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DYNAMIC EQUILIBRIUM AND VOLATILITY IN FINANCIAL ASSET MARKETS

Yacine Aït-Sahalia

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ABSTRACT

This paper develops and estimates a continuous-time model of a financial market where investors' trading strategies and the specialist's rule of price adjustments are the best response to each other. We examine how far modeling market microstructure in a purely rational framework can go in explaining alleged asset pricing "anomalies." The model produces some major findings of the empirical literature: excess volatility of the market price compared to the asset's fundamental value, serially correlated volatility, contemporaneous volume-volatility correlation, and excess kurtosis of price changes. We implement a nonlinear filter to estimate the unobservable fundamental value, and avoid the discretization bias by computing the exact conditional moments of the price and volume processes over time intervals of any length.

Yacine Aït-Sahalia
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, IL 60637-1561
and NBER

1. Introduction

Microeconomic theory is essentially concerned with the study of market equilibrium. Agents make plans, in general as a result of solving individual optimization problems. Then certain variables, typically prices, are assumed to take the values required for those plans to be mutually consistent. The resulting prices are called equilibrium prices. All in all, very little is said about the mechanism(s) that make possible, compute, and finally implement such an equilibrium in the first place. Even less is said about how the economy would return to equilibrium if the equilibrium were shifted for any reason. Equilibrium changes are treated in a comparative statics framework, where the values of some parameters and consequently the equilibrium are shifted. The price moves to the new equilibrium, and trade can only occur in equilibrium.

Casual observation of financial markets, however, reveals that this paradigm may not translate very well in markets where (i) prices move constantly, (ii) trading occurs at very high frequency and at every price and (iii) the volatility of the prices is a crucial element (see Black (1986)). We attempt to model the price formation process to reflect these empirical features. Investors trade for portfolio reasons a speculative asset. The specialist posts a price at which he is required to take the opposite side of the order submitted. After executing the order, the specialist can adjust the price. Observing the price being posted, the investor then acts to rebalance optimally his portfolio, trading occurs, the specialist revises optimally the market price to maximize his expected trading profits and so on. Instead of prices, strategies constitute an equilibrium. An equilibrium is a specialist's pricing rule and a sequence of investors' trading strategies that are mutually best response to each other. The specialist acts as a Stackelberg leader: he receives the investors' demand function before setting his price.

We show that in equilibrium, the specialist (who controls the price adjustment process) finds it optimal to add volatility to the price he posts compared to the exogenous "fundamental value (or price)" of the asset, which is defined as the expected value of the sum of discounted future dividends. The spread between the market and fundamental value of the asset drives the expected return of the asset. By adding volatility, the specialist reduces the holdings of risk-averse investors when high returns are expected, which benefits him since he is taking the opposite side of the investors' trades. The model involves nonlinear strategies but is nevertheless solved in closed form. The price and volume processes determined in equilibrium provide some microeconomic foundations for the statistical specifications of price and volume processes in the literature (see Epps and Epps (1976), Tauchen and Pitts (1983) and Karpoff (1987)).

We then revisit some of the classical empirical asset pricing "anomalies." The model can generate in a fully optimizing world many effects, such as excess volatility or mean reversion in asset prices, which were sometimes interpreted as sure signs of market irrationality (e.g., Shiller (1981), Summers (1986)). There is no exogenous source of noise such as noise traders in Kyle (1985), or pure exogenous noise as in Campbell and Kyle (1993). A nice feature of this model is that these relevant issues in asset pricing can be interpreted as simple hypothesis tests on one or more of the three parameters of the model: α which measures how market prices mean-revert to the stochastic fundamental price, β which measures how the volatility of the market price relates to the spread between market and fundamental prices and finally σ which measures the volatility of the fundamental price itself.

The estimation of the model presents several challenges. A state variable, the fundamental price, is unobservable to the econometrician, and must therefore be estimated along with the parameters. The dynamics of the state variables produced by the model are nonlinear. We are interested in testing hypotheses which involve time series features of the fundamental price: for example, is the fundamental price less volatile than the observed market price? We propose to use the tools of conditionally optimal filtering to achieve this task. Next, we derive closed-form expressions for the conditional moments of the joint processes determining the market and fundamental prices, and the trading volume, conditioned on their history. These conditional moments however cannot be used to form moment conditions in a GMM framework, because the conditioning set contains variables unobservable to the econometrician (the history of the fundamental price process). We therefore compute, again in closed-form, the moments of the joint processes conditioned only on the observable market price and volume processes. These moments are the basis used to construct a GMM estimator of the parameters of the model. Despite the fact that the model is written in continuous-time and the data are sampled at discrete time intervals, the estimator is free of discretization bias.

The paper is organized as follows. Section 2 presents the model and solves for the equilibrium price and volume processes. Section 3 focuses on the estimation strategy. Section 4 examines the empirical implications of the model. Section 5 concludes.

2. Equilibrium Dynamics

2.1 Price Formation

Consider a financial exchange where a stock is traded by a specialist[2] and a price-taker risk-averse investor. The single investor assumption is a proxy for a continuum of small identical investors, each one of them price-taker. The investor can buy and sell the stock as well as lend or borrow at the constant riskfree rate r. There are no constraints on borrowing or short sales, and the stock carries unlimited liability. Throughout the paper Z_i, i=1,2, denote standard Brownian Motions.

Between t and t+dt each share of the stock pays an exogenously-determined stochastic dividend $D_t dt$. We assume that D_t is a martingale following $dD_t = r\sigma dZ_{2t}$ with σ constant. We define the stock's fundamental value or price $\overline{p}_t$ to be the expected value of the sum of discounted future dividends[3]. We have that:

$$\overline{p}_t = E_t\left[\int_t^{+\infty} e^{-r(\tau-t)} D_\tau d\tau\right] = \int_t^{+\infty} e^{-r(\tau-t)} E_t\left[D_\tau\right] d\tau = (1/r)\, D_t \tag{1}$$

since $E_t\left[D_\tau\right] = D_t$. The fundamental price therefore follows the dynamics:

$$d\overline{p}_t = \sigma dZ_{2t} \tag{2}$$

A change in $\overline{p}_t$ reflects the arrival of new information regarding the future cash flows generated by the stock. We attempt in this paper to model microstructure effects and do not assume that the stock necessarily trades for $\overline{p}_t$. Instead let p_t be the market price of the stock at time t. Define the price spread:

$$s_t \equiv \overline{p}_t - p_t \tag{3}$$

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2 In this model, the single specialist enjoys monopoly power on a given stock, a situation typical of organized exchanges. By contrast in dealer markets multiple markets makers are supposed to compete on a given stock. However there exists some empirical evidence to justify modeling the market power of market makers even in a dealer market (for example, collusion and other non-competitive behavior among market makers on NASDAQ is suggested by Christie and Schultz (1994), and Christie, Harris and Schultz (1994)).

3 We may interpret $\overline{p}_t$ as the price that would prevail for the asset in a pure competitive economy with risk-neutral agents (Lucas (1978)). We are not assuming this setup here.

The form of the dynamics (2) is common knowledge, and $\bar{p}_t$ is revealed at every instant to all market participants[4]. At instant t, the investor desires to hold q_t shares, and having observed $\bar{p}_t$, submits his demand function to the specialist. The investor is allowed to condition his trades on price. The specialist then executes the buy or sell order received and takes the opposite side of the trade[5]. He therefore is forced to hold $-q_t$ shares (the stock is assumed to be in zero net supply). He may then revise the price by an amount dp_t. The stock then pays its instantaneous rate of return. We denote by F_t the information set consisting of the sequence of past and present fundamental and market prices[6]. Figure 1 summarizes the price formation in this market.

2.2 Equilibrium Concept

We model this market's microstructure as a stochastic differential game, with the specialist acting as a Stackelberg leader. The investor takes the price adjustment rule as given and determines his optimal holding of the stock, i.e., his best response, by maximizing his expected utility. Knowing the demand function of the investor, i.e., the investor's best response to his choice of price adjustment, the specialist determines the price adjustment by maximizing his expected profit[7]. This is an equilibrium in strategies (not prices or quantities): each player's choice of an optimal strategy is a control problem in which he takes into account the effect of his actions on the state, both directly and indirectly through the influence of the state on the strategies of his opponent. We make all of this more specific below.

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[4] There is no asymmetric information in the model --except that the econometrician does not observe the fundamental value.

[5] NYSE Rule 104 specifies that: "The specialist must take or supply stock as necessary (...)." (NYSE (1995)). In practice, many trades get executed with no formal intervention by the specialist, a fact which is ignored here.

[6] Formally, F_t is the augmentation of the increasing family of σ-fields generated by the stochastic processes $\left\{p_s, \bar{p}_s \,/\, 0 \leq s \leq t\right\}$.

[7] In a Nash Equilibrium, the specialist would not take into account the investor's best response to his action when determining his own optimal strategy.

2.3 Optimization by the Specialist

In deciding how to revise the price, the specialist faces two constraints --like most monopolies, he is regulated. First, he must provide price continuity. The price continuity rate measures the percentage of all trades occurring with no change in price or a one-tick change (1/8 on the NYSE). Second, the exchange expects the specialist to stabilize price movements. The specialist performance is partly assessed through the stabilization rate, the percentage of shares purchased by the specialist at prices below or sold at prices above the last different price[8]. Table 1 reports these two rates for the NYSE.

We incorporate these two requirements in the model by constraining the possible price revision dp_t that the specialist can choose. If the stock's fundamental value were constant, we could possibly model price continuity as a constraint that sets the drift of dp_t to zero. However because $\overline{p}_t$ is stochastic in the model, we interpret the price continuity rule as a requirement that p_t be held as constant as possible --but only after adjusting to the new level of $\overline{p}_t$. Suppose that news affecting the stock are released, that is a large realization of dZ_{2t} occurs, so that the fundamental value changes substantially between t and t+dt. The specialist is still expected to provide price continuity, but it seems natural for him to be allowed (and even, encouraged) to respond to changes in $\overline{p}_t$. The NYSE Specialist's Job Description stipulates that the specialist should "initiate trading in each security as soon as market conditions allow, at a price that reflects a thorough, professional assessment of market conditions at the time." This could be viewed as saying that the specialist, faced with a shock to $\overline{p}_t$, is expected to do whatever possible to move in an orderly fashion the market price in relation to the new value of $\overline{p}_t$. The job description does encourage specialists to learn about the companies (i.e., their $\overline{p}_t$): "in order to establish a positive professional relationship with Exchange-listed companies, [the specialist should] contact during each quarter one or more senior officials [of the company] of the rank of Corporate Secretary or above."

To capture this effect, a tractable assumption is to set the drift of dp_t to have the simple linear form: $\alpha(\overline{p}_t - p_t)dt = \alpha s_t dt$. In order to satisfy price continuity we constrain the specialist to not further adjust the price deterministically. The specialist however controls the price adjustment through its volatility v_t:

[8] The NYSE fixes a minimal monthly stabilization rate for its specialists.

$$dp_t = \alpha(\bar{p}_t - p_t)dt + v_t dZ_{1t} \tag{4}$$

Controlling v_t is how the specialist exploits his monopoly power. The specialist cannot predict the dividend shock between t and t+dt, and hence we assume that $E[dZ_{1t}dZ_{2t}] = 0$.

Given that price adjustment, the instantaneous excess return provided by the stock is given by:

$$dH_t = D_t dt + dp_t - rp_t dt = \{r(\bar{p}_t - p_t)\}dt + dp_t = \beta s_t\, dt + v_t dZ_{1t} \tag{5}$$

where we have defined $\beta \equiv r + \alpha$. The drift of the excess return depends on the price spread s_t, which from (4) has the dynamics:

$$ds_t = -\alpha s_t\, dt + \sigma dZ_{2t} - v_t dZ_{1t} \tag{6}$$

Future excess returns are therefore (partly) predictable, and investors will choose their trading strategies by taking into account (5) and (6).

We further interpret price continuity and stabilization as limiting the specialist's ability to set an arbitrarily large volatility[9]. However instead of setting an artificial upper limit to the admissible volatility choice, we assume that the specialist receives a monetary transfer T from the exchange depending upon the level of volatility that he decides to set. Large volatility choices get penalized (T<0), low volatility ones rewarded (T>0). Other things equal, the transfer is higher when the specialist must react to a large change in $\bar{p}_t$, i.e., when s_t is large in absolute value. This reflects the added difficulty for the specialist of doing business following a large change in what the NYSE Rules call "market conditions." Let $T(v_t, s_t)dt$ be the amount of the transfer from the exchange to the specialist between t and t+dt. We therefore want the transfer function T to satisfy:

$$\partial T / \partial v_t < 0 \tag{7}$$

and:

$$\lim_{v_t \to +\infty} T < 0, \ \lim_{v_t \to 0} T > 0 \tag{8}$$

[9] For example, according to the NYSE Rule 104, "the maintenance of a fair and orderly market implies the maintenance of price continuity with reasonable depth, and the minimization of the effects of temporary disparity between supply and demand" (NYSE (1995), 2104.10).

and be such that in equilibrium the specialist earns zero expected total profit. We give in the Appendix a particular case of a transfer function which has the advantage of allowing for a closed-form solution to the equilibrium and satisfies (7), (8) and the zero profit condition.

The specialist chooses his optimal volatility level, hence his price adjustment rule (3), by maximizing his total expected profit. Total profit consists of both the transfer from the exchange and the specialist's trading revenue:

$$\max_{\{v_\tau, \tau \geq t\}} E_t\left[\int_t^{+\infty} e^{-r\tau}\left\{T(v_\tau, s_\tau)d\tau + d\Pi_\tau\right\}\right] \tag{9}$$

where $d\Pi_t$ is the trading revenue derived from his stock holdings between t and t+dt. As indicated, the specialist recognizes that the number of shares q_t demanded by the investor will depend on his choice of volatility. The specialist is not allowed to trade independently for his own account, but instead must clear the market according to the investor's desires and therefore holds $-q_t$ shares of the stock. Given the investor's demand function $q_t = q(v_t, s_t)$, the specialist's trading revenue $d\Pi_t$ is:

$$d\Pi_t = -q(v_t, s_t)dH_t \tag{10}$$

From (5) and $E_t\left[dZ_{i\tau}\right] = 0$, it follows that:

$$E_t\left[\int_t^{+\infty} e^{-r\tau} d\Pi_\tau\right] = E_t\left[\int_t^{+\infty} e^{-r\tau}\left\{-q(v_\tau, s_\tau)\beta s_\tau\right\}d\tau\right] \tag{11}$$

To summarize: the specialist chooses $\{v_t\}$ to maximize (9), subject to the dynamics of the state variable s_t given by (6). We now describe how the investor determines his demand function $q_t = q(v_t, s_t)$.

2.4 Optimization by Investors

The investor's portfolio holdings q_t and consumption c_t at date t are determined by maximizing his expected utility. In doing so, the investor takes as given the rule of price adjustment (4) determined by the specialist. Assuming exponential utility[10] $u(c_t) \equiv -\exp(-a c_t)$ with Arrow-Pratt coefficient of absolute risk aversion a and discount factor ρ, the investor's objective is:

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[10] Exponential utility makes demand exempt from wealth effects.

$$\max_{\{c_\tau, q_\tau, \tau \geq t\}} E_t\left[\int_t^{+\infty} -\exp(-\rho\tau - a c_\tau)d\tau\right] \tag{12}$$

subject to (6) and the dynamics of his wealth:

$$\begin{aligned} dW_t &= rW_t dt - c_t dt + q_t dH_t \\ &= \{rW_t + \beta s_t q_t - c_t\}dt + q_t v_t dZ_{1t} \end{aligned} \tag{13}$$

When formulating his demand for the stock, the investor will exploit the predictability of the (risky) excess returns (5) to the maximum extent permitted by his own risk-aversion. At time t+dt each investor will trade the quantity dq_t required to insure that his investment q_{t+dt} in the stock is optimal at that point in time, given his holdings q_t an instant earlier, that is: $dq_t = q_{t+dt} - q_t$.

2.5 Computation of the Equilibrium

We first solve the investor's optimization problem, obtaining his best response function to every possible choice of volatility by the specialist. Then we find the optimal volatility choice by the specialist (acting as a Stackelberg leader) given the investor's best response function. The result is summarized as:

Proposition 1:

(i) The investor's optimal stock holdings in response to the specialist's choice of volatility v_t is:

$$q(s_t, v_t) = \frac{s_t}{a}\left[\frac{\beta}{rv_t^2} + \frac{\sqrt{1 + 4\beta^2\sigma^2/r^2v_t^2} - 1}{2\sigma^2}\right] \tag{14}$$

(ii) The specialist, who as a Stackelberg leader takes into account the investor's best response, chooses volatility optimally as:

$$v_t = v(s_t) = \sqrt{\sigma^2 + \beta s_t^2} \tag{15}$$

(iii) Therefore in equilibrium:

$$q_t = q(s_t, v(s_t)) = \frac{s_t}{a}\left[\frac{\beta}{r(\sigma^2 + \beta s_t^2)} + \frac{\sqrt{1 + 4\beta^2\sigma^2/r^2(\sigma^2 + \beta s_t^2)} - 1}{2\sigma^2}\right] \tag{16}$$

Proof: see Appendix.

The basic intuition for the result is the following: when $s_t>0$ (resp. $s_t<0$), positive (resp. negative) excess returns are expected, and the investor desires to hold (resp. short) more shares of the stock: see (14). This is detrimental to the specialist, who must take the opposite side of the trade: see (10). By inputting volatility into the price process when $s_t \neq 0$ (see Figure 2), the specialist manages to reduce the holdings of investors when excess returns are expected. Since investors are risk-averse, they indeed respond to the extra volatility by demanding fewer shares of the stock than they would under constant volatility (see Figure 3). The specialist exploits optimally the rent derived from his monopoly power. In equilibrium he makes zero expected profit. The Exchange exactly compensates him for having to face the trades of an investor who can optimally exploit the predictability of future returns.

2.6 Empirical Implications

We now show that the equilibrium produces the stylized facts cited in the literature as the main characteristics of the joint distribution of price and volume data (see Karpoff (1987) and Gallant, Rossi, Tauchen (1992)): serial correlation in the conditional volatility of price changes (the ARCH effect), contemporaneous correlation between trading volume and absolute changes in prices, and excess kurtosis of price changes[11].

In order to examine these effects, we need to derive some properties of the moments of s_t:

Proposition 2:

(i) The price spread s_t is strictly stationary and the first four conditional moments of $s_{t+\Delta}$ given s_t are given in closed-form for any time interval $\Delta>0$ by:

[11] Skewness of price changes, while less apparent in the distribution estimated from the time series of stock prices, is however typically present in the distribution of stock price changes implicit in the prices of traded stock options (the "smile" effect).

$$\begin{cases} E[s_{t+\Delta} \mid s_t] = s_t e^{-\alpha\Delta} \\ E[s_{t+\Delta}^2 \mid s_t] = \dfrac{2\sigma^2}{2\alpha-\beta} + \left(s_t^2 - \dfrac{2\sigma^2}{2\alpha-\beta}\right)e^{-(2\alpha-\beta)\Delta} \\ E[s_{t+\Delta}^3 \mid s_t] = \left(\dfrac{6\sigma^2 s_t}{2\alpha-3\beta}\right)e^{-\alpha\Delta} + \left(s_t^3 - \dfrac{6\sigma^2 s_t}{2\alpha-3\beta}\right)e^{-(3\alpha-3\beta)\Delta} \\ E[s_{t+\Delta}^4 \mid s_t] = \left(\dfrac{12\sigma^4}{(2\alpha-3\beta)(2\alpha-\beta)}\right) + 12\sigma^2\left(\dfrac{s_t^2}{(2\alpha-5\beta)} - \dfrac{2\sigma^2}{(2\alpha-\beta)}\right)e^{-(2\alpha-\beta)\Delta} \\ \qquad + \left(s_t^4 - \dfrac{12\sigma^2(\sigma^2-(2\alpha-3\beta)s_t^2)}{(2\alpha-5\beta)(2\alpha-3\beta)}\right)e^{-2(2\alpha-3\beta)\Delta} \end{cases} \quad (17)$$

(ii) The stationary unconditional distribution $\pi(s)$ of s_t is given by:

$$\pi(s) = 1 \Big/ \left[\left(2\sigma^2 + \beta s^2\right)^{1+\alpha/\beta} \int_{-\infty}^{+\infty} \left\{ 1 \big/ \left(2\sigma^2 + \beta u^2\right)^{1+\alpha/\beta} \right\} du \right] \quad (18)$$

It admits finite moments up to order n if and only if $2\alpha > (n-1)\beta$, where α and β are treated as independent parameters. Assuming $2\alpha > 3\beta$, the first four unconditional moments of s_t are:

$$E[s_t] = 0,\ E[s_t^2] = 2\sigma^2/(2\alpha-\beta),\ E[s_t^3] = 0,\ E[s_t^4] = 12\sigma^4/((2\alpha-\beta)(2\alpha-3\beta)) \quad (19)$$

Proof: see Appendix.

Serial correlation in price volatility follows from the fact that s_t is serially correlated (from (6)), and thus so is v_t (from (15)). Specifically, by Itô's Lemma, the conditional variance of price changes follows:

$$dv_t^2 = \left((2\alpha+\beta)\sigma^2 - (2\alpha-\beta)v_t^2\right)dt + 2\sqrt{\beta}\sqrt{v_t^2-\sigma^2}\left(\sigma dZ_{2t} - v_t dZ_{1t}\right) \quad (20)$$

and is therefore mean-reverting around its mean value $E[v_t^2] = \sigma^2(2\alpha+\beta)/(2\alpha-\beta)$.

As for the second effect, define the trading volume as the change $|dq_t|$ in the investor's holdings. Because the Brownian increments dZ_i are Gaussian random variables it follows that[12]:

[12] This is a standard result for the correlation of the absolute value of two jointly Gaussian random variables; see e.g., the appendix in Wang (1994) for a derivation.

$$\mathrm{corr}\left(|dp_t|, |dq_t|\right) = \left(1 - \frac{2}{\pi}\right)\left(1 - \sqrt{1 - \mathrm{corr}(dp_t, dq_t)^2}\right) \quad \geq 0 \tag{21}$$

Therefore the magnitude of the price change is positively correlated with the trading volume. According to the Wall Street adage, it "takes volume to move prices" which is what (21) demonstrates for this model.

Finally, to show that the model generates a leptokurtic distribution for price changes, we compute the conditional moments of price changes (see Figure 4). We derive below as part of the estimation procedure the exact expressions of the conditional moments of $(p_{t+\Delta}, \bar{p}_{t+\Delta})$ given $(p_t, \bar{p}_t)$. While the moments are available in closed-form, the excess kurtosis can most easily be seen from the first terms in the Taylor series expansions:

$$\begin{cases} E\left[(p_{t+\Delta} - p_t) \mid \bar{p}_t, p_t\right] = \alpha s_t \Delta + o(\Delta) \\ E\left[(p_{t+\Delta} - p_t)^2 \mid \bar{p}_t, p_t\right] = (\sigma^2 + \beta s_t^2)\Delta + o(\Delta) \\ E\left[(p_{t+\Delta} - p_t)^3 \mid \bar{p}_t, p_t\right] = 3\alpha s_t(\sigma^2 + \beta s_t^2)\Delta^2 + o(\Delta^2) \\ E\left[(p_{t+\Delta} - p_t)^4 \mid \bar{p}_t, p_t\right] = 3(\sigma^2 + \beta s_t^2)^2 \Delta^2 + o(\Delta^2) \end{cases} \tag{22}$$

We then derive the unconditional moments of price changes by applying the law of iterated expectations and using (19):

$$\begin{cases} E\left[(p_{t+\Delta} - p_t)\right] = o(\Delta) \\ E\left[(p_{t+\Delta} - p_t)^2\right] = \sigma^2\left(\dfrac{2\alpha + \beta}{2\alpha - \beta}\right)\Delta + o(\Delta) \\ E\left[(p_{t+\Delta} - p_t)^3\right] = o(\Delta^2) \\ E\left[(p_{t+\Delta} - p_t)^4\right] = 3\sigma^4\left(\dfrac{4\alpha^2 + 3\beta^2}{(2\alpha - 3\beta)(2\alpha - \beta)}\right)\Delta^2 + o(\Delta^2) \end{cases} \tag{23}$$

The price changes exhibit excess kurtosis. Indeed, we can see from (23) that $E\left[(p_{t+\Delta} - p_t)^4\right] \Big/ \left(E\left[(p_{t+\Delta} - p_t)^2\right]\right)^2 > 3$ if and only if $\beta>0$ (i.e., if the specialist optimally adjusts volatility in response to investors' trades). The model therefore reproduces these three major empirical facts: serially correlated price volatility, correlation between trading volume and absolute price changes, and excess kurtosis of price changes.

3. Estimation of the Equilibrium Price and Volume Processes

The objective of this section is to estimate jointly the system of stochastic differential equations specifying the evolution of the price and volume variables. We first concentrate on the price dynamics alone. Recall that $s_t \equiv \overline{p}_t - p_t$ and that Proposition 1 produced the following stochastic dynamics:

$$\begin{cases} dp_t = \alpha(\overline{p}_t - p_t)dt + \sqrt{\sigma^2 + \beta(\overline{p}_t - p_t)^2}\, dZ_{1t} \\ d\overline{p}_t = \sigma\, dZ_{2t} \end{cases} \tag{24}$$

Direct estimation of the parameter vector $\theta \equiv (\alpha, \sigma, \beta)$ in the system (24) is not feasible because $\overline{p}_t$ is unobservable to the econometrician. Only the market price p_t is observed. We propose the following approach to estimate the system. We start by determining the dynamics of the first two conditional moments of $\overline{p}_t$ given p_t (Proposition 3). Second, we derive expressions of the conditional moments of $p_{t+\Delta}$ given p_t and $\overline{p}_t$ (Proposition 4). We then use the law of iterated expectations to obtain the conditional moments of $p_{t+\Delta}$ given p_t, using our estimates of the moments of $\overline{p}_t$ given p_t. Finally we estimate the parameters by the generalized method of moments using the conditional moments of $p_{t+\Delta}$ given p_t that were just derived, and the unconditional moments of p_t. The same procedure is repeated once transaction volume data are introduced --instead of conditioning on the market price alone, we then condition on the full set of observables: market price and volume (Proposition 5).

3.1 Estimation of the Fundamental Value from Data on Market Price Only

The first step is to *filter* the fundamental price at time t from observations of the market price up to that time, assuming full knowledge of the joint dynamics (24), i.e., for a given set of parameter values θ. Intuitively, knowledge of the joint dynamics of the prices, associated with observations of the market price up to t collected in $\vartheta_t \equiv \{ p_s / 0 \le s \le t \}$, should reveal information on the unobservable underlying fundamental price. If we minimize the conditional expected estimation error $R_t \equiv E\left[\left(\hat{\overline{p}}_t - \overline{p}_t\right)^2 | \vartheta_t \right]$, the resulting estimate $\hat{\overline{p}}_t$ of $\overline{p}_t$ is clearly given by the conditional expectation of the fundamental price given the market price observations: $\hat{\overline{p}}_t = E\left[\overline{p}_t | \vartheta_t \right]$.

The theory of nonlinear optimal filtering for diffusion processes is described in detail in Lipster and Shiryayev (1977, Chapter 8). However the equations of optimal filtering can seldom be solved. Other than some specific cases, they can only be solved for linear dynamics, yielding the Kalman-Bucy (1960) filter. Furthermore, the equations can be formulated only when the volatility of the observable process dynamics do not depend on the unobservable process[13]. This is a major hindrance in our problem since the instantaneous volatility $v_t = \sqrt{\sigma^2 + \beta(\overline{p}_t - p_t)^2}$ depends upon both the observable process p_t on the unobservable process $\overline{p}_t$. To address these limitations, multiple suboptimal filtering methods have subsequently been developed. Pugachev and Sinitsyn (1987) present an account of these developments. We use an extended Kalman-Bucy filter[14]. This filter is obtained by expanding the dynamics (24) through a Taylor series expansion of their drift and diffusion for $\overline{p}_t$ in the vicinity of its filtered value $\hat{\overline{p}}_t$. In the extended Kalman-Bucy filter, the expansions of the diffusion terms are limited to the first order term. We solve for the filter and obtain:

Proposition 3:

(i) The extended nonlinear Kalman-Bucy filter follows the stochastic differential equation:

$$d\hat{\overline{p}}_t = \frac{\alpha R_t}{\sigma^2 + \beta\left(\hat{\overline{p}}_t - p_t\right)^2}\left\{dp_t - \alpha\left(\hat{\overline{p}}_t - p_t\right)dt\right\} \tag{25}$$

where the conditional estimation error R_t follows the Ricatti equation:

$$dR_t = \left\{\sigma^2 - \frac{\alpha^2 R_t^2}{\sigma^2 + \beta\left(\hat{\overline{p}}_t - p_t\right)^2}\right\} dt \tag{26}$$

(ii) In the special case where $\beta=0$, i.e., when the specialist does not input additional volatility to the market price relative to the fundamental price, the filter (25) reduces to the linear Kalman-Bucy filter with dynamics: $d\hat{\overline{p}}_t = \left(\alpha R_t/\sigma^2\right)\left\{dp_t - \alpha\left(\hat{\overline{p}}_t - p_t\right)dt\right\}$ where $R_t = \left(\sigma^2/\alpha\right)\left(e^{2\alpha t} - 1\right)/\left(e^{2\alpha t} - 1\right)$. The steady state limit is: $R = \sigma^2/\alpha$.

Proof: See Appendix.

Note that, as required, (25)-(26) make it possible to compute $\hat{\overline{p}}_t$ recursively at any t from a record of observations on the past market price changes. It is interesting to explore

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[13] See equation (8.44) page 307 in Lipster and Shiryayev (1977).

[14] See Pugachev and Sinitsyn (1987), Section 8.3.2.

how the optimal estimator of the fundamental price is updated as new information becomes available. Equation (25) shows that the estimator $\hat{\bar{p}}_t$ is updated in response to the market price change dp_t that was just recorded. But it is only revised in response to the increment in the market price that is uncorrelated with its past values, that is $dp_t - \alpha\left(\hat{\bar{p}}_t - p_t\right)dt$. Indeed $n_t \equiv p_t - \int_0^t \alpha\left(\hat{\bar{p}}_s - p_s\right)ds$ is an innovation process[15] for the market price process p_t. In other words, the estimator of the fundamental price changes only when new unpredictable information arrives. When a change in the market price dp_t is observed, part of it is attributed to a change in the fundamental price (and consequently used to update the fundamental price estimate), and part of it to noise.

Finally, an appealing feature of the estimator is that the estimation error made when replacing the unknown fundamental price by its filtered estimate is bounded above by a finite constant, and therefore does not grow as a power of t when t increases. As could be expected, less noise in the processes (σ smaller) and/or more reversion of p to $\bar{p}$ (α larger) make the error smaller. Note also that if dZ_1 and dZ_2 were correlated with correlation coefficient ρ_{12} then in the case β=0 the conditional estimation error would become $R_t = \left(\sigma^2/\alpha\right)\left(\left(1+\rho_{12}\right)e^{2\alpha t}-1\right)/\left(\left(1+\rho_{12}\right)/\left(1-\rho_{12}\right)e^{2\alpha t}-1\right)$, with steady state limit $R = \left(1-\rho_{12}\right)\left(\sigma^2/\alpha\right)$. Therefore R is smaller when the correlation coefficient ρ_{12} between the stochastic components of the respective fundamental and market price increments is closer to one. Intuitively, as ρ_{12} gets closer to one, movements in the market price tend to reflect more closely movements in the underlying fundamental price process, so it becomes easier to estimate the value of the fundamental price by observing only the market price. In the limit of perfect correlation, observing the market price fully reveals the fundamental price at every instant.

3.2 Parameter Estimation

In this section, we construct a consistent and asymptotically normal estimator of the unknown parameter values in (24).

[15] A random process n_t is an innovation process for a process p_t iff: (i) at any t≥0, n_t is a functional of $\vartheta_t \equiv \{ p_s \ / \ 0 \le s \le t \}$ and (ii) at any 0≤τ≤t<s, the increment $n_s - n_t$ is uncorrelated with p_τ.

Proposition 4:

The conditional means and (co)variances of the pair $(p_{t+\Delta}, \bar{p}_{t+\Delta})$ given $(p_t, \bar{p}_t)$.are given in closed-form for any time interval Δ>0 by:

$$\begin{cases} E[\bar{p}_{t+\Delta} \mid \bar{p}_t, p_t] = \bar{p}_t \\ E[p_{t+\Delta} \mid \bar{p}_t, p_t] = e^{-\alpha\Delta}(p_t - \bar{p}_t) + \bar{p}_t \\ E[\bar{p}^2_{t+\Delta} \mid \bar{p}_t, p_t] = \bar{p}_t^2 + \sigma^2\Delta \\ E[\bar{p}_{t+\Delta} p_{t+\Delta} \mid \bar{p}_t, p_t] = e^{-\alpha\Delta}(\sigma^2/\alpha + \bar{p}_t(p_t - \bar{p}_t)) + (\bar{p}_t^2 - \sigma^2/\alpha) + \sigma^2\Delta \\ E[p^2_{t+\Delta} \mid \bar{p}_t, p_t] = e^{-(2\alpha-\beta)\Delta}((p_t - \bar{p}_t)^2 - 2\sigma^2/(2\alpha-\beta)) + 2e^{-\alpha\Delta}(\sigma^2/\alpha + \bar{p}_t(p_t - \bar{p}_t)) \\ \qquad + (\bar{p}_t^2 - 2(\sigma^2/\alpha)(\alpha-\beta)/(2\alpha-\beta)) + \sigma^2\Delta \end{cases} \tag{27}$$

Proof: see Appendix.

We have obtained in Proposition 3 both $E[\bar{p}_t \mid p_t]$ and $E[\bar{p}_t^2 \mid p_t]$. We now apply the law of iterated expectations to derive the conditional mean and variance of the market price at time t+Δ *conditioned only on the observed variable* at t:

$$E[p_{t+\Delta} \mid p_t] = e^{-\alpha\Delta}(p_t - E[\bar{p}_t \mid p_t]) + E[\bar{p}_t \mid p_t] \tag{28}$$

$$\begin{aligned} E[p^2_{t+\Delta} \mid p_t] &= e^{-(2\alpha-\beta)\Delta}\left(E[(p_t - \bar{p}_t)^2 \mid p_t] - 2\sigma^2/(2\alpha-\beta)\right) \\ &+ 2e^{-\alpha\Delta}(\sigma^2/\alpha + E[\bar{p}_t(p_t - \bar{p}_t) \mid p_t]) \\ &+ (E[\bar{p}_t^2 \mid p_t] - 2(\sigma^2/\alpha)(\alpha-\beta)/(2\alpha-\beta)) + \sigma^2\Delta \end{aligned} \tag{29}$$

The unconditional variance of the price process is given by:

$$E[p_t^2] - E[p_t]^2 = \sigma^2\{2e^{-\alpha t}/\alpha - 2e^{-(2\alpha-\beta)t}/(2\alpha-\beta) - (1/\alpha)(\alpha-\beta)/(2\alpha-\beta) + t\} \tag{30}$$

The parameter vector θ can now be estimated. In (28)-(29), the estimates for $E[\bar{p}_t \mid p_t]$ and $E[\bar{p}_t^2 \mid p_t]$ are given by $\hat{\bar{p}}_t$ and $E[\bar{p}_t^2 \mid p_t] = R_t + \hat{\bar{p}}_t^2$ respectively, which are both functions of p_t and (α,β,σ) as determined in Proposition 3 by (25)-(26). We then use GMM to estimate (α,β,σ) using the moment conditions (28)-(29)-(30). Each conditional moment contributes two orthogonality conditions: itself, and its product with p_t. The estimator is consistent and asymptotically normal from the classical results of Hansen (1982)[16].

3.3 Estimating the Equilibrium Price From Market Price and Transaction Volume Information

When volume data are available, the fundamental price estimator can be improved by filtering out $\bar{p}_t$ from both p_t and q_t, instead of p_t only. By Itô's Lemma:

$$dq_t = \frac{\partial q_t}{\partial s_t} ds_t + \frac{1}{2}\frac{\partial^2 q_t}{\partial s_t^2}\left(\sigma^2 + v_t^2\right)dt \tag{31}$$

where q_t is given in equilibrium by (16), and the price spread follows (6). Note that this expression yields the market depth defined as the order flow required to change the market price by one dollar, i.e., $1/\gamma_t$ in the equation $E[dp_t] = \gamma_t\, E[dq_t]$.

While the model determines exactly the transaction volume equation (31), we estimate the following simplified expression, derived by retaining only in (16) the first order term for small price spreads s_t:

$$dq_t = -\lambda\alpha s_t dt + \lambda\left(\sigma dZ_{2t} - v_t dZ_{1t}\right) \tag{32}$$

Proposition 5:

The conditional mean and variance of the observed transaction volume $q_{t+\Delta} - q_t$ over any discrete time interval of length $\Delta>0$ are:

$$\begin{cases} E\left[(q_{t+\Delta} - q_t) \mid p_t, \bar{p}_t\right] = -\lambda(\bar{p}_t - p_t)(e^{-\alpha\Delta} - 1) \\ E\left[(q_{t+\Delta} - q_t)^2 \mid p_t, \bar{p}_t\right] = \lambda^2\left(e^{-(2\alpha-\beta)\Delta} - 2e^{-\alpha\Delta} + 1\right)(\bar{p}_t - p_t)^2 \\ \qquad\qquad + 2\sigma^2\lambda^2\left(1 - e^{-(2\alpha-\beta)\Delta}\right)/(2\alpha - \beta) \end{cases} \tag{33}$$

Proof: see Appendix.

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[16] We assume that the market price series satisfies the minimal regularity conditions needed to guarantee the existence of the Central Limit Theorem. For example, assume that the price series is β-mixing at an exponential rate of decay strictly greater than one. It is important in practice to choose the GMM weighting matrix optimally because of the difference in scales between the prices and the squared prices (in the empirical application that follows, the mean price level is \$67).

As for the market price process, apply the law of iterated expectations to obtain $E[(q_{t+\Delta} - q_t) \mid p_t]$ and $E[(q_{t+\Delta} - q_t)^2 \mid p_t]$, as functions of p_t and the filtered fundamental price and its conditional estimation error. Then use the resulting moments to form orthogonality conditions, and add them to those derived from (27).

In the extended Kalman-Bucy filter, the fundamental price estimate is now revised by a linear combination of the innovations in both the market price change dp_t and the transaction volume dq_t. It can be shown that the steady state limit of the conditional estimation error is $\tilde{R} < R$. In other words, the estimator of the fundamental price obtained from data on both the market price and transaction volume is always more precise than the estimator obtained from observations on the market price only.

4. Applications and Empirical Implementation

4.1 The Data

The implications of the model are examined on transaction data for AMR, the parent company of American Airlines, during the month of January 1993 (Monday 1/4 to Friday 1/29: 20 trading days). The source for the data is the NYSE TAQ Database, which lists all trades and quotes for the NYSE, Amex and NASDAQ and the regional exchanges. These transactions are submitted by participants on exchanges. We sign trades here as a buy or sell trade based on their relative proximity to the bid and ask quotes prevailing five seconds before[17]. We also aggregate successive trades of the same sign into a single trade. This trade is time-stamped at the average of the individual trade times, weighted by trade size. The aggregation results in a final sample of 2,229 trades. Table 2 reports the descriptive statistics for this sample. Figures 5 and 6 plot AMR's signed transaction volume and the time between trades.

During the sample period, the main events affecting the fundamental value of AMR stock (to be viewed as large realizations of the Brownian Motion Z_2 determining the change in $\bar{p}$) have been gathered from the *Wall Street Journal* and are given in Table 3.

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[17] See Lee and Ready (1991) for various approaches to signing transaction volume.

4.2 Estimation of the Equilibrium Dynamics

The estimation procedure has two distinct steps. All the parameters of the structural system (24) are estimated and the best estimate of the fundamental price is computed at every date. The parameter estimates are reported in Table 4. Figure 7 reports the observed changes in the market price (dp_t) used to construct the fundamental price filter in (25). Figure 8 gives the market price of AMR and the estimated fundamental price. It can be seen on the graph that large differences between the two prices tend to precede an adjustment in the market price that is consistent with the model. When the market price is significantly below the estimated fundamental price at t, the market price tends to go up subsequently, and vice-versa. This suggests graphically some predictive power, a question which is examined more carefully below.

The accuracy of the filter is determined by the conditional estimation error R_t in (26), which we plot in Figure 9. As discussed in Section 3.1, the conditional estimation error is bounded above, a fact which becomes apparent on the graph. We finally report in Figure 10 the stationary distribution of the price spread s_t given by (18). This is the unconditional distribution of the difference between the market and fundamental prices. It measures the likelihood, in steady-state, of observing deviations of any given magnitude between the market and fundamental prices.

A nice feature of this model is that relevant issues or "anomalies" in asset pricing can be interpreted as simple hypothesis tests on one or more of the three parameters of the model, α, β and σ^2, and we now focus on each of them.

4.3 Mean Reversion in Stock Prices

The extent to which stock prices tend to revert to their mean over long horizons has been the subject of long-standing attention in the finance literature, as part of a broader study of departures from the random walk hypothesis. On the empirical side, the investigation of mean-reversion in stock prices has generally focused on the autocorrelation at various frequencies of security returns. The idea that market prices would fluctuate around fundamental values, defined as the discounted cash flows that the stock gives title to, dates back at least to the classical books by Graham and Dodd (1934) and Williams (1938). This line of research includes Cowles (1933), Kendall (1953), Summers (1976), Fama and French (1988), Lo and MacKinlay (1988) and Poterba and Summers (1988).

The empirical findings concentrate on long horizon returns and generally find significant mean reversion.

In this model, the market price can be interpreted as reverting at every instant to the stochastic level $\overline{p}_t$ (recall (24)). Because the fundamental price is itself a stochastic process, we propose two nested definitions of mean-reversion: (i) no mean-reversion in the strong sense, corresponding to the joint hypothesis H_S: $\alpha = 0$ and $\sigma^2 = 0$, and (ii) no mean-reversion in the weak sense, corresponding to the hypothesis H_W: $\alpha = 0$ only. In this framework, define a market price series to be strongly mean-reverting if the joint hypothesis H_S, as well as the two individual hypotheses, are rejected, and weakly mean-reverting if only H_W is rejected. A strongly mean-reverting market price reverts to a non-stochastic value. A weakly mean-reverting series reverts to a randomly changing level. A market price series that is not weakly mean-reverting is essentially a random walk.

The two null hypotheses can be tested in the GMM framework spelled out in Proposition 4. We use the Wald statistics[18]. The test results are in Table 5. Neither H_S nor the two single hypotheses (in particular H_W) are rejected at the 95% level.

4.4 The Predictability of Stock Price Changes

A closely related question is whether stock price changes are predictable. In this model, the fundamental price estimate makes it possible to predict market price changes at every instant, by replacing the unobservable drift of the market price process in (24) with the estimated drift, yielding an estimate at t of the change between t and t+dt:

$$\hat{E}[\, dp_t \mid \vartheta_t \,] = \hat{\alpha}\left(\hat{\overline{p}}_t - p_t\right)dt \tag{34}$$

with the same notation as before: $\hat{\alpha}$ is the unconstrained estimate and $\hat{\overline{p}}_t = E[\overline{p}_t \mid \vartheta_t \,]$. At every instant t (sufficiently far from the beginning of the sample), observations on $\{p_s \,/\, 0 \le s \le t\}$ and the corresponding volume are used to form $\hat{\alpha}$ and $\hat{\overline{p}}_t$. Then the

[18] Asymptotically, the Wald, Likelihood Ratio and Lagrange Multiplier statistics are equivalent, but the Wald statistics does not require that the constrained parameter be computed and is therefore easier to implement. A disadvantage of the Wald statistics, however, is that it is not invariant to reparametrizations of the null hypothesis. This should not be a concern here, since the economic hypothesis to be tested (say, mean-reversion) leaves very little leeway regarding what specification of the statistical hypothesis is natural (α=0 in that case).

expected market price change is computed according to (34). The forecasts can be compared to the random walk forecasts:

$$\hat{\hat{E}}[\, dp_t \mid \vartheta_t \,] = 0 \tag{35}$$

The null hypothesis H_P: $\alpha = 0$ is rejected at the 95% level. An alternative easily interpretable test of forecasting power could be conducted by counting the proportion $\hat{\pi}$ of instants in the sample for which (34) is closer to the actual market price change recorded between t and t+dt than the random walk (35). Under the null that market changes are unpredictable given the past and present market prices in ϑ_t, the probability that (34) be closer to the true market price change than (35) is π=1/2, as the predicted departure from (35) given by (34) is uncorrelated to the actual departure under the null. We would test $H_\pi : \pi = 1/2$ vs. $\overline{H}_\pi : \pi > 1/2$. The sample probability $\hat{\pi}$ would have the binomial distribution $\sqrt{T}(\hat{\pi} - 1/2) \rightarrow N(0, 1/4)$ under the null.

4.5 Do Market Prices Move Too Much?

To examine whether market prices move too much to be justified by changes in the fundamental price, it suffices to test the null hypothesis $H_V : \beta = 0$. Under H_V, the market price is not more volatile than the fundamental price, as both have the same volatility σ. The null is rejected at the 95% level. Note that this is not a statement that the filtered fundamental price is smoother than the market price. At the filtering stage (Proposition 3), the value of β is not yet determined and could well have been zero. The parameters are only estimated at the GMM stage (Proposition 4), and as the parameter values are updated so is the filter.

This evidence is compatible with the classical result of Shiller (1981) and LeRoy and Porter (1981), who found that the volatility of stock prices was too high to be justified by changes in future dividends (see Cochrane (1991) for a critique). Shiller showed that under his assumptions the market price must have a lower volatility than the perfect foresight price, so finding the opposite result yielded the conclusion that markets are inefficient / irrational. An important difference is that there is no constraint here on the relative size of the market and fundamental price volatilities. Rejecting H_V has no implications for market efficiency in this model. Every market participant is rational, yet market prices could be more volatile than fundamental values.

4.6 The Possibility of Fads and Bubbles, and Market Efficiency

This model provides a natural framework to examine the development and growth of fads and bubbles. An efficient (E,e) market can be defined as one where

$$P\left(\left|p_t - \bar{p}_t\right| \geq E\, p_t \mid \vartheta_t\right) \leq e \tag{36}$$

for all dates t. A bubble or fad can similarly be defined as a deviation between the market price and the fundamental price that exceeds a certain arbitrary level (e.g., E=20%) and persists for more than a certain arbitrary amount of time (e.g., a week). All these probabilities can be computed directly from the joint transition densities of the market and fundamental price processes.

5. Conclusions and Extensions

The results suggest that it is possible to replicate, in a fully rational world, some of the empirical findings that have been labeled as market "anomalies." There is no question however that this model is simplistic, and that many other puzzling empirical regularities are beyond its scope. Three extensions can be considered. First, the theoretical model could be extended to incorporate asymmetric information (Detemple (1986), Gennotte (1986), Wang (1993)). For example, the fundamental price could be revealed only to the specialist. The investor would receive a signal on the fundamental price and then trade based on the signal, in addition to the price set by the specialist. In equilibrium, the market price would not be a sufficient statistics for all relevant market information, and the resulting model would have a noisy rational equilibrium flavor (Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981)).

Secondly, the model could incorporate a bid-ask spread, as a source of profit to the specialist (see Glosten and Milgrom (1985)). This could serve as an additional motivation for our assumption that the specialist is constrained in setting the drift of the price adjustment, but controls its volatility. With a bid-ask spread, the specialist would have an incentive to "move prices around," i.e., control the price volatility, as liquidity investors would then readjust their portfolios more often --thereby generating bid-ask profits for the specialist. Thirdly, the model predicts that market price changes exhibit additional volatility compared to those of the fundamental value of the stock. Instead of relying on a particular parametrization, this excess volatility function could be estimated nonparametrically.

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Appendix

Proof of Proposition 1:

(i) The investor's optimization problem has the two control variables c_t and q_t, and the two state variables are W_t and s_t. Let $V(t,W_t,s_t)$ be its value function. The Bellman Equation is:

$$0=\sup_{\{c_t,q_t\}}\left\{-e^{-\rho t-ac_t}+\frac{\partial V}{\partial t}+\frac{\partial V}{\partial W_t}\{rW_t+\beta s_t q_t-c_t\}+\frac{\partial V}{\partial s_t}\{-\alpha s_t\}\right.$$
$$\left.+\frac{1}{2}\frac{\partial^2 V}{\partial W_t^2}\{q_t^2 v_t^2\}+\frac{\partial^2 V}{\partial W_t \partial s_t}\{-q_t v_t^2\}+\frac{1}{2}\frac{\partial^2 V}{\partial s_t^2}\{\sigma^2+v_t^2\}\right\}$$

since $\mathrm{var}(dW_t)=\mathrm{var}(q_t v_t dZ_{1t})=q_t^2 v_t^2 dt$, $\mathrm{var}(ds_t)=\mathrm{var}(\sigma dZ_{2t}-v_t dZ_{1t})=\{\sigma^2+v_t^2\}dt$ and $\mathrm{cov}(dW_t,ds_t)=\mathrm{cov}(q_t v_t dZ_{1t},\sigma dZ_{2t}-v_t dZ_{1t})=-q_t v_t^2 dt$.

The first order conditions give the investor's optimal investment strategy q_t:

$$\frac{\partial V}{\partial W_t}\beta s_t+\frac{\partial^2 V}{\partial W_t^2}q_t v_t^2-\frac{\partial^2 V}{\partial W_t \partial s_t}v_t^2=0$$

and consumption policy c_t:

$$ae^{-\rho t-ac_t}-\frac{\partial V}{\partial W_t}=0$$

Replacing the control variables by their optimal values in the Bellman Equation leads to a partial differential equation to be satisfied by the value function. The solution can be found in the form: $V(t,W_t,s_t)=-(1/r)\exp(-\rho t-arW_t-ah(s_t))$, where $h(s_t)=h_0+h_1 s_t+h_2 s_t^2$ with constant h_i's. We obtain that:

$$q(s_t,v_t)=\frac{s_t}{a}\left[\frac{\beta}{rv_t^2}\right]+\frac{h'(s_t)}{r}\quad\text{and}\quad c_t=rW_t+h(s_t)$$

and so the Bellman Equation evaluated at the optimal policies becomes:

$$0 = r - \rho - ar\{\beta s_t q_t - h(s_t)\} + a\alpha h'(s_t)s_t + \frac{1}{2}a^2r^2q_t^2v_t^2$$
$$- \; arh'(s_t)q_t v_t^2 + \frac{1}{2}\{a^2h'^2(s_t) - ah''(s_t)\}\{\sigma^2 + v_t^2\}$$

The solution for h(.) in the value function is:

$$h_2 = \frac{r}{4a\sigma^2}\left[\sqrt{1+4\beta^2\sigma^2/r^2v_t^2} - 1\right],\; h_1 = 0,\; h_0 = \frac{1}{ar}\left[(\sigma^2 + v_t^2)ah_2 + (\rho - r)\right]$$

It satisfies the transversality condition $\lim_{t\to\infty} E\{V(t, W_t, s_t)\} = 0$. The optimal investment policy simply follows by replacing $h'(s_t)$ by $2h_2s_t$, which yields (2.14).

(ii) The specialist acts as a Stackelberg leader. His objective is to maximize over $\{v_\tau, \tau \geq t\}$:

$$E_t\left[\int_t^{+\infty} e^{-r\tau}\{T(v_\tau, s_\tau) - q(v_\tau, s_\tau)\beta s_\tau\}d\tau\right]$$

taking into account that the investor will react optimally to his choice of volatility: $q_t = q(s_t, v_t)$, given by (2.14). To obtain a closed-form solution to the specialist's problem we specify the transfer function T as:

$$T(v_t, s_t) = \frac{\beta s_t^2}{a}\left[\left[\frac{\sqrt{1+4\beta^2\sigma^2/r^2v_t^2} - 1}{2\sigma^2}\right] + \frac{\beta}{r}\left[\frac{\sigma^2 + v_t^2}{(3\sigma^2 + \beta s_t^2 + v_t^2)^2} + \frac{1}{v_t^2} - \frac{2\sigma^2 + \beta s_t^2}{(4\sigma^2 + \beta s_t^2)^2}\right]\right]$$

This function satisfies the properties (2.7), most notably being decreasing in v_t on R+. Note that if the investor is infinitely risk-averse (a=∞) he never holds the stock, no trade ever takes place, and the transfer function is identically zero. Similarly, if the stock price does not mean-revert to the fundamental price (β=0) the specialist receives no transfer.

Let $V(t, s_t) = e^{-\rho t}k(s_t)$ be the value function. The Bellman Equation is:

$$0 = \sup_{\{v_t\}}\left\{\{T(v_t, s_t) - q(v_t, s_t)\beta s_t\} - \rho k(s_t) + k'(s_t)\{-\alpha s_t\} + \frac{1}{2}k''(s_t)\{\sigma^2 + v_t^2\}\right\}$$
$$= \sup_{\{v_t\}}\left\{\frac{\beta^2 s_t^2}{ar}\left[\frac{\sigma^2 + v_t^2}{(3\sigma^2 + \beta s_t^2 + v_t^2)^2} - \frac{2\sigma^2 + \beta s_t^2}{(4\sigma^2 + \beta s_t^2)^2}\right]\right.$$
$$\left. -\rho k(s_t) + k'(s_t)\{-\alpha s_t\} + \frac{1}{2}k''(s_t)\{\sigma^2 + v_t^2\}\right\}$$

The optimal control v_t to be set by the specialist solves the first order condition:

$$\beta^2 s_t^2 \frac{2v_t(\sigma^2+\beta s_t^2-v_t^2)}{(3\sigma^2+\beta s_t^2+v_t^2)^3}=\frac{ar}{2}k''(s_t)$$

The complete solution is: $v_t^2=\sigma^2+\beta s_t^2$ and $k(s_t)=0$, which satisfies the transversality condition $\lim_{t\to\infty} E\{e^{-rt}k(s_t)\}=0$.

(iii) is immediate given what precedes.

Proof of Proposition 2:

(i) Define $\tau(s)\equiv\exp\left\{\int^s 2\alpha u/(\sigma^2+\beta u^2)du\right\}$, $m(s)\equiv 1/((\sigma^2+\beta s^2)\tau(s))$, the scale measure $T(s)\equiv\int^s\tau(u)du$ and the speed measure $M(s)\equiv\int^s m(u)du$. The process s_t is strictly stationary on D=(-∞,+∞) and both +∞ and -∞ are entrance boundaries if and only if at both boundaries the scale measure T(s) diverges, the speed measure M(s) and the cross-integral $N\equiv\int^s T(v)dM(v)$ both converge (see Karlin and Taylor (1981), 15.6). This properties are satisfied here since near infinity $\tau(s)\propto s^{2\alpha/\beta}$.

Let $p(t,s_t \mid s_0)$ be the conditional density of s_t given s_0. To compute the conditional moments, we define the moment generating function $\phi(t,\theta)\equiv E\left[e^{-\theta s_t}\mid s_0\right]$. For notational simplicity the dependence of ϕ on s_0 is omitted since s_0 is held fixed in what follows. We have that:

$$\begin{aligned}\frac{\partial\phi(t,\theta)}{\partial t}&=\int_{-\infty}^{+\infty}e^{-\theta s_t}\frac{\partial p(t,s_t\mid s_0)}{\partial t}ds_t\\&=\int_{-\infty}^{+\infty}e^{-\theta s_t}\left\{\frac{\partial}{\partial s_t}\left[\alpha s_t p(t,s_t\mid s_0)\right]+\frac{1}{2}\frac{\partial^2}{\partial s_t^2}\left[(2\sigma^2+\beta s_t^2)p(t,s_t\mid s_0)\right]\right\}ds_t\\&=\int_{-\infty}^{+\infty}e^{-\theta s_t}\left\{\theta\alpha s_t+\frac{1}{2}\theta^2(2\sigma^2+\beta s_t^2)\right\}p(t,s_t\mid s_0)ds_t\end{aligned}$$

where we have successively applied the Kolmogorov forward equation (see for example Karlin and Taylor (1981), 15.) and integrated by parts. Therefore, we have obtained that:

$$\frac{\partial\phi(t,\theta)}{\partial t}=-\theta\alpha\frac{\partial\phi(t,\theta)}{\partial\theta}+\frac{1}{2}\theta^2\left(2\sigma^2\phi(t,\theta)+\beta\frac{\partial^2\phi(t,\theta)}{\partial\theta^2}\right)$$

where:

$$(-1)^n \left[\frac{\partial^n \phi(t,\theta)}{\partial \theta^n}\right]_{|\theta=0} = E\left[s_t^n \mid s_0\right] \equiv C_n(t) \text{ and } \phi(t,0)=1.$$

Differentiate both sides of this equality with respect to θ, and evaluate the result at θ=0, to obtain:

$$\begin{cases} C_1'(t) = -\alpha C_1(t) \\ C_2'(t) = -(2\alpha-\beta)C_2(t) + 2\sigma^2 \\ C_3'(t) = -3(\alpha-\beta)C_3(t) + 6\sigma^2 C_1(t) \\ C_4'(t) = -2(2\alpha-3\beta)C_4(t) + 12\sigma^2 C_2(t) \end{cases}$$

Solving these first-order ordinary differential equations with the initial conditions $C_n(0) = s_0^n$ yields the conditional moments $E\left[s_t^n \mid s_0\right]$ for n=1,2,3 and 4.

(ii) In equilibrium, the price spread dynamics are given by (6) with the volatility (15): $ds_t = -\alpha s_t\, dt + \sigma dZ_{2t} - \sqrt{\sigma^2 + \beta s_t^2}\, dZ_{1t}$. The stationary distribution of s_t is determined from the drift $-\alpha s_t$ and the diffusion $(2\sigma^2 + \beta s^2)$, with the normalization constant ξ determined to insure that the density integrates to one:

$$\begin{aligned} \pi(s) &= \frac{\xi}{(2\sigma^2+\beta s^2)} \exp\left\{\int_{-\infty}^{s} \frac{-2\alpha u}{(2\sigma^2+\beta u^2)} du\right\} \\ &= \frac{\xi}{(2\sigma^2+\beta s^2)^{1+\alpha/\beta}} \\ &= \frac{1}{(2\sigma^2+\beta s^2)^{1+\alpha/\beta} \int_{-\infty}^{+\infty} \left\{1/(2\sigma^2+\beta u^2)^{1+\alpha/\beta}\right\} du} \end{aligned}$$

Therefore near infinity $\pi(s) \propto (\beta s^2)^{-1-\alpha/\beta}$ and $\int^{s} u^n \pi(u) du \propto \beta^{-1-\alpha/\beta} s^{n-1-2\alpha/\beta}$ converges if and only if $2\alpha > (n-1)\beta$. The unconditional moments $U_n \equiv E\left[s_t^n\right]$ (independent of t by stationarity) can be computed either directly from the expression of the unconditional density π(s), or more easily by appealing to the ergodicity of the process. That is: $\pi(s) = \lim_{t\to+\infty} p(s,t \mid s_0)$ and hence $U_n = \lim_{t\to+\infty} C_n(t)$. The result for U_n is immediate given the expression of the conditional moments $C_n(t)$.

Proof of Proposition 3:

(i) The Extended Kalman-Bucy filter is derived from the following joint dynamics for the (unobservable) equilibrium and (observable) market prices:

$$d\bar{p}_t = \sigma\, dZ_{2t}$$

$$dp_t = \alpha(\bar{p}_t - p_t)dt + \sqrt{\sigma^2 + \beta(\hat{\bar{p}}_t - p_t)^2}\, dZ_{1t}$$

(see Pugachev and Sinitsyn (1987) page 448).

Our objective is to find the stochastic differential equation followed by: $\hat{f} \equiv E[\, f(\bar{p}_t) \mid \vartheta_t]$ with $\bar{p}_t$ and p_t given above. Using results from the theory of optimal filtering (e.g., Pugachev and Sinitsyn (1987), (15) page 388), it can be shown that $\hat{f}_t \equiv E[\, f(\bar{p}_t) \mid \vartheta_t]$ follows the following stochastic differential equation:

$$d\hat{f}_t = E\left[\frac{1}{2}\frac{d^2 f(\bar{p}_t)}{d\bar{p}_t^{\,2}}\sigma^2 \,\middle|\, \vartheta_t\right]dt + E\left[f(\bar{p}_t)\left\{\alpha(\bar{p}_t - \hat{\bar{p}}_t)\right\} + \frac{df(\bar{p}_t)}{d\bar{p}_t}\rho_{12}\sigma\sqrt{\sigma^2 + \beta(\hat{\bar{p}}_t - p_t)^2} \,\middle|\, \vartheta_t\right].$$
$$\cdot\left(\sigma^2 + \beta(\hat{\bar{p}}_t - p_t)^2\right)^{-1}\left\{dp_t - \alpha(\hat{\bar{p}}_t - p_t)dt\right\}$$

where at this stage we have allowed for the sake of generality the two Brownian Motions to be correlated: $E[dZ_{1t}dZ_{2t}] = \rho_{12}dt$.

Apply this to $f(z) \equiv z$, hence $\hat{f}_t \equiv E[\, f(\bar{p}_t) \mid \vartheta_t] = \hat{\bar{p}}_t$, to obtain the stochastic differential equations (25) for $\hat{\bar{p}}_t$. Then apply it to $f(z) \equiv z^2$, and subtract $\hat{\bar{p}}_t^2$, to obtain the equation (26) for the conditional estimation error R_t.

(ii) In the special case where β=0, the extended Kalman-Bucy filters reduces to the optimal Kalman-Bucy linear filter. The conditional estimation error then follows a deterministic Riccati equation:

$$\frac{dR_t}{dt} = \sigma^2 - \left(\frac{\alpha R_t + \rho_{12}\sigma^2}{\sigma}\right)^2$$

with initial condition $R_0 = 0$. It is immediate to verify that the solution of the Riccati equation is:

$$R_t = (1+\rho_{12})\frac{\sigma^2}{\alpha}\{\exp(2\alpha t)-1\} \Big/ \left\{\left(\frac{1+\rho_{12}}{1-\rho_{12}}\right)\exp(2\alpha t)+1\right\}$$

conditioning on the initial fundamental value $\bar{p}_0$ assumed known, that is $R_0 = 0$. Its steady state solution is:

$$R \equiv \lim_{t\to+\infty} R_t = (1-\rho_{12})\frac{\sigma^2}{\alpha}.$$

Proof of Proposition 4:

Let $p(t, \bar{p}_t, p_t \mid \bar{p}_0, p_0)$ be the conditional density of the pair $(\bar{p}_t, p_t)$ given $(\bar{p}_0, p_0)$ and define the moment generating function $\phi(t, \bar{\theta}, \theta) \equiv E\left[e^{-\bar{\theta}\bar{p}_t - \theta p_t} \mid \bar{p}_0, p_0\right]$. Applying the multidimensional Kolmogorov forward equation, we have that:

$$\frac{\partial\phi}{\partial t} = -\theta\alpha\left(\frac{\partial\phi}{\partial\theta} - \frac{\partial\phi}{\partial\bar{\theta}}\right) + \frac{1}{2}\theta^2\left(\sigma^2\phi + \beta\left(\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial\bar{\theta}^2} - 2\frac{\partial^2\phi}{\partial\theta\partial\bar{\theta}}\right)\right) + \frac{1}{2}\bar{\theta}^2\sigma^2\phi$$

where:

$$(-1)^{n+m}\left[\frac{\partial^n\phi(t,\bar{\theta},\theta)}{\partial\bar{\theta}^n\partial\theta^m}\right]_{\substack{|\bar{\theta}=0\\|\theta=0}} = E\left[\bar{p}_t^n p_t^m \mid \bar{p}_0, p_0\right] \equiv C_{nm}(t) \text{ and } \phi(t,0,0) = 1.$$

Differentiate both sides of this equality with respect to θ, and evaluate the result at $\bar{\theta} = \theta = 0$, to obtain:

$$\begin{cases} C_{10}'(t) = 0 \\ C_{01}'(t) = -\alpha C_{01}(t) + \alpha C_{10}(t) \\ C_{20}'(t) = \sigma^2 \\ C_{11}'(t) = -\alpha C_{11}(t) + \alpha C_{20}(t) \\ C_{02}'(t) = -(2\alpha-\beta)C_{02}(t) + 2(\alpha-\beta)C_{11}(t) + \beta C_{20}(t) + \sigma^2 \end{cases}$$

Solving these first-order ordinary differential equations with the initial conditions $C_{nm}(0) = \bar{p}_0^n p_0^m$ yields the desired conditional moments.

Proof of Proposition 5:

Using the same method as in the proof of Proposition 4, for the two state variables s_t and q_t with respective dynamics (6) and (32), we obtain:

$$\begin{cases} E[q_{t+\Delta} \mid s_t, q_t] = q_t - \lambda s_t (e^{-\alpha\Delta} - 1) \\ E[q_{t+\Delta}^2 \mid s_t, q_t] = q_t^2 - 2\lambda(e^{-\alpha\Delta} - 1) q_t s_t + \lambda^2 \left(e^{-(2\alpha-\beta)\Delta} - 2e^{-\alpha\Delta} + 1\right) s_t^2 \\ \qquad\qquad\qquad\qquad + 2\sigma^2\lambda^2/(2\alpha - \beta) \end{cases}$$

The expressions for the conditional moments of the observed volume, i.e., the change in the investor's holdings between t and t+Δ, $q_{t+\Delta} - q_t$, follow.

Year	Price Continuity Rate	Price Stabilization Rate
1994	97.4%	76.3%
1993	97.1%	77.6%
1992	96.4%	78.3%
1991	95.9%	80.9%
1990	95.8%	83.1%

Table 1: Price Continuity and Stabilization (Source: NYSE 1994 Fact Book).

	Market Price	**Market Price Change**	**Trading Volume**	**Time between Trades**
Unit	Dollars	Dollars	Number of Shares	Minutes
Mean	66.970	-0.001	-86	3.579
Standard Deviation	2.189	0.165	15,350	5.889
Minimum	61.750	-0.875	-162,000	0.017
25% Percentile	65.000	-0.125	-2,100	0.617
50% Percentile	67.375	0.125	100	1.667
75% Percentile	68.750	0.125	2,000	4.200
Maximum	70.250	1.000	173,000	108.517
Lag 1 Autocorrelation	0.996	-0.499	-0.129	0.085

Table 2: Descriptive Statistics

Note: Same-side trades (buy or sell) are aggregated for the purpose of computing trading volume and time between trades.

Date	Trading Day	Event
Jan. 7	4	UAL announces drastic cost-cutting measures. Along with other airline stocks, AMR jumps 7/8 to close at 68 3/4.
Jan. 12	7	Northwest Airlines launched a new fare war. AMR off 1 5/8 at 67 3/4.
Jan. 14	9	Threat of wholesale fare war recedes. AMR leaps 2 to 69.
Jan. 18	11	County NatWest increases its 1992 estimated loss for AMR from continuing operations to $4.50 from $2.80 a share.
Jan. 20	13	AMR reports its worst loss ever ($935m) for 1992 and announces spending cuts of $300m for 1993. AMR stock down 3/4 to 67 1/8.
Jan. 25	16	Possible OPEC production cuts announced. Fuel prices up. AMR loses 1 7/8 to 64 1/8.
Jan. 26	17	AMR announces delayed delivery of eight Boeing jets. AMR down 3/8 to 64.
Jan. 28	19	Delta and UAL announce large 1992 losses. AMR down 1 1/4 to 62 1/4 along with the other airlines' stocks.

Table 3: Potential Shocks Affecting the Fundamental Value of AMR during January 1993

Parameter		Coefficient	95% Confidence Interval
Price Processes	α	1.36 10^{-1}	[0.028,0.25]
	σ^2	6.28 10^{-1}	[0.47,0.79]
	β	8.41 10^{-2}	[0.008,0.16]
Volume Process	λ	4.50 10^{+5}	[17351,72649]

Table 4: Parameter Estimates

Note: The price processes are given by (24). The trading volume process is given by (32). In these estimates the unit of time is one trading day. Prices are measured in dollars, and volume in number of shares traded.

Economic Issue	Null Hypothesis	Wald Statistics
Mean Reversion of Prices?	α=0 and σ^2=0	64.2
	σ^2=0	5.95
Predictability of Price Changes?	α=0	4.62
Do Prices Move Too Much?	β=0	59.1

Table 5: Test Statistics

Note: The Wald Statistics are distributed as χ_2^2 for the joint hypothesis on (α,σ^2), and χ_1^2 for the others. The 95% critical values are respectively 5.99 and 3.84.

Figure 1
Chronology of Events

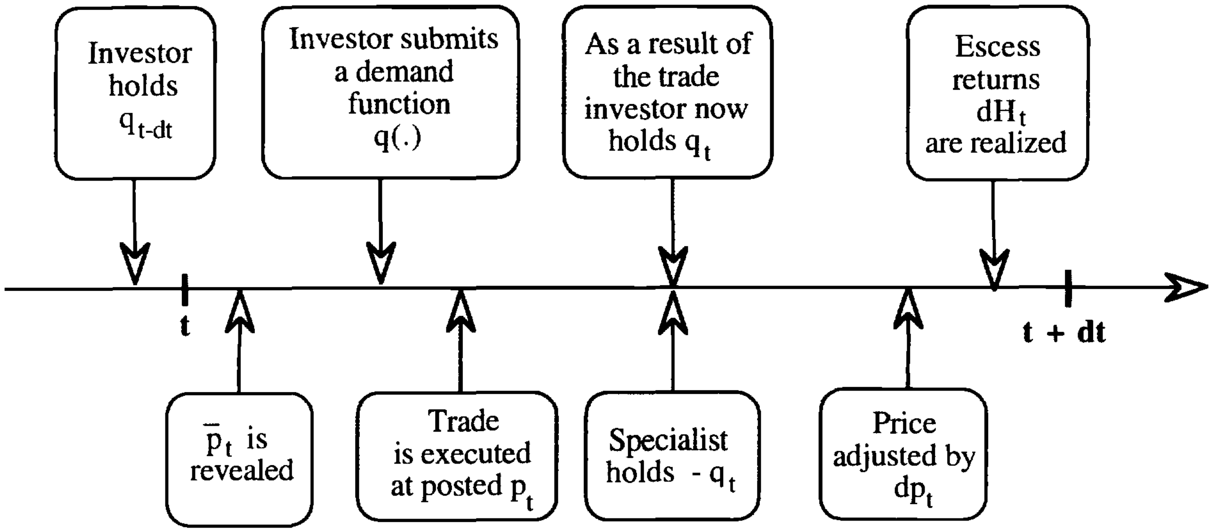

Figure 2
Variance of the Price Adjustment in Equilibrium

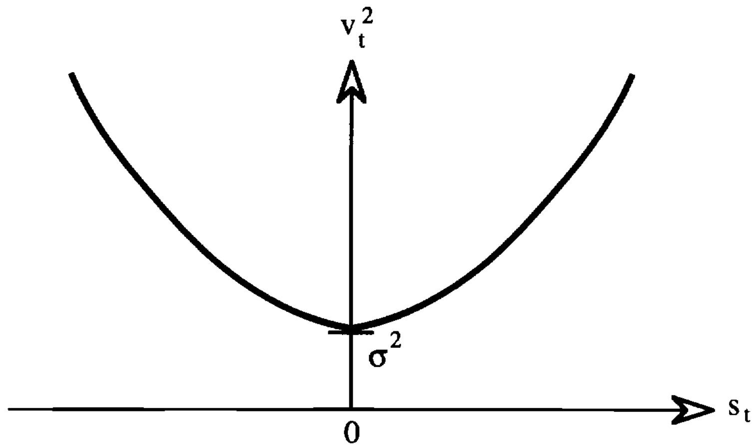

Figure 3
Investor's Asset Holdings in Equilibrium

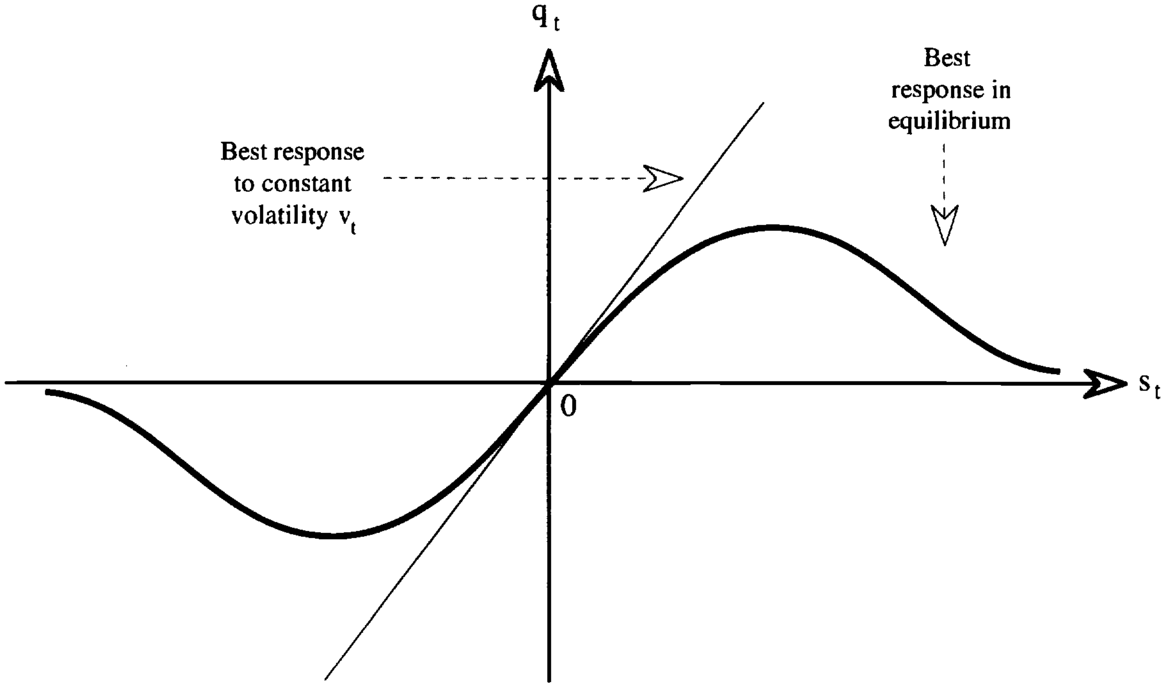

Figure 4
Conditional Moments of Price Changes

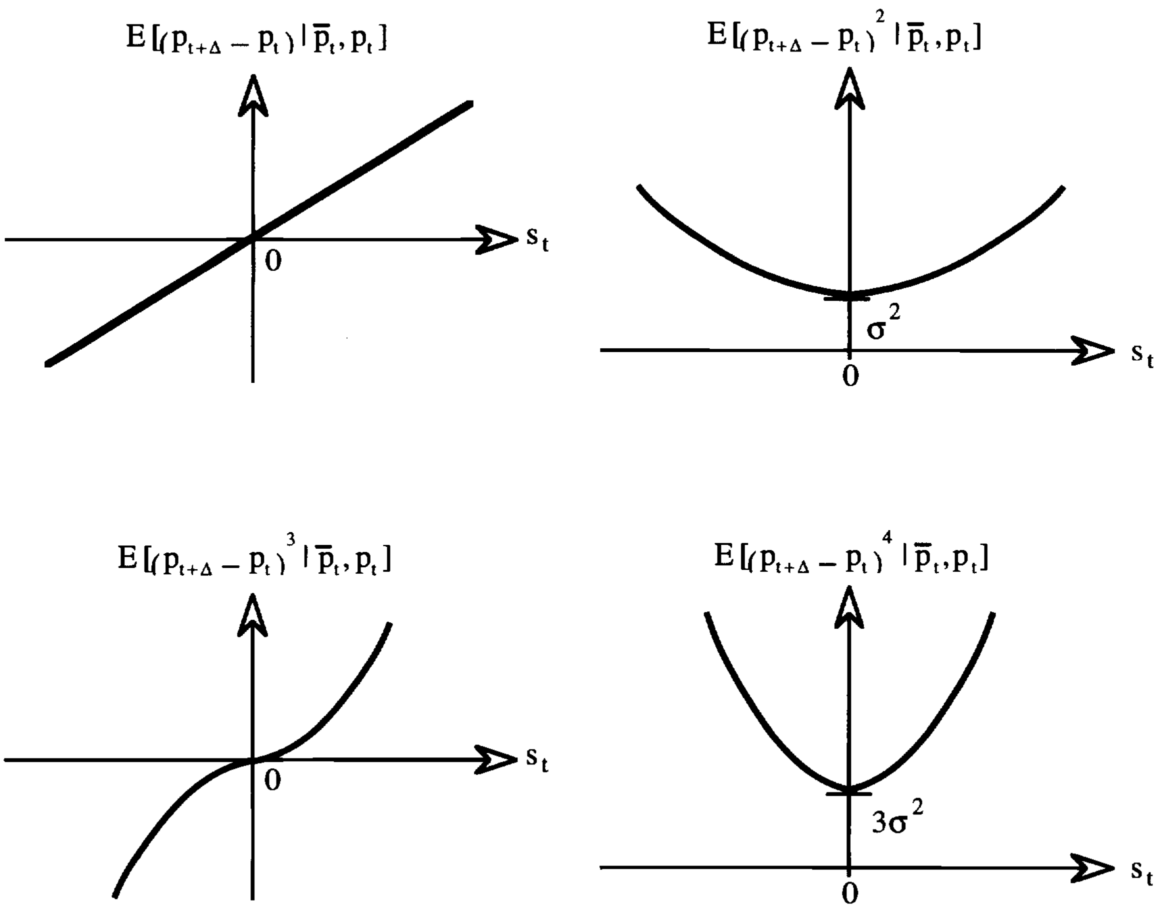

Figure 5
AMR Signed Transaction Volume

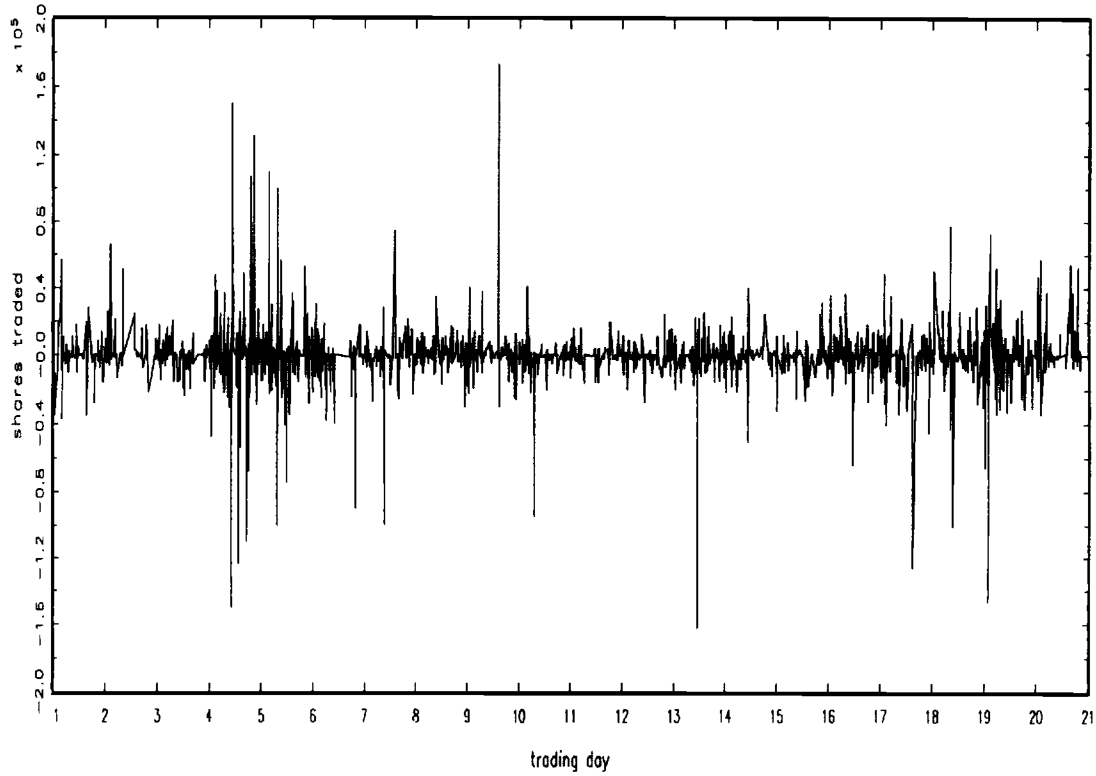

Figure 6
Time Between Trades

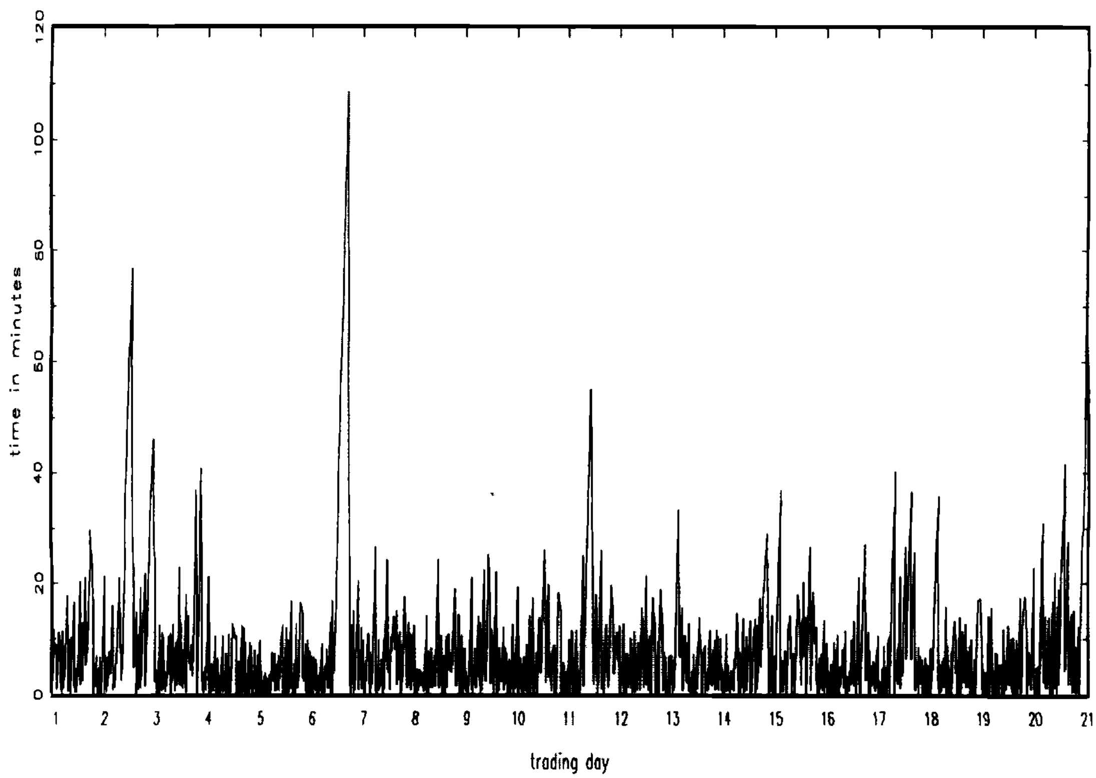

Figure 7
AMR Market Price Change

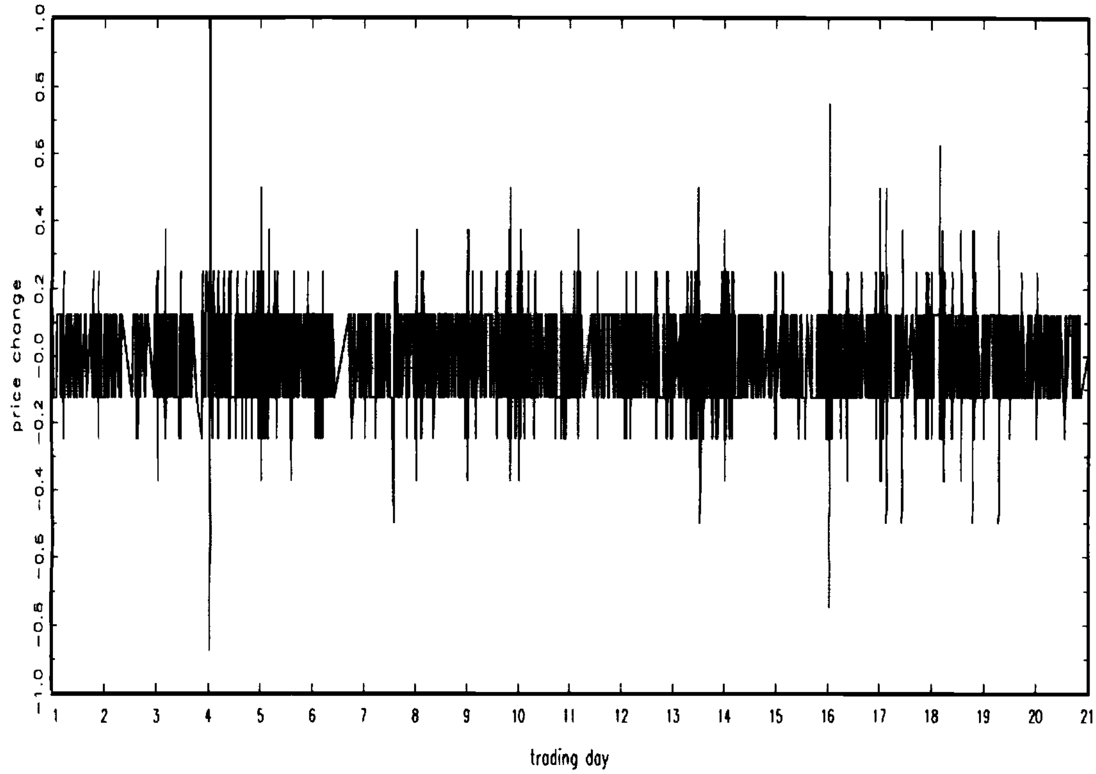

Figure 8
Market and Estimated Fundamental Prices

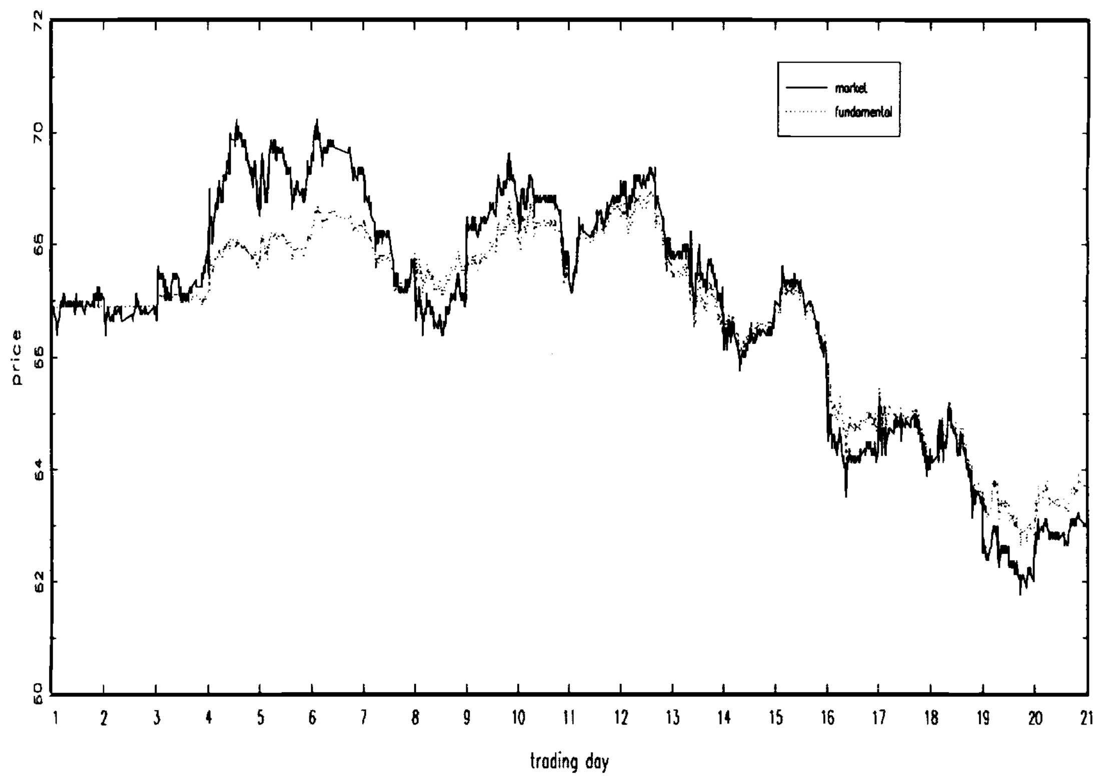

Figure 9
Fundamental Price Conditional Estimation Error

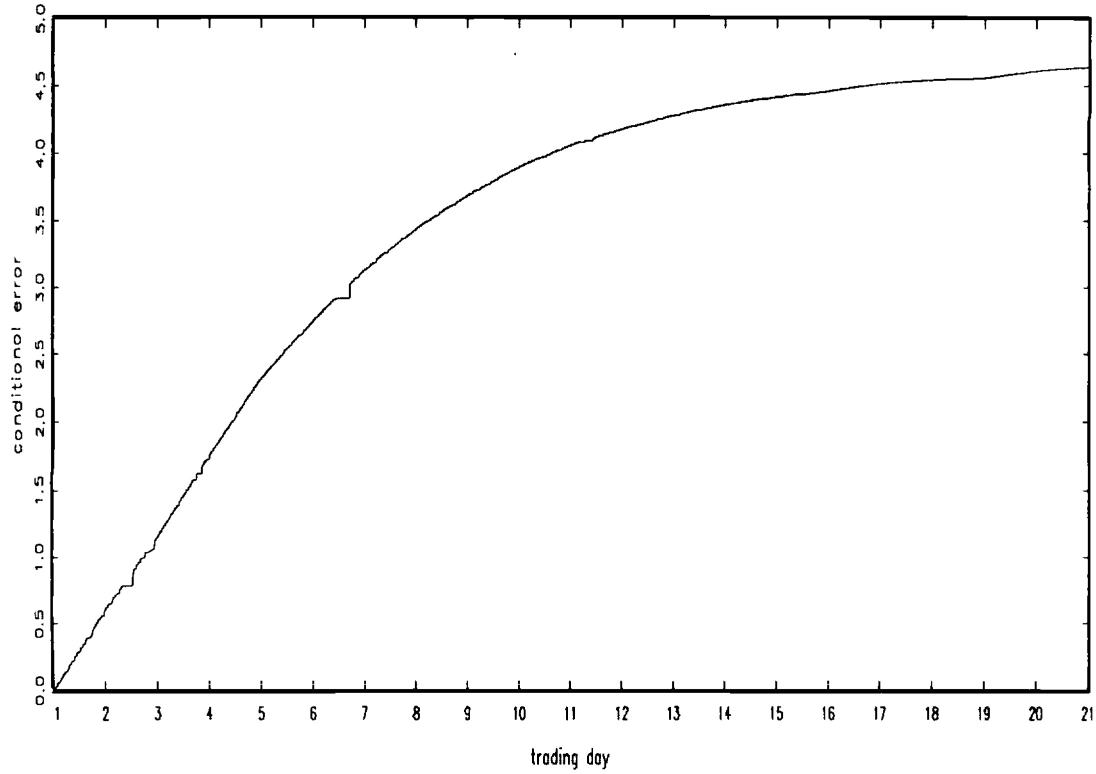

Figure 10
Stationary Distribution of the Price Spread