ex post) to dilute their claims up to the full amount $\rho_0 I$ in order to salvage some of their initial investment. Of course, initial investors take the possibility of dilution into account in the pricing of initial claims.

A firm is unable to meet its liquidity needs by issuing new claims whenever the realized liquidity shock ρ falls in the interval $(\rho_0,\rho^*]$. Can a firm cover the potential shortfall by buying, at date 0, claims issued by other firms and selling these claims at date 1? That is, can the firm assure itself enough liquidity through the financial market? The answer is in general negative.

To develop an intuition for why the financial market may be unable to serve the firms' liquidity needs efficiently, let us assume (for illustration only) that all external claims are equity claims and that they are gathered in a single stock index (a mutual fund), which firms can buy. The absence of aggregate shocks implies that S_1, the value of the stock index at date 1, is deterministic. Suppose that the productive optimum of section 2 obtains; a firm with liquidity shock ρ continues if and only if $\rho \leq \rho^*$. Then, a fraction $F(\rho^*)$ of firms will continue at date 1. The total value of external claims on the productive sector at date 1 (and date 2) is

$$V_1 = F(\rho^*)\rho_0 I. \tag{10}$$

In the absence of credit lines, these external claims need to be diluted by the amount D defined in (9), so, the value S_1 of the stock index at date 1 is

$$\begin{aligned} S_1 &= V_1 - D \\ &= [F(\rho^*)\rho_0 - \int_0^{\rho^*} \rho f(\rho)d\rho]\, I = I - A. \end{aligned} \tag{11}$$

Let us conduct the following thought experiment: Suppose that a fraction $\alpha \in [0,1)$ of the stock index is held by the firms themselves. All firms have the same share in the index.

At date 1, each firm can then withstand liquidity shocks ρ satisfying

$$\rho I \leq \rho_0 I + \alpha S_1. \tag{12}$$

For the right-hand side of (12) to be equal to $\rho^* I$ for some $\alpha < 1$, one must have:

$$\rho^* + \int_0^{\rho^*} \rho f(\rho) d\rho < [1 + F(\rho^*)] \rho_0. \tag{13}$$

Recalling that ρ^* is independent of the extent of moral hazard as measured by B (see (7)), we conclude that for B large enough, that is for ρ_0 small enough, condition (13) is violated in which case firms are unable to withstand liquidity shocks up to ρ^* by holding the stock index. Intuitively, when the firm's insider share R_b is large, an individual firm cannot raise much capital by diluting itself at date 1. Concurrently, the stock index has a low value and the firm cannot raise much by selling the stock index either.

Actually, as the next section demonstrates, the issue is not that the stock market delivers too little liquidity *in the aggregate*. Rather, the ex post *distribution* of liquid assets is inefficient. The lucky firms (those with liquidity shocks $\rho < \rho_0$) hold shares in the stock index, which they do not need. Meanwhile, other firms cannot withstand their larger liquidity shocks because their share in the stock index is confined to be the average share.[12]

It is clear that the previous reasoning does not rely on there being a single financial claim (the stock index) that firms can purchase. More generally, S_1 — the difference between the total value of external claims and the aggregate reinvestment cost when the threshold is ρ^* — is an upper bound on the combined value of *all* external date-0 claims on the productive sector. Hence, in the aggregate, the value of the financial claims held by the firms cannot exceed S_1. Given that dilution cannot raise more than ρ_0 I per firm, some firms (all firms, if financial

---

[12] An example may be helpful. Suppose ρ is uniformly distributed over [0,2] and the parameters are such that $\rho_0 = 1.1$ and $\rho_1 = 3$. Then (7) implies $\rho^* = 2$. The initial investment level is $I = A/(.9)$. If $A = 9$, then $I = 10$, with outside investors investing 1 and the firm 9. Thus, the outside investors' stake in the stock index is worth 1. But $\rho^* - \rho_0 = .9$, implying that there will be realizations of ρ such that firms will need up to 9 shares of the stock index in order to continue. That is nine times as many shares as each can purchase at date 0.

claims are uniformly distributed among firms) cannot withstand shocks exceeding ρ_0 I + S_1.

To conclude, anonymous financial markets cannot in general implement the productive optimum.

3.3 *Intermediation.*

When the financial market fails to supply enough liquidity, the productive optimum can be implemented by introducing an intermediary that offers insurance against liquidity needs by pooling firm risks. The optimal insurance arrangement entails subsidizing firms with a high liquidity demand, allowing them to draw on the market value of firms that experience a low liquidity demand.

We first check that there is enough liquidity in the aggregate to implement the second-best policy. Let all individual firms be pooled together into a single firm, an economy-wide conglomerate. Assuming that the conglomerate follows the second-best policy, where individual firms continue if and only if $\rho \leq \rho^*$, the conglomerate can be sold at date 1 (gross of reinvestments) for the value $F(\rho^*)\rho_0 I$. The conglomerate needs to raise D (see (9)) to cover its liquidity needs. Since $F(\rho^*)\rho_0 I - D = S_1$, which is positive, the conglomerate can raise the necessary funds to implement the second-best policy.

It remains to show that an intermediary can make full use of the potential market value of the private sector. There are many equivalent ways in which this can be done. The simplest is to let the intermediary be a conglomerate as above.[13] At date 0, it issues shares to investors. These shares are claims on the intermediary's financial position at date 2.[14] The intermediary uses the proceeds from its share issue to buy up all the external claims on firms. Firms' securities are priced so that the intermediary breaks even on each firm issue (ex ante). Similarily, the shares of the intermediary are so priced that investors break even. Assuming that the intermediary succeeds in implementing the second-best policy, its shares will now sell for

[13] In our model, the number of intermediaries is indeterminate. We will describe the implementation of the second-best solution as if there were a single intermediary.

[14] As in Jacklin (1987), it is important to avoid claims that can be redeemed at date 1, since such claims can lead to "bank runs".

the amount S_1, since its market value equals the value of its investment portfolio. The intermediary's net "cash flow" at date 0 is zero, guaranteeing that the arrangement is financially feasible.

The intermediary proceeds to grant each firm a credit line equal to $\rho^* I$. Attached to the credit line is the covenant that a firm cannot issue new claims at date 1. A firm can use all or just a portion of its credit line at date 1. There is no charge for using credit (we will return to this point below). However, the firm has no use for excess credit, since the enterpreneur cannot consume funds and any returns from market assets that the firm might purchase at date 1 will go to the intermediary (except for the portion $R_b I$, which goes to the entrepreneur if the project succeeds). Given the credit line, a firm can continue whenever $\rho \leq \rho^*$.

The only remaining issue is whether the intermediary can raise enough funds at date 1 to meet its (deterministic) credit obligations D. But this follows immediately from the fact, argued above, that the private sector can in the aggregate raise $S_1 + D > D$ (assuming $I - A > 0$).

Remark 1: We argued in section 3.1 that the financial market fails to provide adequate liquidity because the value of financial claims held by a firm cannot be contingent on a firm's idiosyncratic liquidity shock. Consequently, firms with low liquidity shocks end up with excessive liquidity, that could be efficiently redistributed to firms with higher liquidity shocks. Such a redistribution however does not occur spontaneously at date 1, since the liquidity-rich firms have no incentives to enter negative NPV arrangements with liquidity-poor ones. One may wonder what it is that an intermediary can do that the financial market cannot do. *Intermediation does not dominate financial markets on informational grounds*. Rather, the critical feature of the credit line arrangement that yields a productive optimum is that it allows negative NPV investments at date 1.[15] By contrast, claimholders can dispose of negative value claims in an anonymous financial market.

Remark 2: Because the private sector is self-sufficient there is no role for government securities.

[15] To offset the date-1 loss, the intermediary receives either an upfront commitment fee or securities in the firm at date 0.

If issued, government securities must yield the market rate of interest, here equal to zero, and they have no impact on the productive sector.

Remark 3: Many alternative arrangements involving intermediation are equivalent to the one described above. For instance, the intermediary could be made to resemble a bank by letting firms issue shares directly to investors and having the intermediary hold only firm debt (claims senior to equity). The intermediary could also charge the firm according to the amount of credit used, rather than just an up-front fixed fee. To illustrate, a credit arrangement could call for a repayment of $(\rho/\rho^*)(R-R_b)I$ at date 2 (which can be met, of course, only if the project succeeds) if the firm uses ρI ($\leq \rho^*$) of credit at date 1. Note that the implied price for credit is below par, as the expected repayment equals $(\rho\rho_0/\rho^*)I$, which is less than ρI, the amount of credit used. In fact, because firms are fully diluted when $\rho > \rho_0$, there is no credit contract that charges an actuarially fair price for all realizations of ρ. An additional up-front fee must be imposed to cover the intermediary's expected loss. In reality, such fees are common, suggesting that use of credit is indeed underpriced ex post.

Variations like these only influence how surplus is redistributed among external claim holders and how initial claims are priced, but they have no effect on real outcomes and therefore we can proceed without pinning down these contractual details.[16]

We summarize this section in:

Proposition 2. *In the absence of aggregate uncertainty:*

(i) The productive optimum cannot always be achieved through an (anonymous) financial market. Ex post, liquidity may be misallocated.

(ii) The productive optimum can be implemented (if $I - A > 0$) by relying on private intermediation of credit. Intermediaries act as insurers against liquidity shocks, cross-subsidizing firms. Intermediaries need not observe individual liquidity shocks in order to implement the optimum.

(iii) There is no liquidity role for government securities. If issued, such securities must be offered

[16] Monitoring and control right considerations could be introduced to reduce contractual ambiguity, but the implications of such embellishments must await further research.

at par (yielding zero interest) and do not affect real allocations.

4. Endogenous Liquidity Supply with Pure Aggregate Uncertainty

4.1 *The demand for government bonds.*

In the previous section liquidity shocks were independent. We now shift to the other extreme, the case where there is only aggregate uncertainty. All firms experience the same liquidity shock.

It is clear that in this case the private sector cannot be self-sufficient. With pure aggregate uncertainty, every firm needs to raise ρI at the same time. Since a firm is worth at most ρ_0I to outsiders at date 1, the aggregate demand for liquidity will exceed the aggregate value of the private sector whenever $\rho_0 < \rho \leq \rho^*$. Intermediation cannot overcome the problem, because at best it can realize the net ex post value of the productive sector, which in this event is zero.

This creates a demand for government supplied liquidity. The government can meet the demand thanks to its assumed ability to commit future consumer endowments via taxation. Suppose the government issues one-period bonds in the amount $(\rho^*-\rho_0)$I at date 0, and sells them at par. We are then back to the environment analyzed in section 2, with government bonds providing a storage facility equivalent to cash. The productive optimum can be achieved by having investors invest $(1+\rho^*)$I - A in each firm at date 0, requiring firms to spend $(\rho^*-\rho_0)$I of this amount on the purchase of bonds.[17]

Since government bonds, issued at par, lead to the productive optimum, Lemma 1 tells us that the introduction of these bonds raises total output and expected aggregate investment (the date-0 investment plus the expected value of date-1 reinvestments). Note, however, that the initial level of investment I decreases as firms save more of their resources to cover date-1 liquidity needs. Thus, issuing government bonds crowds out productive investments at date 0, but increases (expected) reinvestments at date 1.

---

[17] Alternatively, an intermediary can offer each firm a credit line $(\rho^*-\rho_0)$I, backed by the purchase of an equal amount of government bonds. Firms can raise the missing ρ_0I by issuing new shares.

We record these results in:

Proposition 3. *In the presence of aggregate uncertainty:*

(i) The private sector is not self-sufficient.

(ii) Assuming that the net social cost of taxation is zero, the government can ensure the productive optimum by issuing a sufficient amount of bonds at the market rate of interest (here, zero) in which case there is no demand for intermediation.

(iii) The introduction of government bonds reduces date-0 investments and increases date-1 reinvestments (reduces liquidations). Expected output, aggregate investment and the value of firms all increase.

4.2 *The government's objective and liquidity premia.*

By restricting attention to government securities that yield the market rate of interest, section 4.1 made two implicit assumptions. First, that there is no deadweight loss of taxation.[18] Otherwise, the gain in productive efficiency brought about by the bond issue must be traded off against the change in the deadweight loss of taxation. Second, that the government maximizes the firms' profit subject to the consumers breaking even. If either assumption is violated, the government may want to issue securities at a price q above par ($q > 1$) per unit of expected return. We refer to $q-1$ as the liquidity premium. For small premia, there is still a demand for government securities, since the private sector uses them as complementary inputs into the production process.

As is well known, credit rationing models raise some difficult conceptual issues for a welfare analysis. A government that aims at maximizing total surplus (consumers' plus producers') would redistribute all endowments to entrepreneurs as one unit of net worth creates more than one unit of output. On the other hand, a government that only represents consumers might confiscate some or all of the entrepreneurs' initial wealth A. Because this conceptual issue is not specific to our model and lies beyond the limited scope of this paper, we will not

[18] Or, more generally, the marginal deadweight loss of taxation is the same at all dates, so the date-0 deadweight loss due to the income received by issuing government securities is equal to the date-1 (or date-2) deadweight loss associated with the reimbursement of the public debt.

investigate the effects of alternative assumptions on the government's objective function.

Instead, we simply assume that the government arranges for firms to bear the net cost of supplying liquidity, leaving consumers as well off as without government intervention (see, however, the up-coming remark on seignorage). The net tax effect incorporates the benefit of distributing the proceeds from the bond issue at date 0 (or reducing date-0 taxes) and the deadweight loss from taxing consumers to redeem the bonds at date 1. It is apparent that at the social optimum the marginal deadweight loss of taxation must be time increasing. Suppose that the marginal deadweight loss were higher at date 0 than at date 1, say. The government could then issue at date 0 additional one-period bonds at par (at zero rate of interest). The taxpayers would strictly gain; furthermore the firms would be made (at least weakly) better off by the increased supply of liquidity. We conclude that, for the optimal debt policy, $q \geq 1$.[19]

4.3 *The demand for government bonds.*

Let us for the moment rule out cross-shareholdings and intermediation, and run through the analysis of section 2.1 to see how firms respond to the presence of a liquidity premium $q-1 > 0$. For the moment, assume that a firm cannot partially liquidate its investment. (We will later discuss the case of partial liquidation at the firm level.) For a threshold $\hat{\rho}$, the investors' participation constraint becomes

$$F(\hat{\rho})\rho_0 I \geq I - A + \left(\int_0^{\hat{\rho}} \rho f(\rho) d\rho\right) I + (q-1)(\hat{\rho}-\rho_0) I \, , \tag{14}$$

where use is made of the fact that the maximum dilution at date 1 is ρ_0 I, so that $(\hat{\rho}-\rho_0)I$ in government securities is needed to reach the threshold $\hat{\rho}$.

The firm's net utility is now

$$U_f(\hat{\rho},q) \equiv m(\hat{\rho},q) k(\hat{\rho},q) A \tag{15}$$

where

[19] The government cannot gain by reimbursing public debt at date 2, because the deadweight loss of taxation is higher at date 2 than at date 1 by the same reasoning.

$$m(\hat{\rho},q) \equiv F(\hat{\rho})\rho_1 - 1 - \int_0^{\hat{\rho}} \rho f(\rho) d\rho - (q-1)(\hat{\rho}-\rho_0) \tag{16}$$

is the modified margin on per-unit-of-investment profit, and

$$k(\hat{\rho},q) \equiv \frac{1}{1+\int_0^{\hat{\rho}} \rho f(\rho) d\rho + (q-1)(\hat{\rho}-\rho_0) - F(\hat{\rho})\rho_0} \tag{17}$$

is the modified equity multiplier. The firm chooses the threshold $\rho^*(q)$ to minimize the expected unit cost of effective investment:

$$\rho^*(q) \text{ minimizes } \frac{1+\int_0^{\hat{\rho}} \rho f(\rho) d\rho + (q-1)(\hat{\rho}-\rho_0)}{F(\hat{\rho})} \tag{18}$$

over $\hat{\rho} \geq \rho_0$. As a generalization of condition (8) we get[20]

$$\rho_0 < \rho^*(q) + \frac{q-1}{f(\rho^*(q))} < \rho_1. \tag{19}$$

We next ask whether this allocation remains an equilibrium when firms are allowed to issue and trade claims in a financial market. First we argue that, if firms are constrained to issue a *single security* (equity), the equilibrium described above unravels because of free-riding. Suppose that all firms issue shares at date 0. Suppose a contrario that the opening of the financial market does not affect the equilibrium. Shares must then trade at par, else investors would not want to buy them. But then an individual firm could satisfy its liquidity needs more cheaply by buying shares rather than government bonds. To see this, note that the representative firm in the equilibrium without a financial market has an ex post value equal to $(\rho^*(q)-\rho)I > 0$

[20] The firm's optimal cut-off, $\rho^*(q)$, need not be monotone in q. The firm may respond to an increase in q by reducing its date-0 investment and raising the date-1 cut-off. On the other hand, if q is high enough, shutting down all demand for bonds, the firm's date-0 investment will be higher than if q were equal to 1 (see Proposition 3).

when the liquidity shock ρ falls in the interval $(\rho_0, \rho^*(q)]$. This value stems from the excess amount of bonds it holds. Hence, by holding enough shares in the other firms, an individual firm can accommodate shocks arbitrarily close to $\rho^*(q)$ without paying any liquidity premium. Thus, the equilibrium without cross-shareholdings must unravel once firms are allowed to buy each others' shares.

The intuition why the equilibrium breaks down is clear: *Liquidity is a public good if the government charges a premium.* Each firm therefore wants to free ride on firms that purchase government bonds. Of course, all firms cannot follow the free-rider strategy, since then no firm would buy government bonds and there would be no protection at all against liquidity shocks above ρ_0.

4.4 *An analysis of the private sector's optimal policy.*

This section derives the private sector's optimal policy when the government charges a premium. The next section will analyze the implementation of this policy, in particular the private sector's response to the possibility of free-riding. Suppose the private sector buys $(\hat{\rho} - \rho_0)I$ bonds (where I is the investment level of the representative firm) and allocates the funds efficiently at date 1. All firms can withstand the liquidity shock if $\rho \leq \hat{\rho}$, but if $\rho > \hat{\rho}$, some firms must shut down. Instead of shutting down all firms, however, the private sector can resort to *partial liquidation* at the industry level, allowing a fraction $(\hat{\rho} - \rho_0)/(\rho - \rho_0)$ of firms to continue (we can assume that the identity of firms that are allowed to continue is drawn randomly). Whether this is optimal depends on how high the liquidity shock is. If ρ is high enough, it may be better to return the bonds to the investors rather than use the funds for reinvestment.

To determine the private sector's optimal policy, let zI be the face value of the bonds that the private sector buys and let $\lambda(\rho, z)$ be the fraction of firms allowed to continue when the aggregate shock is ρ. Recalling that entrepreneurs get $p_H R_b = \rho_1 - \rho_0$ per unit of investment in case of continuation, an optimal strategy can be found by solving the following program:

$$
\max_{I,\lambda(\cdot)} \quad I\int_0^{\infty}\lambda(\rho,z)(\rho_1-\rho_0)f(\rho)d\rho,
$$

$$
\text{s.t. (i)} \quad I\int_0^{\infty}\lambda(\rho,z)\rho_0 f(\rho)d\rho - I\left[\int_0^{\infty}\lambda(\rho,z)\rho f(\rho)d\rho + (q-1)z\right] \geq I - A, \tag{20}
$$

$$
\text{(ii)} \quad 0 \leq \lambda(\rho,z) \leq \min\left\{1, \frac{z}{\rho-\rho_0}\right\}.
$$

Letting δ be the Lagrangian multiplier for the budget constraint, the optimal choice of λ is:

$$
\lambda(\rho,z) = \begin{cases} \min\left\{1, \dfrac{z}{\rho-\rho_0}\right\}, & \text{if } \rho \leq \bar{\rho} \equiv \dfrac{\rho_1+(\delta-1)\rho_0}{\delta} \\ 0 & , \text{if } \rho > \bar{\rho}. \end{cases} \tag{21}
$$

The choice of z is governed by the first-order condition:

$$
\int_{\rho_0+z}^{\bar{\rho}} \frac{\partial\lambda(\rho,z)}{\partial z}\left[(\rho_1-\rho_0) + \delta(\rho_0-\rho)\right]f(\rho)d\rho - \delta(q-1) = 0. \tag{22}
$$

Note that the lower integration bound is ρ_0+z due to (21). Also, by (21), the term in brackets in the integrand is non-negative. Therefore, (22) implies that $\partial\lambda(\delta,z)/\partial z > 0$ for some values of ρ, as δ and $(q-1) > 0$. In particular, $z < \bar{\rho} - \rho_0$, so that whenever $z + \rho_0 \equiv \hat{\rho} < \rho \leq \bar{\rho}$, we have $0 < \lambda(\rho,z) < 1$, implying that the private sector uses the option of partial liquidation for these ρ-values.

The reason the private sector resorts to partial liquidation should be clear. If the private sector bought enough bonds to allow all firms to continue whenever $\rho \leq \bar{\rho}$, the choice of λ would be unconstrained (λ would equal 0 or 1). The first-order cost of a reduction in z would then be zero, while the first-order benefit would be $q-1 > 0$; it would be better to reduce z. Partial liquidation allows the private sector to economize on the use of bonds. Only when bonds carry no opportunity cost ex post ($q = 1$) will the private sector buy full coverage against adverse liquidity shocks.

We summarize the characteristics of the optimal solution in

Proposition 4. *If $q > 1$, that is, if the net deadweight loss of taxation (which must be non-negative) is strictly positive, the private sector's optimal solution is characterized by two thresholds, $\hat{\rho}$ and $\bar{\rho}$, such that:*

(i) $\rho_0 < \hat{\rho} < \bar{\rho} < \rho_1$.

(ii) The private sector buys $zI = (\hat{\rho}-\rho_0)I > 0$ government bonds.

(iii) All firms continue if $\rho \leq \hat{\rho}$; the fraction $(\hat{\rho}-\rho_0)/(\rho-\rho_0) < 1$ of firms continues if $\hat{\rho} < \rho \leq \bar{\rho}$; no firms continue if $\rho \geq \bar{\rho}$.

This solution is more efficient, privately and socially, than when firms act independently.

The last inequality in part (i) follows because δ represents the marginal value of firm wealth and therefore must be greater than 1. The private sector's solution is better than the solution that individual firms could obtain (as described in section 4.2), because partial liquidation is feasible at the industry level, but not at the firm level. The outcome is also socially more efficient, because consumers are as well off whether firms act alone or collectively. In both cases all the social surplus goes to the entrepreneurs.[21] Note that had partial liquidation been feasible at the firm level, the analysis of section 4.3 would have coincided with that of this section, with the convention that $\lambda(\rho,z)$ would have denoted the fraction of the individual firm's investment that is not liquidated when the aggregate shock is ρ.

4.5 *Implementing the optimal policy. A rationale for multiple securities.*

We now turn to the implementation of the optimal policy derived in section 4.4. There are two distinct issues for implementation.

[21] It is worth noting that the established social value of the private optimum depends on the government's objective function. If the government, instead of letting all the surplus go to the entrepreneurs, chose to maximize its net revenue from selling bonds (that is, consumer surplus), it may well be that restrictions on private arrangements would lead to a higher social utility. The private sector's optimal policy just described has two effects. On one hand it makes bonds more valuable to firms, increasing their demand; on the other hand, it allows firms to bypass the government's mark-up, reducing the demand for bonds. This situation is reminiscent of a government trying to maximize seignorage in a monetary model. Seignorage may be higher if private liquidity creation by way of intermediation and cross-shareholdings is restricted.

First, the private sector's optimal policy involves partial liquidation. If partial liquidation is infeasible at the firm level, it must be performed at the industry level. Financial markets in general cannot implement the partial liquidation policy described in Proposition 4 iii), since the private sector as a whole would need to buy strictly more than $(\hat{\rho}-\rho_0)I$ government bonds. There is thus a need for intermediation. Note that the role of the intermediary is here to implement the partial liquidation policy while economizing on expensive government bonds; by contrast in the absence of aggregate liquidity shock, liquid claims commanded no premium and the role of intermediaries was to dispatch liquidity properly among heterogenous firms. When partial liquidation is feasible at the firm level, intermediation no longer plays a role[22] since the individual firm's program coincides with the private sector's. We conclude that *under a pure aggregate shock intermediation is necessary only when partial liquidation at the firm level is infeasible*.

Second, and more interestingly, whether intermediation is used or not, free-riding is an issue and in either case the solution is the issuing of multiple securities. The rationale for multiple securities is a clientele one. Free-riding may arise because firms that buy costly Treasury securities provide socially valuable liquidity to other firms that invest in them. The purchasers of Treasury securities however cannot internalize the value of this liquidity service if they also want to sell shares to consumers, who have no liquidity needs. Indeed shares must be sold at par to consumers, which precludes charging a liquidity premium to corporate purchasers of shares. Because the industry needs outside funding, we conclude that the optimum derived in section 4.4 cannot be achieved if firms issue a single security. In contrast, the use of multiple securities allows the issuer to price discriminate among investors who value liquidity services differently (this point is highly reminiscent of Gorton-Pennacchi (1990)).

To illustrate the rationale for multiple securities without needing to introduce intermediaries, let us allow *partial liquidation at the firm level*. Let us construct an industry equilibrium that yields the same allocation as that obtained from program (20) as described in Proposition 4. In this equilibrium, all firms invest I (the level given in Proposition 4). They all

[22] Of course, when idiosyncratic and aggregate shocks coexist, intermediation again is strictly preferred by firms as scarce liquidity is wasted by financial markets.

pledge not to finance liquidity shocks exceeding $\bar{\rho}$. A fraction α of the firms, "type-1 firms", purchase $(\bar{\rho}-\rho_0)I$ Treasury securities at price q and hold no shares in other firms. At date 0 they issue a) equity, sold at par, and b) short-term (maturing at date 1) debt with face value $(\bar{\rho}-\rho_0)I$ at price q', with the covenants that a) they must fully dilute their equity before selling Treasury securities to finance liquidity needs and b) that they can sell up to $(\rho-\rho_0)I$ Treasury securities to finance their liquidity need when the aggregate shock is ρ, before servicing the short-term debt.[23] Covenant a) implies that equity in type-1 firms carries no liquidity premium, as it becomes valueless when the aggregate shock exceeds ρ_0. Covenant b) implies that the short-term debt is worth $(\bar{\rho}-\rho)I$ at date 1 when the liquidity shock ρ belongs to the interval $[\rho_0,\bar{\rho}]$.

The remaining fraction 1-α of the firms, the "type-2 firms", do not purchase Treasury securities, but buy all the short-term debt of type-1 firms. Let α be given by

$$\alpha(\bar{\rho}-\hat{\rho})=(1-\alpha)(\hat{\rho}-\rho_0). \tag{23}$$

There is no liquidation in any type of firm when $\rho\leq\hat{\rho}$. Suppose that $\rho_0\leq\rho\leq\hat{\rho}$. Then the value of short-term debt claims held by a representative type-2 firm is equal to $\alpha(\bar{\rho}-\rho)I/(1-\alpha)\geq(\rho-\rho_0)I$, and thus type-2 firms can, by selling these debt claims and by diluting their initial investors (to obtain $\rho_0 I$), withstand their liquidity shock, ρI. On the other hand, type-2 firms are partially liquidated when $\rho\in[\hat{\rho},\bar{\rho}]$. while type-1 firms are not. Type-2 firms can continue to operate a fraction λ_2 of their assets, where

$$\lambda_2(\rho I)=\lambda_2(\rho_0 I)+\frac{\alpha}{1-\alpha}(\bar{\rho}-\rho)I.$$

That is, they can pledge $\lambda_2(\rho_0 I)$ to new investors and further use the return on short-term debt in type-1 firms. The aggregate (type-1 and type-2 firms) fraction λ of assets that are not

[23] The debt contract is contingent on the realization of the aggregate shock for expositional convenience only. Alternatively, type-1 firms can substitute a class of nondilutable long-term (maturing at date 2) senior claim for the short-term debt and dispense with writing a covenant that is contingent on the aggregate shock.

liquidated is therefore

$$\lambda = \alpha+(1-\alpha)\lambda_2 = \alpha+(1-\alpha)\left[\frac{\alpha}{1-\alpha}\frac{\bar{\rho}-\rho_0}{\rho-\rho_0}\right]$$

$$= \frac{\hat{\rho}-\rho_0}{\rho-\rho_0}$$

and coincides with the fraction specified in Proposition 4. Note also that the aggregate demand for Treasury securities is the same as in the intermediated case from Proposition 4ii), as (23) implies that

$$\alpha(\bar{\rho}-\rho_0)\,I = (\hat{\rho}-\rho_0)\,I.$$

Last, the price of short-term senior debt, q', is set so that firms are indifferent between being a type-1 and a type-2 firm. We thus conclude that the optimal policy is implemented by the issuance (issuing ?) of multiple corporate securities when partial liquidation at the firm level is feasible. When partial liquidation at the firm level is infeasible, the optimal policy must be implemented through intermediaries, and these intermediaries must similarly issue multiple securities in order to price discriminate and avoid free-riding.

4.6 *State-contingent bonds.*

While partial liquidation reduces the deadweight loss of taxation and is efficient for the private sector given that the government issues non-contingent bonds, it is not the socially most efficient solution. In this section we show how the government can achieve a better outcome by issuing state-contingent bonds. We begin by characterizing the social optimum when general state-contingent transfers between consumers and producers are allowed.

A social plan is defined by (i) the initial level of investment I per firm, and (ii) a state-contingent continuation policy $\lambda(\rho) \in [0,1]$ that determines the fraction of firms that continue when the aggregate liquidity shock is ρ.

The social program is identical to the program studied in section 4.4, with the exception that the transfers are not constrained by an initial purchase of bonds. Formally, the social program solves:

$$\max_{I,\lambda(\cdot)} \quad I\int_0^\infty \lambda(\rho)(\rho_1-\rho_0)f(\rho)d\rho,$$

$$\text{s.t.} \quad \text{(i)} \quad Iq\int_0^\infty \lambda(\rho)(\rho_0-\rho)f(\rho)d\rho \geq I-A, \tag{24}$$

$$\text{(ii)} \quad 0 \leq \lambda(\rho) \leq 1.$$

Letting δ be the Lagrangian multiplier (again, $\delta > 1$, because it is the shadow value of entrepreneurial wealth), the solution to this program is

$$\lambda^*(\rho) = \begin{cases} 1 & \text{if } \rho \leq \rho^\#, \\ 0 & \text{if } \rho > \rho^\#, \end{cases}$$

where $\rho^\#$ is defined by

$$\rho^\# - \rho_0 = \frac{\rho_1 - \rho_0}{\delta q}. \tag{25}$$

In contrast to the solution in section 4.4, the social optimum specifies either that all firms continue, or no firm continues. The explanation for the difference is that the social program does not have to hoard liquidity (by buying bonds); rather, transfers are provided only as needed.[24] Furthermore, since δ, $q > 1$, we see that the socially optimal cut-off $\rho^\#$ satisfies $\rho_0 < \rho^\# < \rho_1$.

The social optimum $\lambda^*(\rho)$ can be implemented using state-contingent bonds, which pay the holder an amount $x(\rho)$ as a function of the aggregate shock ρ. In particular, let the bonds have the following date-1 payoff:

$$x(\rho) = \begin{cases} 0, & \text{if } \rho \leq \rho_0 \text{ or } \rho > \rho^\#, \\ \lambda^*(\rho)(\rho-\rho_0), & \text{if } \rho_0 < \rho \leq \rho^\#. \end{cases}$$

Let the date-0 price of these bonds be set so that

$$\int_{\rho_0}^\infty \lambda^*(\rho)q(\rho-\rho_0)f(\rho)d\rho.$$

[24] Partial liquidation would become relevant only if investors or entrepreneurs were risk averse or if the government could not use state-contingent transfers.

Note that priced this way, bond revenues exactly compensate the consumers for the deadweight loss of taxation when the policy $\lambda^*(\rho)$ is followed.

It is straightforward to check that with these payoffs and price, firms can replicate the social optimum, if they so desire, by buying I bonds, where I is the socially optimal level of investment. This amount of bonds enables a firm to continue exactly when the social plan calls for continuation. Also, the firm's budget constraint coincides with the social budget constraint so that the firm can attract the required funds from investors at date 0.

Can the firm do better by deviating from the socially optimal plan? Suppose there is a better plan for the firm. In this plan, let $\tilde{I}$ be the date-0 investment level, let $\tilde{z}\tilde{I}$ be the amount of state-contingent bonds the firm purchases, and let $\tilde{\lambda}(\rho)$ be the continuation policy. To be feasible, the plan must satisfy both an ex ante and an ex post budget constraint. The ex ante constraint is

$$\tilde{I} \int_0^\infty \tilde{\lambda}(\rho)(\rho_0-\rho)f(\rho)d\rho \; - \; \tilde{I}\,\tilde{z} \int_0^\infty \lambda^*(\rho)(q-1)(\rho-\rho_0)f(\rho)d\rho \; \geq \; \tilde{I}-A. \tag{26}$$

The ex post constraint is

$$0 \leq \tilde{\lambda}(\rho) \leq \min\{1,\tilde{z}\,\lambda^*(\rho)\}. \tag{27}$$

Substituting the ex post constraint into the ex ante constraint, it is immediate that it would be feasible for a social planner to replicate the firm's plan. Given that the social objective function coincides with the firm's, the firm's plan cannot be better than the social optimum. Hence, the firm prefers the socially optimal plan.

Note that the government need not be concerned by the possibility of free-riding and partial liquidation if bond yields are state contingent, because firms holding bonds do not have excess liquidity when $\rho > \rho_0$, and therefore other firms cannot free-ride by holding their shares.

Remark on seignorage. Suppose that the government aims at maximizing net revenue from issuing government securities, ignoring costs of taxation. Let us compute an upper bound R on the government's net revenue. Let $\bar{U}(=m(\rho_0)k(\rho_0)A)$ denote the "autarky utility" of an entrepreneur with assets A, that is the utility she gets when holding no reserves. Clearly, $\bar{U}$ is a lower bound on entrepreneurial utility. Consider the following program:

$$\begin{aligned}
&\max_{I,\lambda(\cdot)} \quad R \\
&\text{s.t.} \quad \text{(i)} \quad I\int_0^{\infty}\lambda(\rho)(\rho_1-\rho_0)f(\rho)d\rho \;\geq\; \bar{U}, \\
&\qquad\quad \text{(ii)} \quad -R+I\int_0^{\infty}\lambda(\rho)(\rho_0-\rho)f(\rho)d\rho \;\geq I-A, \\
&\qquad\quad \text{(iii)} \quad 0 \leq \lambda(\rho) \leq 1.
\end{aligned} \tag{28}$$

Constraint (ii) reflects the fact that investors must break even. Substituting constraint (ii) (which obviously holds with equality) into the objective function, we see that program (28) is the dual of program (24). That is, the social optimum has the same characterization whether the government maximizes entrepreneurial welfare subject to a deadweight loss of taxation or maximizes seignorage.

4.7 *An interpretation of state-contingent bonds.*

The economic rationale for using state-contingent bonds is clear. Bonds that pay a fixed amount will provide excess liquidity in most states at date 1. Firms (or intermediaries) will cash in the bonds that they do not need for reinvestment, forcing the government to tax consumers and implement a wealth transfer that is unnecessary and socially costly (in the deadweight-loss-of-taxation interpretation). While partial liquidation reduces the tax burden, it still leaves slack and also distorts investment and liquidation decisions. The solution that attacks the root cause of the problem is a state-contingent bond, which dispenses liquidity exactly as needed and eliminates all wasteful transfers of wealth.

Of course, this solution works ideally only in our idealized model. In reality, state-contingent bonds are not used. The most obvious reason why such bonds are not used is that there is no aggregate, measurable state that unequivocally identifies times when firms should be provided more liquidity. There are other shocks than liquidity shocks that move publicly measured variables, and these shocks may call for very different transfers for similar realizations of the aggregate measures. Macroeconomic policy is not a mechanical exercise, because new, unforeseen contingencies keep appearing. Rather than limiting government policy to bonds that are contingent on a few, foreseeable, and verifiable variables, a discretionary policy may be more effective (at least if one ignores the political economy of government policy). Thus, we want to view the use of state-contingent bonds as a metaphor for an active government policy rather than as a serious policy instrument in its own right. The value of state-contingent bonds shows that, besides *creating* liquidity, the government has a potential role in actively *managing* liquidity. Note that this is true even if private contracts could be indexed on government actions or on the aggregate shock.

In our model, *optimal liquidity management entails increasing the value of government bonds in times when liquidity needs are high.*[25] Since we have assumed that bonds mature at date 1, this is accompanied by a corresponding, immediate increase (decrease) in taxes. It may seem odd that taxes are being raised in adverse liquidity states (recessions), as consumers might simultaneously be experiencing negative demand shocks. But this just underscores the importance of multiple shocks, as discussed above. Our specific result is clearly sensitive to the assumption that there is a single liquidity shock with consumers experiencing no shocks at all. If consumers experience an adverse demand shock that requires resources to be shifted to consumption, then taxation and the value of bonds should be reduced rather than increased, notwithstanding negative liquidity shocks. This does not negate the general logic that state-contingent bonds are of value, that is, an active policy can improve the allocation of resources through wealth transfers.

We cannot associate state-contingent bonds with monetary or fiscal policies, since our model does not distinguish between nominal and real claims. But certain analogies can be drawn between the real effects of state-contingent bonds and the alleged effects of fiscal and monetary policies. Note first that we only have a single instrument, corresponding to a case where fiscal and monetary policies are inversely related: when liquidity is tight, fiscal policy is loose and conversely. Some might argue that this is also true in reality, since loose money will either be accompanied by a tight budget, forcing consumers to pay up immediately (as they do in our

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[25] Note that we are not rationalizing continuous government intervention, but rather intervention in severe circumstances.

model), or it imposes an inflation tax, which consumers might anticipate and therefore adjust to immediately, with the same effect as tightening the budget (barring non-linear taxes). A further parallel is that, in the model, liquidity is tight when the value of bonds are low.

Note also that, if one imagined a second generation of entrepreneurs entering at date 1, facing liquidity shocks at date 2, and producing at date 3, then tighter liquidity (lower bond prices) at date 1 would *decrease* their date 1 investments, and increase their date-2 reinvestments (by Proposition 3), as the cost of carrying liquidity into date 2 would become relatively less expensive. In other words, tighter liquidity at date 1 reduces both reinvestments (of date-0 entrepreneurs) and initial investments (of date-1 entrepreneurs), moving aggregate investment unambigously down at date 1. Again, this conforms with standard views on policy effects. Furthermore, in a model with more periods, one could also study bonds with longer maturities. With long-term bonds, wealth transfers between consumers and producers would be accomplished by early redemptions (purchases) of these bonds (adding liquidity) or by issuing new bonds (tightening liquidity).

We do not suggest that these parallels can be used to draw strong conclusions about real policies as our model says nothing specifically about monetary or fiscal interventions. Nevertheless, we find it reassuring that one can associate alleged features of real policies with the abstract state-contingent outcomes in the model. Also, the model provides examples of government interventions intended to accomplish wealth transfers between the producers and consumers in response to unforeseen, aggregate real shocks.

As an illustration, consider the recent collapse of the Soviet Union. In several European countries no attempt was initially made to accommodate this shock (or, if one wants to interpret the central banks' stubborn stance on exchange rates as an active policy, money was made very tight). The result was a massive reduction in entrepreneurial wealth as interest rates soared and asset values plunged. Our analysis supports those who argued that the governments should have responded with policy measures that transferred wealth back to entrepreneurs, by bringing interest rates down. According to received wisdom (and validated by the subsequent events), a reduction in interest rates required a devaluation of the currencies or abandoning fixed exchange rates. In the logic of our model, and we believe in reality, too, such an exchange rate policy should be seen as part of an insurance policy in which the government plays a central role.

5. Relationship to the literature

Several papers have relevance for our work, although they do not address in a unified way the main questions we are interested in, namely corporate liquidity needs, the partial self-sufficiency of the private sector, and the foundations for liquidity supply and management by the government. Let us take up these themes in order.

Corporate liquidity needs

By now, there is a very large literature on credit rationing, making the point that investment will be constrained by the firm's net worth (for a survey, see for instance, Bernanke-Gertler-Gilchrist (1994)). Most of this literature does not deal with liquidity demand as such, but rather with the structure of optimal financial contracts, including the decision to liquidate assets in case of financial hardship. Obviously, decisions to liquidate are closely related to decisions to make further investments, as illustrated in Hart (1995). However, the result that firms face a liquidity covenant in the optimal financial contract is to the best of our knowledge new. On the other hand, the result that investors must provide firms with extra cash (or a credit line) ex ante, is reminiscent of Hart and Moore (1989) (see also Hart (1995)), who show that a given class of financial contracts can be improved upon by using a similar arrangement (the optimality of which is not investigated).

The fact that real assets serve a dual purpose, as productive inputs as well as collateral for debt, has been exploited in recent papers by Kiyotaki and Moore (1993) and Shleifer and Vishny (1992). Both models are broadly concerned with the general equilibrium implications of wealth constrained financing. The latter model, in particular, shares some features with our model. It has two firms, each facing an idiosyncratic demand shock. Firms may try to piggy-back on each others' investment decisions to generate future income in hard times, not directly through security purchases, as we have it, but rather through the ability to sell assets to the other firm ex post. If the firms' liquidity shocks are not perfectly correlated (so that they are not in default simultaneously), then when one firm experiences a liquidity shock but not the other, it can sell its assets for a higher value than if there were no other informed purchaser of its assets. In effect, the model describes a market mediated, albeit imperfect, way of enhancing firm liquidity.

Self-sufficiency and intermediation

The study of the demand for consumer liquidity was originated by Bryant (1980) and Diamond and Dybvig (1983).[26] Diamond and Dybvig demonstrate that intermediaries can provide limited insurance by subsidizing consumers with high liquidity needs. Jacklin (1987), however shows that the associated cross-subsidy is infeasible in the presence of financial markets, as those consumers with low liquidity needs can pretend to have high liquidity needs, pocket the subsidy and reinvest it in financial markets.[27] Thus, financial markets in the Diamond-Dybvig framework both reduce welfare and make intermediation useless. By contrast, in our model, intermediaries bring strict improvements despite the existence of financial markets. The main reason for the difference in conclusions is that our firms cannot misuse credit lines. Entrepreneurs cannot divert corporate funds for their own use other than in the limited fashion captured by B.

Our results on how the government may manage liquidity obtained in section 4, are also quite different from those in Diamond and Dybvig. In their model, government policy can be made contingent on the macroeconomic shock (the number of withdrawing depositors) while bank contracts with depositors cannot be made contingent on this macroeconomic shock.[28] In our model, government policies do not use more aggregate information than private contracts.

Boyd and Prescott (1986), in a related effort to understand the role of intermediation, constructs a model in which financial markets cannot duplicate the services provided by intermediaries. Their model is not about the provision of liquidity as such, but rather about the efficent organization of monitoring. Financial markets are inefficient relative to intermediation, because financial contracts are assumed to be contingent on whether projects are evaluated and

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[26] See also Ramakrishnan and Thakor (1984) and Williamson (1986).

[27] Diamond (1995) shows that intermediaries have a role to play as long as some consumers have imperfect access to financial markets.

[28] The subsequent literature on bank runs for the most part assumes independent shocks, as we do in section 3. Hellwig (1994) introduces macroeconomic uncertainty through a date-1 shock on the the short term interest rate between date 1 and date 2, and shows that depositors who withdraw at date 1 must bear some of the aggregate risk.

also on the outcome of the evaluation. Intermediaries, on the other hand, can offer contracts before an evaluation is made, avoiding the problem of premature information revelation. This seems to be an ad hoc limitation in favor of intermediation.

Bhattacharya and Gale (1987) look at a variation of the Diamond-Dybvig model that is closer in structure to ours. In their model there are several banks (corresponding to our firms), which experience idiosyncratic, "sectoral" liquidity shocks that cancel out in the aggregate. The paper characterizes the socially optimal mechanism for sharing risks across banks, noting that this mechanism cannot be implemented through an interbank lending market. Whether a market in bank shares or some other institution could implement the social optimum is left open. The authors suggest that a central bank might be the right institution for carrying out interbank risk sharing.

Government supplied liquidity

The first paper to argue that government debt can improve the intertemporal allocation of investment is by Diamond (1965). In an overlapping generations model he showed that government debt can crowd out private investment, which in the absence of government debt may be excessive. In his model, government bonds provide no liquidity services and are sold at par. Another difference is that in Diamond public debt crowds out private investment while in our paper public debt boosts private investment by providing aggregate liquidity. Models of crossgenerational insurance (Fisher (1983), Pagano (1988) and Gale (1990)), and of asynchronized investment opportunities (Woodford (1990)) are expressely concerned with the role of government debt as a medium of transfer across periods. Woodford's paper is most closely related to ours. In one version of his model, he assumes that individuals periodically receive investment opportunities, which they cannot take advantage of without storing wealth from one period to the next. Government debt is a vehicle for storing wealth. He shows that in a stationary state, the optimal level of government debt should be positive. The role of government debt is thus the same as in our paper. However, Woodford does not consider the possibility that a private market in claims on output could accomplish the same as government debt. It appears that it could, because there are no agency costs that would limit the sales of such private claims.

In a related vein, Williamson (1992) demonstrates the benefits of fiat money in a model in which adverse selection prevents entrepreneurs from pledging their entire future income.[29] Fiat money plays the role of a store of value (which is the counterpart of government securities in our model, and may have value because Williamson's model is one of overlapping generations). Williamson's focus is distinct from ours; he argues in favor of government securities (in the form of fiat money) and against the trading of private claims. Indeed, Williamson concludes that fiat money has no value if private claims are traded, and that there exists an equilibrium with fiat money that Pareto dominates the purely private equilibrium as long as agents are prevented from trading private claims. Our conclusions are rather different.

Finally, it should be evident that our work has some points of contact with the literature on incomplete markets. Markets in our model are incomplete because individuals cannot borrow against future income and because the value of a firm's claimholdings cannot be contingent on its liquidity shock. The latter problem is effectively dealt with through intermediaries even though the intermediaries do not observe individual liquidity shocks, while the former problem is partly resolved by the government, which creates a new asset that can be used to improve the allocation of investment across periods. The fact that the government can improve on private outcomes by issuing new securities is related to a general result in the incomplete market literature, which states that generically one can add an asset to an incomplete market and make everyone better off (Elul (1995)). Of course, unlike this literature, the source of incompleteness is spelled out in our model.

6. Concluding remarks.

Rather than summarize our results, we wish to address some questions that our paper may raise and indicate directions in which the analysis could be extended.[30] First, the

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[29] On the adverse selection problem and related issues, see also Banerjee-Maskin (1991) and the references in Williamson (1992). For other models on money and government debt, see, for instance, Bewley (1980) and Levine (1991).

[30] We are grateful to Peter Diamond, Peter Howitt and Andrei Shleifer for raising many of these questions.

questions.

What do we mean by liquidity? Liquidity has been given various interpretations in the literature. In models with adverse selection, liquidity is related to the degree of asymmetric information about an asset. Assets about which there is a great deal of asymmetric information are less liquid than assets which everyone knows equally well. Firms can address this illiquidity problem in various ways. For instance, firms can issue claims that place different informational demands on buyers. This is illustrated in the paper by Gorton and Pennacchi (1990), where debt is shown to be informationally less demanding than equity and therefore a firm may be able to raise funds at a lower cost of capital by issuing both.[31]

In our model of moral hazard, assets are not illiquid because people have different assessments about their value. Rather, claims on firms fall into two categories: insider claims, held by entrepreneurs, and outsider claims, held by investors. Insider claims are completely illiquid in the sense that if the entrepreneur were to sell these claims in the market, the value of them (as well as of the external claims) would drop. External claims, on the other hand, are entirely liquid and can be sold at no discount at all times. Because we have assumed that the value of insider claims exceeds the value of the firm's investment (and therefore, insiders must chip in some of their initial endowment to raise funds), there is a potential shortage of liquid assets.

Why do we need an agency model? This question is largely answered by the previous paragraphs. Without an agency problem, all assets would be fully liquid. Firms could sell claims up to the value of their investments at any point in time and could therefore undertake all socially productive investments. It is the inability to transfer all the social surplus to external investors that constrains the firm's financing opportunities. As we have shown, the firm is credit constrained both ex ante and ex post. Therefore, ex post financing is not forthcoming without

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[31] In a related vein, Stein (1995) presents an adverse selection model in which the government can influence the liquidity of bank liabilities by guaranteeing some of the bank debt through deposit insurance. In his model, liquidity coincides with reserves. His paper does not address why the government would like to supply liquidity, nor why it would want to influence liquidity by changing reserve requirements. But his paper, unlike ours, gives one interpretation of the transmission mechanism of monetary policy.

ex ante commitments. Such commitments require assets that allow the firm to hoard liquidity (and pay for it in advance). This is what creates a demand for liquid assets.

What if there were other real private assets in the model? The assumption that there are no other private assets than the investment projects themselves was a simplification. It would be easy to introduce assets like real estate, which could be used to transfer wealth from one period to the next. If there were enough of those assets, they would obviate the need for government securities, but if the aggregate value of the assets fell below the minimum amount of government securities needed to achieve a productive optimum ($(\rho^* - \rho_0)I$), real assets would merely reduce the demand for government bonds, but they would not eliminate it.[32] Similarily, if the liquidation value of firms were positive rather than zero, this would reduce the demand for government bonds.

Is the investment-capital ratio realistic? The model implies that government intervention is needed only when the aggregate demand for liquidity exceeds the value of the private sector. This seems grossly at odds with the empirical fact that the market value of the corporate sector is large relative to investment. Part of the explanation has to do with the interpretation of investment. As we already pointed out, in our model reinvestments include all the cash outlays (wages and working capital, for instance) needed to continue the firm, not just investments in physical capital. This figure is surely an order of magnitude larger than measured aggregate investment.

Nevertheless, it remains true that the private sector can continue on its own as long as $\rho \leq \rho_0$, that is, as long as the value of the initial claimholders' investment has not been driven down to zero. Only when their investment has lost all its value need the government step in. This is an unrealistic feature of the model. It is driven by the equally unrealistic feature that external claims are fully liquid. To the extent that new issues of equity and debt are costly, because of difficulties in valuing these claims correctly, the private sector will run into reinvestment problems before the initial claimholders have lost all of their investment. Similarly,

[32] Real estate would in general carry a liquidity premium for the same reason government bonds do in our model. We derive liquidity premia in a simplified setting in Holmström-Tirole (1996). It should also be noted that real estate prices are likely to move procyclically, making it a worse instrument than government provided insurance.

the holdings of monitors (large shareholder, bank, venture capitalist,...) cannot be diluted without generating further incentive problems.[33] Thus, we think of ρ_0 as the point where the private sector hits a serious debt capacity constraint.

Let us finally suggest some *extensions* of our model. Currently, the link between the producer and the consumer sector is very simple and uninteresting. It would be highly desirable to study a general equilibrium extension that could highlight how consumer and producer shocks would affect the overall economy and feed back on each sector. This might give a more realistic picture of the need for transferring wealth in one direction or the other and therefore on how the government should manage liquidity. In this context, one would also want to be more careful about the deadweight losses caused by taxation and the role played by individual time preferences and risk aversion.

Another important extension would introduce more periods into the model, perhaps by overlapping firms as earlier indicated. In such an extension, the government could issue long-term as well as short-term bonds, which would raise questions about managing the maturity structure of government debt.

While intermediaries play a role in our model, it is a rather passive one. They are merely conduits for forming contracts that are unavailable in a financial market in which anonymous claims on corporate income are traded. In reality, intermediaries play a more active role, monitoring investments and liquidity needs. This role would probably emerge in a multi-period analysis, since one would have to worry about how firms use their excess liquidity. In the present model, excess liquidity was never misused by firms, since they had no use for the extra funds.

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[33] It is straightforward to add monitors into our model, along the lines of Holmström-Tirole (1994).

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