THE TIME VARIATION OF RISK AND RETURN
IN FOREIGN EXCHANGE MARKETS:
A GENERAL EQUILIBRIUM PERSPECTIVE

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ABSTRACT

This paper investigates the statistical properties of high frequency nominal exchange rates and forward premiums in the context of a dynamic two-country general equilibrium model. Primary focus is on the persistence, variability, leptokurtosis and conditional heteroskedasticity of exchange rates and on the behavior of foreign exchange risk premiums. The model combines temporal dependencies in preferences with a transaction cost technology that generates a role for money. Agents in the economy make decisions on a weekly frequency and face shocks which display time-varying uncertainty. Simulations reveal that the model accounts for the statistical properties of exchange rate data much more accurately than previous structural models.

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INTRODUCTION

The increasing globalization of financial markets has intensified the need to understand foreign exchange markets. A large body of empirical research has uncovered many interesting empirical regularities characterizing foreign exchange markets. The following are four well-known stylized facts. First, the forward rate is not an unbiased predictor of the future spot rate. Second, exchange rate changes are highly variable and nearly uncorrelated. Third, forward premiums, which predict exchange rate changes, are less variable and are highly persistent. Fourth, exchange rates display substantial serial dependence in their second moments.[1] Furthermore, conditional heteroskedasticity in exchange rates is only detectable at high frequencies. At the monthly or quarterly frequency, it is no longer statistically significant (Baillie and Bollerslev (1989)). As time-varying second moments imply leptokurtic unconditional distributions, conditional heteroskedasticity may also be an important source behind the fat tails in unconditional distributions of exchange rate changes. Exchange rate changes are the main component of forward market returns. As is well-known, these returns are an important component of the return on *any* foreign currency denominated uncovered investment. The first and fourth empirical facts imply that conditional means and variances of forward market returns vary through time.

This paper develops a two country monetary general equilibrium model that can potentially generate such empirical phenomena. The model builds on the representative agent framework of Lucas (1978). The only friction in the model is the presence of transaction costs which give rise to money being valued in equilibrium and induces variable velocities. Exchange rate movements reflect changes in relative money supplies, velocities and outputs whereas time-variation in expected returns reflects time-varying rewards to consumption and inflation risk.

Previous attempts at explaining the significant rejection of the unbiasedness hypothesis within a fully parameterized general equilibrium model have failed dramatically.[2] The model developed here has several features that are likely to improve its performance with respect to the empirical puzzles. First,

I explicitly introduce time-variation in the conditional variances of the market fundamentals. Second, I assume a *weekly* decision interval for the economic agents.[3] Third, various forms of temporally dependent preferences are explored as in Ferson and Constantinides (1991) and Heaton (1993).

The main novelty of the model here is the combination of time-varying uncertainty in the fundamentals with a high frequency decision interval for the agents. It is well-known that movements in conditional variances of market fundamentals cause movements in both expected returns and conditional variances of asset prices. However, there exists little evidence of time-varying conditional variances in market fundamentals such as monetary shocks and consumption at the monthly or quarterly frequencies at which they are usually measured. As I will explain in more detail below, this might be due to temporal aggregation which causes time-variation in conditional variances to disappear. The same phenomenon causes conditional heteroskedasticity in exchange rates to be much weaker when exchange rates are sampled at lower frequencies.

The discrete-time framework simplifies the introduction of a rich time-nonseparable preference structure. Backus, Gregory and Telmer (1993) and Bansal, Gallant, Hussey and Tauchen (1993) also examine the effects of nonseparabilities in an international model but their preference structure is much less general than the one used in this paper. Backus, Gregory and Telmer (1993) for instance are successful in generating more variable forward market risk premiums by allowing strong habit persistence. However, this comes at the cost of unrealistic values for the autocorrelation structure of forward premiums. By incorporating both durability of consumption and habit persistence of a *long run* nature, I am able to generate more realistic variability and correlation patterns for exchange rates and forward premiums than previous structural models.

The model allows characterization of the endogenous joint distribution of forward market returns, exchange rates and forward premiums as a function of the model's structural parameters and the law of motion of the stochastic forcing variables. I explore the implications of the model through several

simulation experiments. To impose discipline on the simulations, I estimate the conditional mean parameters and unconditional second moments of the *weekly* law of motion for the forcing processes from *quarterly* data on consumption and money growth in the U.S. and the U.K.. The embedded temporal aggregation problem is resolved relying on results in Bekaert (1992).

The simulation experiments are designed to accomplish two goals. First, the generality of the model allows the comparison of the model's predictions for the moments of exchange rate changes, forward premiums and risk premiums to the predictions of simpler models. In particular, I start from a simple cash-in-advance (CIA) model with time-additive preferences, and show the effects of adding transaction costs, durability, habit persistence and time-varying uncertainty in the fundamentals. The importance of time-varying uncertainty in the fundamentals driving foreign exchange markets was stressed by Hodrick (1989), but the idea had surfaced before in the equity pricing literature. Poterba and Summers (1986) derive a positive relationship between the persistence of stock market volatility and stock prices in a simple partial equilibrium model for stock prices whereas Abel (1988) generalizes their results to a general equilibrium Lucas-type (1978) model. He finds that the effect of dividend riskiness on stock prices can be in either direction whereas the magnitude of the effect is an increasing function of the persistence of dividend volatility. I investigate the effect on endogenous moments of both the persistence and the leptokurtic nature of conditional variance shocks in the fundamentals.

The second goal of the paper is to examine the effects of various non-linearities in the economic environment on heteroskedasticity and leptokurtosis in exchange rates and forward premiums. Most research to date has attempted to model these non-linear patterns with purely statistical models. This is one of the first papers to address how volatility clustering of financial returns can arise endogenously in an equilibrium model.[4]

To anticipate the results, the simulations reveal that the model performs very well along a number of dimensions. When consumption exhibits short-run substitutability and long-run complementarity, the

model comes close to matching the autocorrelation structure of the forward premium in the data. The risk premium is many times more variable than in a model with time-additive preferences. The introduction of modestly persistent time-variation in the second moments of the fundamentals at the weekly level, consistent with the lack of heteroskedasticity in these processes at the quarterly level, generates substantial heteroskedasticity and fat tails in both exchange rates and forward premiums. Heteroskedasticity in the forcing processes also substantially increases the variability of the risk premium, without worsening the fit of the model much along other dimensions. Yet, the variability of the risk premium does not exceed that of the forward premium as is implied by the data.

The paper is organized as follows. The first section briefly re-examines the empirical evidence for dollar returns in the Europound market. The second section of the paper presents the model in detail. The third section of the paper discusses the solution procedure and choice of parameters. The fourth section contains the simulation results. In the conclusions of the paper, I point out some possible generalizations of the model.

1. EMPIRICAL REGULARITIES

Definition of variables

The data set consists of weekly observations on dollar-pound rates and one- and three-month Eurodollar and Europound interest rates for the 1975-1990 period, but the empirical results are representative for other major currencies as well. The data are described in more detail in the data appendix. Asterisks indicate British pound variables. Consider an investment in a Europound deposit which carries an interest rate of $i_{t,n}^*$, with n the maturity in weeks. The holding period considered in this paper is one month (a 30 day contract), and one month is approximated by 4 weeks in subsequent analysis.[5] Let S_t be the dollar price of a pound. The uncovered dollar return on a continuously compounded Europound investment is $(S_{t+n}/S_t) \exp(i_{t,n}^*)$. The rate of return is then $[\Delta s_{t+n} + i_{t,n}^*]$, where

$\Delta s_{t+n} = \ln(S_{t+n}) - \ln(S_t)$. Hence, the excess rate of return over a Eurodollar deposit is given by: $r_{t+n} = [\Delta s_{t+n} + i_{t,n}^* - i_{t,n}]$. This return also corresponds to the difference between the future spot rate and the current forward rate. To see this, let $fp_{t,n} = \ln(F_{t,n}) - \ln(S_t)$, where $F_{t,n}$ is the forward rate for a n-week contract in dollars per pound. Consider covered interest rate parity in continuously compounded form:

$$fp_{t,n} = i_{t,n} - i_{t,n}^* \tag{1}$$

It follows that $r_{t+n} = [\ln(S_{t+n}) - \ln(F_{t,n})]$ and hence, r_{t+n} can also be viewed as the logarithmic approximation to the return on a long forward position in the pound scaled by the forward rate, i.e. $\ln(S_{t+n}) - \ln(F_t) \approx (S_{t+n} - F_{t,n})/F_{t,n}$. The ex ante return to forward foreign exchange speculation is denoted by $rp_{t,n}$:

$$rp_{t,n} = E_t[r_{t+n}] = E_t[\sum_{i=1}^{n} \Delta s_{t+i}] - fp_{t,n} \tag{2}$$

with Δs_t referring to *weekly* exchange rate changes.

Time series properties of the variables

Means, standard deviations and autocorrelations of Δs_t and $fp_{t,n}$ are reported in Table 1, Panel A. To summarize the autocorrelation structure of the variables, I report *variance ratios*. The variance ratio of a stationary time series $\{x_t\}_{t=1}^{T}$ for a horizon of k periods is defined as:

$$V_k = \frac{\text{var}(\sum_{j=0}^{k} x_{t-j})}{(k+1)\ \text{var}(x_t)} \tag{3}$$

The variance ratio can be consistently estimated as:

$$\hat{V}_k = 1 + 2\sum_{j=1}^{k} (1 - \frac{j}{k+1})\hat{\rho}_j \tag{4}$$

with $\hat{\rho}_j$ the sample autocorrelation of order j for $\{x_t\}_{t=1}^{T}$. The variance ratio is one for a serially

uncorrelated time series, is less than one if negative autocorrelations dominate and is greater than one if positive autocorrelations dominate.

Panel A in Table 1 also explores the presence of leptokurtosis and heteroskedasticity in the two series. To remove the serial correlation from the forward premium series, it was simply differenced. Autocorrelations and variance ratios are then computed for the squared differenced series. Under the null of conditional homoskedasticity , the autocorrelations equal zero and the variance ratios equal one.

Panel B in Table 1 focusses on cross-correlations. The cross-correlations between Δs_{t+4}, Δs_{t+3}, Δs_{t+2}, Δs_{t+1} and fp_{t} are related to what is known as the unbiasedness issue. When future exchange rate changes are regressed onto a constant and the forward premium, the slope coefficients are typically negative rather than one.[6] The sum of the 4 cross-correlations mentioned above times the ratio of the standard deviation of exchange rate changes to the standard deviation of the forward premium provides an estimate of this slope coefficient. The coefficient is estimated at -2.116 with a standard error of .761.[7] The fitted value of this regression can be used to compute a lower bound to the standard deviation of the risk premium. The lower bound on risk premium volatility is estimated at 10.841 % in annualized terms with a standard error of 2.847%.[8]

From these tables, several regularities emerge that a successful model should match:

(I) Exchange rate changes show some positive persistence but the autocorrelations are generally small. On the other hand, the forward premium is very persistent. I term this regularity the "persistence puzzle".

(II) Exchange rate changes are many times more variable than the forward premium whereas the risk premium in the British pound forward market is also extremely variable. In fact, $\sigma(\Delta s_{t+n}) > \sigma(rp_{t})$ $> \sigma(fp_{t})$. I refer to this relative variability property as the "volatility puzzle".

(III) Forward premiums and exchange rates exhibit marked positive serial dependence in their second moments and substantial leptokurtosis. Somewhat surprisingly, heteroskedasticity and leptokurtosis are

even stronger for forward premiums than they are for exchange rate changes. The predictability of exchange rate changes by the forward premium extends to the second moments (Panel B), but the correlations are generally weaker and less precisely measured.

2. THE ECONOMIC ENVIRONMENT

In this section, I describe the agents' preferences, budget constraints and the Euler equations determining important endogenous variables. I also show how exchange rates are determined and forward contracts priced. In addition, some further intuition is given on why this model might perform better than previous models with respect to the empirical puzzles mentioned above.

Description of the model

The model analyzed in this paper is a generalization of the two country model proposed by Lucas (1982). Each country has its own money, which grows at a stochastic rate, and its own endowment tree, which yields stochastically growing "home" or "foreign" consumption goods. Identical infinitely-lived representative agents in both countries maximize the expected discounted sum of a von Neumann-Morgenstern utility function subject to a sequence of budget constraints.

To derive the budget constraint, consider the period by period problem facing the domestic representative agent. Each period, she purchases home goods x_t^i and foreign goods y_t^i which are priced in the respective currencies with prices P_t^i ($i=x,y$). The exchange rate S_t converts the price of the foreign good into units of the home currency. Whereas in Lucas's model consumption purchases have to be financed with money, I assume that money balances diminish the transaction costs associated with buying consumption goods. In particular, let m_t (n_t) be the level of home (foreign) real money balances held by the household. The transaction cost functions $\Psi^x(x_t^i,m_t)$ and $\Psi^y(y_t^i,n_t)$ are decreasing in own real balances and increasing in the amount of goods bought (see also Marshall (1992) and Bansal, Gallant, Hussey and Tauchen (1993)). The domestic household also chooses the level of home and foreign money balances

M^d_{t+1}, respectively N^d_{t+1}, to be carried over to the next period and acquires asset holdings, summarized in the vector z_{t+1}. Asset prices are stacked in the vector Q_t. All purchases have to be made with current nominal wealth W_t, denominated in the home currency. Wealth consists of the money holdings chosen last period and the current market value of the asset holdings including stochastic payoffs (dividends) D_t. The home consumer's budget constraint is then given by[9]:

$$P^x_t(x_t^d + \Psi^x(x_t^d, m_t)) + S_t P^y_t(y_t^d + \Psi^y(y_t^d, n_t)) + z'_{t+1} Q_t + M^d_{t+1} + S_t N^d_{t+1} \leq W_t$$

$$W_t = z'_t(Q_t + D_t) + M^d_t + S_t N^d_t$$

$$(5)$$

Preferences

As in Eichenbaum and Hansen (1990), preferences are time separable over the service flows s^x_t and s^y_t derived from past consumption purchases of the home good x and the foreign good y:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ u(s^x_t, s^y_t)$$

$$(6)$$

Following Ferson and Constantinides (1991) and Heaton (1993), I assume that goods are durable and that consumers form habits over the services derived from them. Let k^x_t be the stock of home consumption goods which depreciates at the rate $1-\mu_x$ and h^x_t the habit stock defined over past service flows. I assume:

$$k^x_t = \sum_{j=0}^{\infty} \mu^j_x \ x^d_{t-j}$$

$$h^x_t = \eta_x \ (1-\theta_x) \sum_{j=0}^{\infty} \theta^j_x \ k^x_{t-1-j}$$

$$s^x_t = k^x_t - h^x_t$$

$$(7)$$

Note that η_x is the sum of the "habit weights", and that decreasing θ_x will increase habit effects at short lags.

Nonseparabilities embedded in this preference specification are apparent from considering the

expression for marginal utility in this context. The marginal utility derived from a purchase of the home good today is given by:

$$mux_t = E_t[\sum_{\tau=0}^{\infty} \beta^\tau \, a_\tau^x \, \frac{\partial u(s_{t+\tau}^x, s_{t+\tau}^7)}{\partial s_{t+\tau}^x}] \tag{8}$$

where a_τ^x are weights that follow from (7),

$$a_\tau^x = \mu_x^\tau - \eta_x \, (1-\theta_x) \, \frac{\mu_x^\tau - \theta_x^\tau}{\mu_x - \theta_x} \tag{9}$$

$$0 < \mu_x < 1 \quad 0 < \eta_x < 1 \quad 0 < \theta_x < 1$$

It is well-known that nonseparable preferences give rise to more variable intertemporal marginal rates of substitution (henceforth, IMRS) and asset prices (see for instance Hansen and Jagannathan (1991)). Habit forming utility might also account for the "persistence puzzle" mentioned above. In the continuous-time model of Sundaresan (1989), which features exogenous returns and endogenous consumption, habit formation induces very smooth and persistent consumption streams. In my model, the service technology transforms the exogenous endowment shocks into persistent service flows. This implies that endowment shocks have very prolonged effects on future IMRS. As a consequence, they will cause revisions in expectations of IMRS not only now but also in the future. Hence, the predictable parts of IMRS, interest rates (see below), might become very persistent as they are in the data.

I explore two commonly used specifications for the utility function which are special cases of the general preference framework in Eichenbaum and Hansen (1990). The first utility function is homothetic with intratemporal substitution between the "home" and "foreign" service flow equal to one.

$$u(s_t^x, s_t^7) = \frac{[(s_t^x)^\delta (s_t^7)^{1-\delta}]^{1-\gamma}}{1-\gamma} - 1 \qquad \delta \in (0,1) \qquad \gamma \geq 0 \tag{10}$$

The parameter δ is a non-negative share parameter and γ is a curvature parameter. The second specification is the so-called addilog utility function:

$$u(s_t^x, s_t^y) = \frac{(s_t^x)^{1-\gamma}}{1-\gamma} + \frac{(s_t^y)^{1-\gamma}}{1-\gamma} \qquad \gamma \geq 0 \tag{11}$$

Although the combination of a multigood economy with nonseparable preferences complicates the interpretation of the utility parameters, I associate high values of the curvature parameter γ with high risk aversion (low elasticity of intertemporal substitution) and vice versa.[10]

Transaction cost function

The transaction cost function is parameterized as a Cobb-Douglas function in the amount purchased x (y) and real money balances m (n).

$$\Psi^x(x,m) = c_x x^{\xi_x} m^{1-\xi_x} \quad \Psi^y(y,n) = c_y y^{\xi_y} n^{1-\xi_y} \quad c_x, c_y > 0 \qquad \xi_x, \xi_y > 1 \tag{12}$$

The function is increasing in x, decreasing in m and homogeneous of degree 1. The transaction cost technology embeds a CIA (cash-in-advance) constraint (when $\xi_i \to \infty$, i=x,y).

Equilibrium

The home household maximizes (6) over $\{x_t^i, y_t^i, M_{t+1}^i, N_{t+1}^i, z_{t+1}\}$ subject to the budget constraint in (5). An analogous problem is solved by the foreign representative resident. For markets to clear, the money demands in both countries must equal the supplies and the consumption demands including the incurred transaction costs must exhaust the endowments. With the additional assumption of complete markets, the risk averse representative agents of both countries will share all risks. This leads to a tractable *perfectly pooled equilibrium*, introduced in Lucas (1982), in which no wealth re-distributions occur, agents consume constant fractions of the endowments and hold constant fractions of the market portfolio of assets.[11]

Endogenous variables

In our frictionless world, the dollar price of an asset is the expected value of its nominal payoff discounted by the dollar IMRS. Let's denote the dollar IMRS by $mrs_{t+1,t}$. It relates the marginal utility of wealth at the time of the payoff, λ_{t+1}, to the marginal utility of wealth at time t when the asset is

purchased, λ_t:

$$mrs_{t,t+n} = \beta^n \frac{\lambda_{t+n}}{\lambda_t} \tag{13}$$

where ß is the discount factor. An analogous stochastic discount factor applies to assets that pay off in pounds. The λ's are the Lagrange multipliers associated with the sequence of budget constraints. The law of one price implies that the relative value of the marginal utility of pounds versus dollars determines the dollar/pound exchange rate:

$$S_t \lambda_t = \lambda_t^* \tag{14}$$

where asterisks refer to pound variables. This indicates that the dollar will depreciate if its marginal utility decreases relative to the marginal utility of the pound. The home (foreign) interest rate $i_{t,n}$ ($i_{t,n}^*$) is the net return on a nominal bond yielding one dollar (pound) at time $t+n$. Hence,

$$i_{t,n} = -\ln(E_t\{mrs_{t,t+n}\}) \qquad i_{t,n}^* = -\ln(E_t\{mrs_{t,t+n}^*\}) \tag{15}$$

If market fundamentals move so as to increase the expected marginal utility of the dollar, the dollar interest rate decreases, as it is the required return on an asset that pays off dollars when they are relatively valuable. As the forward premium is just the interest differential, the pound is at a discount if the value of the dollar's marginal utility is expected to increase more than that of the pound.

Once expressions for λ_t and λ_t^* are found, equation (14) yields the exchange rate and the domestic and foreign interest rates follow from (15). Using the formulas in equations (1) and (2), the forward premium and risk premium can be computed. Since λ_t is the marginal utility of a dollar, it equals the total expected marginal utility of consumption divided by the transaction cost adjusted price of one unit of the home consumption good:

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$$\lambda = \frac{mux_t}{(1 + \frac{\partial \Psi^x(t)}{\partial x})P_t^x} \tag{16}$$

where $\Psi^x(t)$ is short-hand for $\Psi^x(x_t^i, m_t^i)$ evaluated at the equilibrium and mux_t was given in equation (8).

Define consumption velocity V_t^x as the endowment net of transaction costs times the price level divided by the money supply.[12] In equilibrium, the first derivatives of the transaction cost function only depend on velocity:

$$\frac{\partial \Psi^x(t)}{\partial x} = c_x \xi_x (V_t^x)^{\xi_x - 1} \qquad \frac{\partial \Psi^x(t)}{\partial m} = c_x (1 - \xi_x)(V_t^x)^{\xi_x} \tag{17}$$

Velocity can be solved for from the intertemporal Euler equation. To derive the intertemporal Euler equation heuristically, consider saving one dollar today. The marginal cost of this action equals the marginal utility of a dollar, λ. The marginal benefit of the dollar tomorrow not only includes the marginal utility of wealth tomorrow but also an additional return as the dollar reduces transaction costs. Hence,

$$\frac{mux_t}{(1 + \frac{\partial \Psi^x(t)}{\partial x})P_t^x} = \beta E_t [\frac{mux_{t+1}}{(1 + \frac{\partial \Psi^x(t+1)}{\partial x})P_{t+1}^x}(1 - \frac{\partial \Psi^x(t+1)}{\partial m})] \tag{18}$$

where m denotes real balances at time $t+1$. An analogous procedure yields λ^*.

Specification of the law of motion of the forcing processes

In the simple economy described above, agents are subject to money supply and endowment shocks. Money and endowment growth rates are assumed to be observed *weekly* by the economic agents. Let M_t^* (N_t^*) be the home (foreign) money supply, and let x_t^* (y_t^*) be the home (foreign) endowments. Furthermore, let $X_t = [\ln(M_t^*/M_{t-1}^*), \ln(x_t^*/x_{t-1}^*), \ln(N_t^*/N_{t-1}^*), \ln(y_t^*/y_{t-1}^*)]'$. I assume the law of motion for X_t to follow:

$$A(L)X_t = a_0 + M(L)\epsilon_t$$

$$\epsilon_t | \mathcal{F}_{t-1} \sim N(0, H_t)$$

$$h_t^i = w_i + b_i h_{t-1}^i + c_i (\epsilon_{t-1}^i)^2 \quad i = 1, 2, 3, 4$$

$$h_t^{ij} = \rho_{ij} (h_t^i)^{.5} (h_t^j)^{.5} \quad i \neq j$$

(19)

with a_0 a vector of constants, $A(L)$, $M(L)$ polynomials in the lag-operator, ϵ_t a 4x1 vector of innovations with conditional covariance matrix H_t and $\mathcal{F}_t$ the information set at time t. The time variation in the second moments is parsimoniously modelled by a constant correlation GARCH (1,1) model (see Bollerslev (1990)). This model has been very successful in capturing volatility clustering in financial series (see the survey in Bollerslev, Chou and Kroner (1992)).

The conditional mean specification is motivated, estimated and tested below. Time-varying second moments in the exogenous forcing processes are potentially important determinants of risk premiums in the forward market. Previous attempts at modelling the time-series behavior of exchange rates (Hodrick (1989)) or the risk premium (Canova and Marrinan (1993), Kaminsky and Peruga (1990)) by explicitly allowing for conditionally heteroskedastic forcing processes, have not been very successful. These models assume that the decision interval of the representative agents coincides with the monthly or quarterly interval at which the data were sampled. The failure to produce a variable enough risk premium is then due to the lack of sufficient time-variation in the second moments of consumption and money measures.

Time aggregation is an important issue here for two reasons. First of all, using highly aggregated data to estimate a conditional variance model might lead to poor estimates of the true heteroskedasticity in the data. A nice illustration is contained in Drost and Nijman (1993), who estimate GARCH models for various exchange rate series at different frequencies. They find that a monthly model implied by a daily or weekly model contains strong conditional heteroskedasticity although the homoskedasticity assumption could not be rejected by direct estimation of the monthly model. Second, the fact that time-

averaged consumption data are used instead of spot consumption changes, might severely distort the measurement of consumption risk and the covariance between consumption and money. Both are important components of the risk premium in this model.[13]

3. SOLVING AND CALIBRATING THE MODEL

In this section, I first define the parameter and state vector for the model and briefly discuss how the model is solved numerically. Since the model is too complex to allow estimation of all the parameters, I also discuss the choice of a set of benchmark parameter values. Finally, I describe the estimation technique for a sub-set of the parameters.

Numerical solution of the model

The economic environment can be summarized by the parameters Φ, the state vector Θ_t and a set of Euler equations that the endogenous variables must satisfy. I partition the parameter space as $\Phi = [\Xi, \Omega]$, where Ξ stacks the parameters of the law of motion of the forcing processes and $\Omega = [\beta, \gamma, q, c_x, c_y, \xi_x, \xi_y, \mu_x, \eta_x, \theta_x, \mu_y, \eta_y, \theta_y]$ with γ denoting the curvature parameter of the utility function and $q = \gamma$ for addilog utility, and $q = \delta$ for homothetic utility. It will be useful to further partition the parameter space as follows. Let Ξ_1 denote the parameters governing the conditional mean equation and the unconditional variances and correlations of the four exogenous processes and let Ξ_2 denote the parameters governing the time variation in the conditional variances $[b_i, c_i]$ $(i = 1,2,3,4)$. Hence, $\Xi = [\Xi_1', \Xi_2']'$. Furthermore, let $\Omega = [\Omega_1', \Omega_2']'$, with $\Omega_1 = [\beta, \alpha, \gamma, c_x, c_y, \xi_x, \xi_y]$ and $\Omega_2 = [\mu_x, \eta_x, \theta_x, \mu_y, \eta_y, \theta_y]$.

Conditional on estimates for $[\Xi, \Omega_2]$, the *state vector* Θ_t of the model can be determined. The growth rates for the exogenous state variables are denoted by, for example, $gm_t = M_t^s / M_{t-1}^s$ (analogous definitions apply to gx_t, gn_t, gy_t). The conditional variances $h_{t+1,t}$ are in the time t information set and also part of the state vector. Because of the constant correlation GARCH model, they suffice to forecast future conditional variances and co-variances. The stock of durables and the habit stock are introduced in

stationary format by dividing by consumption levels as is done in Heaton (1993). Hence,

$$\Theta_t = [1, gm_t, gx_t, gn_t, gy_t, h_{t+1,11}, h_{t+1,22}, h_{t+1,33}, h_{t+1,44}, k_t^1/x_t^1, k_t^2/y_t^2, h_t^1/x_t^1, h_t^2/y_t^2]'$$

The crucial endogenous variables to be solved for are the velocities (equation (18)) and the marginal utilities (equation (8)). Once these variables are solved for, the other endogenous variables follow straightforwardly. The solution technique proceeds in two steps. First, a stationary representation of the relevant Euler equations is obtained where all variables depend only on the stationary state vector defined above. Then the endogenous variables are approximated by polynomials in the state variables and the Euler equations are numerically solved using the approach of Judd (1990). To evaluate expectations, I use Monte Carlo integration as in Heaton (1993) with 4,000 observations. A detailed description of the solution technique can be found in the appendix.

Parameter Calibration

For the homothetic utility specification I let $\Omega_1 = [.99^{(1/52)}, 1.1, .5, .09, .09, 2.5, 2.5]'$, for the addilog utility specification I let $\Omega_1 = [.975^{(1/52)}, .2, .2, .09, .09, 2.5, 2.5]'$. As described in the appendix, these parameters are calibrated to imply reasonable velocity and interest rate behavior in the non-stochastic steady state of the model and to imply low transaction costs. Previous estimates of the curvature parameter in the utility function vary wildly but they are often quite low.[14] Although the current specification of Ω_2 imposes discipline on the simulations, I explore the implications of the model for a setting with more curvature in the utility function as well.

The Ω_2-parameters are chosen as follows: $\eta_x = \eta_y = .75$ $\theta_x = \theta_y = .95$ $\mu_x = \mu_y = .76$. With $\mu_x = .76$, the half-life of durability is 2.5 weeks and three quarters of consumption vanishes within the month. This is consistent with the parameter estimates of Dunn and Singleton (1986) and Eichenbaum and Hansen (1990), since, as in these papers, consumption here is also measured as Nondurables and Services.[15] The sum of the habit weights η_x is based on the parameter estimates of Ferson and Constantinides (1991) and Heaton (1993), which vary between .60 and .95. Estimates of simple nonseparability parameters typically

typically favor durability when monthly data are used and favor habit persistence when quarterly or annual data are used. As Heaton (1993) stresses, these two findings can be plausibly reconciled by a parameter configuration in which durability dominates in the short run and habit effects in the long run. Or, in the notation of equation (9), $a_r > 0$ for small r and $a_r < 0$ for large r. This requires:

$$\eta_x(1-\theta_x) \leq \mu_x \leq \theta_x + \eta_x(1-\theta_x) \qquad (20)$$

My choice of $\theta_x = .95$ implies positive a_r within the quarter, and negative a_r from the 9th month onwards. As I do not know of any similar estimation exercises for the U.K., I fix these parameter values at the same levels as chosen for the U.S..

The parameters Z are partially estimated using data on empirical proxies to the endowment and money supply shocks. I interpret the endowments as per capita consumption of nondurables and services in the U.S. (the home country) and the U.K. (the foreign country), respectively. The monetary shocks are assumed to be shocks to a broad money concept in these countries.[16] These data are sampled quarterly, which introduces a *temporal aggregation* problem into the estimation. In general, an attempt to identify parameters from a weekly model with quarterly data is plagued with the *aliasing problem* (see for instance Hansen and Sargent (1983), Nijman and Palm (1990)). There could be many models that yield observationally equivalent laws of motion for the quarterly data. Below I motivate a particular form for the general law of motion in equation (19) which has the implication that Z_1 is identifiable from quarterly data. The parameter vector Z_1 can then be estimated by the General Method of Moments (Hansen (1982)).

Diebold (1986) proves that GARCH processes converge to unconditional normality under temporal aggregation, which might make it practically difficult to identify Z_2 from quarterly data.[17] Therefore, I do not attempt to estimate these parameters from the data. I further simplify interpretation of the results by assuming $b_i = b_j$, and $c_i = c_j$, $i,j = 1,..4$. Note that this does not imply that the conditional variances of the forcing processes are identical or that they are perfectly correlated, only that they move similarly

over time, e.g. they have identical persistence. In the benchmark case, I put $b_i = .80$, $c_i = .13$, for all i. Taken as a univariate GARCH-model, this parameterization implies a kurtosis coefficient of .91 and a half-life of conditional variance shocks of 3 weeks. This is about half the kurtosis and persistence of conditional variance shocks implied by a univariate GARCH(1,1) model estimated from the weekly pound/$ exchange rate data used in this paper.[18] The conditional heteroskedasticity assumed at the weekly level is also weak enough so that it disappears when the data are time-aggregated.[19] In simulation exercises below, I explore various patterns of time variation in the second moments of the forcing processes and their implications for exchange rate behavior.

This completes the discussion of the parameter calibration. In what follows, I first further discuss the estimation of Z_t. In the next section, I report all empirical results, including the estimation results for Z_t and extensive simulation results.

Estimating Z_t

I estimate the following simple parameterization of the general conditional mean specification in (19):

$$A(L) = I - \begin{bmatrix} a_{1,1}L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,1}L & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

$$M(L) = I$$

In words, the money processes follow an ARIMA(1,1,0), whereas the consumption processes are martingales but the innovations of all four series are assumed to be correlated. While this model seems very simple, I will show that it generates sufficiently rich dynamics at the quarterly frequency to be consistent with the data.

To motivate this simple model, consider the time-series properties of the forcing processes in Table 2.[20] The autocorrelations of the first differenced series, reported in Panel A, show that it would be hard to reject the hypothesis that the first autocorrelation of both consumption series is .25 and that the second

autocorrelation is zero. The monetary aggregates are much more persistent than the consumption series. Panel B contains the results of a VAR on the 4 variables. Tests on the VAR order support a first order VAR. For each equation, I perform a Wald test for the joint significance of the coefficients on the variables other than the lagged left hand side variable. Note the significant cross-effects, particularly in the U.K. equations. A test of the hypothesis that the innovation covariances are jointly zero also rejects. Diagnostic tests on the residuals of the first order VAR detect little evidence of significant serial correlation, heteroskedasticity or deviations from normality, with the exception of some remaining serial correlation in the U.S. consumption equation residuals.

I argue that these data patterns can be consistent with the simple law of motion specified in (19) because of temporal aggregation. The effects of time-averaging flow variables are well-known. In my model an additional complication arises because of the log-transformation. I choose to approximate logs of arithmetic averages with geometric averages as in Hall (1988) and Heaton (1993), i.e. I assume:

$$\ln(\sum_{\tau=0}^{12} x_{t-\tau}^{\cdot}) - \ln(\sum_{\tau=0}^{12} x_{t-13-\tau}^{\cdot}) \approx \frac{1}{13} \sum_{\tau=0}^{12} [\ln(x_{t-\tau}^{\cdot}) - \ln(x_{t-13-\tau}^{\cdot})] \tag{22}$$

A time-averaged random walk implies a first order autocorrelation of .25 (Working (1960)). For money, a stock variable, temporal aggregation will introduce an additional MA-component. As is shown in Bekaert (1992), the implied quarterly law of motion of the ARIMA(1,1,0) specification is ARIMA(1,1,1), which is potentially consistent with the data patterns.

The VAR tests show that the forcing variables are correlated which is accommodated through the assumption of correlated residuals in (19). The significant cross-effects in the conditional means might also be due to a time-aggregation effect. Bekaert (1992) gives a detailed description of the restrictions that the weekly model imposes on quarterly data and provides an identification proof for a model that embeds the parameterization in equation (19). Hence, the estimation of Z_t is not subject to the aliasing critique.

The model specified in (19) and (21) has 12 parameters: the two autocorrelation coefficients of the money processes and the distinct elements of the unconditional covariance matrix of the innovations, Σ_{e}. To guarantee positive semi-definiteness of Σ_{e}, I estimate its Cholesky-decomposition. I estimate the parameters by standard GMM. Let X_{t} contain the de-meaned quarterly observations on log-differences of money and consumption. Consider the function $f(X_{t})$ that maps a subset of the data sample into a vector-valued stochastic process such that $(1/T) \Sigma f(X_{t})$ is a vector of sample moments at time t. Let $E[f(Z_{1},X_{t})]$ represent the corresponding vector of population moments. The parameters Z_{1} are estimated with 16 orthogonality conditions, using the moments $E[f(X_{t},Z_{1})] = [vech(E(X_{t} X_{t}')', E[X_{t}^{i} X_{t-1}^{i}], E[X_{t}^{i} X_{t-2,t}^{j}]]'$ for all i and for j=1,3 with the indexing being the same as for X_{t}. As these moments are analytically known, construction of the orthogonality conditions is trivial. The estimation is done with a weighting matrix put equal to the inverse of a consistent estimate of the spectral density at frequency zero of the orthogonality conditions. The latter is estimated using a Bartlett kernel with an "optimal" bandwidth of 3 (Andrews (1991)). The estimation imposes 4 over-identifying restrictions which can be tested. To test the validity of the model, various other restrictions of the model will be tested using the methodology from Eichenbaum, Hansen and Singleton (1988).

4. EMPIRICAL RESULTS

Estimation of Z_{1}

Table 3 contains the estimation results for Z_{1}. The estimation yields very persistent money supply rules. The parameters of the innovation covariance matrix are reasonably precisely estimated. The table reports the resulting standard deviations and correlations. The $\chi^{2}(4)$ test statistic of the over-identifying restrictions is 4.157 with a p-value of .385, indicating that there is no evidence against the restrictions.

The C-statistics, reported in the table, test some other restrictions implied by the model. The first set of restrictions tested is the implication of the model's weekly law of motion for consumption that the

second autocovariance of quarterly consumption data ought to be zero. The second set of restrictions concerns the cross-moments of the quarterly variables. In general, the first order covariance of the quarterly measured data is non-zero but restricted by the weekly model. I test 6 cross-moment restrictions: the covariance between U.S. money and lagged U.K. money (and vice versa), between U.S. consumption and lagged U.K. consumption (and vice versa) and between U.S. (U.K.) consumption and lagged U.S. (U.K.) money. All these covariances were among the strongest in the VAR estimated on quarterly data. As the table shows, none of the tests performed rejects the restrictions imposed by the simple weekly model.

Simulation results: Variability and persistence puzzle

Simulation results are reported for a CIA-model, the model with time-additive preferences, the model with durability only, the model with durability and habit persistence and finally for the complete model incorporating time-varying uncertainty in the fundamentals.

(1) A CIA-Model

First, consider the case of addilog preferences in Table 4. Panel A contains simulations for the benchmark parameter values, whereas Panel C increases the curvature parameter for the utility function to 2. In a CIA-model with time-additive preferences, expressions for Δs_{t+1} and fp_{t} in terms of the exogenous processes are given by:

$$\Delta s_{t+1} = (\gamma - 1)[\ln\frac{x_{t+1}^{*}}{x_{t}^{*}} - \ln\frac{y_{t+1}^{*}}{y_{t}^{*}}] + \ln\frac{M_{t+1}^{*}}{M_{t}^{*}} - \ln\frac{N_{t+1}^{*}}{N_{t}^{*}} \tag{23}$$

$$fp_{t} = (\gamma - 1)[u_{t}^{x} - u_{t}^{y}] + u_{t}^{M} - u_{t}^{N} + v \tag{24}$$

where u^{j} indicates the conditional mean of $\ln(j_{t+1}^{*}/j_{t}^{*})$, $j = M, N, x, y$ and v is a constant depending on the (co-) variances of the forcing processes. When $\gamma = 1$ (log-utility), exchange rates only depend on money and are consequently highly persistent and not very variable. Likewise, forward premiums are very

persistent and show considerable variance because future money supplies are highly predictable. When γ differs from 1, exchange rates also depend on the supply shocks, which are uncorrelated (see above). Therefore, their variability increases and their persistence drops, which is apparent from both Panels A and C.

For the homothetic utility case, exchange rates only depend on the money processes and inherit their persistence. The CIA model, in general, seems to over-predict the persistence of exchange rates and under-predict its variability. The simple model implies realistic forward premium volatility and under-predicts its persistence at long horizons.

(2) Introducing Transaction Costs

Intuition on the effects of the different features of the model is best served by focussing on Panel C. The transaction cost technology induces variable velocities. Variable velocity acts as an additional demand shock and both increases the variability of currency depreciation and dilutes the persistence that is injected by the highly autocorrelated money rules. However, it also implies less predictable IMRS, making their predictable parts, interest rates, less variable and less persistent.

(3) Introducing Durability

Introducing durability potentially provides a remedy to this persistence puzzle. Suppose the agent expects high home endowment growth. Because she desires to smooth consumption, she attempts to consume part of the bumper crop now. As she is constrained by her present income, she attempts to sell bonds hence driving up interest rates which makes her willing to buy her share of the current endowment. With durability, a positive shock builds into the service flows and is likely to cause a revision of expectations in the next period as well, hence injecting more persistence in interest rates. The high depreciation rate $1-\mu_x$, and the lack of any correlation in the original endowment growth rates reduce the potential impact of durability. Also note that the IMRS are just about as variable here as they were in the time-additive model, so that exchange rate variability is virtually unaffected.

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(4) Introducing Habit Persistence

The next column reports the results for the benchmark parameter configuration but with Z_2 still set equal to 0. It offers the most satisfactory match with the data. The economic intuition behind the results is straightforward. With habit formation, consumers are more reluctant to diverge from smooth consumption streams. One could interpret the habit stock as a subsistence level of which it is very painful to deviate. This leads to more variable IMRS, exchange rates and forward premiums: when the fundamentals change or expectations get revised, bigger price movements are needed to induce agents to hold the endowments. Although the model matches forward premium volatility, exchange rate variability is still somewhat too low. This specification also comes close to explaining the "persistence puzzle". The persistent service flows generated under this preference specification generate substantial persistence in endogenous interest rates, but the model still fails to deliver the persistence observed in forward premium data. On the other hand, the supply shocks cause a large enough forecast error in the IMRS to keep the persistence of exchange rates low. Strikingly, exchange rates show significant mean reversion at long horizons.[21] This is also due to the long-run habit persistence built in. A positive home supply shock weakens the dollar by reducing its marginal utility. The endowment adds to the stock of durables and hence affects the marginal utility of the dollar next period as well. Of course, new shocks mitigate this effect and the positive persistence induced by this is not very high. The higher service flow today also adds to the habit stock tomorrow and the higher habit stock eventually increases the marginal utility of the dollar causing the negative correlations. As expected, negative correlations start to dominate after two months.

When γ is dropped to .2 in the benchmark case, the performance of all models is dismal. They all severely under-predict both the persistence and variability of exchange rate changes and forward premiums. Clearly, there has to be some curvature in the utility function for habit persistence to generate any significant effects.

With homothetic preferences (Panel B), exchange rate changes and the forward premium become less variable and the forward premium somewhat less persistent. Agents have utility over a geometric average of the two service flows which implies smoother marginal utilities and IMRS.

(5) Introducing Time-varying Uncertainty in the Fundamentals

As all three panels show, the model under-predicts the variance of the risk premium by several orders of magnitude. Although habit persistence increases the standard deviation of the risk premium somewhat, the biggest effects are observed when time-varying uncertainty in the forcing processes is allowed for. Risk premium volatility is larger by a factor of over 2 (Panel A) to over 4 (Panel C), compared to the case without heteroskedasticity in the forcing processes. Time-varying uncertainty also increases the variability of both exchange rates and forward premiums. Still, even with $\gamma = 2$, risk premium variability in the data is more than 50 times larger than in the model!

To sum up, the benchmark model performs better than previous models do but stills provides a very poor fit with the data. With risk aversion modestly higher than log-utility however, the model performs relatively well with respect to the persistence puzzle although it produces too much mean reversion in exchange rates and somewhat under-predicts the persistence of forward premiums. On the other hand, it fails drastically with respect to the variability premium puzzle, producing $\sigma(\Delta s_{t+n}) > \sigma(fp_{t}) > \sigma(rp_{t})$ instead of $\sigma(\Delta s_{t+n}) > \sigma(rp_{t}) > \sigma(fp_{t})$.

Simulation results: Risk, uncertainty and exchange rates

The previous section has shown, that introducing conditional heteroskedasticity in the model's market fundamentals substantially increases the variability of the risk premium. In this section, I want to address two additional questions. One is whether the model can produce fat tails and/or heteroskedasticity in weekly exchange rates and the forward premium and how much of these endogenous non-linearities are due to the heteroskedasticity in the forcing processes. Secondly, I want to further explore the effect on equilibrium exchange rate moments of different patterns of heteroskedasticity in the forcing processes.

Table 5 shows the kurtosis coefficient of exchange rate changes and the differenced forward premium, and variance ratios for the squared series. The additional non-linearities introduced by variable velocity, durability and habit persistence help to generate leptokurtosis and conditional heteroskedasticity in the endogenous variables. Nevertheless, the non-linearities induced in exchange rates are fairly weak whereas there is already quite some fat tails and serial dependence in the second moments of the differenced forward premium. In fact, for both benchmark specifications the model with habit persistence generates moments for the differenced forward premium that are almost within a two standard error band of the observed moments.

When time-varying uncertainty is introduced, all moments in all three specifications are within or very near a two standard error band of the observed moments. The moment that seems the hardest to match and is under-predicted is the leptokurtosis in exchange rate changes.

Next I examine the importance of both the persistence of conditional variance shocks to the fundamentals and the leptokurtic nature of the shocks. The former is governed by the coefficient on past variances in the GARCH specification whereas the latter is primarily governed by the coefficient on past residuals. In a first experiment, the kurtosis coefficient is fixed at its benchmark value and I vary b_i so as to generate a half-life of conditional variance shocks varying between 1 and 20 weeks. In a second experiment, I fix the persistence of conditional variance shocks (i.e. $b_i = .80$), but I adapt c_i so as to generate kurtosis coefficients varying between 0.01 and 6.0. To conserve space, I do not report the full results but offer a brief discussion. First, at the benchmark parameters, varying the conditional variance properties has only minor effects on the variability of the endogenous variables. When γ is set equal to 2 however, increasing the exogenous kurtosis coefficient from 0.01 to 6.0 doubles the variability of the risk premium, although it only leads to modest increases in exchange rate and forward premium variability. Increasing the persistence of conditional variance shocks mainly increases the persistence of the forward premium without having significant effects on the variability of the endogenous variables.

Second, not surprisingly, there are substantial effects on the non-linear behavior of the endogenous variables. Generally, a small degree of nonlinearities in the forcing process, coupled with the nonlinearities in the model, generates non-linear patterns in exchange rate changes and forward premiums that are consistent with what is observed in the data. For instance, there are different ways to generate a kurtosis coefficient of around 2.0 for exchange rate changes, which is less than 1 standard error below the sample moment of 3.0. In particular, the exogenous kurtosis coefficient can be increased to 4.0 or it can be kept at its benchmark value of .91 and the half life of conditional variance shocks can be increased from 3 weeks to about 10 weeks. As I will show below, the same effect can be accomplished through changing preference parameters as well.

Simulation results: Sensitivity analysis

To further explore the performance of the model, I vary respectively the curvature parameter, the habit weight parameter and the curvature parameter in the transaction cost technology, keeping the other parameters in the benchmark specifications constant. These three parameters are important determinants of the level of non-linearity in the model. Figure 1 shows the effect on 9 moments of changing the curvature parameter in the addilog specification. Similar graphs for the other experiments are available from the author on request. The moments involved are the standard deviation of exchange rate changes, the forward premium and the risk premium, the variance ratio (horizon 13) for exchange rate changes and the forward premium, the kurtosis coefficient for exchange rate changes and the forward premium and the variance ratio (horizon 13) for squared exchange rate changes and the squared differenced forward premium. The full horizontal line represents the sample value whereas the dotted horizontal lines are two standard error bands around the sample value. The parameter γ is varied between 0 and 5.5 whereas the other parameters are fixed at their benchmark values.

In discussing the results of the sensitivity analysis, let us first focus on the volatility and persistence puzzles (Figures 1a and 1b). Increasing the curvature parameter, the sum of the habit weights or the

curvature parameter in the transaction cost technology has similar effects, albeit in differing degrees. The variability of IMRS and hence of exchange rates, the forward premium and the risk premium alike increases. Risk premium variability becomes only more realistic at a relatively high level of curvature in the utility function or high values for the habit weight parameter. For instance, for $\gamma > 4$, risk premium variability is within the two standard error band. Exchange rate variability and forward premium volatility can be matched, but not *simultaneously*. The effect of more habit persistence is minimal in the addilog case, as there is not enough curvature in the utility function. Increasing the habit weight also has ambiguous effects on the persistence puzzle. Forward premium persistence goes up, but the model generates more mean reversion in exchange rates. Finally, the model does very well in generating realistic non-linear patterns in the forward premiums and exchange rate changes over a quite broad parameter range (see Figure 1c). Note how increasing γ also increases the endogenous leptokurtosis and heteroskedasticty in exchange rate changes. When γ is larger than 5, leptokurtosis in exchange rates becomes unrealistically high. Generally, the effects of changing the transaction cost technology parameter are less pronounced.

In changing other preference parameters, the most favorable effect occurs when θ is increased. This increases the long memory in the service flows. The effects on the addilog utility case with $\gamma = 2$ of increasing θ to .97 can be seen in the EXP I-column in Table 4, Panel C. The variability of exchange rates is now within one standard error of the sample moment, whereas the persistence of the forward premium also increases. Although the variability of the forward premium is too high, it does increase percentage wise less than the variability of the risk premium does.

Lastly, it is possible to specify parameter configurations for which the standard deviation of the risk premium is within 2 standard errors of the sample standard deviation, without increasing the curvature parameter too much. As shown in the EXP III column of Table 4, Panel C, this comes at the cost of far too variable exchange rates and forward premiums. In the EXP II column, the same parameters are used

but without heteroskedasticity. This once again shows the dramatic effect of adding time-varying uncertainty in the fundamentals to the model. Relative to EXP II, time-varying uncertainty almost doubles the variability of forward premiums and exchange rate changes and leads to a more than tenfold increase in the volatility of the risk premium.

The failure of the model to generate sufficient risk premium volatility also surfaces in the correlations between future spot changes and the current forward premium. Not a single experiment in all my simulations yields negative correlations!

CONCLUSIONS

Standard frictionless monetary general equilibrium models fail to explain the relative variability and persistence of exchange rates, and the forward and risk premium. They also fail to capture their non-linear behavior. In this paper, I maintained a frictionless rational expectations model but introduced a more realistic decision interval for the representative agents and more realistic preferences incorporating various forms of time nonseparabilities. The combination of time-varying uncertainty in the fundamentals and time aggregation was shown to be an important factor in explaining the risk premium puzzle. Simulation results indicated that the risk premium can be made several orders of magnitude more variable than in previous models without implying very unrealistic endogenous moments for exchange rate changes and the forward premium. The non-linearities embedded in the model coupled with weak forms of time-varying uncertainty in the fundamentals endogenously generate substantial heteroskedasticity and leptokurtosis in exchange rates and the forward premium. Still, the model has the tendency to over-predict the variability of the forward premium and severely under-predict the variability of the risk premium. Although the persistence of forward premiums can be matched, it requires levels of risk aversion that imply slightly too variable forward premiums and exchange rates that are too mean-reverting.

Several generalizations of the present model are potentially useful. Bekaert, Hodrick and Marshall

(1994) for instance, explore the predictability of asset returns, including forward market returns, in a recursive preference framework with first-order risk aversion. Although first-order risk aversion also substantially increases the variance of the risk premium relative to time-additive models, the model substantially under-predicts the variability of exchange rates, the forward premium and the risk premium.

The empirical results of Bekaert and Hodrick (1993) suggest another useful direction for further research. In an empirical re-examination of the unbiasedness hypothesis, we find that unbiasedness holds for the British pound during the 1975-1980 period. In fact, the slope coefficient in a typical unbiasedness regression test is slightly larger than 1 for that sub-period. In the turbulent eighties, the rejection of unbiasedness is very severe. Interestingly, for the 1973-1976 period, i.e. before the system of floating exchange rates was formally implemented, we also find negative slope coefficients. It is possible that rational agents, faced with an array of policy signals, need time to recognize or "believe" changes in policy regimes. Such rational "learning" can lead to systematic forecast errors (see for instance Lewis (1989)) and partially explain the negative slope coefficients.

A third generalization is to break the complete markets set-up. In that case, the IMRS need not be equalized across countries. As agents can still self-insure through asset trading, asset prices might not differ very much from the complete market case. However, the perfect risk sharing assumed in this paper has several counterfactual implications. First of all, the extent of international portfolio diversification is actually very limited. Second, the set-up implicitly imposes Purchasing Power Parity (PPP), which is grossly violated in the data. In fact, when PPP-deviations exist, investors in different countries measure their real returns differently and generally desire to hold different portfolios (see Adler and Dumas (1983)). An additional channel to break PPP, is to explicitly model non-tradables. When utility is nonseparable in tradables and non-tradables, the IMRS's which are used to discount asset payoffs depend on non-tradables as well. Lastly, the large current account imbalances between major industrial countries indicate the usefulness of a general equilibrium model that allows wealth redistributions. I intend to explore a heterogeneous agent economy with non-tradable goods in the near future.

FOOTNOTES

1. See the survey on conditional heteroskedasticity in asset prices by Bollerslev, Chou and Kroner (1992).

2. See for instance Bansal, Gallant, Hussey and Tauchen (1993), Bekaert (1994), and Canova and Marrinan (1993).

3. This frequency strikes a balance between the belief that periods of turbulence in asset prices at that frequency coincide with turbulent movements in market fundamentals such as money growth, productivity shocks and policy shifts, whereas at higher frequencies market micro structure effects might play a predominant role.

4. Bansal, Gallant, Hussey and Tauchen (1993) is a related, contemporaneous effort.

5. The analysis in Bekaert and Hodrick (1993) indicates that this is a harmless assumption.

6. For a recent assessment of these tests, see Bekaert and Hodrick (1993).

7. All the moments used in computing this coefficient are estimated in a joint General Method of Moments System (GMM).

8. This is consistent with the Vector Autoregressive Regression (VAR)-based estimates of Bekaert (1993).

9. The formulation implicitly assumes that there exist assets paying out the monetary transfers and endowments.

10. Constantinides (1990) for instance shows in a single good economy that habit persistence drives a wedge between the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution.

11. Relaxing this assumption can only occur at considerable computational cost and would make it impossible to solve the model with time nonseparable preferences and conditional volatility shocks. Preliminary results of general equilibrium analysis with heterogeneous agents and incomplete markets, suggest that agents manage to smooth consumption very well with only a limited number of available assets (see e.g. Lucas (1991) and Marcet and Singleton (1990)). Consequently, asset prices do not differ very much from the complete markets case.

12. The empirical proxy for the endowments will be taken to be net of transaction costs.

13. A lognormal example that links the risk premium to the second moments of the forcing processes is worked out in an appendix available from the author.

14. See for instance Bansal, Gallant, Hussey and Tauchen (1993) and Canova and Marrinan (1993) for parameter estimates lower than 1 in an international framework.

15. Whereas they did not allow for lags beyond two months, my specification implies that less than 10% of the stock remains after 2 months.

16. See the Data Appendix for more details.

17. Results on the time-aggregation of GARCH-processes are scarce. Drost and Nijman (1993) restrict themselves to univariate models and show that strong forms of GARCH are not closed under temporal aggregation.

18. The parameter on the past conditional variance equals .882, the coefficient on the past squared innovation .094, implying a half-life of conditional variance shocks of 5.54 and a kurtosis coefficient of 1.82.

19. For a small Monte Carlo experiment demonstrating this, see Bekaert (1992).

20. The log-difference specification of (19) is justified by the unit root and cointegration tests reported in Bekaert (1992).

21. Huizinga (1987) actually finds evidence for mean reversion in real exchange rates.

DATA APPENDIX

The exchange rate data are daily data from Citicorp Data Services. Bob Korajzyck supplied DRI Eurocurrency interest rates running until mid 1988 which were obtained at INSEAD. All rates are sampled each Friday and averages of bid and ask rates. When Friday was a holiday, the Thursday rate was picked.

Quarterly consumption on nondurables and services is taken from the OECD Quarterly National Accounts. The U.K. semi-durables category is included as it is comprised of consumption items that are included in the nondurables category for the United States. The series are seasonally adjusted and in 1982 dollars, resp. 1985 pounds.

Money is measured as end-of-the-quarter M2 money stocks. For the U.S., M2 is taken from Citibase. Due to the introduction of MMDAs and super NOW accounts, there is an outlier in the U.S. data in the first quarter of 1983. The money growth rate for that quarter is replaced with a weighted average of past and future growth rates, incorporating 18 quarters of data. Published monetary aggregates in the U.K. cannot be used as there were several definitional changes that make it virtually impossible to deduce a consistently defined series over the whole sample. Therefore, I obtained data on the concept M4 directly from the Bank of England. It is a broad aggregate comparable to M2 in the U.S. except that it also includes deposits (including Certificates of deposits) with building societies. In November 1981, a big financial institution (the trustees savings bank) turned into a bank, increasing the money supply by 7.5% overnight (see Bank of England Quarterly Bulletin, Dec. 1981). This outlier was corrected for in a similar fashion as the U.S. outlier. The money series are also seasonally adjusted. Both the money and real consumption data were divided by total population (series 99z in the International Financial Statistics data set) to arrive at per capita data. The population data are mid-year estimates which are linearly interpolated to obtain quarterly data.

APPENDIX 1: THE CALIBRATION OF Ω_1

In the non-stochastic steady state of the model, the expressions for velocity and interest rates only depend on the parameters in Ω_1. In the case of addilog utility, steady state home consumption velocity is given by

$$\overline{V}^x = [c_x(\xi^x - 1)]^{\frac{1}{\xi^x}} [\frac{\overline{gm}\,\overline{gx}^{\gamma-1}}{\beta} - 1]^{\frac{1}{\xi^x}}. \tag{1}$$

with upper bars denoting steady state values. Likewise, the four week steady state interest rate is defined by

$$(1 + \overline{I})^{-1} = \beta^4 \frac{\overline{gx}^{4(1-\gamma)}}{\overline{gm}^4}. \tag{2}$$

The expressions for steady state interest rates and velocity for the homothetic utility case are

$$\overline{V}^x = [c_x(\xi^x - 1)]^{\frac{1}{\xi^x}} [\frac{\overline{gm}\,(\overline{gx}^\delta\,\overline{gy}^{(1-\delta)})^{\gamma-1}}{\beta} - 1]^{\frac{1}{\xi^x}} \tag{3}$$

$$(1 + \overline{I})^{-1} = \beta^4 \frac{(\overline{gx}^\delta\,\overline{gy}^{(1-\delta)})^{4(1-\gamma)}}{\overline{gm}^4} \tag{4}$$

Total transaction costs as a proportion of total consumption, TC_x, in steady state are given by

$$TC_x = c_x(\overline{V}^x)^{\xi^x - 1} \tag{5}$$

The expressions for the foreign good are similar.

To obtain empirical estimates of the steady state values, I obtain monthly interest rate means from the interest rate data used in the paper, and quarterly velocity means from the OECD data on consumption and consumption deflators and the money supply data described in the Data Appendix. To link quarterly (V_q) and weekly (V_w) velocity, I assume:

$$\overline{V}_q^x = \overline{V}_w^x \frac{1 - \overline{gm}^{11}}{1 - \overline{gm}}. \tag{6}$$

An analogous expression holds for foreign consumption velocity.

I consider the following parameter range:

$\beta \in \{.95, .955, .96,..., 1.0\}$

$\gamma \in \{0, .1, .2, .3,..., 2.9, 3.0, 3.25, 3.50, ...,10.25\}$

$c_x = c_y \in \{.0001, .001, .002,..., .01, .02,..., .1\}$

$\xi_x = \xi_y \in \{1.25,..., 2.0, 2.5, 3.0, 4.0, 5.0\}$.

To find a "reasonable" parameter configuration for Ω_1, the following procedure is performed:

(1) Compute mean growth rates of home and foreign consumption and money from the simulated observations according to the law of motion specified in equation (19) in the text, i.e. Z_1 is set equal to its estimated values but Z_2 is set equal to 0.

(2) For each parameter configuration, compute the steady states for velocity, the monthly interest rates and transaction costs TC_x and TC_y using equations (1)-(5).

(3) Discard parameter configurations which yield TC_x or $TC_y > .01$ and $< .0001$.

(4) Discard parameter configurations which yield too high or too low velocity. The lower bound is computed as the weekly equivalent from the quarterly mean, computed with the data used in the paper. As I used a broad monetary aggregate, I consider these means to over-estimate the mean of velocity consistent with the transaction cost technology modelled in the paper. I also compute an "adjusted " mean by multiplying all the velocities with the ratio of consumption to GNP as in Marshall (1992). I use this *quarterly* mean as the upper bound for weekly velocities. Hence, the velocity mean requirements are weak.

(5) Of the remaining parameter configurations, these parameters are chosen that come closest to matching the interest rate means. For both utility functions, the resulting parameter configurations are nearly within a 3 standard error band of the data means.

APPENDIX 2: SOLVING THE MODEL

As indicated in the text, the marginal utilities mux_t, muy_t and the consumption velocities V_t^x, V_t^y must be determined. The expression for mux_t follows from equations (8) and (9) in the text. It can be rewritten as:

$$
\begin{aligned}
& mux_t = w_1\, dmux_t + w_2\, hmux_t \\
& w_1 = 1 - \frac{\eta_x(1-\theta_x)}{\mu_x - \theta_x} \qquad w_2 = \frac{\eta_x(1-\theta_x)}{\mu_x - \theta_x} \\
& dmux_t = E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \mu_x^\tau \frac{\partial u(s_{t+\tau}^x, s_{t+\tau}^y)}{\partial s_{t+\tau}^x}\right] \\
& hmux_t = E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \theta_x^\tau \frac{\partial u(s_{t+\tau}^x, s_{t+\tau}^y)}{\partial s_{t+\tau}^x}\right]
\end{aligned}
\tag{7}
$$

A remaining problem is that, consistent with what is observed, the endowments and money supplies are assumed to grow over time. Therefore, I solve for a *stationary equilibrium*, in which stationary endogenous variables such as exchange rate changes, inflation and velocity depend only on stationary state variables, including growth rates of money supplies and endowments. To induce stationarity in the stocks of consumption goods and the habit stocks, I divide by the level of consumption. This normalization is used by Heaton (1993) and leads to a natural stationary representation of the state vector. Define a scaled marginal utility as:

$$
smux_t = \frac{mux_t}{\dfrac{\partial u(x_t^d, y_t^d)}{\partial x_t^d}}
\tag{8}
$$

where the denominator is the particular period-utility function evaluated at present equilibrium consumption purchases. Define sdmux_t analogously by replacing mux_t in (8) by $dmux_t$. Then $sdmux_t$ solves the following difference equation:

An analogous equation applies to $hmux_t$. Once market clearing is imposed, the marginal utility ratios in (9) will be direct functions of the state vector and the difference equation can be solved for $sdmux_t$. Note that: $smux_t = w_1\, sdmux_t + w_2\, shmux_t$. With the solution for $smux_t$, the Euler equation determining

$$sdmux_t = \frac{\frac{\partial u(s_t^x, s_t^y)}{\partial s_t^x}}{\frac{\partial u(x_t^d, y_t^d)}{\partial x_t^d}} + \beta \mu_2 E_t [\, sdmux_{t-1} \frac{\frac{\partial u(x_{t-1}^d, y_{t-1}^d)}{\partial x_{t-1}^d}}{\frac{\partial u(x_t^d, y_t^d)}{\partial x_t^d}} \,] \qquad (9)$$

velocity can be re-written in stationary format:

$$\frac{smux_t}{1 + \frac{\partial \Psi^x(t)}{\partial x}} = \beta E_t [\, smux_{t-1} \frac{\frac{\partial u(x_{t-1}^d, y_{t-1}^d)}{\partial x_{t-1}^d}}{\frac{\partial u(x_t^d, y_t^d)}{\partial x_t^d}} (DP_{t-1}^x)^{-1} \frac{1 - \frac{\partial \Psi^x(t+1)}{\partial m}}{1 + \frac{\partial \Psi^x(t+1)}{\partial x}} \,] \qquad (10)$$

As the right hand side depends on state variables at time t, equation (10) describes the solution for $\partial \Psi^x / \partial x$ and hence for velocity.

Let $\Theta_t \in R^K$ be the vector of stationary state variables which completely spans the information set and $\varphi_t = [V_t^x, V_t^y, smux_t, smuy_t]$, the set of endogenous variables to be solved for. The other endogenous variables are trivial functions of φ_t. As the model is dichotomous in $\varphi_t^x = [V_t^x, smux_t]$ and $\varphi_t^y = [V_t^y, smuy_t]$, I can restrict the discussion to the solution method for φ_t^x. The two basic Euler equations that must be solved in this model are (9) and (10). They are of the form:

$$\Lambda(\varphi_t, \Theta_t) = E_t[\Gamma(\varphi_{t-1}, \Theta_{t-1})] \qquad (11)$$

where $\Lambda(.,.)$ and $\Gamma(.,.)$ are known functions. As φ_t is a continuous function mapping R^K, the space of the state variables Θ_t, into R, it can be approximated arbitrarily well by polynomials. The polynomial coefficients are found by minimizing the approximation error over some norm. The technique I use follows Heaton (1993) in employing Monte Carlo integration to evaluate expectations. Marcet and Marshall (1993) actually formally prove that, when the sample size and the polynomial order, approach infinity, the numerical equilibrium solution converges to the Rational Expectations Equilibrium.

More specifically, I first solve for the scaled marginal utilities. Let $p_n(\Theta_t)$ be a vector of polynomial elements in Θ_t of degree $\le n$. Denote $f_n(\Theta_t; q_n) = [\mathcal{P}_n(\ln(\Theta_t))' q_n]$, where q_n indicates the polynomial

coefficients. Let sdmux$_t$ $\approx$ $f_s(\Theta_t; q_s)$. Substitute this in the difference equation (8) to find the approximation error $u(\Theta_t; q_s)$:

$$u(\Theta_t; q_s) = f_s(\Theta_t; q_s) - \frac{\frac{\partial u(s_t^1, s_t^2)}{\partial s_t^1}}{\frac{\partial u(x_t^4, y_t^4)}{\partial x_t^4}} - \beta \mu E_t [f_s(\Theta_{t+1}; q_s) \frac{\frac{\partial u(x_{t+1}^4, y_{t+1}^4)}{\partial x_{t+1}^4}}{\frac{\partial u(x_t^4, y_t^4)}{\partial x_t^4}}] \qquad (12)$$

The parameters q_s are chosen such as to make $u(\Theta_t, q_s)$ orthogonal to the polynomial elements $p(\Theta_t, q_s)$[1] or:

$$E[u(\Theta_t; q_s) p_s(\Theta_t)] = 0 \qquad (13)$$

By the law of iterated expectations, this can be rewritten as:

$$E[\{ f_s(\Theta_t; q_s) - \frac{\frac{\partial u(s_t^1, s_t^2)}{\partial s_t^1}}{\frac{\partial u(x_t^4, y_t^4)}{\partial x_t^4}} - \beta \mu f_s(\Theta_{t+1}; q_s) \frac{\frac{\partial u(x_{t+1}^4, y_{t+1}^4)}{\partial x_{t+1}^4}}{\frac{\partial u(x_t^4, y_t^4)}{\partial x_t^4}} \} p_s(\Theta_t)] = 0 \qquad (14)$$

The expectation can be taken by Monte Carlo integration, i.e. by drawing a long time series for Θ_t and taking sample averages of the expression in equation (14). The coefficients q_s then follow from solving a system of non-linear equations. The difference equation for shmux$_t$ is solved analogously and smux$_t$ is obtained as in equation (7).

To solve for velocity, approximate V_t^x by polynomials and substitute the approximate velocity function into equation (10). In this case the polynomials will be exponentiated to guarantee positive velocities. The marginal utility smux$_t$ is replaced by its approximation previously solved for. This will give rise to

---

[1] This is the Galerkin method, one of the Minimum Weighted Residuals (MWR) methods, described by Judd (1990).

an approximation error that is projected onto the polynomial elements in order to solve for the polynomial

coefficients.[2]

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Table 1
Time series properties of exchange rates, the forward premium and term spreads

The sample period is January 1975 to December 1990. Observations are sampled weekly on Δs_t the weekly logarithmic exchange rate change, $fp_{t,4}$, measured as the difference between the U.S. one month interest rate and the U.K. one month interest rate. All interest rates are annualized, i.e. they are multiplied by 1200. Weekly currency depreciation is also multiplied with 1200. The symbol σ always denotes the standard deviation, ac_n denotes the nth autocorrelation and v_i the variance ratio including i autocorrelations, estimated as in Equation (4) in the paper. In the third row, the normalized kurtosis coefficient ku is computed for (de-meaned) currency depreciation and for the differenced forward premium. The standard errors are derived by GMM. The mean, standard deviation, kurtosis coefficient and the moments reported in Panel B are estimated jointly using 13 Newey-West (1987) lags in calculating the variance matrix. The standard errors for the variance ratios follow from the joint estimation of 52 autocorrelations and their variance matrix using 51 Newey-West lags. The cross-correlations in Panel B are between the forward premium and future exchange rate changes and between the differenced forward premium and squared demeaned depreciation respectively.

Panel A: Univariate properties

	Δs_t	$fp_{t,4}$	$(\Delta s_t)^2$	$(\Delta fp_{t,4})^2$
mean	-.319	-2.637		
	(.702)	(.426)		
σ	17.668	3.481		
	(1.069)	(.343)		
ku	3.000			8.635
	(1.290)			(2.495)
ac_1	.025	.972	.171	.145
	(.037)	(.007)	(.073)	(.057)
ac_2	.006	.949	.160	.228
	(.054)	(.012)	(.055)	(.083)
ac_3	.044	.925	.194	.199
	(.034)	(.019)	(.047)	(.031)
ac_5	-.033	.873	.197	.208
	(.036)	(.031)	(.104)	(.070)
ac_{13}	-.008	.678	-.005	.072
	(.039)	(.075)	(.021)	(.022)
v_4	1.110	4.794	1.663	1.702
	(.070)	(.049)	(.229)	(.210)
v_{13}	1.304	12.350	2.419	2.718
	(.133)	(.390)	(.427)	(.413)
v_{26}	1.460	21.097	3.177	3.746
	(.195)	(1.295)	(.495)	(.634)
v_{52}	1.725	32.798	4.101	5.957
	(.314)	(3.667)	(.717)	(1.174)

Panel B: Cross correlations

	$\Delta s_{.,1}/(\Delta s_{.,1})^2$	$\Delta s_{.,2}/(\Delta s_{.,2})^2$	$\Delta s_{.,3}/(\Delta s_{.,3})^2$	$\Delta s_{.,4}/(\Delta s_{.,4})^2$
$f_{p,4}$	-.097	-.104	-.104	-.112
	(.038)	(.038)	(.039)	(.037)
$(\Delta f_{p,4})^2$	.051	.-.036	.020	-.020
	(.068)	(.019)	(041)	(.024)

Table 2
Table 2
Time series properties of the forcing processes

The standard errors of the autocorrelations are computed by GMM, using the heteroskedasticity and autocorrelation consistent covariance estimator described in Andrews (1991) with a Bartlett kernel and optimal bandwidth of 2. The appropriate lag length for the VAR minimizes the Akaike or Schwarz criterion in Panel B. Parameter estimates are obtained by OLS and reported in Panel C with heteroskedasticity consistent standard errors. The symbol Δ_m stands for the m-weeks difference operator and the subscript a for time-averaged data. The $\chi^2(3)$ statistic in Panel C tests whether the 3 coefficients other than the one on the lagged left hand side variable are jointly equal to zero. A test statistic on the joint significance of the 6 innovation covariances is also reported. The test statistic is derived from the joint distribution of the VAR parameters and the innovation matrix and has a $\chi^2(6)$ distribution.In Panel D, the Cumby-Huizinga (1992) l-test for serial correlation of the residuals is robust to conditional heteroskedasticity and lagged dependent variables. The Q2-test is the Ljung-Box test statistic applied to squared residuals. The ARCH-test is the standard Lagrange multiplier test for serial correlation in the squared residuals, as proposed by Engle (1982). All tests are $\chi^2(n)$ with n the number of lagged squared residuals included in the test. Ku is the normalized kurtosis coefficient and Sk the normalized skewness coefficient. Their asymptotic distribution is N(0,24/T), N(0,6/T) respectively, with T the sample size, under the null of normality. BJ is the Bera-Jarque (1982) test for normality and is $\chi^2(2)$. P-values, based on the χ^2-distribution, are reported for all test statistics.

Panel A: Autocorrelations

	$\Delta_m \ln(M^r_{i,a})$	$\Delta_m \ln(x_{i,a})$	$\Delta_m \ln(N^r_{i,a} 1)$	$\Delta_m \ln(y_{i,a})$
ac_1	.464 (.102)	.296 (.128)	.322 (.167)	.376 (.119)
ac_2	.255 (.080)	.067 (.104)	.439 (.120)	.143 (.095)
ac_3	.345 (.078)	.139 (.100)	-.002 (.119)	-.102 (.192)
ac_4	.118 (.163)	.284 (.117)	.238 (.122)	-.130 (.150)

Panel B: Selection criteria for the VAR order

VAR order	Akaike criterion	Schwarz criterion
1	-40.34	-39.80
2	-40.03	-38.95
3	-40.28	-38.64

Panel C: Parameter coefficients for VAR of order 1

	$\Delta_{\_}\ln(M^r_{t\text{-}1,a})$	$\Delta_{\_}\ln(x^c_{t\text{-}1,a})$	$\Delta_{\_}\ln(N^r_{t\text{-}1,a})$	$\Delta_{\_}\ln(y^c_{t\text{-}1,a})$	R^2	Wald
$\Delta_{\_}\ln(M^r_{t,a})$	.537 (.121)	-.297 (.194)	-.068 (.123)	-.066 (.102)	.212	6.008 (.111)
$\Delta_{\_}\ln(x^c_{t,a})$	.158 (.068)	.204 (.142)	.087 (.100)	.050 (.065)	.091	7.078 (.069)
$\Delta_{\_}\ln(N^r_{t,a})$	-.012 (.137)	-.286 (.175)	.289 (.149)	.097 (.083)	.097	14.749 (.002)
$\Delta_{\_}\ln(y^c_{t,a})$	-.425 (.164)	.656 (.300)	.275 (.164)	-.092 (.135)	.152	16.535 (.001)

Test on innovation covariances: 17.793 (.007)

Panel D: Residual Diagnostics

	1(4)	Q2(4)	ARCH(4)	Ku	Sk	BJ
Eq. 1	6.825 (.145)	3.876 (.423)	3.922 (.417)	.265 (.665)	-.137 (.655)	.386 (.824)
Eq. 2	12.462 (.014)	7.902 (.095)	8.746 (.068)	.973 (.112)	.296 (.333)	3.459 (.177)
Eq. 3	1.371 (.849)	3.081 (.544)	3.938 (.415)	.631 (.303)	-.370 (.227)	2.520 (.284)
Eq. 4	10.613 (.031)	4.912 (.296)	5.704 (.222)	.140 (.819)	.005 (.986)	.052 (.974)

Table 3
GMM estimation of the law of motion for the forcing processes

The parameters estimated govern the unconditional moments of the law of motion specified in Equation (23) in the text. Although the Cholesky-decomposition of the unconditional covariance matrix of the innovations is estimated, I report the resulting correlation matrix of the innovations with standard deviations on the diagonal. The standard errors are obtained from the standard errors of the estimated parameters using the Mean Value Theorem. By estimating the Cholesky-decomposition directly, positive semi-definiteness of the covariance matrix is automatically imposed. The J_T statistic tests the over-identifying restrictions of the original estimation with 16 moments. P-values are given in parentheses. The C-tests test various restrictions implied by the weekly model for quarterly data. These restrictions are discussed in the text. The test methodology follows Eichenbaum, Hansen and Singleton (1988). First, the parameters are re-estimated using the 8 moment conditions to be tested in addition to the 16 original moments. This yields a $\chi^2(12)$ test statistic for the over-identifying restrictions, which is denoted by $J_T(12)$. Suitable partitions of the resulting weighting matrix are then used to estimate the model parameters with less moment conditions. The difference of the $J_T(12)$ test statistic and the new test of the over-identifying restrictions is denoted by $C_T(n)$ with n the number of moment conditions not used in the final estimation. Hence, $C_T(6)$ is the test statistic for the 6 cross-moment restrictions, $C_T(2)$ the test statistic for the consumption autocovariance restrictions and $C_T(8)$ tests all 8 restrictions simultaneously. All test statistics have χ^2-distributions with degrees of freedom equal to the number indicated between brackets.

Autocorrelation coefficients
$a_{1,1}$ = .893 (.031)
$a_{3,1}$ = .865 (.038)

Correlation matrix

.0003	.291	-.198	-.247
(.00006)	(.128)	(.161)	(.107)
	.0015	-.055	.255
	(.00014)	(.149)	(.128)
		.0004	.465
		(.00007)	(.114)
			.0003
			(.00024)

Tests of the model

$J_T(4)$	$C_T(8)$	$C_T(6)$	$C_T(2)$
4.157	5.110	3.996	1.114
(.385)	(.746)	(.677)	(.573)

Table 4
Simulation results for exchange rate changes and the forward premium

The simulated sample used in computing the endogenous model moments has 4000 observations. Second order polynomials are used in approximating the endogenous variables. The acronyms are understood as: CIA = cash-in-advance, AL = addilog, HO = homothetic, T = time-additive, D = Durability, DH = durability and habit persistence. The benchmark parameters for DHAL are $\beta = .97^{1/12}$, $\eta_x = \eta_y = .75$, $\theta_x = \theta_y = .95$, $\mu_x = \mu_y = .76$, $c_x = c_y = .090$, $\xi_x = \xi_y = 2.5$, $\gamma = 0.2$. The benchmark parameters for DHHO are $\beta = .99^{1/12}$, $\eta_x = \eta_y = .75$, $\theta_x = \theta_y = .95$, $\mu_x = \mu_y = .76$, $c_x = c_y = .090$, $\xi_x = \xi_y = 2.5$, $\gamma = 1.1$, $\delta = .5$. The first column repeats the data moments and their GMM based standard errors. The next 3 columns of the table contain simpler variations of the benchmark specification, for example TAL in Panel A denotes a time-additive model ($\eta_x = \eta_y = 0.0$, $\mu_x = \mu_y = 0.0$) with addilog preferences and a curvature parameter equal to 2. The b_i and c_i parameters ($i = 1,2,3,4$) in the far-right column are the parameters governing the time-variation in the second moments of the forcing processes as specified in Equation (19) in the text. The forcing processes are generated according to the estimated parameters of Table 3 and with conditional heteroskedasticity as determined by b_i and c_i. For any simulation with conditional heteroskedasticity, polynomial elements with a higher than .99 correlation with other polynomial elements are discarded in order to reduce the dimensionality of the system. Panel A reports results for addilog preferences, Panel B for homothetic preferences and Panel C for addilog preferences at the benchmark parameters but with γ set equal to 2. The last three columns in Panel C report simulations for more extreme parameter values. In EXP I, θ_x and θ_y are set equal to .97, but the other parameters are the ones from the full model with $\gamma = 2.0$. In EXP II, there is no conditional heteroskedasticity ($b_i = c_i = 0$), but $\gamma = 2.2$; $\eta_x = \eta_y = .80$ and $\theta_x = \theta_y = .98$. In EXP III, the parameters are the same as in EXP II but with the benchmark case conditional heteroskedasticity.

Panel A: Addilog preferences

	DATA	CIA	TAL	DAL	DHAL	DHAL $b_i = .80$ $c_i = .13$
$\sigma(\Delta s_t)$	17.668 (1.069)	2.825	4.091	4.180	4.280	4.366
$v_{11}(\Delta s_t)$	1.304 (.133)	1.653	1.084	1.059	1.077	1.050
$v_{21}(\Delta s_t)$	1.725 (.314)	1.995	1.044	1.006	.978	1.084
$\sigma(fp_{t,t})$	3.481 (.343)	3.812	.446	.646	.909	1.019
$v_{11}(fp_{t,t})$	12.350 (.390)	8.142	1.176	2.761	3.905	5.433
$v_{21}(fp_{t,t})$	32.798 (3.667)	12.320	1.245	3.242	4.926	8.858
$\sigma(rp_{t,t})$	10.841 (2.847)	.002	.005	.003	.005	.011

Panel B: Homothetic preferences

	DATA	CIA	TAHO	DAL	DHAL	DHAL $b_i = .80$ $c_i = .13$
$\sigma(\Delta s_t)$	17.668 (1.069)	1.304	4.933	4.901	6.385	6.911
$v_{13}(\Delta s_t)$	1.304 (.133)	8.140	1.037	1.041	.948	.992
$v_{22}(\Delta s_t)$	1.725 (.314)	12.439	1.015	1.024	.762	.746
$\sigma(fp_{t,t})$	3.481 (.343)	3.757	.474	.674	1.794	2.246
$v_{13}(fp_{t,t})$	12.350 (.390)	8.115	1.132	3.168	9.303	8.558
$v_{22}(fp_{t,t})$	32.798 (3.667)	12.345	1.221	3.532	20.552	17.242
$\sigma(rp_{t,t})$	10.841 (2.847)	.001	.003	.005	.009	.025

Panel C: Addilog preferences with $\gamma = 2$

	DATA	CIA	TAL	DAL	DHAL	DHAL $b_1 = .80$ $c_i = .13$	EXP I	EXP II	EXP III
$\sigma(\Delta s_t)$	17.668 (1.069)	4.159	7.544	7.541	12.650	13.923	17.583	27.831	41.617
$v_{13}(\Delta s_t)$	1.304 (.133)	2.314	1.038	1.032	.907	.970	.987	.978	.995
$v_{22}(\Delta s_t)$	1.725 (.314)	3.111	1.048	1.045	.622	.614	.661	.700	.592
$\sigma(fp_{t,t})$	3.481 (.343)	3.731	.685	.865	3.903	4.953	6.279	9.540	14.781
$v_{13}(fp_{t,t})$	12.350 (.390)	7.952	1.176	3.421	10.009	9.049	9.139	11.126	8.670
$v_{22}(fp_{t,t})$	32.798 (3.667)	12.161	1.224	3.825	22.620	18.683	20.712	30.057	21.201
$\sigma(rp_{t,t})$	10.841 (2.847)	.003	.006	.011	.041	.183	.363	.376	4.608

Table 5
Heteroskedasticity and fat tails in the endogenous variables

See Tables 1 and 4 for an explanation of the symbols used in this table.

Panel A: Addilog Preferences

	DATA	CIA	TAL	DAL	DHAL	DHAL $b_1 = .80$ $c_1 = .13$
$ku(\Delta s_t)$	3.000 (1.290)	.012	-.027	-.042	.156	.652
$v_{13}((\Delta s_t)^2)$	2.419 (.427)	1.242	.956	.912	1.126	1.835
$v_{52}((\Delta s_t)^2)$	4.101 (.717)	1.185	.871	.816	1.262	2.771
$ku(\Delta fp_{t,4})$	8.635 (2.495)	.052	.731	.589	3.118	12.401
$v_{13}((\Delta fp_{t,4})^2)$	2.718 (.413)	1.157	1.664	1.713	1.814	2.350
$v_{52}((\Delta fp_{t,4})^2)$	5.957 (1.174)	1.335	1.574	1.743	2.182	2.869

Panel B: Homothetic preferences

	DATA	CIA	TAL	DAL	DHAL	DHAL $b_1 = .80$ $c_1 = .13$
$ku(\Delta s_t)$	3.000 (1.290)	.040	.010	-.010	.030	.280
$v_{13}((\Delta s_t)^2)$	2.419 (.427)	7.374	1.085	1.068	1.058	1.761
$v_{\Omega}((\Delta s_t)^2)$	4.101 (.717)	10.150	.937	.948	.885	2.574
$ku(\Delta fp_{t,4})$	8.635 (2.495)	-.011	3.015	3.347	3.056	5.198
$v_{13}((\Delta fp_{t,4})^2)$	2.718 (.413)	1.030	1.934	1.693	1.793	2.569
$v_{\Omega}((\Delta fp_{t,4})^2)$	5.957 (1.174)	.968	1.990	1.889	2.028	3.531

Panel C: Addilog preferences with more curvature

	DATA	CIA	TAL	DAL	DHAL	DHAL $b_1 = .80$ $c_1 = .13$
$ku(\Delta s_t)$	3.000 (1.290)	.031	.020	.047	.128	.772
$v_{13}((\Delta s_t)^2)$	2.419 (.427)	1.429	1.068	1.071	1.032	2.625
$v_{\Omega}((\Delta s_t)^2)$	4.101 (.717)	1.373	.869	.909	.901	4.461
$ku(\Delta fp_{t,4})$	8.635 (2.495)	.249	3.215	2.528	1.543	6.363
$v_{13}((\Delta fp_{t,4})^2)$	2.718 (.413)	1.289	2.508	1.713	1.864	2.385
$v_{\Omega}((\Delta fp_{t,4})^2)$	5.957 (1.174)	1.158	2.946	1.934	1.933	3.224

Figure 1:   Sensitivity to γ (Addilog Utility)

Figure 1a: Effect on Variability
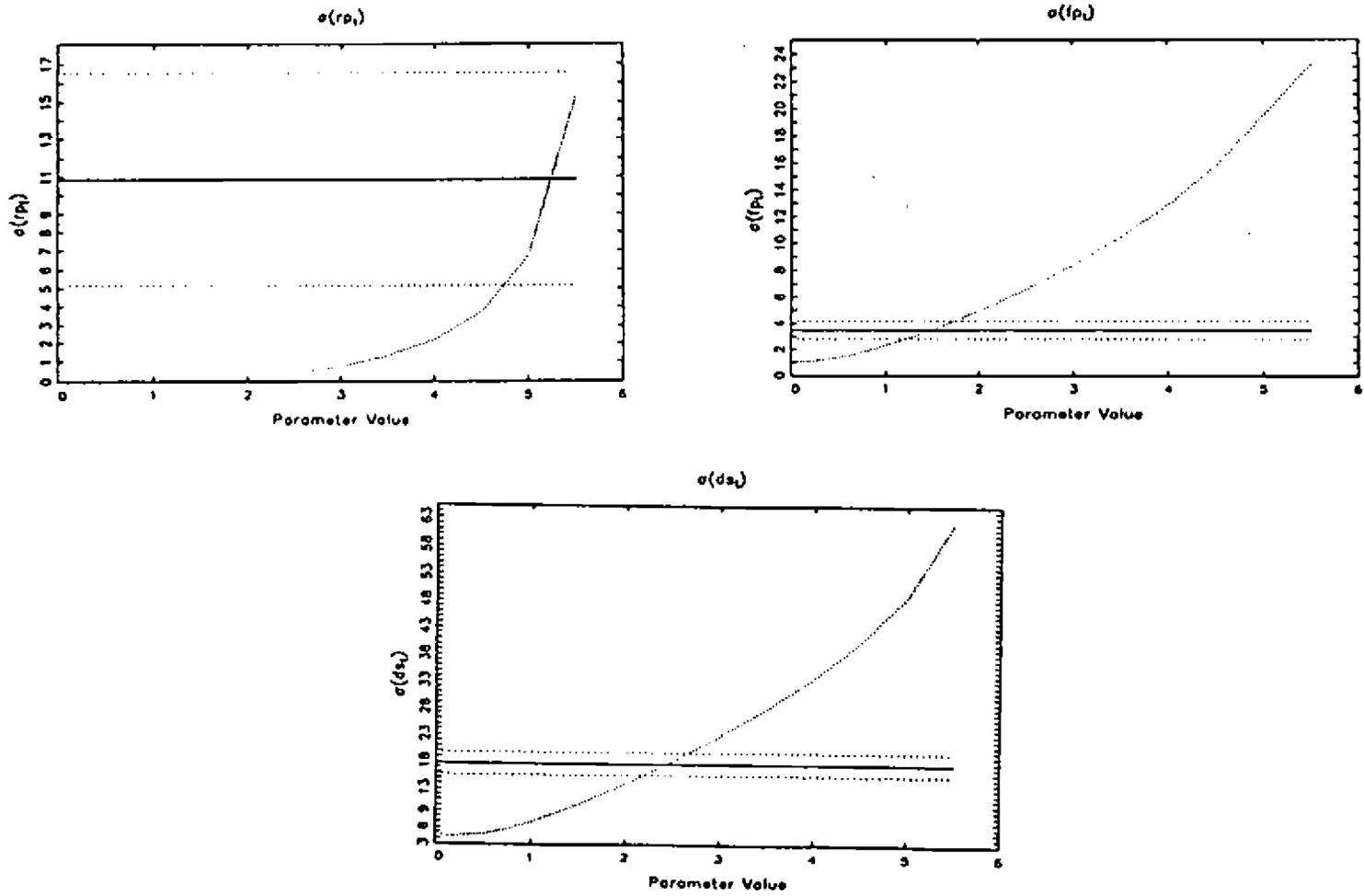
Figure 1b: Effect on Persistence

$v_{13}(ds_t)$

$v_{13}(fp_t)$
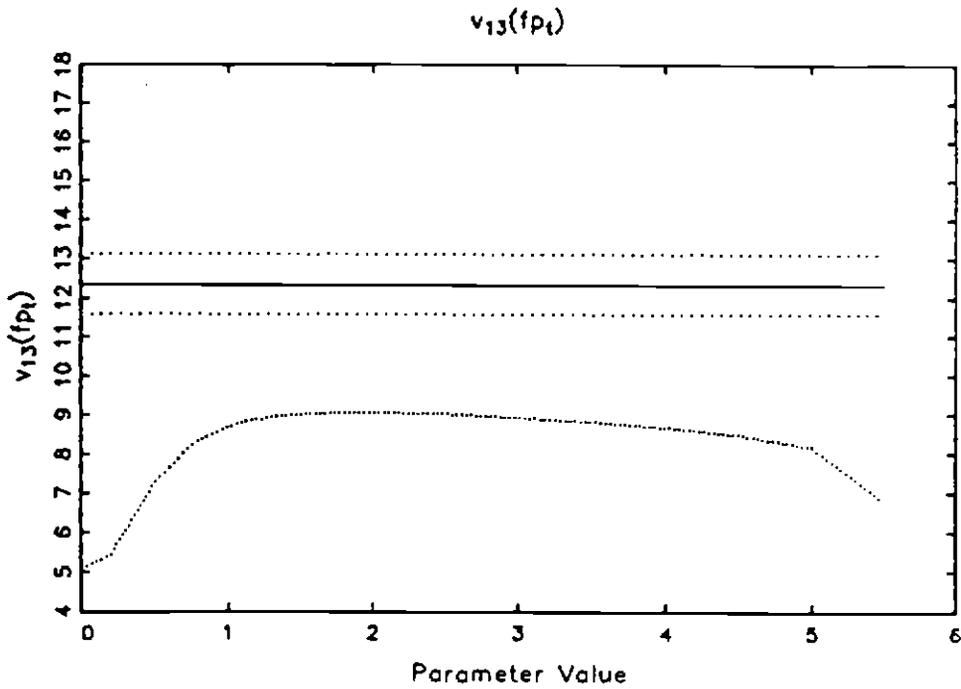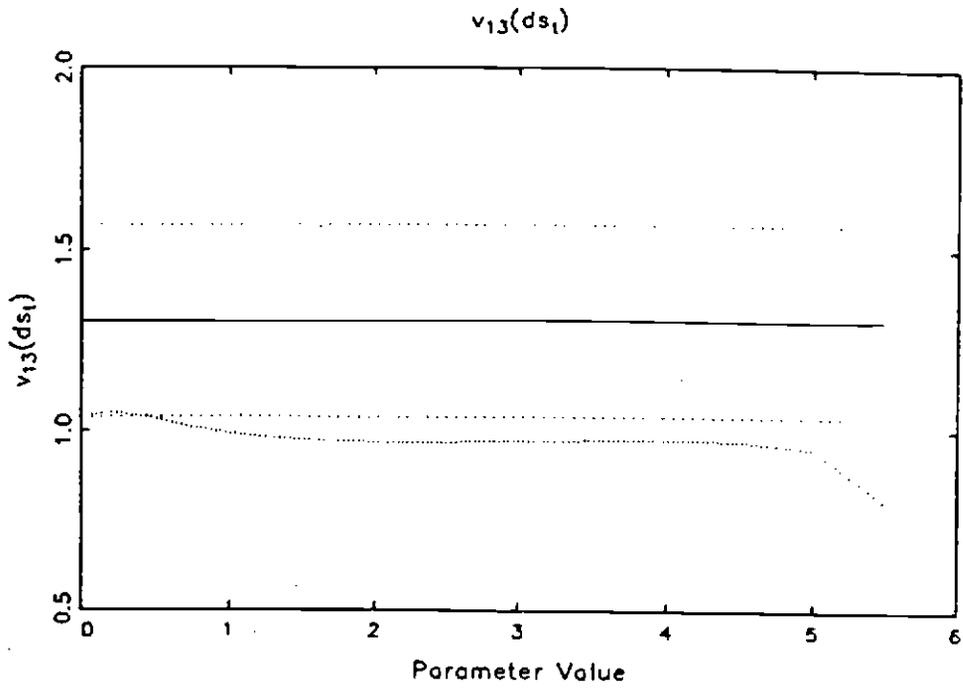
Figure 1c: Effect on Non-Linearities