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BANK CAPITAL REGULATION IN  
GENERAL EQUILIBRIUM

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ABSTRACT

We study whether the socially optimal level of stability of the banking system can be implemented with regulatory capital requirements in a multi-period general equilibrium model of banking. We show that: (i) bank capital is costly because of the unique liquidity services provided by demand deposits, so a bank regulator may optimally choose to have a risky banking system; (ii) even if the regulator prefers more capital in the system, the regulator is constrained by the private cost of bank capital, which determines whether bank shareholders will agree to meet capital requirements rather than exit the industry.

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## I. Introduction

Bank failures and panics are thought to involve large harmful externalities. As a result, most banking systems throughout the world are heavily regulated. Common restrictions include limitations on bank asset choices, periodic examinations of bank asset quality, and required minimum bank capital/assets ratios, all with the aim of reducing the chance of individual bank failure and thus enhancing the overall stability of the overall banking system. Furthermore, most governments implicitly or explicitly insure bank deposits so as to prevent panics and maintain confidence in the payments system. This may in turn increase the need for the regulations just described, since fixed-rate deposit insurance increases banks' incentives to increase their risk of failure either by choosing riskier assets or by increasing leverage. Taking this well-known moral hazard problem as given, our paper explores the difficulties facing regulators in choosing capital adequacy policy for banks.

Bank regulation has changed dramatically in the last decade, becoming increasingly focused on capital requirements. This emphasis is new: while a minimum amount of capital was required for entry into the banking business, before 1981 U.S. banking regulators did not enforce specific, uniform, capital guidelines.<sup>1</sup> U.S. bank regulators began enforcing explicit capital requirements in 1982; regulators proposed risk-based requirements in 1986, and these requirements became effective in 1990; finally, with the passage of FDICIA in 1991, regulators were required to force banks to meet capital requirements. Similar trends have developed internationally. In 1988, regulators of the G10 countries plus Switzerland and Luxembourg coordinated their bank regulatory policies under the auspices of the Bank for International Settlements, focusing on capital requirements; now, banks in these twelve countries are required to have the same capital ratios. (This was the Basle Agreement -- see Committee on Banking Regulations and Supervisory Practices (1987).)

This paper analyzes the public policy implications of capital requirements. In contrast to earlier work on bank capital, we analyze a general equilibrium model in which banks perform two unique activities: information production about borrowers and liquidity production for investors. Information

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<sup>1</sup> The Federal Reserve Act of 1913 established minimum capital requirements for bank membership in the system, but did not require or enforce subsequent capital levels. The Banking Act of 1933 (and the revisions of 1935 and 1936) required regulators to consider the "adequacy of the capital structure." During the 1950s and 1960s, the Federal Reserve System calculated a type of risk-based capital standard called the "Form for Analyzing Bank Capital," or ABC Form, but no attempt was made to enforce capital requirements based on this report. See Morgan (1992) and Baer and McElravey (1992).

production about borrowers leads to the creation of an intangible asset ("charter value") which is lost if the bank fails or otherwise exits the industry. Liquidity production refers to the role of demand deposits as a transactions medium for which there are imperfect substitutes. We examine two scenarios: one in which the banking system already exists, and the regulator must decide whether to increase capital requirements, and one in which the regulator is free to design the ex ante requirements of the banking system. We also discuss how the prospect that bank risk may change influences the ex ante choice of capital requirements.

In all cases our general equilibrium framework imposes the constraint that bank capital comes from agents within the model. Moreover, explicitly modeling the unique functions of banks shows that the cost of bank capital is also unique. Finally, general equilibrium is necessary for addressing the social welfare impact of capital regulation.

The bank regulator faces a tradeoff between bank capital and deposits. Increasing bank capital reduces the risk of bank failure, which is both privately and socially desirable because charter value is saved more often. On the other hand, deposits have a liquidity advantage because their value is less informationally sensitive than other securities, such as bank capital; agents using securities other than deposits for trading face a "lemons cost." Thus, bank capital and bank deposits are not perfect substitutes, and increasing bank capital at the expense of deposits is costly.

The regulator is also constrained by the fact that, because banks are privately owned, they can always opt to leave the banking industry or simply not enter it in the first place. Thus, an enforceable capital requirement must be one which banks meet voluntarily. In deciding whether to comply with capital requirements or exit, a bank's incumbent shareholders also weigh costs of bank capital against potential loss of charter value, but the private costs may differ from the social costs. In part this is because higher capital reduces the subsidy on outstanding insured deposits, creating a variant of the well-known debt overhang problem of Myers (1977) and causing the private cost of capital to exceed the social cost. Also, to the extent bank failure involves negative externalities, social charter value may exceed private charter value. Both effects make bank shareholders less willing to raise capital than the regulator. Of course, both the regulator and the existing shareholders are constrained by the fact that new shareholders are also private agents and must voluntarily agree to buy bank shares.

Our analysis leads to some striking results. For example, even though we assume that the bank regulator maximizes social welfare, this selfless regulator may find it optimal to pursue policies that resemble "forbearance" -- i.e., the regulator does not close banks that have a low or negative net worth. Specifically, if charter value is sufficiently low, a banks will not wish to raise additional capital, as both debt overhang from existing deposits and the lemons cost of additional capital more than offset the gain from improving the odds of keeping the bank alive and capturing charter value. Since exit causes loss of charter value for certain, the regulator generally goes along with this behavior. Imposing higher ex ante capital requirements may not solve the problem either, since agents may simply not open banks at all if they find the initial costs of capital too high. In fact, in some cases the regulator might not wish to change banks' decisions even if she could: social welfare may be maximized by allowing risky banks rather than raising additional costly capital. Thus, what looks like forbearance may in fact be optimal.

As already mentioned, our work differs from existing literature in this area by our emphasis on general equilibrium, social welfare, and time consistent behavior by bank regulators. Earlier theoretical work on bank capital focuses on whether bank shareholders faced with an increase in the required capital/assets ratio increase the risk of their bank's asset portfolio; the answers to this question are conflicting and depend heavily on specific modeling assumptions.<sup>2</sup> The earlier work largely neglects special features of banks (an exception is Genotte and Pyle (1991)) and focuses on partial equilibrium, taking the cost of capital as exogenous and assuming that the capital is always forthcoming. Santomero and Watson (1977) do point out that regulators may ignore the social costs of setting capital requirements too high, but they do not analyze these costs.<sup>3</sup>

Our paper proceeds as follows. Section II discusses the unique activities of banks that we incorporate in our model. Section III outlines our modeling assumptions. Section IV analyzes optimal capital requirements and equilibrium when the banking system already exists and initial deposit and capital

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<sup>2</sup> See in particular Pringle (1974), Pringle (1975), Kahane (1977), Koehn and Santomero (1980), Lam and Chen (1985), Kim and Santomero (1988), Furlong and Keeley (1989), Keeley and Furlong (1990), and Genotte and Pyle (1991). Also related are Peltzman (1970), Sharpe (1978), and Buser, Chen and Kane (1981).

<sup>3</sup> Mailath and Mester (1993) also analyze the time consistency problem for bank regulators, but they focus on the design of optimal deposit insurance premia in a partial equilibrium setting.

levels are given exogenously. Section V analyzes optimal capital requirements and equilibrium at "the beginning of the world," when the entire banking system is starting. Section VI discusses how this initial equilibrium would be affected if agents knew that the risk of bank assets could vary stochastically in the future, leading to a role for subsequent changes in capital requirements. Section VII concludes.

## II. The Uniqueness of Banking

The argument that banks (unlike other firms) require special regulation and guarantees presumes that banks provide unique services. As already mentioned, we focus on two special features of banks: (i) they issue a unique form of debt (checkable demand deposits) as a liability, and (ii) they originate a unique form of debt (bank loans) as an asset. We now discuss what distinguishes these two financial claims from other securities, and why these unique claims produce a need for government intervention.

### A. Bank Activities on the Liability Side

We follow Gorton and Pennacchi (1990) in assuming that the uniqueness of demand deposits lies in their role as a desirable medium of exchange.<sup>4</sup> An uninformed investor with uncertain future consumption timing faces some chance that she will have to sell any securities she holds before their cash flows are realized in order to finance unexpected consumption. To the extent other investors are better informed, the uninformed investor is exposed to trading losses at this interim date. If she holds the riskless security, she will not suffer a trading loss because there cannot be any private information about this security's payoff. Thus there is a demand for privately produced riskless trading securities; banks meet this demand by issuing demand deposits.

Since Gorton and Pennacchi model a simple exchange economy with risk neutral agents, the role of banks as providers of a medium of exchange simply redistributes welfare. By extending this model to include an initial investment decision, Qi (1993) shows that banks can be welfare-improving. Fearing trading losses, uninformed investors underinvest in risky projects; by funding the projects and issuing less

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<sup>4</sup> This model contrasts with that of Diamond and Dybvig (1983), where banks' liabilities provide a vehicle for consumption-smoothing by risk averse consumers. In their model, early liquidation of real assets causes an exogenous loss; in Gorton and Pennacchi, the cost of early liquidation is entirely due to informational asymmetries.

risky demand deposits, banks alleviate this problem by reducing the information sensitivity of the uninformed investors' holdings. To the extent that privately-issued demand deposits are not riskless, government deposit insurance can further improve welfare.

To incorporate these results into our model in a simple way, we assume that uninformed investors face a discount  $1-\gamma$  ( $\gamma < 1$ ) if they try to sell risky securities at any time before the actual payout on these securities. This discount implicitly reflects the cost imposed by the existence of informed traders: those uninformed investors who buy the securities do not know if they are dealing with a liquidity-constrained uninformed investor or an informed trader, and so they impose a "lemons" discount. Since bank equity is risky, this discount makes raising additional capital costly. For simplicity, rather than model the informed traders or underinvestment costs directly, we assume that the welfare losses from trading in bank equity equal  $1-\gamma$  per dollar of bank equity traded at the interim date.<sup>5</sup>

We also assume that individual banks cannot produce completely riskless demand deposits, creating a need for government deposit insurance as in Qi (1993).<sup>6</sup> As we will see, such insurance exacerbates the potential for moral hazard by banks, which in turn motivates regulatory capital requirements.

### B. Bank Activities on the Asset Side

On the asset side, we follow Diamond (1984), Ramakrishnan and Thakor (1984), and Boyd and Prescott (1986) in assuming that banks serve a special role as delegated monitors of borrowers.<sup>7</sup> By originating and holding loans to borrowers, banks have an incentive to produce and act upon private

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<sup>5</sup> Since higher lemons discounts would reduce risky investment by the uninformed, the welfare loss from lemons effects should be an increasing function of the discount; we have simplified matters by assuming this linear form. Also, in a more complete model, the lemons discount would vary directly with the bank's leverage (riskier equity leads to higher gains to having private information).

<sup>6</sup> Though we do not model panics, it should be noted that this is an additional motivation for deposit insurance; see Diamond and Dybvig (1983). Such panics could arise in our model if deposit withdrawals followed a first-come-first-served rule.

<sup>7</sup> Of these, Boyd and Prescott's model is closest in spirit to ours in that the private information concerns the ex ante distribution of borrower returns, and the bank acts on behalf of a coalition of shareholders. Sharpe (1990), Diamond (1991), and Rajan (1992) model other variations on the special role of banks as monitors.

information where free rider problems prevent dispersed creditors from doing so. Our key assumption is that this private information is valuable and cannot be salvaged if the bank fails, so that bank failure leads to significant dissipative costs. James (1991) and Slovin, Sushka and Polonchek (1993) provide empirical evidence that these costs are significant. Effectively, private information about borrowers is an intangible asset linked to the bank's continued operation; we call this asset the bank's charter value.<sup>8</sup>

Our definition of charter value concerns the bank's private valuation of the information about borrowers that it has acquired through a lending relationship, but a bank failure may also result in losses above and beyond the loss of bank-specific information. One possibility is the commonly cited "domino effect" -- the notion that a large bank failure could cause difficulties for other financial institutions. Some evidence supporting this notion is provided by Bernanke (1983), who studies the macroeconomic effects of the U.S. bank holiday declared during the Great Depression. To the extent such effects do exist, the continued operation of a bank may have net social value in excess of the private charter value already mentioned. Bank regulators have repeatedly invoked this logic as a justification for bank rescue efforts; notable examples include Continental Illinois (U.S., 1984), Johnson Matthey Bankers (U.K., 1984), Schroder, Munchmeyer, Hengst & Co. (Germany, 1983), Banco Ambrosiano (Italy, 1982), and Al Saudi Banque (France, 1976).

For simplicity, rather than model bank information acquisition or domino effects directly, we exogenously specify values for the private and social gains to continued bank operation. If the bank is solvent at the model's horizon, these charter values are preserved; otherwise, they are dissipated. Note that in our model charter value does not emanate from restricted entry into banking; we discuss this later.

### III. The Model

We now turn to the specific assumptions and features of our model.

*Preferences and Endowments.* The economy exists over four dates, 0, 1, 2, 3. There is a continuum of risk neutral agents with total mass 1. All are endowed with a unit of the one consumption

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<sup>8</sup> Bank charters may have value to bank shareholders for other reasons as well. If entry into banking is restricted, bank charters carry monopoly rents; if deposit insurance is underpriced, charters carry a government subsidy. We discuss this in the conclusion.



good at date 0, and  $G$  more units at date 2.<sup>9</sup> All agents either consume or invest at date 0, rebalance their portfolios at date 1, and consume at date 2 or 3. As in Diamond and Dybvig (1983), agents vary in how likely they are to be forced to consume early ("suffer a liquidity shock"). At date 0, each agent knows her type, which is the probability  $p$  with which she must consume at date 2; however, the shock itself is not realized until the beginning of date 2. Types and the outcome of shocks are private information. There is a continuum of agents with total mass 1 who have each type  $p$ , and the total distribution of types is uniform over  $[0,1]$ ; thus, the mass of agents of type  $p$  who are forced to consume early is precisely  $p$ .

Given this framework, an agent of type  $p$ 's expected utility at date 0 is:

$$U(C) = C_0(p) + pC_2(p) + (1 - p)C_3(p)$$

where  $C_t(p)$  is type  $p$ 's consumption at date  $t$ .

*Investment possibilities and banks.* At date 0, the only physical investment possibilities in the economy are risky assets that mature at date 3 and cannot be liquidated earlier. Because, as noted above, research suggests that banks play a unique role in choosing and monitoring investments, we assume that agents must form coalitions ("banks") in order to obtain the specialized technology for finding and maintaining illiquid assets. For simplicity, we take bank size as given. Implicitly, we are assuming that the technology involves a fixed cost, and that agents must form measurable coalitions if they are to cover this cost. Since costs of coalition formation should increase with the measure of agents involved, tradeoffs between the cost of the technology and the cost of coalition formation should lead to an optimal bank size. We also focus on symmetric outcomes, so that all banks pursue the same equilibrium policies at date 0.

We assume that at date 3, physical assets return either  $R+\alpha$  or  $R-\alpha$  ( $\alpha > 0$ ) with equal probabilities, so that their expected return is  $R$ ; we also assume  $R \geq 1$ , and  $\alpha \leq R$ . To distinguish between bank-specific and systematic risk in a simple way, we assume that the date 3 outcome is

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<sup>9</sup> The purpose of the date 2 good endowment is to guarantee consumption for agents at date 2 and to simplify taxation to support deposit insurance. Thus, we assume that  $G$  is large enough to pay for deposit insurance and early consumption. In an overlapping generations model, these needs would be met out of additional production.

independent across different banks, but that a given bank's assets all have the same realization.

At date 0, banks finance themselves by issuing demand deposits and equity, investing the proceeds in the physical asset. The total amount invested by a bank is normalized to 1. Demand deposits are government-insured; at date 1 they can be withdrawn at face value, and at dates 2 or 3 they can be withdrawn for their face value times  $R_D$ . The initial amount of deposits issued by a bank is denoted  $D_0$ . The initial amount of equity issued is  $1-D_0$ , consisting of  $N_0$  shares at price  $P_0$  (in terms of the single good). We assume that those consumers who buy shares at date 0 make the bank's date 1 decisions, particularly the decision at date 1 as to whether to raise any required additional capital or to exit the industry. The goal of these initial shareholders is to maximize their overall expected consumption.

Since the government provides insurance for deposits, it appoints a bank regulator to oversee the banking system. In our model, this regulator maximizes social welfare by choosing bank capital requirements and forcing banks that do not meet these requirements to exit. Capital requirements are set at date 0, and may be revised at date 1 when new information about bank asset risk becomes available. Deposit insurance is supported by lump-sum taxation at date 2; the proceeds are invested risklessly until date 3, and yield zero return.<sup>10</sup> In this economy, the *aggregate* level of bank default losses is known with certainty at date 2, even though *which* individual banks have losses is not known until date 3. Thus, the government can impose the right amount of tax at date 2.

*Sequence of events.* At date 0 agents form banks, which issue claims and invest the proceeds.

At date 1, bank assets' risk level (risky or riskless) is realized. The bank regulator may announce new risk-based capital ratios based on bank portfolio risk. The banks' initial shareholders decide whether to raise the additional capital or to exit the industry, as described below. A security market opens in which bank equity may be issued by banks or traded by investors in exchange for demand deposits.

At date 2, agents receive an additional  $G$  units of the consumption good and learn whether they have experienced a liquidity shock. The security market reopens. If an early consumer owns deposits or

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<sup>10</sup> For simplicity, we assume there is a risk-free zero-return storage facility available at date 2. Since demand deposits also offer zero return at date 2, the facility's only function is to simplify taxation.

Focusing on lump-sum taxation avoids incentive effects of deposit insurance premia, etc. Imposing the tax at date 2 avoids two complications: if the tax were imposed at date 3, late consumers would have incentive to consume all their wealth early so as to avoid the tax; if the tax were imposed at dates 0 or 1, it would affect aggregate investment in the risky physical asset.

equity, she can trade these to another consumer in return for some of the date 2 good endowment. Also, we implicitly assume that at date 2, (unmodeled) traders become informed about the date 3 realization of each bank's assets. The presence of these traders means that bank equity now trades at the lemons discount  $1-\gamma$  previously discussed. The government calculates aggregate expected losses on deposits and levies a lump sum tax  $T_2$  from all agents, investing the proceeds risklessly at zero return until date 3.

Finally, at date 3, bank asset values are realized. Banks distribute the proceeds to their claimants, who consume them. Solvent banks receive any bank charter value at this point. The government makes good on any bank defaults on deposits, having already raised the necessary funds through date 2 taxation.

Figure 1 summarizes this sequence of events.

*Bank charters and exit.* At date 3, solvent banks receive the private value of the bank charter,  $C_b$ ; as explained above, this is simply a device for capturing the value of the bank's private information about investment opportunities. Unless otherwise indicated, we assume that the social value of the bank charter is the same as its private value.

At date 1 bank shareholders can decide to leave the banking industry rather than meet new capital requirements. Since the bank's portfolio is invested in illiquid assets, banks which exit remain in operation until all assets pay off and depositors can be paid. Thus, the effect of exit is that bank shareholders lose the charter value  $C_b$  regardless of whether or not all date 3 claims can be paid.

*Discussion.* Although our model effectively does not allow bank shareholders to make a portfolio choice, there is little loss of generality. As discussed in the next section, banks can shift risk onto the deposit insurer by increasing leverage. Since allowing banks to alter their asset risk while holding leverage constant would have the same qualitative effect, focusing on the leverage form of moral hazard saves considerable complexity.

The timing of the model is such that the regulator can revise capital requirements at date 1 after a bank's portfolio and initial capital are already in place, but before agents know whether they are early or late consumers and before risky bank portfolio values (i.e.,  $\pm\alpha$ ) have been realized. Since bank default leads to loss of charter value, the regulator may wish to decrease the risk of such default by increasing bank capital. On the other hand, by requiring more consumers to hold bank equity at date 1, an increased capital requirement forces more consumers to bear the lemons cost when equity is sold by early

consumers at date 2. This trade-off between the chance of bank failure and the lemons cost of capital is critical to our analysis.

We solve the model backwards. Since behavior at dates 2 and 3 is a simple function of the situation at the end of date 1, we effectively begin with the date 1 behavior of consumers, regulators, and banks, given a banking system with initial deposit and capital levels. Once we have found the date 1 equilibrium, we turn to date 0 behavior, when banks have yet to form and the regulator is faced with the problem of setting initial capital requirements.

#### **IV. Capital Requirements and Equilibrium Given an Existing Banking System**

In this section, we solve for the date 1 equilibrium behavior of consumers, banks, and the regulator, given that banks begin with a mix of deposits and equity inherited from date 0. Equilibrium requires market clearing in the bank equity market given a capital requirement such that consumers maximize their utility and the regulator maximizes social welfare.

After detailing the situation at the start of date 1 and discussing the potential moral hazard problem regulators would face if they imposed no capital requirements at all, we begin our analysis with the behavior of consumers when faced with bank equity issues. This gives us the price of new equity relative to deposits as a function of the additional capital requirement, which we use in analyzing the regulator's objectives and the banks' choice between complying with capital requirements and exiting the industry. The section ends with a brief discussion of our results.

##### **A. Risky Bank Assets and the Moral Hazard Problem**

Suppose that, at the start of date 1, agents with early consumption probability (type) less than some  $p^*$  hold only equity, while the remaining agents hold only deposits. In later sections we show that this assumption is consistent with agents' risk neutrality and the expected lemons cost of selling equity to finance early consumption. Also suppose that at date 0 the regulator made each bank raise  $N_0$  shares at a market-determined price of  $P_0$ . Since the marginal shareholder is assumed to have probability of early consumption  $p^*$ ,  $N_0 = n_0 p^*$ , where  $n_0$  is the number of shares purchased by each agent with type  $p$  less than  $p^*$ . Also, the initial level of deposits must equal  $1-p^*$ , the measure of consumers who didn't choose

to buy shares. Table 1 provides a complete list of our notation.

Recall that a bank's date 1 decisions are made by its initial shareholders. If the regulator took no further actions at date 1, these shareholders could freely decide whether to do nothing, issue additional equity to reduce deposits, or issue additional deposits to repurchase equity. In fact, Appendix A demonstrates that there is a potential for moral hazard: so long as the bank charter value is not above a critical point, the shareholders would seek to increase their leverage by swapping deposits for equity.

The proof proceeds in two steps. First, we show that, keeping its chance of default fixed, a bank always prefer the highest possible leverage. The intuition is that, since bank assets are risky, any bank shareholder faces a lemons cost if she is forced to sell her shares and consume at date 2. If an increase in leverage doesn't increase the chance of default, the bank's chance of getting its charter value is not reduced, yet some shareholders get to exchange shares, which bear a lemons cost, for deposits, which do not bear such a cost; thus, the shareholders who are most likely to consume early are strictly better off. In addition, a bank which does run some risk of default takes advantage of deposit insurance by offering risky deposits at a risk-free interest rate.

Since there are only two possible asset returns, the bank basically chooses between just being solvent all the time, or just being able to make payments when returns are  $R + \alpha$  and defaulting when returns are  $R - \alpha$ . If the bank moves from being default-free to defaulting half the time, it loses charter value  $C_B$  half the time, but it gains  $2\alpha$  in government subsidies on its deposits half the time; it also increases its deposits relative to its equity, saving lemons costs for some consumers. Thus, if a bank is to forego moral hazard, its charter value must exceed  $2\alpha$  times a mark-up related to lemons costs and the marginal shareholder's chance of being forced to consume early. The precise details may be found in Proposition A1 in Appendix A.

Thus, as in Marcus (1984), a sufficiently high charter value prevents banks from engaging in moral hazard. At the very least, the bank regulator should wish to prevent this by restricting share repurchases at date 1; the regulator may also wish to require additional capital at date 1 or set stricter capital requirements at date 0. The rest of this paper addresses these issues.

### B. The Consumer's Problem

At date 1, each consumer must decide whether to rebalance her portfolio, given that banks may be issuing more equity. At date 1, let  $n_1(p)$  be the number of shares a type  $p$  consumer purchases,  $D_1(p)$  be a type  $p$  consumer's total deposits at the end of date 1,  $P_1$  be the current price of a share of bank equity,  $N_1$  be the number of new shares issued by each bank, and  $E_1$  be the date 1 conditional expected value of date 3 bank equity. Since date 0 consumption is sunk, a consumer of type  $p$  chooses  $n_1(p)$  to maximize  $pC_2(p) + (1-p)C_3(p)$ , taking  $P_1$  as given, subject to the following constraints:

- (i)  $C_2(p) = R_D D_1(p) + \gamma[(n_0(p) + n_1(p))/(N_0 + N_1)]E_1 + G - T_2$
- (ii)  $C_3(p) = R_D D_1(p) + [(n_0(p) + n_1(p))/(N_0 + N_1)]E_1 + G - T_2$
- (iii)  $n_1(p)P_1 = D_0(p) - D_1(p)$
- (iv)  $n_1(p) \geq 0$
- (v)  $D_1(p) \geq 0$

Constraint (i) is an early consumer's budget constraint, which holds with equality since she must consume all her holdings at date 2. The consumer finances her consumption with deposits, which will have earned  $R_D$ , the selling value of her bank equity, and the endowment  $G$  that she receives at date 2, less any taxes  $T_2$ . Her share of bank equity equals the shares that she purchased at dates 0 and 1 ( $n_0(p) + n_1(p)$ ) divided by the total number of shares issued at those dates ( $N_0 + N_1$ ); its selling value is this fraction times the expected value of bank equity times  $\gamma$ , which represents the lemons cost of selling equity at date 2. Constraint (ii) is the budget constraint for a late consumer; it is the same as that for an early consumer except that, since the bank liquidates at date 3, a late consumer does not face a lemons problem and receives the full value of her share of equity.<sup>11</sup> Constraint (iii) says that purchases of new equity at date 1 must be financed out of existing deposits. Constraints (iv) and (v) are nonnegativity constraints.<sup>12</sup>

Substituting constraint (iii) into (i) and (ii), and substituting constraints (ii) and (iii) into the

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<sup>11</sup> Although the late consumer's portfolio will change at date 2 if she sells some consumption goods to an early consumer, she receives fair value for what she sells, leaving the budget constraint unaltered.

<sup>12</sup> This assumes that the capital requirement modification at date 1 is either an increase or no change.

objective function, the first order condition for a maximum is

$$-P_1 R_D + [p\gamma + (1-p)][E_1/(N_0+N_1)] + (\psi-\phi)P_1 = 0,$$

where  $\psi$  and  $\phi$  are the Lagrange multipliers associated with constraints (iv) and (v), respectively. Let  $p'$  be the type of consumer for which  $\psi = \phi = 0$ . Then the first order condition becomes:

$$P_1(N_0+N_1) = E_1[1-(1-\gamma)p']/R_D \quad (1)$$

Consumers with  $p > p'$  hold no equity ( $n_1(p)=0$ ), so  $D_1(p) = D_0(p) = 1$ . Consumers with  $p \in (p^*, p')$  sell all their deposits for equity, so  $D_1(p)=0$  and  $n_1(p)P_1 = D_0(p) = 1$ . Consumers with  $p < p^*$  continue to hold their original shares  $n_0(p)$  and do not hold any deposits.

Equation (1) says that, for the marginal shareholder (type  $p'$ ), the market price of the bank stock ( $P_1$  times total number of shares,  $N_1 + N_0$ ) equals the discounted expected value of the bank equity ( $E_1/R_D$ ) reduced by a factor reflecting the marginal consumer's expected lemons cost  $(1-(1-\gamma)p')$ . Because equity is risky and faces this lemons cost, the effective cost of bank capital is higher than the return on deposits.

### C. The Regulator's Objective Function

The regulator's problem is to choose date 1 bank capital requirements so as to maximize expected social welfare, taking into account the pricing of new equity issues and the fact that banks may choose to exit rather than comply. If the regulator requires an increase in capital (decrease in deposits) of  $\Delta D$ , the bank must issue  $N_1$  new shares at price  $P_1$  so that  $N_1 P_1 = \Delta D$ . Since the regulator chooses capital requirements for all banks simultaneously, she knows that the price  $P_1$  reflects the marginal consumer's probability  $p'$  of being forced to sell shares early at a discount, as just discussed. Assuming that the regulator does not permit a decrease in capital, we have  $p' \geq p^*$  and  $N_1 \geq 0$ .<sup>13</sup> Social welfare is

$$W = \int_0^1 [pC_2(p) + (1-p)C_3(p)] dp$$

<sup>13</sup> The regulator can prevent decreases by refusing to insure any deposits issued to repurchase equity.

Using our results on consumers' optimal portfolio choices by type, this becomes

$$W = \int_0^{p'} \left[ \frac{[1-p(1-\gamma)] \cdot E_1(\cdot)}{P_0(N_0+N_1)} \right] dp + \int_{p'}^{p''} \left[ \frac{[1-p(1-\gamma)] \cdot E_1(\cdot)}{P_1(N_0+N_1)} \right] dp + R_D(1-p') + G - T_2(\cdot)$$

The first, second, and third terms are the expected consumption value of the investments of those who buy bank equity at date 0, those who buy bank equity at date 1, and those who hold deposits through date 1, respectively. The fourth term is the total consumption obtained from the date 2 endowment and the fifth term is the date 2 tax needed to cover losses on government deposit insurance. Both the expected value of the bank's equity ( $E_1$ ) and the date 2 tax ( $T_2$ ) depend on the level of deposits at the end of date 1, the risk of the banks' assets, and whether banks comply with new capital requirements.

Let  $\tilde{I}_C$  be an indicator variable that equals 1 if the bank complies with capital requirements and 0 otherwise. Since bank assets are risky,

$$\begin{aligned} E_1(\cdot) &= \begin{cases} R - R_D(D_0 - \tilde{I}_C \cdot \Delta D) + \tilde{I}_C \cdot C_B, & \text{if } R - \alpha \geq R_D(D_0 - \tilde{I}_C \cdot \Delta D), \\ \frac{1}{2}[(R + \alpha) - R_D(D_0 - \tilde{I}_C \cdot \Delta D) + \tilde{I}_C \cdot C_B], & \text{if } R + \alpha \geq R_D(D_0 - \tilde{I}_C \cdot \Delta D) > R - \alpha, \\ 0 & \text{otherwise.} \end{cases} \\ T_2(\cdot) &= \begin{cases} 0, & \text{if } R - \alpha \geq R_D(D_0 - \tilde{I}_C \cdot \Delta D), \\ \frac{1}{2}[R_D(D_0 - \tilde{I}_C \cdot \Delta D) - (R - \alpha)], & \text{if } R + \alpha \geq R_D(D_0 - \tilde{I}_C \cdot \Delta D) > R - \alpha, \text{ and} \\ R_D(D_0 - \tilde{I}_C \cdot \Delta D) - R & \text{otherwise.} \end{cases} \end{aligned} \quad (2)$$

It is easy to show that  $E_1(\cdot) - T_2(\cdot) = R - R_D(D_0 - \tilde{I}_C \cdot \Delta D)$  plus either  $\tilde{I}_C \cdot C_B$ ,  $\frac{1}{2}\tilde{I}_C \cdot C_B$ , or 0. This leads to the following result (from here on, all proofs are given in Appendix B unless otherwise noted).

**Lemma 1.** *Even if banks would willingly comply with a higher capital requirement ( $l_C = 1$ ), the regulator only increases capital (decreases deposits) if this change reduces a bank's chance of default.*

The intuition is straightforward. Increasing capital requirements means that more equity is held; this increases total lemons costs, decreasing aggregate welfare. Although increased capital requirements reduce the size of losses when banks default, in the aggregate this is a wash, since deposit insurance is simply a transfer from all agents to depositors of failed banks. The only net gain to increasing capital occurs when the chance of bank failure is reduced, in which case bank charter value is preserved more often.



Lemma 1 simplifies the regulator's problem considerably: since she only chooses capital increases that just move banks to a lower default probability, there are three cases she must consider.

$R + \alpha - R_D \cdot D_0 < 0$ . The bank always defaults at date 3 unless additional capital is raised. The regulator may require either  $N_1 \cdot P_1 = \Delta D^L \equiv [R_D \cdot D_0 - (R + \alpha)] / R_D$  or  $N_1 \cdot P_1 = \Delta D^H \equiv [R_D \cdot D_0 - (R - \alpha)] / R_D$ .

$R - \alpha - R_D \cdot D_0 < 0 \leq R + \alpha - R_D \cdot D_0$ . The bank only defaults if date 3 returns are low ( $R - \alpha$ ). The regulator may require additional capital in the amount  $\Delta D^H$ .

$R - \alpha - R_D \cdot D_0 \geq 0$ . The bank never defaults at date 3. There is no need for increased capital.

Market clearing requires that  $\Delta D$  equal the measure of consumers who switch from deposits to equity; i.e.,  $D_0 - D_1 = p' - p^*$ . This implies that, if  $\Delta D = \Delta D^L$ ,  $p' = p^L \equiv (R_D - R - \alpha) / R_D$ , while if  $\Delta D = \Delta D^H$ ,  $p' = p^H \equiv (R_D - R + \alpha) / R_D$ . Note that  $p^H > p^L$ . With these results in hand, we can now state when the regulator wants to require more bank capital. (In the following results, the first argument in  $C_{reg}(\cdot)$  refers to target capital level, while the second refers to the initial chance of default.)

**Proposition 1:** *Suppose that bank assets are risky, and banks comply with any capital requirement the regulator chooses. (i) If banks would always default without additional capital ( $R_D \cdot D_0 > R + \alpha$ ), define*

$$C_{reg}(L, 1) \equiv \left[ 1 - \frac{p^*(1-\gamma)}{2} \right]^{-1} \cdot \left[ \frac{p^L(1-\gamma)}{1-p^L(1-\gamma)} \cdot R_D \Delta D^L \right] \quad \text{and}$$

$$C_{reg}(H, 1) \equiv \left[ 1 - \frac{p^*(1-\gamma)}{2} \right]^{-1} \cdot \left[ \frac{p^H(1-\gamma)}{1-p^H(1-\gamma)} \cdot \frac{R_D \Delta D^H}{2} + \frac{p^*(1-\gamma)}{2} \alpha \right]$$

The regulator wants additional capital  $\Delta D^L$  only if  $C_B \geq C_{reg}(L, 1)$ ; she wants additional capital  $\Delta D^H$  only if  $C_B \geq C_{reg}(H, 1)$ . If both conditions are met, she chooses  $\Delta D^H$  over  $\Delta D^L$  if  $C_B \geq 2C_{reg}(H, 1) - C_{reg}(L, 1)$ , and vice versa.

(ii) If banks would sometimes default without additional capital ( $R + \alpha \geq R_D \cdot D_0 > R - \alpha$ ), then the regulator wants to increase bank capital by  $\Delta D^H = [R_D \cdot D_0 - R + \alpha] / R_D$  if and only if

$$C_B \geq C_{reg}(H, 1/2) \equiv \left[ 1 - \frac{p^*(1-\gamma)}{2} \right]^{-1} \cdot \left[ \frac{p^H(1-\gamma)}{1-p^H(1-\gamma)} + \frac{p^*(1-\gamma)}{2} \right] \cdot R_D \Delta D^H.$$

Thus, the regulator only wants additional capital if bank charter value is sufficiently high, so that the reduced chance of defaulting and losing the charter value offsets the increased lemons costs that new bank shareholders must bear. The precise bound on the charter value depends on the risk of default and the lemons cost of the future value of additional capital (the terms in  $R_D \cdot \Delta D$ ), modified by the fact that existing shareholders will pay a lemons cost on any additional charter value (the terms in  $p^*(1-\gamma)$ ). It is easy to show that, when there are no lemons costs ( $\gamma = 1$ ), the lower bounds on charter value equal zero.

In part (i) of the proposition, the bank would default for certain without additional capital, and the regulator can choose between two possible capital levels. It is possible that the regulator might find low additional capital ( $\Delta D^L$ ) attractive but not high additional capital ( $\Delta D^H$ ), or vice versa. This is because the expected costs of losing charter value are linear in the chance of default, while the lemons costs of bank capital depend on the expected value of equity, which is not linear in default probability. For example, if the bank would barely default for good returns, little additional equity is needed to secure charter value some of the time, but a great deal is needed to secure charter value for certain. In part (ii) of the proposition, the bank would default half the time without additional capital, and the regulator only needs to consider one possible capital level.

Proposition 1 describes what the regulator would do if left to her own devices, but she must also consider whether or not banks will comply with the requirement that she sets. It is easy to show that the regulator never chooses a capital requirement that banks would *not* comply with:

*Proposition 2: It is never optimal for the regulator to require capital such that banks choose to exit.*

The proof is as follows. Lemma 1 showed that increasing capital requirements is a wash as far as deposit insurance is concerned; the only gain is a higher chance of preserving bank charter value, and the only loss is the increased lemons costs from increased holdings of bank equity. If the regulator chooses a capital requirement that banks refuse to meet, then banks exit, and bank charter value is lost for sure. Thus, forcing banks to exit decreases date 1 expected social welfare.

Note that Proposition 2 would not change even if the regulator attached a higher charter value to bank operations than did private agents, e.g., if bank failure entailed important externalities; the

regulator would still prefer to preserve charter value some of the time rather than force exit for sure. Since the bank shareholders make the compliance decision as private agents, private charter value determines the feasible set of capital requirements. This means that we must examine the bank compliance decision in order to fully solve for the regulator's capital requirement.

#### D. The Bank's Compliance Decision

Since the regulator never wishes to force banks to exit, it follows that the regulator's threat to revoke the charters of banks that don't comply is not credible; banks know that they will be allowed to continue operating even if they *don't* raise additional capital. This alters the value of bank equity slightly:

$$E_1(\cdot) = \begin{cases} R - R_D(D_0 - \bar{I}_C \cdot \Delta D) + C_B, & \text{if } R - \alpha \geq R_D(D_0 - \bar{I}_C \cdot \Delta D), \\ \frac{1}{2}[(R + \alpha) - R_D(D_0 - \bar{I}_C \cdot \Delta D) + C_B], & \text{if } R + \alpha \geq R_D(D_0 - \bar{I}_C \cdot \Delta D) > R - \alpha, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Thus, if the regulator requires additional capital  $N_1 \cdot P_1 = \Delta D > 0$ , a bank's current shareholders choose between the status quo (no additional capital) and issuing additional equity at the price  $P_1$ ; since all current shareholders are fully invested in bank stock at the start of date 1, they will choose the option that maximizes the expected consumption value of their equity. If they comply, their equity's expected consumption value is

$$S(\bar{I}_C - 1) = \int_0^{p^*} [p\gamma + (1-p)] \left[ \frac{n_0(p) \cdot E_1(\bar{I}_C - 1)}{N_0 + N_1} \right] dp = \left[ p^* - \frac{p^{*2}}{2}(1-\gamma) \right] \left[ \frac{E_1(\bar{I}_C - 1)}{P_0(N_0 + N_1)} \right].$$

where  $E_1(\bar{I}_C = 1)$  is given in equation (3), and we have made use of the fact that, since the initial shareholders invest all their date 0 endowment in bank stock,  $n_0(p)P_0 = 1$  for  $p \leq p^*$ .

Similarly, if shareholders don't comply, their equity's expected consumption value is

$$S(\bar{I}_C - 0) = \int_0^{p^*} [p\gamma + (1-p)] \left[ \frac{n_0(p) \cdot E_1(\bar{I}_C - 0)}{N_0} \right] dp = \left[ p^* - \frac{p^{*2}}{2}(1-\gamma) \right] \left[ \frac{E_1(\bar{I}_C - 0)}{P_0 N_0} \right].$$

Comparing these last two equations, it follows that bank shareholders comply with the capital requirement if and only if

$$E_1(\bar{I}_c - 1) \left[ \frac{N_0}{N_0 + N_1} \right] \geq E_1(\bar{I}_c - 0). \quad (4)$$

Recall that, in equilibrium, if all banks raise capital,  $P_1$  satisfies equation (1), where the superscript indicates risky bank assets. Using this and equation (3), it is easy to derive the following conditions for whether or not bank shareholders comply with any given capital requirement:

**Proposition 3:** *Suppose bank assets are risky. (i) If banks would default for certain without additional capital ( $R_D \cdot D_0 > R + \alpha$ ), and the capital requirement is  $\Delta D^L$ , banks only comply if*

$$C_B \geq \frac{2R_D \Delta D^L}{1 - p^L(1 - \gamma)} \equiv C_{\text{bank}}(L, 1)$$

*If the capital requirement is  $\Delta D^H$ , banks only comply if*

$$C_B \geq \frac{R_D \Delta D^H}{1 - p^H(1 - \gamma)} - \alpha \equiv C_{\text{bank}}(H, 1)$$

*If both conditions are met, banks raise  $\Delta D^H$  instead of  $\Delta D^L$  if and only if  $C_B \geq 2C_{\text{bank}}(H, 1) - C_{\text{bank}}(L, 1)$ .*

*(ii) If  $R + \alpha \geq R_D \cdot D_0 > R + \alpha$ , and the capital requirement is  $\Delta D^H$ , banks only comply if*

$$C_B \geq R_D \Delta D^H \cdot \left[ \frac{1 + p^H(1 - \gamma)}{1 - p^H(1 - \gamma)} \right] \equiv C_{\text{bank}}(H, 1/2)$$

*(iii) For any capital requirement and initial chance of default,  $C_{\text{reg}}(\cdot) < C_{\text{bank}}(\cdot)$ . Also,  $2C_{\text{bank}}(H, 1) - C_{\text{bank}}(L, 1) > 2C_{\text{reg}}(H, 1) - C_{\text{reg}}(L, 1)$ .*

Not surprisingly, a bank with risky assets only complies with increased capital standards if its charter value is sufficiently high relative to the amount of extra capital that absorbs losses in default, modified for the lemons costs of attracting additional shareholders. This lower bound on charter value is higher than that required by the regulator because, in addition to increasing lemons costs, raising

additional capital causes a transfer from bank shareholders (old and new) to taxpayers: additional capital reduces losses on insured deposits that would otherwise have been covered through future taxes. In fact, if there were no lemons costs ( $\gamma = 1$ ), each lower bound in the proposition is exactly equal to the reduction in expected losses on deposits caused by the additional capital.

In case (i), where the bank would default for certain without additional capital, the bank's current shareholders are indifferent -- their equity is worthless unless more capital is raised. The requirements on charter value come down to the feasibility of getting new investors to buy additional bank stock. In case (ii), where the bank would only default half the time without additional capital, the bank's current shareholders' position is worth something, and they may not wish to pay the lemons cost needed to attract more capital.

*Corollary 1* *The regulator's constrained optimal capital choice equals the bank shareholders' choice as outlined in Proposition 3.*

This follows immediately from Proposition 3(iii): whenever bank shareholders want a given capital level, the regulator wants *at least* that much capital. This result would not necessarily hold if the regulator's threat to close non-complying banks was credible, since failing to comply would cost the bank shareholders more in terms of foregone charter value.

*Corollary 2 (Comparative Statics of the Banks' Compliance Decision).* (i) *Banks are less willing to raise capital (or to choose a higher capital requirement over a lower one) as the deposit rate  $R_D$ , the initial deposit level  $D_0$ , or the lemons cost of trading equity  $1-\gamma$  increase.* (ii) *They are more willing to raise capital if the mean return  $R$  on bank assets increases.* (iii) *If the risk of bank assets increases, banks are more willing to raise a low amount of capital ( $\Delta D^L$ ) when facing certain default, but they are less willing to raise enough capital to make themselves default-free.*

These results follow from the comparative statics of the various cut-off levels for charter value given in Proposition 3. Increases in the deposit rate or initial level of deposits force banks to raise more

capital to reduce default risk to any given level, which is more expensive because of higher debt overhang and a higher probability of early consumption (and thus higher expected lemons costs) for the marginal shareholder. Increases in the lemons cost of bank capital obviously make capital more costly to raise. On the other hand, a higher mean return on bank assets reduces the amount by which banks default, reducing the amount of capital that needs to be raised and thus reducing both debt overhang and the marginal shareholder's expected lemons costs.

Increasing bank asset risk tends to increase the amount of capital banks must raise in order to reduce default risk to any given level, except in the case where banks are certain to default without capital and are striving to reduce the default chance to one half. This corresponds to the more general result that, for any symmetric distribution of bank returns, a symmetric increase in the distribution's risk increases the chance of default for deposit rates below the mean return, and decreases the chance of default for deposit rates above the mean. Thus, the result that increasing asset risk  $\alpha$  may make banks more willing to raise low amounts of capital only applies to banks with an extremely high chance of default.

#### E. Discussion

Because bank equity involves a lemons cost, it is more expensive than deposits. Whether bank shareholders view raising capital as a positive NPV project depends on the tradeoff between this lemons cost and the portion of new equity that absorbs losses on deposits on the one hand, and the increased chance of retaining charter value, on the other hand. Although the regulator only cares about the tradeoff between increased lemons costs and preservation of charter value, she is still constrained by the bank shareholders' preferences: she does not want to force banks to exit at date 1, because this results in the loss of charter value for certain. In fact, this completely destroys the credibility of her threat to close banks that do not comply with increased capital requirements. Thus, even banks with a high risk of failure may be allowed to continue.

Our results have a somewhat negative cast: acting to maximize social welfare, the regulator is more likely to want a safe banking system than are the bank shareholders, but she cannot implement her wishes through capital requirements. Also, our analysis does not distinguish between private and social charter values. To the extent that bank failure involves significant negative externalities, the social charter

value of bank continuation may be greater than the private value, widening the gap between what the regulator wants and what she can accomplish. We return to this in the conclusion.

Nevertheless, even if the regulator could credibly threaten to close risky banks that did not raise additional capital, she might not choose to do so. If lemons costs are sufficiently high relative to charter value, the reduction in lemons costs from high deposit volumes might well outweigh the increased chance of defaulting and losing charter value. In this case, it seems somewhat misleading to refer to the regulator's decision not to increase capital requirements as "forbearance." It is true that the regulator is not closing risky banks or forcing them to raise capital, but in this case, letting risky banks continue maximizes social welfare.

## V. Capital Requirements and Equilibrium When the Banking System is Being Formed

By date 1 banks' funding mix is fixed, and, as we have seen, it may be difficult for the regulator to get private agents to agree to increase bank capital. We now examine whether the regulator can overcome the problem of being unable to raise capital at date 1 by enforcing high capital standards at date 0, *before* banks have outstanding capital and deposit levels.

At date 0 the regulator must choose an initial capital requirement and consumers must choose amounts of debt and equity to hold. An equilibrium at date 0 is a capital requirement  $N_0 \cdot P_0$  together with a price  $P_0$ , a marginal shareholder  $p^*$ , and a deposit rate  $R_D$  such that: (i) consumers maximize utility, (ii) the regulator maximizes social welfare, and (iii) markets clear ( $D_0 \leq 1 - N_0 \cdot P_0$ ). Since potential shareholders can freely decide not to form a bank if they feel capital requirements are too onerous, once again the regulator's choices may be constrained.

### A. The Consumer's Problem

At date 0, a consumer of type  $p$  chooses  $n_0(p)$  to maximize  $C_0(p) + pC_2(p) + (1-p)C_3(p)$  subject to the following constraints:

$$(i) \quad C_2(p) = R_D D_1(p) + \gamma[(n_0(p) + n_1(p))/(N_0 + N_1)]E_1 + G - T_2,$$

$$(ii) \quad C_3(p) = R_D D_1(p) + [(n_0(p) + n_1(p))/(N_0 + N_1)]E_1 + G - T_2,$$

- (iii)  $n_0(p)P_0 + D_0(p) + C_0(p) = 1$ ,  
 (iv)  $n_0(p) \geq 0$ ,                      (v)  $D_0(p) \geq 0$ ,                      (vi)  $C_0(p) \geq 0$ ,  
 (vii)  $n_1(p)$  and  $D_1(p)$  are chosen optimally at date 1, given  $n_0(p)$ ,  $D_0(p)$ , and  $P_1$ .

Constraints (i)-(vi) are analogous to the constraints given in Section IV, with the addition of the possibility that agents choose to consume their endowment at date 0 rather than invest. Constraint (vii) says that, since agents are rational, they take into account how they will behave at date 1.

Analysis is simplified considerably by the following result:

*Lemma 2: Suppose that banks raise capital  $N_0 \cdot P_0$  at date 0, and that, based on this capital level and the resulting level of deposits  $D_0 = 1 - N_0 \cdot P_0$ , additional capital requirements at date 1 are  $N_1 \cdot P_1 = \Delta D$ . Then: (i) the equilibrium date 0 price of a share of equity,  $P_0$ , equals  $P_1$ ; (ii) if  $p'$  is the date 1 marginal shareholder's probability of early consumption, then at date 0 any consumer with  $p$  less than  $p'$  is willing to buy shares at  $P_0$ .*

The proof is as follows. At date 0, no one will pay more than  $P_1$  for shares -- by waiting, they can always buy them at date 1 at price  $P_1$ . Furthermore, at this price, a consumer with probability of early consumption  $p'$  is just willing to buy shares at date 0, since  $P_1$  will be her marginal value of shares at date 1. It follows that any consumer with  $p$  less than  $p'$  is strictly willing to buy shares at this price, so the price need not be lower to attract shareholders. Thus, the equilibrium price  $P_0$  equals the expected date 1 equilibrium price  $P_1$ .<sup>14</sup>

Since consumers with  $p < p'$  are indifferent between buying shares at date 0 or date 1, for simplicity we assume that only those with the lowest probability  $p$  of early consumption buy shares at date 0. This implies that the "marginal" shareholder at date 0 has  $p$  equal to  $p^*$ , where  $p^* = N_0 \cdot P_0$ .

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<sup>14</sup> The same logic would also hold if the regulator allowed a capital *decrease* at date 1 when bank assets were risky. Although the marginal shareholder at date 0 would have  $p = p^* > p'$  (the marginal shareholder at date 1), at date 1 she could sell her shares at a price reflecting  $p'$ . Thus, at date 0 she would be willing to pay a price that reflected the *eventual* marginal shareholder's probability  $p'$ .



### B. The Regulator's Date 0 Choice of Initial Capital and Deposit Levels

The regulator's problem is to choose an initial capital requirement so as to maximize expected welfare, where expected welfare equals  $W$  as given in the previous section (unless autarky prevails). Notice that, through market clearing, choosing an initial capital requirement  $N_0 \cdot P_0$  also sets  $D_0 = 1 - N_0 \cdot P_0 = 1 - p^*$  and (as we will see) the equilibrium deposit rate  $R_D$ . Since the regulator's decision on date 1 capital requirements depends only on  $R_D$ ,  $D_0$ , and the parameters  $R$ ,  $C_B$ ,  $\alpha$ , and  $\gamma$ , the initial capital requirement also fully determines behavior at date 1.

From our previous analysis, we know that  $D_0$  and  $R_D$  lead to three general regions, corresponding to whether  $R_D \cdot D_0$  is less than  $R - \alpha$ , between  $R - \alpha$  and  $R + \alpha$ , or greater than  $R + \alpha$ . In the first region, there are no additional capital requirements at date 1; in the second region, the additional capital requirement is either  $\Delta D^H$  or zero; in the third region, the additional capital requirement is either  $\Delta D^H$ ,  $\Delta D^L$ , or zero. Thus, there might be as many as six possible banking equilibria, plus another (autarky) where no banks form and all consumers simply consume their date 0 endowment. In fact, as the next proposition shows, effectively there are only three distinct equilibria in which banks form.

**Proposition 4 (Possible Equilibria).** *The following equilibrium outcomes are possible.*

(i) *Equilibrium ND (banks never default): the equilibrium deposit rate  $R_D$  equals*

$$R_D^{ND} = \frac{1}{2} \cdot [(\alpha + C_B)\gamma + R - \alpha] + \frac{1}{2} \cdot \sqrt{[(\alpha + C_B)\gamma + R - \alpha]^2 + 4(\alpha + C_B)(R - \alpha)(1 - \gamma)}.$$

*The date 1 deposit level  $D_1$  is  $(R - \alpha)/R_D^{ND}$ ; the regulator is indifferent between setting  $D_0$  at this level, or setting deposit levels somewhat higher and then requiring additional capital at date 1. Social welfare is*

$$W^{ND} = R + C_B + G - \frac{p^H(1 - \gamma)}{2}(\alpha + C_B).$$

*This equilibrium is feasible whenever  $R_D^{ND} \geq 1$ .*

(ii) *Equilibrium SD (banks sometimes default): banks default when asset returns are poor ( $R - \alpha$ ).*

*The equilibrium deposit rate equals*

*The date 1 deposit level  $D_1$  is  $(R + \alpha)/R_D^{SD}$ ; as in (i), the regulator is indifferent as to how this is achieved.*

$$R_D^{SD} = \frac{1}{2} \cdot \left[ \frac{C_B \gamma}{2} + R + \alpha \right] + \frac{1}{2} \cdot \sqrt{\left[ \frac{C_B \gamma}{2} + R + \alpha \right]^2 + 2C_B(R + \alpha)(1 - \gamma)} .$$

*Social welfare is*

$$W^{ND} = R + \frac{C_B}{2} + G - \frac{p^L(1 - \gamma)}{2} \cdot \frac{C_B}{2} .$$

*This equilibrium is always feasible.*

(iii) *Equilibrium AD (banks always default). The equilibrium deposit rate is  $R_D^{AD} > R + \alpha$ , and the initial deposit level  $D_0$  equals 1. Social welfare  $W^{AD}$  is  $R + G$ . This equilibrium is always feasible.*

(iv) *Autarky (no banks form). Social welfare  $W^*$  is  $1 + G$ . This equilibrium is always feasible.*

The intuition is as follows. From Lemma 2, we know that the price of bank equity does not change between dates 0 and 1. As a result, bank capital can be raised on the same terms at both dates, and so the regulator is indifferent as to how the final capital level is achieved (of course, she must select an initial level from which the target level is in fact optimal at date 1). From Lemma 1, we know that the regulator only targets the minimum capital level for any given chance of bank default; since each equilibrium corresponds to one of the three possible levels of bank default, this leads to the three target deposit levels  $(R - \alpha)/R_D$ ,  $(R + \alpha)/R_D$ , and 1. Market clearing then determines the equilibrium deposit rate  $R_D$ , and social welfare can be found by straight substitution. Feasibility follows from the fact that any consumer has a reservation return of one from consuming her endowment, whereas the return to entering the banking system is at least  $R_D$  (and more for inframarginal bank shareholders).<sup>15</sup>

Notice that the regulator's ability to choose the risk of the banking sector is far less restricted at date 0, when banks have yet to form, than at date 1, when banks are already in operation. At date 0, consumers that don't take part in the banking system can only fall back on autarky, whereas date 1 bank

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<sup>15</sup> Although the deposit rate in Equilibria SD and AD is partly subsidized by taxes, all consumers are taxed, whether or not they hold deposits. Thus a consumer can compare the return to consuming at date 0 with the return on deposits or shares, ignoring the effect of taxes.

shareholders' equity may have greater than autarkic value even if no additional capital is raised. Furthermore, at date 0, there is no debt overhang, deposits not yet having been raised. Both factors move the private cost of bank capital closer to the social cost, relaxing the constraints on the regulator.<sup>16</sup>

*Corollary 3 (Comparative Statics of Equilibrium Deposit Rates).* (i)  $R_D^{ND}$  is decreasing in bank asset risk  $\alpha$  and lemons costs  $1-\gamma$ , and increasing in bank assets' mean return  $R$  and bank charter value  $C_B$ . (ii)  $R_D^{SD}$  is decreasing in lemons costs and increasing in bank asset risk, bank assets' mean return, and bank charter value.

Intuitively, increases in lemons costs make bank equity less attractive, so banks can offer a lower return on deposits and still attract depositors. Conversely, increases in bank asset return or charter value make bank equity more attractive, so the deposit rate must be higher to attract depositors. In contrast to these results, which do not depend on the type of equilibrium, the effect of increases in bank asset risk do vary with the type of equilibrium. In a system where banks never default, higher asset risk reduces the volume of deposits that can be offered, decreasing the deposit rate; the opposite is true in a system where banks sometimes default.<sup>17</sup> Combined with Proposition 4 (i), these results imply that default-free banking is less likely to be feasible when asset risk or lemons costs increase, while the opposite is true when mean asset returns or charter value increases.

The regulator's choice of equilibrium follows by comparing social welfare for each outcome:

*Corollary 4 (Choice of Banking System).* (i) The regulator never chooses autarky or a system where banks always fail (AD) -- a system with some risk of bank default (SD) strictly dominates both. (ii) When a system where banks never default (ND) is infeasible, the regulator chooses the system where banks sometimes default (SD). Otherwise, she chooses ND if and only if

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<sup>16</sup> We are implicitly assuming that the regulator's required capital choice serves as a coordination device. Otherwise, autarky is always possible: to be viable, banks require a measurable coalition of consumers, so if everyone believes everyone else will consume their own endowment, autarky dominates.

<sup>17</sup> As in Corollary 2 (iii), in the latter case the deposit rate is above the banks' mean asset return, so an increase in risk reduces the amount of capital required to reduce default risk by a small amount.

$$C_B + \left(1 - \frac{R + \alpha}{R_D^{SD}}\right) \left(\frac{1 - \gamma}{2}\right) C_B - \left(1 - \frac{R - \alpha}{R_D^{ND}}\right) (1 - \gamma)(\alpha + C_B) > 0.$$

In autarky or a system where banks always fail, charter value is never realized. By contrast, even though the system where banks sometimes fail involves some bank capital and thus positive lemons costs, bank equity is positive and equal to the expected charter value. Since lemons costs are a fraction of equity value, the net gain to having charter value realized some of the time rather than none of the time is always positive. This accounts for part (i) of the corollary. Part (ii) is more ambiguous. If charter value is very high and bank asset risk very low, the condition is met: the amount of capital needed to remove any risk of bank default is small, the charter value gained is high, and it pays to have a default-free banking system. When charter value is low and bank asset risk is high, default-free banking is too expensive relative to the gain in charter value. Similarly, when lemons costs are low ( $\gamma$  is close to one), additional capital is cheap, and it pays to have a default-free banking system.

Note that, since the marginal shareholder is indifferent between shares and deposits, and all depositors have the same expected consumption, inframarginal shareholders are strictly better off than depositors in equilibrium. As a result, even if overall welfare is higher in a default-free banking equilibrium, high- $p$  consumers might prefer the equilibrium where banks sometimes default. In the second equilibrium, deposit rates (net of taxes to support deposit insurance) might be higher, leaving high- $p$  consumers (who hold deposits in both equilibria) better off than in the default-free equilibrium. Of course, their gain is more than offset by the bank shareholders' loss.

## VI. Bank Capital Requirements and Equilibrium When Bank Asset Risk Can Change

The results of the previous section suggest that, when regulators are free to set initial requirements for a banking system, many of the constraints they might face if confronted with an arbitrary existing banking system can be avoided. Nevertheless, our analysis assumed that the underlying distribution of bank asset returns does not change between the date at which the banking system is formed and the date at which regulators decide whether to require additional capital. In this section, we sketch

the how our results would be affected if bank asset's return distribution evolved stochastically between these two dates.

Suppose that the mean return  $R$  on bank assets is itself random, and can be either  $R_1$  or  $R_2 < R_1$ ; furthermore, the actual realization of  $R$  becomes known at date 1 but not before. Then the regulator must choose initial bank capital levels without knowing what  $R$  will be, but she can require additional capital at date 1 if need be. It is clear that, once the initial capital level is chosen, date 1 equilibrium will be determined as in Section IV of this paper; thus, as before, the date 0 capital choice completely determines behavior at date 1.

Since the amount of additional bank capital required to achieve any given chance of default varies with bank assets' mean return  $R$  (see the discussion following Corollary 2), the marginal shareholder at date 1 and the price of bank equity at date 1 are both likely to vary with  $R$ 's date 1 realization. This has two important complicating effects on analysis. First, it may be the case that consumers with relatively low probabilities of early consumption prefer to wait until date 1 before they purchase shares.<sup>18</sup> Since many eventual shareholders would wish to defer purchase, this effect constrains the amount of initial bank capital that the regulator can feasibly require.<sup>19</sup> Second, since the price of bank equity will vary between date 0 and date 1, the regulator will no longer be indifferent to the way in which a given target level of date 1 bank capital is achieved: in some cases, she may prefer to raise all capital initially, while in others, she may wish to raise some capital initially and more at date 1. Finally, the number of equilibria will increase, both because of the complication just noted and because the greater number of asset return

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<sup>18</sup> To see why, note that these consumers are inframarginal shareholders at the end of date 1: they get higher value out of their shares than the marginal shareholder, yet it is the marginal shareholder's early consumption probability  $p$  that is reflected in the date 1 price of bank equity. Thus the relative advantage of being an inframarginal shareholder at date 1 also varies with  $R$ .

One can show that, if the date 0 price of bank equity  $P_0$  makes low- $p$  consumers (inframarginal shareholders) indifferent between buying shares at date 0 and buying shares at date 1, it is unlikely to equal the expected date 1 price  $E_R[P_1]$ . However, if the date 0 price of bank shares  $P_0$  was less than the expected date 1 price  $E_R[P_1]$ , high- $p$  consumers who didn't plan to hold shares at the end of date 1 would have incentive to buy shares at date 0 for resale at date 1. This would put upward pressure on the date 0 price, and so low- $p$  consumers would strictly prefer to wait until date 1 to buy shares.

<sup>19</sup> It also means that, at the start of date 1, shareholders may be "high- $p$ " consumers who plan to sell shares by the end of date 1 to "low- $p$ " consumers. The bank's objective function would now be to maximize the date 1 share price, which equals the welfare of the (eventual) marginal shareholder.

realizations increases the number of possible bank default states.

Despite these complications, some general implications are fairly clear. Given any initial capital level, banks' chance of default is higher when the date 1 realization of mean asset return is low ( $R_1$ ) than when it is high ( $R_2$ ). It also follows that the amount of additional capital required at date 1 to reduce bank default risk to any given level is higher when the realization of  $R$  is low. This means that both debt overhang and marginal shareholder's chance of bearing lemons costs will both be very high when  $R$  is low, making capital difficult to raise. In addition, although we have assumed that the lemons discount factor  $\gamma$  on selling shares at date 2 is constant,  $\gamma$  is likely to be lower when bank assets returns are low: the risk of bank returns ( $\pm\alpha$ ) is a higher proportion of mean return, increasing the relative advantage of informed traders. This would further increase the relative cost of capital at date 1 when  $R$  is low.

One could argue that the regulator should anticipate this by requiring higher capital at date 0. Nevertheless, if the ex ante chance of high  $R$  is much greater than the chance of low  $R$ , then, since capital is expensive to raise, the regulator is likely to gear initial capital requirements to reflect the situation when  $R$  is high, and less bank capital is needed. This means that, if bank assets prove to have low average yields -- e.g., in a recession -- banks will face high default probabilities, and yet neither they nor regulators will find it optimal to increase capital levels.

## VII. Conclusions

Our paper highlights several problems facing the designers of bank capital regulation. Earlier work has emphasized banks incentives to shift risk to depositors or a bank insurance fund when bank "charter values" -- rents or quasi-rents from continued bank operations -- are low; we show that the same situation creates a debt overhang problem that makes bank shareholders reluctant to raise additional capital when charter values are low or existing capital is relatively low. Nevertheless, if bank deposits served no special function, this problem could be solved ex ante by requiring banks to finance themselves entirely with equity. It is for this reason that our general equilibrium framework is critical: in it, banks not only create informational charter value on the lending side, but also offer a unique mechanism (deposits) for liquidity provision on the funding side.

This changes the motivations of the regulator. In standard models, the regulator is assumed to

want to minimize losses to a bank insurance fund, or perhaps to maximize a social welfare function that includes some notion of charter value. In our model, losses to the deposit insurance fund are simply a transfer, and the regulator focuses on the tradeoff between preserving charter value and reducing lemons costs associated with using bank equity for liquidity purposes. This means that, even ignoring whether or not banks will comply with capital requirements, the regulator may not wish to raise additional capital: the lemons cost of new capital may outweigh the benefit of a lower chance of bank failures.

Bank shareholders also care about this tradeoff: they receive some private charter value from their bank's continued operation, and additional capital must be raised at prices that reflect lemons costs. Nevertheless, we have argued that externalities connected with bank failure suggest that social charter value may well be higher than private charter value, and debt overhang certainly makes the private cost of bank capital higher than its social cost; in our model, the debt overhang effect alone is enough to guarantee that banks are less willing to raise additional capital than is the regulator.

Because the regulator prefers that banks continue in business and have some chance of preserving their charter value rather than exit and lose it for sure, the bank shareholders' view of the charter value/lemons cost tradeoff is always binding on the regulator -- ex post. Ex ante, the regulator can largely overcome this constraint by setting initial capital requirements when the banking system is being formed. Even here, a banking system with low risk of default may be socially optimal yet infeasible, and in any case the regulator may still prefer a banking system with significant chance of bank failure if bank asset risk or equity lemons costs are high, or if charter values or bank assets' mean returns are low. Furthermore, if the distribution of bank asset returns changes over time, as it almost certainly does, the regulator may well choose ex ante capital requirements that are too low ex post in those situations where bank returns are worse than expected.

The recent experience of Japan's banking industry illustrates these issues. Over the last few years, it has become increasingly obvious that Japanese banks' loan portfolios are in far worse shape than was thought at the end of the 1980s. As a result, many observers have called for massive writeoffs and capital infusions, yet neither the banks nor their regulators have responded. Given the massive debt overhang that such banks face, their shareholders' reluctance to raise more capital is understandable. Furthermore, continuing revelations of bad loans have caused many commentators to opine that there are even more

bad loans to be claimed, suggesting a fair amount of informational asymmetry; if so, potential shareholders may perceive high lemons costs to trading shares, increasing the banks' effective cost of capital. Finally, banks play a more central role in Japan than in the United States, where regulators felt that the failure of large banks entailed externalities high enough to justify intervention to save both Continental Illinois in 1984 and Citicorp in 1991; thus, Japanese regulators may view the costs of enforcing the Basle Accords' capital guidelines and forcing some banks to exit as unacceptably high.

There are two caveats to our approach which bear further discussion. First, our result that bank regulators are completely unable to override the interests of existing bank shareholders is admittedly quite strong. In a more dynamic context, regulators might find it time consistent to close some banks so as to send other banks a signal of "toughness." Furthermore, if banks could switch into assets that were not only riskier but negative net present value investments (as many have argued that the American savings & loans industry did in the 1980s), regulators might have incentive to close banks before they mount up greater social losses. Finally, although we have assumed that deposit insurance is a costless transfer, dissipative costs to taxation should make the regulator more inclined to raise capital and minimize losses on deposits. Nevertheless, in an earlier draft of this paper, we showed that, even if banks view the regulator's threat of closure as credible, their threat of exit still constrains the regulator in some cases. Also, a more credible bank closure policy would not change the fact that bank capital is socially costly.

The second caveat concerns the relationship between several of the key parameters of our model. Although we have taken bank asset risk  $\alpha$ , charter value  $C_B$ , and the lemons cost  $1-\gamma$  as exogenous, in fact they should be endogenous and positively related: greater risk increases both the informational charter value created by banks and the potential lemons costs faced by those trading bank shares; also, any increase in asymmetric information should enhance both charter value and lemons costs. Nevertheless, these factors needn't be perfectly correlated: asset risk is partly systematic (and thus less conducive to asymmetric information), charter value can be linked to various institutional constraints in either the lending or deposit-taking arenas (as in Japan, where banks once had something of a protected monopoly), and lemons costs caused by informed traders are influenced by various institutional arrangements in the stock markets (for example, the relative ease or difficulty of short-selling).

The last point brings up an important issue for policymakers. Our work suggests that bank



shareholders require a higher charter value than do regulators before raising additional capital (or forgoing risk-shifting), and the value of bank charters may well be even higher than the banks' private value. Thus, policy makers should be concerned with the extent to which increased competition in financial services has eroded private charter values, particularly since social charter value linked to the viability of the payments system may not have declined nearly as much as private and social charter values linked to lending. One issue for future research concerns whether it is socially desirable to create private charter value by restricting entry into banking so as to implement the socially desired level of risk in banking.

### Appendix A: The Bank Moral Hazard Problem

*Lemma A1.* If a bank only considers leverage changes that do not affect its chance of default, it issues as many deposits as possible: if (i)  $R - \alpha < R_D \cdot D_0 \leq R + \alpha$ , it sets  $D_1 = (R + \alpha)/R_D$ , and if (ii)  $R - \alpha \geq R_D \cdot D_0$ , it sets  $D_1 = (R - \alpha)/R_D$ .

*Proof:* (i) Suppose a bank repurchases  $N_1$  shares at price  $P_1$  in exchange for deposits  $\Delta D = D_1 - D_0$ , keeping the chance of default unchanged (so  $R + \alpha \geq R_D \cdot D_1$ ). Let  $N_0 - N_1 = n_0 \cdot p'$ , so that  $p'$  is the new marginal shareholder. The expected value of equity before the repurchase is  $E_1 = \frac{1}{2}[R + \alpha - R_D \cdot D_0 + C_B]$ ; afterwards, it is  $E'_1 = \frac{1}{2}[R + \alpha - R_D \cdot D_1 + C_B] = E_1 - \frac{1}{2}R_D \cdot \Delta D$ . If the marginal shareholder is indifferent,  $P_1$  satisfies two equations:  $P_1 = [E'_1 / (R_D(N_0 - N_1))] \cdot [1 - p'(1 - \gamma)]$ , and  $\Delta D = P_1 \cdot N_1 = P_1 \cdot n_0(p^* - p')$ . We combine these to get a quadratic equation for  $p'$ :

$$(1 - \gamma)p'^2 + \left[ -1 - p^*(1 - \gamma) - \frac{R_D \cdot \Delta D}{E'_1} \right] \cdot p' + p^* - 0$$

A positive solution always exists. Before the repurchase, the welfare of a shareholder of type  $p$  is  $(E_1 \cdot n_0 / N_0) \cdot [p(\gamma - 1) + 1]$ ; afterwards, it is  $[(E_1 - \frac{1}{2}R_D \cdot \Delta D) \cdot n_0 / (N_0 - N_1)]$  if  $p \leq p'$ , and  $R_D \cdot (n_0 \cdot P_1)$  otherwise. Since  $N_0 = n_0 \cdot p^*$  and  $N_0 - N_1 = n_0 \cdot p'$ , total shareholder welfare before the repurchase is

$$S = \left[ \frac{E_1 \cdot n_0}{n_0 \cdot p^*} \right] \int_0^{p^*} (1 - (1 - \gamma)p) dp - E_1 \cdot \left[ 1 - \frac{p^*(1 - \gamma)}{2} \right]$$

and total shareholder welfare afterwards is

$$\begin{aligned} S'(p') &= \left[ \frac{(E_1 - \frac{1}{2}R_D \cdot \Delta D) \cdot n_0}{n_0 \cdot p'} \right] \int_0^{p'} (1 - (1 - \gamma)p) dp + R_D \cdot (n_0 P_1) \int_{p'}^{p^*} dp \\ &= (E_1 - \frac{1}{2}R_D \cdot \Delta D) \cdot \left[ 1 - \frac{p'(1 - \gamma)}{2} \right] + R_D \cdot \Delta D. \end{aligned}$$

Since  $p^*(\gamma - 1)/2 < p'(\gamma - 1)/2 < 0$ ,  $S'(p') > S$ . In fact,

$$\frac{\partial S'}{\partial p'} = -(E_1 - \frac{1}{2}R_D \cdot \Delta D) \cdot \frac{1 - \gamma}{2} + R_D \cdot \frac{\partial \Delta D}{\partial p'} \cdot \left[ \frac{2 + p'(1 - \gamma)}{4} \right]$$

which is negative, so the bank sets  $D_1$  as high as possible within this region.

(ii) Suppose a bank repurchases  $\Delta N$  at price  $P_1$  for  $\Delta D$  deposits (defined as before), leaving the chance of default unchanged (so  $R - \alpha \geq R_D \cdot D_1$ ). Define  $p'$  and  $N_1$  as in (i). Expected equity value before the repurchase is  $E_1 = R - R_D \cdot D_0 + C_B$ ; afterwards, it is  $E'_1 = R - R_D \cdot D_1 + C_B = E_1 - R_D \cdot \Delta D$ . Once more, we can use the two equations for  $P_1$  to get the same quadratic equation for  $p'$ . More importantly, total shareholder welfare before the repurchase is  $S$  as in (i), and total shareholder welfare afterwards is

$$S'(p') = (E_1 - R_D \cdot \Delta D) \cdot \left[ 1 - \frac{p'(1-\gamma)}{2} \right] + R_D \cdot \Delta D$$

Since  $p^*(1-\gamma)/2 > p'(1-\gamma)/2 > 0$  and  $1 - p'(1-\gamma) < 1$ ,  $S'(p') > S$ ; also,  $\partial S'/\partial p' < 0$ . *Q.E.D.*

**Proposition A1.** (i) Suppose  $R - \alpha \geq R_D \cdot D_0$ , so that, given initial leverage, the bank never defaults. The bank sets  $D_1 = (R - \alpha)/R_D$  or  $(R + \alpha)/R_D$  depending on whether or not

$$C_B > \left[ \frac{4 + 2 \cdot p^H \cdot (1-\gamma)}{2 - 2p^H \cdot (1-\gamma) + p^L \cdot (1-\gamma)} \right] \cdot \alpha > 2\alpha$$

where  $p^H$  is the marginal shareholder when  $D_1 = (R - \alpha)/R_D$ , and  $p^L$  is the marginal shareholder when  $D_1 = (R + \alpha)/R_D$ . (ii) Suppose  $R - \alpha < R_D \cdot D_0 \leq R + \alpha$ , so that, given initial leverage, the bank only defaults in the bad state. The bank sets  $D_1 = (R - \alpha)/R_D$  or  $(R + \alpha)/R_D$  depending on whether or not

$$C_B > \left[ \frac{4\beta + 2 \cdot p^* \cdot (1-\gamma)}{2 - 2p^* \cdot (1-\gamma) + p^{RL} \cdot (1-\gamma)} \right] \cdot \alpha - \left[ \frac{4(\beta-1)}{2 - 2p^* \cdot (1-\gamma) + p^{RL} \cdot (1-\gamma)} \right] \cdot [R - R_D D_0] > 2\alpha$$

$$\text{where } \beta = \frac{1 - \frac{1}{2} p^* \cdot (1-\gamma)}{1 - p^{RH} \cdot (1-\gamma)}$$

The lower bound on  $C_B$  is greater than  $2\alpha$ .

*Proof:* (i) Note that the expected value of equity after increasing deposits to  $(R - \alpha)/R_D$  is  $E'_1 = \alpha + C_B$ , while its value after increasing deposits to  $(R + \alpha)/R_D$  is  $E'_1 = \frac{1}{2} C_B$ . The marginal shareholders are  $p^H$  and  $p^L$  respectively, as defined by the quadratic equation in Lemma A1's proof.<sup>20</sup> Given these results, it is easy to show that the initial shareholders' total welfare after the repurchase is  $S'(p')$  as in Lemma A1, where  $p' = p^H$  if  $D_1 = (R - \alpha)/R_D$ , and  $p' = p^L$  if  $D_1 = (R + \alpha)/R_D$ . (Since welfare for *all* choices

<sup>20</sup> This is easily seen for  $p^H$ . For  $p^L$ , it follows because the marginal investor depends on  $\Delta D$  and  $P_1$ ;  $P_1$  is the price of equity *after* the repurchase, which depends on  $\hat{E}_1$  only. The fact that there was no initial chance of default enters into the solution only through its effect on  $\Delta D$ .

with the same default chance have similar form, Lemma A1 implies that the bank can restrict its choices to  $D_1 = (R+\alpha)/R_D$  and  $(R-\alpha)/R_D$ . The bank chooses whichever deposit level gives greater shareholder welfare. Substituting in yields the following expressions:

$$S'(p^H) = (\alpha + C_B) \cdot \left[ 1 - \frac{p^H(1-\gamma)}{2} \right] + (R-\alpha) - R_D \cdot D_0$$

$$S'(p^L) = \frac{C_B}{2} \cdot \left[ 1 - \frac{p^L(1-\gamma)}{2} \right] + (R+\alpha) - R_D \cdot D_0$$

Taking  $S'(p^H) > S'(p^L)$ , gathering terms in  $C_B$  and  $\alpha$ , and simplifying gives the desired condition.

(ii) Equity values after setting  $D_1$  to  $(R-\alpha)/R_D$  or  $(R+\alpha)/R_D$  are as given in (i). As before, setting  $D_1$  to  $(R+\alpha)/R_D$  requires a share repurchase, so that initial shareholder welfare afterwards is  $S'(p^L)$  as in (i). However, if  $D_1 = (R-\alpha)/R_D$ , deposits *decrease*, so the bank issues  $N_1$  additional shares to depositors with  $p$  between  $p^*$  and  $p^H$ ; afterwards, original shareholder welfare is

$$S'(p^H) = \frac{(\alpha + C_B)n_0}{N_0 + N_1} \cdot \int_0^{p^*} (1-p(1-\gamma)) dp = \frac{(\alpha + C_B)n_0}{N_0 + N_1} \cdot p^* \left[ 1 - \frac{p^*(1-\gamma)}{2} \right]$$

Each new shareholder buys  $n_1$  shares where  $n_1 \cdot P_1 = D_0(p) = 1$ .  $P_1 = [1-p^H \cdot (1-\gamma)] \cdot (\alpha + C_B) / (R_D(N_0 + N_1))$ . Also,  $p^H - p^* = D_0 - D_1$ , so  $N_1 = n_1 \cdot (p^H - p^*) = n_1 \cdot (D_0 - D_1)$ . The expression for  $P_1$  implies that  $(\alpha + C_B) / (N_0 + N_1) = R_D \cdot [n_1 \cdot (1 - p^H \cdot (1-\gamma))]^{-1}$  this into  $S'(p^H)$  and using  $n_0 p^* = N_0$  yields:

$$S'(p^H) = \frac{R_D \cdot N_0 \cdot [1 - \frac{1}{2} p^* (1-\gamma)]}{n_1 \cdot (1 - p^H (1-\gamma))}$$

Also,  $n_1 = 1/P_1$ ; substituting  $N_1 = n_1(D_0 - D_1)$  into  $P_1$  and solving for  $n_1$ , we have  $n_1 = R_D \cdot N_0 / [(\alpha + C_B)(1 - p^H(1-\gamma)) - R_D(D_0 - D_1)]$ . Now substitute for  $n_1$  and  $D_1 = (R-\alpha)/R_D$  to obtain

$$S'(p^H) = (\alpha + C_B) \left[ 1 - \frac{p^*(1-\gamma)}{2} \right] + [R - \alpha - R_D D_0] \beta$$

Thus,  $S'(p^H) > S'(p^L)$  if and only if:

$$(\alpha + C_B) \left[ 1 - \frac{p^*(1-\gamma)}{2} \right] + [R - \alpha - R_D D_0] \beta > \frac{C_B}{2} \cdot \left[ 1 - \frac{p^L(1-\gamma)}{2} \right] + R + \alpha - R_D D_0.$$

Collecting terms in  $C_B$  and  $\alpha$  and simplifying gives the condition in the proposition. Also,  $R - R_D \cdot D_0 > \alpha$ , and  $\beta > 1$  since  $p^H > p^*$ , so the RHS of this condition exceeds

$$\left[ \frac{4 + 2 \cdot p^* \cdot (1-\gamma)}{2 - 2p^* \cdot (1-\gamma) + p^L \cdot (1-\gamma)} \right] \cdot \alpha$$

which exceeds 2 since  $p^* > p^L$ . The only remaining question is whether the bank might wish to increase its equity further by reducing  $D_1$  below  $(R-\alpha)/R_D$ . For any deposit level  $D'_1 < (R-\alpha)/R_D$ , original shareholder welfare  $S' = E'_1 \cdot N_0 \cdot \delta / N_1$ , where  $\delta = 1 - \frac{1}{2}p^*(1-\gamma)$  and  $E'_1 = C_B + R - R_D D'_1$ . Thus:

$$\begin{aligned} \frac{\partial S'}{\partial D'_1} &= \frac{\partial E'_1}{\partial D'_1} \cdot \frac{\delta N_0}{N_1} - \frac{\partial N_1}{\partial D'_1} \cdot \frac{E'_1 \cdot \delta \cdot N_0}{N_1^2} - \frac{R_D \cdot \delta N_0}{N_1} + \frac{n_1 \cdot E'_1}{N_1} \cdot \frac{\delta \cdot N_0}{N_1} \\ &= \left[ \frac{1}{1 - p'(1-\gamma)} \right] \cdot \frac{R_D \delta N_0}{N_1} \end{aligned}$$

where the last equality follows from  $E'_1/N_1 = R_D/[n_1 \cdot (1-p'(1-\gamma))]$ . Since  $1 - p'(1-\gamma) < 1$ ,  $\partial S'/\partial D_1 > 0$ , and any decrease in deposits below  $(R-\alpha)/R_D$  reduces original shareholder welfare. *Q.E.D.*

### Appendix B

*Proof of Lemma 1:* first, use  $P_0 \cdot N_0 = p^*$ ,  $P_1 \cdot N_1 = p' \cdot p^*$ , and  $1 - p' = D_0 - \Delta D$  to substitute for  $P_0$ ,  $P_1$ , and  $1 - p'$  in the regulator's objective function  $W(\cdot)$ ; then integrate to obtain:

$$\begin{aligned} & - \left[ 1 - \frac{p^*(1-\gamma)}{2} \right] \left[ \frac{N_0 \cdot E_1(\cdot)}{N_0 + N_1} \right] + \left[ 1 - \frac{(p' + p^*)(1-\gamma)}{2} \right] \left[ \frac{N_1 \cdot E_1(\cdot)}{N_0 + N_1} \right] + R_D(D_0 - \Delta D) + G - T_2(\cdot) \\ & - E_1(\cdot) - T_2(\cdot) + R_D(D_0 - \Delta D) + G - \frac{p^*(1-\gamma)}{2} \cdot E_1(\cdot) - \frac{p'(1-\gamma)}{2(1-p'(1-\gamma))} \cdot R_D \Delta D \end{aligned}$$

where we have used  $\Delta D = N_1 \cdot P_1 = \{N_1 \cdot E_1(\cdot) / (R_D \cdot (N_0 + N_1))\} \cdot \{1 - p'(1-\gamma)\}$  from equation (1) in the text. Also from the text,  $E_1(\cdot) - T_2(\cdot) + R_D(D_0 - \Delta D) = R + C_B \cdot [1 - \text{banks' chance of default}]$ . Since  $p' = p^* + \Delta D$ , and  $E_1(\cdot)$  increases in  $\Delta D$ , the other terms in the objective function are decreasing in  $\Delta D$ . This proves the lemma. *Q.E.D.*

*Proof of Proposition 1:* (i) The regulator chooses the alternative that maximizes social welfare. Let  $W(\Delta D)$  denote welfare when the additional capital requirement is  $\Delta D$ ; then, using the expression for  $W$  given in the proof of Lemma 1, we have  $W(0) = R + G$ ,

$$W(\Delta D^L) = R + \frac{C_B}{2} + G - \frac{p^*(1-\gamma)}{2} \cdot \frac{C_B}{2} - \frac{p^L(1-\gamma)}{2(1-p^L(1-\gamma))} \cdot R_D \Delta D^L, \text{ and}$$

$$W(\Delta D^H) = R + C_B + G - \frac{p^*(1-\gamma)}{2} \cdot (\alpha + C_B) - \frac{p^H(1-\gamma)}{2(1-p^H(1-\gamma))} \cdot R_D \Delta D^H.$$

The regulator wants additional capital  $\Delta D$  rather than no additional capital if and only if  $W(\Delta D) \geq W(0)$ . Rearranging  $W(\Delta D^L) \geq W(0)$  yields  $C_B \geq C_{reg}(L, 1)$ , and rearranging  $W(\Delta D^H) \geq W(0)$  yields  $C_B \geq C_{reg}(H, 1)$ . If both conditions are met, the regulator chooses  $\Delta D^H$  if  $W(\Delta D^H) > W(\Delta D^L)$ , and  $\Delta D^L$  if  $W(\Delta D^H) < W(\Delta D^L)$ ; these conditions can be rearranged to give  $C_B > (<) 2C_{reg}(H, 1) - C_{reg}(L, 1)$ .

(ii) As before, the regulator compares  $W(\Delta D)$  and  $W(0)$ , but now the only  $\Delta D$  considered is  $\Delta D^H$ .  $W(\Delta D^H)$  is identical to the expression in (i) above, but  $W(0)$  is now

$$W(0) = R + \frac{C_B}{2} + G - \frac{p^*(1-\gamma)}{2} \cdot \left[ \frac{R + \alpha - R_D D_0 + C_B}{2} \right].$$

First, note that  $R + \alpha - R_D \cdot D_0 = 2\alpha - R_D \cdot \Delta D^H$ . Substitute this into  $W(0)$ ; then  $W(\Delta D^H) \geq W(0)$  can be rearranged to yield  $C_B \geq C_{reg}(H, 1/2)$ . *Q.E.D.*

*Proof of Proposition 3:* (i) First, note that  $N_0/(N_0+N_1) = 1 - N_1/(N_0+N_1)$ , and, using equation (1),

$$\left[ \frac{N_1}{N_0+N_1} \right] \cdot E_1(\bar{I}_C - 1) = \frac{(N_1 P_1) R_D}{1 - p'(1-\gamma)} = \frac{R_D \Delta D}{1 - p'(1-\gamma)}.$$

Substitute these results and the values of  $E_1$  given in equation (3) into equation (4) in the text; rearranging yields the expressions  $C_B \geq C_{bank}(L, 1)$  when the capital requirement is  $\Delta D^L$ , and  $C_B \geq C_{bank}(L, 1)$  when the capital requirement is  $\Delta D^H$ . If both conditions are met, banks choose  $\Delta D^H$  over  $\Delta D^L$  if  $E_1(\bar{I}_C = 1) \cdot [N_0/(N_0+N_1)]$  when  $\Delta D = \Delta D^H$  exceeds the same expression evaluated at  $\Delta D = \Delta D^L$ ; rearranging this condition yields  $C_B \geq 2C_{bank}(H, 1) - C_{bank}(L, 1)$ .

(ii) The proof is identical to that of (i).

(iii) Note that  $C_{reg}(L, 1) = 1/2 C_{bank}(L, 1) \cdot p^L(1-\gamma) \cdot [1 - p^*(1-\gamma)/2]^{-1}$ . Since  $p^L$ ,  $p^*$ , and  $1-\gamma$  are all positive and less than one, it follows that  $p^L(1-\gamma) \cdot [1 - p^*(1-\gamma)/2]^{-1} < 2$ , so  $C_{reg}(L, 1) < C_{bank}(L, 1)$ .

Similarly,  $C_{reg}(H, 1) < C_{bank}(H, 1)$  if and only if

$$\left[ 1 - \frac{p^*(1-\gamma)}{2} \right]^{-1} \cdot \left[ \frac{p^H(1-\gamma)}{1 - p^H(1-\gamma)} \cdot \frac{R_D \Delta D^H}{2} + \frac{p^*(1-\gamma)}{2} \alpha \right] < \frac{R_D \Delta D^H}{1 - p^H(1-\gamma)} - \alpha$$

Multiplying both sides by  $1 - p^*(1-\gamma)/2$  and collecting terms in  $R_D \cdot \Delta D^H$  and  $\alpha$ , this is equivalent to

$$\frac{R_D \Delta D^H}{1 - p^H(1-\gamma)} \cdot \left[ 1 - \frac{p^*(1-\gamma)}{2} - \frac{p^H(1-\gamma)}{2} \right] > \alpha.$$

Since  $p^* < p^H$ , the LHS of this condition is greater than  $R_D \cdot \Delta D^H$ . Since  $R_D \cdot D_0 > R + \alpha$ ,  $R_D \cdot \Delta D^H = R_D \cdot D_0 - R + \alpha > 2\alpha > \alpha$ , so the condition holds, and  $C_{reg}(H, 1) < C_{bank}(H, 1)$ .

Take the expression for  $C_{reg}(H, 1/2)$  in the text; since  $p^*(1-\gamma) < 1$ , we have

$$C_{reg}(H, 1/2) < 2 \cdot \left[ \frac{p^H(1-\gamma)}{1 - p^H(1-\gamma)} + \frac{1}{2} \right] \cdot R_D \Delta D^H = \left[ \frac{1 + p^H(1-\gamma)}{1 - p^H(1-\gamma)} \right] \cdot R_D \Delta D^H = C_{bank}(H, 1/2).$$

Finally,  $2C_{\text{reg}}(\text{H},1) - C_{\text{reg}}(\text{L},1)$  equals

$$\begin{aligned} & \left[ 1 - \frac{p^*(1-\gamma)}{2} \right]^{-1} \cdot \left[ \frac{p^{\text{H}}(1-\gamma)R_{\text{D}}\Delta D^{\text{H}}}{1-p^{\text{H}}(1-\gamma)} + p^*(1-\gamma)\alpha - \frac{p^{\text{L}}(1-\gamma)R_{\text{D}}\Delta D^{\text{L}}}{1-p^{\text{L}}(1-\gamma)} \right] \\ & < 2 \cdot \left[ \frac{p^{\text{H}}(1-\gamma)R_{\text{D}}\Delta D^{\text{H}}}{1-p^{\text{H}}(1-\gamma)} + \alpha - \frac{p^{\text{L}}(1-\gamma)R_{\text{D}}\Delta D^{\text{L}}}{1-p^{\text{L}}(1-\gamma)} \right] \\ & = 2 \cdot \left[ -R_{\text{D}}\Delta D^{\text{H}} + \frac{R_{\text{D}}\Delta D^{\text{H}}}{1-p^{\text{H}}(1-\gamma)} + \alpha + R_{\text{D}}\Delta D^{\text{L}} - \frac{R_{\text{D}}\Delta D^{\text{L}}}{1-p^{\text{L}}(1-\gamma)} \right]. \end{aligned}$$

Since  $R_{\text{D}}\Delta D^{\text{L}} - R_{\text{D}}\Delta D^{\text{H}} = -2\alpha$ , the last expression equals  $2C_{\text{bank}}(\text{H},1) - C_{\text{bank}}(\text{L},1)$ . *Q.E.D.*

*Proof of Corollary 2:* (i) Since  $p^{\text{H}}$ ,  $p^{\text{L}}$ ,  $R_{\text{D}}\Delta D^{\text{H}}$ , and  $R_{\text{D}}\Delta D^{\text{L}}$  are all increasing in  $R_{\text{D}}$ , and  $R_{\text{D}}\Delta D^{\text{H}}$  and  $R_{\text{D}}\Delta D^{\text{L}}$  are both increasing in  $D_0$ , it immediately follows that  $C_{\text{bank}}(\text{L},1)$ ,  $C_{\text{bank}}(\text{H},1)$ , and  $C_{\text{bank}}(\text{H},\frac{1}{2})$  are increasing in  $R_{\text{D}}$  and  $D_0$ . It is also immediate that these three expressions are increasing in  $1-\gamma$ . If we can show that  $2C_{\text{bank}}(\text{H},1) - C_{\text{bank}}(\text{L},1)$  is also increasing in  $R_{\text{D}}$ ,  $D_0$ , and  $1-\gamma$ , we are done. For convenience, we work with  $F \equiv C_{\text{bank}}(\text{H},1) - \frac{1}{2}C_{\text{bank}}(\text{L},1)$ , which equals

$$\frac{R_{\text{D}}\Delta D^{\text{H}}}{1-p^{\text{H}}(1-\gamma)} - \alpha - \frac{R_{\text{D}}\Delta D^{\text{L}}}{1-p^{\text{L}}(1-\gamma)}$$

Since  $p^{\text{H}} > p^{\text{L}}$ ,  $1-p^{\text{H}}(1-\gamma) < 1-p^{\text{L}}(1-\gamma)$ , and it is easy to show that  $\partial F/\partial D_0$  and  $\partial F/\partial(1-\gamma)$  are positive. Also, using  $\partial p^{\text{H}}/\partial R_{\text{D}} = (R-\alpha)\cdot R_{\text{D}}^{-2}$  and  $\partial p^{\text{L}}/\partial R_{\text{D}} = (R+\alpha)\cdot R_{\text{D}}^{-2}$ , we have

$$\begin{aligned} \frac{\partial F}{\partial R_{\text{D}}} &= \frac{\Delta D^{\text{H}}}{1-p^{\text{H}}(1-\gamma)} + \frac{\Delta D^{\text{H}}(1-\gamma)(R-\alpha)/R_{\text{D}}}{(1-p^{\text{H}}(1-\gamma))^2} - \frac{\Delta D^{\text{L}}}{1-p^{\text{L}}(1-\gamma)} - \frac{\Delta D^{\text{L}}(1-\gamma)(R+\alpha)/R_{\text{D}}}{(1-p^{\text{L}}(1-\gamma))^2} \\ &= \frac{\Delta D^{\text{H}}[1-p^{\text{H}}(1-\gamma)+(1-p^{\text{H}})(1-\gamma)]}{(1-p^{\text{H}}(1-\gamma))^2} - \frac{\Delta D^{\text{L}}[1-p^{\text{L}}(1-\gamma)+(1-p^{\text{L}})(1-\gamma)]}{(1-p^{\text{L}}(1-\gamma))^2} \end{aligned}$$

Since  $1+(1-2p)(1-\gamma) = (1-p(1-\gamma))^2 + (1-\gamma)(1-p(1-\gamma))$  for  $p = p^{\text{H}}$  and  $p = p^{\text{L}}$ , this equals

$$\Delta D^{\text{H}} \cdot \left[ 1 + \frac{1-\gamma}{1-p^{\text{H}}(1-\gamma)} \right] - \Delta D^{\text{L}} \cdot \left[ 1 + \frac{1-\gamma}{1-p^{\text{L}}(1-\gamma)} \right] > 0.$$



Thus  $2C_{\text{bank}}(H,1) - C_{\text{bank}}(L,1)$  is increasing in  $R_D$ ,  $D_0$ , and  $1-\gamma$ .

(ii) Since  $p^H$ ,  $p^L$ ,  $R_D \cdot \Delta D^H$ , and  $R_D \cdot D^L$  are all decreasing in  $R$ , it follows that  $C_{\text{bank}}(L,1)$ ,  $C_{\text{bank}}(H,1)$ , and  $C_{\text{bank}}(H,1/2)$  are decreasing in  $R$ . Defining  $F = C_{\text{bank}}(H,1) - 1/2 C_{\text{bank}}(L,1)$  as in (i), similar calculations show that  $\partial F/\partial R$  is negative, so  $2C_{\text{bank}}(H,1) - C_{\text{bank}}(L,1)$  also decreases in  $R$ .

(iii)  $p^H$  and  $R_D \cdot \Delta D^H$  are both increasing in  $\alpha$ , while  $p^L$  and  $R_D \cdot \Delta D^L$  are both decreasing in  $\alpha$ . It follows that  $C_{\text{bank}}(L,1)$  decreases in  $\alpha$ , while  $C_{\text{bank}}(H,1)$  and  $C_{\text{bank}}(H,1/2)$  increase in  $\alpha$ . These results imply that  $2C_{\text{bank}}(H,1) - C_{\text{bank}}(L,1)$  increases in  $\alpha$ . *Q.E.D.*

*Proof of Proposition 4:* (i)-(iii) Assuming that a banking system forms, social welfare is given by the regulator's objective function  $W$  from Section IV of the text:

$$W = \int_0^{p'} \left[ \frac{[1-p(1-\gamma)] \cdot E_1(\cdot)}{P_0(N_0+N_1)} \right] dp + R_D(1-p') + G - T_2(\cdot)$$

where we have used Lemma 2's result that  $P_0 = P_1$ . Since  $P_0 \cdot N_0 = 1-p^*$  and  $P_1 \cdot N_1 = p'-p^*$ ,  $P_0(N_0+N_1) = p'$ . Substituting into  $W$  and integrating gives  $W = E_1 - T_2 + R_D(D_0 - \Delta D) + G - E_1 \cdot p'(1-\gamma)/2$ . We know that the first three terms equal  $R + C_B[1 - \text{banks' chance of default}]$ , so  $W$  only depends on  $E_1$ ,  $p'$  (the eventual marginal shareholder's  $p$ , which also equals  $1-D_1$ ) and the banks' chance of default, which depends on  $R_D$  and  $D_1 = 1-p'$ .

Market clearing requires  $D_0 = 1 - N_0 \cdot P_0 = 1 - N_0 \cdot P_1 = 1 - (N_0+N_1)P_1 + N_1 \cdot P_1$ . From equation (1) in the text,  $(N_0+N_1)P_1 = E_1 \cdot (1-p'(1-\gamma))/R_D$ , and  $N_1 \cdot P_1 = \Delta D$ ; substituting in and rearranging, we have  $1-D_0 - \Delta D = 1-D_1 = E_1 \cdot (1-p'(1-\gamma))/R_D$ , so  $R_D(1-D_1) = R_D \cdot p' = E_1 \cdot (1-p'(1-\gamma))$ . Note that  $E_1$  is decreasing in  $R_D$  and  $D_1 = 1-p'$ ; using the Implicit Function Theorem, it follows that  $R_D$  is decreasing in  $p'$ , so  $E_1$  is increasing in  $p'$ . This implies that, for any given chance of bank default, social welfare  $W$  is decreasing in  $p'$ .

It follows that, for a given chance of default, the regulator prefers the highest deposits (lowest  $p'$ ) consistent with that default chance:  $p' = p^H$  for no default,  $p^L$  for sometimes default, and 0 for always default. Thus, there are effectively only three banking equilibrium outcomes. (As noted in the text, the regulator must choose the initial capital level so that the final capital level is feasible, but simply setting  $D_0 = D_1$  works.) Substituting each value of  $p'$  and the corresponding value of  $E_1$  ( $\alpha + C_B$ ,  $1/2 C_B$ , or 0) into  $W$  gives the values of social welfare given in (i)-(iii).

The only thing left is to determine the equilibrium deposit rate  $R_D$  and check for feasibility in each case:

*No Default.* Market clearing implies  $R_D \cdot p^H = (\alpha + C_B)(1-p^H(1-\gamma))$ , where  $p^H = 1-(R-\alpha)/R_D$ . Substituting in and rearranging gives the following quadratic equation for  $R_D$ :

$$(R_D)^2 - [(\alpha + C_B)\gamma + R - \alpha] \cdot (R_D) - (\alpha + C_B)(R - \alpha)(1 - \gamma) = 0$$

Applying the quadratic formula, this equation always has one positive and one negative root; the positive root is the formula for  $R_D$  given in the text. Since  $R_D$  and thus  $p^H$  have been determined to be consistent with market clearing, etc., the only feasibility condition is that consumers prefer deposits to autarky, which is equivalent to  $R_D \geq 1$ . (As noted in the text, an individual consumer ignores taxes, since they are lump sum.)

*Sometimes Default.* Market clearing implies  $R_D \cdot p^L = \frac{1}{2}C_B(1 - p^L(1 - \gamma))$ , where  $p^L = 1 - (R + \alpha)/R_D$ . Substituting in and rearranging gives the following quadratic equation for  $R_D$ :

$$(R_D)^2 - [\frac{1}{2}C_B \cdot \gamma + R + \alpha] \cdot (R_D) - \frac{1}{2}C_B(R + \alpha)(1 - \gamma) = 0$$

Again, this always has one positive and one negative root; the positive root is the solution given in the text. That solution is clearly larger than  $2(\frac{1}{2}C_B \cdot \gamma + R + \alpha)$ ; since  $R \geq 1$ , it follows that  $R_D > 1$ , and consumers always prefer this outcome to autarky.

*Always Default.* Since  $p' = E_t = 0$ , market clearing is clearly satisfied.  $R_D \cdot D_1$  must exceed  $R + \alpha$ ; since  $D_1 = 1 - p' = 1$ , this is satisfied whenever  $D_1 \geq R + \alpha$ . (To be precise, an individual bank would have incentive to set deposit rates infinitely high, but if all banks do this, the basic equilibrium outcome does not change.) Since deposit rates exceed  $R \geq 1$ , this is always feasible relative to autarky.

(iv) The proof is straightforward and is omitted. *Q.E.D.*

*Proof of Corollary 3:* both  $R_D^{ND}$  and  $R_D^{SD}$  can be written as  $\frac{1}{2}[A + (A^2 + 4B)^{1/2}]$ . Factoring out  $\frac{1}{2}$ , it follows that, for any parameter  $x$ ,  $\partial R_D / \partial x$  equals  $\partial A / \partial x + (A^2 + 4B)^{-1/2}[A \cdot \partial A / \partial x + 2 \cdot \partial B / \partial x]$ .

$$(i) A = (\alpha + C_B)\gamma + R - \alpha, B = (\alpha + C_B)(R - \alpha)(1 - \gamma).$$

$\partial A / \partial \alpha = -(1 - \gamma) < 0$ ;  $\partial B / \partial \alpha = (R - \alpha - \alpha - C_B)(1 - \gamma)$ . Since  $A \cdot \partial A / \partial \alpha + 2 \cdot \partial B / \partial \alpha = (1 - \gamma)[R - \alpha - (2 + \gamma)(\alpha + C_B)] < (1 - \gamma)(R - \alpha) < (A^2 + 4B)^{1/2}$ ,  $\partial R_D / \partial \alpha < 0$ .

$\partial A / \partial \gamma = \alpha + C_B$ ;  $\partial B / \partial \gamma = -(\alpha + C_B)(R - \alpha)$ .  $A \cdot \partial A / \partial \gamma + 2 \cdot \partial B / \partial \gamma = (\alpha + C_B)[(\alpha + C_B)\gamma - (R - \alpha)]$ ; the absolute value of this is less than  $(\alpha + C_B)A$ , and  $A$  is less than  $(A^2 + 4B)^{1/2}$ , so  $\partial R_D / \partial \gamma > 0$ , and  $R_D$  decreases in  $1 - \gamma$ .

Finally, both  $A$  and  $B$  are increasing in  $R$  and  $C_B$ , so  $R_D$  is increasing in both parameters.

(ii)  $A = \frac{1}{2}C_B \cdot \gamma + R + \alpha$ ,  $B = \frac{1}{2}C_B(R + \alpha)(1 - \gamma)$ . Since  $A$  and  $B$  are both increasing in  $\alpha$ ,  $R$ , and  $C_B$ ,  $R_D$  is increasing in all three parameters. This leaves  $\gamma$ :

$\partial A / \partial \gamma = \frac{1}{2}C_B$ ;  $\partial B / \partial \gamma = -\frac{1}{2}C_B(R + \alpha)$ .  $A \cdot \partial A / \partial \gamma + 2 \cdot \partial B / \partial \gamma = \frac{1}{2}C_B[\frac{1}{2}C_B \cdot \gamma - (R + \alpha)]$ ; the absolute value of this is less than  $\frac{1}{2}C_B \cdot A$ , and  $A$  is less than  $(A^2 + 4B)^{1/2}$ , so  $\partial R_D / \partial \gamma > 0$ , and  $R_D$  decreases in  $1 - \gamma$ . *Q.E.D.*

*Proof of Corollary 4:* (i) Since  $R \geq 1$ ,  $W^{AD} = R+G \geq 1+G = W^K$ , so the "always default" equilibrium weakly dominates autarky.  $W^{SD} = W^{AD} + \frac{1}{2}C_B[1-\frac{1}{2}p^L(1-\gamma)]$ ; since  $p^L$  and  $1-\gamma < 1$ ,  $W^{SD} > W^{AD}$ , and the "sometimes default" equilibrium strictly dominates the "always default" equilibrium.

(ii) Part (i) implies that the regulator chooses the "sometimes default" equilibrium if "never default" is infeasible. Otherwise, she chooses "never default" over "sometimes default" if  $W^{ND} > W^{SD}$ , and vice versa. Substituting for  $p^H$  in  $W^{ND}$  and  $p^L$  in  $W^{SD}$ , rearranging terms, and multiplying by one half gives the condition in the text. *Q.E.D.*

### References

- Baer, Herbert and John McElravey (1992), "The Changing Impact of Capital Requirements on Bank Growth: 1975 to 1991," Federal Reserve Bank of Chicago, working paper.
- Bernanke, Ben (1983), "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," American Economic Review 73: 257-276.
- Boyd, John and Edward Prescott (1986), "Financial Intermediary Coalitions," Journal of Economic Theory 38: 211-221.
- Buser, Stephen, Andrew Chen and Edward Kane (1981), "Federal Deposit Insurance, Regulatory Policy, and Optimal Bank Capital," Journal of Finance 35: 51-60.
- Committee on Banking Regulations and Supervisory Practices (1987), Proposals for International Convergence of Capital Measurement and Capital Standards (December), Bank for International Settlements.
- Diamond, Douglas (1984), "Financial Intermediation and Delegated Monitoring," Review of Economic Studies 51: 393-414.
- Diamond, Douglas (1991), "Monitoring and Reputation: The Choice Between Bank Loans and Directly Placed Debt," Journal of Political Economy 99: 689-721.
- Diamond, Douglas and Philip Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy 91: 401-19.
- Furlong, F. and M. Keeley (1989), "Capital Regulation and Bank Risk-Taking: A Note," Journal of Banking and Finance 13: 883-891.
- Genotte, Gerard and David Pyle (1991), "Capital Controls and Bank Risk," Journal of Banking and Finance 15: 805-824.
- Gorton, Gary and George Pennacchi, "Financial Intermediaries and Liquidity Creation," The Journal of Finance 45: 49-72.
- James, Chris (1991), "The Losses Realized in Bank Failures," Journal of Finance 46: 1223-1242.
- Kahane, Y. (1977), "Capital Adequacy and the Regulation of Financial Intermediaries," Journal of Banking and Finance 1: 207-218.
- Keeley, M. and F. Furlong (1990), "A Re-examination of Mean-Variance Analysis of Bank Capital Regulation," Journal of Banking and Finance 14: 69-84.
- Kim, Daesik and Anthony Santomero (1988), "Risk in Banking and Capital Regulation," Journal of Finance 43: 1219-1233.

- Koehn, M. and A. Santomero (1980), "Regulation of Bank Capital and Portfolio Risk," Journal of Finance 35: 1235-1244.
- Lam, Chun H. and Andrew H. Chen (1985), "Joint Effects of Interest Rate Deregulation and Capital Requirements on Optimal Bank Portfolio Adjustments," Journal of Finance 40: 563-75.
- Mailath, George and Loretta Mester (1993), "A Positive Analysis of Bank Closure," Journal of Financial Intermediation 3: 272-299.
- Marcus, Alan (1984), "Deregulation and Bank Financial Policy," Journal of Banking and Finance 8: 557-65.
- Morgan, George (1992), "Capital Adequacy," in The New Palgrave Dictionary of Money and Finance, Peter Newman, Murray Milgate, and John Eatwell, eds. (Macmillan Press).
- Myers, Stewart (1977), "Determinants of Corporate Borrowing," Journal of Financial Economics 5: 147-175.
- Peltzman, Saro (1970), "Capital Investment in Commercial Banking and Its Relationship to Portfolio Regulation," Journal of Political Economy 78: 1-26.
- Pringle, John (1974). "The Capital Decision in Commercial Banks," Journal of Finance 29: 779-95.
- Pringle, John (1975), "Bank Capital and the Performance of Banks as Financial Intermediaries," Journal of Money, Credit, and Banking 7: 545-559.
- Rajan, R. (1992), "Insiders and Outsiders: The Choice Between Informed and Arm's Length Debt," Journal of Finance 47: 1367-1400.
- Qi, Jianping (1993), "Efficient Investment and Depository Intermediaries," University of South Florida, working paper.
- Ramakrishnan, R. and Anjan Thakor (1984), "Information Reliability and a Theory of Financial Intermediation," Review of Economic Studies 52: 415-432.
- Santomero, Anthony and Ron Watson (1977), "Determining the Optimal Capital Standards for the Banking Industry," Journal of Finance 32: 1267-82.
- Sharpe, S. (1990), "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships," Journal of Finance 45: 1069-1087.
- Sharpe, William (1978), "Bank Capital Adequacy, Deposit Insurance and Security Values," Journal of Financial and Quantitative Analysis 13: 701-18.
- Slovin, Myron, Marie Sushka, and John Polonchek (1993), "The Value of Bank Durability: Borrowers as Bank Stakeholders," Journal of Finance 48: 247-266.

# Table 1

## Notation

|             |   |
|-------------|---|
| $\gamma$    | Discount factor on securities other than demand deposits, which are traded at date 2. |
| $p$         | Probability of consuming at date 2; a consumer's type.                                |
| $C_t$       | Consumption at date $t$ .   |
| $R$         | The average return on bank assets.  |
| $\pm\alpha$ | The shock to bank asset returns; equally likely.                                      |
| $D_t$       | Demand deposits of a bank at time $t$ .   |
| $N_t$       | The number of shares issued by a bank at time $t$ .                                   |
| $n_t(p)$    | The number of shares purchased by a consumer of type $p$ at date $t$ .                |
| $\Delta D$  | The reduction in deposits which equals additional capital raised at date 1.           |
| $P_t$       | The price of a bank share at time $t$ .   |
| $R_D$       | Interest factor earned on demand deposits from date 1 to either date 2 or date 3.     |
| $E_1$       | The expected value of bank equity at date 1.  |
| $G$         | Date 2 endowment of the consumption good.   |
| $T_2$       | Lump sum tax levied at date 2 to finance government deposit insurance at date 3.      |
| $C_B$       | Bank charter value.   |

## Figure 1 Sequence of Events

