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**DETERMINISTIC VS. STOCHASTIC  
TREND IN U.S. GNP, YET AGAIN**

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**Working Paper 5481**

**NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
March 1996**

**Craig Hakkio, Andy Postlewaite, Glenn Rudebusch, Chuck Whiteman, two referees and a co-editor provided insightful and constructive comments, as did numerous seminar participants, but all errors remain ours alone. We gratefully acknowledge support from the National Science Foundation, the Sloan Foundation, and the University of Pennsylvania Research Foundation. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.**

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**ABSTRACT**

A sleepy consensus has emerged that U.S. GNP data are uninformative as to whether trend is better described as deterministic or stochastic. Although the distinction is not critical in some contexts, it *is* important for point forecasting, because the two models imply very different long-run dynamics and hence different long-run forecasts. We argue that, even for the famously recalcitrant GNP series, unit root tests over long spans *can* be informative. Our results make clear that uncritical repetition of the "we don't know, and we don't care" mantra is just as scientifically irresponsible as blind adoption of the view that "all macroeconomic series are difference-stationary," or the view that "all macroeconomic series are trend-stationary." There is simply no substitute for serious, case-by-case analysis.

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## **Deterministic vs. Stochastic Trend in U.S. GNP (Yet Again)**

Fifteen years after the seminal work of Nelson and Plosser (1982), the question of deterministic vs. stochastic trend in U.S. GNP (and other key aggregates) remains open. The surrounding controversy certainly isn't due to lack of professional interest -- the literature on the question is huge. Instead, the low power of tests of stochastic trend (or "difference stationarity" in the parlance of Cochrane, 1988) against nearby deterministic-trend ("trend-stationary") alternatives, together with the fact that such nearby alternatives are the relevant ones, explains the lack of consensus.

In an important paper, Rudebusch (1993) contributes to the "we don't know" literature initiated by Christiano and Eichenbaum (1990) by arguing that unit-root tests applied to U.S. quarterly real GNP per capita lack power even against *distant* alternatives. First, Rudebusch shows that the best-fitting trend-stationary and difference-stationary models imply very different medium- and long-run dynamics. Then he shows with an innovative procedure that, regardless of which of the two models obtains, the exact finite-sample distributions of the Dickey-Fuller (e.g., 1981) test statistics are very similar. Thus, Rudebusch concludes that unit root tests are unlikely to be capable of discriminating between deterministic and stochastic trend.

The distinction between trend stationarity and difference stationarity is not critical in some contexts. Often, for example, one wants a broad gauge of the persistence in aggregate output dynamics, in which case one may be better informed by an interval estimate of the dominant root in an autoregressive approximation. Hence the importance of Stock's (1991) clever procedure for computing such intervals. But the distinction between trend stationarity

and difference stationarity *is* potentially important in other contexts, such as economic forecasting, because the trend- and difference-stationary models may imply very different dynamics and hence different point forecasts, as argued by Stock and Watson (1988) and Campbell and Perron (1991).

Motivated by the potential importance of unit roots for the forecasting of aggregate output, as well as other considerations that we discuss later, we extend Rudebusch's analysis to several long spans of annual U.S. real GNP data, and we examine the robustness of all results to variations in the sample period and the particular GNP measure employed. As we shall show, the outcome is both surprising and robust.

#### **I. Construction of Annual U.S. Real GNP Series, 1869-1993**

Three annual "raw" data series underlie the annual series used in this paper. We create the first two, which are real GNP series, by splicing the Balke-Gordon and Romer 1869-1929 real GNP series to the 1929-1993 real GNP series reported in Table 1.10 of the National Income and Product Accounts of the United States, measured in billions of 1987 dollars. The two historical real GNP series are measured in billions of 1982 dollars, which we convert to 1987 dollars by multiplying by 1.166, which is the ratio of the 1987 dollar value to the 1982 dollar value in the overlap year 1929, at which time both the Balke-Gordon and Romer series are in precise agreement.

The third series is total population residing in the United States (in thousands of people), as reported by the Bureau of the Census. For 1869-1970, we take the data from Table A-7 of *Historical Statistics of the United States*. For 1971-1993, we take the data from the Census Bureau's *Current Population Reports*, Series P-25.

From these underlying series we create and use:

**GNP-BG ("GNP, Balke-Gordon"):** Gross national product, pre-1929 values from Balke-Gordon;

**GNP-R ("GNP, Romer"):** Gross national product, pre-1929 values from Romer;

**GNP-BGPC ("GNP, Balke-Gordon, per capita"):** Gross national product per capita, pre-1929 values from Balke-Gordon;

**GNP-RPC ("GNP, Romer, per capita"):** Gross national product per capita, pre-1929 values from Romer.

The post-1929 values of GNP-BG and GNP-R are identical, as are the pre-1929 values of GNP-BGPC and GNP-RPC. Pre-1929, the series differ because of the differing assumptions underlying their construction.

As a guide to subsequent specification, we first estimate an extensive battery of conventional Dickey-Fuller regressions,

$$y_t = \hat{\mu} + \hat{\gamma} t + \hat{\delta} y_{t-1} + \sum_{j=1}^{k-1} \hat{\phi}_{t-j} \Delta y_{t-j} + \hat{\epsilon}_t$$

for each of the four real GNP variables discussed above and for  $k=1$  through  $k=6$ .<sup>1</sup> A unit root corresponds to  $\delta = 1$ , and the Dickey-Fuller statistic is  $\hat{\tau} = (\hat{\delta}-1)/SE(\hat{\delta})$ , where

$SE(\hat{\delta})$  is the standard error of the estimated coefficient,  $\hat{\delta}$ . The common sample period for

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<sup>1</sup> We gave particular care to the determination of  $k$ , the augmentation lag order, because it's well-known that the results of unit-root analyses may vary with  $k$ . A number of authors have recently addressed this important problem, exploring the properties of various lag-order selection criteria. For example, Hall (1994) establishes conditions under which the Dickey-Fuller test statistic converges to the Dickey-Fuller distribution when data-based procedures are used to select  $k$ , and he verifies that the conditions are satisfied by the popular Schwarz information criterion. Ng and Perron (1995), however, argue that  $t$  and  $F$  tests on the augmentation lag coefficients in the Dickey-Fuller regression are preferable, because they lead to less size distortion and comparable power.

all variables and for all values of  $k$  is 1875 to 1993. The selected lag order in the Dickey-Fuller regression for all four variables is  $k=2$ , regardless of whether we used the Schwarz criterion, the Akaike criterion, or conventional hypothesis-testing procedures to determine  $k$ . More precisely, all diagnostics indicate that  $k=1$  is grossly inadequate and that  $k > 2$  is unnecessary and therefore wasteful of degrees of freedom. Thus, in terms of a "reasonable range" in which to vary  $k$ , we focus throughout the paper on  $k=2$  through  $k=4$ , and our attention centers on  $k=2$ .

## II. Evidence From Rudebusch's Exact Finite-Sample Procedure

We use Rudebusch's procedure throughout. In Table 1 we display the full-sample estimates of the selected trend-stationary and difference-stationary models for each of the four GNP series. For each series, the two models fit equally well but they imply very different dynamics. This can be seen by comparing the forecasts shown in Figure 1, in which we show GNP per capita using Romer's pre-1929 values (GNP-RPC), 1869-1933, followed by the forecasts from the best-fitting trend- and difference-stationary models, 1934-1993, made in 1933. 1932 and 1933 are years of severe recession, so the forecasts are made from a position well below trend. The forecasts from the trend-stationary model revert to trend quickly, in sharp contrast to those from the difference-stationary model, which remain permanently lower.

For each series, we compute the exact finite-sample distribution of  $\hat{t}$  under the best-fitting difference-stationary model and the best-fitting trend-stationary model, and then we check where the value of  $\hat{t}$  actually obtained (call it  $\hat{t}_{\text{sample}}$ ) lies relative to those distributions. The p-values  $\text{Prob}(\hat{t} \leq \hat{t}_{\text{sample}} | f_{\text{DS}}(\hat{t}))$  and  $\text{Prob}(\hat{t} \leq \hat{t}_{\text{sample}} | f_{\text{TS}}(\hat{t}))$ , where  $f_{\text{DS}}(\hat{t})$  is the distribution of  $\hat{t}$  under the difference-stationary model and  $f_{\text{TS}}(\hat{t})$  is the distribution of  $\hat{t}$  under the trend-

stationary model, summarize the relevant information. In Table 2 we show the p-values for  $k=2$  through  $k=4$ . The results provide overwhelming support for the trend-stationary model. For each value of  $k$  and each aggregate output measure, the p-value associated with  $\hat{t}$  under the difference-stationary model is very small, while the p-value associated with  $\hat{t}$  under the trend-stationary model is large. In the leading case of  $k=2$ , to which all diagnostics point, the p-value under the difference-stationary model is consistently less than .01, while that under the trend-stationary model is consistently greater than .62. To illustrate the starkness of the results, we graph in Figure 2 the exact distributions of  $\hat{t}$  for the best-fitting difference-stationary and trend-stationary models for GNP-RPC with  $k=2$ . It is visually obvious that  $\hat{t}_{\text{sample}}$  is tremendously unlikely relative to  $f_{\text{DS}}(\hat{t})$  but very likely with respect to  $f_{\text{TS}}(\hat{t})$ .

We check the robustness of our results by varying the starting and ending dates over a wide range. We find no qualitative change in any result. In Figure 3, for example, we show the exact finite-sample p-values of  $\hat{t}$  under the best-fitting difference-stationary model for GNP-RPC, computed using the Rudebusch procedure over samples ranging from  $t_1$  through  $t_T$ , with  $t_1 = 1875, \dots, 1895$  and  $t_T = 1973, \dots, 1993$ . The p-value always lies below .05 and typically falls well below .01.

It is of particular interest to examine carefully the robustness to starting date, because the quality of the data deteriorates as one moves backward in time. Thus, in Figure 4 we show the exact finite-sample p-values of  $\hat{t}$  under the best-fitting difference-stationary model for each of the four aggregate output measures, computed using the Rudebusch procedure over samples ranging from  $t_1$  through 1993, with  $t_1 = 1875, \dots, 1945$ . Again, the p-values are typically very small.

Finally, we wish to reconcile our results with those of Nelson and Plosser. In Figure 5, we show U.S. real GNP per capita using Romer's pre-1929 values (GNP-RPC), together with a fitted linear trend. Nelson and Plosser use only the shaded subsample, 1909-1970. Two issues are relevant. First, the Nelson-Plosser sample is obviously much shorter than ours, and on that ground alone Nelson and Plosser had less power to detect deviations from difference stationarity. Second, Figure 5 makes clear that the only prolonged, persistent deviation of output from trend is the Great Depression and the ensuing World War II, which sit squarely in the center of the Nelson-Plosser sample. If we restrict our analysis to the Nelson-Plosser years, we obtain  $\hat{t} = -3.26$  and we would not reject the difference-stationary model at conventional levels. If we trim fifteen years from each end of the Nelson-Plosser sample, using only 1924-1955, we obtain  $\hat{t} = -2.71$ , corresponding to even less evidence against the difference-stationary model. Conversely, as we expand the sample to include years both earlier and later than those used by Nelson and Plosser, the evidence against difference-stationarity grows quickly, because the earlier and later years are highly informative about the question of interest, as output clings tightly to trend. By the time we use the full sample, 1875-1993, we obtain  $\hat{t} = -4.57$  and we reject difference-stationarity at any reasonable level.

### III. Concluding Remarks

There is no doubt that unit root tests *do* suffer from low power in many situations of interest. Rudebusch's analysis of postwar U.S. quarterly GNP illustrates that point starkly. We have shown, however, that Rudebusch's procedure produces evidence that distinctly favors trend-stationarity using long spans of annual data. Moreover, allowing for breaking trend in the spirit of Perron (1989) would only strengthen our results. Thus, the U.S. aggregate output



data are not as uninformative as many believe. Interestingly, the same conclusion has been reached by very different methods in the Bayesian literature (e.g., DeJong and Whiteman, 1992) and in out-of-sample forecasting competitions (e.g., Geweke and Meese, 1984; DeJong and Whiteman, 1993).

We have already stressed the importance of our results for forecasting aggregate output. They are also important for macroeconomic modeling more generally. For example, recent important work by Elliott (1995) points to the non-robustness of cointegration methods to deviations of variables from difference-stationarity. More precisely, Elliott shows that even very small deviations from difference-stationarity can invalidate the inferential procedures associated with conventional cointegration analyses. Our results suggest that U.S. aggregate output is not likely to be difference-stationary -- the dominant autoregressive root is likely close to, but less than, unity. This points to the desirability of additional work on inference in macroeconomic models with dynamics that are long-memory but mean-reverting, as in Diebold and Rudebusch (1989), or short-memory with roots local to unity, as in Elliott, Rothenberg and Stock (1992).

Finally, let us summarize our contribution as we see it. We do not take issue with Rudebusch -- we in fact study very different aggregate output series, and certainly we are not the first to notice that the use of longer GNP samples may produce sharper unit root inference. (See, for example, Stock and Watson, 1986.) Rather, we are skeptical of blanket generalizations of the Christiano-Eichenbaum-Rudebusch results, prevalent in the recent oral tradition. We have argued that, even for the famously recalcitrant aggregate output series, unit root tests over long spans *can* be informative, and that they may be important for point

forecasting, among other things. Our results make clear that uncritical repetition of the "we don't know, and we don't care" mantra is just as scientifically irresponsible as blind adoption of the view that "all macroeconomic series are difference-stationary," or the view that "all macroeconomic series are trend-stationary." There is simply no substitute for serious, case-by-case, analysis.

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**Table 1**  
**Estimated Best-Fitting Trend- and Difference-Stationary Models**

Regressor:	c	t	y(-1)	y(-2)	$\Delta y(-1)$	SER
<b><u>Trend-Stationary</u> (dependent variable y)</b>						
GNP-R	.88 (.18)	.58 (.12)	1.33 (.08)	-.51 (.08)	--	.043
GNP-RPC	-1.07 (.24)	.31 (.07)	1.33 (.08)	-.51 (.08)	--	.043
GNP-BG	.90 (.20)	.59 (.14)	1.21 (.09)	-.39 (.09)	--	.050
GNP-BGPC	-1.14 (.27)	.33 (.08)	1.20 (.09)	-.39 (.09)	--	.051
<b><u>Difference-Stationary</u> (dependent variable <math>\Delta y</math>)</b>						
$\Delta$ GNP-R	.02 (.01)	--	--	--	.43 (.08)	.046
$\Delta$ GNP-RPC	.01 (.004)	--	--	--	.42 (.08)	.046
$\Delta$ GNP-BG	.02 (.01)	--	--	--	.30 (.09)	.054
$\Delta$ GNP-BGPC	.01 (.01)	--	--	--	.30 (.09)	.054

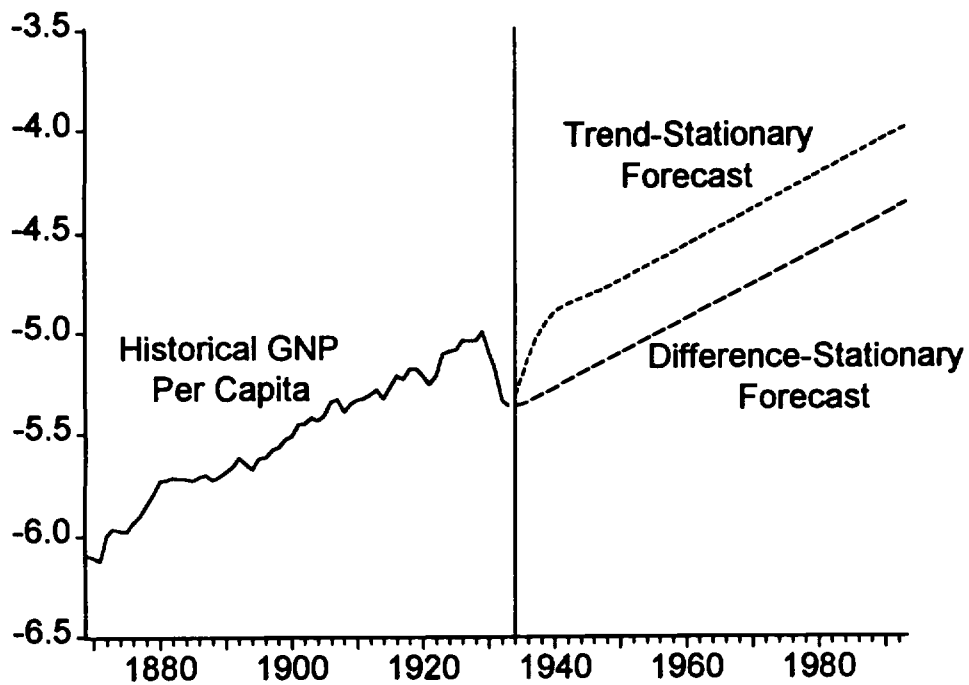
Notes to table: c is a constant term, t is a linear trend, and y is the log of Romer's gross national product (GNP-R), the log of Romer's gross national product per capita (GNP-RPC), the log of Balke and Gordon's gross national product (GNP-BG), or the log of Balke and Gordon's gross national product per capita (GNP-BGPC). All samples are annual, 1875-1993. Standard errors appear in parentheses. For the trend-stationary models, the trend coefficients and their standard errors have been multiplied by 100. The last column reports the standard error of the regression (SER).

**Table 2**  
**Exact p-Values of the Dickey-Fuller Statistic**

	k = 2	3	4
<b>Variable:</b>			
<b>GNP-R</b>			
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{TS}}(\hat{t})$ )	.625	.690	.592
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{DS}}(\hat{t})$ )	.001	.005	.012
<b>GNP-RPC</b>			
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{TS}}(\hat{t})$ )	.668	.673	.695
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{DS}}(\hat{t})$ )	.001	.007	.017
<b>GNP-BG</b>			
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{TS}}(\hat{t})$ )	.651	.616	.657
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{DS}}(\hat{t})$ )	.005	.002	.020
<b>GNP-BGPC</b>			
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{TS}}(\hat{t})$ )	.692	.656	.711
Prob( $\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{DS}}(\hat{t})$ )	.006	.003	.021

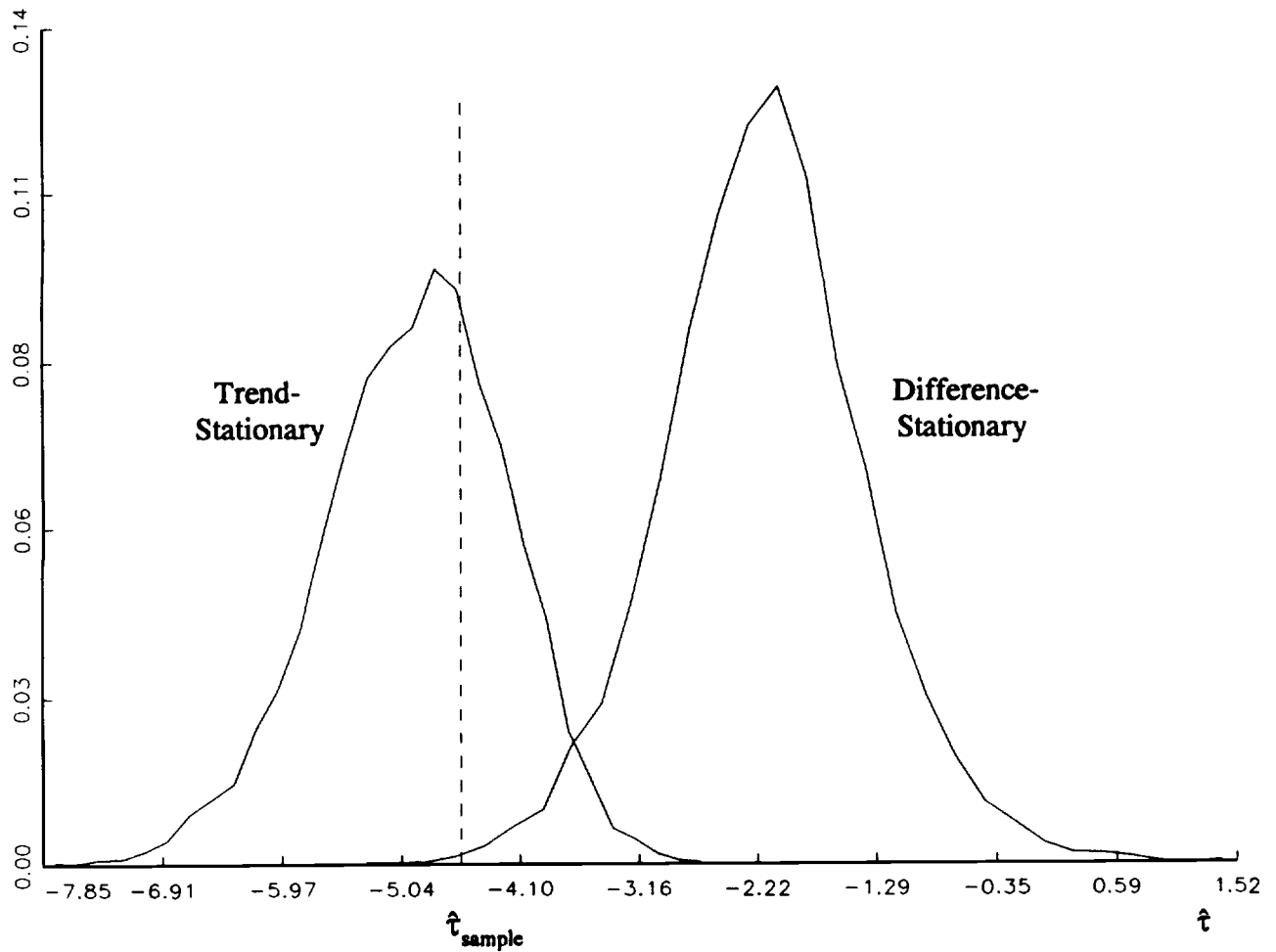
Notes to table:  $\hat{t}$  is the Dickey-Fuller statistic,  $f_{\text{TS}}(\hat{t})$  is the empirical distribution of  $\hat{t}$  conditional on the trend-stationary model,  $f_{\text{DS}}(\hat{t})$  is the empirical distribution of  $\hat{t}$  conditional on the difference-stationary model,  $k$  is the augmentation lag order in the Dickey-Fuller regression, and  $\text{Prob}(\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{TS}}(\hat{t}))$  and  $\text{Prob}(\hat{t} \leq \hat{t}_{\text{sample}} \mid f_{\text{DS}}(\hat{t}))$  are the probabilities of obtaining  $\hat{t} \leq \hat{t}_{\text{sample}}$  under the trend-stationary and the difference-stationary models. The variables are the log of Romer's gross national product (GNP-R), the log of Romer's gross national product per capita (GNP-RPC), the log of Balke and Gordon's gross national product (GNP-BG), or the log of Balke and Gordon's gross national product per capita (GNP-BGPC). All samples are annual, 1875-1993.

**Figure 1**  
**GNP Per Capita, Historical and Two Forecasts**



Notes to figure: We plot U.S. real GNP per capita, together with the optimal 1934-1993 forecasts (made in 1933) from the best-fitting trend-stationary model (short dash) and from the best-fitting difference-stationary model (long dash). Pre-1929 GNP data are from Romer (1989).

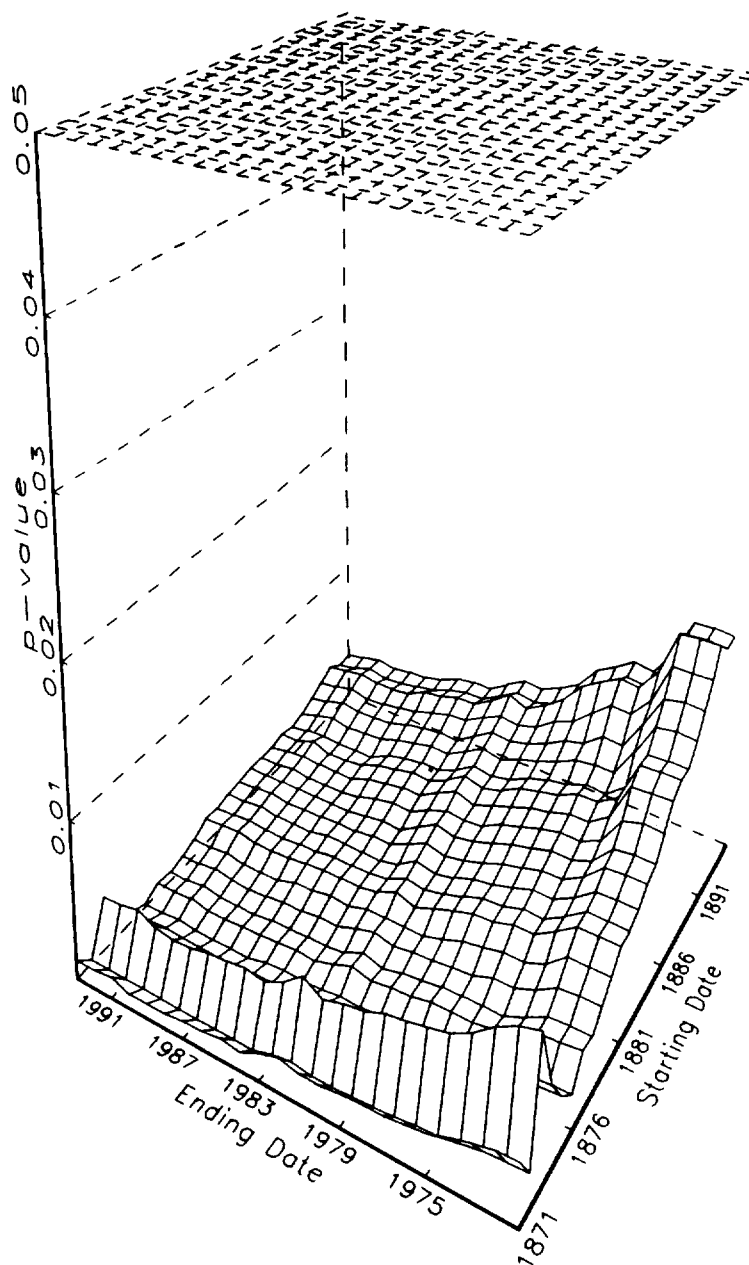
**Figure 2**  
**Exact Distributions of the Dickey-Fuller Statistic**



Notes to figure: We show exact finite-sample distributions of the Dickey-Fuller statistic under the best-fitting trend-stationary and difference-stationary models for U.S. real GNP per capita. The sample period is 1875-1993. Pre-1929 GNP data are from Romer (1989).  $\hat{\tau}_{\text{sample}}$  is the value of the Dickey-Fuller statistic actually obtained.

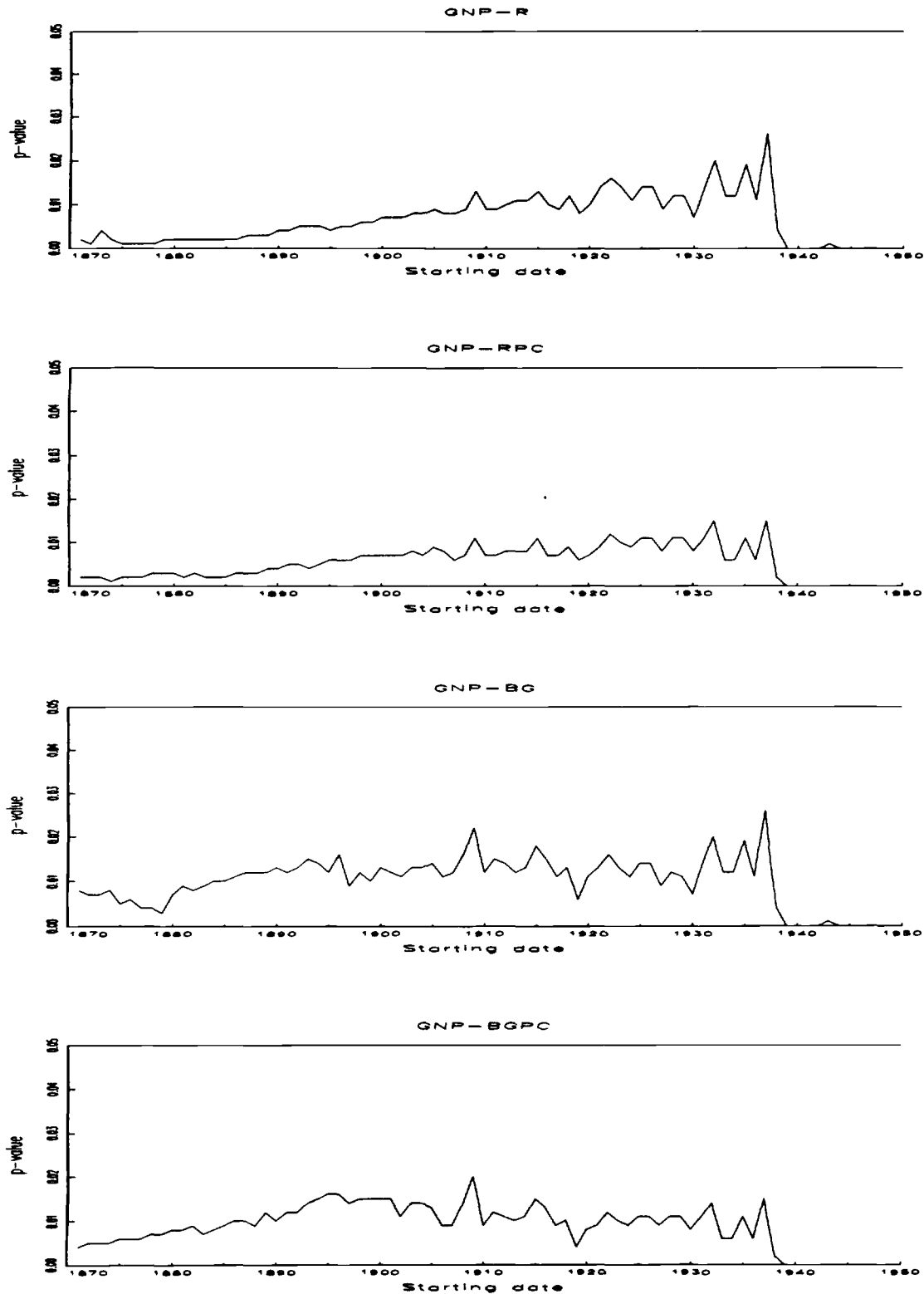


**Figure 3**  
**Exact p-values of the Dickey-Fuller Statistic**



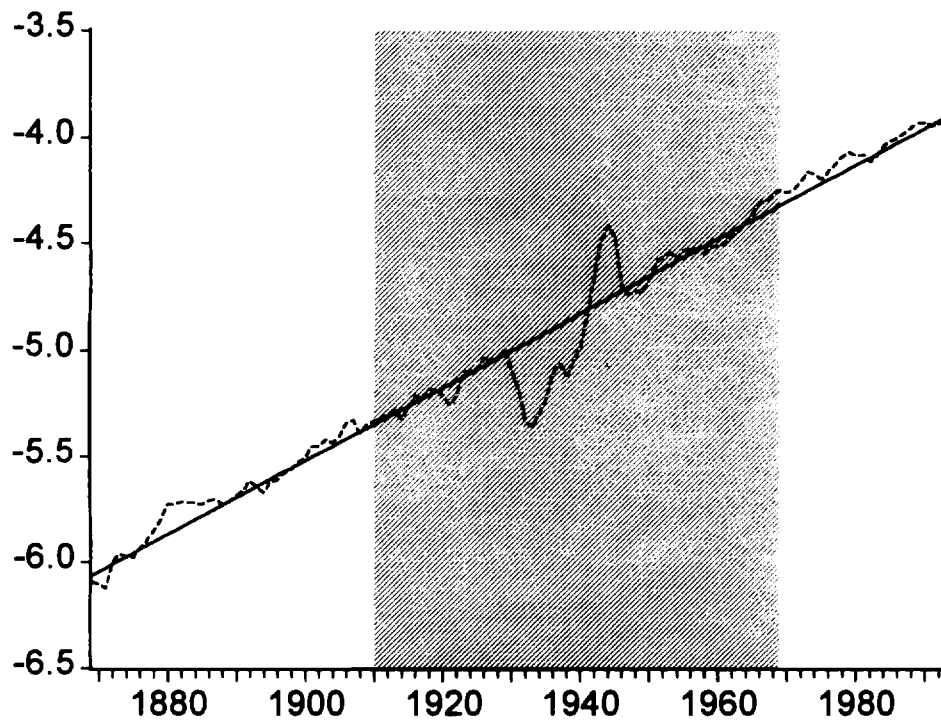
Notes to figure: We show exact finite-sample p-values of the Dickey-Fuller statistic under the best-fitting difference-stationary model for U.S. real GNP per capita. Samples range from  $t_1$  through  $t_T$ , with  $t_1 = 1875, \dots, 1895$  and  $t_T = 1973, \dots, 1993$ . Pre-1929 GNP data are from Romer (1989). For visual reference, we also show the plane corresponding to  $p = .05$ .

**Figure 4**  
**Exact p-values of the Dickey-Fuller Statistic**



Notes to figure: We show exact finite-sample p-values of the Dickey-Fuller statistic under the best-fitting difference-stationary model for the log of Romer's gross national product (GNP-R), the log of Romer's gross national product per capita (GNP-RPC), the log of Balke and Gordon's gross national product (GNP-BG), or the log of Balke and Gordon's gross national product per capita (GNP-BGPC). Samples range from  $t_1$  through 1993, with  $t_1 = 1875, \dots, 1945$ . For visual reference, each sub-plot is scaled to have a height of .05.

Figure 5  
GNP Per Capita, Actual and Trend



Notes to figure: We show the log of U.S. real GNP per capita, together with a fitted linear trend. The Nelson-Plosser subsample, 1909-1970, is shaded. Pre-1929 GNP data are from Romer (1989).