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THE SIZE AND TIMING OF DEVALUATIONS IN CAPITAL-CONTROLLED DEVELOPING ECONOMIES

Robert Flood
Nancy Marion

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ABSTRACT

A developing country often pegs its exchange rate to a single currency, such as the U.S. dollar, even though it faces a higher inflation rate than the country to which it is pegged. As a consequence, it experiences real exchange-rate misalignments and a series of easily-anticipated devaluations. While the chaotic capital market events surrounding anticipated devaluations are avoided through quantitative capital controls, the country is left with the classic devaluation problem: when should it devalue, and by how much?

In this paper, we consider a policymaker who pegs the nominal exchange rate and adjusts the peg periodically so as to minimize a set of costs. The control problem is made difficult by the fact that the future times for devaluations are currently unknown stochastic variables. Characterizing the real exchange rate as regulated Brownian motion permits the cost minimization problem to be solved explicitly. The size and timing of devaluations are jointly determined outcomes of optimizing behavior.

The framework yields insights into how changes in the stochastic environment affect both the size and timing of devaluation. These insights provide guidance about the determinants of devaluation episodes even when Brownian motion is not the relevant stochastic process for real exchange rates. Using cross-sectional data on 80 peg episodes from seventeen Latin American countries over the 1957-1990 period, we find empirical support for the model's main predictions.

Robert Flood
Research Department
International Monetary Fund
700 19th Street
Washington, DC 20431
and NBER

Nancy Marion
Department of Economics
Dartmouth College
Hanover, NH 03755

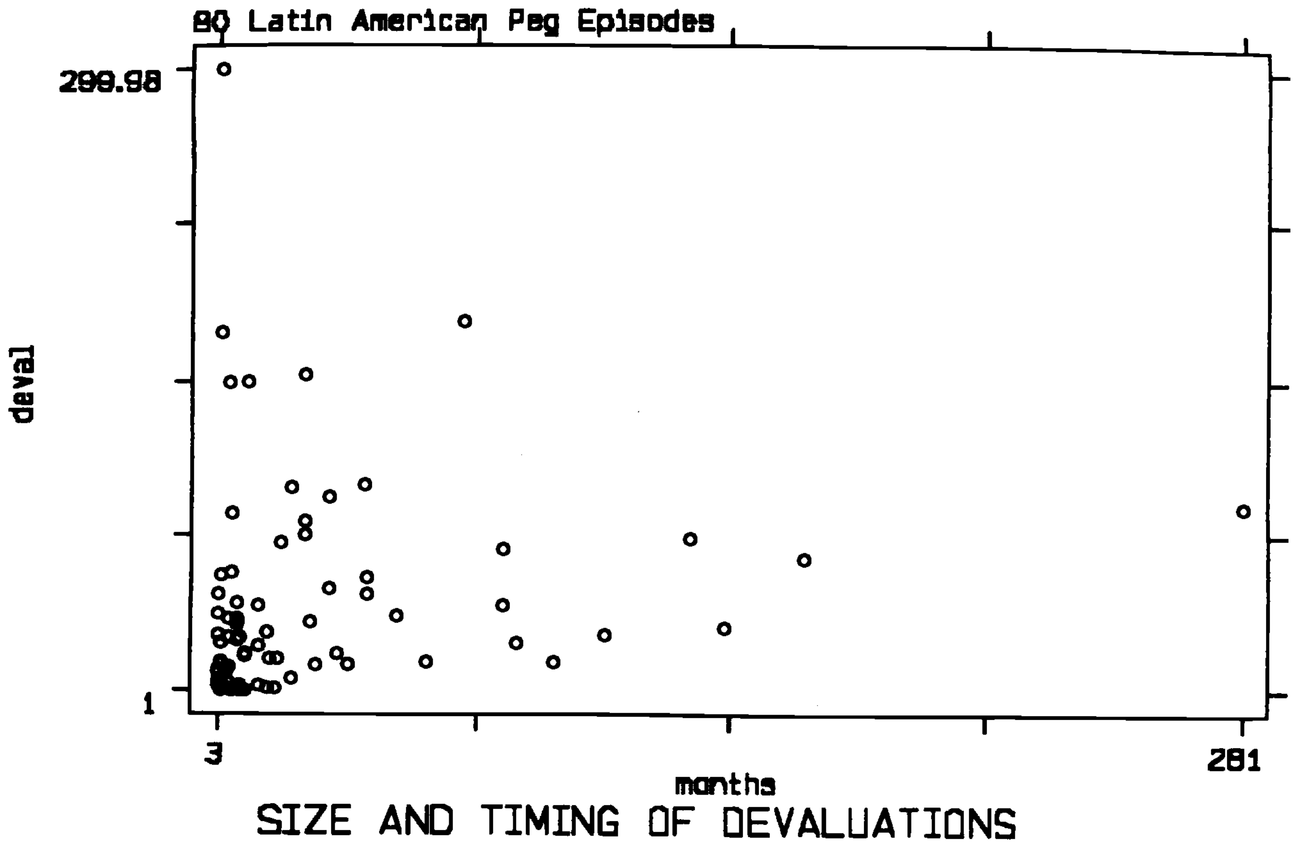
1. Introduction

A developing country often pegs its exchange rate to a single currency, such as the U.S. dollar, even though it faces a higher inflation rate than the country to which it is pegged. As a consequence, it experiences real exchange-rate misalignments and a series of easily-anticipated devaluations. It is typically the case in such a developing country that the chaotic capital market events surrounding anticipated devaluations are avoided through quantitative capital controls.[1] The country is left, however, with the classic devaluation problem: When is the optimal time to readjust the peg and what is the optimal size of the adjustment? It is generally believed that there is a trade-off between the size and timing of devaluations, with the country choosing a series of small and frequent devaluations or a set of large and infrequent ones.

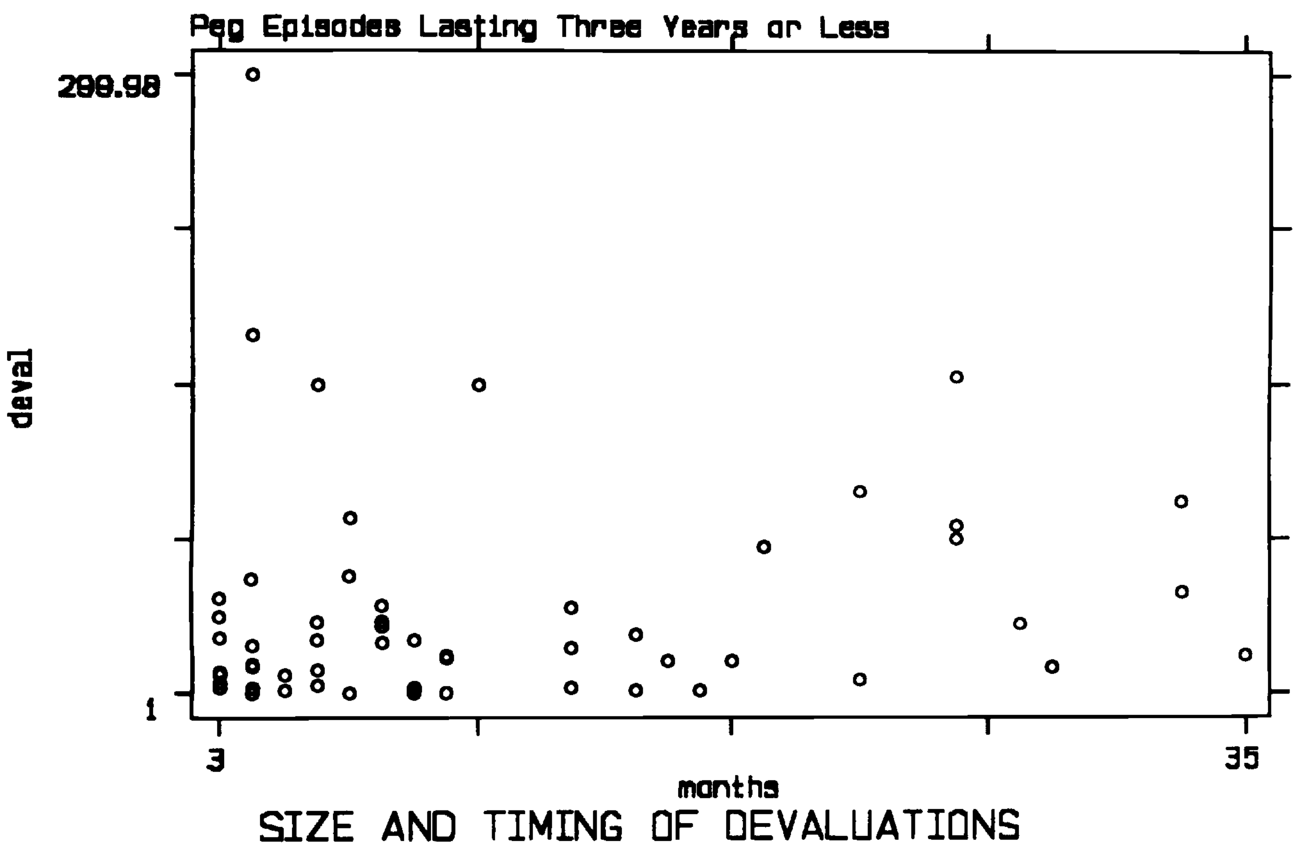
This paper is motivated by the considerable dispersion seen in the size and timing of devaluations across developing countries. The top panel of Figure 1 illustrates this dispersion for a sample of dollar pegs in Latin America over the 1957-1990 period. The bottom panel of Figure 1 shows the dispersion only for the pegs that lasted three years or less. In each panel, the size of devaluation is measured by the percentage change in the nominal exchange rate on the day the peg is abandoned. The time of devaluation is measured by the number of months spent on a particular peg. The scatterplots reveal a variety of outcomes for the size and timing of devaluations. How does a policy authority go about choosing the optimal mix of devaluation size and frequency, given a set of costs attached to the devaluation decision?

Our purpose is to develop a simple framework that identifies and tests for some key determinants of the size and timing of devaluations. Section 2 develops a model of the policymaker responsible for devaluation and presents some empirical predictions implied

[1]The consequences of anticipated realignments in economies with relatively free capital movements are discussed in Goldstein *et al* (1993). Capital controls are discussed in Edwards (1989).



80 Latin American Peg Episodes
299.98
1
deval
3
281
months
SIZE AND TIMING OF DEVALUATIONS



Peg Episodes Lasting Three Years or Less
299.98
1
deval
3
35
months
SIZE AND TIMING OF DEVALUATIONS

FIGURE 1

by the model. An attractive feature of the model is that the size and timing of devaluations are jointly determined outcomes of optimizing behavior. Previous work has taken the size of devaluation as exogenous when calculating the time that a fixed exchange rate ends. Another interesting feature of the model is that the policymaker faces a problem in which the future times for devaluations are currently unknown stochastic variables. Typical maximization methods are not helpful for such control problems. By characterizing the real exchange rate as "regulated Brownian motion," we are able to use methods developed by Harrison (1985) to solve the policymaker's control problem despite the stochastic timing issue.

Section 3 presents some empirical evidence in support of the model, using data on exchange-rate pegs in Latin America. One interesting finding is that higher variance of the real exchange rate does not mean that devaluations, on average, must occur more frequently. For our sample of Latin American pegs, higher variance of the real exchange rate actually increases the time spent on the peg. This outcome is consistent with the theory we develop, because when size and timing of devaluations are jointly determined, the effect of increased variance on timing can go in either direction. Higher variance reduces time spent on a peg for a *given* devaluation size. But higher variance also increases the amount of devaluation the policymaker is willing to undertake at the end of the peg and so may delay the actual devaluation time. Section 4 offers some conclusions and directions for further research.

2. Theoretical model

In order to proceed with the analysis, we start with a simplified case and examine to what extent it can shed light on the factors that influence the size and timing of devaluations. We consider the case of a developing country with capital controls, where the decisions about the peg rest with the authorities. We also identify the deviation of the real exchange rate from its desired level--what is often termed "exchange-rate misalignment"--as the single distortion that influences the exchange-rate decision.

When a developing country fixes the value of its currency to that of another country, its real exchange rate nevertheless changes over time. The movement in the real exchange rate comes about because the two countries usually experience different trend inflation rates and because domestic and foreign prices are subject to random shocks. At some point the real exchange rate may depart so much from its desired level that an adjustment in the peg is appropriate.

Figure 2 illustrates the stochastic path of the real exchange rate for a typical developing country that pegs its currency to that of the U.S. dollar. The real exchange rate is q_t, where $q_t = e_t + p^*_t - p_t$. In the notation, e is the log of the nominal exchange rate (home currency/foreign currency), p* is the log of the foreign price level and p is the log of the domestic price level. When the peg starts at time zero, the real exchange rate is set at q_0

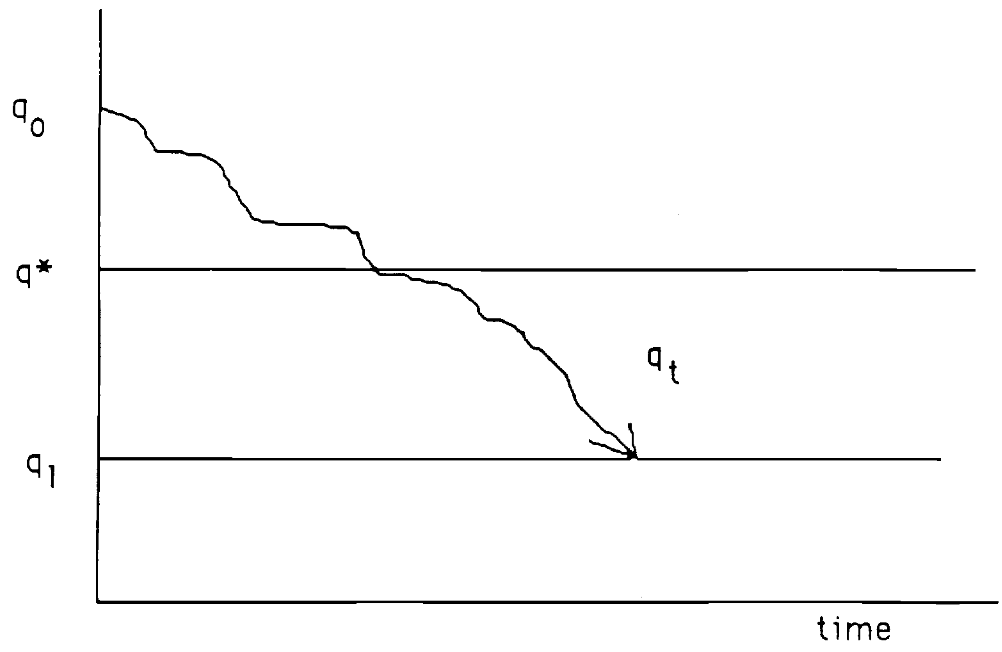


FIGURE 2

by fixing the nominal exchange rate at e_o. The desired, or equilibrium, real exchange rate is the variable, q^*. It is taken as exogenous with respect to the devaluation decision.

The real exchange rate follows a stochastic process. We assume the policymaker can only influence its behavior by adjusting the nominal exchange rate. In light of the data set we study in the next section, we assume that the real exchange rate process has a negative trend.

Given this trend, the real exchange rate eventually will hit some policy-determined lower barrier q_ℓ at which time the peg is abandoned. A devaluation of the home currency instantaneously moves the real exchange rate back above the lower barrier.[2]

The policymaker can determine the optimal amount of real appreciation to incur over the peg spell by optimally setting the initial value of the real exchange and the lower barrier. If the real exchange rate continues to be characterized by a single stochastic process, then the exchange-rate band bounded by q_o and q_ℓ represents the optimal size of devaluation. With this band optimally set, the stochastic process characterizing the real exchange rate gives a distribution of times when the real exchange rate will hit the lower barrier. The expected time when the real exchange rate hits the lower barrier can then be determined. This gives the expected time of devaluation.

To keep the problem comparable to the stochastic flow problems analyzed in Harrison (1985), we normalize by setting the lower barrier at zero. We set:

$$0 = q_\ell - q_\ell \qquad x^* = q^* - q_\ell$$

(1)

$$x_o = q_o - q_\ell \qquad x_t = q_t - q_\ell$$

[2]One can extend the problem to the case where there is both a lower barrier q_ℓ and an upper barrier q_u, with $q_\ell \leq q_o \leq q_u$. The home currency is devalued when q_t hits q_ℓ and it is upvalued when q_t hits q_u. In the data set we study, pegs always end with devaluations, so we focus on the case of a single, lower barrier.

Now zero is the normalized lower barrier, x_o is the normalized starting state for the real exchange rate, **x*** is the normalized equilibrium real exchange rate, and x_t is the normalized real exchange rate at time t.

In the absence of intervention by the policymaker, let the real exchange rate follow (μ, σ) Brownian motion:

$$(2) \qquad x_t = x_o + \mu t + \sigma y_t$$

where y is a Wiener process with independent incremental shocks that are normally distributed with mean zero and unit variance. It follows that μ and σ^2 are the non-stochastic drift and the variance of x, respectively.

Letting $x_o > 0$, we consider the processes (ℓ, z) which are obtained from x by imposing a lower control barrier at zero. The variables ℓ and z have the following properties:

(3) ℓ is increasing and continuous with $\ell_o = 0$,

(4) $z_t \equiv x_t + \ell_t \geq 0$ for all $t \geq 0$, and

(5) ℓ increases only when $z = 0$.

In the terminology of Harrison (1985), z_t is a *regulated* Brownian motion while x_t is *unregulated* Brownian motion. We can interpret ℓ_t as the cumulative increase in the real exchange rate effected by the policymaker up to time t.

In fixing the nominal exchange rate, the policymaker is concerned with a trade-off between the cost of real exchange-rate misalignment and the cost associated with adjusting the peg. The policymaker's problem can be characterized by means of a loss function. The policymaker's objective is to minimize the expected present value of costs incurred over an infinite planning horizon by fixing the nominal exchange rate and periodically readjusting it. We consider two types of costs, a flow cost and a fixed cost. The flow cost of pegging is the misalignment cost, and it is measured by the square of the deviation of the actual real exchange rate from its desired level, $(z_t - x^*)^2$. The fixed cost of pegging is incurred whenever the peg is readjusted. It is measured by a fixed transaction cost, δ, times the size of the readjustment, $d\ell$. When discounting is continuous at interest rate λ, the policymaker's problem amounts to the minimization of:

$$(6) \qquad k(x_o) \equiv E_{x_o}\{\int_0^{\infty} e^{-\lambda t}[(z_t - x^*)^2 dt + \delta d\ell]\}; \quad x_o > 0$$

In terms of notation, $k(\cdot)$ is the expected present value of costs and E_{xo} is the expectations operator conditional on setting the real exchange rate at x_o at the start of the peg, where x_o is above the lower barrier of zero. The policymaker can at any time increase the real exchange rate by any amount desired but is obliged to keep $z_t \geq 0$.

The policymaker has rational expectations and minimizes (6) with respect to x^* and x_o. Recalling the normalization in (1), this means that the policymaker chooses an initial value for the real exchange rate (q_o) and a lower barrier (q_ℓ) so as to minimize expected discounted costs, given the equilibrium real exchange rate q^*. The chosen exchange-rate band ($x_o = q_o - q_\ell$) represents the optimal real appreciation over a peg spell. If the real exchange rate follows the same stochastic process over the infinite planning horizon of the policymaker, the band x_o represents the optimal size of devaluation at the end of each peg episode. We shall see that the optimal band width will depend on the characteristics of the path of the real exchange rate, the fixed cost of adjustment and the interest rate.

To calculate the expected discounted cost in (6), it suffices to solve the differential equation $\lambda k(x_o) - \mu k'(x_o) - (1/2)\sigma^2 k''(x_o)$ subject to the boundary requirement that $k'(0) = -\delta$. The solution is:

(7) $$k(x_o) = g(x_o) + k(0)e^{-\alpha(\lambda)x_o}$$

where

(8) $$g(x_o) \equiv E_{x_o}\{ \int_0^T e^{-\lambda t}(x_t - x^*)^2\, dt \} \quad ,$$

(9) $$k(0) = \frac{g'(0) + \delta}{\alpha(\lambda)} \quad ,$$

(10) $$\alpha(\lambda) \equiv (\frac{1}{\sigma^2})[(\mu^2 + 2\sigma^2\lambda)^{1/2} + \mu] \quad ,$$

and T is the first time the real exchange rate hits the lower boundary.[3]

If T were known with certainty, the calculation of $g(x_O)$ in (8) and hence the calculation of expected discounted costs in (7) would be straightforward. The difficulty arises from the fact that T is stochastic. Fortunately, the Ito stochastic calculus provides a solution strategy for calculating $g(x_O)$. (See the appendix for details.) We can use our solution for $g(x_O)$ to rewrite the expected discounted costs as:

(11) $$k(x_o) = \frac{(x_o - x^*)^2}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{2(x_o - x^*)\mu + \sigma^2}{\lambda^2} + \frac{1}{\alpha(\lambda)}[\frac{2\mu}{\lambda^2} - \frac{2x^*}{\lambda} + \delta]\, e^{-\alpha(\lambda)x_o}$$

The policymaker minimizes (11) with respect to x* and x_O. The first-order conditions are:

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[3]See Harrison (1985), pp. 36-48, 92-93.

(12) $$\frac{\partial k}{\partial x^*} = -(x_o - x^*) - \frac{\mu}{\lambda} - \frac{e^{-\alpha(\lambda)x_o}}{\alpha(\lambda)} = 0$$

and

(13) $$\frac{\partial k}{\partial x_o} = (x_o - x^*) + \frac{\mu}{\lambda} - [\frac{\mu}{\lambda} - x^* + \frac{\delta\lambda}{2}]\, e^{-\alpha(\lambda)x_o} = 0$$

From the first-order conditions we find that the optimal values for x^* and x_o are:

(14) $$x^* = \frac{\mu}{\lambda} + \frac{\lambda\delta}{2} + \frac{1}{\alpha(\lambda)}$$

and

(15) $$x_o + \frac{e^{-\alpha(\lambda)x_o}}{\alpha(\lambda)} = \frac{1}{\alpha(\lambda)} + \frac{\lambda\delta}{2}$$

The second-order conditions confirm that these solutions for x^* and x_o minimize expected discounted costs.[4]

The optimal band width (the optimal amount of real appreciation over a peg spell, or the optimal size of devaluation) is given by $x_o = q_o - q_\ell$. From (10) and (15), we see that

[4]The second-order conditions for a global minimum require that $\alpha > 0$. The second-order conditions are:

$$\frac{\partial^2 k}{\partial x^{*2}} = \frac{2}{\lambda} > 0$$

$$\frac{\partial^2 k}{\partial x_o^2} = (\frac{2}{\lambda})[1 - e^{-\alpha x_o}] > 0 \quad \text{if } \alpha > 0 \text{ since } x_o > 0$$

the optimal band width depends in a nonlinear way on the drift and variance of the real exchange rate path, the interest rate and the cost of peg adjustment. Note that band width does not depend on the level of the equilibrium real exchange rate. An anticipated change in the equilibrium real exchange rate causes an identical change in q_0 and q_ℓ, leaving the band width unchanged.

We next calculate the expected time of devaluation. Let T(0) denote the first time $t > 0$ at which $x_t = 0$, that is, the first time that the real exchange rate hits the lower barrier, triggering a devaluation.

Calculating the Laplace transform $E_{x_0}[e^{-\lambda T(0)}]$ yields:[5]

$$(16) \qquad E_{x_0}[e^{-\lambda T(0)}] = e^{-\alpha(\lambda)x_0}, \qquad x_0>0 \quad ,$$

where $\alpha(\lambda)$ is defined in (10) and x_0 is given in (15). Note that the expected time of devaluation is conditional on the optimal band width. In the previous literature on collapsing pegs, (e.g. Flood and Marion, 1982; Blanco and Garber, 1986), the size of devaluation is taken as exogenous when determining the expected date of devaluation. The present model appropriately treats the expected size and time of devaluation as jointly determined. The drift and variance of the real exchange rate path, the interest rate and the cost of peg adjustment jointly determined the optimal size and time of devaluation.

We now develop some predictions of the theoretical model that can be tested using data on exchange-rate pegs. In particular, we examine how a change in the drift and variance of the real exchange rate affect the optimal size and timing of devaluations.

First, we examine the effect of the drift and variance parameters on the optimal band for the real exchange rate, which gives the optimal amount of real appreciation over the peg

[5]See Harrison (1985), pp. 38-44, for details.

spell, or the optimal devaluation size. Given our solutions for α and x_O in (10) and (15), it follows that :[6]

$$\frac{\partial x_o}{\partial \mu} = (\frac{\partial x_o}{\partial \alpha})(\frac{\partial \alpha}{\partial \mu}) < 0 \quad (17)$$

$$\qquad (-) \quad (+)$$

and

$$\frac{\partial x_o}{\partial \sigma^2} = (\frac{\partial x_o}{\partial \alpha})(\frac{\partial \alpha}{\partial \sigma^2}) > 0 \quad (18)$$

$$\qquad (-) \quad (-)$$

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[6]From (15), let $F(\alpha, x_O(\alpha)) = \quad x_o + \frac{e^{-\alpha(\lambda)x_o}}{\alpha(\lambda)} - \frac{1}{\alpha(\lambda)} - \frac{\lambda\delta}{2} = 0.$

Then using implicit differentiation, we find that

$$\frac{\partial x_o}{\partial \alpha} = \frac{-\partial F/\partial \alpha}{\partial F/\partial x_o} = \frac{-[1 - e^{-\alpha x_o}(1 + x_o\alpha)]}{\alpha^2(1 - e^{-\alpha x_o})} < 0$$

since $\alpha x_O > 0$ and $0 < e^{-\alpha x_o}(1 + x_o\alpha) < 1$.

From (10) it follows that

$$\frac{\partial \alpha}{\partial \mu} = \frac{1}{\sigma^2}[\frac{(\mu^2 + 2\sigma^2\lambda)^{1/2} + \mu}{(\mu^2 + 2\sigma^2\lambda)^{1/2}}] > 0$$

To satisfy the second-order conditions for minimization of the loss function, $\alpha(\lambda)$ in (10) must be positive, implying that the numerator and denominator of $d\alpha/d\mu$ must each be positive, even when $\mu<0$. It also follows that:

$$\frac{\partial \alpha}{\partial \sigma^2} = \frac{-(\mu^2 + \sigma^2\lambda) - \mu(\mu^2 + 2\sigma^2\lambda)^{1/2}}{(\mu^2 + 2\sigma^2\lambda)^{1/2}(\sigma^2)^2} \gtrless 0 \quad \text{for} \quad \mu < 0$$

For plausible parameter values $d\alpha/d\sigma^2$ is negative.

Equation (17) indicates that the policymaker who faces a more negative drift will start the peg at a higher real exchange rate, given a normalized lower barrier of zero. The wider band within which the real exchange rate can fluctuate without triggering a devaluation implies that the policymaker accepts a greater real exchange-rate appreciation over the peg spell. Alternatively, we can interpret equation (17) as stating that more negative drift increases the optimal devaluation size.

Equation (18) shows that an increase in the variance of the real exchange rate increases the optimal band within which the real exchange rate can fluctuate without setting off a devaluation. Thus holding drift and other determinants constant, higher variance in the real exchange rate should increase the optimal devaluation size.

In order to calculate the effect of drift and variance on the optimal time of devaluation, we again invoke rational expectations so that the expected time when the real exchange rate hits the lower barrier is equal to the actual time it hits the lower barrier adjusted for an error term, ε_t, that is uncorrelated with x_o:

$$(19) \quad E_{x_o}[e^{-\lambda T(0)}] = e^{-\lambda T(0)} e^{\varepsilon_t}$$

Substituting (19) into (16), taking logs of both sides and rearranging terms yields

$$(20) \quad T(0) = \frac{\alpha(\lambda)x_o}{\lambda} + \frac{\varepsilon_t}{\lambda} \quad .$$

The effect on the timing of devaluation of a change in the drift component of the real exchange rate process is :

$$(21) \quad \frac{\partial T}{\partial \mu} = (\frac{1}{\lambda})[\alpha(\frac{\partial x_o}{\partial \alpha})(\frac{\partial \alpha}{\partial \mu}) + x_o(\frac{\partial \alpha}{\partial \mu})] \gtreqless 0$$

$$\qquad (+)(+)\ (-)\ \ (+)\quad (+)\ (+)$$

and the effect of a change in the variance of the real exchange rate is:

$$\frac{\partial T}{\partial \sigma^2} = (\frac{1}{\lambda})[\alpha(\frac{\partial x_o}{\partial \alpha})(\frac{\partial \alpha}{\partial \sigma^2}) + x_o(\frac{\partial \alpha}{\partial \sigma^2})] \gtrless 0 \tag{22}$$

$$(+) \quad (+)(-) \quad (-) \quad (+) \quad (-)$$

A more negative drift or a larger variance of the real exchange rate has an uncertain impact on the timing of devaluation. The ambiguity with respect to drift is due to opposing forces. For a *given* band size (a given x_o), a larger negative drift always *reduces* the time spent on a peg since the real exchange rate hits the lower barrier more quickly. However, a larger negative drift also increases the optimal band size. The same ambiguity arises with respect to real exchange-rate variance. For a *given* band size, a higher variance of the real exchange rate *reduces* time on the peg. But higher variance also increases the optimal size of the band. Thus the effects of drift and variance on the timing of devaluations is ultimately an empirical question.[7]

3. Empirical Evidence

In this section we use data from capital-controlled fixed-exchange rate episodes in Latin America to test some predictions of the model. The Latin American countries were chosen because they have easily identifiable dollar pegs and because they are most subject to the type of real exchange rate misalignment that is highlighted by our choice of loss function.

[7]Even though the policymaker can calculate the expected time of devaluation, private agents may not be able to do so if they lack complete information about the policymaker's fixed cost of devaluation or the discount rate used to calculate the present value of costs. Even if private agents can extract this information and infer the expected timing of devaluation, they cannot initiate a speculative run on the currency due to the presence of controls. It is conceivable that private agents with market power could alter their pricing policies in anticipation of a devaluation, thus affecting the behavior of the real exchange rate and the timing of devaluation. In our framework we do not model pricing behavior explicitly. Relative prices are treated as Brownian motion.

Recall that the model assumes that the authorities peg the nominal exchange rate so that the value of the real exchange rate at the start of a peg episode and the lower barrier for the real exchange rate that triggers a devaluation are optimally chosen. When the real exchange rate follows regulated Brownian motion with negative drift, the stochastic properties of the real exchange rate process are determinants of the optimal band within which the real exchange rate fluctuates without triggering a devaluation. The band yields the optimal real appreciation over a peg spell. If the real exchange rate follows the same stochastic process over the policymaker's planning horizon, the characteristics of the process determine the optimal devaluation size at the end of each peg spell. Since the timing of devaluation is conditional on the optimal band width, the properties of the real exchange rate also help determine the optimal time of devaluation.

The exact nonlinear specifications for the optimal size and expected time of devaluation are given in (15) and (16). The partial effects of the drift and variance of the real exchange rate on the optimal band width are given by (17) and (18), and their effects on the timing of devaluation are given by (22) and (23).

We start by examining the appropriateness of our assumption about the real exchange rate process during fixed exchange rate episodes. We then test whether the drift and variance of the real exchange rate process affect the size and timing of devaluation in the ways predicted by the model. We conduct these tests without taking the model specifications too seriously at first. At this point we are interested in whether regressions based on linear and nonlinear approximations will show the predicted effects of drift and variance. Since these initial tests are promising, we then consider the exact model specifications and test how well they hold using our data.

We use monthly data from actual dollar peg episodes in Latin America. The data come from the IMF's *International Financial Statistics* compact disk dated September 1993. Since we want observations over the entire peg spell, data availability requires us to consider peg episodes that start on or after January, 1957, and end on or before January,

1990. A country must maintain a fixed nominal exchange rate for at least three months to be included in the sample. All peg spells in the sample meet the requirement that the initial value of the real exchange rate is greater than its value at the end of the peg episode. Based on these criteria, we end with a sample of 80 peg episodes from 17 countries in Latin America.[8]

To construct the real exchange rate, we use a bilateral real exchange rate index based on end-of-the month nominal exchange rates (home currency/dollar) and monthly average consumer price indices for the home country and the U.S. The real exchange rate series is normalized so that the index is always set at 100 at the end of the month in which the exchange rate is initially fixed. The observed real exchange rate at the end of the last full month on the peg is used to proxy the lower barrier. Thus the actual band width, or actual real appreciation over a peg spell, is measured by mapping x_0 back into $(q_0 - q_\ell)$, where $q_0 = 100$ and q_ℓ is the observed lower barrier described above. Because real exchange rate data are available only at monthly frequencies, this measure of band width is somewhat imprecise. If real exchange rates are characterized by negative drift over the entire peg spell, this measure of band width probably understates the true band width since the actual starting date of a peg can precede the measured starting date and the actual stopping date of the peg can follow the measured stopping date.

The time on a peg is measured by the number of complete months during which the nominal exchange rate is fixed.

Some information about band width and time spent on the peg for our sample is summarized in Figure 3. The top histogram in the figure shows the variation in real exchange-rate appreciation (band width) over peg episodes. The average real appreciation

[8]The countries are Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Jamaica, Mexico, Nicaragua, Panama, Peru, Uruguay, and Venezuela. In all peg episodes, the home currency weakens when the peg is abandoned. Most peg episodes end with a devaluation, though some end with a move to a crawl or a float.

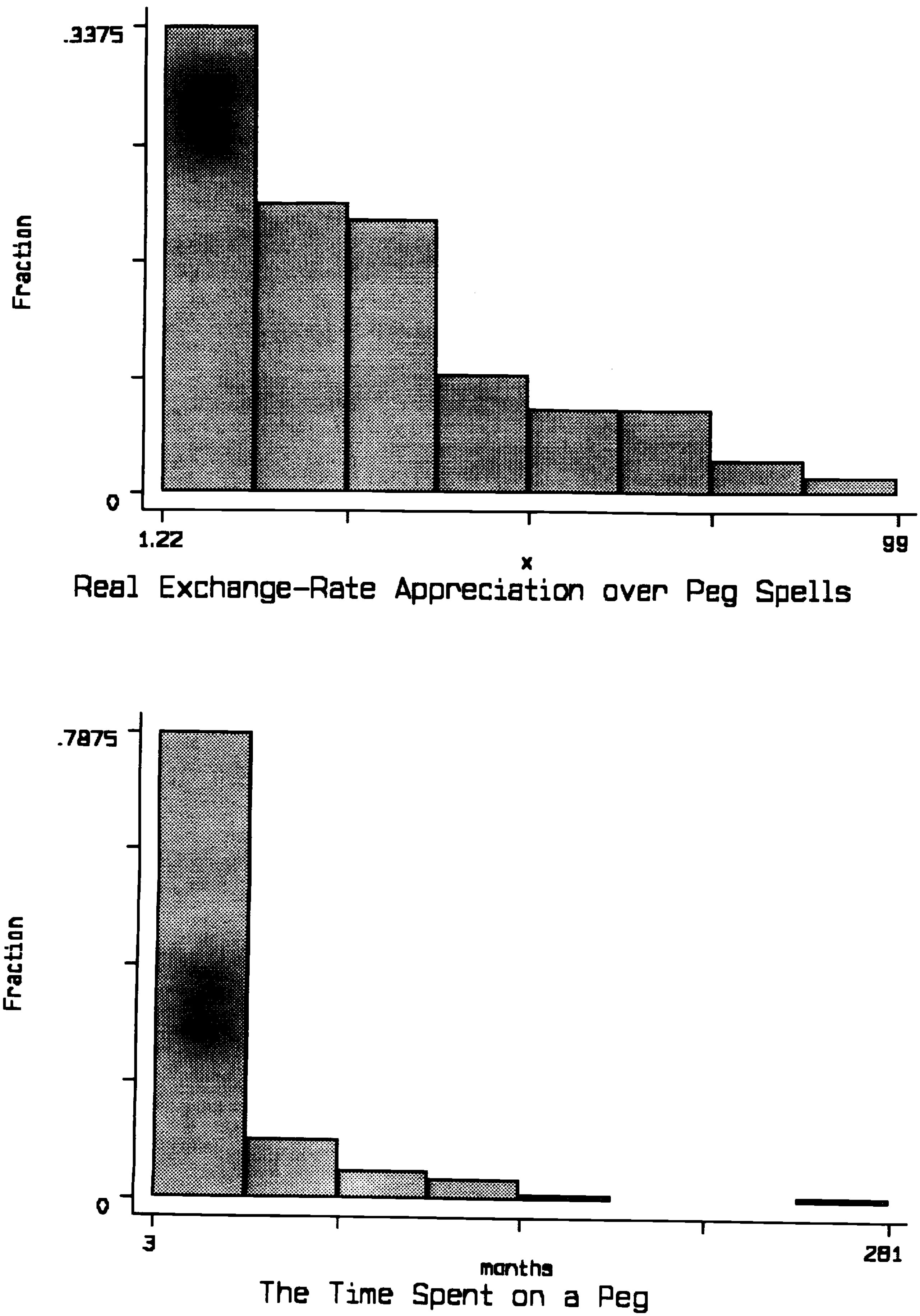

.3375
Fraction
0
1.22
99
x
Real Exchange-Rate Appreciation over Peg Spells
.7875
Fraction
0
3
281
months
The Time Spent on a Peg

FIGURE 3

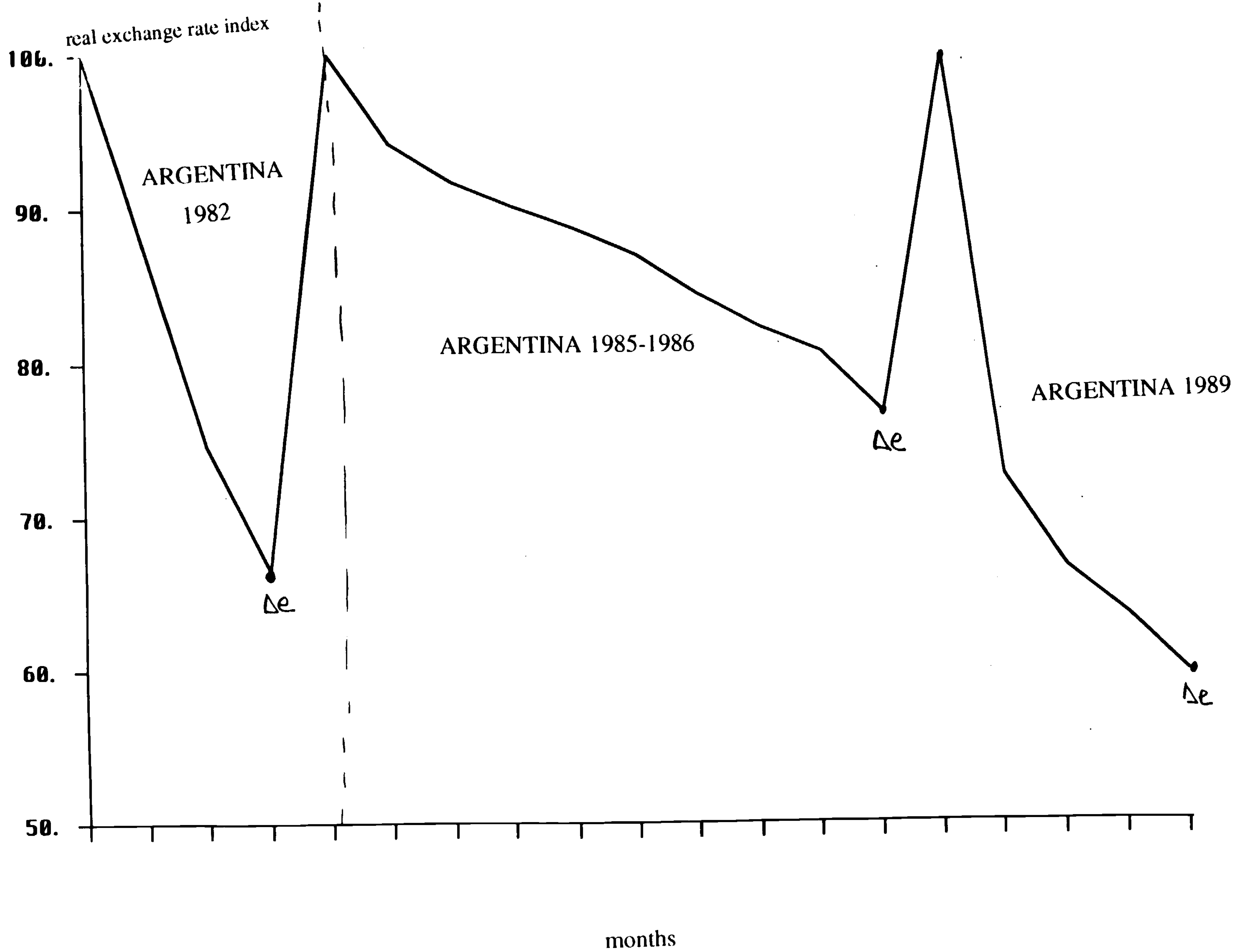
is 28 percent and the standard deviation around the mean is 21.75 percent. The median real appreciation is 23 percent. The range is considerable. Several countries exited a peg after experiencing real appreciations under 5 percent, while others permitted real appreciations in excess of 75 percent before ending their pegs. The histogram reveals that a third of the peg episodes had a real appreciation of 13 percent or less, while three-fourths of the episodes had a real appreciation under 38 percent.

The bottom histogram in Figure 3 illustrates the variation in the time spent on the peg. The average time spent on a dollar peg is 28 months, while the median duration is 10 months. The standard deviation around the mean and the range of the sample are substantial. The standard deviation of the duration is about 44 months. The range of peg durations is 3 months (by construction) to 281 months (the longest peg episode represents Paraguay's peg of 126 guaranies to the dollar between 1960 and 1984). The bottom histogram reveals that over three-fourths of the pegs in the sample end within three years.

For each peg spell, the drift of the real exchange rate is obtained by regressing the first difference of the monthly real exchange rate index on a constant and using the estimated value of the constant. The variance of the residuals from this regression is the measured variance of the real exchange rate over the course of the peg spell.

Figure 4 illustrates the behavior of the bilateral real exchange rate during selected peg episodes. The graphs support our notion that the real exchange rate is characterized by negative drift and periodic shocks. Latin American countries have generally had larger price level increases than the United States during fixed exchange-rate episodes, generating real appreciations. Some of the figures show a more moderate drift at the end of the spell. This phenomenon may call into question the model's assumption that the drift is always exogenous with respect to the behavior of the policy-maker or private agents.

Table 1 shows the effects of real exchange rate drift and variance on the size of the band as measured by the actual real appreciation over the peg spell. The estimation results in Table 1 are based on a simple cross-section linear regression using White's correction to



real exchange rate index
100.
90.
80.
70.
60.
50.
ARGENTINA
1982
ARGENTINA 1985-1986
ARGENTINA 1989
Δe
Δe
Δe
months

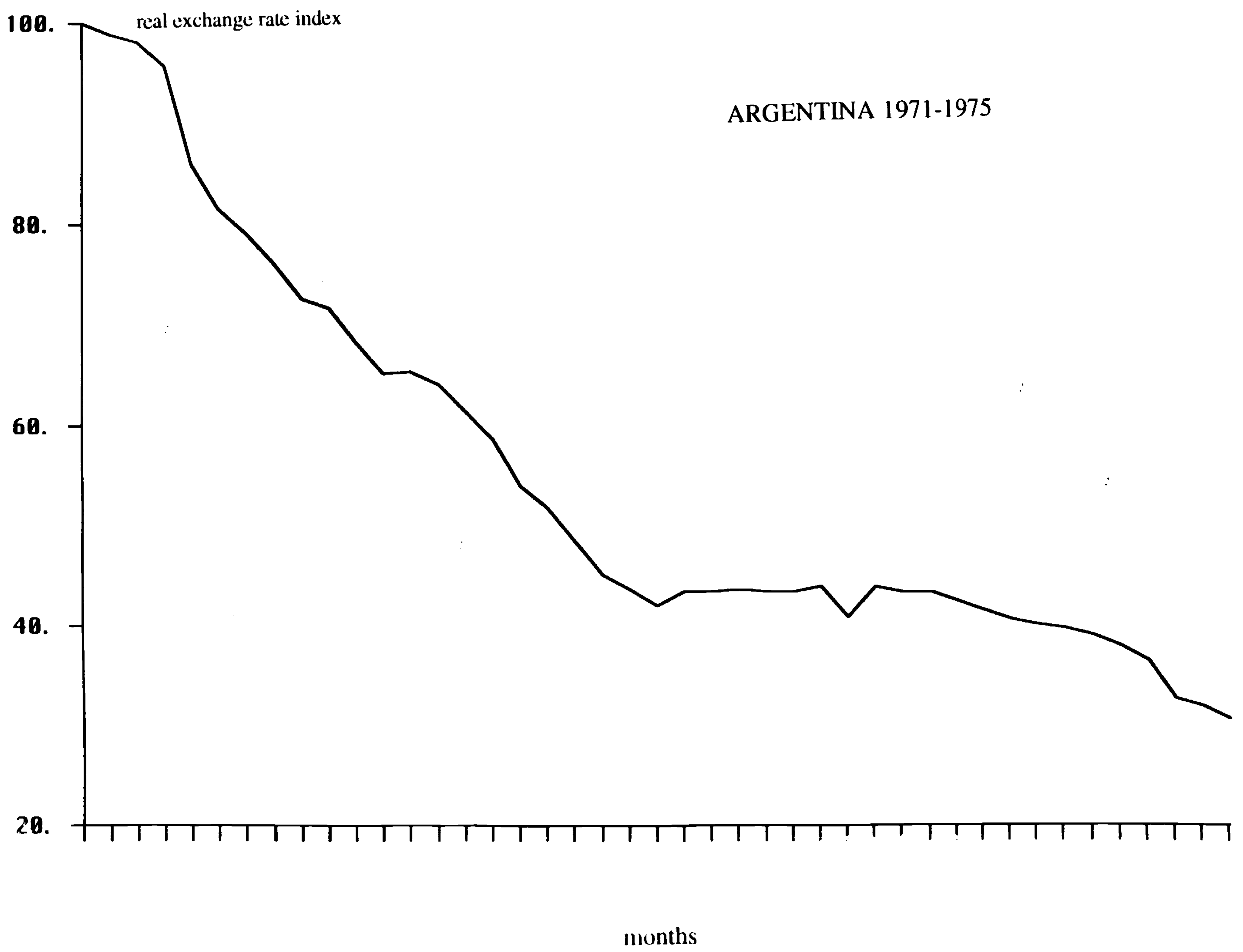
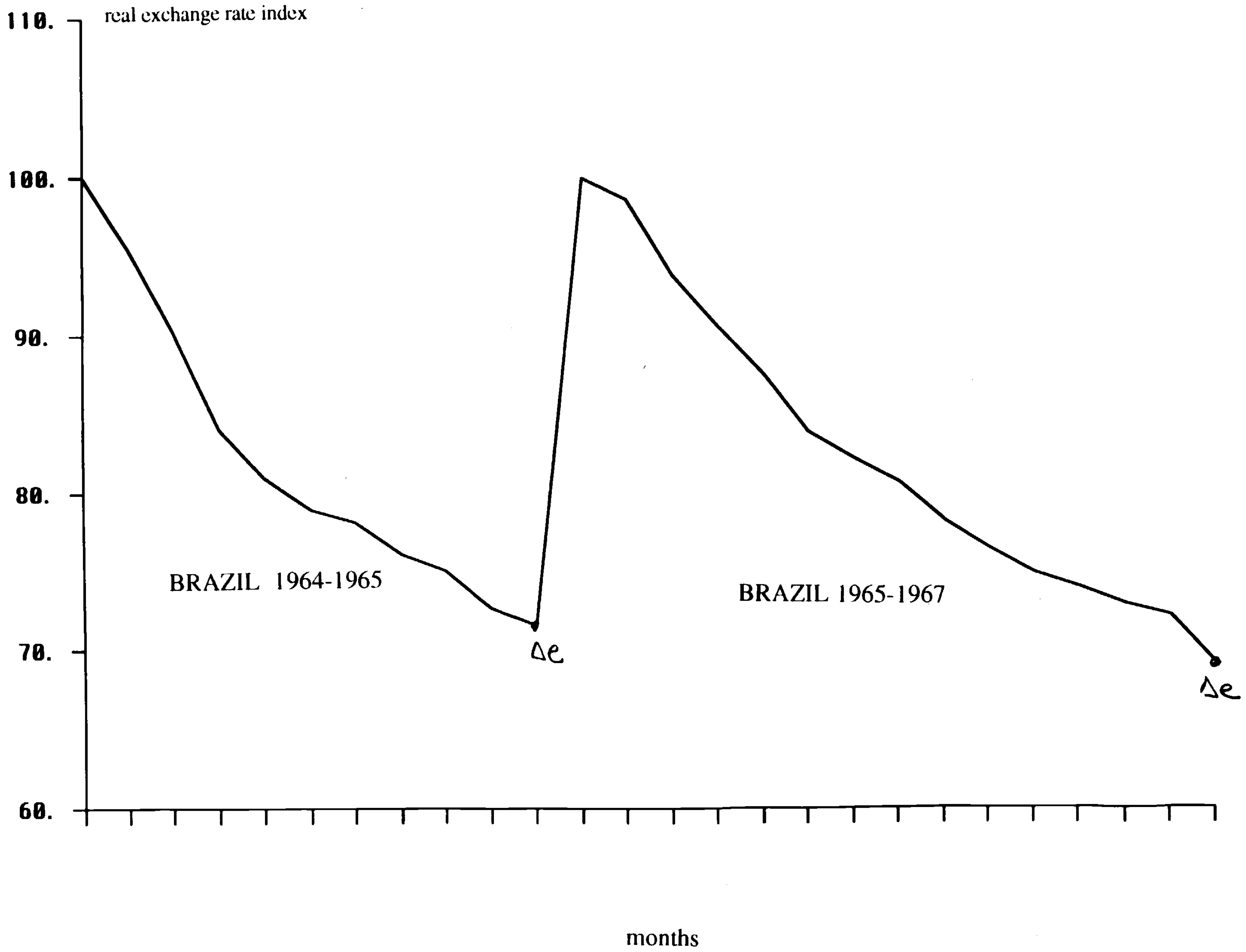
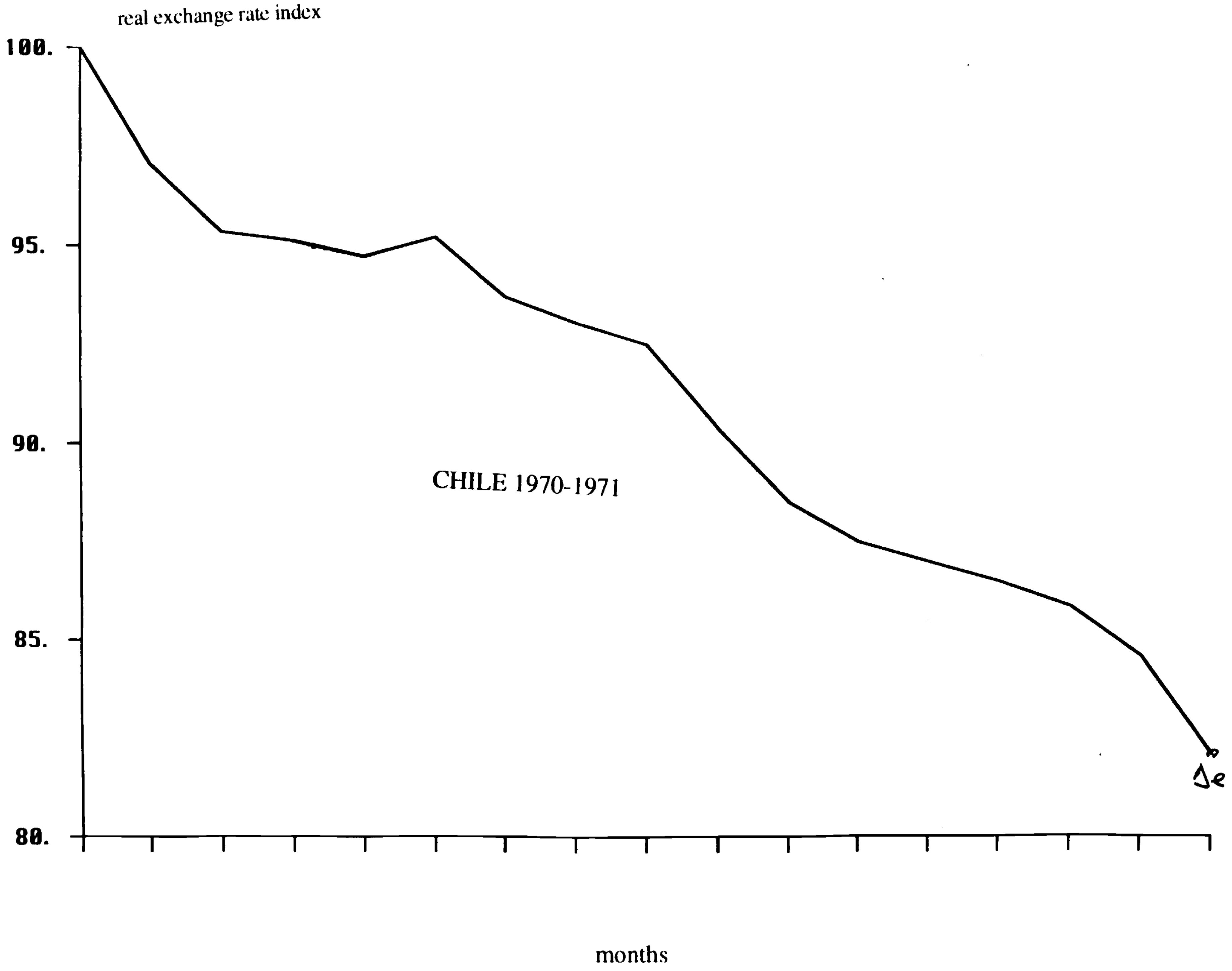
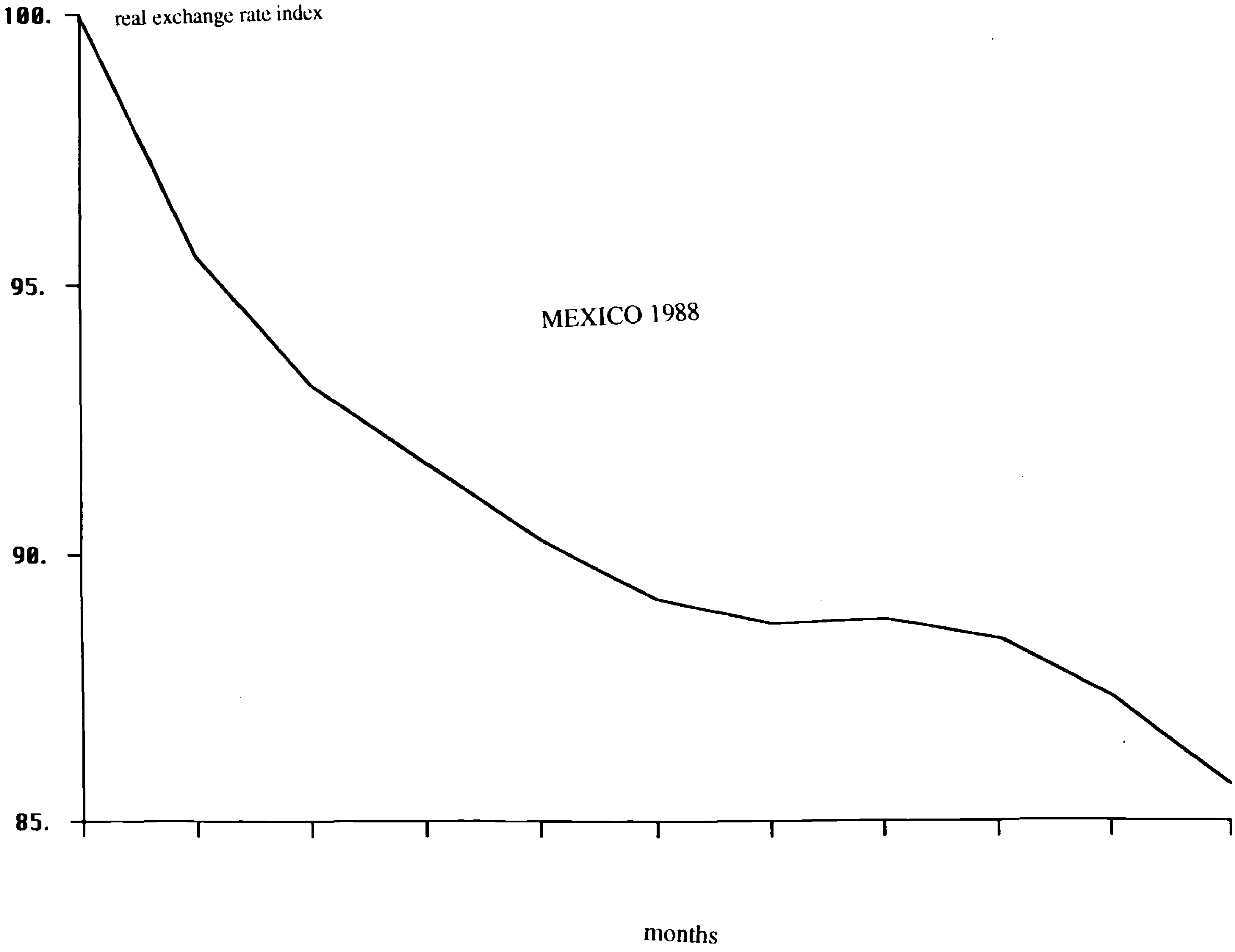

100.
real exchange rate index
ARGENTINA 1971-1975
80.
60.
40.
20.
months

FIGURE 4



real exchange rate index
110.
100.
90.
80.
70.
60.
BRAZIL 1964-1965
BRAZIL 1965-1967
Δe
Δe
months


real exchange rate index
100.
95.
90.
85.
80.
CHILE 1970-1971
months



100.
real exchange rate index
95.
MEXICO 1988
90.
85.
months

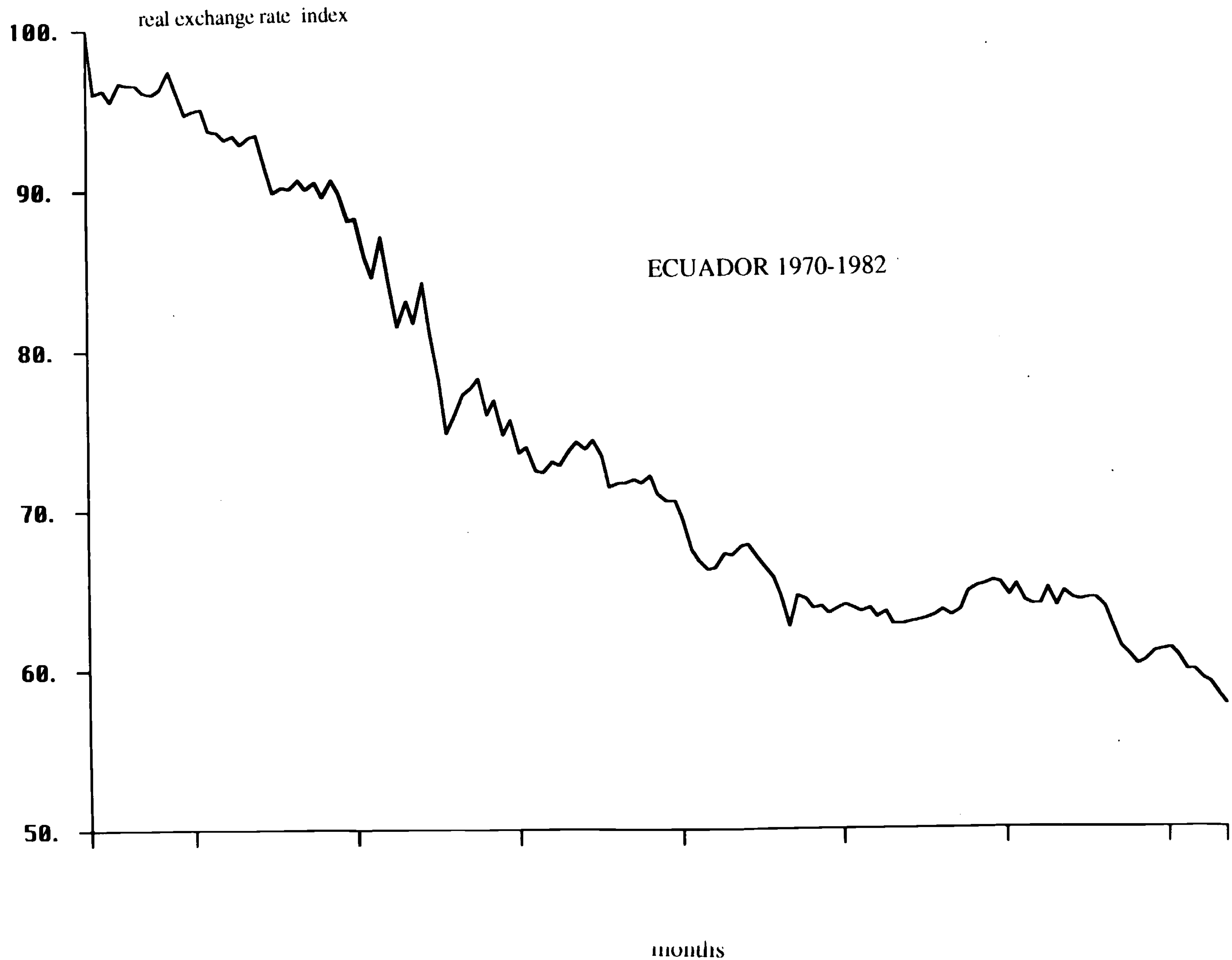

real exchange rate index
100.
90.
80.
70.
60.
50.
ECUADOR 1970-1982
months

TABLE 1: Estimation Results on Band Width

	(1)	(2)	(3)	(4)	(5)
n	80	80	80	80	80
constant	20.5894**	24.6468**	21.6141**	12.7892**	11.4727**
	(2.39)	(2.12)	(2.80)	(5.62)	(5.85)
drift of the real exchange rate	-2.7366**	---	-1.8290*	-1.6770**	-0.9733
	(0.46)		(1.02)	(0.81)	(0.74)
variance of the real exchange rate	---	0.3054**	0.1293	0.0108	0.0889
		(0.08)	(0.14)	(0.07)	(0.08)
$\overline{R^2}$	0.220	0.198	0.222	0.488	0.541

Standard errors are in parentheses. The values are based on White's (1980) heteroskedasticity-consistent covariance matrix.

** (*) Significant at the 95 (90) percent confidence level.

Equations (1) - (3) have no fixed effects; equation (4) includes country dummy variables; equation (5) includes country dummies and a set of individual calendar year dummies for the post-Bretton Woods years, 1971-1990.

Regressions based on a cross-section of peg spells. Dependent variable is band width (x_o).

Source: IMF, *International Financial Statistics*, compact disk September 1993.

achieve heteroskedastic-consistent estimates. The results are consistent with the theoretical predictions. Regression (1) shows that the simple correlation between drift and band width is negative and highly significant. A more negative drift therefore increases the size of the band and hence the amount of real appreciation incurred over the peg spell. Regression (2) shows that the simple correlation between variance and band width is positive and highly significant. Regression (3) examines the effects of both drift and variance on band width. Along with the constant term, drift and variance explain about 22 percent of the variation in band width. The coefficient on the drift variable is still negative but only significant at the 90 percent confidence level. The coefficient attached to variance is insignificant.

If drift and variance are highly correlated, then multicollinearity might make it difficult to obtain statistically significant estimates on both drift and variance. Indeed, the correlation between drift and variance is -0.82 and highly significant. It is well known that higher inflation levels are positively correlated with higher inflation variance, so something of the sort may be going on here since the real exchange rate process is driven primarily by the domestic price process.

The constant term in the regressions captures the fixed cost of peg readjustment. It may also include other spell-specific elements that influence band size. There is no reason to believe that this constant term should be the same for all peg spells. We thus reestimate regression (3) with various fixed effects. Regression (4) in Table 1 reports results using a set of country dummies, and regression (5) reports results using both country dummies and a set of calendar year dummies for the year in which a devaluation took place. When country dummies are incorporated, the coefficient on drift becomes significant at the 95 percent confidence level and the adjusted R^2 rises from .222 to .488. When both country and calendar year dummies are added, the coefficient on drift becomes insignificant.

Since our first set of regressions relies on an approximation for band size rather than the exact nonlinear specification in (15), we also experiment with various log-linear approximations. We report one version in Table 2, regressions (1)-(3). In this version,

TABLE 2: Additional Estimation Results on Band Width

	(1)	(2)	(3)	(4)
n	80	80	80	80
constant	19.8180** (2.03)	14.0385** (6.09)	10.0243* (5.61)	20.3171** (1.98)
drift of the real exchange rate	-0.7336 (0.53)	-0.8241* (0.42)	-0.7555 (0.55)	---
log of the variance of the real exchange rate	6.9646** (4.17)	4.0294** (1.33)	3.6481** (1.25)	---
drift of the real exchange rate (revised)				-1.4888** (0.52)
log of the variance of the real exchange rate (revised)				4.3181** (1.41)
$\overline{R}^2$	0.384	0.538	0.567	0.366

Standard errors are in parentheses. The values are based on White's (1980) heteroskedasticity-consistent covariance matrix.

** (*) Significant at the 95 (90) percent confidence level.

Equations (1) and (4) have no fixed effects; equation (2) includes country dummy variables; equation (3) includes country dummies and a set of individual calendar year dummies for the post-Bretton Woods years, 1971-1990.

Regressions based on a cross-section of peg spells. Dependent variable is band width (x_0).

Source: IMF, *International Financial Statistics*, compact disk September 1993.

band width is regressed on a constant, the drift and the log of the variance. The fit is better in this version. Together with the constant term, the drift and the log of the variance explain almost 40 percent of the variation in band width. Both drift and log variance enter with appropriate signs, but only the log of the variance is highly significant. Introducing fixed effects does not change the key results, although the inclusion of country dummies improves the explanatory power of drift.

Each peg spell in our sample ends with a devaluation. Therefore the expected value of shocks toward the end of a spell is negative rather than zero. To check for the biases introduced, we constructed a Brownian motion with negative drift and some variance and simulated its movement until it hit a lower barrier. The simulation results indicate that the estimated value of the drift is biased upwards (in absolute terms) and the estimated variance is biased downwards. In the absence of specific measures for the amount of bias, we follow a common-sense approach. We construct measures of drift and variance using only the first 75 percent of observations from each peg spell. While this strategy introduces some small sample bias, it should reduce somewhat the biases generated by having a non-random sample of peg spells. Moreover, this strategy may limit a possible problem concerning the endogeneity of the real exchange rate with respect to the policymaker's behavior. For a few spells, the drift moderates as the real exchange rate approaches the lower barrier, suggesting that domestic inflation was deliberately reduced to preserve the peg.[9] The revised measures should exclude these periods. The revised measures are similar to the original measures of drift and variance, but the correlation between drift and variance is now somewhat reduced. The effects of drift and log variance on band width using the revised measures is reported in regression (4) of Table 2. The results support the

[9] Examples include the last months of the following peg episodes: Bolivia, 1959-1972; Bolivia 1972-1979; Chile, 1979-1982; and Paraguay 1985-1986.

predictions of the model. Both drift and log variance enter with appropriate signs and are highly significant. [10]

While band width is an acceptable proxy for devaluation size, we also considered the percentage change in the nominal exchange rate on the day the peg is abandoned. These data are obtained from *Pick's Currency Yearbook* and supplemented by various issues of the IMF's *Annual Report on Exchange Arrangements and Exchange Restrictions*. Ideally we would like to have a measure of the change in the *real* exchange rate at the time the peg is adjusted, but relative price data are unavailable on a daily basis. The correlation between band width and the percentage change in the nominal exchange rate is +0.46 and highly significant.

When we regressed the percentage change in the nominal exchange rate on the drift and the variance of the real exchange rate, neither drift nor variance had any explanatory power when entered in levels. Multicollinearity did not appear to be the culprit since the joint explanatory power of the two components of the real exchange rate process was negligible as well. When the variance was entered in logs, the coefficient on variance was positive and significant at the 95 percent confidence level, but the coefficient on drift was still insignificant. The explanatory power of the regression was also quite low. Overall, the percentage change in the nominal exchange rate proved to be a less satisfactory proxy for the dependent variable.

Table 3 shows that both drift and variance significantly affect the time of devaluation. Recall that *a priori* their effects can be of either sign. The coefficient on the drift term is positive and highly significant so that a more negative drift shortens the time spent on a peg. The coefficient on the variance term is also positive and highly significant, suggesting that a higher variance increases time on a peg. While this latter outcome may seem surprising, recall that higher variance decreases time on a peg for a *given* band, but it

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[10] For further discussion on the problem of using constructed regressors measured with error, see Pagan (1984).

TABLE 3: Estimation Results for Time on the Peg

	(1)	(2)	(3)
sample size	80	80	80
constant	41.4049** (7.41)	31.1565** (10.68)	41.4358** (15.25)
drift of the real exchange rate	6.6318** (1.87)	7.9568** (2.25)	7.8079** (2.12)
variance of the real exchange rate	0.4492** (0.18)	0.4668** (0.20)	0.7467** (0.22)
$\overline{R}^2$	0.105	0.106	0.359

Standard errors are in parentheses. The values are based on White's (1980) heteroskedasticity-consistent covariance matrix.

** (*) Significant at the 95 (90) percent confidence level.

Equations (1) has no fixed effects; equation (2) includes country dummy variables; equation (3) includes country dummy variables and a set of individual calendar year dummies for the post-Bretton Woods years, 1971-1990.

Regressions based on a cross-section of peg spells. Dependent variable is time on the peg (T), expressed in months.

Source: IMF, *International Financial Statistics*, compact disk, September, 1993.

also increases the size of the band within which the real exchange rate fluctuates without triggering a realignment. In the Latin American peg episodes, the latter effect apparently dominates. The results are robust to the introduction of various fixed effects. The results are also similar when the log of the variance is used in the regression or when the revised measures of drift and variance are used. In summary, a more negative drift in the real exchange rate shortens the life of a peg, while a larger variance actually increases the time spent on a peg because it increases the amount of real appreciation the policymaker is willing to tolerate over the peg spell.

We now estimate parameters of the actual specifications for the optimal band width and time of devaluation given in (15) and (20). Table 4 shows the results of the estimations for cross-section data on Latin American peg spells. Specification (1) reports the results of estimating the fixed cost of peg adjustment, δ, when the monthly real interest rate is set at 0.004 (an annual real rate of 5 percent). *A priori,* we want δ to be positive, but we have no view about its magnitude. The fixed cost is positive and precisely estimated, with $\delta = 843$ and the standard error at 163. The log likelihood function is -198 for 80 observations.

The theoretical model predicts that relatively long spells have a bigger fixed cost of adjustment, *ceteris paribus.*[11] To test how well this prediction holds up, we replace δ with (δ+β1*dum) in the estimation of the band equation, where dum is a dummy variable that takes on a value of one for those peg episodes that last longer than the median sample time and zero otherwise. If longer spells are indeed characterized by a larger fixed cost, then the coefficient $\beta 1$ should be positive and significantly different from zero. Specification (2) in Table 4 shows that this is indeed the case.

Specification (3) examines whether longer spells have a larger fixed cost of adjustment and, simultaneously, whether the coefficient attached to the ($1/\alpha$) term is indeed

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[11] $\frac{\partial T}{\partial \delta} = \frac{\alpha}{\lambda}\frac{\partial x_o}{\partial \delta} > 0$ since $\frac{\partial x_o}{\partial \delta} = \frac{\lambda/2}{1 - e^{-\alpha x_o}} > 0$

TABLE 4: Estimations of Parameters in Exact Specifications for Band Width and Time on a Peg

---

Band width (x_o)

$$x_o + \frac{1}{\alpha(\lambda)} e^{-\alpha(\lambda)} = \frac{1}{\alpha(\lambda)} + \frac{\lambda\delta}{2} + \varepsilon_i$$

---

1. 80 observations
   Log Likelihood = -198.41

δ = 843.442	s.e. = 162.56

Band width (x_o)

$$x_o + \frac{1}{\alpha(\lambda)} e^{-\alpha(\lambda)} = \frac{1}{\alpha(\lambda)} + \frac{\lambda}{2}(\delta + \beta_1 * dum) + \varepsilon_i$$

---

2. 80 observations
   Log Likelihood = -186.22

δ = 121.03	s.e. = 196.22
β_1 = 1481.86	s.e. = 281.03

Band width (x_o)

$$x_o + \frac{1}{\alpha(\lambda)} e^{-\alpha(\lambda)} = \beta_2 \left(\frac{1}{\alpha(\lambda)}\right) + \frac{\lambda}{2}(\delta + \beta_1 * dum) + \varepsilon_i$$

---

3. 80 observations
   Log likelihood = -186.18

δ = 164.37	s.e. = 255.82
β_1 = 1453.32	s.e. = 302.35
β_2 = 0.9999	s.e.= 0.0003

Band width (x_o)

$$x_o + \frac{1}{\alpha(\lambda)} e^{-\alpha(\lambda)} = (\frac{1}{\alpha(\lambda)}) + \frac{\lambda\delta}{2} + \varepsilon_i$$

4. 80 observations
   Log likelihood = 615.854

$\delta = 901.150$	s.e. = 479.70
$\lambda = 0.0002$	s.e. = 0.00001

Time on Peg

$$e^{-\lambda T} = e^{-\alpha(\lambda)x_o} + \omega_i ; \qquad x_o = 100 - q_\ell$$

5. 80 observations
   Log likelihood = 82.51

$q\ell = 81.2593$	s.e. = 1.67

Time on Peg

$$e^{-\lambda T} = e^{-\alpha(\lambda)x_o} + \omega_i ; \qquad x_o = 100 - q_\ell$$

6. 80 observations
   Log likelihood = 83.52

$q\ell = 83.1254$	s.e. = 2.08
$\beta_1 = 0.9811$	s.e. = 0.01

Time on Peg

$$e^{-\lambda T} = e^{-\alpha(\lambda)x_0} + \omega_i$$

7. 80 observations
Log likelihood = 572.10

$\lambda = 0.0004$	s.e. = 0.00002

close to one. The results indicate that both conjectures are appropriate. The coefficient on the dummy variable is positive and is precisely and plausibly estimated. It indicates that longer spells have a bigger fixed cost associated with devaluation. The coefficient attached to the $(1/\alpha)$ term is precisely and plausibly estimated at 0.9999, with a standard error of 0.0003. The estimate conforms with our prior that the coefficient should be about one.

So far the model is linear in the parameters that we have estimated. We now consider a case where the model is nonlinear in the estimated parameters. We estimate jointly the fixed cost of peg adjustment, δ, and the monthly real interest rate, λ, in the band equation. Specification (4) shows that δ is a reasonably estimated positive number. Its value is 901 and its standard error is 479. The estimate is moderately precise. The monthly real interest rate is precisely estimated. Its value is 0.0002 and its standard error is 0.00001. The estimate is positive but smaller than one would expect.

Specification (5) reports the estimation for equation (20), which gives the number of months on the peg. The estimation is a test of the joint null hypothesis that time spent on the peg is the outcome of cost minimization and that the policy-maker forms expectations rationally. Since $x_o = (q_o - q_\ell)$, we estimate the lower barrier q_ℓ given a starting value for the real exchange rate of $q_o = 100$. We would expect that $0 < q_\ell < 100$. We also set the monthly real interest rate at 0.004. The lower barrier is precisely and plausibly estimated at $q_\ell = 81.2593$ with a standard error of 1.67. The sample median for q_ℓ is $q_\ell = 76.05$, and the sample mean is $q_\ell = 71.85$.

Specification (6) estimates the lower barrier without constraining the coefficient attached to $\exp(-\alpha x_o)$ to be one. The lower barrier is precisely and plausibly estimated at q_ℓ=83.1254 with a standard error of 2.08, and the $\beta 1$ coefficient is close to one. Indeed, it is estimated to be 0.9811 with a standard error of .01.

Specification (7) estimates the monthly real interest rate parameter in the time equation. The parameter λ is precisely estimated, with $\lambda = 0.0004$ and a standard error of

0.00002. The estimated monthly real interest rate is positive but smaller than one would expect.

Overall, then, our estimates indicate that the data are supportive of the nonlinear specifications of the size and timing of devaluations as a function of the stochastic process of the real exchange rate, the interest rate, and the fixed cost of adjustment.

4. Conclusion

When developing economies maintain controls on capital and currency flows, the size and timing of devaluation are jointly determined by the policy authorities. In this paper, we consider the policymaker who pegs the nominal exchange rate and adjusts the peg periodically so as to minimize the flow cost of real exchange-rate misalignment and the fixed cost of peg readjustment. The optimization problem is made difficult by the fact that the time of devaluation is a stochastic variable. Hence the policymaker has a stochastic horizon for making decisions about the peg. Fortunately for the case where the real exchange rate follows regulated Brownian motion, the cost minimization problem can be solved explicitly for the optimal band within which the real exchange rate can fluctuate without triggering a devaluation and for the time when the real exchange rate hits the lower barrier, causing a devaluation.

The framework yields insights into how changes in the stochastic environment jointly affect the size and time of devaluation. These insights may provide guidance about the determinants of devaluation episodes even when Brownian motion is not the relevant stochastic process for real exchange rates. Using data on 80 peg episodes from seventeen Latin American countries over the 1957-1990 period, we find that the stochastic components of the real exchange rate are indeed predictors of the size and timing of devaluation. Moreover, their effects can best be understood within a framework where the optimal size and timing of devaluation are jointly determined decision variables. For example, one would expect that higher variance of the real exchange rate would generate more frequent devaluations. But in our sample of Latin American pegs, higher variance increases the time spent on a peg. Such an outcome is consistent with the theory presented here, for when size and timing are jointly determined, the effect of increased variance on timing can go in either direction. Higher variance reduces the time spent on a peg for a given band size, but it also increases the optimal band and hence the amount of real appreciation the policymaker is willing to tolerate before adjusting the peg .

We are able to obtain explicit solutions for the optimal size and timing of devaluations by claiming that the policymaker monitors one state variable in order to make decisions about the peg. However, it is more realistic to believe that the policymaker tracks several state variables in deciding the size and timing of devaluation. For example, the policymaker may decide to terminate the peg the first time that *either* the real exchange rate *or* international reserves hits some policy-determined lower barrier.[12] If all of the monitored variables follow Brownian motion, then their linear combination also follows Brownian motion. Incorporating into the theoretical framework a composite variable which is a linear combination of the monitored variables is a straightforward exercise. Such an extension shows that the stochastic process of each monitored variable as well as the covariances among them influence the size and timing of devaluation. More complex interrelationships among the various monitored state variables are also possible. While these more complicated processes do not lend themselves to analytical methods, their implications can be explored through simulations. We plan to pursue such a strategy in future work.

[12]Bilson (1979) identifies two variables that can be leading indicators of the time of currency devaluation in developing countries: (1) a monetary indicator based on the monetary approach to the exchange rate that includes a role for the difference between the actual and equilibrium values of the real exchange rate, and (2) the ratio of international reserves to high-powered money. Klein and Marion (1994) also find that the real exchange rate and the level of international liquidity help predict the monthly probability of devaluation in a set of developing countries.

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Appendix

Using Harrison (1985), pp. 44-48, the $g(x_o)$ function in (8) is equal to:

$$\text{(A1)} \quad g(x_o) = f(x_o) - f(0)e^{-\alpha(\lambda)x_o}; \qquad x_o \geq 0$$

where, using Fubini's theorem,

$$\text{(A2)} \quad f(x_o) \equiv E_{x_o}[\int_0^\infty e^{-\lambda t}(x_t - x^*)^2 dt] = \int_0^\infty e^{-\lambda t} E_{x_o}[(x_t - x^*)^2]dt \quad .$$

To calculate (A2), use the expression in (2) for x_t and derive the expected squared deviation of the real exchange rate from its equilibrium value:

$$\text{(A3)} \quad E_{x_o}(x_t - x^*)^2 = (x_o - x^*)^2 + \mu^2 t^2 + 2(x_o - x^*)\mu t + t\sigma^2$$

Substituting (A3) into (A2) gives:

$$\text{(A4)} \quad f(x_o) = (x_o - x^*)^2 \int_0^\infty e^{-\lambda t} dt + \mu^2 \int_0^\infty e^{-\lambda t} t^2 dt + [2(x_o - x^*)\mu + \sigma^2] \int_0^\infty e^{-\lambda t} t\, dt$$

Integrating (A4) yields the following expression for $f(x_o)$:

$$\text{(A5)} \quad f(x_o) = \frac{(x_o - x^*)^2}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{2(x_o - x^*)\mu + \sigma^2}{\lambda^2} \quad .$$

Substituting (A5) into (A1) and also evaluating the function $f(x_o)$ at $x_o=0$ gives:

$$\text{(A6)} \quad g(x_o) = \frac{(x_o - x^*)^2}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{\{2(x_o - x^*)\mu + \sigma^2\}}{\lambda^2} - [\frac{x^{*2}}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{(\sigma^2 - 2x^*\mu)}{\lambda^2}]e^{-\alpha x_o}$$

To get the complete expression for expected discounted costs, $g'(0)$ must also be calculated. Note that

$$\text{(A7)} \quad g'(x_o) = \frac{2(x_o - x^*)}{\lambda} + \frac{2\mu}{\lambda^2} + \alpha[\frac{x^{*^2}}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{(\sigma^2 - 2x^*\mu)}{\lambda^2}]e^{-\alpha x_o} \quad ,$$

so that

$$\text{(A8)} \quad g'(0) = \frac{-2x^*}{\lambda} + \frac{2\mu}{\lambda^2} + \alpha[\frac{x^{*^2}}{\lambda} + \frac{2\mu^2}{\lambda^3} + \frac{(\sigma^2 - 2x^*\mu)}{\lambda^2}]$$