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NONTRANSFERABLE LICENSES:  
WHAT IS THE DIFFERENCE?**

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**ABSTRACT**

This paper questions the presumption that transferable quota licenses are worth more and result in higher welfare. We show that the price of a transferable license will tend to be higher than that of its nontransferable counterpart only if the underlying quota is quite restrictive. Despite this, if consumer surplus and license revenue have equal weight in the welfare function, transferability is preferable to nontransferability. If their weights are unequal, then the comparison could go either way. We also show that increased uncertainty, in the form of a mean preserving spread, does not affect the license price under nontransferability and could raise or lower the level of the license price with transferability depending on the restrictiveness of the quota.

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## **1. Introduction**

Quantitative restrictions are usually implemented by means of issuing quota licenses and allocating these licenses in some way to various agents in the economy. What are the consequences of making these quota licenses transferable as opposed to nontransferable? Most economists would argue that transferable licenses result in higher license prices and improved welfare. The economic intuition behind this seems compelling: if transferability is allowed, licenses will go to agents who value them the most, and this will result in a higher license price. Moreover, since transferable licenses will be allocated "properly" by the market under perfect competition while nontransferable ones need not be, welfare, defined as the sum of license revenue and surplus, will necessarily be higher under transferability, irrespective of the weight attached to revenue relative to surplus.

The difference between transferable licenses and nontransferable licenses is of considerable importance in the policy arena. For example, there is a continuing active debate in the environmental economics literature over the idea of allowing industrial polluters to transfer pollution rights granted to them by the standard-setting authorities. In international trade, the exporting country participants of the Multi-Fibre Arrangement vary significantly in the degree of license transferability permitted; despite the general belief that transferable licenses are somehow preferable to nontransferable ones, a surprisingly large number of countries prohibit or actively discourage license trading. Quota transferability is also an important issue in many other supply-control schemes such as fishery quotas, tobacco production quotas, and taxicab medallions. Among these, the recent auction of personal communication services licenses by the FCC has attracted much academic attention -- a feature

of these sales is that the licenses are transferable, yet none of the work on these auctions respects the implications of transferability (namely, the link between ex post and ex ante prices) as is done in this paper.

In contrast to the usual presumptions, we show that the price of a transferable license will tend to be higher than the price of an otherwise identical nontransferable license only if the underlying quota is quite restrictive. If the quota is not very restrictive, the price of a transferable license will tend to be lower than its nontransferable counterpart. Despite this, transferability is still preferable to nontransferability if consumer surplus and license revenue have equal weight in the welfare function. If their weights are unequal, however, then the comparison could go either way. In other words, if the underlying quota is binding enough, and license revenue is accorded a greater weight than consumer surplus in the welfare function, nontransferability can dominate transferability. Of course, these conditions constitute a special case and we do not advocate nontransferability on these grounds.

Our results differ from the standard ones for two reasons. First, we do not compare an existing, possibly arbitrary, allocation of licenses under nontransferability with that under transferability. Instead, we compare the allocations and outcomes that will result endogenously under the two systems. Second, we explicitly introduce uncertainty so that there is a reason why agents who obtain licenses ex ante may wish to sell them ex post. After all, if the agents' valuations are known and fixed, and the ex ante allocation is endogenous, there will be no reason for them to wish to trade ex post.

Our models are the simplest ones necessary to make our points. We use competitive models throughout to abstract from the complications which arise when agents behave

strategically.<sup>1</sup> Section 2 outlines the traditional analysis. In Section 3, we develop our general approach and analyze the equilibrium under the two systems, transferability and nontransferability. We also examine the effects on the license price as the quota level varies, and compare the welfare consequences of the two systems. In Section 4, we perform some comparative statics on the effect of increases in the level of uncertainty. We show that increasing uncertainty could raise or lower the license price with transferability depending on the restrictiveness of the quota. In Section 5, we relax our assumption of identical agents and provide an example to show that this does not invalidate our basic result regarding the comparison of transferable and nontransferable license prices. Section 6 concludes.

## **2. The traditional model**

The basic intuition behind the traditional presumption that transferable licenses are superior to nontransferable ones is nicely summarized in Faini et al (1992), where it is argued that constraints on license transferability are a source of inefficiency.<sup>2</sup> We recapitulate their results below.

Suppose there are two firms in a country producing a homogeneous good for export. Let the supply functions of the firms be  $S_1$  and  $S_2$  in Figure 1. Suppose the good is subject to a quota  $Q$  and that the initial allocation of this quota led to the firms being awarded  $Q_1$  and  $Q_2$  licenses respectively, such that their marginal costs were equalized with  $Q_1 + Q_2 = Q$ . Now suppose the firms' marginal costs change after this allocation is made. This could happen if, for example, Firm 1 invests in capital which lowers its marginal cost, and hence its supply curve, to  $S'_1$  while Firm 2 does not. If the licenses are not transferable, Firm 1 will continue to produce  $Q_1$  at a total cost given by the area  $OEAQ_1$  and Firm 2 will continue to produce  $Q_2$  at a total cost given by the area  $OFDQ_2$ . If the licenses are transferable, however, Firm 2, the higher marginal cost firm, will sell quota to Firm 1, the lower marginal cost firm, until their marginal costs are again equalized. Firm 1 will now increase its production to  $Q'_1$  and its total cost to  $OEBQ'_1$ , while Firm 2 reduces its output to  $Q'_2$  and its total cost to  $OFCQ'_2$ . Thus, the area  $Q'_2CDQ_2 - Q_1ABQ'_1$  represents the net gain in welfare from permitting transferability.

Faini et al (1992) do not compare the license prices under transferability with those under nontransferability. However, it should be clear that if demand is given, and if there is no uncertainty regarding the supply curves of the two firms (i.e., if the supply curves are fixed at  $S_1$  and  $S_2$ ), then as long as the initial allocation of licenses is efficient in the sense that the

marginal costs of the two firms are equalized, there will be no need for transfers to occur. In Figure 1, this license price would be given by GH, the difference between the demand price (where the demand curve intersects the quota level) and the supply price (where the horizontal sum of the supply, or marginal cost, curves intersects the quota level.)

### **3. The Basic Model**

In order to compare transferability and nontransferability in a meaningful way, we have first to specify the underlying uncertainty in the valuations of licenses, and then to compare the two systems in the context of this uncertainty. We do so next, first in what looks like a special model, but which we show is really quite general.

Let  $v(s)$  denote the willingness to pay, or valuation, of an individual agent in state  $s$  for a license. Each agent demands only one license at most. In the case of export licenses, for example,  $v(\cdot)$  can be interpreted as the difference between the price in the restricted market and the marginal cost of the restricted suppliers.<sup>3</sup> At this time we assume that  $v(\cdot)$  is independent of the quota level. This is possible, for example with export licenses when the exporting country is small so that world prices don't change with the quota; but are random and the domestic supply price is constant. In what follows, we show that our results are valid even if valuations depend on the restrictiveness of the quota as seems reasonable.

Assume there are two time periods. In the first period, the state of the future,  $s$ , is unknown so each agent is uncertain about his valuation; in the second period, this valuation is realized. Licenses are sold in Period 1. If the licenses are nontransferable, then an agent who purchases a license in Period 1 (when  $s$  is unknown) cannot sell it in the second period (when  $s$  is realized.) If the licenses are transferable, then the license holder may, if he wishes, sell his license in the second period at whatever the market clearing price happens to be at that time. The license price in the first period is called the ex ante price, and the license price in the second period is called the ex post price.

Assume for simplicity that all agents are identical ex ante, and that they share a common



distribution of realizations ex post. Further, assume that these realizations are independent across all agents. Let  $f(v)$  be the density function associated with the valuations of each agent, and  $F(v)$  be the cumulative density function. Let  $v^U$  and  $v^L$  be the upper and lower supports of this distribution. We shall assume that  $v^L$  is positive so that licenses will always be used ex post, even if they are nontransferable.

### 3.1 Nontransferable licenses

What is the market clearing price of a nontransferable license in this model? Since all agents are identical ex ante, they would all be willing to pay the same amount, namely, the expected value of  $v$ . Hence, the demand curve for licenses would be horizontal at this price. Of course, the supply of licenses is fixed at  $Q$ . Thus, as long as the quota is binding, that is, as long as there are more agents than quota, the equilibrium price of a license equals the expected valuation if the license is nontransferable.

Who actually receives a license is irrelevant since all agents are identical ex ante. Note that there is zero surplus from obtaining a license -- agents pay exactly the amount they are willing to pay for a license. Note also that the license price is independent of the quota level as long as the quota is binding.

### 3.2 Transferable licenses

Now consider what happens if the licenses are transferable. For simplicity, we will assume that there is an infinite number of agents and treat them as a continuum. Since all agents are identical and their realizations are independent, the ex post distribution of

realizations among the agents will be the same as the ex ante distribution of valuations for each agent. Thus, we can also interpret  $F(v)$  as the proportion of the  $N$  agents who have a valuation less than or equal to  $v$  in the second period.

What determines the ex ante price with transferability? The value of a license in Period 1 depends on the outcome in Period 2, hence the model needs to be solved backwards and we must first find out the price will be in Period 2. In general, this price could be stochastic, although this turns out not to be the case given our assumptions. The ex post price is determined by equating total supply with total demand. This is equivalent to ensuring that the fraction of agents with valuations exceeding the ex post price equals the quota ( $Q$ ) relative to the size of the markets ( $N$ ). Denoting  $Q/N$  by  $\alpha$ , we can thus define the ex post price,  $P_2$ , implicitly in the following way:

$$1 - F(P_2) = \alpha. \tag{1}$$

Note that although each agent's realization is stochastic, the ex post price denoted by  $P_2$  is non-stochastic. Moreover, the ex post price depends on the level of the quota -- the more restrictive the quota, the higher the ex post price. This is the key to our results.

Now let us determine the price in Period 1. Since licenses are transferable, the surplus that an agent would obtain if he purchased the license in Period 1 and sold it in Period 2 (if it was in his interests to do so) must equal the surplus which he could obtain if he waited and purchased the license in Period 2 (if it was in his interests to do so). In other words, the payoff from buying a license in Period 1 must be the same as that from not buying at that time. Intuitively, this has to be the case because given that the quota is binding and all agents are

identical ex ante, if the payoff from buying exceeds that from not buying in Period 1, there would be an excess demand for licenses in Period 1. Conversely, if the payoff from not buying exceeded that from buying in Period 1, there would be an excess demand in Period 2. Therefore, such license prices could not be market clearing prices.

Although the ex post price is non-stochastic in our model, for the sake of generality we shall use notation which allows it to be stochastic. Let  $P_{2s}$  be the equilibrium price in Period 2 when the state is  $s$ . Let  $P_1$  be the price in Period 1. Thus:

$$E_s \{ \max[v_s, P_{2s}] - P_1 \} = E_s \{ \max[(v_s - P_{2s}), 0] \} \quad (2)$$

The left hand side of equation (2) represents the agent's surplus from buying a license in Period 1 and either using it (if his realized valuation exceeds the price in Period 2) or selling it (if his realized valuation falls short of the Period 2 price.) The right hand side represents the surplus the agent would obtain from buying a license in Period 2 only if his realized valuation exceeds the price at that time. Now note that since:

$$\max[v_s, P_{2s}] = P_{2s} + \max[(v_s - P_{2s}), 0] \quad (3)$$

Equation (3) implies that:

$$\begin{aligned} E_s \{ P_{2s} + \max[(v_s - P_{2s}), 0] - P_1 \} &= E_s \{ \max[(v_s - P_{2s}), 0] \} \\ \Rightarrow E_s(P_{2s}) &= P_1. \end{aligned} \quad (4)$$

Thus, the ex ante price,  $P_1$ , must equal the expected ex post price,  $E_s(P_{2s})$ . This should not be unexpected since the ex ante price depends on the outcome in Period 2. In the absence of

transactions costs and the rigidities induced by them, if the expected license price in Period 2 exceeds the Period 1 price, then everyone would have the incentive to wait until Period 2 to purchase a license. Conversely, if the ex ante price exceeds the expected Period 2 price, then everyone would want to purchase a license in Period 1. Therefore, in equilibrium, the ex ante price has to equal the expected ex post price.

A graphical depiction is useful to formalize the above argument. The ex ante price of a transferable license,  $P_1$ , is equal to its ex post price,  $P_2$ , with  $P_2$  implicitly defined by Equation (1). In Figure 2, where  $F(v)$  is the cumulative density function of  $v$ ,  $P_2$  is the point on the horizontal axis at which the distance between  $F(v)$  and the value 1 is equal to  $\alpha$ . The price of a nontransferable license, on the other hand, is  $E(v)$ , the expected value of  $v$ . Recall that  $\alpha = Q/N$ . In Figure 2,  $\alpha$  is drawn for a quota that is not very restrictive. Note that in this case,  $P_2 < E(v)$ , i.e., the transferable license is cheaper than the nontransferable license. The reverse would be true if the quota were very restrictive, all else the same. For example, at  $\alpha' = Q'/N$ , where  $Q' < Q$ ,  $P_2' > E(v)$ , i.e., the transferable license is more expensive than the nontransferable license. In order to clarify what we mean by "restrictive," let us define  $\alpha^*$  as:

$$1 - F(E(v)) = \alpha^* . \quad (5)$$

Then transferability results in a higher license price than nontransferability if  $\alpha < \alpha^*$ , and a lower price if  $\alpha > \alpha^*$ .

What if agents' valuations depend on the restrictiveness of the quota? The following argument shows that although ex ante and ex post prices are affected, their difference remains unchanged. Assume that all agents are identical ex ante as done so far. Let the valuation of an

agent be denoted by  $T(v, \alpha)$  where  $\alpha = Q/N$  so that the level of valuations is affected by the restrictiveness of the quota. Further assume that  $T(v, \alpha) = h(\alpha) + v$ , where  $h'(\cdot) < 0$ . In other words, a decrease in the restrictiveness of the quota, that is, an increase in  $\alpha$  reduces valuations. As before,  $v$  is uncertain for each agent ex ante, and is distributed according to  $f(\cdot)$ . With  $v$  being independently and identically distributed over the continuum of agents, the ex post distribution of  $v$  is the same as that of each agent ex ante. Thus, ex ante, the expected value of  $T(\cdot)$  is given by  $h(\alpha) + E(v)$  and this is the license price ( $L^{NT}$ ) under non transferability. With transferability, the price ex post is such that the proportion of agents with  $T$  above the license price, ( $L^T$ ), equals  $\alpha$ . That is,  $1 - F(L^T - h(\alpha)) = \alpha$ , because if  $T = h(\alpha) + v > L^T$ , then  $v > L^T - h(\alpha)$  and the proportion of agents with  $T > L^T$  is given by  $1 - F(L^T - h(\alpha))$ . Thus,  $L^T(\alpha) = F^{-1}(1 - \alpha) + h(\alpha)$ . However,  $L^T(\alpha) - L^{NT}(\alpha) = F^{-1}(1 - \alpha) - E(v)$ , which is the same as earlier when valuations did not depend on  $\alpha$  so that our earlier result still goes through. Allowing the level of valuations to depend on the restrictiveness of the quota thus alters the equilibrium price under transferability and under non transferability by the same amount and leaves their difference unchanged. For this reason, we choose to restrict attention from here on to the simpler model where valuations are independent on the restrictiveness of the quota without loss of generality.

The results so far are summarized in Proposition 1.

*Proposition 1:*

*The price of a license with transferability can be higher or lower than that without transferability. Given the market size,  $N$ , there exists a critical quota level,  $Q^*$ , which corresponds to the critical level of  $\alpha$ ,  $\alpha^*$ , such that if the quota lies below  $Q^*$ , (that is, if the quota is restrictive enough,) the ex ante price with transferability will exceed that without*

*transferability. However, if the quota level exceeds  $Q^*$ , (that is, if the quota is not very restrictive,) the reverse will be true.*

An interesting feature of this equilibrium is that unlike the case with nontransferability, rents are not all competed away despite the assumption that all agents are identical. The reason is that ex ante, an agent's bid is determined by equating his payoff from purchasing the license in Period 1 with his payoff from not purchasing the license in Period 2. If the license is not transferable, then his payoff from not buying the license in Period 1 is zero; hence, the ex ante price of a nontransferable license will rise to the point where it reflects exactly the ex ante valuation of the license,  $E(v)$ . If the license is transferable, however, then the agent's payoff from not buying the license in Period 1 is not zero, since he has the opportunity to purchase the license on the secondary market in Period 2. In other words, we can think of his payoff from not buying the license in Period 1 as his payoff from waiting and buying the license in Period 2 if it is in his interest to do so. The ex ante price in this case is restrained by the ex post price, and will not rise to the point where all surplus is eliminated. Ex ante, no agent would be willing to bid more than the ex post price since he would be better off waiting and buying the quota in the second period. The ex post price does not depend on the ex ante price since it is optimal to let bygones be bygones. Ex post, since licenses are transferable, we can think of the entire supply of licenses being put on the market. Agents who own licenses ex ante and end up having a realization higher than the ex post price buy back their licenses. The remainder sell their licenses to agents who do not have licenses ex ante but desire them ex post.

### 3.3 Welfare Comparisons

Our next step is to analyze the welfare implications of Proposition 1. Does a higher license price necessarily imply higher welfare? Is transferability always better than nontransferability? We show that the answer to the first question is no and the answer to the second is yes, if license revenue and surplus are given equal weight in the welfare function. If unequal weights are permitted, the answer to the second question is no. It makes intuitive sense for transferability to dominate nontransferability when equal weight is given to revenue and surplus. After all, license revenue comes at the expense of surplus, and transferability allows agents with higher surplus ex post to obtain the licenses.

Both surplus and revenues from license sales enter the welfare function. In the case of nontransferable licenses, there is zero surplus as it is all competed away; therefore welfare equals license revenue only. With transferable licenses, welfare includes surplus which is positive.

Since all agents are ex ante identical, national welfare is proportional to that of the typical agent. Consider the welfare of a typical agent ex ante. Assume that license revenues are returned in a lump sum manner to all agents. Thus the welfare of an agent equals his surplus ex ante plus his share,  $1/N$ , of the license revenue of  $QE(v)$ . Since surplus is zero, the welfare of the representative agent under nontransferability is given by:

$$W_N = \frac{Q}{N} E(v) = \alpha E(v). \quad (6)$$

What about welfare with transferability? If transfers are allowed, the representative agent will purchase the license whenever his valuation exceeds the ex ante price and he will earn

surplus from this action. In addition, he receives his share of the license revenue,  $(1/N)QP_2$ . Thus, welfare with transferability equals the sum of surplus and revenue accruing to the typical agent. This is given by:

$$W_T = \int_{P_2}^v (v - P_2) f(v) dv + \alpha P_2. \quad (7)$$

Therefore, subtracting (6) from (7) gives:

$$W_T - W_N = \int_{P_2}^v (v - P_2) f(v) dv + \alpha [P_2 - E(v)] \quad (8)$$

It is obvious from (8) that if the price under transferability is more than that under nontransferability (i.e., if  $P_2 - E(v) > 0$ ), then transferability must yield higher welfare than nontransferability since both terms on the right hand side of (8) are positive in this case.

On the other hand, if the reverse is true and  $P_2 - E(v) < 0$ , then the sign of (8) is not apparent from examination of the first line of the equation as the first term is positive and the second is negative. However, by manipulating (8) and using the fact that  $1 - F(P_2) = \alpha$ , and then integrating by parts, we obtain:

$$W_T - W_N = (1 - \alpha) [E(v) - P_2] + \int_{v^L}^{P_2} F(v) dv. \quad (9)$$

This is positive if  $P_2 - E(v) < 0$ , since both terms in (9) are positive in this event. Thus, if



both surplus and revenue are given equal weight in welfare, transferability is better than nontransferability.

What if revenue and surplus receive unequal weights? From (8) it is clear that if the weight on revenue rises enough, and  $P_2 - E(v) < 0$ , that is, if the quota is not very restrictive, then nontransferability becomes better than transferability. If  $P_2 - E(v) > 0$ , that is, if the quota is restrictive enough, increasing the weight on revenue makes transferability even more attractive than in the case with equal weights. However, raising the weight on surplus above unity can never make nontransferability better than transferability since there is no surplus under nontransferability. These results are summarized in Proposition 2.

*Proposition 2:*

*When surplus and revenue are given equal weight in welfare, transferability yields higher welfare than nontransferability. If the weight on revenue is high enough and the quota is not very restrictive, that is, the license price under transferability is less than that under non-transferability, then nontransferability can result in higher welfare.*

#### **4. The effect of increased uncertainty**

Let us now consider how the results of our basic model are affected by exogenous changes in the distribution of  $v$ . In particular, we want to see how increasing uncertainty, modelled as a mean-preserving spread, affects the comparison between transferable and nontransferable licenses.

We shall first summarize the implications of a mean-preserving spread as established in Rothschild and Stiglitz (1970), and then apply them to our problem. A mean preserving spread affects the density function by moving weight to the tails of the distribution while leaving the mean unchanged. If two density functions,  $g(\cdot)$  and  $f(\cdot)$  differ by a single mean preserving spread,  $s(\cdot)$ , so that  $g(\cdot) - f(\cdot) = s(\cdot)$ , and  $G(\cdot)$ ,  $F(\cdot)$  and  $S(\cdot)$  are the corresponding cumulative density functions, then by definition: (1)  $S(v) = G(v) - F(v)$  for all  $v$ ; (2)  $S(v^L) = S(v^U) = 0$ ; (3) there exists  $Z$  such that  $S(v) \geq 0$  if  $v \leq Z$ , and  $S(v) \leq 0$  if  $v > Z$ ; (4) if  $T(y) = \int_{v^L}^y S(v) dv$ , for  $v^L \leq y \leq v^U$ , then (a)  $T(v^L) = T(v^U) = 0$ ;<sup>4</sup> and (b)  $T(y) \geq 0$  for  $v^L \leq y \leq v^U$ .<sup>5</sup>

These properties are illustrated in Figure 3, where  $G(\cdot)$  is obtained by adding a mean preserving spread,  $S(\cdot)$ , to  $F(\cdot)$ . Thus, below  $v^{**}$ ,  $G(\cdot)$  lies above  $F(\cdot)$  and above  $v^{**}$  it lies below  $F(\cdot)$ . The distance between  $G(\cdot)$  and  $F(\cdot)$  is  $S(\cdot)$ . This distance is zero at both end points. The area between  $G(\cdot)$  and  $F(\cdot)$  to each side of  $v^{**}$  is the same.

Now consider the effect on the license price of such a change in uncertainty. With the distribution function  $F(\cdot)$ ,  $P_2$  was the ex ante price with transferability as explained earlier, with  $F(P_2) = 1 - \alpha$ . With a mean preserving spread, if  $P_2$  lies below  $v^{**}$ , then the ex ante price falls to  $P_2'$  where  $G(P_2') = 1 - \alpha$ . If  $P_2$  lies above  $v^{**}$ , then the ex ante price rises to  $P_2''$  where  $G(P_2'') = 1 - \alpha$ . Since  $P_2$  is low when  $\alpha$  is high or the quota is not very restrictive, and

is high when the quota is quite restrictive we have the following result.

Proposition 3:

*Let  $\alpha^{**}$  be defined by  $F(v^{**}) = 1 - \alpha^{**}$ . Let  $Q^{**}$  be the quota level corresponding to  $\alpha^{**}$ . If the quota level,  $Q$ , exceeds  $Q^{**}$  then an increase in uncertainty lowers the ex ante price with transferability. If  $Q$  is less than  $Q^{**}$ , then an increase in uncertainty raises the ex ante price with transferability.*

Since the price without transferability is unaffected by a mean preserving spread, and since the ex ante price with transferability exceeds the price without transferability only when the quota is quite restrictive, we can deduce from the above that increasing uncertainty is likely to accentuate this difference in either case.

## 5. Heterogeneous agents

The results we have developed so far rest on the special assumptions in our model, e.g. identical agents with independently and identically distributed valuations. However, it is easy to see that Proposition 1 is also generally applicable in the case of heterogeneous agents.

Suppose now that there are different types of agents in the economy. Let  $x$  denote agent type, such that the higher the value of  $x$ , the greater the willingness of the agent to pay for a license. The distribution of agent types in the population is given by  $g(x)$ , with upper and lower supports denoted  $x^u$  and  $x^l$  respectively. Each agent knows his own type,  $x$ , ex ante. As before, suppose that each agent's valuation,  $v$ , also depends on the state of nature,  $s$ , in the next period. Ex ante, each agent only knows the distribution of  $v$ , denoted  $f(v)$ . This distribution is identical for, and independent across all agents, so it also represents the distribution of realizations ex post. Hence, each agent's ex post valuation of a license is given by  $x+v(s)$ . Also as before, assume each agent demands at most one license.

If the licenses are nontransferable, an agent will be willing to pay  $x+E(v)$  for the license. Note that  $E(v)$  is a constant, hence the ex ante distribution of valuations in the economy is given simply by  $g(x)$ . Letting  $\alpha$  denote the restrictiveness of the quota,  $Q/N$ , as before, and arranging agents in ascending type, equilibrium will obtain when the fraction of agents with ex ante valuations exceeding  $x^*$  is exactly equal to  $\alpha$ , i.e.:

$$1 - G(x^*) = \alpha \tag{10}$$

where  $G(\cdot)$  denotes the cumulative distribution function of  $x$ . The marginal agent in this case will be the one of type  $x^*$ , where  $x^* = G^{-1}(1-\alpha)$ . The nontransferable license price is thus:

$$L^{NT} = G^{-1}(1 - \alpha) + E(v). \quad (11)$$

This is bounded below by  $x^L + E(v)$  and above by  $x^U + E(v)$ . Once we allow for heterogeneous agents, therefore, the nontransferable license price is no longer independent of the restrictiveness of the quota. Note that the variability of the nontransferable license price depends only on the ex ante variability of the agents and not on the variability of states, which is averaged to  $E(v)$  by all agents.

What if the licenses are transferable? We know that the equilibrium will obtain where the proportion of agents with ex post valuations exceeding the license price is exactly equal to  $\alpha$ . Let  $L^T$  denote the transferable license price. Then for an agent of type  $x$ , his ex post valuation,  $x + v$ , will be larger than  $L^T$  only if  $v > L^T - x$ . The probability of this occurrence is given by  $1 - F(L^T - x)$ . Hence:

$$\int_{x^L}^{x^U} g(x) [1 - F(L^T - x)] dx = \alpha \quad (12)$$

implicitly defines the transferable license price. Unlike the nontransferable license price which only depends on the distribution of agent types and the mean of the states, the transferable license price depends on the both the distribution of agent types and the distribution of states. As long as the range of agent types is not too large relative to the range of possible states of nature in the second period, the argument underlying Proposition 1 will still hold. Introducing heterogeneous agents will most certainly make the ex ante license price a function of the quota, but should not change the flavor of the results in Proposition 1 as long as the variability

across agent types is not too great.

Figure 4 provides a sketch of our argument. An example with uniform distributions for  $x$  and  $v$  is worked out in detail in the appendix. Agent types,  $x$ , are plotted along the vertical axis, with  $x^L$  and  $x^U$  being respectively the lower and upper limits on the individuals' willingness to pay for a license. The state-dependent valuations,  $v$ , are plotted along the horizontal axis, with  $v^L$  and  $v^U$  as the lower and upper limits respectively. Let us denote the joint distribution of  $v$  and  $x$  by  $f(v, x)$ . The roughly circular area in Figure 4 represents the support of  $f(v, x)$ . For each  $x$ , the conditional distribution of  $v$  is  $f(v|x)$ , and the conditional mean  $E(v|x)$  is also drawn in Figure 4.

Under nontransferability, the marginal agent type,  $x^*$ , is defined such that the proportion of agents with  $x > x^*$  is equal to  $\alpha$ , the restrictiveness of the quota. In Figure 4, the proportion of the area of the joint distribution above the horizontal line  $x = x^*$  is exactly equal to  $\alpha$ . The nontransferable license price is simply  $L^{NT} = x^* + E(v|x^*)$ .

With transferability, assuming a continuum of agents of type  $x$ , and with valuations  $v$  independently and identically distributed ex ante for each agent, the ex post distribution of realizations  $v$  (conditional on agent type  $x$ ) across all agents is the same as the ex ante distribution of valuations  $v$  for each agent. Thus, the fraction of the population willing to pay, say,  $x^* + E(v|x^*)$  ex post is given by the proportion of the area of the joint distribution above the downward sloping 45 degree line  $x + v = x^* + E(v|x^*)$ . Equilibrium obtains when the proportion of agents with ex post valuations,  $x + v$ , exceeding the license price,  $L^T$ , reflects the restrictiveness of the quota. In Figure 4, the transferable license price is found when the proportion of the area of the joint distribution above the  $x + v$  line is exactly equal to  $\alpha$ .

Let us first consider the most restrictive quota:  $\alpha=0$ . In this case, the nontransferable license price would have to be  $L^{NT} = x^U + E(v|x^U)$  since there are no points and zero density above the upper limit,  $x^U$ , of agent types. However, it can be seen from Figure 4 that the transferable license price has to be higher than that, since the proportion of the area of  $f(v,x)$  above the line  $x+v = x^U + E(v|x^U)$  is not zero! In other words, at the price of  $x^U + E(v|x^U)$ , there would be an excess demand if the licenses are transferable.

Next, consider the opposite extreme of a nonrestrictive quota:  $\alpha=1$ . In this case, the nontransferable license price would have to be  $L^{NT} = x^L + E(v|x^L)$  since the proportion of agents willing to pay at least  $x^L$  is equal to one. However, it can be seen from Figure 4 that the transferable license price has to be lower than that, since the proportion of the area of  $f(v,x)$  above the line  $x+v = x^L + E(v|x^L)$  is not one, i.e., there would be an excess supply at price  $x^L + E(v|x^L)$  if the licenses were transferable.

To summarize, for very restrictive quotas ( $\alpha \rightarrow 0$ ), we have the result that  $L^T > L^{NT}$  but for not very restrictive quotas ( $\alpha \rightarrow 1$ ), we have  $L^T < L^{NT}$ . Hence, the basic result of Proposition 1 still holds when we allow for heterogeneous agents.

## **Conclusion**

In this paper we developed a simple model which allows us to look at the difference between transferable and nontransferable licenses in the presence of uncertainty. Our basic result is that the price of a transferable license tends to exceed that of a nontransferable license only if the underlying quota is quite restrictive. Despite this, if consumer surplus and license revenue have equal weight in the welfare function, transferability is still preferable to nontransferability. We also demonstrated how increased uncertainty, in the form of a mean-preserving spread, will not affect the nontransferable license price but could raise or lower the transferable license price depending on the restrictiveness of the quota. Furthermore, allowing for heterogeneous agents does not affect the general flavor of our results.

Our work has obvious implications and extensions in a number of areas. First, this model is applicable to a variety of settings, from environmental permits to quota licenses. Each application has a different microeconomic structure from which the agents' valuations are derived, and it should be modelled fully in the relevant context. Second, since the existence and/or size of surplus is very different with transferability than without, it follows that entry considerations are also likely to differ under the two regimes. Future work on this topic will include modelling the microstructure underlying the agents' valuations, and comparing the policy implications toward entry under the two regimes.



Figure 1: The traditional model

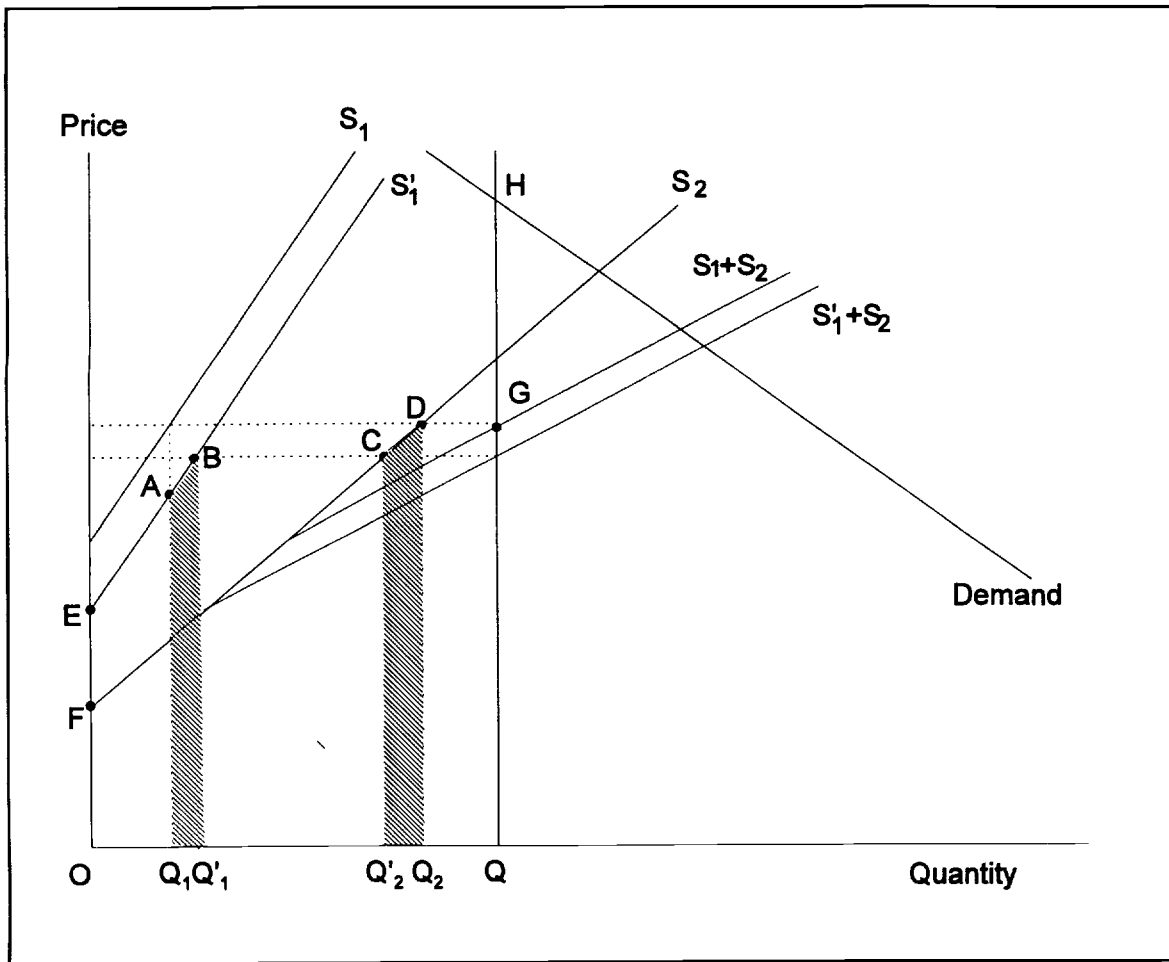


Figure 2: Transferable and nontransferable license prices

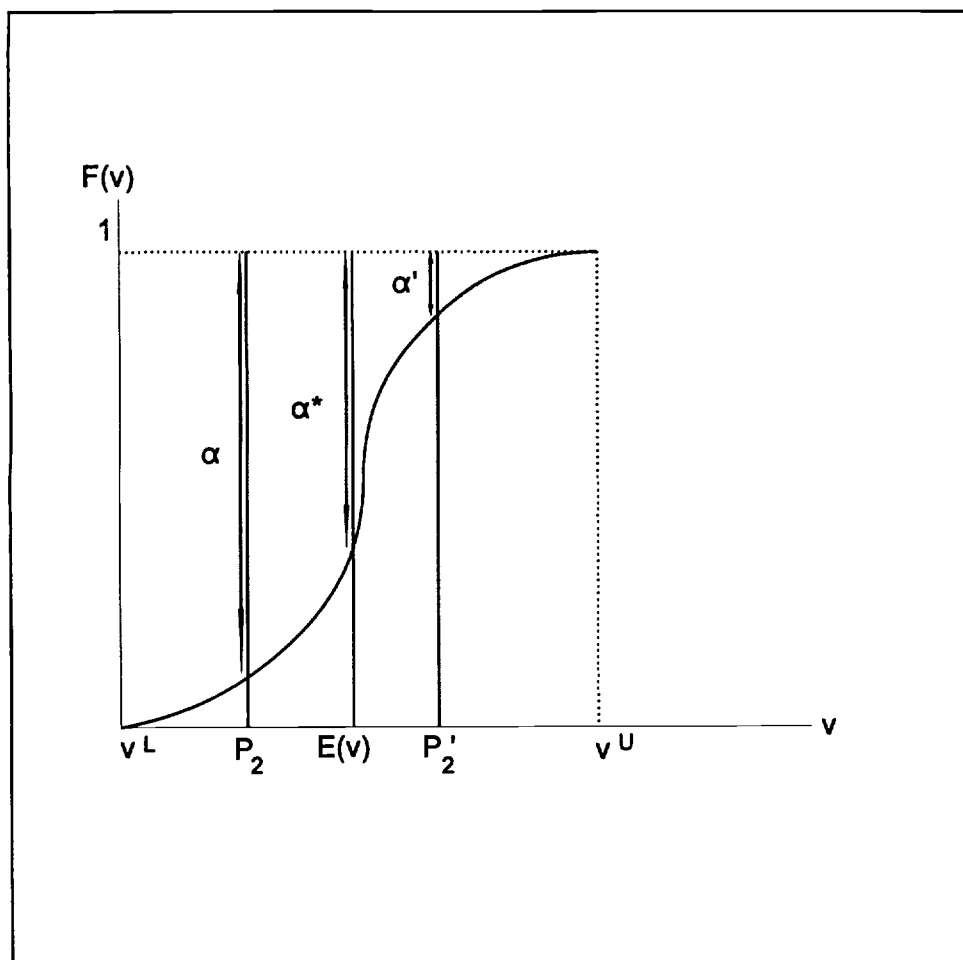
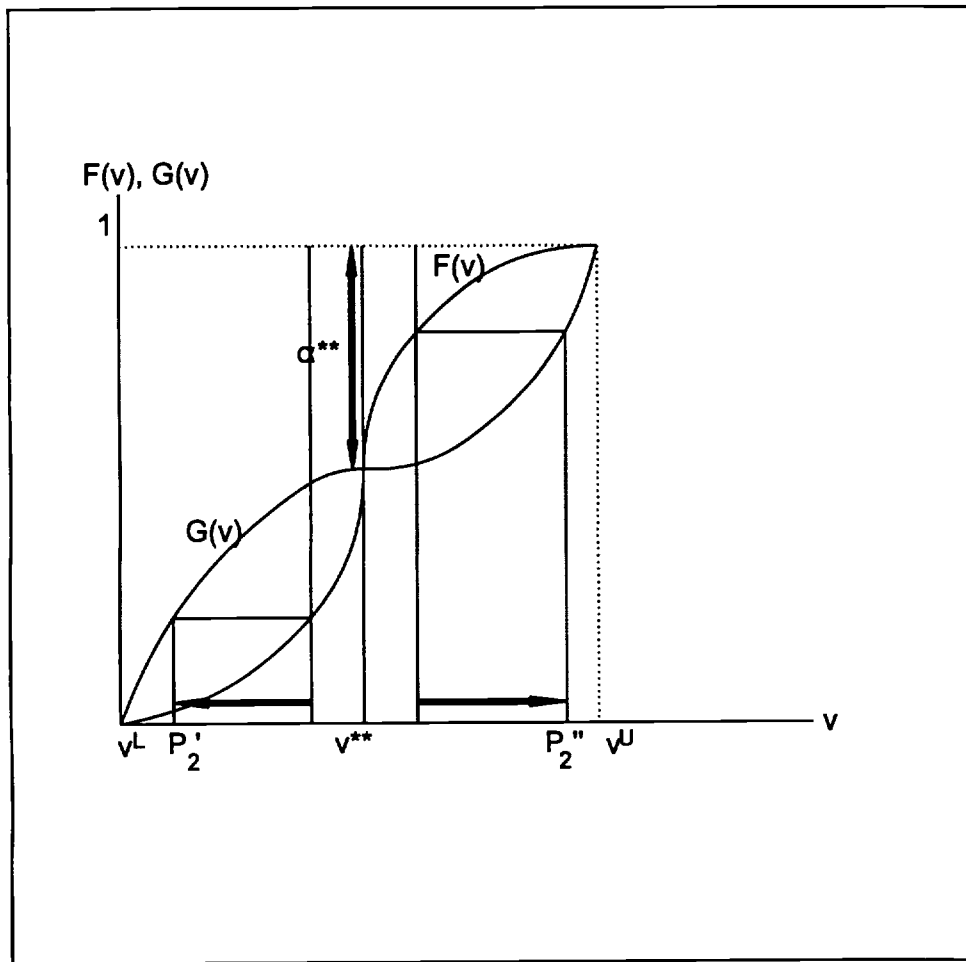
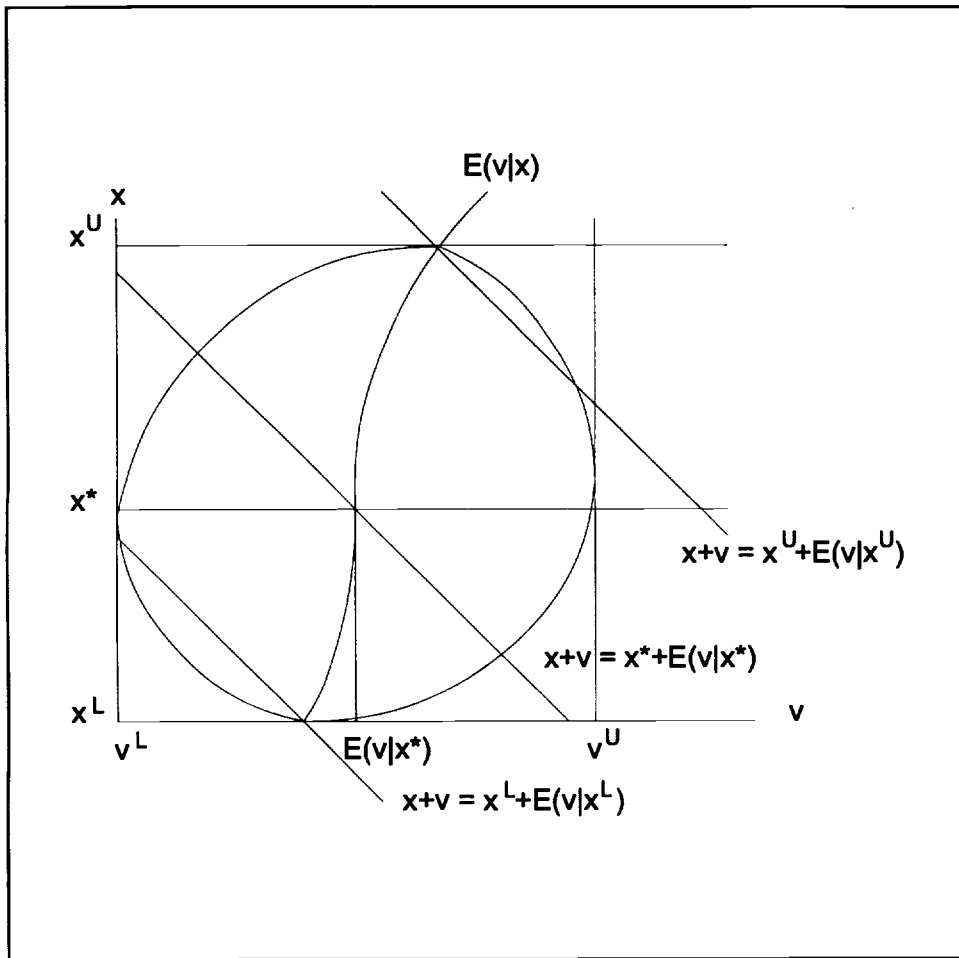


Figure 3: The effect of uncertainty



**Figure 4: Transferable vs nontransferable license prices**



## References

- Faini, R., J. de Melo and W. Takacs. 1992. "A Primer on the MFA Maze." Brussels: European Centre for Advanced Research in Economics. Xerox.
- Lott, J.R. 1987. "Licensing and Nontransferable Rents." *American Economic Review* 77: 453-455.
- Rothschild, M. and J.E. Stiglitz. 1970. "Increasing Risk: 1. A Definition." *Journal of Economic Theory* 2: 225-243.
- Tan, L.H. 1992. "Market Structure and the Implementation of Quotas." Ph.D dissertation. Harvard University.

## End notes

1. Our motivation for studying this issue came from an empirical puzzle that was studied in Tan (1992). She found that in the market for vehicle ownership permits in Singapore, transferability did not seem to raise their value. She then provided a discrete example that could explain her results.
2. Lott (1987) presents a similar argument in the context of professional licensing to show that non-transferable licenses result in greater social losses than transferable licenses.
3. In the case of import licenses,  $V(\cdot)$  can be interpreted as the consumer's willingness to pay for the licenses, and is based on the difference between the consumer's valuation of the import and its price in the world market.
4.  $T(v^L) = 0$  follows by definition.  $T(v^U) = 0$  is less obvious. By definition of a mean preserving spread,

$$\int_{v^L}^{v^U} v s(v) dv = 0.$$

Integrating by parts gives:

$$\begin{aligned} \int_{v^L}^{v^U} v s(v) dv &= v S(v) \Big|_{v^L}^{v^U} - \int_{v^L}^{v^U} S(v) dv \\ &= - \int_{v^L}^{v^U} S(v) dv \\ &= -T(v^U) \\ &= 0. \end{aligned}$$

5. This follows from points (3) and (4a).

### Appendix: An example with uniform distributions

Suppose  $x$  is uniformly distributed over the interval  $[0, X]$ , so that  $g(x) = 1/X$  and  $G(x) = x/X$ , and  $v$  is uniformly distributed over the interval  $[0,1]$ , with  $f(v) = 1$  and  $F(v) = v$ . Then  $E(v) = 1/2$ , and from (11), we have:

$$L^{NT} = (1 - \alpha)X + \frac{1}{2}. \quad (\text{A-1})$$

$L^{NT}$  is bounded below by  $1/2$ , and above by  $X + 1/2$ . Note that  $L^{NT}$  is a negative function of  $\alpha$  -- the more restrictive the quota (i.e., the smaller  $\alpha$  is), the higher is the license price under nontransferability.

The derivation of the transferable license price is slightly trickier. We know that  $L^T$  is determined such that the proportion of agents with ex post valuations  $x+v$  exceeding  $L^T$  is exactly equal to  $\alpha$ . In the  $(v, x)$  space shown in Figure A.1, with  $X \leq 1$ , the area of relevance is bounded by the two axes, the line  $v=1$ , and the line  $x=X$ . Combinations of  $x$  and  $v$  such that  $x+v=L^T$  are represented by a straight line of slope  $-1$ . The line AB represents  $x+v=L^T$  for a value of  $L^T$  less than  $X$ , the line CD for  $L^T$  between  $X$  and  $1$ , and the line EF for  $L^T$  greater than  $1$ .

Let us consider the case of  $L^T < X$  first. The fraction of agents with  $x+v > L^T$  is given by the area above the line AB, which is  $X - 1/2(L^T)^2$ , relative to the area  $X$ . Setting this equal to  $\alpha$ , we have:

$$L^T = \sqrt{2X(1 - \alpha)} \quad (\text{A-2})$$

As the restrictiveness of the quota is relaxed ( $\alpha \rightarrow 1$ ), the license price falls ( $L^T \rightarrow 0$ ).

If  $X < L^T < 1$ , then it is clear from line CD in Figure A.1 that the proportion of agents with

$x+v > L^T$  is  $[(1-L^T)X + \frac{1}{2}X^2]/X$ . The license price for which this proportion is equal to  $\alpha$  is:

$$L^T = 1 + \frac{1}{2}X - \alpha \quad (\text{A-3})$$

Finally, if  $L^T > 1$ , then it can be deduced from line EF in Figure A.1 that the proportion of agents with  $x+v > L^T$  is  $\frac{1}{2}[1-(L^T-X)]^2/X$ . Again, setting this equal to  $\alpha$ , we have:

$$L^T = 1 + X - \sqrt{2\alpha X} \quad (\text{A-4})$$

The more restrictive the quota ( $\alpha \rightarrow 0$ ), the higher the license price ( $L^T \rightarrow 1+X$ .)

Figure A.2 compares  $L^{NT}$  and  $L^T$  as functions of  $\alpha$ , in the case where  $X \leq 1$ .  $L^{NT}$  is depicted as a downward sloping line with slope  $-X$ . For values of  $\alpha$  below  $\frac{1}{2}X$ ,  $L^T$  is given by (A-2), for values of  $\alpha$  between  $\frac{1}{2}X$  and  $(1-\frac{1}{2}X)$ ,  $L^T$  is given by (A-3), and for values of  $\alpha$  exceeding  $(1-\frac{1}{2}X)$ ,  $L^T$  is given by (A-4). Note that in this example,  $L^T = L^{NT}$  at  $\frac{1}{2}$ . For more restrictive values of  $\alpha$  (i.e.,  $\alpha < \frac{1}{2}$ ),  $L^T > L^{NT}$ , but for less restrictive values of  $\alpha$  (i.e.,  $\alpha > \frac{1}{2}$ ),  $L^T < L^{NT}$ . Hence the basic result regarding the comparison of transferable and nontransferable license prices stated in Proposition 1 still holds.

Figure A.3 illustrates the  $(v, x)$  space for  $X > 1$ . As before, the area of relevance is bounded by the two axes and the lines  $v=1$  and  $x=X$ . Line AB represents combinations of  $x+v=L^T$  for  $L^T$  less than 1, line CD for  $L^T$  between 1 and  $X$ , and line EF for  $L^T$  exceeding  $X$ . Following the same procedure as before, it can be shown that the equilibrium  $L^T$  is given by (A-2) in the first case, and (A-4) in the third case. If  $1 < L^T < X$ , then the proportion of agents with  $x+v > L^T$  is  $[(X-L^T) + \frac{1}{2}]/X$ , i.e., the area above the line CD relative to the total area  $X$  in Figure A.3. The license price for which this proportion is equal to  $\alpha$  is:



$$L^T = \frac{1}{2} + X(1 - \alpha) \quad (\text{A-5})$$

Figure A.4 compares  $L^{NT}$  and  $L^T$  as functions of  $\alpha$ , in the case where  $X > 1$ . Again,  $L^{NT}$  is depicted as a downward sloping line with slope  $-X$ . For values of  $\alpha$  below  $\alpha^*$  (where  $\alpha^* = 1/(2X)$ ),  $L^T$  is given by (A-4), and for values of  $\alpha$  exceeding  $\alpha^{**}$  (where  $\alpha^{**} = 1 - \alpha^*$ ),  $L^T$  is given by (A-2). For values of  $\alpha$  between  $\alpha^*$  and  $\alpha^{**}$ ,  $L^T$  is given by (A-5) -- note that in this region,  $\bar{L} = \bar{L}^T$ . Again, the basic result of Proposition 1 continues to hold.

Figure A.1: Combinations of  $x$  and  $v$  with  $x+v=L^T$ , when  $X \leq 1$

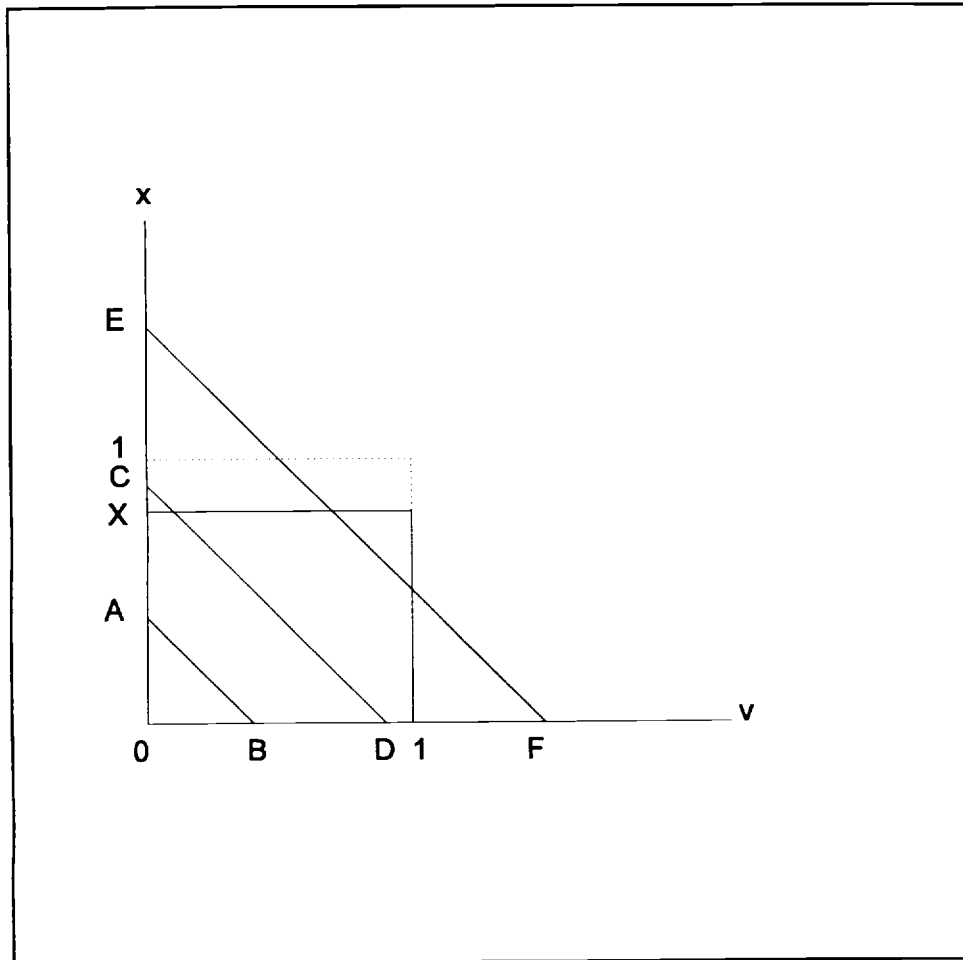


Figure A.2: Transferable vs nontransferable license prices,  $X \leq 1$

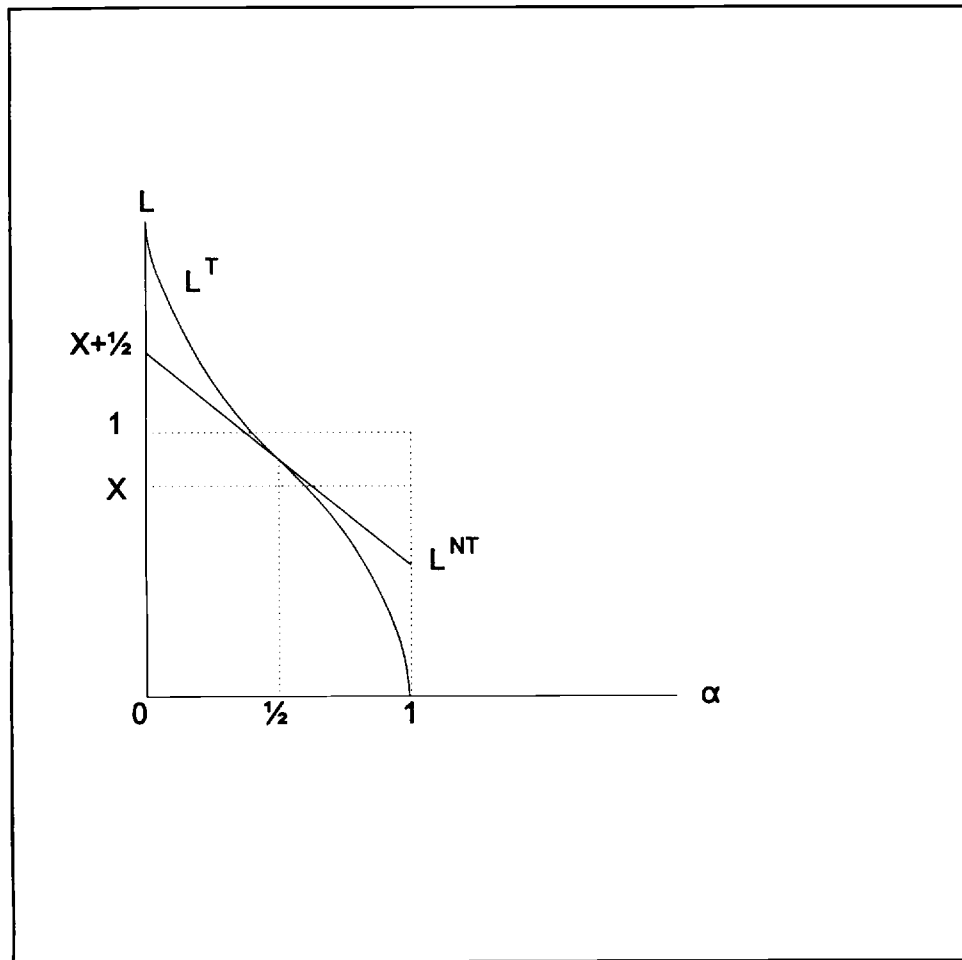


Figure A.3: Combinations of  $x$  and  $v$  with  $x+v=L^T$ , when  $X > 1$

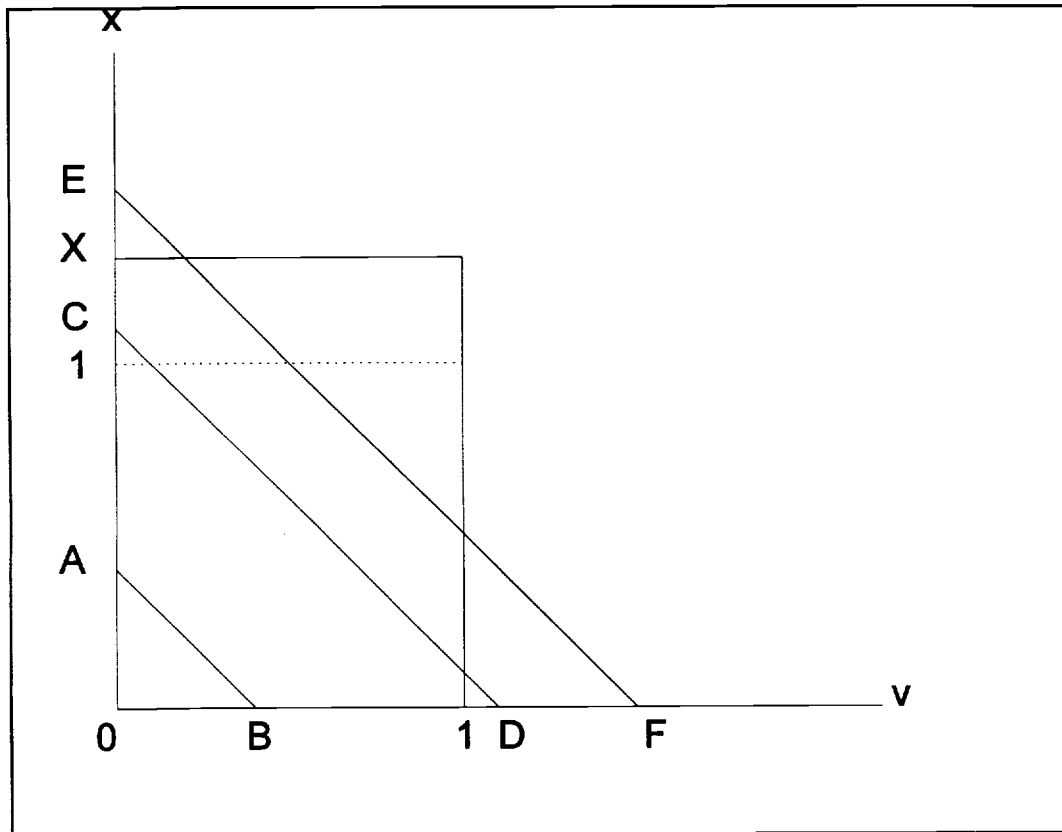


Figure A.4: Transferable vs nontransferable license prices,  $X > 1$

