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ELEPHANTS

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ABSTRACT

Existing models of open-access resources are applicable to non-storable resources, such as fish. Many open-access resources, however, are used to produce storable goods. Elephants, rhinos, and tigers are three prominent examples. Anticipated future scarcity of these resources will increase current prices, and current poaching. This implies that, for given initial conditions, there may be rational expectations equilibria leading both to extinction and to survival. Governments may be able to eliminate extinction equilibria by promising to implement tough anti-poaching measures if the population falls below a threshold. Alternatively, they, or private agents, may be able to eliminate extinction equilibria by accumulating a sufficient stockpile of the storable good.

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I. Introduction

Twenty-nine percent of threatened birds worldwide and more than half the threatened mammals in Australasia and the Americas are subject to over-harvesting [Goombridge, 1992]. Most models of open-access resources assume that the good is non-storable [Clark, 1976; Gordon, 1954; Schaefer, 1957]. While this may be a reasonable assumption for fish, it is inappropriate for many other species threatened by over-harvesting, as illustrated in Table I. Although 30% of threatened mammals are hunted for presumably non-storable meat, 20% are hunted for fur or hides, which are presumably storable, and approximately 10% are threatened by the live trade [Goombridge, 1992].

African elephants are a prime example of a resource which is technologically difficult to protect as private property, and is used to produce a storable good. From 1981 to 1989, Africa's elephant population fell from approximately 1.2 million to just over 600,000 [Barbier, et al., 1990]. Dealers in Hong Kong stockpiled large amounts of ivory [*New York Times Magazine*, 1990]. As the elephant population decreased, the constant-dollar price of uncarved elephant tusks rose from $7.00 a pound in 1969 to $52.00 per pound in 1978, and $66.00 a pound in 1989 [Simmons and Freuteo., 1989]. The higher prices increased incentives for poaching.

Recently, governments have toughened enforcement efforts with a ban on the ivory trade, shooting of poachers on sight, strengthened measures against corruption, and the highly publicized destruction of confiscated ivory.[4] This crackdown on poaching has been accompanied by decreases in the price of elephant tusks [Bonner, 1993]. Since these policy changes reduce short-run ivory supply as well as demand, it is not clear that the fall in price

[4] In September 1988, Kenya's president ordered that poachers be shot on sight, and in April 1989 Richard Leakey took over Kenya's wildlife department.

would have been predicted under a static model, and indeed most economists did not predict this decline. However, the fall in price is consistent with the dynamic model set forth in this paper, under which improved anti-poaching enforcement may increase long-run ivory supply by allowing the elephant population to recover.

Under the model, anticipated future scarcity of storable resources leads to higher current prices, and therefore to more intensive current exploitation. For example, elephant poaching leads to expected future shortages of ivory, and thus raises future ivory prices. Since ivory is a storable good, current ivory prices therefore rise, and this creates incentives for more poaching today. Because poaching creates its own incentives, there may be multiple rational expectations paths of ivory prices and the elephant population for a range of initial populations.

In order to gain intuition for why there may be multiple rational expectations equilibria, it is useful to consider the following two period example, for which we thank Marty Weitzman. Suppose that each year there is a breeding season during which population grows by an amount $B(x)$ given an initial population of x. Following the breeding season, an amount h is harvested. Denote the elephant population at the beginning of the harvest season in year one as x_0. Then the population at the end of the harvest in year one will be $x_0 - h_1$, and the population at the end of the harvest in year two will be $x_0 - h_1 + B(x_0 - h_1) - h_2$. To keep the model as simple as possible, we assume that the world ends after two years.

Figure II

Time Line for Two Period Example

Time	Population
Initial (year 1)	x_0
After harvest, h_1, in year 1	$x_0 - h_1$
After breeding in year 2	$x_0 - h_1 + B(x_0 - h_1)$
After harvest, h_2, in year 2 (end of world)	$x_0 - h_1 + B(x_0 - h_1) - h_2$

Let c denote the cost of harvesting an animal, and denote the amount of the good demanded at a price of p as $D(p)$. Assume $D' < 0$ and $D(\infty) = 0$. The interest rate, which is assumed to be the only cost of storage, is denoted r.

There will be an equilibrium in which the animal is hunted to extinction in year 1 if the initial population is less than enough to satisfy demand during the first year at a price of c, plus demand during the second year at a price of $(1+r)c$. Algebraically, this can be written as: $x_0 < D(c) + D((1+r)c)$.

There will be an equilibrium in which the animal survives if the initial population, minus the amount required to satisfy first-year demand at price c, plus the births in the breeding season, is more than enough to satisfy second period demand at price c. This will be the case if $x_0 - D(c) + B(x_0 - D(c)) > D(c)$.

If both conditions hold, then there will be two equilibria. In one, the animal survives. In the other, the price is high enough that the population is eliminated in the first period, and the breeding that would have satisfied second-period demand never takes place. There will be multiple equilibria if the initial population is such that

$$D((1+r)c) + D(c) > x_0 > 2D(c) - B(x_0 - D(c)).$$

Note that as the interest rate increases, there will be an extinction equilibrium for a diminishing range of initial population levels. For sufficiently high interest rates, there will only be a single equilibrium path of population for any initial stock, just as in non-storable fisheries models.

Note that the example above implicitly assumes that the good is not destroyed when it is consumed. It thus applies to goods such as rhino horn, which is consumed in traditional Asian medicines. We will call such goods *storable* and distinguish them from *durable* goods, which are not used up when they are consumed. (Ivory is often considered an example of a durable good.) In an earlier version of this paper, we showed that there could be multiple equilibria in a two-period model of durable goods. This paper models storable, but we believe that except where noted, the results would be qualitatively similar for durable goods.

In the remainder of the paper we use a continuous time, infinite-horizon model, which allows us to solve for steady-state population and prices, and to examine cases in which extinction is not immediate following a shift in expectations, or the path of population and prices is stochastic.

The model may be relevant for policy. It suggests that even if the population level is steady, so that standard models would predict the continued survival of the species, the species could still be vulnerable to a switch to an extinction equilibrium. One way to eliminate the extinction equilibrium would be to increase the population of the animal by providing additional habitat. This is, however, likely to be expensive.

If governments have credibility, they may be able to eliminate the extinction equilibrium, and coordinate on the high population equilibrium, merely by promising to implement tough anti-poaching measures if the population falls below a threshold. This suggests a theoretical possibility that laws which provide little protection to non-endangered species, and practically unlimited protection to endangered species may be justified in some cases.

Finally governments or conservation organizations may be able to eliminate the extinction equilibria by building sufficient stockpiles of the storable good, and threatening to sell the stockpile if the animal becomes endangered or the price rises beyond a threshold. This is somewhat analogous to central banks using foreign exchange reserves to defend an exchange rate (see, for example, [Obstfeld 1986; 1994]). Stockpiles could be built either by deliberately harvesting animals, or by storing confiscated contraband taken from poachers, rather than either destroying or selling it.

A number of other papers find multiple equilibria in models of open-access resources with small numbers of players [Lancaster, 1971; Haurie and Pohjola, 1987; Levhari and Mirman, 1980; Reinganum and Stokey, 1984; and Benhabib and Radner, 1992]. In these models, each player prefers to grab resources immediately if others are going to do so, but to leave resources in place, where they will grow more quickly, if others will not consume them immediately. Tornell and Velasco [1992] introduce the possibility of storage into this type of model.

The effects examined in the previous models are unlikely to lead to multiple equilibria if there are many potential poachers, each of whom assumes that his or her actions have only an infinitesimal effect on future resource stocks, and on the actions chosen by other players. This paper argues there may nonetheless be multiple equilibria for open-access renewable resources used in the production of storable goods, because if others poach, the animal will become scarce, and this will increase the price of the good, making poaching more attractive.

Because poaching transforms an open-access renewable resource into a private exhaustible resource, this paper can be seen as helping unify the Gordon-Schaefer analysis of open-access renewable resources with Hotelling's [1931] analysis of optimal extraction of private non-renewable resources.

The remainder of the paper is organized as follows. Section II presents the standard Gordon-Schaefer fisheries model, in which storage is impossible. Section III shows how the model can be adapted to allow for storage, and classifies the possible equilibria. Section IV discusses equilibria in which people believe there is some probability that the economy will coordinate on extinction and some probability the economy will coordinate on survival. Section V concludes with a discussion of policy implications.

II. The Standard Gordon-Schaefer Model With No Storage

In the standard Gordon-Schaefer model, as set forth by Clark [1976],

$$\frac{dx}{dt} = B(x) - h, \qquad \text{II.1}$$

where x denotes the population, h is the harvesting rate, and $B(\cdot)$, the net-births function, is the rate of population increase in the absence of harvesting.[5] $B(0) = 0$, since if the population is extinct, no more animals can be born. We will measure the population in units of the habitat's carrying capacity, so $B(1) = 0$, and $B(x)$ is strictly negative for $x > 1$. B is strictly positive if population is positive and less than 1. This implies that, without harvesting, the unique stable steady state for the population is 1.

The rate of harvest will depend on the demand and the marginal cost faced by

[5]This is often taken to be the logistic function $B(x) = x(1 - x)$.

poachers. The marginal cost of poaching, c, is a decreasing function of the population x, so that $c = c(x)$, with $c'(x)<0$. We assume that $c'(x)$ is bounded and that there is a maximum poaching marginal cost of c_m, so that $c(0) = c_m$.

Given price, p, consumer demand is $D(p)$, where D is continuous, decreasing in p, and zero at and above a maximum price p_m. We will restrict ourselves to the case in which $p_m > c_m$, so that some poaching will be profitable, no matter how small the population. This condition is necessary for extinction to be a stable steady-state.

Since the good is open-access, and storage is assumed to be impossible, its price must be equal to the marginal poaching cost. Algebraically, $p = c(x)$. The harvest must be exactly equal to consumer demand, so $h = D(c(x))$. The evolution of the system in which storage is impossible is thus described by:

$$\frac{dx}{dt} = B(x) - D(c(x)) \equiv F(x) \qquad \text{II.2}$$

We assume that B, D, and c are differentiable. Since $B(0) = 0$, and $p_m > c_m$, $D(c(0)) > 0$, so that $F(0) < 0$, as illustrated in Figure II.1. Thus, zero is a stable steady state of II.2. $F(1) < 0$ since $B(1) = 0$, and $D(c(1)) > 0$. We will consider the case in which F is positive at some point in (0,1), so that extinction is not inevitable. Assuming that F is single-peaked[6], there will generically be points X_S and X_U so that F is negative and increasing on $(0, X_U)$, positive on (X_U, X_S), and negative and decreasing on $(X_S, 1]$. Hence, if population is between 0 and X_U, it will become extinct, whereas if it starts above X_U, it will tend to the high steady state, X_S. Thus, if storage is impossible, there will be multiple steady states, but a unique equilibrium given initial population.

[6] For most of the sequel, we don't strictly need F to be single peaked, but this requirement simplifies the analysis and the notation, and is not too restrictive.

III. Equilibria with Storage

This section introduces the possibility of storage into a Gordon-Schaefer type model. We assume that storage is competitive, that there is no intertemporal substitution in demand for the good, and that the cost of storage is an interest cost, with rate r.

We will look for rational expectations equilibria, or paths of population, stores, and price in which poachers, consumers, and storers are behaving rationally at all times. This section considers perfect foresight equilibria, in which the path is deterministic; Section IV considers equilibria in which the path is stochastic. The steady states of the model with storage are the same as those in the model without storage, as we show below.[7] Indeed, the stable steady states of the last section comprise the entire stable limit set of the system with storage (i.e. there are no cycles or chaotic attractors).

We analyze the fairly general model introduced in the last section with two stable steady states, one at zero and the other at X_S. In fact, the propositions of this section can be easily generalized to cover much more general models in which there are many stable steady states, or in which extinction is not stable.

Our strategy for finding equilibria is as follows. Simple accounting arithmetic and the absence of arbitrage opportunities in poaching and storage yield *local equilibrium conditions* on the possible equilibrium paths. Because there may or may not be storage or poaching, it turns out that there are three possible different dynamic régimes: *no storage, storage,* and *no poaching*. Using the local equilibrium conditions, we derive differential equations for the equilibrium paths in each régime. The steady states give terminal or boundary conditions

[7]We will make a distinction between a steady state, which is a stationary value of population and stores, and an equilibrium.

which allow us completely to determine the equilibrium paths, which we represent using phase diagrams in population-stores space. The steady states provide a terminal condition that allow us completely characterize the equilibrium paths.

LOCAL EQUILIBRIUM AND FEASIBILITY CONDITIONS

The local equilibrium conditions are determined by the absence of arbitrage opportunities for both poachers and storers of the good:

The Storage Condition The possibility of storage introduces constraints on the path of prices. As in [Hotelling 1931], in order to rule out arbitrage,

$$\frac{dp}{dt} = rp \text{, if } s > 0, \qquad \text{III.1}$$

where s denotes the amount of the good that is stored. If the price were rising less quickly, people would sell their stores, and if the price were rising more quickly, people would hold on to their stores or poach more. This "storage condition" is slack when stores are zero. In this case, $dp/dt \leq rp$, because otherwise people would find it profitable to hold stores.

The Poaching Condition Because poaching is competitive, if there is poaching at all, the price of the good must be equal to the marginal cost of poaching another unit of the good, which is $c(x)$ if the population is x. Thus the "poaching condition" is that

$$p = c(x) \text{, if there is poaching.} \qquad \text{III.2}$$

This condition is slack if there is no poaching, in which case $p \leq c(x)$.

Note that, in addition to the local equilibrium conditions above, there are some feasibility conditions:

"Conservation of Elephants" At all times, the increase in stores plus the increase in population must equal the net births minus the amount consumed, or

$$\dot{s} + \dot{x} = B(x) - D(p). \qquad \text{III.3}$$

Note that, as mentioned earlier, we assume that the good is destroyed when it is consumed. Note also that animals which die naturally cannot be turned into the storable good.[8]

Finally, both population, x, and stores, s, must be non-negative at all times.

The above conditions imply that, once on an equilibrium path, population, stores, and price, must be a continuous function of time. This is because, with perfect foresight, jumps would be anticipated and arbitraged. See Appendix A, proposition A.1 for a more formal proof. As we discuss below, there may be an initial jump to get to the equilibrium path.

These conditions must be satisfied at all points on a rational expectations equilibrium path. There are four conceivable dynamic régimes for the system, depending on which of the storage and poaching conditions (III.1 and III.2) are binding at any time, but only three of these potential régimes are actually possible:

No Storage Régime Stores are zero, but there is poaching. The zero profit condition for poaching implies that $p = c(x)$. The storage condition restricts the rate at which the price can rise and not induce storage ($\dot{p} \leq rp$). Because the price is inversely related to the population,

[8] We write the conservation condition as an equality. Because the price is positive, no one would throw the good away voluntarily.

it is possible to translate this condition that prices may not rise too fast into a condition that the population may not fall too fast: differentiating $p = c(x)$, the condition that no one wants to hold positive stores becomes

$$\frac{dx}{dt} \geq r\frac{c(x)}{c'(x)}. \qquad \text{III.4}$$

In the No Storage Regime, the dynamics are the same as Section II, the model with no storage:

$$\begin{aligned} \dot{x} &= B(x) - D(c(x)) \\ s &= 0 \\ p &= c(x) \end{aligned} \qquad . \qquad \text{III.5}$$

Storage Régime Stores are positive and there is poaching, so $dp/dt = rp$, and $p = c(x)$. Here, the exponential path of the price translates into a differential equation for population: differentiating $p = c(x)$ gives the same expression, but with equality, that we had for the No Storage régime (III.4). Given the path of population and, hence, price and consumption, the dynamics of stores are determined by "conservation of elephants" (III.3), and we can express all the local equilibrium dynamics in terms of the population, x:

$$\begin{aligned} \dot{x} &= r\frac{c(x)}{c'(x)} \\ \dot{s} &= B(x) - D(c(x)) - \dot{x}\,. \\ \dot{p} &= rc(x) \end{aligned} \qquad \text{III.6}$$

No Poaching Régime Stores are positive, but there is no poaching. Without poaching, the rate of change of population is just the net birth rate. All demand is being satisfied from stores, so stores must be falling at the same rate as demand. For stores to be positive, price must be rising exponentially at rate r. The dynamics can thus be summarized by:

$$\dot{x} = B(x)$$
$$\dot{s} = -D(p)\,. \qquad \text{III.7}$$
$$\dot{p} = rp$$

Note that since there is no poaching, it is *not* possible to substitute $c(x)$ for p.

No Storage, No Poaching This is impossible if population is positive, since it would imply that there is no consumption, so the price must be p_m, but p_m is greater than c_m, which is the maximum marginal cost of poaching, so there would have to be poaching, which contradicts the assumption that there was no poaching.

To be in steady state, stores must be zero because, when they are positive, price must be rising exponentially. This means that there are only two stable steady states: extinction, in which population and stores are zero, and what we will call the "high steady state", in which population is X_S, stores are zero, and price is $c(X_S)$. If stores are zero, then the system must be in the No Storage régime, and will thus have the same steady states as the model with no storage in Section II, *i.e.* $x = 0$ or X_S.

DYNAMICS WITHIN THE STORAGE AND NO STORAGE REGIMES

We shall begin by looking at the two régimes in which there is poaching: No Storage and Storage.

Equilibrium Paths in the No Storage Régime.

For the system to be in the no storage régime in equilibrium, people must not want to hold positive stores, so price must not be rising faster than rp. Since the price is determined by the population, $p = c(x)$, storage implies that the population cannot fall too fast. Specifically, from III.4 and III.5,

$$B(x) - D(c(x)) \geq \frac{rc(x)}{c'(x)}. \qquad \text{III.8}$$

As is clear from Figure III.1, for small enough r, III.8 will hold if and only if $x \in [X_U^*, X_S^*]$, where X_S^* and X_U^* are the two critical points at which the storage condition is just binding, *i.e.* $B - D = r\,c\,/\,c'$. Moreover, $0 < X_U^* < X_U < X_S < X_S^*$.

If the system starts with population in $(X_U, X_S^*]$ and no stores, then it is an equilibrium to follow the No Storage Régime dynamics to X_S, the stable steady state. If the system starts with no stores and a population of exactly X_U, the unstable steady state, the system will stay there. Here, as elsewhere, for the sake of clarity we shall not discuss measure zero cases like this in any detail.

If the system starts with no stores and with population in $[X_U^*, X_U)$, then the No Storage dynamics will eventually take population to a point less than X_U^*. At some point, therefore, the system must leave the No Storage Régime and enter the Storage Régime. We discuss this after we have found the equilibrium paths in the Storage Régime.

Equilibrium Paths in the Storage Régime

The dynamics of population are determined by the price, which is rising exponentially. The dynamics of stores are determined by "conservation of elephants": what is harvested and not consumed must be stored. We may rewrite III.6 as a differential equation for the trajectory of stores, s, in terms of x:

$$\frac{ds}{dx} = \frac{c'(x)}{rc(x)}\left\{B(x) - D(c(x)) - r\frac{c(x)}{c'(x)}\right\}. \qquad \text{III.9}$$

$dx/\mathrm{d}t$ is still just $rc(x)/c'(x)$, which is strictly negative, and bounded above.

Equation III.9 implies that rational expectations trajectories in population-stores space must have stores decreasing with population, x, if $x < X_U^*$, or $x > X_S^*$. Stores must be an increasing function of population if $x \in (X_U^*, X_S^*)$. There is a maximum of stores at X_U^*, and a minimum at X_S^*. To see the intuition for this, note that if population is very high or very low, population would tend to fall rapidly without stores, and as may be seen from Figure III.1, it would fall rapidly enough that price would be rising faster than rate r. In order to prevent population from falling too rapidly, part of demand must be satisfied out of stores, and so stores must decrease with time. X_U^* and X_S^* are the points at which, in the absence of stores, the population would fall just fast enough that price would rise at rate r. Between X_U^* and X_S^*, the price would rise more slowly than rate r with no storage. For an equilibrium with stores, therefore, more than current demand must be being harvested and stores must increase to make the population fall fast enough so that price rises at exactly rate r.

Equation III.9 is the differential equation for the trajectories of equilibria in population-stores space. The equilibria are now to be determined by boundary conditions. One possibility is that stores run out while population is still positive, and the system enters the No Storage Régime. The only place at which this can possibly happen is where population is exactly X_S^*. To see why, consider the following: to be in the No Storage Régime, $x \in [X_U^*, X_S^*]$. Because population, stores and price are continuous in equilibrium, the system must leave the Storage régime at the same point at which it enters the No Storage régime. As explained above, stores are *decreasing* as a function of x, so strictly *increasing* as a function of time (x is falling) if $x \in (X_U^*, X_S^*)$, and at a maximum at $x = X_U^*$. But stores have to run out at the point of transition from the Storage to No Storage Régime, so stores must have been falling, (or at least not increasing or at a maximum) immediately before the transition. The only point at which stores could run out is, therefore, X_S^*.

The other possible boundary condition is that population becomes extinct before stores run out. Since x is decreasing at a rate which is bounded below while stores are positive, the population must become extinct in finite time if stores do not run out. After that, stores will be consumed until they reach zero as well. It turns out that the quantity of stores remaining when the population becomes extinct is uniquely determined in a rational expectations equilibrium. To see this, note that the price charged for the last unit of stores must be p_m, the maximum price people are willing to pay for the good, or a storer would profit by waiting momentarily to sell his or her stock. The zero profit condition in poaching implies that the price when the population becomes extinct must be $c(0) = c_m$. Price is rising exponentially while stores are positive, so we can calculate the amount, $U(p)$, consumed from the time when price is p until price reaches p_m:

$$U(p) = \int_0^{\frac{1}{r}\ln\left(\frac{p_m}{p}\right)} D\left(p\, e^{rt}\right) dt . \qquad \text{III.10}$$

The amount of stores remaining at the moment of extinction must, therefore, be $U(c_m)$.

We have shown that there can only be two equilibrium paths in the Storage Régime (See Figure III.2)

1 *High Steady State Storage Equilibrium* In this equilibrium, population starts at $x \geq X_S^*$. The system evolves until stores run out when population is X_S^*, and then enters the No Storage Régime. The equations $p = c(x)$, and $dp/dt = r\,p$ determine the path of population and price. Stores are given by $s = s_+(x)$, where

$$s_+(x) = \int_{X_S^*}^{x} \frac{c'(q)}{rc(q)} \left\{ B(q) - D(c(q)) - r\frac{c(q)}{c'(q)} \right\} dq. \qquad \text{III.11}$$

2 *Extinction Storage Equilibrium.* In this equilibrium, population becomes extinct, and at that moment, stores = $U(c_m)$. The equations $p = c(x)$ and $dp/dt = rp$ determine the path of population and price. Stores are given by $s = s_e(x)$, where

$$s_e(x) = U(c_m) + \int_{0}^{x} \frac{c'(q)}{rc(q)} \left\{ B(q) - D(c(q)) - r\frac{c(q)}{c'(q)} \right\} dq. \qquad \text{III.12}$$

For this to be an equilibrium, stores must stay positive at all times along this path. If stores would have to become negative at some point in the future, this path is not an equilibrium. If $s_e(x)$ is ever negative, we define X_{max} to be the smallest positive root of $s_e(x)$. If there is none such, we say that $X_{max} = \infty$. To be an equilibrium, the starting population must be less than X_{max}.

$s_e(x)$ and $s_+(x)$ are parallel. Both have a minimum at X_S^*. It is clear from Figure III.2 that X_{max} is finite if and only if $s_e(x)$ lies below $s_+(x)$. If X_{max} is finite, it must lie between X_U^* and X_S^*.

Transitions Between Storage and No Storage Régimes

We now examine under which circumstances an equilibrium path can move from the No Storage to the Storage Régime. If the initial population is small enough, an equilibrium path can move to the Storage régime and, thence, to extinction. It may have to do this: if $X_U^* > 0$ and the system starts in the No Storage régime with population less than X_U, then the system *must* eventually move to the Storage régime because if it didn't, the population would fall fast enough to violate the storage condition once it had fallen past X_U^*. On an equilibrium path, the system must move to the Storage régime before that point is reached. The system may also move to the Storage régime when it doesn't strictly have to. By continuity of stores, the system must make the transition from the No Storage to the Storage régime where $s_e(x) = 0$, *i.e.* at X_{max}[9]. If the path in the No Storage régime crosses X_{max}, then the system can move to the Storage régime path s_e leading to extinction. At such a transition, the rates of change of population, stores, and price will jump, but the storage and poaching conditions are not violated because the levels will not jump.

We may thus define two sets of points on equilibrium paths in the Storage and No Storage régimes: A_e, the set of points leading to extinction, and A_+, the set of points leading to the high steady state, as illustrated in Figure III.3.

The system must end up on one of these paths, A_+ or A_e. Given arbitrary initial values of population and stores (x_0,s_0), there can either be an initial cull, or there can be an interlude when there is no poaching, as discussed below.

MOVING TO EQUILIBRIUM

If the initial population and stores are not on one of the equilibria identified above, then one of two things will happen. If the initial point in population-stores space is below the equilibrium paths described above, then the system may jump instantaneously to one of the equilibrium paths *via* a cull. If the initial point is above an equilibrium, demand may be satisfied from stores with no poaching for a while until the path meets A_e or A_+.

[9] The transition cannot happen at X_S^*, because the population would be falling there in the No Storage régime, so the system could never reach that point.

Culling

If the system starts below an equilibrium path in population stores space, there may be an instantaneous harvest, which we shall call a "cull". In this case, the price starts high enough that it is above the marginal cost implied by the initial population, $c(x_0)$, and there will be instantaneous poaching up to the point at which the price is equal to the marginal cost.[10] Although continuity of price, population, and stores is required by rationality, such a jump is allowed if it is unanticipated, or at the "beginning of time", as it is in this case. We will make a distinction between "initial" values of population and stores and "starting" values, which are the values just after the initial cull. When we need to indicate this, we will write (x_0, s_0) for initial population and stores, and $(x(0), s(0))$ to denote starting (*i.e.* at time 0 on the equilibrium path) values.

In a cull, live elephants are killed and turned into dead elephants one-to-one. This means that, in population-stores space, the system moves up a downward sloping diagonal, and the total quantity of elephants, dead or alive, is conserved. We call this quantity $Q = x + s$. For a cull to be rational, it must take the system to a point on one of the equilibrium paths we identified above, A_e or A_+.

To get to the high steady state equilibrium path by culling, initial population and stores must lie below the line $s = s_+(x)$, and $x_0 < X_S^*$.

To get to the path leading to extinction, if X_{max} is infinite, the initial point must lie below $s = s_e(x)$. If X_{max} is finite, points below $s_e(x)$ can also cull to the equilibrium, but there may also be other points from which this is feasible. In particular, if the curve $s = s_e(x)$ has a

[10] If the marginal cost of poaching became sufficiently great as the instantaneous rate of poaching became great enough, the harvest would take place over time, rather than instantaneously. Structurally, though, there is little real difference in the two approaches: the rational expectations equilibria are determined by the boundary conditions (where people anticipate the system must end up), and these are essentially the same in both cases.

tangent of gradient -1, then, as illustrated in Figure III.4, points above the curve, but below the tangent can also reach the extinction equilibrium by culling. A quick look at III.9 shows that the points at which $s_e(x)$ has gradient -1 are X_U and X_S, but only the tangent at X_U lies above the curve. The value of Q at this tangency, Q_{max}, is the maximum value Q may have so that the extinction equilibrium may be reached *via* culling. If $X_{max} < X_U$, then this tangency doesn't exist, and only points below s_e can cull to the extinction equilibrium.

No Poaching

If there are sufficient initial stores, there will be equilibria in which the starting price is below $c(x)$, and there is no poaching for a time while demand is satisfied out of stores. Eventually poaching must resume, at a point on A_e or A_+. While there is no poaching, population will be rising, and stores falling as they are consumed. Price is rising exponentially, at rate r. In population-stores space, trajectories with no poaching must be downward sloping and population must be increasing so long as population is less than one.

When poaching resumes at a point on one of the A_i paths, price, population, and stores are all determined. Given the end point, there is a unique, downward sloping no poaching trajectory leading to it. In order for no poaching to be rational, and for an initial point to end up on one of the A_i, the initial point must lie on one of these trajectories (Figure III.5). To get to the path leading to the high steady state, the initial point must lie to the right of the boundary of the set of points on trajectories leading to A_+, which we denote L_+, and above the curve $s = s_+(x)$. To get to the path leading to extinction, the initial point must lie to the left of the boundary of the set of points on trajectories leading to points on A_e, which we denote L_e. We include a more formal treatment of this in the Appendix A, proposition A.3.

We have now found all the possible equilibria of the model with storage. As illustrated in Figure III.6, population-stores space may be divided into at most three regions depending on whether there exist equilibria leading to extinction, the high steady state, or both. In the first region, there is no equilibrium path leading to extinction. This will be the case if the initial population and stores are high enough, so that killing and storing enough to get to extinction would mean that stores would have to be held long enough that the storers would lose money. In the second region, there is no equilibrium path leading to the high steady state. This is the case if population and stores are low enough that, even if poaching were temporarily to cease and demand were to be satisfied from stores until they should run out, the population cannot recover enough to guarantee species survival. The third region is where there are multiple possible equilibria, some to extinction, and some to the high steady state. In this deterministic, perfect foresight model, which equilibrium is chosen is determined by exogenously formed, self-fulfilling expectations.

Depending on parameter values, some of these regions may be empty. It is possible that there will be no region in which survival is assured. If X_{max} is infinite, any point can get to the extinction set A_e, either through a cull if it lies below s_e, or by an interlude with no poaching if it lies above s_e.

If, on the other hand, X_{max} is small enough (less than X_U), then there will be no region of multiple equilibria, and the fate of the system will be entirely determined by its initial point, and not by expectations.

Note that, if there is an initial no poaching interlude, the population will be rising to start with even if the eventual fate of the system is extinction. There will often be over-shooting with No Poaching equilibria, and one should not, therefore, become complacent if elephant populations are increasing.

It turns out that X_{max} and Q_{max} are both decreasing in r, the storage cost. For proofs, see Appendix A, proposition A.2. This should not come as a surprise. Q_{max} tells us the largest population can be and still reach extinction via culling and a storage equilibrium path. The larger the population, the longer stores have to be held before extinction. This is clearly going to be less desirable with higher storage costs. Increasing the storage cost thus always reduces the region of phase space from which extinction is possible. Governments could increase storage costs by threatening prosecution of anybody found to be storing the good. The international ban on ivory trade may have had this effect.

For sufficiently large r, X_{max} will be less than X_U, and there will be no region of multiple equilibria at all; the ultimate fate of the species is the same as in the model with no storage possible, given the same initial conditions. In this sense, our model converges to the standard Gordon-Schaefer model as storage cost rises.

If $Q_{max} > X_S$, then even starting from the high steady state with no stores, the population will be vulnerable to coordination on the extinction equilibrium. This highlights another possible policy response to limit the possibility of extinction: the government or private conservation organizations may increase the size of habitat available to the species. Increasing the habitat, while leaving demand unchanged, will increase the steady state population, X_S, more than proportionally. At the same time, Q_{max} will fall. We show, in Appendix A, proposition A.4 that, for sufficient habitat, X_S will be above Q_{max}, and the species will then be safe from speculative attacks leading to extinction when it is in the high steady state.

IV. Non-Deterministic Equilibria

So far, we have focused on perfect foresight equilibria, in which all agents believe that the economy will follow a deterministic path. This section considers a broader class of rational expectations equilibria in which agents may attach positive probability to a number of future possible paths of the economy. One reason to consider this broader class of equilibria is that the perfect foresight equilibrium concept has the uncomfortable property that there may be a path from A to B, and from B to C, but not from A to C. To see this, note that if Q_{max} is greater than X_S, then for sufficient initial population, the only equilibrium will lead to the high steady state. For a system that starts in the high steady state, however, an extinction storage equilibrium would also be possible.

Note also that the concept of no poaching regimes is also much more relevant when stochastic paths are admissible, since in order to have an equilibrium with no poaching, there must be stores, and the only way stores can be generated within the model is through a storage equilibrium. However, within the limited class of perfect foresight equilibria, people must assign zero weight to the possibility that there might be a switch from an storage regime to a no poaching regime.

While we have not fully categorized the extremely broad class of equilibria with stochastic rational expectations paths, we have been able to describe a subclass of equilibria, which we conjecture illustrates some more general aspects of behavior. We consider equilibria in which agents believe there is a constant hazard that a sunspot will appear and that, when this happens, the economy will switch to the extinction storage equilibrium, with no possibility of any other switches.[11] Thus all agents know that the economy will switch to the extinction equilibrium eventually with probability one, but they are unsure when. We divide time into two parts: before the sunspot (B.S.), and after the sunspot (A.S.). The equilibrium behavior A.S. is simple: it is just the extinction equilibrium we found in the last section. In this section, we look for equilibria B.S. in which the population does not become extinct.

We first derive a stochastic analogue of the storage condition. We then show that there are equilibria with a small switching hazard in which the behavior is similar to that seen in section III: the B.S. steady state population is X_S, and no stores are held in this steady state. There are also equilibria with a higher switching hazard in which positive stores are held in B.S. steady-state equilibrium in anticipation of a switch to the extinction equilibrium. There cannot be equilibria with a hazard rate above a certain threshold, because in this case extinction would become so likely that it would become certain and the system would have to jump immediately to the extinction equilibrium.

We also show that while the species can survive a series of small increases in the hazard rate of switching by building up stores after each increase, it might not be able to sustain the same increase in the hazard rate if it took place in a single jump, because the required increase in storage would be so great as to drive the species into extinction.

In this section, it is mathematically more convenient to work with the total of stores and population, Q, rather than stores, s. Since $Q = s + x$, working with (x,Q) is equivalent to working with (x,s).

For the sake of clarity and brevity, we relegate all proofs to Appendix B, where we

[11]In some cases, extinction is instantaneous after the switch, so there is no way that agents could rationally ascribe positive probability to any further switch.

derive the results of this section more formally.

B.S. Equilibrium Conditions

Let the B.S. state be (x, Q, p), and the A.S. state be (x_e, Q, p_e). Note that, since the switch happens instantaneously by culling, Q doesn't change when the sunspot appears. If there are positive stores, the expected profit from storage must be zero, so that, if we denote by π the hazard rate that the sunspot will appear:

$$\dot{p} + (p_e - p)\pi = rp \quad \text{if } s > 0. \qquad \text{IV.1}$$

If stores are zero, it must be because expected profits from storage are not positive, and so

$$\dot{p} + (p_e - p)\pi \leq rp \quad \text{if } s = 0. \qquad \text{IV.2}$$

IV.1 & IV.2 are just generalizations of the storage condition when the price change is stochastic, and not necessarily continuous.

We shall consider equilibria in which, once the system is in the extinction equilibrium, there is no possibility of further change. (x_e, Q) must, therefore, lie on the extinction storage equilibrium path derived in section III. In some cases, there may be more than one point on this path to which the system could jump. We will consider equilibria in which the system jumps to the lowest possible population on this path. Thus the population after the switch is a function of Q: $x_e(Q)$ is the smallest population such that $s_e(x_e(Q)) + x_e(Q) = Q$. This is illustrated in Figure IV.1. Because harvesting cannot increase the population, for people to believe in the possibility of a switch to the extinction equilibrium we must have $x \geq x_e(Q)$. We will also only formally consider cases in which Q_{max} is bigger than X_S. The system must, therefore, be in the region where $Q_{max} \geq Q \geq x > x_e(Q)$.

By assumption, the system jumps to the extinction equilibrium path if there is a sunspot. We may, therefore, determine the A.S price as a function of Q, the total stores plus population at the time of the sunspot. Q is conserved during the switch. p_e is a *decreasing* function of Q (as we prove in Appendix B, proposition B.1), and it is continuous on $[0, Q_{max}]$. When $Q \leq U(c_m)$, the system jumps straight to extinction, and $p_e = U^{-1}(Q)$. When $Q > U(c_m)$, the population jumps to $x_e(Q)$, and $p_e = c(x_e(Q))$.

We first consider the system dynamics when there are positive stores, so that IV.1 holds. By an argument analogous to that used in Section III, while stores are positive (*i.e.* $Q \geq x$), the system evolves before the sunspot according to the differential equations:

$$\begin{aligned} \dot{Q} &= B(x) - D(c(x)) \equiv F(x) \\ \dot{x} &= \frac{1}{c'(x)}\left[(r+\pi)c(x) - \pi p_e(Q)\right] \end{aligned} \qquad \text{IV.3}$$

B.S. Steady States

It is rather easy to solve for the steady states of IV.3. The first equation tells us that, for total stores and population to be constant, population must be either X_S, or zero (we ignore X_U), just as was the case in the deterministic case in the last section. We are interested in the steady state at X_S. Given the population X_S, the B.S. price will be $c(X_S)$. To determine the steady state level of stores, note that the more stores, the lower the A.S. price will be, and so the less profitable it will be to speculate on the sunspot's appearance. There will thus be a unique level of stores plus population, Q_S, that satisfies the storage condition with equality. The second equation of IV.3 allows us to solve for this level of stores in terms of the interest rate, the sunspot hazard, and the characteristics of the extinction equilibrium after the sunspot:

$$Q_S = p_e^{-1}\left(\frac{r+\pi}{\pi}c(X_S)\right). \qquad \text{IV.4}$$

Because stores must be positive, this is only a feasible B.S. steady state if $Q_S \geq X_S$. This will be the case for $\pi \geq \pi_l$, where

$$\pi_l = \frac{rc(X_S)}{p_e(X_S) - c(X_S)} \quad . \qquad \text{IV.5}$$

If $\pi < \pi_l$, the storage condition cannot be satisfied with equality at X_S, and there must be zero stores in steady state.

Because it must be possible to reach the A.S. equilibrium path *via* culling, it is also the case that we must have $Q_S \leq s_e(X_U) + X_U$. If Q_{max} is finite, the right hand side of IV.5 is just Q_{max}. Even if Q_{max} is infinite, this equation still holds, as if stores are too large at X_S, one cannot cull to a point on the extinction equilibrium. For this to hold, π must be below π_h, where

$$\pi_h = \frac{rc(X_S)}{c(X_U) - c(X_S)} \, . \qquad \text{IV.6}$$

In summary, then, if $\pi < \pi_l$, then there is no B.S. steady state equilibrium with positive stores. In this case, X_S is a steady state with no stores, just as it was in the perfect-foresight case of section III: the sunspot probability is low enough that the costs of holding positive stores outweigh the expected profit when the sunspot happens.

If $\pi_l < \pi < \pi_h$, then there is a B.S. steady state with positive stores, (X_S, Q_S). The possibility of the sunspot causes agents to hold positive stores in anticipation, so the more likely the sunspot's occurrence, the higher the stores held in anticipation of it (see Appendix B, proposition B.2).

If $\pi > \pi_h$, there exists no B.S. steady state at X_S. This is because if the sunspot probability is high enough, extinction becomes self-fulfilling even before the sunspot happens, and there is no steady state equilibrium before the sunspot apart from extinction.

B.S. Equilibrium Dynamics

We now summarize the main features of the B.S. equilibrium dynamics. Details of this are in Appendix B, propositions B.2 - B.7. Figure IV.2 illustrates a possible phase diagram in x - Q space.

Steady states for population X_U are always totally unstable, so we ignore them.

If $\pi < \pi_l$, then the dynamics are basically the same as for the perfect foresight case. Along the equilibrium path, there will be some level of positive stores $s_+^\pi(x)$ when population is above a critical value X_S^π. In this case, population decreases towards X_S over time, and stores are falling and run out at X_S^π, the stochastic analogue of X_S^*, where the storage condition IV.2 is just binding. The system continues to the B.S. steady state $x = X_S$, $s = 0$ in the No Storage régime exactly as in section III. As we show in the Appendix B, proposition B.4, X_S^π is decreasing with π. Obviously $X_S^0 = X_S^*$, and $X_S^{\pi_l} = X_S$.

If $\pi_l < \pi < \pi_h$, the B.S. equilibrium path is the saddle path of the fixed point (X_S, Q_S). This saddle path rises with increasing π. This means, as is not surprising, that if the probability of a sunspot is higher, then higher stores will be held for all population levels, not just at the steady state. We illustrate the dynamics in Figure IV.3.

Comparative Statics and Unanticipated Changes in π

Now we consider unanticipated changes in π, the transition hazard. If π increases[12],

[12] If π decreases, of course, the price would have to fall to get back to the stable manifold, and poaching would stop for a while in a non-deterministic version of a no poaching equilibrium.

the B.S. equilibrium path moves up, and there will be a cull to get to the new equilibrium path from the old one, so long as the increase in π is not too big. If the increase in π is too big the required cull will be so large that the new equilibrium path cannot be reached, and the system will switch immediately to the extinction equilibrium. Note that a large increase in π need not necessarily lead to extinction if it happens gradually, in a series of small, unanticipated steps. If the increase in π is slow enough, then the equilibrium is sustainable up to the point at which the equilibrium ceases to exist altogether (*i.e.* π_h).

Thus, if a policy maker knew that there had to be a shift in expectations towards a higher probability of extinction, and somehow had control over the timing of that shift, it would be best to make the shift gradual, rather than rapid. This hints that if policy makers have access to continuously changing information about the state of the population, it might be best for them to release this information on a regular basis, rather than simply trying to cover up bad news about the availability of the resource and hope that the situation repairs itself before people find out. This can only be conjectured, however, because in the model, there is no uncertainty about the population, only about what other agents are thinking.

We conjecture that if there was a chance of switching to a no poaching equilibrium at any point on the A.S. trajectory, the rate of growth of prices in the extinction equilibria would have to be higher. Similarly, the possibility of switching back to an extinction equilibrium from a no poaching equilibrium would mean that the rate of growth of prices would have to be lower in the no poaching equilibrium. Note that the possibility of switching to a no poaching equilibrium makes it harder to have a storage equilibrium, just as increasing storage costs would.

V. Conclusion

This paper has argued that there may be multiple possible rational expectation paths of population and prices for open-access resources used in the production of storable goods. Expectation of future poaching will increase future prices, and this will increase current prices, thus rationalizing the initial increase in poaching. Note that this argument does not apply to non-storable goods, such as fish, because the price of fish depends only on current supply and demand, and not on expectations of prices. It also does not apply to privately held goods, such as oil, since anticipation of higher prices will lead people to postpone extracting the resource.

It is becoming cost-effective for people to assert property rights to elephants in a few areas of Africa [Simmons and Kreuteo, 1989]. Most elephants, however, continue to live in open-access areas, and only a fraction of the elephant population can profitably be protected as private property. (It is expensive to protect elephants as private property, since they naturally range over huge territories and ordinary fences cannot contain them [Bonner, 1993].) If elephants can only be supported as private property above a certain price, then there may be one equilibrium in which they are a plentiful open-access resource at a low price, and another equilibrium in which they are a scarce private resource at a high price.

The analysis carries several policy implications. First, it indicates that in order to assure the survival of a species, it may be necessary to preserve a large enough herd not only to allow the species to survive at current equilibrium poaching levels, but also to prevent an equilibrium with a higher level of poaching. If $Q_{max} > X_S$, then the population may appear safe, but may in fact be vulnerable to a switch in equilibrium. One way to rule out the extinction equilibrium is to increase the habitat for the animal, so that the steady-state population becomes greater than Q_{max}.

It may be possible for governments and international organizations to avoid the extinction equilibrium if they can commit to drastic measures to prevent extinction. This could keep prices down and reduce the incentive to poach. If a government or international organization could credibly announce that it would spend a large amount on elephant protection if the herd fell below a certain critical size, it might never actually have to spend the money, whereas if the same government spent a moderate amount on elephant protection each year, the herd might become extinct. The model thus suggests a rationale for conservation laws that extend little protection to a species until it is declared endangered, and then provide extensive protection with almost no regard to cost.[13] Whether this is important empirically is another matter.

While conservationists and governments may wish to coordinate on low-poaching equilibria, people who hold stores will prefer to coordinate on a high-poaching equilibrium, in which the species becomes extinct. In fact, although game officials in Zimbabwe removed the horns of some rhinos in order to protect them from poaching, poachers killed the rhinos anyway. The *New York Times* [July 11 or 12, 1994], quotes a wildlife official as explaining their behavior by saying "If Zimbabwe is to lose its entire rhino population, such news would increase the values of stockpiles internationally."[14]

Note that if there were a "George Soros" of elephants who had sufficient resources, or were not subject to credit constraints, he or she could use his or her market power to coordinate on the extinction equilibrium simply by offering to buy enough of the good at a

[13] On the other hand, the model suggests that if the government plans to impose such strong anti-poaching enforcement that the long-run harvest will decline, and announcing these regulations ahead of time may lead to a rush to poach.
[14] It is also possible that the poachers killed the rhinos to obtain the stumps of their horns, or to make rhino poaching easier in the future.

high enough price. A speculator who already owned some of the good would make substantial profits by inducing coordination on the extinction equilibrium, so this equilibrium may be more likely in the absence of government intervention, assuming the parameters are such that both extinction and survival equilibria exist.

The model also suggests that it may be possible to eliminate the extinction equilibrium by accumulating a sufficient stockpile of the storable good, and threatening to release it onto the market if the animal goes extinct or becomes sufficiently endangered. (Note that this policy is more likely to be time consistent than policies which promise to spend arbitrary amounts of resources to preserve an animal. If the animal is already going extinct, there is no reason not to sell the stockpile.) As illustrated in Figure III.6, if Q_{max} is finite, but greater than the high steady state, X_S, then an extinction equilibrium will exist in steady state if the government does not stockpile stores. If the government or a conservation organization holds stores greater than the boundary of the region where the extinction equilibria cease to exist, and credibly promises to release them onto the market if the population falls below a threshold, the extinction equilibrium will be eliminated. The organization holding stores would have to pay the interest costs on the stores, and this would entail a financial loss, but the price might be worth paying if the organization valued conservation, and the stores eliminated the extinction equilibrium.

If Q_{max} is infinite, stores cannot eliminate the extinction equilibrium, but they can extend the range of the survival equilibrium. For example, suppose that the initial stock is X_s, and the initial stores are zero, but that there is an exogenous shock to population, for example due to disease. If there are no stores, then the species will be driven to extinction if the population dips below X_U^*. However, if there are sufficient stores, there will be a no poaching equilibrium in which demand is satisfied by stores and the population can recover.

Bergstrom [1990] has suggested that confiscated contraband should be sold onto the market. This analysis suggests that an alternative policy would be to hold confiscated supplies of goods such as rhino horn and released them on onto the market only if it appears that the market is coordinating on the extinction equilibrium. For example, a rule might be adopted that confiscated rhino horn would be sold only if the rhino population dipped below a certain level, or the price rose above a certain level.

Stores could be built up not only by confiscating contraband, but also by harvesting. Sick animals could be harvested, and animals could be harvested during periods when population is temporarily above its steady state level, due, for example, to a run of good weather.

Building up stores will reduce the population, but only temporarily. Once the target stockpile has been accumulated, harvesting to build up the stockpiles can be discontinued, and the live population will return to the same level as in the absence of stockpiling. The presence of the stockpiles, however, will permanently eliminate or reduce the chance of a switch to the extinction equilibrium. If Q_{max} were less than X_S it is particularly important to build up stockpiles gradually, so as to prevent the population from falling below Q_{max}, and thus creating an opportunity for coordination on the extinction equilibrium.

Many conservationists oppose selling confiscated ivory on the market, for fear that it would legitimize the ivory trade. Building stores achieves the same goal of depressing prices, but without the disadvantage of legitimizing the ivory trade. Stores could potentially be held until scientists develop cheap and reliable ways of marking or identifying "legitimately" sold animal products so they can be distinguished from illegitimate products.

While stockpiles may help promote conservation of animals which are killed for goods which are storable but not durable, such as rhino horn, this analysis does not strictly apply to durable goods, utility from them. Elephant ivory is often considered an example of such a durable good. The government has no reason to wait before selling confiscated durable goods, since in any case, private agents will store any durable goods sold on the market. In practice, however, there are few completely durable goods. Even ivory is not perfectly durable, since it depreciates, and uncarved ivory is not perfectly substitutable for carved ivory, due to changing styles and demand for personalized ivory seals.

In the perfect foresight model of Section III, no private stores were held by speculators in the high steady-state. However, if the price is stochastic, either due to sunspot coordination, as in Section IV, or to exogenous shocks, such as weather or disease, then speculators may hold stores, and government stores may crowd these out. In the example considered in Section IV, government stores would crowd out private stores one for one, until the government accumulated greater stores than would be held by private agents. Any further accumulation by the government would reduce the range of equilibria in which agents could anticipate extinction.

Finally, it is worth noting that this model suggests that if one country reduces the price of ivory by protecting its elephants, this reduces the incentive to poach in other countries. In conventional models of non-storable resources, increased anti-poaching efforts in one country will initially drive up the price of the good, encouraging extra poaching in other countries. Under this model, increased anti-poaching efforts in one country may reduce poaching in other countries, both in the short run and in the long run.

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Appendix A Proofs for Section III

Proposition A.1 The path of population, x, stores, s, and price, p, is continuous on an equilibrium path.

Proof Together, the storage and poaching conditions imply that the equilibrium price path must be continuous in time. A jump up in price would violate the storage condition, and a jump down in price would imply an instantaneous infinite growth rate of the population, which is impossible.

While there is poaching, $p = c(x)$, which is continuous and monotonic, so population, x, must be continuous. In the no poaching régime, population develops as III.7, so is continuous. Population cannot jump suddenly across régime changes, either, as that would require a jump in price so there is an instantaneous harvest. Population is thus continuous. Stores are differentiable within régimes, and so are continuous. For there to be a jump in stores across régimes, there would have to be an instantaneous harvest, which would require a jump in price, which is impossible. Hence stores are continuous.

Proposition A.2

(a) The maximum initial value of population plus stores the system may have and still get to the storage equilibrium path $s_e(x)$ is Q_{max}, where

$$Q_{max} = \max\{ X_{max}, s_e(X_U) + X_U \}. \qquad \text{A.1}$$

(b) Q_{max} is decreasing in storage cost, r.

Proof (a) Q_{max} must either be X_{max}, or the point lying on the $s = 0$ axis and the tangent to $s_e(x)$ of gradient -1. These tangencies occur at X_U or X_S ($F(x) = 0$ in equation III.9 gives $ds/dx = -1$). $s_e(x)$ is concave at X_S, so Q_{max} cannot be associated with X_S.

(b) If $Q_{max} = X_{max}$, then $s_e(Q_{max}) = 0$. Differentiating with respect to r,

$$s_e'(Q_{\max})\frac{\partial Q_{\max}}{\partial r}+\frac{\partial s_e}{\partial r}(Q_{\max})=0 \qquad \text{A.2}$$

$s_e'(x)$ is just (III.9), and $\partial s/\partial r$ is:

$$\begin{aligned}\frac{\partial s_e}{\partial r}(x) &= \frac{\partial}{\partial r}\left\{\int_{X_S^*}^{x}\frac{c'(u)}{rc(u)}\left(F(u)-\frac{rc(u)}{c'(u)}\right)du\right\} = \\ &= -\frac{1}{r}\left(s_e(x)+x-X_S^*\right)-\frac{\partial X_S^*}{\partial r}\frac{c'(u)}{rc(u)}\left(F(u)-\frac{rc(u)}{c'(u)}\right)\Bigg|_{u=X_S^*} \qquad \text{A.3} \\ &= -\frac{1}{r}\left(s_e(x)+x-X_S^*\right)\le 0\end{aligned}$$

since $x > X_S^*$, and $s_e(x)$ must be non-negative. The second term is zero, because at X_S^*, the storage condition is satisfied with equality, and that is precisely what is in the parentheses. Because $s_e(x)$ must be strictly decreasing at X_{max},

$$\frac{\partial Q_{\max}}{\partial r}=-\frac{\partial s_e}{\partial r}(X_{\max})\frac{1}{s_e'(X_{\max})}\le 0. \qquad \text{A.4}$$

If $Q_{max} = s(X_U) + X_U$, then, because X_U is independent of r, the result follows in the same way, but then the equivalent of A.3 holds because the second term vanishes because X_U is independent of r.

Proposition A.3 If initial population and stores are (x_0, s_0) then if, and only if

$$(x_0,s_0)\in\bigcup_{t=0}^{\infty}\mathbf{P}\phi_{-t}(A_i)=E_i, \qquad \text{A.5}$$

where $\mathbf{P}$ is the projection operator $\mathbf{P}(x,s,p) = (x,s)$, $i = +$ or e, and ϕ_t is the time evolution operator mapping $\{x(0), s(0), p(0)\}$ to $\{x(t), s(t), p(t)\}$, there is a starting price p_0 and poaching resumption time t_p so that $\phi_t(x_0,s_0,p_0)$ is an no poaching equilibrium leading to the point

$(x, s_i(x), c(x))$ at time t_p for some x. These equilibria are not, in general, unique. There may be equilibria leading to A_e and A_+. There may also be cases where the equilibrium passes through **P**A_+ or **P**A_e on its way to another point. If there are multiple equilibria from the same point (x_0, s_0), then the one with the lower starting price must have a steeper trajectory in s-x space, since stores will be consumed faster with a lower price.

In other words, there is a no poaching equilibrium ultimately leading to the steady state X_S if and only if $L_+(x_0) < s_0$ and $s_0 > s_+(x_0)$, where L_+ is the left boundary of the set E_+ defined in A.5. Likewise, there is an no poaching equilibrium leading to extinction if and only if $L_e(x_0) < s_0$, and $s_0 > s_e(x_0)$. See Figure III.5. L_i are downward sloping. L_e and L_+ will be the same line if $X_{max} \leq X_U$.

Proof By Figure III.5. L_i are downward sloping, because they are possible no poaching paths, and so stores are decreasing, while population is increasing.

Proposition A.4 If the habitat available to the population is increased sufficiently, it is always possible to make $X_S > Q_{max}$.

Proof Denote the available habitat by K, and the total population, in real units by ϕ. Thus $x = \phi/K$. We assume that demand, measured in real units, is independent of habitat, and that the poaching marginal cost, c, is a function only of population relative to habitat. Thus $c(x) = c(\phi/K)$. The dynamics of the population in real units without storage will be:

$$\dot{\phi} = KB(\phi / K) - D(c(\phi / K)), \qquad \text{A.6}$$

which implies that

$$\dot{x} = B(x) - \frac{1}{K} D(c(x)) . \qquad \text{A.7}$$

The steady states X_S, and X_U will be functions of habitat, K, and are such that the RHS of A.7 is zero. Differentiating, we find that

$$\frac{\partial X_S}{\partial K} > 0 \text{, and } \frac{\partial X_U}{\partial K} < 0 . \qquad \text{A.8}$$

This means that increasing the habitat more than proportionally increases the population in the high steady state. This is not unsurprising, given that demand has not changed.

In the region where Q_{max} is close to X_S, $Q_{max} = X_U + s_e(X_U)$. We may write this as:

$$Q_{\max} = X_U + \frac{U}{K} + \int_0^{X_U} \left(B(q) - \frac{D(c(q))}{K} - \frac{rc(q)}{c'(q)} \right) \frac{c'(q)}{rc(q)} dq . \qquad \text{A.9}$$

When we differentiate this expression with respect to K,

$$\frac{\partial Q_{\max}}{\partial K} = \frac{\partial X_U}{\partial K} - \frac{U}{K^2} + \int_0^{X_U} \frac{D(c(q))c'(q)}{K^2 rc(q)} dq - 1 < -1 . \qquad \text{A.10}$$

Thus Q_{max} is falling with K at a rate bounded away from zero. X_S is rising with K. It must be, therefore, that we can find K large enough that $Q_{max} < X_S$.

Appendix B Proofs for Section IV

Proposition B.1 p_e is a decreasing function of Q, continuous on $[0, Q_{max}]$. For $Q \in [0, U(c_m)]$, $p_e(Q) = U^{-1}(Q)$. For $Q \in [U(c_m), Q_{max}]$, $p_e(Q) = c(x_e(Q))$.

Proof If $Q > U(c_m)$, the system jumps to $x_e(Q)$, and the price is then $c(x_e(Q))$. If $Q < U(c_m)$, the price will be $U^{-1}(Q)$. $x_e(Q)$ is decreasing in Q, and continuous as required. U^{-1} is decreasing in Q. $U^{-1}(U(c_m)) = c_m = c(0) = c(x_e(U(c_m)))$, so p_e is continuous at $U(c_m)$.

Proposition B.2 $dQ/dt = 0$ when $x = X_U$ or X_S. The line where $dx/dt = 0$, $Q_0(x)$, is increasing in x, and increasing in π. For π in a suitable region (see below), there are two steady states, (X_U, Q_U), and (X_S, Q_S), and

$$Q_i = p_e^{-1}\left(\frac{g+\pi}{\pi}c(X_i)\right), \text{ where } i \text{ is } U \text{ or } S. \qquad \text{B.1}$$

Both Q_i are increasing with π. The line $x = x_e(Q)$ is a trajectory of the system. See Figure IV.2.

Proof Since $dQ/dt = F(x)$, $dQ/dt = 0$ iff $x = X_S$ or X_U. From IV.3, the line where $dx/dt = 0$ satisfies

$$(r+\pi)c(x) = \pi p_e(Q_0(x)). \qquad \text{B.2}$$

Differentiating with respect to x,

$$(r+\pi)c'(x) = \pi p_e'(Q_0(x))\frac{\partial Q_0}{\partial x}. \qquad \text{B.3}$$

c' and p_e' are both negative, so Q_0 must be increasing with x, for given π. Differentiating the same equation with respect to π,

$$c(x) = p_e(Q_0) + \pi p_e'(Q_0)\frac{\partial Q_0}{\partial \pi}. \qquad \text{B.4}$$

So that, on rearranging,

$$\frac{\partial Q_0}{\partial \pi} = \frac{c(x) - p_e(Q_0)}{\pi p_e'}. \qquad \text{B.5}$$

When the system jumps, population cannot rise, so price cannot fall. Hence, $c(x) < p_e$, and B.5 is positive. Steady states are where $dQ/dt = dx/dt = 0$. Substituting X_S or X_U into B.2 quickly yields B.1

Proposition B.3 (X_U, Q_U) is totally unstable; there may or may not be oscillatory behavior. (X_S, Q_S) is hyperbolic for all π. The stable manifold is thus a line, upward sloping, and passing through (X_S, Q_S). See Figure IV.3

Proof Consider x and Q near the steady states, $(x,Q) = (X_i + \xi, Q_i + \theta)$, where ξ and θ are small. Using Taylor's theorem on IV.3 (and assuming that we're allowed to), to first order,

$$\begin{pmatrix} \dot{\xi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} g+\pi & -\dfrac{\pi p_e'(Q_i)}{c'(X_i)} \\ F'(X_i) & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \theta \end{pmatrix} \qquad \text{B.6}$$

The eigen-values of this linearized system are:

$$\lambda_i^{\pm} = \frac{1}{2}\left\{ (g+\pi) \pm \sqrt{\left[(g+\pi)^2 - \frac{4\pi F'(X_i) p_e'(Q_i)}{c'(X_i)} \right]} \right\}. \qquad \text{B.7}$$

p_e' and c' are always negative. $F'(X_U) > 0$, and $F'(X_S) < 0$. Thus at (X_S, Q_S) the discriminant is strictly larger than $(r + \pi)^2$, and so one eigen-value is strictly positive, the other is strictly negative. This means that, locally, there exist 1 dimensional stable and unstable manifolds for this fixed point. At (X_U, Q_U), the discriminant is less than $(r + \pi)^2$, and may be negative. Both eigen-values have, therefore, strictly positive real parts, and the steady state is totally unstable.

Proposition B.4 X_S^π is decreasing with π. $X_S^0 = X_S^* > X_S$. There always exists π_l at which $X_S^{\pi l} = X_S$, so that for $0 < \pi < \pi_l$, $X_S^\pi > X_S$, and for $\pi > \pi_l$, $X_S^\pi < X_S$.

$$\pi_l = \frac{rc(X_S)}{p_e(X_S) - c(X_S)} \qquad \text{B.8}$$

Proof X_S^π is the point at which:

$$F(X_S^\pi) = \frac{1}{c'(X_S^\pi)}\left[(r+\pi)c(X_S^\pi) - \pi p_e(X_S^\pi)\right] \qquad \text{B.9}$$

When $\pi = 0$, this is the same as the relation defining X_S^*. Figure IV.4 shows X_S^π. Let $A(x,\pi) = [\ (r+\pi)c(x) - \pi p_e(x)\ ] / c'(x)$. Since the line $A(x,\pi)$ for constant π crosses $F(x)$ from below, $\partial A(X_S^\pi)/\partial x > F'(X_S^\pi)$. Hence,

$$\frac{\partial X_S^\pi}{\partial \pi}\frac{\partial A}{\partial x} + \frac{\partial A}{\partial \pi} = \frac{\partial X_S^\pi}{\partial \pi}F' \quad \Rightarrow \quad \frac{\partial X_S^\pi}{\partial \pi} = \frac{\partial A}{\partial \pi}\Big/\left(F' - \frac{\partial A}{\partial x}\right) < 0 \qquad \text{B.10}$$

If $X_S^\pi = X_S$, then $F(X_S) = 0$, so π_l, if it exists, satisfies $(r+\pi)c(X_S) = \pi p_e(X_S)$. Because $p_e > c$, a solution does exist.

Proposition B.5 If stores run out, they must do so at X_S^π. This is only possible if $X_S^\pi > X_S$, in which case it is a minimum of stores. If $X_S^\pi < X_S$ it is a maximum.

Proof The rate of change of stores goes from - to 0 to + as x falls across X_S^π. But if $X_S^\pi < X_S$, then $A(X_S, \pi) > 0$, and so x is increasing, not decreasing. Thus stores are at a maximum, as stated.

Proposition B.6 For given parameters, there is only one equilibrium path (but see Proposition IV.8) in the storage régime. If $\pi < \pi_l$, there is a path $Q = s_+^\pi(x) + x$ where stores run out at $X_S^\pi > X_S$ and the system reverts to the no storage régime. If $\pi \geq \pi_l$, the equilibrium path is the stable manifold of the fixed point (X_S, Q_S). See Figure IV.5

Proof If $\pi < \pi_l$, then stores may run out at X_S^π, and we get exactly the same equilibrium structure as in section III. The system cannot go to extinction before the switch, because that path would be the one the system would switch to. In that case, assuming a π hazard of switching is meaningless. The system cannot follow the stable manifold of (X_S, Q_S). Why not? Proposition IV.3 proves that $\pi p_e(Q_S) = (r + \pi)c(X_S)$. If $\pi < \pi_l$ then $(r+\pi)c(X_S) < \pi p_e(X_S)$, so we must have $X_S > Q_S$. This would mean that, if the system were on the stable manifold, it would have to tend to a point with strictly negative stores, which is not allowed. Thus it is not rational ever to be on the stable manifold. If $\pi > \pi_l$, then the opposite happens: stores may not run out at X_S^π, but the system may move along the stable manifold in equilibrium.

Proposition B.7 If $\pi > \pi_h$, then the system must be in the extinction equilibrium, where π_h is the hazard rate at which $Q_S = Q_{max}$, or

$$\pi_h = \frac{gc(X_S)}{c(X_U) - c(X_S)} \quad \text{B.11}$$

Proof Q_S is increasing in π and, once past π_l, (X_S, Q_S) is the only stable steady state before the switch to extinction. As discussed above, $Q \leq Q_{max}$ for all points in equilibrium before the switch. Thus if $Q > Q_{max}$, the stable manifold to (X_S, Q_S) cannot be an equilibrium. If there is a π at which $Q_S = Q_{max}$, then it satisfies: $\pi p_e(Q_{max}) = (r + \pi)c(X_S)$. But $p_e(Q_{max})$ is just $c(X_U)$ (recall Figure IV.1). Solving this for π, such a π_h does exist, and is as claimed.

Table I: Some Species Used for Storable Goods or by Collectors

Sources: [Goombridge 1992], [Life 1994], [Wall Street Journal 1994] and others

Bears Giant Panda Asiatic Black Bear Grizzly Bear South American Spectacled Bear Malayan Sun Bear Himalayan Sloth Bear **Cats** Tiger Cheetah *Lynx felis* *Lynx canadensis* Ocelot *Felis pardalis* Little spotted cat *F.* *tigrina* Margay *F. wiedii* Geoffroy's Cat *F.* *geoffroyi* Leopard Cat *F.* *bengalensis* **Other Mammals** Black Rhino Amur Leopard Caucasian Leopard Markhor Goat Saiga Antelope Cape Fur Bull Seal Sea Otter African Elephant Chimpanzees	**Lizards** Horned Lizard Latin American Spectacled Caiman *Caiman crocodilus* Tegus Lizard Monitor Lizard *Varanus niloticus* *V. exanthematicus* *V. salvator* *V. bengalensis* *V. flavenscens* **Snakes** *Python reticulatus* *P. molurus* *P. curtus* *P. sebae* *Eunectes spp.* Boa Constrictor Rat snake *Ptyas* *mucosus* Dog-faced Water Snake *Cerberus rhynchops* Sea snakes (genus *Lapemis* and *Homalopsis*) **Toads** Colorado River Toad **Turtles** Hawsbill Sea Turtle Egyptian Tortoise American Box Turtle **Butterflies** Schaus Swallowtail Homerus Swallowtail Birdwing Queen Alexandra's Birdwing *Ornithoptera* *alexandrae* **Birds** Red and Blue Lorry Parrots Quetzal *Pharomachrus* *mocinno* Roseate Spoonbill *Ajaia* *ajaia* Macaws *Ara spp.* Hyacinth Macaw	**Medicinal Plants** species of *Dioscorea* species of *Ephedra* *Dioscorea deltoidea* *Rauvolfia serpentina* *Curcuma spp.* *Parkia roxburghii* *Voacanga gradifolia* *Orthosiphon aristasus* *Rauvolfia* species of *Aconitum* **Rattan** *Calamus caesius* *C. manan* *C. optimus* **Orchids** *Dendrobium aphyllum* *D. bellatulum* *D. chrysotoxum* *D. farmeri* *D. scabrilingue* *D. senile* *D. thrysiflorum* *D. unicum* **Trees** *Astronium urundeuva* *Aspidosperma polyneuron* *Ilex paraguaiensis* *Didymopanax morotoni* *Araucaria hunsteinii* *Zeyhera tuberculose* *Cordia milleni* *Atriplex repanda* *Cupressus atlantica* *Cupressus dupreziana* *Diospyros hemiteles* *Aniba duckei* *Ocotea porosa* *Bertholetia excelsa* *Dipterix alata* *Abies guatemalensis* *Tectona hamiltoniana* Mahogany Teak	**Other Plants** Himalayan Yew Green Pitcher Plant Sm. Begonia Chisos Mt. Hedgehog's Cactus Key Tree Cactus Nellie Cory Cactus

ERRATUM

Working Paper #5674

Enclosed are Figures II.1 through IV.3 for Working Paper 5674, "Elephants," by Michael Kremer and Charles Morcom which were inadvertently omitted from the original paper. Please insert these sheets in your copy of the paper.

Dynamics of the Gordon-Schaefer Model with no Storage

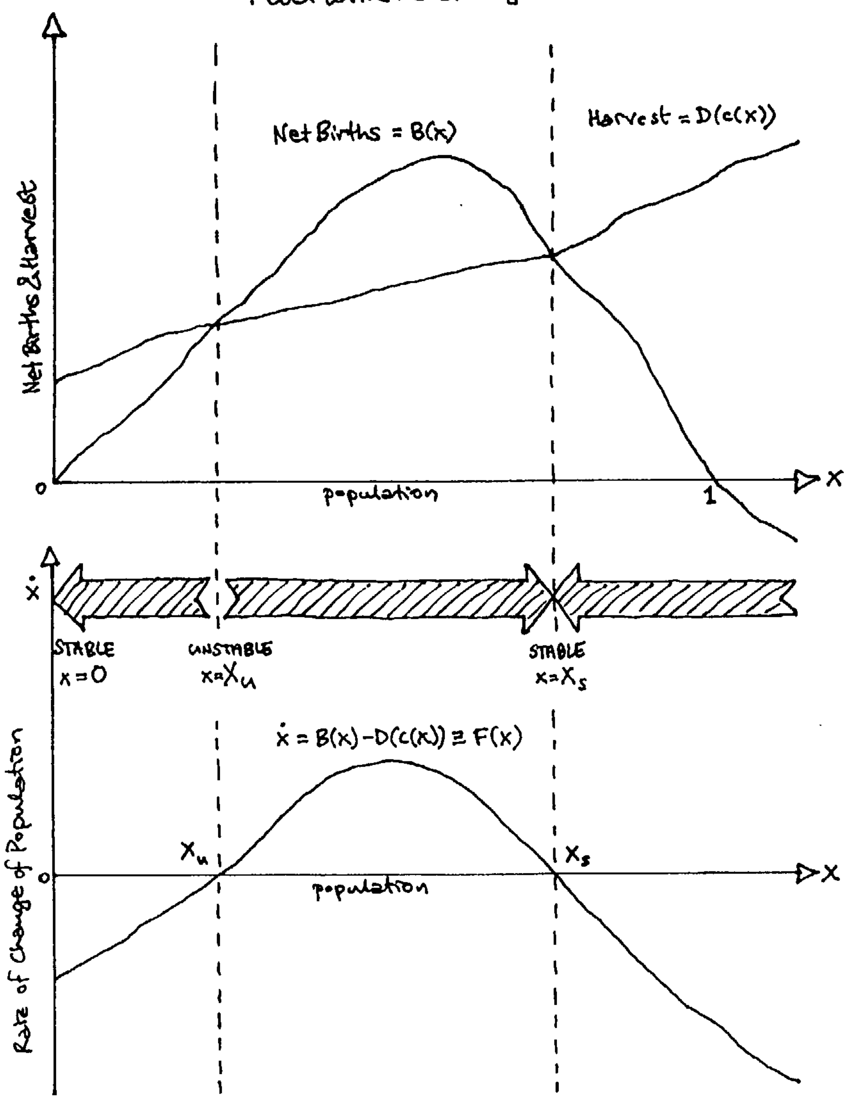

Figure II.1

The Storage Condition in the No Storage Régime : Definition of X_u^* & X_s^*

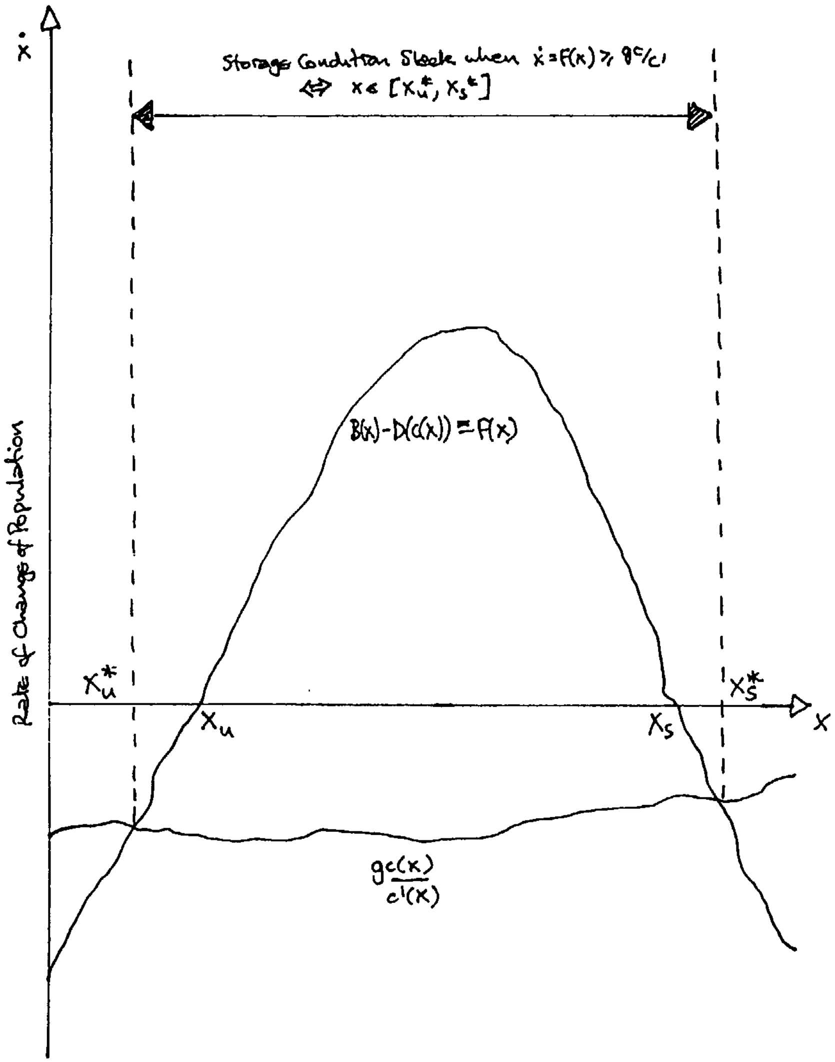

Figure III·1

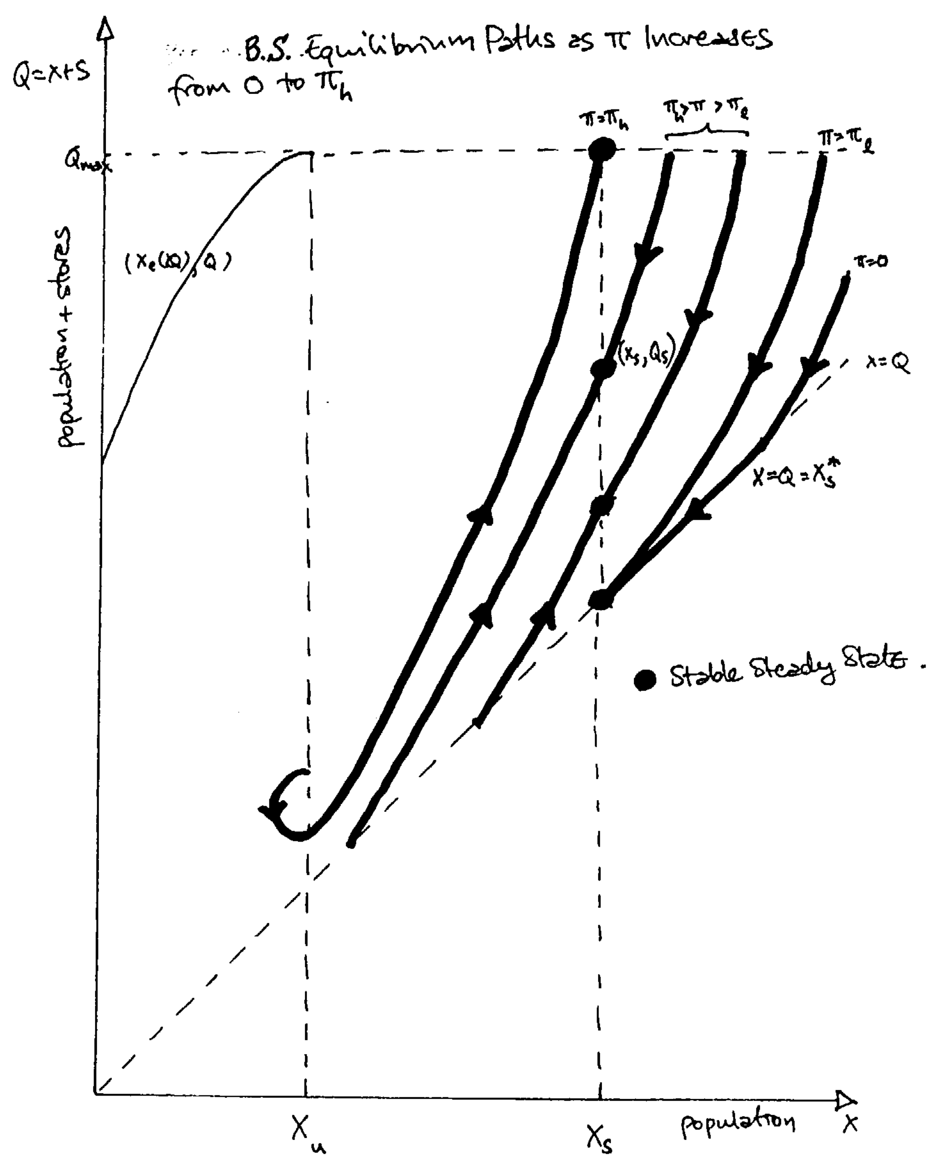

Figure IV.3

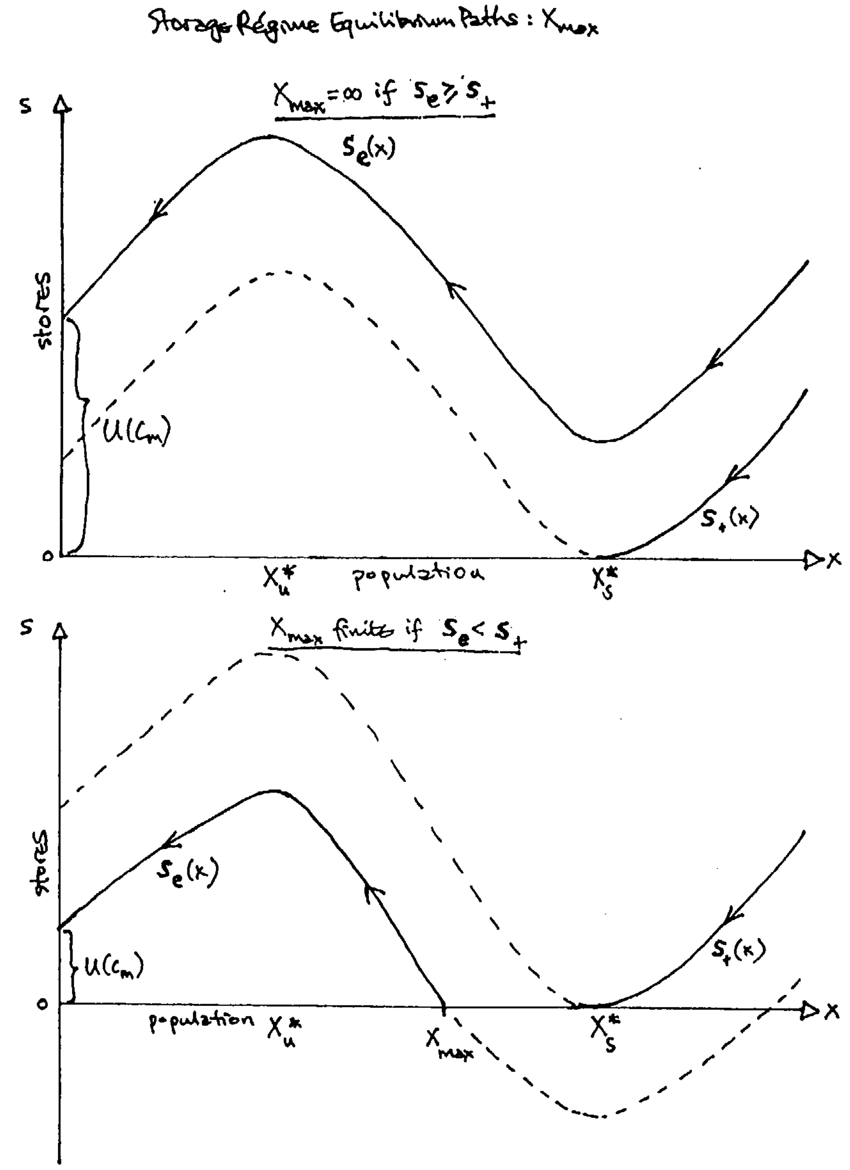

Figure III.2

The Storage and No Storage Regime Equilibrium Sets A_+ and A_e

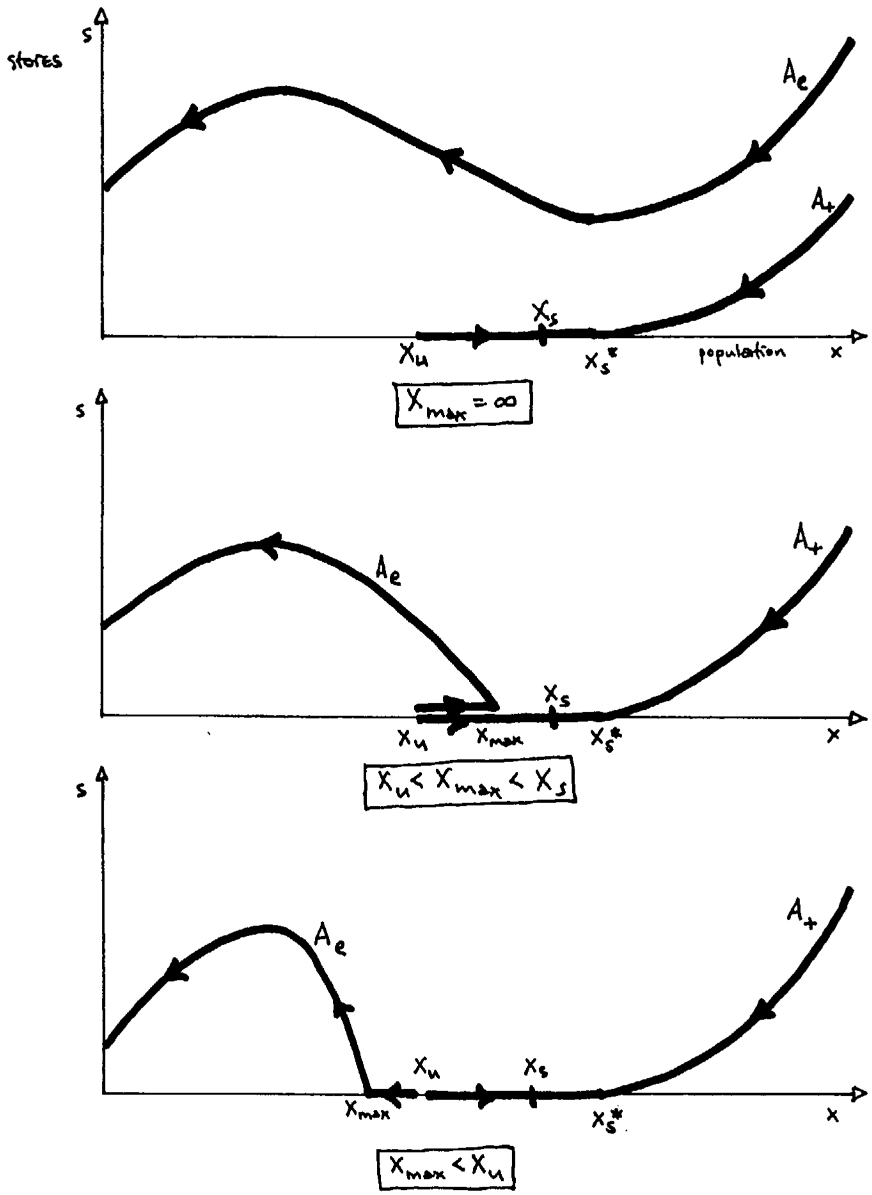

Figure III·3

The Equilibrium Stable-Manifold Path

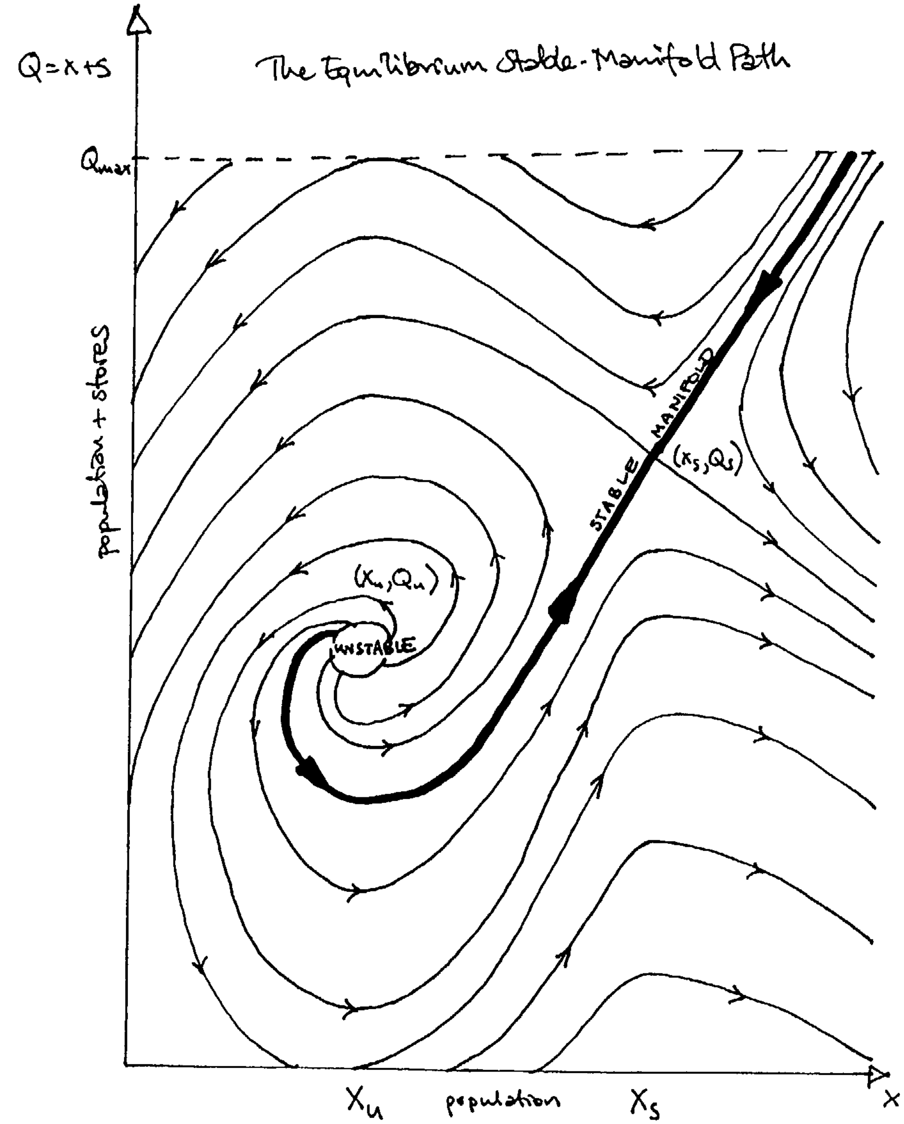

Figure IV·2

$x_e(Q)$: Switching to the Extinction Equilibrium

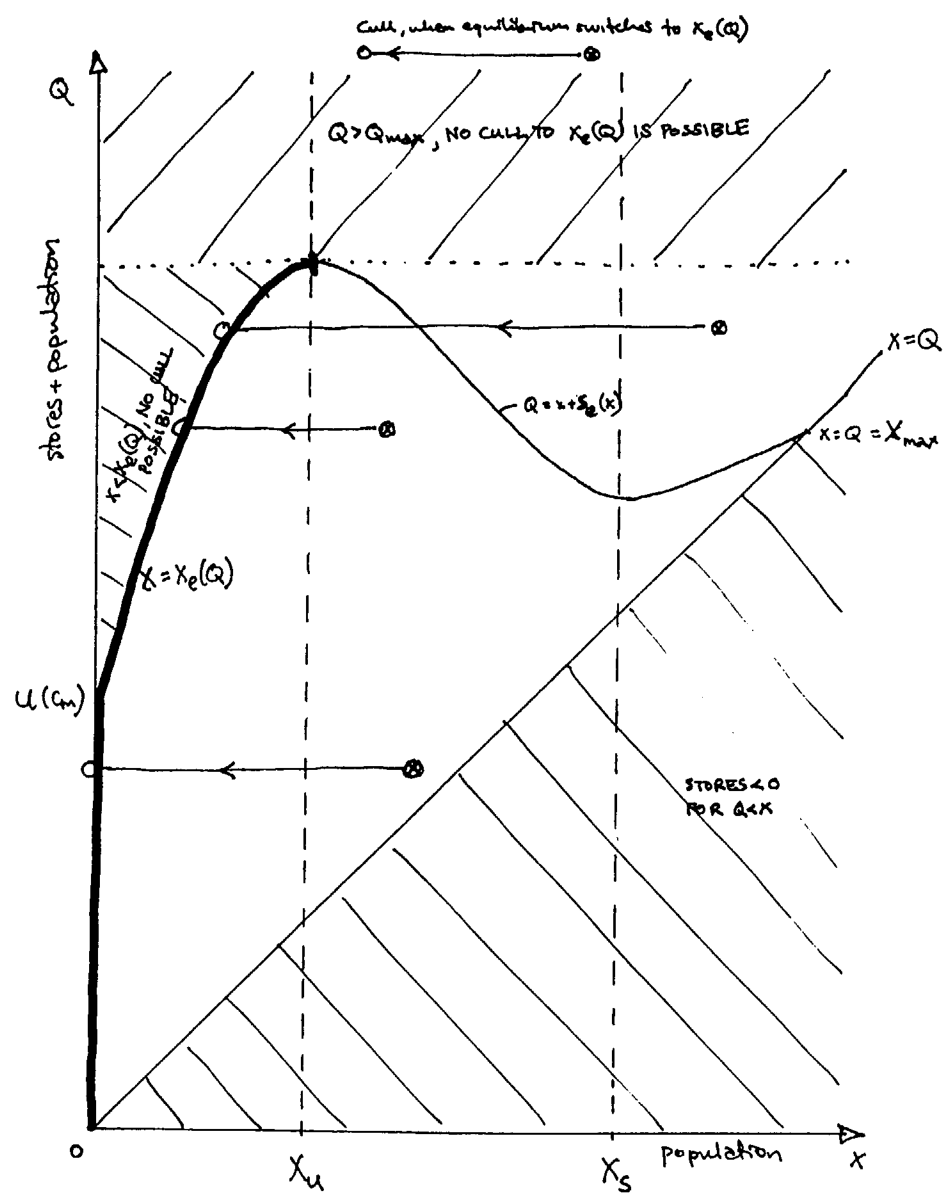

Figure IV.1

Definition of Q_{max}

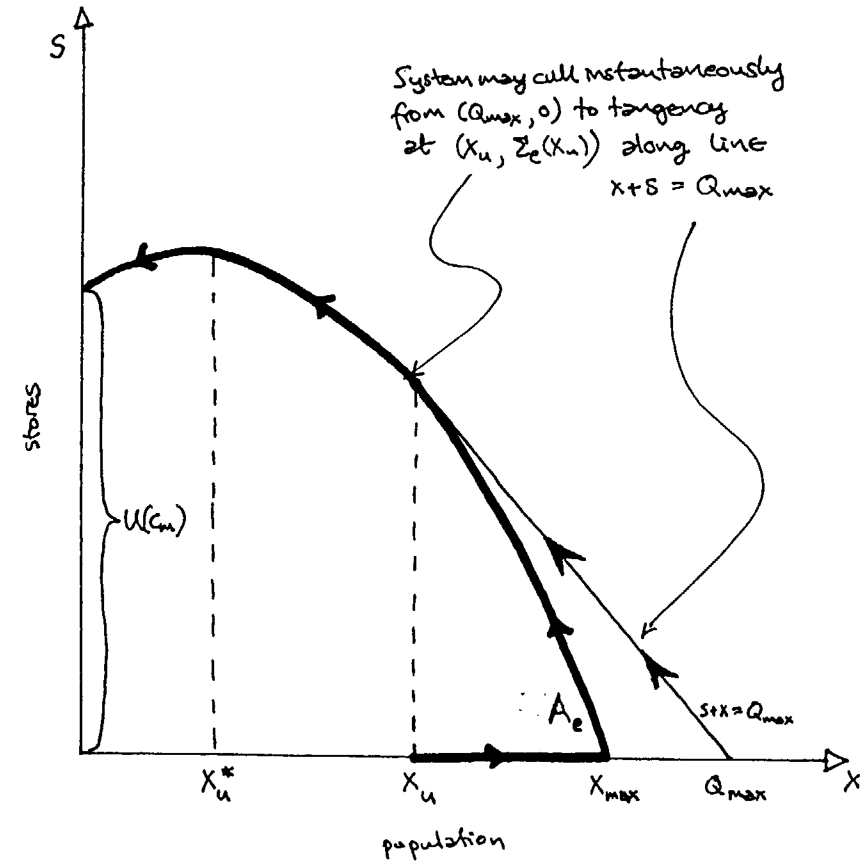

Figure III.4

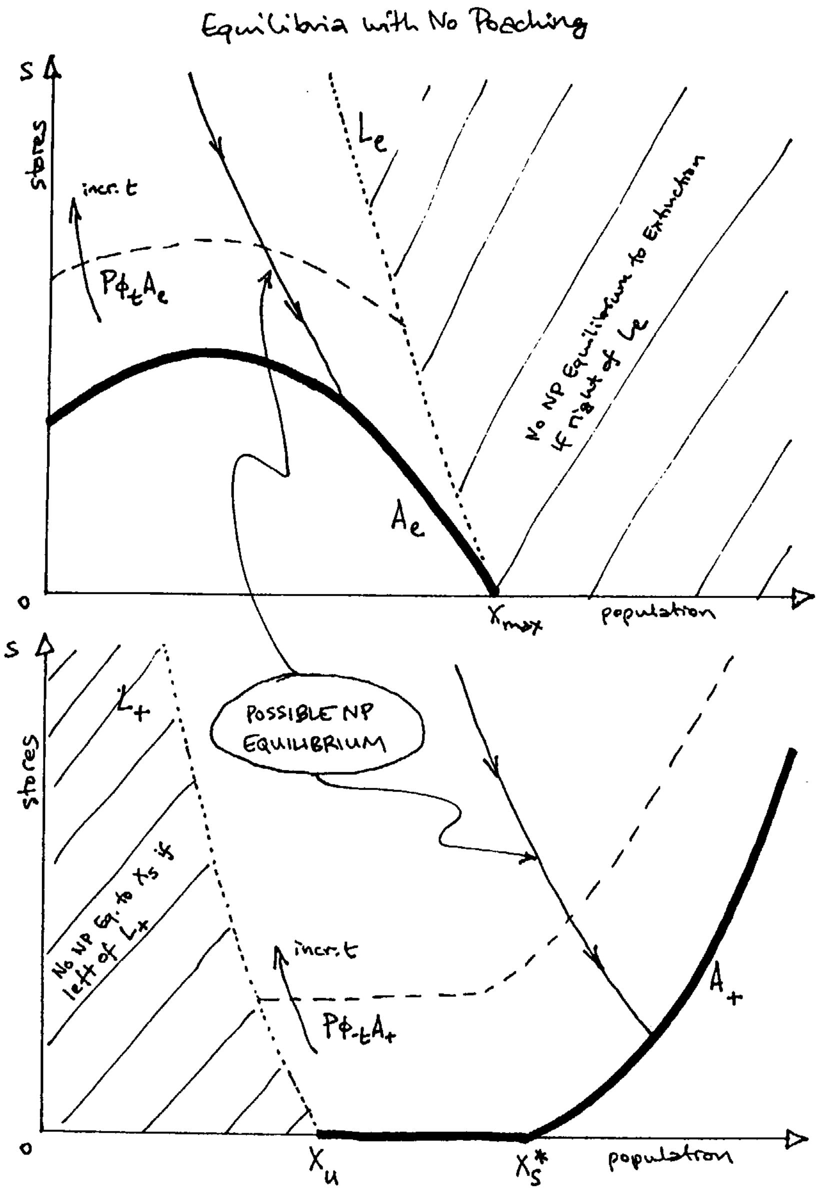

Figure III.5

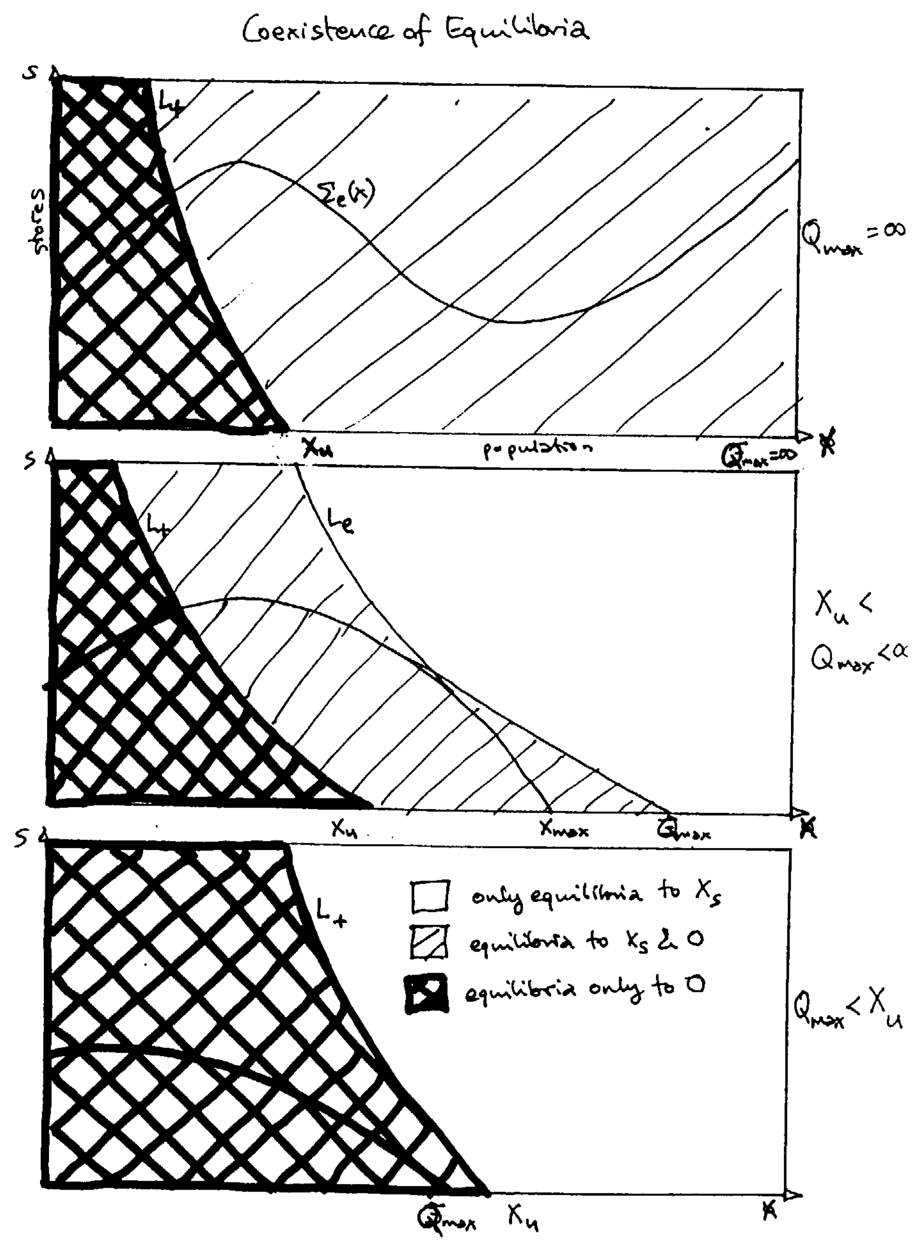

Figure III.6