NBER WORKING PAPER SERIES

THE COMOVEMENTS BETWEEN REAL ACTIVITY AND PRICES AT DIFFERENT BUSINESS CYCLE FREQUENCIES

Wouter J. den Haan

Working Paper 5553

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1996

I am especially grateful to Valerie Ramey for her insightful ideas and the many fruitful discussions on this topic. I also would like to thank Jeff Campbell, Timothy Cogley, Graham Elliott, Marjorie Flavin, Clive Granger, Bob Hall, Jim Hamilton, Tom Sargent, John Taylor, and Bharat Trehan for useful comments. This project is supported by NSF grant SBR-9514813. This paper is part of NBER's research program in Economic Fluctuations and Growth. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper 5553
April 1996

THE COMOVEMENTS BETWEEN REAL ACTIVITY AND PRICES AT DIFFERENT BUSINESS CYCLE FREQUENCIES

ABSTRACT

In this paper, I present two different methods that can be used to obtain a concise set of descriptive results about the comovement of variables. The statistics are easy to interpret and capture important information about the dynamics in the system that would be lost if one focused only on the unconditional correlation coefficient of detrended data. The methods do not require assumptions about the order of integration. That is, the methods can be used for stationary as well as integrated processes. They do not require the types of assumptions needed for VAR decompositions either. Both methods give similar results. In the postwar period, the comovement between output and prices is positive in the "short run" and negative in the "long run." During the same period, the comovement between hours and real wages is negative in the "short run" and positive in the "long run." I show that a model in which demand shocks dominate in the short run and supply shocks dominate in the long run can explain the empirical results, while standard sticky-price models with only demand shocks cannot.

Wouter J. den Haan
Department of Economics
University of California, San Diego
La Jolla, CA 92093-0508
and NBER

1. INTRODUCTION.

The observed correlation between macroeconomic variables plays an important role in the development and testing of business cycle theories. Macroeconomic models are judged on their ability to reproduce key correlations in the data, such as the comovement of output with wages, prices, and hours. Using these kind of empirical results to judge theories presupposes that there is a set of correlations upon which everyone can agree. In fact, the strength and even the sign of these empirical correlations are often very sensitive to the methods used to calculate them.

Nowhere is this problem more evident than in the case of the comovement between output and prices. For decades, it was generally believed that prices and output exhibited a positive correlation.[1] Often, some type of Phillips-curve effect was mentioned to rationalize a positive comovement between real activity and either prices or inflation. Recently, however, the sign of the relationship between output and prices has been called into question. In a noteworthy study, Cooley and Ohanian (1991) find that, although the correlation between output and prices is positive from 1870 to 1975, it appears to be negative during the postwar period. The correlation they study is the unconditional correlation coefficient of detrended GNP and prices, using various methods of detrending. In contrast, support for a positive correlation is given in Chadha and Prasad (1993), who argue that prices and GNP have to be detrended using different methods. The correlation between hours and real wages has also received considerable attention in the literature. Most empirical studies in the literature have shown that real wages are, if anything, procyclical rather than countercyclical.[2]

In this paper, I argue that an important source of the disagreement in the literature is the focus on only one correlation coefficient. By focusing on only the unconditional correlation, one is losing valuable information about the dynamic aspects of the comovement of variables. Moreover, since the unconditional correlation coefficient is only defined for stationary variables, the researcher has to transform the data to render it stationary, and there are many ways of doing this. I present two different types of methods that can be used to obtain a concise set of descriptive results about the comovement of variables. The statistics are easy to interpret and capture important information about the dynamics in the system that would be lost if one focused on only the unconditional correlation coefficient of detrended data. The first method calculates the correlations of VAR forecast errors at different horizons.[3] Identifying assumptions, usually needed for VAR decompositions, or assumptions about the order of integration of the variables are not required. That is, the methods can be used for stationary as well as

[1] Cf. Abel and Bernanke (1994), Blanchard and Fischer (1989), King and Watson (1994), Lucas (1972,1977), and Mankiw (1989).

[2] Cf. Dunlop (1938), Sargent (1978), Bils (1985), Bernanke and Powell (1986), Blanchard and Fischer (1989), and Abraham and Haltiwanger (1995).

[3] Following a suggestion by Jim Hamilton, Engel (1993) uses the forecast errors from a VAR to analyze the variance of several relative price indices.

integrated processes. The second method uses band-pass filters from the frequency domain proposed by Baxter and King (1994). In this paper, I prove, using spectral analysis for integrated processes, that the properties of these type of frequency filters are equivalent for stationary and first and second-order integrated processes. This result contradicts several claims in the literature that the properties of frequency domain filters depend on the order of integration of the input series.

I apply these methods to two examples: the comovement of output with the general price level, and the comovement of hours with real wages. Both methods give very similar results. For the postwar period, the comovement between GNP and prices is positive in the "short run" and negative in the "long run". In the complete sample from 1875 to 1993, the correlation between prices and GNP is positive at most frequencies. For the comovements between real wages and hours, I find a similar difference between the short run and the long run during the postwar period. In the "short run", the correlation between hours and real wages is negative, and in the "long run", it is positive. The results are robust to changes in the specification of the VAR, the data frequency, the particular price index used, and the particular postwar period considered.

Hansen and Heckman (1996) criticize the calibration approach for not matching a full set of dynamics of the model to the dynamics in the data. The statistics that are proposed in this paper do incorporate information about the dynamics but are still very intuitive. I argue that the type of statistics proposed in this paper is very effective in providing information to aid a researcher who wants to build a structural model. I support this claim by showing that the reported empirical findings have important implications for economic theories that have not been mentioned in the literature. For example, Lucas (1977) concluded from the empirical finding that real wages are acyclical that *"any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure"*. Boldrin and Horvath (1995) use the same empirical finding to motivate a model of real wage contracts. However, I find significant cyclical patterns when I distinguish between short-run and long-run movements, which suggest that cyclical movements in real wages should play an important role. The dynamic pattern found in the comovement of prices and output also has important implications for economic theories. Kydland and Prescott (1990) conclude that *"any theory in which procyclical prices figure crucially in accounting for postwar business cycle fluctuations is doomed to failure"*. My results indicate that a theory in which prices do not have some procyclical feature is, at best, missing an important part of the explanation of US business cycle fluctuations. A related question is whether the sign of the correlation coefficients can reveal the importance of demand versus supply shocks. One might think that a negative correlation between prices and output implies the presence of supply shocks. However, Chadha and Prasad (1993) and Judd and Trehan (1995) document that a standard "sticky-price" model with only demand shocks is capable of generating a negative unconditional correlation coefficient between prices and output for some detrending methods. The methods proposed in this paper reveal important information that is relevant to this question. In particular, I show that, in contrast to the results in Chadha and Prasad (1993) and Judd and Trehan (1995), a standard sticky-price model with only demand shocks cannot explain the complete set of empirical results reported in this paper. I argue that a model in which demand shocks dominate in the short run and supply shocks dominate in the long run is a plausible explanation for the empirical results.

The remainder of this paper is organized as follows. In section 2, I show how to use a VAR to measure the correlation between output and prices at different forecast horizons. In section 3, I show how to use band-pass filters to measure the correlation between output and prices at different frequencies. In section 4, I present the empirical results for the US economy regarding the comovements between prices and output. In section 5, I present the empirical results regarding the comovements between real wages and hours. In section 6, I discuss the implications of the empirical results for economic theories. The last section concludes.

2. MEASURING CORRELATIONS AT DIFFERENT FORECAST HORIZONS.

In section 2.1, I show how to use forecast errors to calculate correlation coefficients at different forecast horizons. In section 2.2, I analyze the relationship between this procedure and impulse response functions.

2.1 Using forecast errors to calculate correlation coefficients.

Consider an N-vector of random variables, X_t. The vector X_t is allowed to contain any combination of stationary processes and processes that are integrated of arbitrary order. If one wants to describe the comovement between prices, P_t, and output , Y_t, then X_t has to include at least P_t and Y_t. Estimating the VAR involves estimating:

$$(2.1) \qquad X_t = \mu + Bt + \sum_{l=1}^{L} A_l X_{t-l} + \varepsilon_t$$

where A_l is an $N\times N$ matrix of regression coefficients, μ and B are N-vectors of constants, ε_t is an N-vector of innovations, and the total number of lags included is equal to L. The estimated VAR can be used to construct a time series of k-period ahead forecast errors for each of the elements of X_t. I denote the k-period ahead forecast and the k-period ahead forecast error of the variable Y_t by $Y^{f}_{t+k,k}$, and $Y^{ue}_{t+k,k}$, respectively. I do the same for P_t. The forecast errors are used to calculate the covariances or correlation coefficients between the two series. I denote the covariance between the two random variables $P^{ue}_{t+k,k}$ and $Y^{ue}_{t+k,k}$ by $COV(k)$ and the correlation coefficient between these two variables by $COR(k)$. If the series are stationary, then the correlation coefficient of the forecast errors will converge to the unconditional correlation coefficient of the two series as k goes to infinity. In appendix E I show that a consistent estimate of $COV(k)$ and $COR(k)$ does not require any assumptions on the order of integration of X_t. For example, it is possible that X_t contains stationary as well as integrated processes. An important assumption for the derivation of this consistency results is that equation (2.1) is correctly specified. In particular, the lag order must be large enough to guarantee that ε_t is not integrated. That is, if X_t contains I(1) stochastic processes, then the lag order has to be at least equal to 1, and when X_t contains I(2) stochastic processes, then the lag order has to be at least equal to 2.

Using an unrestricted VAR in levels leads to consistent estimates of the covariances of the k-period ahead forecast errors both when X_t does and when X_t does not include integrated processes. Alternatively, one can estimate a VAR in first differences or an error-correction system. When the restrictions that lead to these systems are correct, then imposing them may or may not lead to more efficient forecasts in a finite sample.[4] Asymptotically, there is no efficiency gain.[5] However, if they are not correct, then they lead to inconsistent estimates of the correlation coefficients. This suggests that, in practice, one would want to estimate the VAR in levels.

2.2 The relationship with impulse response functions.

There is an alternative way to use the VAR to construct measures of comovements at different forecast horizons. The alternative method clarifies the relationship between this procedure and impulse response functions. The k-period ahead forecast error, $Y^{ue}_{t+k,k}$, can be written as follows:

(2.2)
$$Y^{ue}_{t+k,t} = \left(Y_{t+k} - Y^{f}_{t+k,t+k-1}\right) + \left(Y^{f}_{t+k,t+k-1} - Y^{f}_{t+k,t+k-2}\right) + \cdots + \left(Y^{f}_{t+k,t+1} - Y^{f}_{t+k,t}\right)$$

In this equation, the k-period ahead forecast error is written as the sum of the updates in the forecast of Y_{t+k}, starting at period $t+1$. The first term on the right hand side is just the one-period ahead forecast error realized at period $t+k$. The second term is the update of the two-period ahead forecast. I denote the update in the k-period ahead forecast at period t by $Y^{\Delta f}_{t+k,k}$. Also, I denote the covariance between $Y^{\Delta f}_{t+k,k}$ and $P^{\Delta f}_{t+k,k}$ by $COV(k^{\Delta})$. Since the terms on the right hand side of equation 2.1 are serially uncorrelated, there is a simple relation between $COV(k^{\Delta})$ and $COV(k)$. That is,

(2.3)
$$COV(k) = \sum_{\kappa=1}^{k} COV(\kappa^{\Delta}).$$

When k is equal to one, the two covariances are identical. The "$COV(k^{\Delta})$" covariances, therefore, contain the same information as the "$COV(k)$" covariances. The "$COV(k^{\Delta})$" covariances clarify the relation between these statistics and impulse response functions. Suppose that $X_t = A\, Z_t$, where A is an $N \times M$ matrix of coefficients and Z_t is an M-vector of (independent) fundamental shocks. Let $Y^{imp,m}_{k}$ be the

[4] See Hamilton (1994, page 516) for a discussion.

[5] See Fuller (1976).

effect on output of a one standard deviation shock in the m^{th} element of Z_t after k periods. Thus, $Y_k^{imp,m}$ is the impulse response of the m^{th} element of Z_t on Y_t. I define $P_k^{imp,m}$ in the same way. Then, $COV(k^{\Delta})$ is equal to the sum of the products of the k-step impulse responses across all fundamental shocks. That is,

$$(2.4) \qquad COV(k^{\Delta}) = \sum_{m=1}^{M} Y_k^{imp,m} P_k^{imp,m}.$$

Thus, $COV(k^{\Delta})$ measures the comovement of output and prices after k periods in response to a *typical* shock. $COV(k)$ accumulates the effects over k periods. The impulse response functions give complete information about the comovements of output and prices after any type of shock. Estimating impulse response functions, however, requires making identifying assumptions. The results often depend on the particular type of identifying assumptions, and the assumptions are often ad hoc. The advantage of the procedure proposed in this paper is that it does not require making these type of assumptions. The disadvantage of this procedure is that it only gives the dynamic effect of a typical shock.

3. MEASURING CORRELATIONS AT DIFFERENT FREQUENCIES.

In this section, I show how to use spectral analysis to decompose series by frequency and to measure the correlations of two series at different frequencies. In section 3.1, I assume that the variables are stationary. In section 3.2, I show that the procedures remain valid if the series are first or second-order integrated processes, or when the series contain a deterministic linear or quadratic time trend.

3.1 Frequency-domain filters for stationary processes.

From the Wold-theorem, I know that any covariance stationary series has a time-domain representation. Equivalently, any covariance stationary series has a frequency-domain representation. Informally, the variable x_t can be represented as a weighted sum of periodic functions of the form $\cos(\omega t)$ and $\sin(\omega t)$, where ω denotes a particular frequency. The frequency domain representation is given by

$$(3.1) \qquad x_t = \mu + \int_0^{\pi} \alpha(\omega)\cos(\omega t)\,d\omega + \int_0^{\pi} \delta(\omega)\sin(\omega t)\,d\omega$$

Here, $\alpha(\cdot)$ and $\delta(\cdot)$ are random processes. The spectrum of a series x_t is given by

$$(3.2) \qquad S_x(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} \gamma_j\, e^{-i\omega j}, \qquad -\pi \le \omega \le \pi,$$

where γ_j is the j^{th} autocovariance and $i^2 = -1$. The spectrum is useful in determining which frequencies are important for the behavior of a stochastic variable. If the spectrum has a peak at frequency $\omega = \pi/3$, then the cycle with periodicity 6 (= $2\pi / (\pi/3)$) periods is quantitatively important for the behavior of this stochastic variable. Consider the following examples. If x_t is white noise, then the spectrum is flat. A flat spectrum means that all cycles are equally important for the behavior of the variable x_t. This is intuitive since the existence of cycles implies forecastibility, and white noise is, by definition, unforcastable. Next, suppose that x_t is an AR(1) with coefficient ρ, where $0 < \rho < 1$. The spectrum for this random variable has a peak at $\omega = 0$ and is monotonically decreasing with $|\omega|$. Since the periodicity of a cycle with zero frequency is infinite, this stochastic process does not have an observable cycle. If the stochastic variable x_t has a unit root, then the spectrum would be infinite at frequency zero.

Baxter and King (1994) show how to construct filters that isolate specific frequency bands, while removing stochastic and deterministic trends. Suppose one wants to isolate that part of a stochastic variable x_t that is associated with frequencies between ω_1 and ω_2, with $0 < \omega_1 < \omega_2 \leq \pi$. If $\omega_2 = \pi$, then the filter is called a high-pass filter since all frequencies higher than ω_1 are included. If $\omega_2 < \pi$, then the filter is called a band-pass filter. The filters are two-sided symmetric linear filters and can be expressed as follows.

(3.3) $$x_t^F = B(L)\, x_t$$

where x_t^F is the filtered series, L is the lag operator, and

(3.4) $$B(L) = \sum_{h=-\infty}^{\infty} b_h\, L^h, \qquad \text{with } b_h = b_{-h}.$$

Let the Wold-representation for x_t be given by

(3.5) $$x_t = C(L)\, \varepsilon_t .$$

Then,

(3.6) $$x_t^F = B(L)\, C(L)\, \varepsilon_t ,$$

A useful result in spectral analysis is that the spectrum of x_t^F is given by

(3.7) $$S_{x^F}(\omega) = |B(e^{-i\omega})|^2\, S_x(\omega),$$

where $|B(e^{-i\omega})|$ is the gain of the filter $B(L)$. The spectrum of the filtered series x_t^F has to be equal to S_x if $|\omega| \in [\omega_1, \omega_2]$ and equal to zero if ω is outside this set. Therefore, the gain of the filter has to be equal to one if $|\omega| \in [\omega_1, \omega_2]$ and equal to zero otherwise. Using the converse of the Riesz-Fischer theorem, one can find the time-series representation, i.e. $B(L)$, that corresponds to these conditions on the gain of the filter. The formulas are as follows

$$b_0 = \frac{\omega_2 - \omega_1}{\pi}$$

(3.8)

$$b_h = \frac{\sin(\omega_2 h) - \sin(\omega_1 h)}{\pi h}. \qquad h = \pm 1, \ldots$$

The ideal filter is an infinite moving average and cannot be applied in practice. In practice one has to truncate $B(L)$. This gives an approximate filter $A(L)$, where

(3.9) $$A(L) = \sum_{h=-K}^{K} a_h L^h,$$

and K is the truncation parameter. I set this parameter equal to 20 for the experiment using postwar data, and equal to 40 for the experiment using the whole sample. These numbers are much higher than the values used in Baxter and King (1994), but I find that some of the estimated correlations are sensitive to the choice of K for lower values of K.[6] [7] Note that a higher value of K means a more accurate band-pass filter but, also, the loss of more data points. The ideal filter $B(L)$ has the property that $B(1)=0$. To ensure the same property for the feasible filter $A(L)$, I adjust the coefficients of $A(L)$ in such a way that they add up to zero as well. Let θ be equal to

(3.10) $$\theta = \frac{-\sum_{h=-K}^{K} b_h}{2K+1}.$$

As in Baxter and King (1994), I adjust the coefficients as follows

(3.11) $$a_h = b_h + \theta$$

Note that the distortion introduced by this restriction goes to zero if K goes to infinity.

3.2 Frequency-domain filters for non-stationary processes.[8]

In this section, I analyze the properties of frequency-domain filters for more general stochastic processes. In particular, I show that the properties derived in section 3.2 remain valid when the input

6 See appendix B.

7 Other approaches in the frequency domain also have to deal with the endpoints. The HP filter deals with the endpoints by using a different filter for each observation in the sample. Band spectrum regressions assume that the last observation is at the same point in the cycle as the first observation. (See Engle (1974)).

8 This section has benefited a lot from discussions with Clive Granger.

series are integrated stochastic processes or when the input series have a linear or quadratic time trend. To prove this, I have to define the spectrum of a non-stationary random process. Although most of the literature on spectral analysis focuses on stationary processes, there are some exceptions.[9] In fact, Hannan (1970) and Priestley (1988) consider much more general non-stationary processes than the ones considered in this section.

First, I will discuss first-order integrated processes. When the series x_t is integrated, then the covariances used to define the spectrum in equation (3.2) are not well defined. Therefore, I will define the spectrum of an integrated process as the limit of the spectrum of a stationary stochastic process. The motivation for this definition is the following. According to the Beveridge-Nelson decomposition, one can, under mild regularity conditions, write an I(1) process as the sum of a random walk, initial conditions, and a stationary process.[10] Thus,

$$(3.12) \qquad x_t = c + x_{t-1} + e_t,$$

where e_t is a stationary process. The assumption is made that the spectrum of e_t is continuous in a neighborhood around zero. This means that the number of deterministic sinusoidal components with frequencies close to zero is finite. Consider the following "AR(1)-type" process:[11]

$$(3.13) \qquad x_t = c + \rho x_{t-1} + e_t.$$

Equation (3.12) can be written as:

$$(3.14) \qquad x_t = c + \lim_{\rho\to 1} \rho x_{t-1} + e_t.$$

Note that, as long as $|\rho| < 1$, the process defined in (3.13) is stationary and has a well-defined spectrum, which I denote by $S_\rho(\omega)$. Equation (3.14) motivates the following definition of the spectrum of an I(1) process:

$$(3.15) \qquad S_x(\omega) \equiv \lim_{\rho\to 1} S_\rho(\omega) = \lim_{\rho\to 1} \left|\frac{1}{1-\rho e^{-i\omega}}\right|^2 S_e(\omega).$$

Note that $S_x(\omega)$ is finite for all frequencies except zero. Similarly, I can define the spectrum of an I(2) stochastic process as

$$(3.16) \qquad S_x(\omega) = \lim_{\rho\to 1} \left|\frac{1}{1-2\rho e^{-i\omega}+\rho^2 e^{-2i\omega}}\right|^2 S_e(\omega)$$

---

9 For example, Hannan (1970) and Priestley (1988).

10 See, for example, Hamilton (1994).

11 This process is not necessarily an AR(1), since e_t could be serially correlated.

The frequency-domain filter $B(L)$ satisfies the following properties:

$$(3.17)\qquad \begin{aligned} \left|B(e^{-i\omega})\right| &= 1 \qquad \text{if} \quad \omega_1 \le \omega \le \omega_2, \quad \text{and} \\ \left|B(e^{-i\omega})\right| &= 0 \qquad \text{if} \quad \omega < \omega_1, \omega > \omega_2. \end{aligned}$$

Moreover, $B(1) = 0$. In appendix F, I show that these properties imply that $B(L)$ can be written as[13]

$$(3.18)\qquad B(L) = (1\text{-}L)\,\overline{B}(L)\ ,$$

where $\overline{B}(1) < \infty$. Consequently, $B(L)$ has the property that it can make first-order integrated processes stationary. Let $x_t^F = B(L)\,x_t$. When x_t is a stationary process, then the following holds:

$$(3.19)\qquad \begin{aligned} S_{x^F}(\omega) &= S_x(\omega) \quad \text{if} \quad \omega_1 \le \omega \le \omega_2, \quad \text{and} \\ S_{x^F}(\omega) &= 0 \qquad\quad \text{if} \quad 0 \le \omega < \omega_1, \ \omega_2 < \omega \le \pi. \end{aligned}$$

I have to show that these properties also hold when x_t is an I(1) process. In this case,

$$(3.20)\qquad x_t^F = \frac{B(L)}{(1-L)} e_t\,.$$

Because of (3.18), $B(L)/(1\text{-}L)$ is well defined. This means that the spectrum of x_t^F is well defined and is given by

$$(3.21)\qquad S_{x^F}(\omega) = \left|\frac{B(e^{-i\omega})}{1-e^{-i\omega}}\right|^2 S_e(\omega) = \lim_{\rho\to 1}\left|\frac{B(e^{-i\omega})}{1-\rho e^{-i\omega}}\right|^2 S_e(\omega) = \left|B(e^{-i\omega})\right|^2 S_x(\omega)$$

Recall that $S_x(\omega)$ is well defined for all values of ω bigger than zero. This, together with (3.17), implies that S_{x^F} is equal to zero for $0 < \omega < \omega_1$. It remains to be shown that S_{x^F} is equal to zero when ω is equal to zero. Equation (3.18) implies that $B(e^{-i\omega})/(1\text{-}\ e^{-i\omega})$ is well-defined when ω equals zero. Since S_e and $B(e^{-i\omega})/(1\text{-}\ e^{i\omega})$ are continuous in a neighborhood around zero, S_{x^F} is continuous at zero. This, together with the fact that S_{x^F} is equal to zero for values of ω close to zero, means that S_{x^F} is equal to zero for ω equal to zero.

Harvey and Jaeger (1993), Cogley and Nason (1995), and Osborn (1995) argue that the properties of frequency-domain filters depend on the order of integration of the input series. This clearly contradicts the analysis above. These papers reach a different conclusion because they focus on the

---

13 This property is well-known for finite moving-average representations. However, for infinite representations, (3.18) does not automatically follow from the property that $B(1) = 0$. In appendix F, I show that (3.18) holds for symmetric infinite representations. An example of an infinite-order polynomial for which (3.18) does not hold is the filter $D(L)=(1\text{-}L)^d$, with $0 < d < 1/2$. This filter has the property that $D(1) = 0$, but if $\overline{D}(L)$ is defined by $D(L) = \overline{D}(L)\,(1\text{-}L)$, then $\overline{D}(1) = \infty$.

stationary part of the series, although the filter is always applied to the level. Consider, for example, the process

$$(3.22) \qquad x_t = \rho x_{t-1} + e_t,$$

where e_t is an arbitrary stationary process. When $|\rho| < 1$, these papers compare $B(L)\, x_t$ with the stationary part of the series, i.e. x_t. But, when $\rho = 1$, they compare $B(L)\, x_t$ with the stationary part of the series, i.e. $(1\text{-}L)x_t$. Thus, when $|\rho| < 1$, they analyze the properties of the filter $B(L)$, and when $\rho = 1$, they analyze the properties of the filter $B(L)/(1\text{-}L)$. This means that there is a discontinuity in the focus of the analysis when ρ equals 1. I prefer the analysis above that uses the definition of the spectrum for integrated processes. Note that if a researcher is interested in the first-difference of an integrated process instead of the level he can, of course, apply the filter to Δx_t as opposed to x_t.

Next, I discuss second-order integrated processes. The analysis given above can be easily generalized to second-order processes when the filter $B(L)$ can be written as

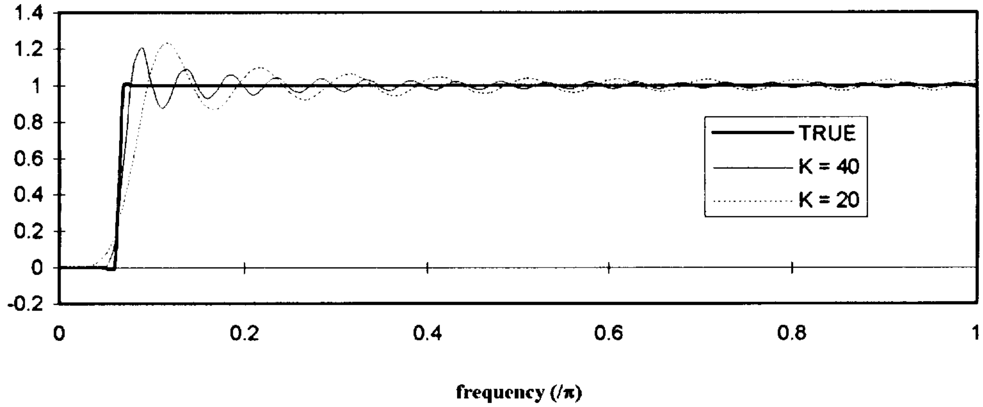
$$(3.23) \qquad B(L) = (1\text{-}L)\,(1\text{-}L^{-1})\; \overline{B}(L) = -L^{-1}\,(1\text{-}L)^2\; \overline{B}(L)\,,$$

where $\overline{B}(1) < \infty$. Equation (3.23) is satisfied for finite-order symmetric filters with the property $B(1) = 0$. In appendix F, I show that a symmetric infinite-order filter with $B(1)$=0 only satisfies equation (3.23) when the coefficients vanish at a sufficiently fast rate. The coefficients of the infinite-order frequency domain filters discussed in this section do not vanish fast enough. However, as long as ω is not equal to zero, then $B(e^{-i\omega})/(1 - 2\, e^{-i\omega} + e^{-2i\omega})$ is still well-defined. Thus, for the infinite-order filter, the properties in (3.19) hold for second-order integrated processes when $\omega \neq 0$. When one uses the finite-order approximation, then equation (3.23) is satisfied. Thus, the finite-order filters discussed in this paper have the property that the spectrum of the filtered series is equal to the squared gain of the approximate filter times the spectrum of the input series, for stationary and for first and second-order integrated processes.

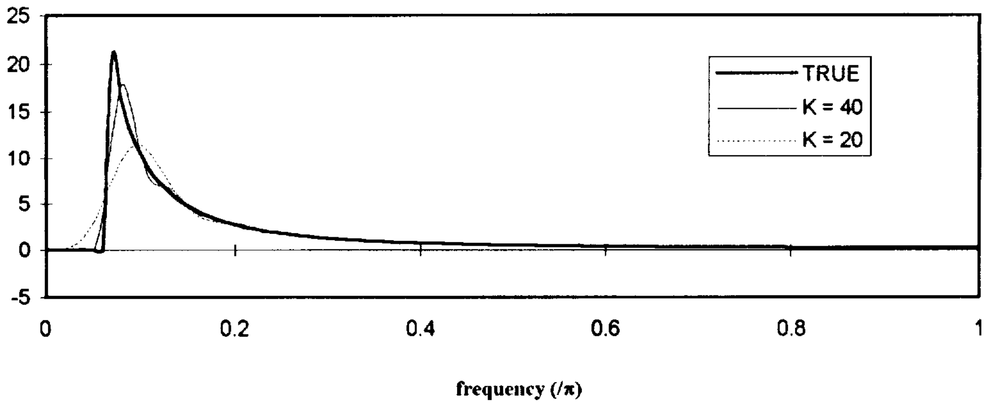
However, for finite-order filters, there is another reason why it may matter whether the input series are integrated or not. Consider the approximation of the filter that eliminates all frequencies below ω_l. The squared gain of the approximate filter is not exactly equal to zero for frequencies less than ω_l, and not exactly equal to 1 for frequencies bigger than ω_l. Since the spectrum of the filtered series is equal to the squared gain times the spectrum of the input series, the approximation error may be bigger for processes for which the value of the spectrum goes to infinity as the frequency goes to zero, i.e. integrated processes.

FIGURE 1: SPECTRA OF FILTERED RANDOM PROCESSES ($\omega_1 = \pi/16$, $\omega_2 = \pi$).

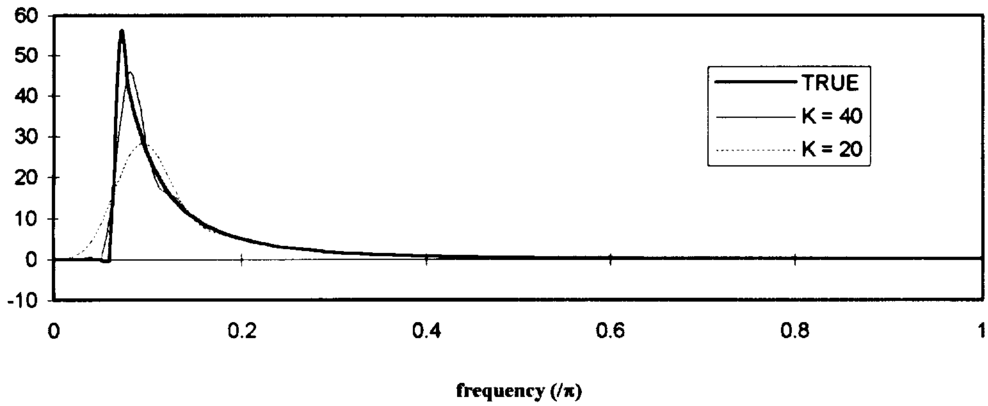
A: White Noise (Squared Gain)



B: AR(1) with Coefficient equal to 0.95



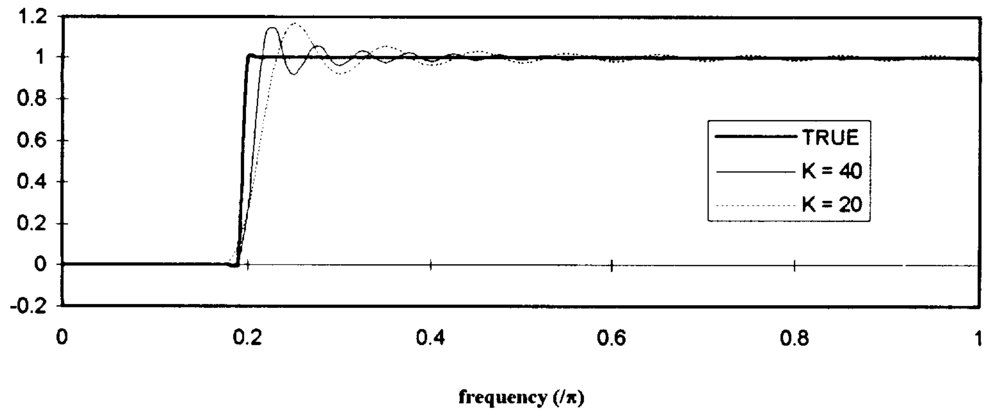
C: Integrated AR(1) with Coefficient equal to 0.4



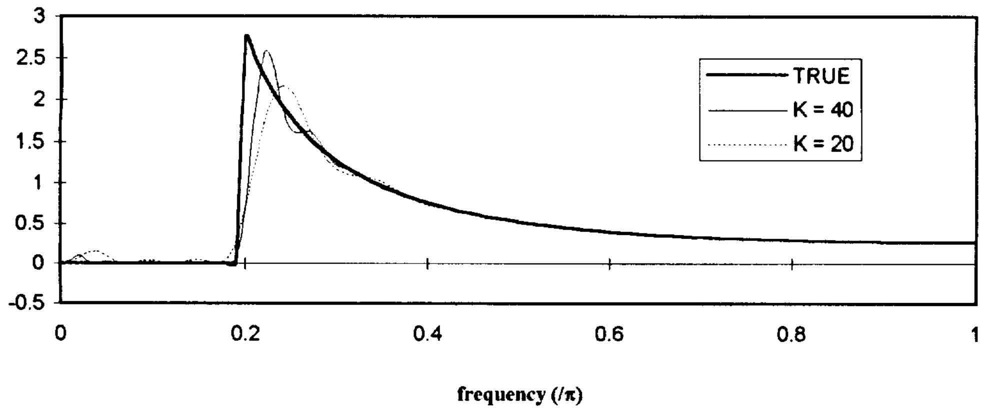
Note: The variance of the white noise process in panel A is chosen in such a way that the graph also represents the squared gain. Using equation (3.7), one can calculate the spectrum of the filtered series as the squared gain times the spectrum of the original random variable.

FIGURE 2: SPECTRA OF FILTERED RANDOM PROCESSES ($\omega_1 = \pi/4$, $\omega_2 = \pi$).

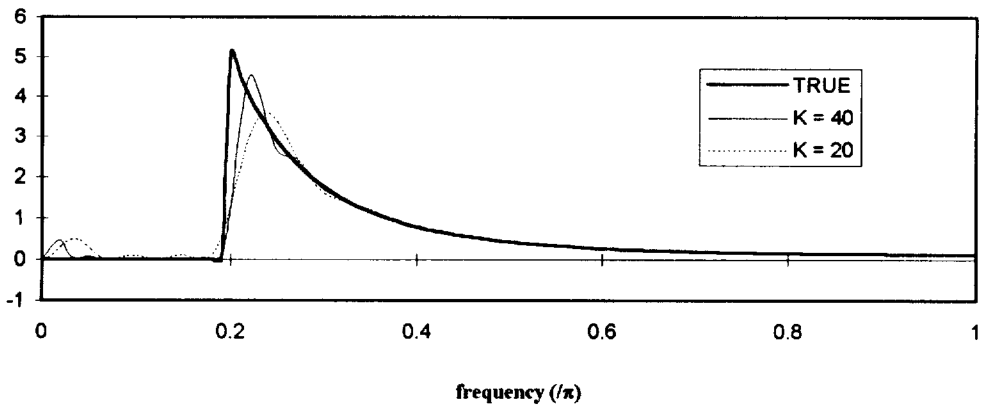
A: White Noise (Squared Gain)



B: AR(1) with Coefficient equal to 0.95



C: Integrated AR(1) with Coefficient equal to 0.4



Note: The variance of the white noise process in panel A is chosen in such a way that the graph also represents the squared gain. Using equation (3.7), one can calculate the spectrum of the filtered series as the squared gain times the spectrum of the original random variable.

To address this question, I calculate the spectrum of some filtered processes. The filters that I consider are two high-pass filters that eliminate all cycles associated with periods bigger than 8 periods and 32 periods, respectively. Besides the ideal infinite-order high-pass filter, I also consider two approximate filters with truncation parameters equal to 40 and 20, respectively. I consider three stochastic processes. The first is a white noise process, where the variance is chosen in such a way that the spectrum of the filtered series is equal to the squared gain. The second process is an AR(1) with a coefficient equal to 0.95. The third process is an integrated AR(1) with a coefficient equal to 0.4. The results are given in figures 1 and 2. As documented in the graph, for all stochastic processes the approximation is better for the frequencies that are less than ω_l than for the frequencies just above ω_l. Also, the approximation errors are bigger for the two serially correlated processes than for the white noise process. This suggests that the truncation parameter that one uses should depend on the persistence of the underlying process; i.e., a higher truncation parameter is needed for more persistent processes. The graph also shows that the study of approximation errors does not reveal a fundamental difference between the persistent stationary process and the integrated process studied here. For example, for K equal to 40 and ω_l equal to $\pi/16$, the peak of the approximated spectrum is 14% and 12% less than the peak of the true spectrum for the stationary persistent process and the integrated process, respectively.

Now suppose that the series x_t has a linear time trend. That is, x_t can be written as

(3.24) $$x_t = a\,t + y_t,$$

where y_t is a stationary or integrated process. When one applies the filter $B(L)$ and uses the results shown in (3.18), then

(3.25) $$x_t^F = B(L)\,x_t = \overline{B}(1)\,a + B(L)\,y_t$$

and

(3.26) $$\begin{aligned} S_{x^F}(\omega) &= S_y(\omega) \quad \text{if} \quad \omega_1 \le \omega \le \omega_2, \text{ and} \\ S_{x^F}(\omega) &= 0 \quad \text{if} \quad \omega < \omega_1, \omega > \omega_2. \end{aligned}$$

Thus, the filter removes a linear trend, and (3.19) holds for the non-deterministic part of the series. When one uses a finite-order filter to approximate $B(L)$, then one can use the results in (3.23), and the filter will also take out a quadratic time trend.

4. THE COMOVEMENTS BETWEEN US PRICES AND OUTPUT.

In this section, I use the procedures from sections 2 and 3 to analyze the comovements between prices and output in the US economy. In section 4.1, I estimate spectra of the series. In section 4.2, I use the k-period ahead forecast errors to calculate correlation coefficients, and in section 4.3, I use the frequency-domain filters to calculate correlation coefficients. I use quarterly data from 1875 till 1993 for the log of GNP and the log of GNP's implicit price deflator.[14] I analyze both the full sample and the postwar period from 1954 to 1993. In appendix B, I show that the pattern of results discussed in this section is extremely robust. In particular, I show that the results do not change when I

- impose unit root specifications in the estimation of the VAR,
- add monetary indicators to the VAR,
- use monthly instead of quarterly data,
- use a different price index, or
- divide the postwar period into different subsamples.

4.1 The spectra of US prices and output.

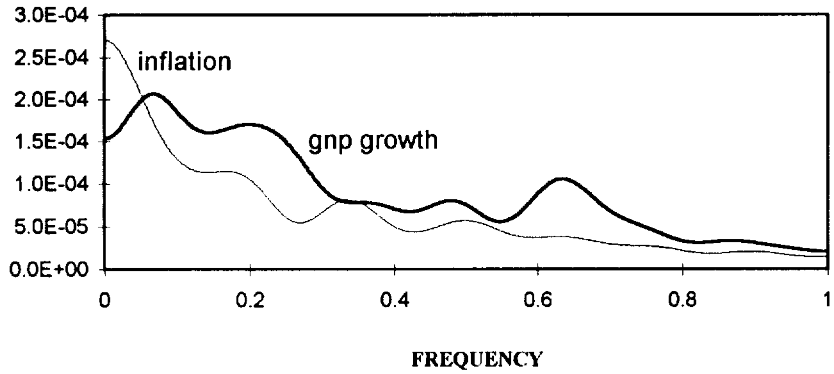
In this section, I estimate the spectra of detrended measures of output and prices using a selection of detrending procedures. The point I want to make is that the time-series properties of some of these detrended series are very different. In this case, the unconditional correlation coefficient is a misleading statistic. Consider the correlation between the two random variables X_t and Y_t. Let X_t be an AR(1) process with an AR coefficient equal to 0.99 and innovation ε_t. Let Y_t be white noise that is perfectly correlated with ε_t. In this case, the unconditional correlation coefficient between X_t and Y_t is equal to 0.14. This low number obscures the perfect correlation between the innovations of the two series. The method using the forecast errors from a VAR would have revealed this perfect correlation at the one-period forecast horizon. A high-pass filter would similarly document the high correlation. For this case, the correlation coefficient for series associated with cycles less than four periods would be equal to 0.89. Both methods would approach 0.14 if the forecast horizon, or the periodicity of the cycles included, goes to infinity.

---

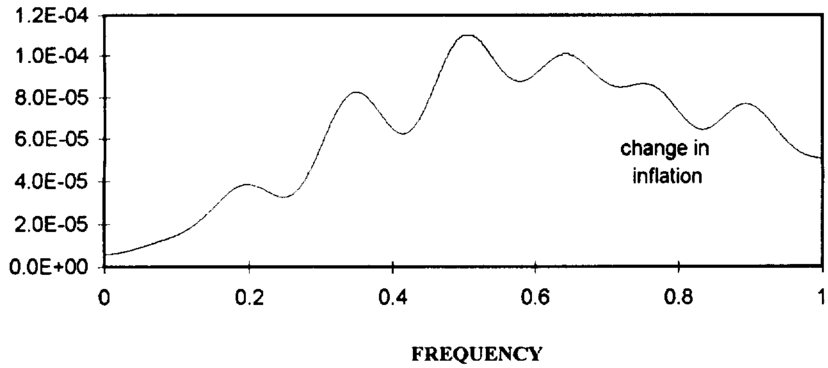
[14] A detailed description of the data is given in appendix A.

FIGURE 4: SPECTRA OF DETRENDED PRICES AND OUTPUT.

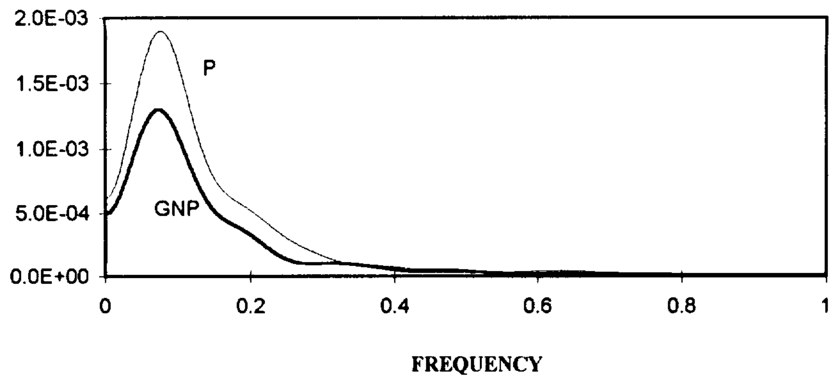
A: First Differences



B: Second Differences



C: HP-Filter



Note: Figures 4A, 4B, and 4C display the estimated spectra using quarterly US data from 1875 to 1993, using the indicated filter. The estimator for the spectrum is given in footnote 12.

In figure 4A, I plot the spectra of first-differenced output and prices. In figure 4B, I plot the spectrum of first-differenced inflation.[15] Note that the spectra of GNP growth differs somewhat from the spectrum of inflation and is very different from the spectrum of the change in inflation. Consistent with this finding is that the autocorrelation coefficient of inflation is substantially higher than the autocorrelation coefficient of GNP growth. In figure 4C, I plot the spectra of output and prices filtered using the Hodrick-Prescott (HP) filter.[16] Note that the HP filter generates, by construction, series with a similar spectrum. Chadha and Prasad (1994) propose to use the correlation between inflation and HP filtered GNP as a measure of the comovement between prices and real activity. As documented in figure 4, the spectra of HP filtered GNP and inflation are very different, which indicates that the time-series properties will be different. As documented by the example in the last paragraph, the unconditional correlation coefficient between two series may be very hard to interpret when the time-series properties are so different. The time-series properties of the series obtained with the two methods proposed in this paper are similar by construction. Series that are detrended using first differences, using the Beveridge-Nelson decomposition, or by taking out a deterministic time trend do not have this property.

4.2 Correlation between US prices and output at different forecast horizons.

To calculate the forecast errors, I estimate a bivariate fourth-order VAR including the log of GNP, the log of the implicit price deflator, a constant, and a linear time trend. In appendix B, I show that the results are robust to several changes in the specification of the VAR. In figure 5A, I document the correlation between the k-period ahead forecast errors. During the postwar period, the correlation is positive for forecast horizons less than five years, and the correlation is negative for forecast horizons equal to five years and higher. Within the complete sample, however, the correlation is positive at any forecast horizon. In figure 5B, I plot the covariance between the changes in the k-period ahead forecasts of GNP and prices. During the postwar period, this covariance is positive for forecast horizons less than 3 years. As explained in section 2, the covariance is better than the correlation coefficient in revealing the quantitative importance of the comovements. As documented in figure 5b, the positive covariances for the low forecast horizons, are quantitatively less important than the negative correlations at the higher forecast horizons. Also, the covariance between the one-quarter ahead forecast errors of prices and output is small. This is true for the postwar period as well as for the complete sample. The results reported in this section are consistent with King and Watson (1994). They document that there is a positive

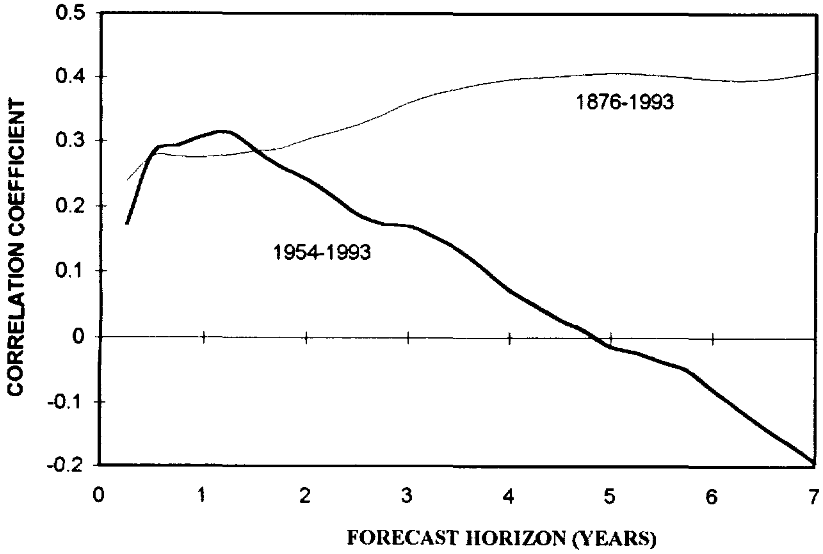
[15] The estimator of the spectrum is equal to: $\hat{S}_x(\omega) = \frac{1}{2\pi}\left\{\hat{\gamma}_0 + 2\sum_{j=1}^{20}\left(1-\frac{j}{21}\right)\hat{\gamma}_j \cos(\omega j)\right\}$. The general shape of the spectrum is insensitive to the truncation parameter. See Christiano and Den Haan (1996) and Den Haan and Levin (1994) for a discussion on alternative spectral density estimators.

[16] See Prescott (1986) and King and Rebello (1993) for a discussion of the HP filter.

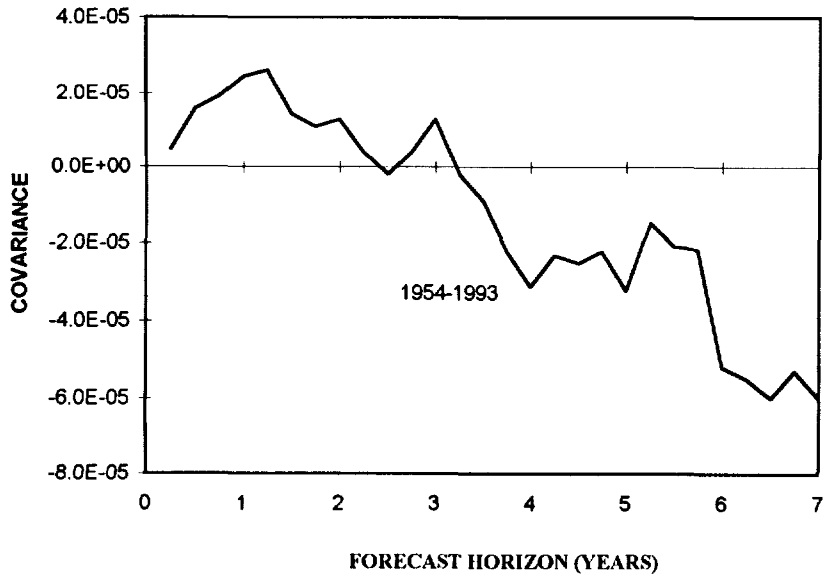
correlation between the "trend" values of the unemployment and inflation rates during the postwar period and a negative correlation between the "business-cycle" components of the two series.

FIGURE 5: COMOVEMENT BETWEEN PRICES AND OUTPUT.

A: Comovement Between Forecast Errors



B: Comovement Between the Updates in the Forecasts



Note: Figures 5A and 5B show the indicated measure of comovement between GNP and its deflator using US data for the indicated period.

In table D in appendix D, I report the numerical values of the estimates and their significance levels for the postwar period.[17] As documented in table D1, most of the positive estimates for the correlation between the forecast errors are significant at conventional significance levels and none of the negative estimates, found at the long forecast horizons, are significant. Note that the calculation of standard errors is difficult for the longer forecast horizons since the correction for serial correlation becomes more important.[18] I repeat the exercise using monthly data for the deflator of nondurable consumption and the index for industrial production. The results are reported in table D3. With this data set, I find a similar pattern for the sign of the correlation coefficients. Using the monthly data set, I find that the positive estimates for the short forecast horizon are highly significant both for the bivariate VAR and for the VAR that includes monetary indicators. The negative estimates at the long forecast horizons are insignificant for the forecast errors from a bivariate VAR but significant for the forecast errors from the VAR with the monetary indicators.

4.3 Correlation between US prices and output at different frequencies.

In figures 6A and 6B, I plot the correlation coefficients of the filtered series of GNP and prices using the high-pass and band-pass filter, respectively. Since the filter is a two-sided moving average, one loses some observations at the beginning and end of the period. For the complete sample, I set K equal to 40, and, for the postwar period, I set K equal to 20. These high numbers are necessary since, at lower values, increases in K would change the results quantitatively.[19] As shown in figure 6A, the correlation between the series associated with a cycle less than or equal to 5 years is positive during the postwar period. The band-pass filter, that also eliminates higher frequency cycles with an increase in the periodicity, becomes negative at a lower periodicity. For the complete sample, the correlation coefficients calculated with the high-pass filter are always positive. The correlation coefficients reach a minimum when the periodicity is around 5 years. At this periodicity, the correlation coefficient of prices and output filtered with the band-pass filter is negative. The popular Hodrick-Prescott filter is an approximation to a high-pass filter that includes that part of the series associated with cycles less than 8 years. As documented in figure 6A, the correlation coefficients of output and prices are indeed very similar for these two filters.

In appendix D, I report the estimates for the correlation coefficients and their significance levels for the postwar period. As documented in table D2, some of the positive estimates at the low periodicities are significant, and several of the negative estimates at the high periodicities are significant.

---

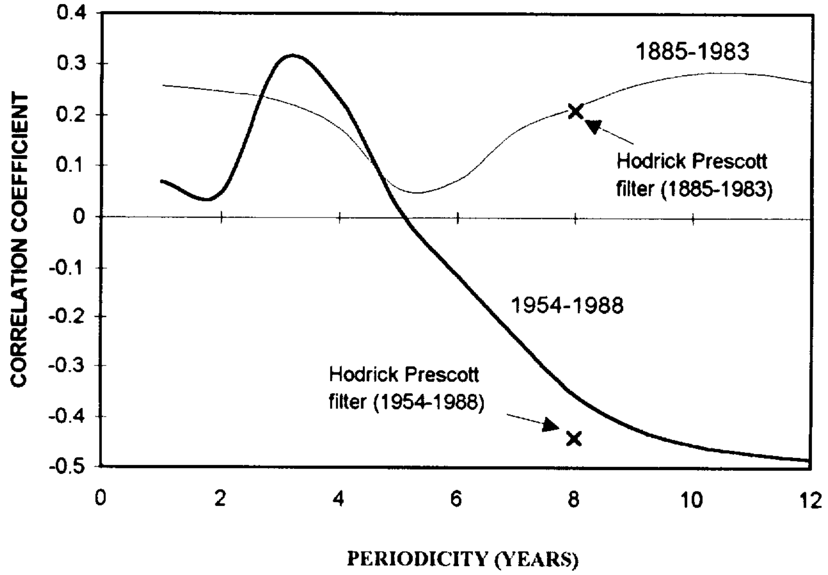
[17] For the complete sample, the estimates are almost always significantly positive. These results are available on request.

[18] To correct for serial correlation, I use the VARHAC procedure from Den Haan and Levin (1994). Similar significance levels are found when I use the automatic bandwidth procedure from Newey and West (1993). There are several studies that document the difficulty of obtaining accurate standard errors in small samples in the presence of serially correlated errors. See, for example, Christiano and Den Haan (1996) for a study using typical business cycle statistics.

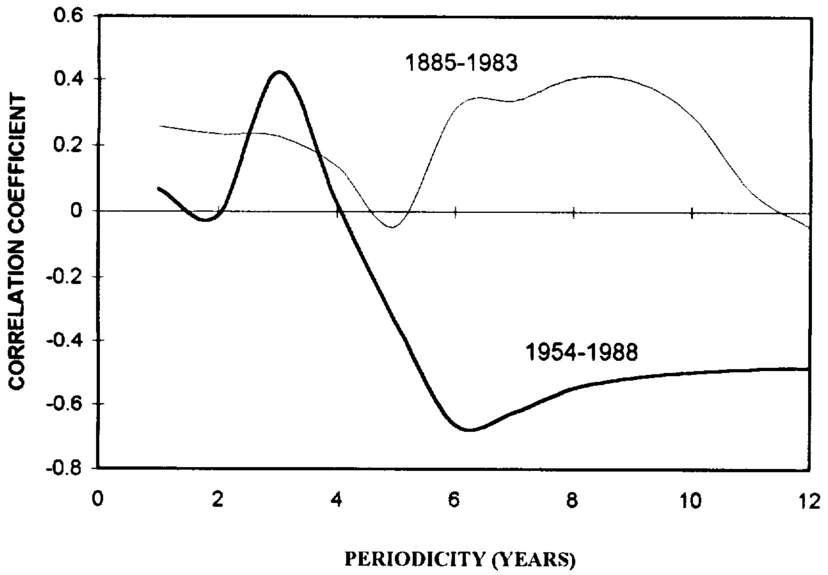
[19] See appendix B for a sensitivity analysis of the results to changes in the truncation parameter.

FIGURE 6: COMOVEMENT BETWEEN PRICES AND OUTPUT.

A: High-Pass Filter



B: Band-Pass Filter



Note: Figures 6A and 6B show the correlation coefficient between filtered US GNP and prices over the indicated time period, where the indicated filter is used to filter out cycles with the indicated periodicity. The high-pass filter removes all cycles with a period higher than the indicated periodicity. The band-pass filter removes all cycles with a period higher than the indicated periodicity as well as the cycles with a period less than the indicated periodicity minus one year.

5. THE COMOVEMENTS BETWEEN US REAL WAGES AND HOURS.

In this section, I analyze the comovement between real wages and hours in the postwar period. I use quarterly data from 1954 to 1993 for real compensation per hour in the business sector and per capita hours of all employees in the business sector.[20] In appendix C, I perform a sensitivity analysis of the results reported in this section. I find the same results in several postwar subsamples and for several wage deflators.

The debate over the comovement between real wages and hours in US data seems to focus on the question of whether the (unconditional) correlation coefficient of the real wage rate and the amount of hours worked is zero or slightly positive.[21] I will show that the data are much more informative when one does not focus entirely on one correlation coefficient. With the methods proposed in this paper, I find support for a negative as well as a positive correlation coefficient. That is, I find that, in the "short run", the correlation between real wages and hours is negative, and, in the "long run", it is positive. Figure 7A displays the results of the procedure that uses the forecast errors from the VAR. Again, a bivariate VAR with four lags, a constant, and a linear time trend is used. As documented in the figure, the correlation between forecast errors with a forecast horizon less than two and a half years is negative. At higher forecast horizons, the correlation is positive, although, at some point, it gets close to zero again. Figure 7B displays the results using the frequency domain filters. When the filter removes all cycles with a period just above two years, the correlation between real wages and hours is almost zero. When the cutoff is chosen to be a lower number, the correlation is negative, and, when the cutoff is higher, the correlation is positive. The estimates and their significance are reported in tables D1 and D2 in appendix D. I find that there are highly significant negative "short run" correlation coefficients and highly significant positive "long run" coefficients for the filtered series. Using the VAR forecast errors, I find that one of the negative "short run" coefficients and two of the positive "long run" coefficients are significant.

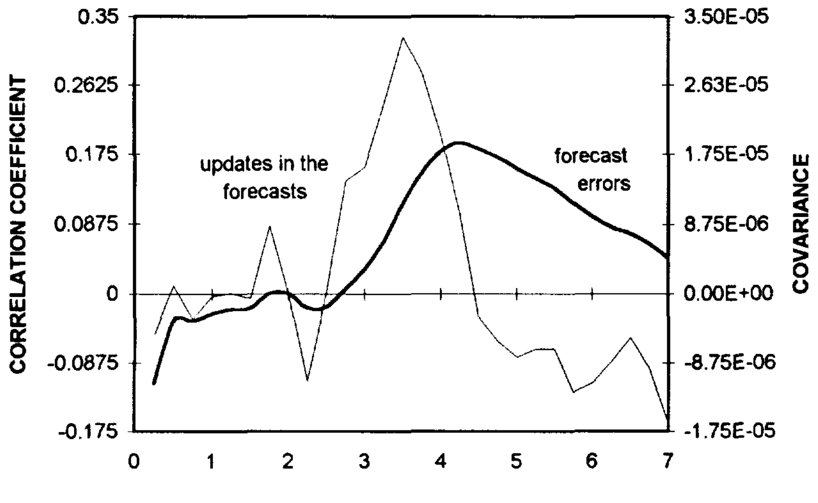
---

[20] A detailed description of the data is given in appendix A.

[21] Cf. Dunlop (1938), Sargent (1978), Bils (1985), Bernanke and Powell (1986), Blanchard and Fischer (1989), and Abraham and Haltiwanger (1995).

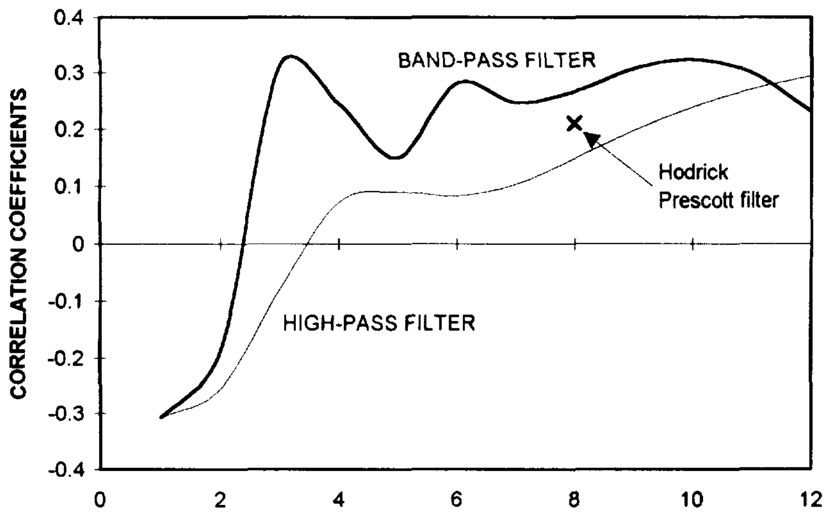
FIGURE 7: COMOVEMENT BETWEEN REAL WAGES AND HOURS.

A: VAR FORECASTS



Note: This figure displays the covariance of the updates of the forecasts of real wages and hours and the correlation between the forecast errors of real wages and hours. A bivariate VAR is used to construct the forecasts. The VAR is estimated using quarterly data from 1954 to 1993.

B: FREQUENCY DOMAIN FILTERS



Note: This figure displays the correlation coefficient between filtered real wages and hours over the period from 1954 to 1988, where the indicated filter is used to filter out cycles with the indicated periodicity. The high-pass filter removes all cycles with a period higher than the indicated periodicity. The band-pass filter removes all cycles with a period higher than the indicated periodicity as well as the cycles with a period less than the indicated periodicity minus one year.

6. IMPLICATIONS FOR ECONOMIC THEORY.

In this section, I show that the results reported in the last two sections have important implications for economic theories. The examples in this section illustrate that the proposed statistics are very effective in revealing the type of information that is important for economists who try to build a structural model. In section 6.1, I discuss the implications of the results for the comovement of prices and output. In section 6.2, I discuss the implications of the results for the comovement of real wages and hours.

6.1 Economic theory and the comovements between prices and output.

An interesting question is whether the sign of the correlation coefficients can reveal the importance of demand versus supply shocks. For example, one might think that a negative correlation between prices and output implies the presence of supply shocks. However, as pointed out by Judd and Trehan (1995), the sign of the unconditional correlation coefficient also depends on dynamic responses in the model. In particular, Chadha and Prasad (1993) and Judd and Trehan (1995) document that a standard "sticky-price" model with only demand shocks is capable of generating a negative unconditional correlation coefficient between prices and output for some detrending methods. Without additional assumptions, the methods proposed in this paper also do not identify the underlying fundamental shocks of the system. Nevertheless, they reveal important additional information that is relevant to this question. First, in section 6.1.1 I show that, in contrast to the results in Chadha and Prasad (1993) and Judd and Trehan (1995), a standard sticky-price model with only demand shocks cannot explain the complete set of empirical results reported in this paper. To be precise, I show that one can generate the negative correlation between VAR forecast errors only under implausible conditions on the demand shock. The second reason why the empirical results reported in this paper have information about the relevance of demand and supply shocks is the following. In a static model, the most plausible consequence of a positive demand shock is an increase in output and the price level, and the most plausible consequence of a positive supply shock is an increase in output and a decrease in the price level. Short run effects in a model are not affected by internal propagation mechanisms. Therefore, I think it would be hard to develop a model without demand shocks in which the "short-run" correlation between prices and output is positive. I argue that a model in which demand shocks dominate in the short run and supply shocks dominate in the long run is a plausible explanation for the empirical results. As an example, I show that a standard "sticky-price" model with empirically plausible demand and supply shocks can explain the pattern of correlation coefficients observed in the data.

6.1.1 A standard sticky-price model with only demand shocks.

The sticky-price model that I analyze in this section is very similar to the model analyzed in Chadha and Prasad (1993). The output in this economy is determined by demand. The demand for output is given by

$$(6.1) \qquad y_t = y_t^d = M_t - P_t + \overline{D}_t = D_t - P_t,$$

where y_t denotes the logarithm of output at time t, the superscript d denotes demand, M_t denotes the logarithm of the nominal money stock, P_t represents the logarithm of the aggregate price level, $\overline{D}_t$ is a non-monetary nominal demand term, and D_t combines the monetary and the non-monetary demand terms. The demand term D_t behaves according to

$$(6.2) \qquad (1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)D_t = \varepsilon_t, \qquad \lambda_3 < \lambda_2 < \lambda_1 \le 1, \text{ and } |\lambda_i| \le 1.$$

Note that D_t could have a unit root , in which case $\lambda_1 = 1$. The logarithm of the supply of output is exogenous and given by

$$(6.3) \qquad y_t^s = a + b\,t.$$

Without loss of generality, I set the parameters a and b equal to zero. I define $\widetilde{P}_t$ as the price level at which aggregate demand equals supply. Thus,

$$(6.4) \qquad \widetilde{P}_t = D_t - y_t^s.$$

I assume that prices adjust gradually towards the equilibrium price level. In particular,

$$(6.5) \qquad P_t = (1-\beta)P_{t-1} + \beta\widetilde{P}_t, \qquad \text{where} \quad 0 < \beta \le 1.$$

The law of motion for prices in this model is given by

$$(6.6) \qquad P_t = \frac{\varepsilon_t}{\left(1-(1-\beta)L\right)\left(1-\lambda_1 L\right)\left(1-\lambda_2 L\right)\left(1-\lambda_3 L\right)},$$

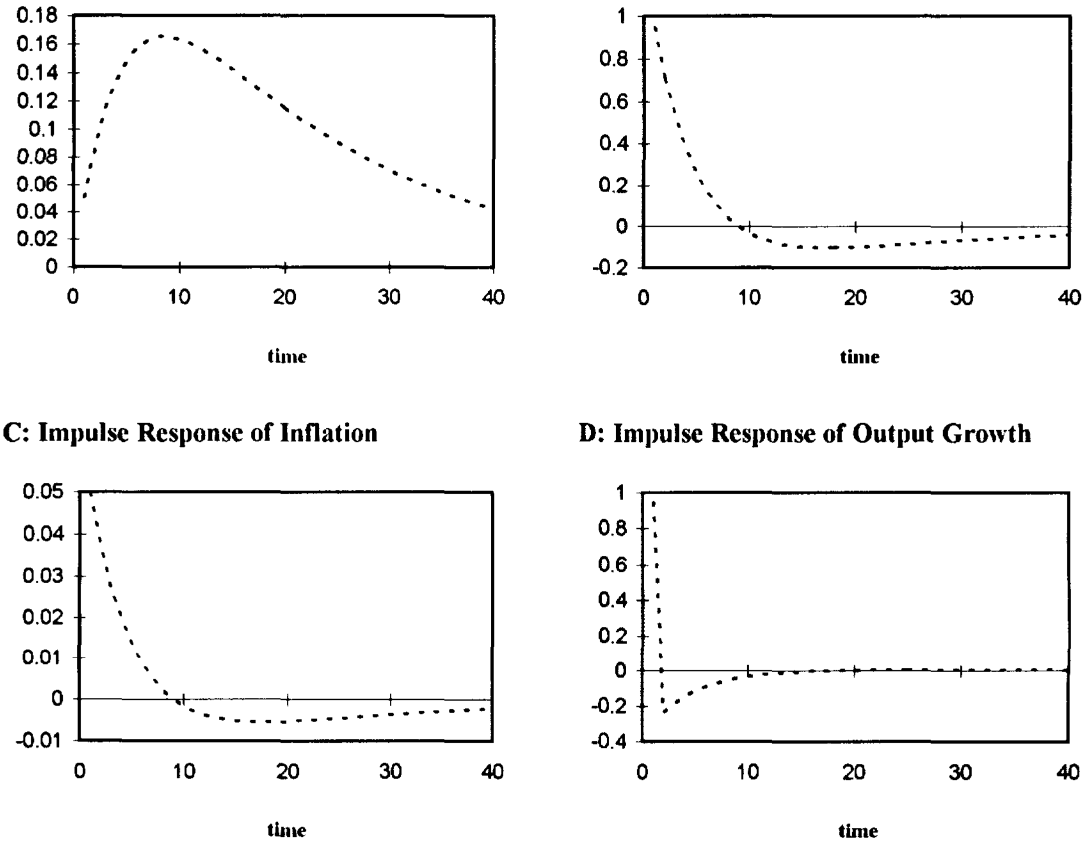
and the law of motion for output is given by

$$(6.7) \qquad y_t = \frac{(1-\beta)(1-L)\varepsilon_t}{\left(1-(1-\beta)L\right)\left(1-\lambda_1 L\right)\left(1-\lambda_2 L\right)\left(1-\lambda_3 L\right)}.$$

Note that, even when the nominal demand shock has a unit root, i.e. $\lambda_1 = 1$, output is still stationary. To illustrate the dynamics of the model, I plot the impulse response functions of several variables in response to a demand shock. I consider two cases. In the first case, the demand shock is stationary ($\lambda_1 = 0.8$, $\lambda_2 = \lambda_3 = 0$), and the speed of the adjustment parameter, β, is equal to 0.05. In the second case, the (nominal) demand shock is permanent ($\lambda_1 = 1$, $\lambda_2 = 0.5$, $\lambda_3 = 0$), and the speed of adjustment parameter, β, is equal to 0.05. As documented in figures 8 and 9, there are several ways in which (detrended) measures of output and prices are negatively correlated in this economy with only demand shocks. For example, when one compares panel A and panel B in figure 8, one can see that there are several periods in which the price level is above its long-run value and output is below. Also, as documented by panel C and panel D in figure 9, there is a negative correlation between inflation and output growth, except during the initial periods.

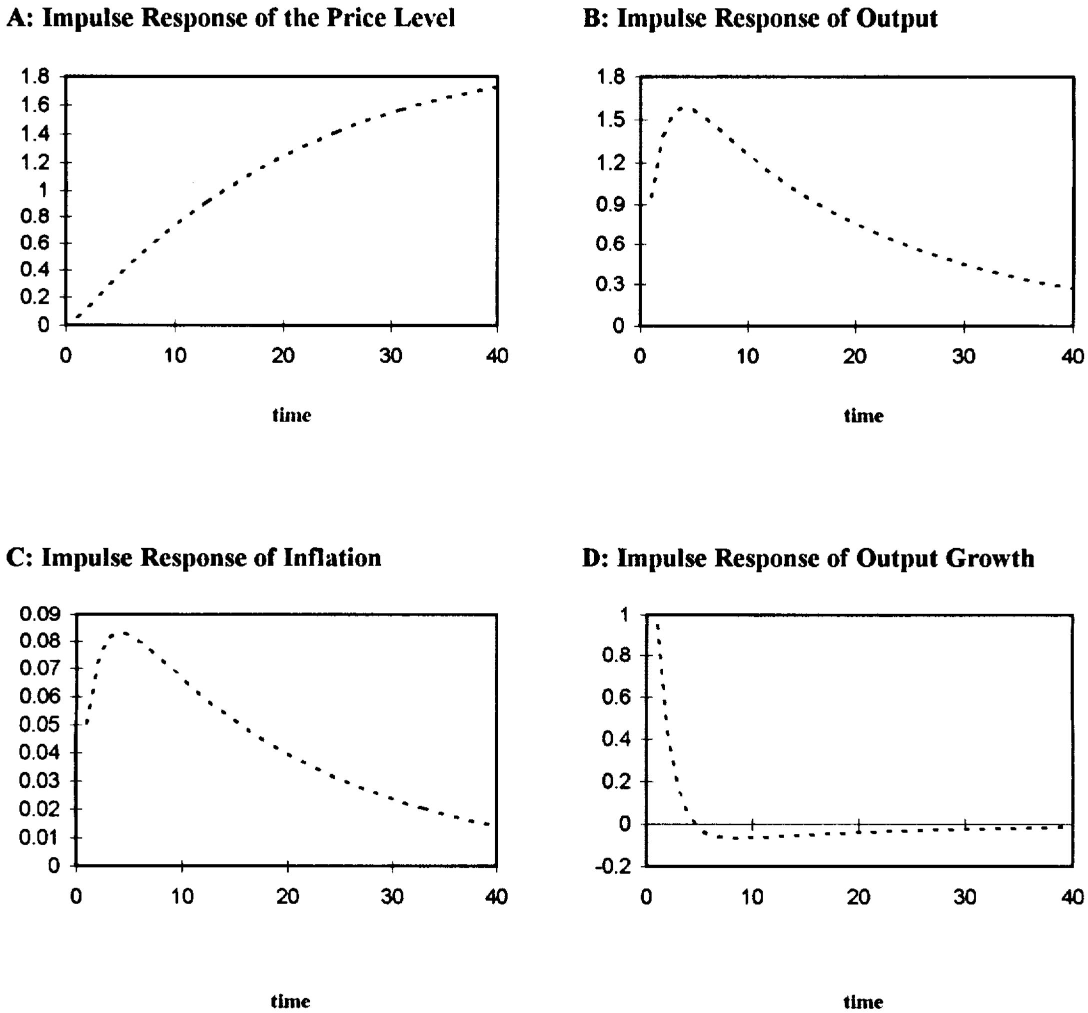
FIGURE 8. THE EFFECTS OF A STATIONARY DEMAND SHOCK.

A: Impulse Response of the Price Level **B: Impulse Response of Output**



Note: These figures plot the impulse response functions of the indicated variables in response to a demand shock when the demand shock is an AR(1) with λ_1 = 0.8 and the speed of adjustment parameter β = 0.05.

FIGURE 9. THE EFFECTS OF A PERMANENT DEMAND SHOCK.



Note: These figures plot the impulse response functions of the indicated variables in response to a demand shock when the demand shock is an integrated AR(1) with λ_1 = 1.0 and λ_2 = 0.5 and the speed of adjustment parameter β = 0.05.

Next, I address the question of whether this type of model can generate the types of results discussed in the empirical sections of this paper. I will argue that it cannot. In particular, the following proposition shows that the correlation coefficients are guaranteed to be positive unless very implausible assumptions on the law of motion of the demand shock are made.

Proposition: Let $COR(k)$ be the correlation coefficient between the k-th period ahead forecast errors of prices and output. Without loss of generality let λ_1 be the largest root. If

$$\begin{aligned} &\lambda_1 && > 0 \\ (6.8) \qquad &\max\{\lambda_1, 1-\beta\} && > \max\{-\lambda_2, -\lambda_3\}, \text{ and} \\ &\min\{\lambda_1, 1-\beta\} && > \min\{-\lambda_2, -\lambda_3\}. \end{aligned}$$

Then, $COR(k) > 0$ for all k.

Proof: Recall from the discussion in section 2, that the covariance of the k- period ahead forecast error is the *sum* of the product of the k impulse responses. Therefore, I define the variable S_t as

$$(6.9) \qquad S_t = S_{t-1} + p_t y_t .$$

The law of motion for S_t is given by

$$(6.10) \qquad S_t = \frac{(1-\beta)\,\varepsilon_t^2}{\left(1-(1-\beta)L\right)^2 (1-\lambda_1 L)^2 (1-\lambda_2 L)^2 (1-\lambda_3 L)^2}.$$

The proof of this proposition follows directly from (6.10). Under the conditions of the proposition, any negative roots are dominated by the positive roots, which means that the impulse response of S_t is always positive.[22]

Intuitively, the assumptions of the proposition say that if there are any negative roots in the law of motion of the demand shock, then they cannot be to big. Note that these assumptions are sufficient to rule out negative correlation coefficients. It does not say that if these assumptions are violated that the correlation coefficients will be positive for some forecast horizon. Also note that if all roots are positive, then the assumptions are satisfied. Another way to see how weak the assumptions are is to consider the case in which the demand shock is integrated ($\lambda_1 = 1$). In this case, the assumptions reduce to

$$(6.11) \qquad 1-\beta > \min\{-\lambda_2, -\lambda_3\}.$$

This assumption is, for example, satisfied as long as $\lambda_3 = 0$. This assumption can only be violated if there are two negative roots and both are larger than $(1-\beta)$ in absolute value. Chadha and Prasad (1993) set β equal to 0.05, which means that you need two negative roots with values bigger than 0.95. The reason that such strong conditions are necessary to generate negative comovements between forecast errors is the following. Since the forecast errors accumulate the errors, one needs the *accumulated* effect of a demand shock on output to be negative. It is more likely that the impulse response function of output

---

[22] A proof of this last statement is given in appendix G.

is negative at one particular point, but this is not enough to generate negative comovements between forecast errors. Nevertheless, there are many plausible *dgps* for which the impulse response function for output in response to a demand shock is always positive. For example, it is easy to show that when the impulse response of an innovation in demand on D_t is monotonically increasing then the impulse response of output is always positive.

Next, I empirically test how sensible the assumption of the proposition are. The question arises which variable to use for the demand shock. Under the null hypothesis that there are no supply shocks, negative roots that are present in the nominal demand shock should also be present in output and the price level. Therefore, I test whether there are negative roots in the law of motion for prices and output using postwar data. As an example of a demand shock, I use nominal *M2*. First, I test for the presence of a unit root in the level and the first difference of the series. The results are reported in table 1. For *GNP*, its deflator, and *M2*, I cannot reject the null of a unit root. For the first-difference in *GNP* and *M2*, I clearly reject the null of a unit root. The results are mixed for inflation. Using the augmented Dickey-Fuller test, I do not reject the null of a unit root in inflation, and, using the Phillips-Perron test, I do. I continue the analysis under the assumption that inflation is stationary. In table 2, I report the estimates of the two autoregressive roots in the specification[23]

(6.12) $$(1 - \lambda_2 L)(1 - \lambda_3 L)\ \Delta z_t = \varepsilon_t.$$

As documented in table 2, I find real roots for all three variables. More importantly, for none of the three variables do I find two negative roots. Moreover, when I do find negative roots, then the value is small.

TABLE 1: UNIT ROOT TESTS.

Variable	sample period	Phillips-Perron	Augmented Dickey-Fuller (4)	Augmented Dickey-Fuller (6)
$M2$	1960:3 - 1991:2	-5.07	-1.68	-2.06
P	1954:1 - 1993:4	-10.60	-0.79	-0.79
GNP	1954:1 - 1993:4	-10.42	-7.91	-8.34
$\Delta M2$	1960:3 - 1991:2	-45.62**	-34.60**	-38.29**
ΔP	1954:1 - 1993:4	-82.67**	-17.24	-16.93
ΔGNP	1954:1 - 1993:4	-99.00**	-119.91**	-115.31**
θ	1960:3 - 1991:2	-12.27	-10.27	-10.07
$\Delta\theta$	1960:3 - 1991:2	-173.90**	-105.09**	-114.45**

Note: In this table, I report the results of the Phillips-Perron Z_ρ test and the augmented Dickey-Fuller (Q) test for a unit root using the estimated OLS autoregressive coefficient. The spectral density in the Phillips-Perron test is calculated using the VARHAC estimator from Den Haan and Levin (1994). Q stands for the number of lags included in the regression. When the null of a unit root is rejected at the 10% (1%) level, then I indicate this with a * (**). See Hamilton (1994) for a description of these tests.

[23] The analysis can easily be extended to specifications with more than two roots. The results are not sensitive to this extension.

TABLE 2: ESTIMATED ROOTS OF AR(2) SPECIFICATION

Variable	sample period	λ_2	λ_3
$\Delta M2$	1960:3 - 1991:2	0.62 (0.12)	0.03 (0.15)
ΔP	1954:1 - 1993:4	0.90 (0.03)	-0.47 (0.07)
ΔGNP	1954:1 - 1993:4	0.48 (0.11)	-0.18 (0.12)

Note: In this table I report the two autoregressive roots estimated using least-squares. The standard errors (in parentheses) correct for heteroskedasticity.

The proposition does not show that a model with only demand shocks that generates the type of results that are reported in the empirical section of this paper cannot exist. The purpose of the proposition is to show that the statistics proposed in this paper to describe the comovement of two variables are much more powerful in distinguishing between different models than the unconditional covariance between two detrended variables.

6.1.2 A model with demand and supply shocks.

As was shown above, a model with only demand shocks cannot replicate the negative correlation coefficients between forecast errors observed in the postwar period. I will now address the question of how well the model described above does when I use empirically plausible demand *and* supply shocks. For the demand shock, I use (the logarithm of) nominal $M2$, and, for the supply shock, I use the logarithm of the Solow residual.[24] For both variables, I cannot reject the null of a unit root in the level, but I can reject the null of a unit root in the first-difference.[25] I use a model selection criterion to find the best (univariate) model for the change in the two variables. Both the Akaike information criterion (AIC) and the Schwartz information criterion choose a second-order process. Using the results from the least-squares regressions, we get the following system that is used to generate the demand and the supply shocks[26]

$$\text{(6.13)}\qquad \begin{aligned} \Delta D_t &= 0.6425\,\Delta D_{t-1} - 0.0054\,\Delta D_{t-2} + 0.00605\,\varepsilon_t^d, \\ \Delta \theta_t &= -0.091\,\Delta \theta_{t-1} + 0.251\,\Delta \theta_{t-2} + 0.00678\,\varepsilon_t^\theta. \end{aligned}$$

In addition we assume that ε_t^d and ε_t^θ are i.i.d. N(0,1) distributed variables. In every replication, I simulate an economy with 260 observations. To randomize initial conditions, I disregard the first 100 observations which leaves us with a typical postwar sample size. For each economy, I calculate the covariances at different forecast horizons and at different frequencies. I repeat the experiment 500 times. I consider 5 different values for the speed of the adjustment parameter β. Those are 0.1, 0.2, 0.3, 0.4, and 0.5. In figure 10, I plot the averages of the correlation coefficients across 500 replications for the 5

[24] The data for the Solow residual were kindly provided by Christian Zimmermann and are described in Zimmermann (1994).

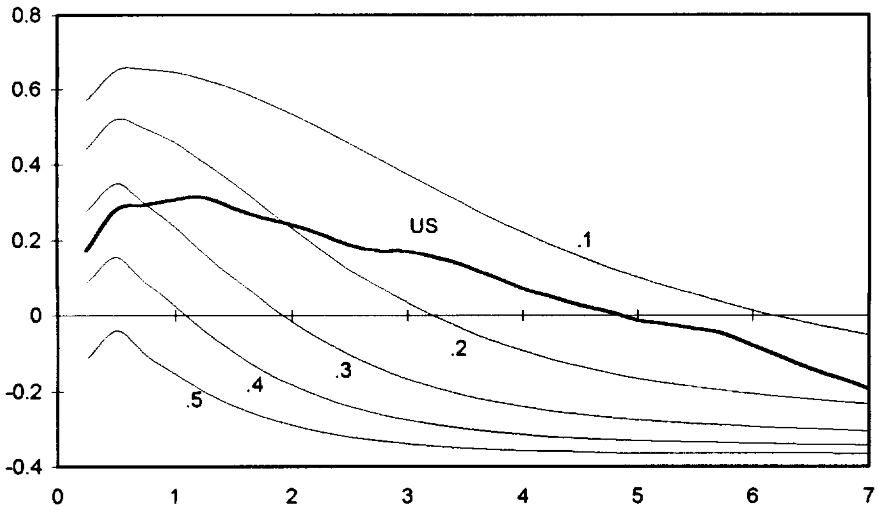
[25] See table 1.

[26] Without loss of generality, I set the constant equal to zero.

different values of β, as well as the empirical counterpart.[27] As documented in the graphs, as long as prices do not adjust too quickly, then I observe the same pattern for the comovement of prices and output as I observe in US postwar data. That is, in the long run, supply shocks dominate since demand shocks only have a temporary effect on output. In the short run, demand shocks dominate since an innovation in the supply shock only effects output by changing the price level, which happens gradually. A nominal demand shock, on the other hand, affects output directly. As documented in the graphs, to generate positive correlation coefficients at the forecast horizons and frequencies at which the correlation coefficients are positive in the data, one needs the speed of adjustment parameter to be between .1 and .2. These values are a little bit higher than the values reported in Rotemberg (1982).[28] The model does not fit the empirical findings in several other respects. For example, although for some values of the speed of adjustment the qualitative pattern of the correlation coefficients is similar to the one observed for US data, the quantitative pattern is different. In addition, results not reported here show that the model is capable of replicating the pattern of the US postwar covariances of forecast errors both qualitatively and quantitatively, but that the model's covariances at the high frequencies are much too low.

FIGURE 10: COMOVEMENT BETWEEN PRICES AND OUTPUT IN THE MODEL

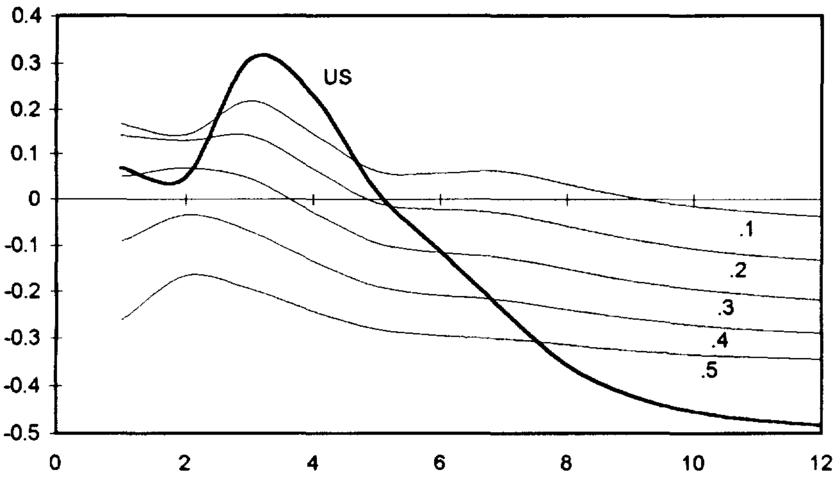
A: VAR Forecast Errors



[27] These results are very similar to the correlation coefficients of one long realization.

[28] Rotemberg's preferred estimates vary from 0.05 to 0.08.

B: High-Pass Filter



PERIODICITY (YEARS)

Note: These graph plot the average correlation coefficient across 500 replications of the model economy described in the text as well as the empirical counterpart using postwar quarterly data from 1954 to 1993. The numbers in the graph indicate the speed of adjustment.

The purpose of this example is not to propose an ideal model to describe the behavior of US prices and output. The example is meant as a simple example to illustrate how informative the proposed correlation measures are and how they can be used to judge different theories or identify parameters in a model. The example shows that the proposed sticky price model with only demand shocks cannot generate the negative correlation coefficients of forecast errors as long as some very plausible conditions are satisfied. In addition, it shows that a model in which demand shocks have a (permanent) positive effect on prices and a temporary effect on output, and supply shocks have a negative effect on prices and persistent positive effect on output, can replicate the qualitative pattern of correlation coefficients observed for US postwar data.

6.2 Economic theory and the comovements between real wages and hours.

Previous empirical studies of the correlation between real wages and output indicate that real wages are either acyclical or slightly procyclical. Standard real business cycle models, however, imply a strong positive correlation between real wages and hours because an increase in productivity causes the firm's labor demand curve to shift upward. Most undergraduate textbooks contain a version of an IS-LM model in which the amount of labor is determined by the firm's labor demand curve. This, together with diminishing returns to scale, implies a negative correlation between hours and real wages. Therefore, neither theory is supported by a small positive correlation coefficient. In fact, the results reported by

Dunlop (1938) led Keynes (1939) to doubt the assumption of the *General Theory* that firms are always on a stable downward sloping demand curve. Lucas (1977) concluded from the empirical finding that real wages are acyclical that *"any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure"*. Boldrin and Horvath (1995) use the fact that the unconditional correlation coefficient between real wages and hours is close to zero to support a theory of sticky real wages. Rotemberg and Woodford (1991) add countercyclical markups to eliminate countercyclical real wages in a model in which firms do operate on a downward sloping labor demand curve. Christiano and Eichenbaum (1992) emphasize intertemporal labor-leisure substitution to eliminate the strong procyclical behavior of real wages in standard real business cycle models. The results on the comovement of real wages and hours reported in the last section indicate how misleading the unconditional correlation coefficient is. First, they document that there is an important cyclical role of real wages. In particular, the correlation is negative in the "short run" and positive in the "long run". Moreover, these results question whether standard Keynesian models or real business cycle models need to be adjusted, since the results are consistent with a standard Keynesian model in the "short run" and a standard real business cycle model in the "long run". For example, the results in the short run are consistent with a Keynesian model with sticky *nominal* wages. It would be an interesting exercise to examine, using models with sticky nominal wages, the relation between the length of nominal contracts and the forecast horizons and frequencies for which the correlation between real wages and hours are negative.

7. CONCLUDING COMMENTS.

In this paper, I have presented two different types of methods to measure the comovements between prices and output at different business cycle frequencies. The first method uses the k-period ahead forecast errors from a VAR. The second method uses filters constructed in the frequency domain. The methods discussed in this paper can be used for non-integrated as well as for integrated variables. Thus, the researcher does not have to take a stand on what is the best way to detrend the data. The two methods proposed in this paper have relative advantages and disadvantages. Implementation of the frequency-domain filters requires fewer choices. The researcher need only check sensitivity of the results to the truncation parameter K. Estimation of the VAR involves making choices about the maximum lag-order and about which variables to include. However, the results of the frequency domain filter may be harder to interpret because most models are formulated in the time domain. For example, in section 6, I showed that it was very easy to use the results from the VAR to reject a certain class of models. Finally, the frequency domain filters are less sensitive to simple structural breaks like a change in the deterministic time trend, especially when the truncation parameter is small. If the structural break is not modeled in the VAR, then the estimated correlation coefficients are inconsistent.

Both methods indicate a very clear pattern for the comovement of prices and real activity during the postwar period. During this period, prices and GNP are positively correlated in the "short run" and negatively correlated in the "long run". During the same period, real wages and hours are negatively correlated in the "short run" and positively correlated in the "long run". These procedures illustrate that useful information is lost when one focuses only on the unconditional correlation coefficient. Unconditional correlation coefficients of detrended output and prices do not reveal the positive correlation found in the "short run". The unconditional correlation coefficient of real wages and hours does not reveal the negative correlations in the "short run" nor the positive correlations in the "long run".

Recently, Hansen and Heckman (1996) criticized the calibration approach for not matching a full set of dynamics of the model to the dynamics in the data. The statistics that are proposed in this paper do incorporate dynamics but are still very intuitive. I argue that the statistics proposed in this paper are very effective in providing information that is useful for a researcher who wants to build a structural model. I have supported this claim by showing that the reported empirical results have important implications for economic theories that have not been mentioned in the literature. I was able to show this despite the fact that I did not use a new data set and that the empirical results reported here are implicit in, for example, the estimated VAR coefficients or the cross spectrum. I think that these other statistics are just more difficult to interpret. For example, the number of VAR coefficient becomes large very easily when there are many variables included. Interpreting the cross spectrum is also difficult since it has a real and an imaginary part.

REFERENCES.

Abel, A.B., and B.S. Bernanke, 1994, *Macroeconomics*, second edition, Addison-Wesley Publishing Company.

Abraham, K.C., and J.C. Haltiwanger, 1995, Real wages and the business cycle, *Journal of Economic Literature* 33, pp. 1215-1264.

Baxter, M., and R.G. King, Measuring business cycles: Approximate band-pass filters for economic time-series, manuscript University of Virginia.

Bernanke, B., and J. Powell, 1986, The cyclical behavior of industrial labor markets: A comparison of the prewar and postwar eras, in Robert Gordon (ed.), *The American business cycle: Continuity and change*. NBER and University of Chicago Press, pp. 583-621.

Bils, M., 1985, Real wages over the business cycle: Evidence from panel data, *Journal of Political Economy* 93(4), pp. 660-689.

Blanchard, O.J., and S. Fischer, 1989, *Lectures on macroeconomics*, MIT press, Cambridge.

Boldrin M. and M. Horvath, 1995, Labor contracts and Business Cycles, *Journal of Political Economy* 103, pp. 972-1004.

Chadha, B. and E. Prasad, 1993, Interpreting the cyclical behavior of prices, *IMF Staff Papers* 40, pp. 266-298.

Chadha, B. and E. Prasad, 1994, Are prices countercyclical? Evidence from the G-7. *Journal of Monetary Economics* 34(2), pp. 239-258.

Christiano, L.J., and M. Eichenbaum, 1992, Current real-business-cycle theories and aggregate labor-markt fluctuations, *American Economic Review* 82, pp. 430-450.

Christiano, L.J., and W.J. den Haan, 1996, Small sample properties of GMM for business cycle analysis, *Journal of Business and Economic Statistics*, forthcoming.

Cogley, T., and J.M. Nason, 1995, Effects of the Hodrick-Prescott filteron trend and difference stationary time series: Implications for business cycle research, *Journal of Economic Dynamics and Control* 19, pp. 253-278.

Cooley, T.F. and L.E. Ohanian, 1991, The cyclical behavior of prices, *Journal of Monetary Economics* 28, pp. 25-60.

Den Haan, W.J., and A. Levin, Inferences from parametric and non-parametric spectral density estimation procedures, UCSD working paper 94-30.

Dunlop, J., 1938, The movement of real and money wages, *Economic Journal*, pp. 413-434.

Engel, C., 1993, Real exchange rates and relative prices: An empirical investigation, *Journal of Monetary Economics* 32(1), pp. 35-50.

Engle, R.F., 1974, Band spectrum regression, *International Economic Review* 15, 1-11.

Fuller,

Gordon, R., 1986, *The American business cycle: continuity and change*, NBER and University of Chicago Press.

Hamilton, J.D., 1994, *Time Series Analysis*, Princeton University Press.

Hannan, E.J., 1970, *Multiple time series*, Wiley Press.

Hansen, L.P. and J.J. Heckman, 1996, The empirical foundations of calibration, *Journal of Economic Perspectives* 10, pp. 87-104.

Harvey, A.C. and A. Jaeger, 1993, Detrending, stylized facts and the business cycle, *Journal of Applied Econometrics* 8, pp. 231-247.

Judd, J.P., and B. Trehan, 1995, The cyclical behavior of prices: Interpreting the evidence, *Journal of Money, Credit, and Banking* 27, pp. 789-797.

Keynes, J.M., 1939, Relative movements of real wages and output, *Economic Journal* 49, pp. 34-51.

King, R.G., and S.T. Rebelo, 1993, Low frequency filtering and real business cycles, *Journal of Economic Dynamics and Control* 17, pp. 207-232.

King, R.G., and M.W. Watson, 1994, The postwar U.S. Phillips curve: A revisionist econometric history, *Carnegie-Rochester Conference Series*, forthcoming.

Kydland, F.E., and E.C. Prescott, 1990, Business cycles: real facts and a monetary myth, *Quarterly Review* Spring, Federal Reserve Bank of Minneapolis, pp. 3-18.

Lucas, R.E., Jr. 1972, Expectations and the neutrality of money, *Journal of Economic Theory* 4, pp. 103-124.

Lucas, R.E., Jr., 1977, Understanding business cycles, Stabilization of the domestic and international economy, in: K. Brunner and A.H. Meltzer eds., *Carnegie-Rochester Conference Series on Public Policy*, pp. 7-29.

Mankiw, N.G., 1989, Real business cycles: A new Keynesian perspective, *Journal of Economic Perspectives* 3(3), pp. 79-90.

Newey, W.K., and K.D. West, 1994, Automatic lag selection in covariance matrix estimation, *Review of Economic Studies* 61(4), pp. 631-653.

Osborn, D., 1995, Moving average detrending and the analysis of business cycles, manuscript, University of Osborn.

Park, J.Y., and P.C.B. Phillips, 1988, Statistical inference in regressions with Integrated Processes: Part 1, *Econometric Theory* 4, pp. 468-498.

Park, J.Y., and P.C.B. Phillips, 1989, Statistical inference in regressions with Integrated Processes: Part 2, *Econometric Theory* 5, pp. 95-132.

Prescott, K.J, 1986, Theory ahead of business cycle measurement, *Carnegie-Rochester Conference Series on Public Policy* 25: pp. 11-66.

Priestley, 1988, *Non-linear and non-stationary time series analysis*, Academic Press.

Rotemberg, J.J., 1982, Sticky Prices in the United States, *Journal of Political Economy* 90, 1187-1211.

Rotermberg, JJ. and M. Woodford, 1991, Markups and the business cycle, *NBER macreconomics annual*, eds. O. Blanchard and S. Fischer, MIT Press, pp. 63-129.

Sargent, T., 1978, Estimation of dynamic labor demand schedules under rational expectations, *Journal of Political Economy* 86(6), pp. 1009-1044.

Sims, C.A., J.H. Stock, M.W. Watson, 1990, Inference in linear time series models with some unit roots, *Econometrica* 58, pp. 113-145.

Zimmermann, C., 1994, Technology innovations and the volatility of output: An international perspective, working paper no. 34, Research on Employment and Economic Fluctuations, University of Quebec at Montreal.

APPENDIX A. DATA SOURCES.

In this appendix, we describe the data sources. The reported sample indicates the period necessary to duplicate all the results in this paper. The actual period for which correlation coefficients are calculated is shorter, since the estimation of the VAR and the use of the frequency filters reduces the length of the sample period.

Output:

- Per capita GNP from the appendix in Gordon (1986), updated using GNPQ/GPOP from CITIBASE. Quarterly data from 1875 to 1993.
- Industrial production, IP, from CITIBASE. Monthly data from 1959 to 1994:6.

Hours:

- Per capita hours of all employees in the business sector, LBHE/GPOP, from CITIBASE. Quarterly data from 1949 to 1993.

Nominal Wages:

- Compensation per hour in the business sector, LBCP, from CITIBASE. Quarterly data from 1949 to 1993.

Prices:

- GNP deflator from the appendix in Gordon (1986), updated using GNP/GNPQ from CITIBASE. Quarterly data from 1875 to 1993.
- Wage deflator, LBCP/LBCP7, from CITIBASE. Quarterly data from 1949 to 1993.
- Non-durable price deflator, GMCN/GMCNQ, from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.
- Consumer price index, PUNEW (CPI-U), from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.
- Producer price deflator, PW, from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.

Monetary Indicators:

- Federal funds rate, FYFF, from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.
- Total reserves adjusted for reserve requirements, FMRRA, from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.
- Non-borrowed reserve mix adjusted for extended credits and reserve requirements, FMRNBC/FMRRA, from CITIBASE. Monthly and quarterly data from 1959 to 1994:6.
- Money Stock, FM2, from CITIBASE. Quarterly data from 1960 to 1991:2.

Solow Residual:

- log (output) - 0.636*log(hours) - (1 - 0.636)*log(capital) from Zimmermann (1994)

APPENDIX B. SENSITIVITY ANALYSIS FOR PRICES AND OUTPUT.

In this appendix, we analyze whether the results reported in the paper for the comovement between prices and output are sensitive to

- the truncation parameter K of the two-sided MA-filter,
- the sample period,
- whether or not a unit root is imposed in the estimation of the VAR,
- the number of variables included in the VAR,
- whether monthly or quarterly data are used, and
- the choice of the price index.

Although, quantitatively, we find some differences, qualitatively, the results are extremely robust. In all but one case, we find that, during the postwar period, the correlation coefficients are positive in the "short run" and negative in the "long run". The only exception occurs for the comovement between monthly industrial production and the producer price index when a bivariate VAR is used to construct the forecast errors. In this case, the correlation coefficients are positive at all forecast horizons considered. Even in this case, however, prices are less procyclical in the long run since the correlation coefficients decline to values close to zero if the forecast horizon increases.

(1) *Sensitivity to the truncation parameter of the frequency-domain filter.*

Implementation of the frequency-domain filters discussed in the text requires the choice of a truncation parameter, K. In Figure B.1, we plot the correlation coefficients for different values of K using the sample from 1895 to 1973, and in figure B.2, we plot the correlation coefficients for different values of K using the sample from 1954 to 1983. The data series used are quarterly GNP and its deflator. We see that, in the post-war period, the general pattern is not very sensitive to the choice of K. Therefore, we set K equal 20 in the post-war period. This means that 5 years of data are lost at the beginning and at the end of the sample. As documented in figure B.1, there is much more sensitivity to the choice of K for the complete sample. However, since the results do not change very much if we increase K from 40 to 80, we set K equal to 40.

Figure B.1. Sensitivity to the truncation parameter of the high-pass filter (1895-1973).

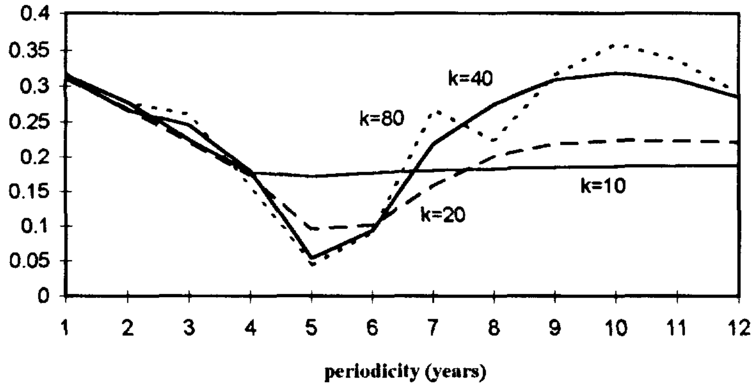
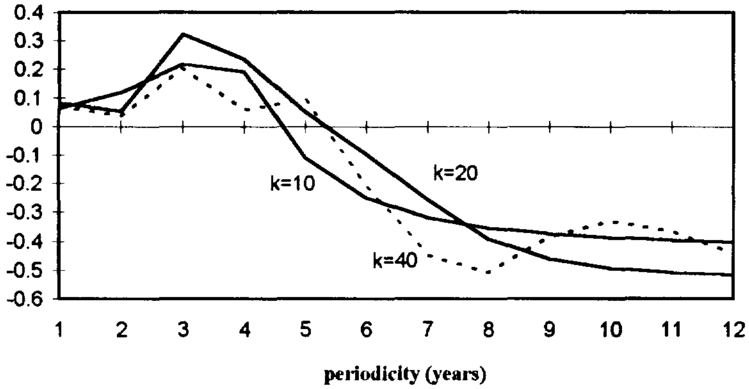


Figure B.2. Sensitivity to the truncation parameter of the high-pass filter (1954-1983).

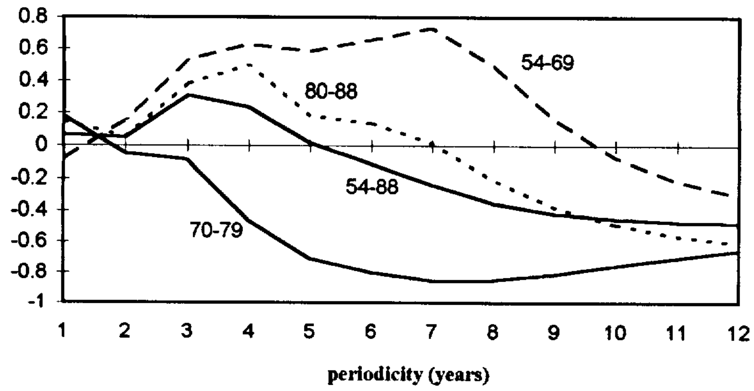


Note: These figures plot the correlation coefficients between quarterly GNP and its deflator, both filtered using a high-pass filter. The high-pass filter is approximated using a two-sided moving-average filter with truncation parameter *K*.

(2) *Sensitivity to the sample period.*

To study the sensitivity of the results to the sample period, we divide the post-war period into three subperiods. In particular, we study the period from 1954 to 1969, the period from 1970 to 1979, and the period from 1980 to 1988. In figure B.3, we plot the correlation coefficients of filtered GNP and prices. The data are filtered with a high-pass filter. We see that the pattern in all three subperiods is similar to the pattern observed in the complete post-war period. Note, however, that during the seventies prices are more countercyclical. In particular, the correlation coefficient is already negative when cycles associated with a periodicity of less than two years are included.

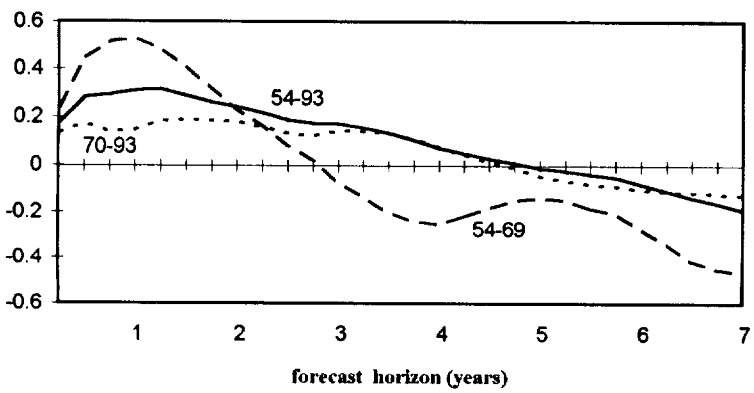
Figure B.3. Sensitivity to the sample period, high-pass filter.



Note: This figure plots the correlation coefficients between filtered quarterly GNP and its deflator for the indicated sample period.

The sample periods considered above are too short to study the correlation of forecast errors since we include long forecast horizons. Therefore, we split the postwar period into just two subsamples. The first is from 1954 to 1969, and the second is from 1970 to 1993. The results are plotted in figure B.4. We see that, in both subsamples, the pattern of correlation coefficients is very similar to the one observed for the complete postwar period.

Figure B.4. Sensitivity to the sample period, VAR forecast errors.



Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of quarterly GNP and its deflator. A bivariate VAR is used to estimate the forecast errors.

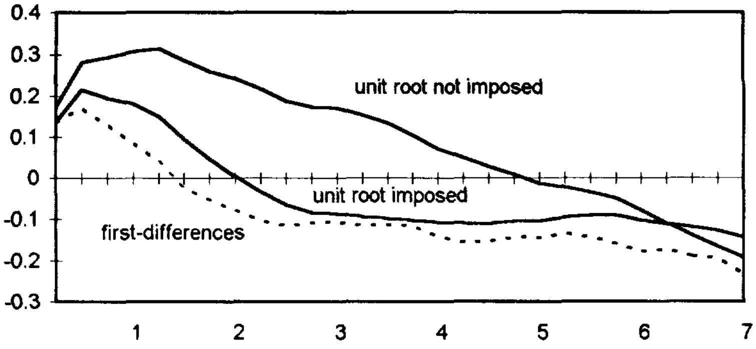
(3) *Sensitivity to imposing unit roots.*

As shown in Appendix E, the correlation coefficients of the estimated VAR residuals are consistent even in the presence of unit roots in the time-series in the VAR. However, in the presence of unit roots, efficiency gains are possible when they are imposed. It is often argued that GNP has a unit root and the price level has at least one unit root.[1] Here, we check whether imposing a unit root affects the

[1] See, for example, King, Plosser, Stock and Watson (1991).

results for the postwar sample. To do this, we first estimate a VAR in first differences.[2] Next, we calculate the forecast errors for the *levels* of output and prices. The correlation coefficients between these series are plotted in figure B.5, together with the correlation coefficients obtained from the VAR estimated in levels.

Figure B.5. Sensitivity to imposing unit roots, VAR forecast errors.



Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of quarterly GNP and its deflator for the period from 1954 to 1993. A bivariate VAR is used to estimate the forecast errors.

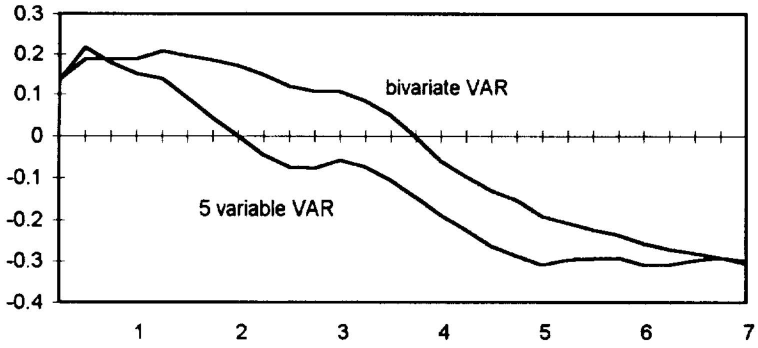
Although there are some quantitative differences between the two series, the pattern is robust to whether or not unit roots are imposed. Using the VAR estimated in first differences, we also calculate the correlation between the forecast errors for the first-differenced series. We find that the correlation coefficients are positive at the short forecast horizon and become negative at the longer forecast horizons. In particular, the correlation coefficient for the forecast errors of the changes becomes negative at a forecast horizon equal to 6 quarters. Note that, at this forecast horizon, the correlation coefficients for the forecast errors for the levels are still positive. Note that there is no reason why the correlation coefficient for the change would equal the correlation coefficient for the level, except in the one-period ahead forecast error.

(4) *Sensitivity to including additional variables in the VAR*

To check whether the results change when other variables are included in the VAR, we also use a VAR with three monetary indicators. These indicators are the federal funds rate, the amount of total reserves, and the ratio of non-borrowed reserves to total reserves. The correlation coefficients of the forecast errors for the two systems are reported in figure B.6. Although there are some quantitative differences, the general pattern is very similar for both systems.

[2] Note that, if one had reason to believe that prices and output are cointegrated, then one would have to estimate an error-correction mechanism.

Figure B.6. Sensitivity to including other variables in the VAR.



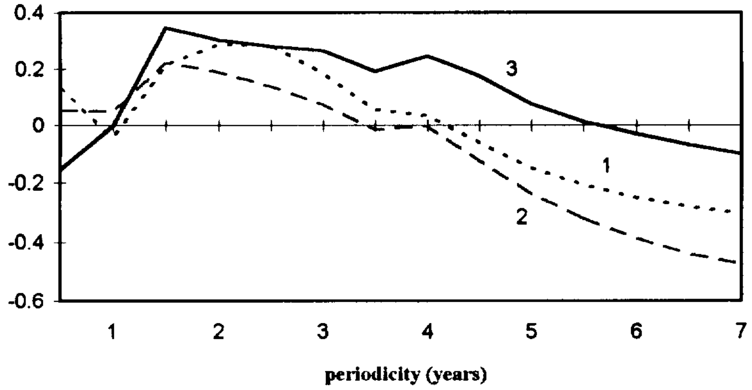
Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of quarterly GNP and its deflator for the period from 1959 to 1993, where the indicated VAR is used to estimate the forecast errors.

(5) *Sensitivity to data frequency, price index, and the number of variables in the VAR.*

Finally, we check whether the same pattern can be observed for monthly data. For the indicator of real activity, we use industrial production. For the price index, we consider the deflator of non-durables, the consumer price index, and the producer price index. The truncation parameter is set equal to 60, which corresponds to 5 years of data. In figure B.7, we plot the correlation coefficients of the series filtered with a high-pass filter for the period from January 1964 to June 1988. Although we observe a similar pattern for all three deflators, the producer price index is (at most frequencies) more procyclical than the other two deflators.

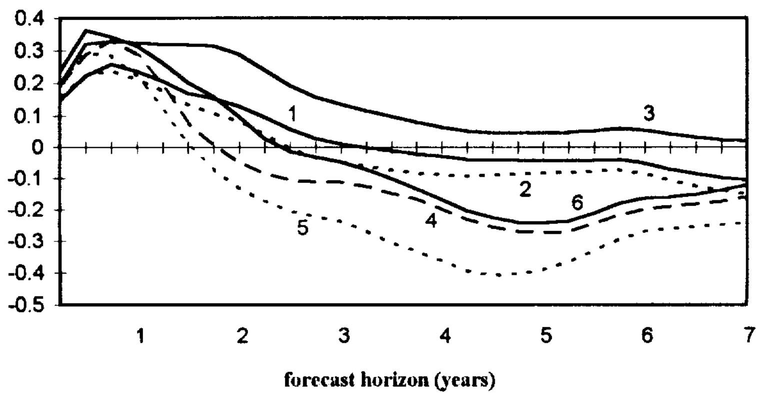
Next, we consider the correlation coefficients of the VAR forecast errors for the period from 1960 to 1993. We use a bivariate VAR and a VAR that also includes the three monetary indicators used above. As documented in figure B.8, we observe the typical pattern for all sequences of correlation coefficients, except one. When we use the producer price index, and the forecast errors are estimated with a bivariate VAR, then the correlation coefficients are positive for all forecast horizons. Even in this case, however, prices are less procyclical in the long run since the correlation coefficients decline to values close to zero if the forecast horizon increases.

Figure B.7. Sensitivity to data frequency and the price index, high-pass filter



Note: This figure plots the correlation coefficients between filtered monthly industrial production and a deflator using a two-sided moving-average high-pass filter for the period from 1964 to June 1989. The series numbered 1 correspond to the deflator of nondurables. The series numbered 2 correspond to the CPI. The series numbered 3 correspond to the PPI.

Figure B.8. Sensitivity to data frequency, the price index, and the number of variables included in the VAR.



Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of monthly industrial production and prices for the period from January 1960 to June 1994. The series numbered 1 through 3 correspond to a bivariate VAR and the series numbered 4 through 6 correspond to a VAR that also includes three monetary indicators. The series numbered 1 and 4 correspond to the deflator of nondurables. The series numbered 2 and 5 correspond to the CPI. The series numbered 3 and 6 correspond to the PPI.

APPENDIX C. SENSITIVITY ANALYSIS FOR REAL WAGES AND HOURS.

In this section, we analyze whether the results reported in the paper for the comovement between real wages and hours are sensitive to

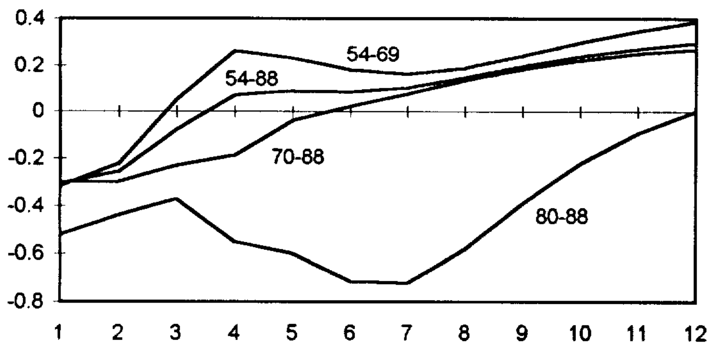
- the sample period, and
- the choice for the nominal wage deflator.

(1) *Sensitivity to the sample period.*

It is well-known that the behavior of wages has changed sometime around 1970. For example, the average annual growth in the real wage rate was equal to 3% in the period from 1954 to 1969, and only 0.7% in the period from 1970 to 1993. The pattern of the correlation coefficients, however, is similar in the two subsamples. That is, we find negative comovements in the "short run" and positive comovements in the "long run". This is documented in figure C.1 for the comovements of data filtered with high-pass filters. When we only include data from 1980 to 1988, however, the correlation coefficients are negative for all frequencies. It is still true, however, that the comovement is less countercyclical at the low frequencies. There are two ways in which we can document this. First, note that the correlation coefficients are increasing. Second, when we filter the data with band-pass filters, we find that the comovement between hours and real wages is positive for cycles associated with a period that is larger than 6 years and negative for cycles associated with a period that is less than 6 years.

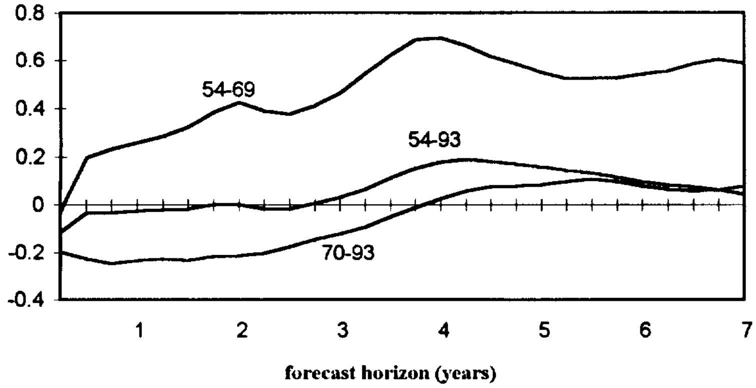
As documented in figure C.2, the pattern of correlation coefficients across different sample periods is also similar when the forecast errors from a VAR are used. Just as in figure C.1, we see that real wages are more procyclical in the earlier samples. The VAR used to calculate the forecast errors in the subsamples is the same as the VAR used to calculate the forecast errors in the combined sample. The same pattern is observed, however, when the VAR is reestimated.

Figure C.1. Sensitivity to the sample period, high-pass filter.



Note: This figure plots the correlation coefficients between filtered quarterly hours and real wages for the indicated sample period.

Figure C.2. Sensitivity to sample period, VAR forecast errors.

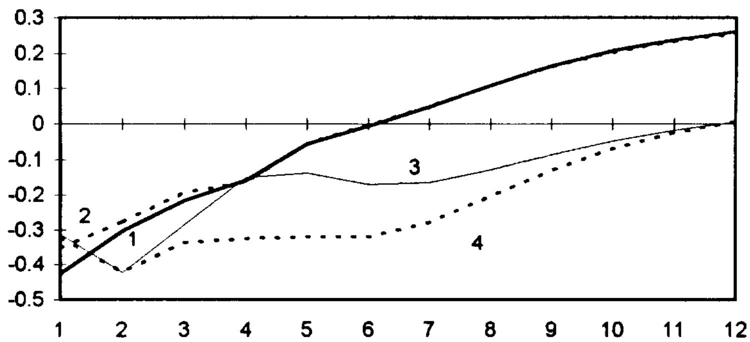


Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of quarterly hours and real wages. A bivariate VAR is used to estimate the forecast errors.

(2) *Sensitivity to the deflator.*

We consider four different deflators for the nominal wage rate. First, we use the one used in CITIBASE to deflate nominal wages. Second, we use the consumer price index. These two deflators are very similar. Third, we use the producer price index. Fourth, we use the deflator for non-durables. As documented in figures C.3 and C.4, we find the same pattern for all four deflators. Note that the producer wage displays the least procyclicality. Theoretically, this is what one would expect when the labor supply curve is upward sloping and the labor demand curve is downward sloping.[3] Using the VAR forecast errors we observe both negative and positive estimates for all deflators. For the high-pass filter, we find the same results for the first two consumer price indices, but, using the producer price index and the non-durable deflator, we find that all correlation coefficients (except for the one corresponding to a periodicity equal to 12 periods) are less than zero.

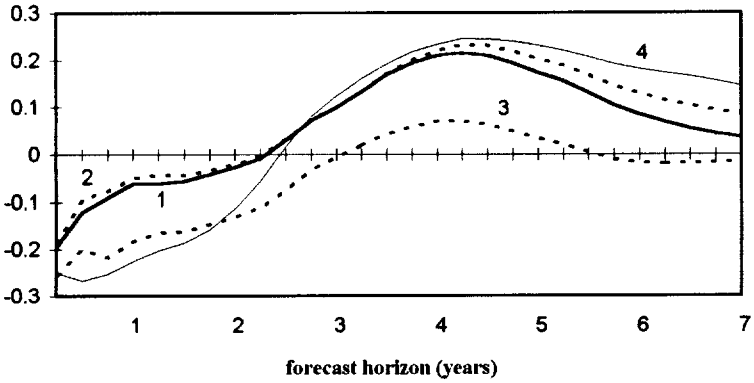
Figure C.3. Sensitivity to wage rate deflator, high-pass filter.



Note: This figure plots the correlation coefficients between filtered quarterly hours and real wages for the period from 1964 to 1988. The deflators used are the deflator used by CITIBASE to deflate nominal wages (1), the consumer price index (2), the producer price index (3) and the deflator for non-durables (4).

[3] See Abraham and Haltiwanger (1995).

Figure C.4. Sensitivity to wage rate deflator, VAR forecast errors.



Note: This figure plots the correlation coefficient of the k-period ahead forecast errors of quarterly hours and real wages for the period from 1960 to 1993. A bivariate VAR is used to estimate the forecast errors. The deflators used are the deflator used by CITIBASE to deflate nominal wages (1), the consumer price index (2), the producer price index (3) and the deflator for non-durables (4).

APPENDIX D. SIGNIFICANCE LEVELS.

Table D.1. Correlation coefficient of *k*-period ahead forecast errors
(quarterly data from 1954:1 to 1993:4)

Forecast horizon (quarters)	Prices and GNP	Wages and Hours
1	0.173*	-0.114*
2	0.281**	-0.034
3	0.293*	-0.034
4	0.308*	-0.025
5	0.314*	-0.020
6	0.286*	-0.018
7	0.261*	0.001
8	0.242	0.001
12	0.171	0.030
16	0.071	0.178**
20	-0.013	0.158*
24	-0.079	0.0975
28	-0.193	0.0439

Note: Prices are the log of the GNP deflator. Real wages and hours are the variables LBCP7 and LBHE/GPOP from CITIBASE. The forecast errors are constructed using a bivariate VAR including four lags. We test whether the estimated coefficient is equal to zero using a one-sided test. Rejections at the 10%, the 5%, and the 1% significance level are indicated by *, **, and ***, respectively. To correct for serial correlation, the standard errors for COR(k) are calculated using the VARHAC estimator from Den Haan and Levin (1994).

Table D.2. Correlation coefficient of filtered prices and real activity (high-pass filter)
(quarterly data from 1954:1 to 1988:4)

Periodicity (years)	Prices and GNP	Wages and Hours
1	0.069	-0.308***
2	0.048	-0.255***
3	0.310**	-0.077
4	0.232	0.072
5	0.018	0.090
6	-0.113	0.083
7	-0.240	0.104
8	-0.356**	0.148
9	-0.421***	0.197**
10	-0.454***	0.239***
11	-0.473***	0.271***
12	-0.483***	0.295***

Note: The high-pass filter retains that part of the series that is associated with cycles with a period less than the indicated periodicity. Prices are the log of the GNP deflator, and real wages and hours are the variables LBCP7 and LBHE/GPOP from CITIBASE. The filters are two-sided moving averages with 20 lags and leads. We test whether the estimated coefficient is equal to zero using a one-sided test. Rejections at the 10%, the 5%, and the 1% significance level are indicated by *, **, and ***, respectively. To correct for serial correlation, the standard errors are calculated using the VARHAC estimator from Den Haan and Levin (1994).

Table D.3. Correlation coefficient of *k*-period ahead forecast errors
(monthly data from 1964:1 to 1994:6)

Forecast horizon (months)	Prices and Industrial Production	
	Bivariate VAR	5 variable VAR
3	0.157***	0.203***
6	0.224***	0.292***
9	0.238***	0.285***
12	0.214**	0.225***
18	0.136**	0.010
24	0.083	-0.130
30	-0.002	-0.202
36	-0.048	-0.237
42	-0.073	-0.306**
48	-0.089	-0.362***
54	-0.091	-0.409***
60	-0.084	-0.390**
66	-0.077	-0.331**
72	-0.086	-0.270*
78	-0.125	-0.253*
84	-0.149	-0.243

Note: The deflator used is CPI-U. The forecast errors are constructed using a bivariate VAR including twelve lags. We test whether the estimated coefficient is equal to zero using a one-sided test. Rejections at the 10%, the 5%, and the 1% significance level are indicated by *, **, and ***, respectively. To correct for serial correlation, the standard errors for COR(*k*) are calculated using the VARHAC estimator from Den Haan and Levin (1994).

Table D.4. Correlation coefficient of filtered prices and real activity (high-pass filter)
(monthly data from 1964:1 to 1989:6)

Periodicity (Years)	Non-Durable Consumption Deflator and Industrial Production	Consumption Price Index and Industrial Production	Producer Price Index and Industrial Production
0.5	0.134 ***	0.055	-0.157**
1	-0.043	0.051	-0.003
1.5	0.215***	0.222***	0.347***
2	0.289 ***	0.188***	0.303***
2.5	0.285***	0.138	0.280***
3	0.186***	0.074	0.266***
3.5	0.057	-0.015	0.190**
4	0.036	-0.005	0.245***
4.5	-0.059	-0.123	0.175*
5	-0.148	-0.236	0.078
5.5	-0.208	-0.319*	0.013
6	-0.249	-0.386**	-0.031
6.5	-0.279*	-0.439***	-0.068
7	-0.302**	-0.475***	-0.100

Note: The high-pass filter retains that part of the series that is associated with cycles with a period less than the indicated periodicity. The deflator used is CPI-U. The filters are two-sided moving averages with 60 lags and leads. We test whether the estimated coefficient is equal to zero using a one-sided test. Rejections at the 10%, the 5%, and the 1% significance level are indicated by *, **, and ***, respectively. To correct for serial correlation, the standard errors for COR(*k*) are calculated using the VARHAC estimator from Den Haan and Levin (1994).

APPENDIX E. CONSISTENCY OF COVARIANCE ESTIMATES USING FORECAST ERRORS

In this appendix, we show that the covariances calculated using the estimated VAR residuals converge to their population values as the sample size goes to infinity. The variables that enter in the VAR can be any mix of stationary and integrated processes. The proof is a straightforward application of the techniques developed in Park and Phillips (1988,1989) and Sims, Stock, and Watson (1990). Therefore, we just give a simple example. In particular, we focus on a first-order VAR in which one variable is an I(1) stochastic process and one variable is an I(0) stochastic process. In particular, we assume that the stochastic processes p_t and y_t are generated by the following VAR:

(E.1)
$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & a_3 \\ b_1 & -b_1 & b_3 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ p_{t-2} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \nu_t \end{bmatrix},$$

where ε_t and ν_t are each i.i.d. random variables with zero mean that may be correlated.

Thus, y_t is an I(0) process, and p_t is an I(1) process. The following VAR is estimated estimated with least-squares:

(E.2)
$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ p_{t-2} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \nu_t \end{bmatrix}.$$

Let $\hat{\varepsilon}_t$ and $\hat{\nu}_t$ be the estimates of the residuals from the least-squares regression.

<u>Proposition:</u> If y_t and p_t are generated by equation (E.1), then

$$\frac{\sum_{t=3}^{T} \hat{\varepsilon}_t \hat{\nu}_t}{T} \Rightarrow \mathrm{E}\varepsilon_t \nu_t .$$

With $\Rightarrow$ we indicate convergence in probability.

<u>Proof:</u>

Using Theorems 3.1 and 3.2 in Park and Phillips (1989) or the results in section 3 in Sims, Stock and Watson (1990), we know that the least-squares estimators of the parameters satisfy:

(E.4)

$$T^{1/2}(\hat{a}_1 - 1) = O_p(1)$$
$$T^{1/2}(\hat{a}_2) = O_p(1)$$
$$T(\hat{a}_1 + \hat{a}_2 - 1) = O_p(1)$$
$$T^{1/2}(\hat{a}_3 - a_3) = O_p(1)$$

$$T^{1/2}(\hat{b}_1 - b_1) = O_p(1)$$
$$T(\hat{b}_2 + \hat{b}_1) = O_p(1)$$
$$T^{1/2}(\hat{b}_3 - b_3) = O_p(1)$$

The idea is that the coefficients associated with the stationary canonical regressor converge at rate $T^{1/2}$, and the coefficients that are associated with the nonstationary canonical regressor converge at rate T. To understand the mapping between the regressors and the canonical regressors, it is useful to write p_{t-1} as $p_{t-2} + u_{t-1}$, where u_{t-1} is a stationary random variable. Then, p_{t-2} is the only I(1) variable in both regressions. The coefficients in front of this variable converge at rate T. Using the definition of the estimated residual, we get the following expression for the estimate of the covariance:

$$\frac{\sum_{t=3}^{T} \hat{\varepsilon}_t \hat{v}_t}{T} =$$

$$\frac{\sum_{t=3}^{T}\left((1-\hat{a}_1)u_{t-1} + (1-\hat{a}_1-\hat{a}_2)p_{t-2} + (a_3-\hat{a}_3)y_{t-1} + \varepsilon_t\right)\left((b_1-\hat{b}_1)u_{t-1} - (\hat{b}_1+\hat{b}_2)p_{t-2} + (b_3-\hat{b}_3)y_{t-1} + v_t\right)}{T}$$

To show that this estimator converges to E ($\varepsilon_t\ v_t$), we use the following results from Park and Phillips (1989), Sims, Stock and Watson (1990) or Hamilton (1994):

(E.5)

$$\frac{\sum_{t=3}^{T} p_{t-2}^2}{T^2} = O_p(1)$$
$$\frac{\sum_{t=3}^{T} p_{t-2}\, y_{t-1}}{T} = O_p(1)$$
$$\frac{\sum_{t=3}^{T} p_{t-2}\, \varepsilon_t}{T} = O_p(1)$$

$$\frac{\sum_{t=3}^{T} p_{t-2}\, u_t}{T} = O_p(1)$$
$$\frac{\sum_{t=3}^{T} p_{t-2}\, v_t}{T} = O_p(1)$$

For example, we get

$$\frac{\sum_{t=3}^{T}\left((1-\hat{a}_1-\hat{a}_2)(\hat{b}_1+\hat{b}_2)p_{t-2}^2\right)}{T} = \frac{\left(T(1-\hat{a}_1-\hat{a}_2)\,T(\hat{b}_1+\hat{b}_2)\right)\dfrac{\sum_{t=3}^{T}(p_{t-2})^2}{T^2}}{T}.$$

Both terms in the numerator are $O_p(1)$, and thus, the ratio is $o_p(1)$. The other terms are $o_p(1)$ for similar reasons. This completes the proof.

This proposition can be extended to more general processes, but the mapping between the regressors and the canonical regressors becomes more complicated. These techniques can also be used to show consistency for covariances of k-step ahead forecast errors for finite k. The proposition can also be generalized to include higher-order integrated processes or deterministic time trends. For example, suppose that p_t is an I(2) stochastic process. In this case,

$$\frac{\sum_{t=3}^{T} p_{t-2}^2}{T^4} = O_p(1) .$$

The proof still goes through since the least-squares coefficients associated with the canonical second-order regressor converge at rate T^2.

APPENDIX F. UNIT ROOTS IN INFINITE-ORDER FILTERS.

Proposition F1: Suppose that $B(L)$ satisfies the following properties:

(1) $B(L)$ is symmetric, and

(2) $B(L)$ can be expressed as $\sum_{j=-\infty}^{\infty} b_j\, L^j$, and

(3) $B(1) = 0$.

Then $B(L)$ can be written as follows:

$B(L) = (1\text{-}L)\ \overline{B}(L)$, with $\overline{B}(1) = 0$.

Proof:

$$\text{(F.1)}\quad B(L) = \sum_{j=-\infty}^{\infty} b_j\, L^j = b_0 + \sum_{j=1}^{\infty} b_j \left(L^j + L^{-j}\right) =$$

$$= \sum_{j=1}^{\infty} b_j \left(L^j - 2 + L^{-j}\right) = -\sum_{j=1}^{\infty} b_j \left(1 - L^j\right)\left(1 - L^{-j}\right) =$$

$$= -\sum_{j=1}^{\infty} b_j\, (1 - L)\left(1 - L^{-1}\right)\left(\sum_{i=0}^{j-1} L^i\right)\left(\sum_{i=0}^{j-1} L^{-i}\right)$$

Thus,

$$\text{(F.2)}\quad \begin{aligned} \overline{B}(L) &= -\sum_{j=1}^{\infty} b_j \left(1 - L^{-1}\right)\left(\sum_{i=0}^{j-1} L^i\right)\left(\sum_{i=0}^{j-1} L^{-i}\right), \text{ and} \\ \overline{B}(1) &= -\sum_{j=1}^{\infty} b_j\, (1 - 1)\, j^2 = 0. \end{aligned}$$

Proposition F2: Suppose that $B(L)$ satisfies the following properties:

1. $B(L)$ is symmetric,
2. $B(L)$ can be expressed as $\sum_{j=-\infty}^{\infty} b_j L^j$,
3. $B(1) = 0$, and
4. $\sum_{j=1}^{\infty} j^2 b_j < \infty$.

Then $B(L)$ can be written as follows:

$B(L) = (1-L)(1-L^{-1})\ \overline{\overline{B}}(L)$, with $\overline{\overline{B}}(1) < \infty$.

Proof

The proof is identical to the proof for proposition 1. In this case, we get

$$\overline{\overline{B}}(1) = \sum_{j=1}^{\infty} b_j j^2 < \infty.$$

It is easy to verify that the frequency domain filter given in equation (3.8) satisfies the conditions of proposition F1, but does not satisfy the additional condition of proposition 2.

APPENDIX G. POSITIVE IMPULSE RESPONSE FUNCTIONS[1]

Let y_t be a finite AR process of order $2n$. Thus,

$$(G.1)\qquad \left(1-\lambda_1^h L\right)\left(1-\lambda_1^l L\right)\left(1-\lambda_2^h L\right)\left(1-\lambda_2^l L\right)\cdots\left(1-\lambda_n^h L\right)\left(1-\lambda_n^l L\right)\; y_t \;=\; \varepsilon_t,$$

where ε_t is an i.i.d. random variable. We assume that

$$(G.2)\qquad \begin{array}{l} 0 \;\leq\; \lambda_i^h \;<\; 1, \text{ and} \\ \left|\lambda_i^l\right| \;\leq\; \left|\lambda_i^h\right|. \end{array}$$

Proposition G1: Suppose that the law of motion for y_t satisfies (G.1) and (G.2). Then the coefficients in the MA representation

$$(G.3)\qquad y_t \;=\; \varepsilon_t \;+\; \sum_{j=1}^{\infty}\theta_j\,\varepsilon_{t-j}$$

are all positive.

Proof: Since the roots are bounded away from one, we know that the AR process is invertible. Equation (G.1) can be written as follows:

$$(G.4)\qquad y_t = \left[\frac{1}{\lambda_1^h-\lambda_1^l}\left(\frac{\lambda_1^h}{1-\lambda_1^h L}-\frac{\lambda_1^l}{1-\lambda_1^l L}\right)\right]\cdots\left[\frac{1}{\lambda_n^h-\lambda_n^l}\left(\frac{\lambda_n^h}{1-\lambda_n^h L}-\frac{\lambda_n^l}{1-\lambda_n^l L}\right)\right]\varepsilon_t$$

It is not hard to see that all coefficients inside the square brackets are positive. Note that the ratios inside the round brackets are the impulse response functions of AR(1) processes with coefficients equal to λ^h_j and λ^l_j, respectively. Since $|\lambda^h_j| \geq |\lambda^l_j|$, we know that the first ratio always dominates the second. Thus the MA coefficients of the process in round brackets are positive. The term in front of the round brackets is positive as well.

Note that this proposition can easily be extended to $2n+1$-th order AR processes as long as the additional root is positive.

---

[1] Thanks to Tom Sargent the reader does not have to go through the much longer proof that I came up with.