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TOWARD A MODERN MACROECONOMIC MODEL USABLE FOR POLICY ANALYSIS

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ABSTRACT

This paper presents a macroeconomic model that is both a completely specified dynamic general equilibrium model and a probabilistic model for time series data. We view the model as a potential competitor to existing ISLM-based models that continue to be used for actual policy analysis. Our approach is also an alternative to recent efforts to calibrate real business cycle models. In contrast to these existing models, the one we present embodies all the following important characteristics:

i) It generates a complete multivariate stochastic process model for the data it aims to explain, and the full specification is used in the maximum likelihood estimation of the model; ii) It integrates modeling of nominal variables -- money stock, price level, wage level, and nominal interest rate -- with modeling real variables; iii) It contains a Keynesian investment function, breaking the tight relationship of the return on investment with the capital-output ratio; iv) It treats both monetary and fiscal policy explicitly; v) It is based on dynamic optimizing behavior of the private agents in the model.

Flexible-price and sticky-price versions of the model are estimated and their fits are evaluated relative to a naive model of no-change in the variables and to an unrestricted VAR. The paper displays the model's implications for the dynamic responses to structural shocks, including policy shocks, and evaluates the relative importance of various shocks for determining economic fluctuations.

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Toward a Modern Macroeconomic Model Usable for Policy Analysis

Introduction

This paper presents a macroeconomic model that is both a completely specified dynamic general equilibrium and a probabilistic model for time series data. We view the model, perhaps with future refinements, as a potential competitor to existing ISLM-based models that continue to be used for actual policy analysis in institutions such as the Federal Reserve Board, the Congressional Budget Office, or the International Monetary Fund. Our approach is also an alternative to recent efforts to calibrate real business cycle models. In contrast to these existing models, the one we present embodies all the following characteristics:

i) It generates a complete multivariate stochastic process model for the data it aims to explain, and the full specification is used in fitting the model.

ii) It integrates modeling of nominal variables -- money stock, price level, wage level, and nominal interest rate -- with modeling real variables.

iii) It allows for increasing costs in the production of capital goods, breaking the tight relationship of the return on investment with the capital-output ratio.

iv) It treats both monetary and fiscal policy explicitly.

v) It is based on dynamic optimizing behavior of the private agents in the model.

The paper displays results of fitting the model that are encouraging, though still highly preliminary. A restricted version of the model fit only to data on 3 real variables performs approximately as well as an unrestricted VAR, attributing most cyclical variability to real shocks, but not to shocks in the "Solow residual". A 10-variable flexible price version of the model has not yet been successfully fitted, possibly because of fundamental problems with matching data on prices and real variables together with that type of model. A 10-variable sticky price model fits worse than an unrestricted VAR, though the structural model is much more tightly parameterized. It fits about as well as a naive no-change model. The estimated structure implies extreme price stickiness and effective monetary policy, but attributes little of observed cyclical variability to monetary policy shocks (and none to fiscal shocks).

Now we discuss in more detail each of the five aspects of the model listed above.

i) We regard the fact that we have a complete stochastic process model as important because it allows us to bring all aspects of the data to bear in generating estimates, improving efficiency relative to instrumental variables approaches that in effect use a narrow band of the available information in estimation. We also can investigate any desired aspect of the discrepancy between our model's implications and the behavior of the data, because we can simulate solutions of it. Any use of a model to trace out the impacts of policy intervention will require use of its full set of dynamic implications. If the model has been estimated by single-equation methods, important aspects of its dynamic structure may never have been confronted with the data, and its policy implications may be correspondingly unreliable. Of course all these remarks apply a *fortiori* to a comparison of this model with ones that are largely calibrated informally rather than estimated.

ii) Though our model in many respects follows the spirit of real business cycle (RBC) modeling exercises as pioneered by Kydland and Prescott (1982), we do not follow that literature in paying attention almost exclusively to the behavior of real, rather than nominal aggregates. The RBC approach may be motivated in part by the fact that it has usually started from models in which real and price behavior dichotomizes, so that a complete model for the real variables in the system is possible without any reference to nominal variables. Of course such models have few interesting implications for monetary policy -- indeed are in a sense aimed at showing that monetary policy is unimportant. Thus one reason for our emphasis on including nominal variables is our aim of eventually exploring specifications in which price stickiness generates stronger nominal-real interactions. But even for models that do dichotomize, there is information about the model structure in price as well as real data. RBC modelers sometimes invoke the idea that nominal aggregates are less likely to correspond to the theoretical constructs that appear in their models than are real aggregates. The wage, for example, is claimed not to be a true market-clearing price as assumed in the theory. But since it is left unspecified what mechanisms produce the results of a market-clearing model in a world where measured price variables are not market-clearing prices, we find the argument for ignoring price variables unconvincing. There are, after all, strong reasons to suppose that measured quantity variables are distant from the corresponding theoretical constructs as well.

iii) The curvature in the transformation curve relating output of investment to output of capital is important in matching some aspects of observed cyclical behavior. We would like the technology to be able to generate fluctuations in real rates of return without corresponding large shifts in the level of the capital stock or output. With a relative price of C and I goods in the model, this can occur, with high rates of return generating above-normal I, but only smooth growth in K. In sticky-price models, we would like to allow the possibility that a monetary of fiscal expansion could lower nominal interest rates. Increasing costs of capital goods production provide a mechanism that could contribute to this behavior in the model. Also, as recently shown by Nakornthob (1993), models that make prices and wages sticky in the sense that they cannot move discontinuously may easily have no equilibrium solution unless they include a variable relative price of C and I.

iv) Those few models in the RBC style that have considered "aggregate demand" policy variables have generally focused on monetary or fiscal policy alone. In fact, monetary and fiscal policy are intimately related, as Leeper (1991) and Sims (1992a) show theoretically and as becomes evident in the estimation of a model like this. As we explain below, the parameters of monetary and fiscal policy equations must, in order to guarantee existence of a unique equilibrium, lie in a set of complicated geometry. While there are certain conditions under which monetary and fiscal policy dichotomize, with most of the equilibrium derivable from the monetary policy specification alone, these conditions are not generic. Even a model that aims mainly at guiding analysis of monetary or fiscal policy alone needs to treat both together to give reliable results. This is especially true in a period like the recent history of the U.S., in which there have probably been changes in beliefs about the political feasibility of keeping taxation in line with government commitments to spending and debt service, at all levels of government. Such shifts in beliefs could have impacts on prices, and, in a model with sluggish prices, on real variables, not captured by models that ignore monetary-fiscal interactions.

v) Recognizing the implications of dynamic optimizing behavior is important in macroeconomic modeling. But because this point has been emphasized first by natural rate theorists, who aimed to show that dynamic optimization by private agents undermined the effectiveness of aggregate demand management policies, and then by RBC modelers, who model a world without demand managers, Keynesians and monetarists have not embraced it. To be sure, Keynesian textbooks now include treatments of forward-looking theories of consumption and (sometimes) investment accelerator effects arising out of expectations, but these treatments of forward-looking behavior are in themselves incomplete. Furthermore they are seldom integrated into the general-equilibrium versions of Keynesian models, which usually fall back instead on the ISLM framework, without even a clear distinction of real and nominal interest rates. Expectational elements are, paradoxically, emphasized more in the wage and price-setting components of such models than in their asset accumulation sectors. Yet the Keynesian and monetarist notion of nominal aggregate demand is at its root a theory about the relation of supply to demand for nominal government assets -- debt and money. Demand for these assets depends critically on the public's beliefs about future government monetary and fiscal policy. Any model that is to be used to trace out effects of monetary and fiscal policy needs to consider the implications of dynamic optimization, and this is especially true of a model that intends to explore the implications of price stickiness.

Previous successful attempts to use maximum likelihood to estimate a maximizing equilibrium model, such as Altug (1989), dealt with much simpler theoretical structures. Altug estimates a version of Kydland and Prescott's (1982) model, for which the simpler social planner's solution can be supported by a competitive equilibrium. The introduction of money requires solving the decentralized problem and brings with it the difficulties inherent in ensuring that a determinate equilibrium exists. Though we, like Altug, impose stationarity, her approach is to extract deterministic trends from the data, while ours is to allow unit and near-unit roots that imply long-term deviations from steady state. Most importantly, her approach postulates the existence of a measurement error in the data with well-defined stochastic properties but no economic interpretation. Our model's stochastic disturbances are all structural. This means in particular that we have a higher-dimensional vector of structural disturbances than in any previous model of this type. While we may eventually need to introduce something like measurement error in the model, we will do so reluctantly. The allocation of any substantial part of observed variation to such an uninterpreted source raises serious difficulties in using the model for prediction and policy analysis.

McGrattan (1992) and McGrattan, Rogerson, and Wright (1993) use maximum likelihood to estimate general equilibrium models with distorting taxes, so they also cannot rely on solving the social planner's problem. Like Altug, these authors introduce measurement errors as additional sources of uncertainty.

Watson (1993) adds measurement error that is by construction a linear function of the systematic component of the model. Though this unattractive perfect collinearity between error and systematic components is introduced in order to minimize the size of the error, Watson finds that in matching a standard RBC model to data, 40 to 60 percent of the variance must be attributed to economically uninterpreted sources.

The Model

Consumers

Consumers are endowed each period with one unit of time, which they divide between work, L, and leisure. They derive utility from consumption net of transactions costs, C^*, and leisure, 1-L, and discount utility at the rate of time preference, β. Consumers can hold two types of nominal assets, non-interest bearing money, M, and interest bearing government debt, B, and one real asset, capital, K. Income is earned from the capital and labor they rent to firms (which together make up factor income Y) and from the interest received from holding government debt, IB/P. In the sticky-price version of the model firms make temporary pure profits and losses, and these are assumed to be returned to consumers as dividends. We assume they maximize:[1]

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[1]Note that, though β and (as we will see below) π vary through time, this objective function does not run afoul of the well-known result that time-varying discount rates generate time-inconsistency. The usual inconsistency result depends on the discount rate being thought of as indexed by the number of periods into the future at which the discounting is done. We think of consumers as understanding in advance that they will at the absolute date s discount at rate $\beta(s)$, so no time-inconsistency is involved.

$$E\left[\int_0^\infty \exp\left(-\int_0^t \beta(s)ds\right) \frac{(C^{*\pi}(1-L)^{1-\pi})^{1-\gamma}}{1-\gamma}\, dt\right] \qquad (1)$$

subject to

$$\lambda: \qquad XC + QI + \tau + \frac{\dot{M}+\dot{B}}{P} = Y + \frac{iB}{P} \qquad (2)$$

$$\nu: \qquad XC^* + \phi VY = XC \qquad (3)$$

$$\omega: \qquad \dot{K} = I - \delta K \qquad (4)$$

$$\xi: \qquad Y = rK + wL + S \qquad (5)$$

$$\psi: \qquad V = \frac{PY}{M}\, . \qquad (6)$$

We assume that $\pi \in (0,1)$ and $\gamma > 0$. The Lagrange multiplier associated with each constraint is listed at the left. The agents' choice variables are C^*, C, L, I, M, B, K, Y and V.

The first equation is the consumers' budget constraint, where C is gross consumption, I is investment, and τ is the level of lump-sum taxes. Consumption goods and investment goods are distinct from the output good, so they have prices X and Q relative to the output good. Government bonds earn the nominal rate of return i and P is the general price level, i.e. the price in dollars of the output good.

The second equation defines consumption net of transactions costs, with total output serving as a measure of the level of transactions at a given point in time. Costs are assumed to be increasing in the volume of transactions and in velocity, V. This simple transactions technology implies that costs approach zero as the level of real money balances approaches infinity, pushing C^* toward C. As the level of real balances approaches zero, transactions costs are unbounded, which implies that no non-monetary equilibrium exists. This feature of the model could easily be modified by specifying a transactions technology that places an upper bound on transactions costs.

Firms

The third and fourth equations are standard. Equation (4) specifies the law of motion for capital, and (5) defines income as the sum of dividends S with factor payments to capital and labor, which consumers receive from the firm. Equation (6) defines the income velocity of money.

Firms rent factor inputs from consumers, transform them into "output" according to a production technology, convert "output" into salable C-goods and I-goods according to another part of the production technology, and sell the C and I goods to consumers and the government. Because the technology is linear homogenous, there are no profits in a market-clearing competitive equilibrium and we need not keep track of who receives the profits in the flexible price version of the model. In the sticky-price version we need treat profits as part of consumer income and must keep track of the difference between income and factor payments. The net profits from the firms' buying, selling and transforming activities are their maximand, S (written assuming that the number of firms matches the number of consumers so we can again avoid separate explicit market-clearing equations):

$$\max \left\{X(C+g) + QI^{*} + A(\alpha K^{\sigma}+L^{\sigma})^{1/\sigma} - rK - wL - ((C+g)^{\mu}+\theta I^{*\mu})^{1/\mu}\right\} \tag{7}$$

with C+g, I^{*} (defined below), K, and L as choice variables. To keep the production technology concave, $\sigma\leq1$, and to keep the costs-of-adjustment convex, $\mu\geq1$. The borderline case $\sigma=0$ corresponds to Cobb-Douglas production and $\sigma<0$ corresponds to low elasticity of substitution. The g term that appears in this problem is the level of government purchases, which shares a relative price with the consumption good. Since government purchases are exogenous, their appearance does not affect the firms' optimization problem.[2]

[2]The first-order conditions for the consumers and the firms appear in an appendix.

Government

The government uses consumption goods in amount g for a purpose that yields no utility to individuals. It also takes responsibility for endowing newly born agents with the same wealth as existing agents and for redistributing equally to living agents the wealth of all those that die. Population grows at the (possibly varying) proportional rate n. With this device, the population can fluctuate while the model can maintain the assumption that there is an infinitely lived representative agent. Thus the government operates with the budget constraint

$$\frac{\dot{M}+\dot{B}}{P} + \tau = \frac{iB}{P} + gX + QnK \ , \tag{8}$$

where QnK is net transfers of capital denominated in terms of output goods. The variables in this equation are in per capita terms to avoid having to introduce additional market-clearing equations for the policy variables. Aggregate money and debt grow or shrink not only through the $\dot{M}$ and $\dot{B}$ terms in this equation, but also through net issuance of government paper to newborns. Thus only the real capital that is demanded by the newly born (or turned over by the newly dead) creates a resource drain (or inflow) for the government. Wealth in the form of government paper can be created or destroyed without any effect on per capita M and B simply by the government's issuing or retiring new paper.

Investment goods produced by the firm, I^*, are both those bought by the existing population and those purchased by the government for distribution to the newborns. Thus a market-clearing condition is

$$I^* = I + nK \ . \tag{9}$$

The social resource constraint, which is redundant if both firm and government budget constraints are in the system, is

$$X(C+g) + Q(I+nK) = Y \ . \tag{10}$$

We treat g and n as determined exogenously and P, K and i as determined by market conditions. Thus the government has as choice variables M, B, and τ. Besides its constraints, it needs two policy equations.

The monetary and tax policy rules are motivated by two considerations: the need to satisfy the intertemporal government budget identity and the pursuit of countercyclical policy objectives. In addition, monetary and fiscal policies must interact to determine the price level. Leeper (1991) and Sims (1992a) use simple rules in equilibrium models to compute the regions of the policy parameter space where unique equilibria exist. In those models, the monetary authority obeys a nominal interest rate rule that depends on inflation or money growth and the fiscal authority adjusts lump-sum taxes in response to the level of real debt held by the public. With systematic policy behavior summarized by the two parameters in the policy rules, there are four qualitatively distinct regions of the policy parameter space: in two of these there exist unique equilibria, in one real debt explodes generically to violate transversality or feasibility, and in one the price level is indeterminate.

If monetary policy fixes the stock of high-powered money or raises the nominal interest rate strongly in response to increases in nominal variables like inflation or money growth, then a unique equilibrium exists only if fiscal policy behaves compatibly by matching any increase in real debt with an equivalent increase in the present value of direct taxes. This policy environment produces the usual monetarist/Ricardian propositions that underlie many economists' views of monetary and tax policy effects. Of course, given the hypothesized monetary policy behavior, refusal of the fiscal authority to increase direct taxes when real debt rises violates the government's intertemporal budget identity, placing the model in the region of the parameter space where no equilibrium exists.

Alternatively, when the fiscal authority refuses to change taxes in the face of real debt expansions, then monetary policy must prevent nominal interest rates from rising and generating explosive growth in government debt. Such a policy mix implies that total government liabilities -- high-powered money plus government debt -- determine the price level, while the ratio of money-to-debt is determined by the nominal interest rate. Shocks to lump-sum taxes influence nominal magnitudes when prices are flexible and both real and nominal variables when prices are sticky. The choice between debt- or tax-financing of government spending, therefore, *is relevant* although the models are "Ricardian" in the sense that private agents fully discount the future tax -- direct and inflation -- liabilities associated with increases in government debt. This result emerges because future lump-sum taxation and future inflation taxation represent different marginal sources of debt financing and elicit different private responses to a given cut in current direct taxes. Moreover, monetary policy shocks can have unexpected effects in this region of the policy parameter space. For example, a disturbance that unexpectedly raises the nominal interest rate can be deflationary or inflationary, depending on the assumed fiscal behavior.[3]

Finally, a policy combination where taxes respond strongly to real debt and nominal interest rates respond weakly to nominal variables leaves the price level indeterminate. Although the ratio of money-to-bonds is determined, their sum is not: at each date, many different levels of total government liabilities are consistent with an equilibrium and associated with each level is a different price level. Explicitly modeling fiscal behavior generalizes the well-known Wicksellian view that a pegged nominal interest rate does not determine prices. A pegged rate coupled with direct taxes that do not rise sufficiently when real debt increases can uniquely determine the price level.

Actual policy behavior also contains strong countercyclical components. Monetary policy tends to lower interest rates as unemployment rises and employment falls and to raise rates when inflationary pressures mount. Fiscal policy, through both discretionary tax changes and automatic stabilizers, tends to lower revenues when employment and inflation decline. The policy rules are parameterized to embody both of these reasons for monetary and tax policy changes. In addition to these systematic responses of policy to observable data, the rules contain disturbances reflecting policy responses to unmodelled economic or political developments.

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[3]Leeper (1993) simulates a wide range of policy effects in a simple model with interest rate and tax rules.

The policy rules in this model are more complex than those studied in earlier papers, so the relevant regions of the parameter space cannot be derived analytically. The underlying economic intuition carries over, however. The monetary policy rule is

$$\frac{\dot{i}}{i} = a_p \log(P/\bar{P}) + a_{inf}\frac{\dot{P}}{P} + a_i \log(i/\beta) + a_L \log(L/\bar{L}) + \varepsilon_i, \tag{11}$$

and the tax policy rule is

$$\frac{d}{dt}\left(\frac{\tau}{C}\right) = b_\tau\left(\frac{\tau}{C} - \overline{\frac{\tau}{C}}\right) + b_L \log(L/\bar{L}) + b_{inf}\frac{\dot{P}}{P} + b_x\left(\frac{B}{PY} - \overline{\frac{B}{PY}}\right) + \varepsilon_\tau . \tag{12}$$

The overscored variables denote steady state values. Note that the steady state price level, $\bar{P}$, and the steady state debt-to-GNP level, $\bar{B}/\bar{Y}$ are free parameters of (11) and (12) just as are the a's and b's.

Policy Analysis

The specification of policy behavior in equations (11) and (12) leads to two types of policy experiments one might conduct with the fitted model: interventions on the out-of-sample paths of the disturbances, ε_i and ε_τ, and once-and-for-all changes in the policy parameters, the a's and b's. We view the first type of experiment as useful for policy analysis of the sort conducted in preparation for Federal Open Market Committee meetings or Congressional debates about fiscal policy. The analyst would choose several candidate paths for, say, ε_i, and present the model's predictions for each path to the policy-makers. Most commonly, the ε_i paths themselves would be generated to provide responses to questions such as, "What would happen if the Federal Reserve held interest rates below 4% for the next six months?" To do so, one would solve for ε_i sequences that make the time path of interest rates behave as desired. Because the model implies that there are many potential stochastic influences on interest rates, this kind of projection is generally quite different from simply forecasting conditional on a given time path of the interest rate.

A menu of options generated along these lines would form the basis for the policy discussion. Choices of paths of such shocks in this manner is not a trivial exercise, and the process closely mimics actual policy practices, as has been argued in detail elsewhere [Sims 1982, 1987; Cooley, LeRoy, Raymon 1984, and LeRoy 1993]. Of course, repeated use of the model in this way for policy choice could eventually be seen by the public as changing the a's and b's, but this is not necessarily true and is in any case likely to take a long time. Even if policy-makers announce that they are making permanent changes in the way policy variables are set, the public will for good reason wait for the announcement to be backed by sustained action before accepting the change as even approximately permanent. And in any case for systematic use of the model to eventually change the a's and b's, it would have to be true both that users of the model had a strong impact on policy debates and that the use of the model changed their conclusions about good policy, rather than simply letting them reach those conclusions more quickly and cheaply.

By construction, changes in policy parameters are rare and permanent events -- not the stuff of regular cyclical policy debates. Thus while it may be interesting to use the model to see whether an alternative set of the a's and b's would deliver a better equilibrium than the historically estimated one, it is internally contradictory to evaluate policy "rules" in this way as if the result were a contribution to the usual year-to-year or decade-to-decade ebb and flow of macroeconomic policy arguments.

The Sticky-Price Version

The sticky-price formulation we are working with does not have the flexible price model as a special case or even as a limit point. In particular, we drop equations (A4) and (A14) in the appendix, corresponding to the workers' matching of marginal utility of leisure to the wage and the firm's matching of marginal productivity of labor to the wage. In their place we postulate differential equations relating the rates of growth of $\dot{P}/P$ and W to the discrepancies between the left- and right-hand sides of (A4) and (A14):

$$\frac{d^2}{dt^2}\log(P) = -\chi_p \log\left[A^{\sigma}\left(\frac{Y}{L}\right)^{1-\sigma}\cdot w^{-1}\right]. \qquad \text{(A4')}$$

$$\frac{\dot{w}}{w} = -\chi_w\left(\log\left((1-2\phi V)\frac{w}{X}\right) - \log\left(\frac{1-\pi}{\pi}\,\frac{C^*}{(1-L)}\right)\right) \qquad \text{(A14')}$$

These can be thought of as "markup" and "Phillips Curve" equations. They have no rational expectations elements and can therefore be criticized as old-fashioned. However, any element of stickiness is going to violate canons of purity on the rationality froot. We are using a modeling framework in which the inflation rate is stationary and do not regard it as unrealistic to treat price dynamics formulated as in (A4') and (A14') as "structural" relative to policy disturbances that come as a stationary time series of shocks. Note also that because (A4') is formulated with the second derivative on the left, it implies that when inflation is at a constant rate, there is no persistent gap between the wage and the marginal product of labor. And since (A14') is in terms of the real wage w, it again does not require any gap between the wage and the marginal utility of labor in the presence of steady inflation.

Io principle the speed-of-adjustment parameters χ_w and χ_P can be positive or negative. When negative, they fit an interpretation that requires P and w to have continuous time paths and be predetermined. When positive they are interpreted as allowing discontinuity in P and w paths, with the levels of these variables set to match average expected future values of the right-hand-side forcing variables. We actually do oot impose these interpretations directly, but instead simply treat (A4') and (A14') as containing endogenous expectational disturbance terms or exogenous disturbances, respectively, according to whether the model as a whole has the right number and location of unstable roots to support the interpretation.[4]

We do not tie (A4') and (A14') to a particular institutional story. They are consistent with many of the stories that have been told in the literature about price adjustment mechanisms. For example, if we denote the right-hand side of (A4') as

[4]As the number of unstable roots in the system formed by the constraints and first order conditions increases, there are correspondingly increased numbers of stability conditions imposed on the model. A new exact relation among the variables required to suppress an additional unstable root will in general conflict with the specification of the model, unless an additional equation is specified as having an endogenously determined disturbance.

Z(t), then a continuous-time adaptation of John Taylor's[5] overlapping-contracts pricing mechanism would postulate

$$\dot{p} = \theta(p^{*}-p) \ , \tag{13}$$

where p is log price, θ is the rate at which the stock of firms are selected to adjust per unit time and p^{*} is the value to which they adjust p when the adjust. Then, recognizing that the price they set now will hold over a long future span, they form p^{*} as

$$p^{*} = \phi\, e^{\phi t}\, E\left[\int_{t}^{\infty} e^{-\phi s}(p(s)-\gamma Z(s))\ ds\right]. \tag{14}$$

If $\theta=\phi$, as is reasonable if adjusters average over a time span in the future that corresponds to their actual hazard rate for being selected for price adjustment, exactly (A4'1 emerges, though with $\chi_p<0$.

Another possible story to back up the price adjustment mechanism would be built on search. By explicitly modeling choice of search activity levels by firms and workers we could create a foundation for modeling unemployment and job vacancy data along with the rest of the model.

Notice that our formulation, though it treats the price adjustment mechanism as an institutional datum, involves no ad hoc or suboptimal behavior by firms or workers. The firms and workers simply treat both the price and quantity of labor as beyond their control. Given the prices and quantities turned out by the labor market mechanism, savings and investment are carried out completely optimally, and with full accounting for expected future paths of inflation and output.

One might think that as the speed with which P and w react to gaps between the left- and right-hand sides of (A4) and (A14) increases, we would smoothly move from a

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[5](1979). Another example of a rational expectations sticky-price model is that in "Money and Business Cycles" [1991] by Robert King. He uses a Taylor-style adjustment mechanism for wages, but assumes firms always to be on their labor demand curves and observes that this creates a strongly positive reaction of the nominal interest rate to demand expansions.

sticky-price world to a flexible-price world. However, as the speeds of adjustment of P and w increase, we converge to flexible-price behavior in a complicated way. This point was shown a year ago by Don Nakornthab in a somewhat simpler version of this model. Its intuitive economic explanation traces back to a point that can be found in Keynes *General Theory*: making wages or prices more flexible will not eliminate Keynesian unemployment, indeed it may make it worse. In a sticky-price model like that we have set up here, quantities respond strongly to fiscal and monetary policy shifts and to technological shocks. But if prices cannot jump downward in response to an initial contractionary demand shock, for example, the effect on real interest rates of a more rapid deflationary response of prices may actually make the initial impact on quantities greater. The result is that as the speed of adjustment of wages and prices increases, the sensitivity of quantities to shocks increases rather than decreases. What Nakornthab shows that is not obvious from Keynes's informal discussion is that the duration of the quantity responses decreases as their speed increases. The amount of time quantity variables spend away from steady state following a shock decreases with increased price and wage flexibility, but the amplitude of their movement stays about the same and the initial speed of their reaction increases.[6]

Stochastic Specification

The model contains 11 sources of uncertainty in its neo-classical version and 11 to 13 (depending on whether the price-adjustment equations are backward or forward-looking) in the sticky-price version. Besides the serially uncorrelated policy disturbances, ε_i and ε_τ, the model is driven by stochastic behavior of some of the parameters and exogenous variables: n, g, π, β, δ, θ, α, A, and ϕ. Each of these is a logarithmic first-order AR in continuous time, except for β, which is a logarithmic first-order AR in unlogged form. Each has a steady-state parameter[7], a decay parameter and a variance parameter associated with it. The shocks ε_i, ε_τ, n, g, δ, and ϕ are independent, the preference shocks π and β are correlated with each other, but

[6]In a sticky-price model that differs from that here in a number of important ways, DeLong and Summers [1986] have also made the point that increasing rates of price adjustment does not necessarily reduce volatility of real quantities.

[7]As we see below, these "steady-state" parameters need not, if dynamics are non-stationary, correspond to actual means or steady-states.

not with any other shocks, and the technology disturbances θ, α, and A are correlated with each other, but independent of the other shocks.

Our approach is to carry out inference as an exploration of the shape of the likelihood function. We do not follow the procedure, common in time series inference, of using the likelihood conditional on initial observations, as that loses information and may generate fits that embody large "transients" at the beginning of the sample that are attributed to initial conditions. (See Sims [1992b]). Models that build in non-stationarity may easily absorb into the statistical "trend" much of what economists think of as business cycle variation. This is true whether the trend is modeled as difference-stationarity, deterministic trend components, or as what is removed by some high-pass filter. At an earlier stage of this work we used unconditional likelihood based on a stationarity assumption, arguing that a stationary model near enough to the non-stationary boundary of the parameter space could generate arbitrarily strong persistence and thus fit even apparently non-stationary data. However, numerical instabilities near the non-stationary boundary were difficult to handle, so we now generate likelihood conditional on an assumption that the model started up from its "steady-state" value 100 years before the initial date of the sample. Well away from the nonstationary boundary, the likelihood formed this way is essentially identical to the unconditional likelihood formed under an assumption of stationarity, but at the boundary it does not have the sharp singularity of the stationary likelihood. It can still show numerical problems if the parameters imply rapidly explosive behavior, but the problems are rarer and easier to deal with than those of the stationary unconditional likelihood.

If the coefficient a_p on the price level in the monetary policy equation is zero, it becomes possible to write the entire model and its first-order conditions as functions of real money balances M/P, real debt B/P, and the inflation rate in place of the variables M, B and P. In this form, of course, P is by construction non-stationary. In earlier work we did not force a_p to zero and fit to levels of the data. While we have recently switched over to relying mainly on the $a_p=0$ version, the results we report below for a 3-variable fit are for the $a_p\neq0$ version of the model. The results reported for the 10-variable sticky-price model are for the $a_p=0$ version.

Solving and Estimating the Model

The first-order conditions, the constraints, and the policy rules of the model form a system of nonlinear stochastic differential equations. We set the model's disturbances to zero and solve for the steady state as a function of the fixed parameters and the mean values of the disturbances. Since the stochastic specification does not require stationarity, this "steady state" may not correspond to an actual steady state of the deterministic version of the model. It is what the steady state would be if all the exogenously evolving stochastic parameters were fixed at their "steady-state" values, despite the possibility that the dynamic specification may imply that these exogenous parameters do not tend to stay near their "steady-state" values. We then compute a first-order Taylor expansion of the first-order conditions and the constraints around the steady state.

From the resulting system of first-order linear differential equations, we calculate the eigenvalues and eigenvectors. For a determinate equilibrium to exist, the system must have a number of unstable roots[8] that matches the number of forward-looking first-order conditions. The left eigenvectors associated with the unstable roots are the coefficients of linear constraints on the model's variables that must hold to suppress the unstable component of the solution. Combining these relationships with the remaining equations in the model produces a complete linearized solution. Our algorithm allows interpreting the price and wage adjustment equations in the sticky-price version to be either forward or backward looking, depending on the number of unstable roots.

Once the linearized model has been solved, the matrix-valued autocovariance function can be derived as a function of the matrices of coefficients in the linearized system. The autocovariance function is aggregated over time from the continuous time theoretical structure to correspond with quarterly time series observations. The likelihood function for the data can then be computed and estimated over

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[8] What qualifies as unstable here in principle depends on the detailed structure of the model. Roots that are slightly explosive may still be consistent with transversality conditions. We allow "slightly" explosive roots and hope that our guess at a dividing line between stable and unstable regions does not affect results. If we had guessed wrong, we might have expected to find maxima at the boundary of the allowable region of the parameter space, and this seems not to have occurred.

the free parameters of the model. The model consists of 17 endogenous variables and 11 to 13 exogenous shocks. Private and policy behavior and the statistical properties of the exogenous disturbances are summarized by 46 parameters in the flexible-price model and 50 parameters in the sticky-price version. Before being sent through the estimation algorithm, the parameters are transformed to ensure the estimates satisfy some simple a priori bounds (mostly log transformations to ensure positivity).

The model is fit to two different sets of quarterly data over the sample period 1959:1 to 1992:3. Initially, three U.S. time series on real personal consumption expenditures, hours, and real gross private domestic investment less inventory accumulation are used to estimate a subset of the parameters associated with preferences, technologies, and the real exogenous disturbances. These estimates are used to judge the model's ability to replicate some of the calibration exercises performed on real business cycle models. The present work, however, applies a more stringent set of measures of fit to the data than is typically employed in calibration exercises.

The second data set adds to these three series real wages, the rate of inflation in the GDP deflator, real total government purchases, real total government revenues, the real monetary base, the three-month Treasury bill rate, and working age population. At this stage the estimation is extended to include all the model's parameters, including those in the policy equations. All series are converted into per capita terms and in the current version of the model, all series are logged except tax revenues, the inflation rate and the interest rate.[9]

Our approach to inference is based on the likelihood principle -- that is, on the idea that the information in the data about the model is completely captured in the shape of the likelihood function. From this point of view, we need not be concerned with the fact that for parameter values close to the nonstationary boundary of the parameter space the distribution of maximum likelihood estimators is not well approximated by the usual asymptotic theory for stationary models. The maximum of the log likelihood function and the second-order Taylor expansion of it around the

[9]Detailed descriptions appear in a data appendix.

maximum carry the same sort of information about the function's shape regardless of the presence of near-non-stationary behavior. This point is elaborated in a simple example model in Sims and Uhlig [1991].

Numerical Considerations

Estimation of this model presents some numerical difficulties. In addition to the usual requirement that most economic variables be positive, a determinate solution requires there to be the proper number of unstable roots in the linearized system of differential equations and that they generate a well-behaved mapping between exogenous disturbances and expectational error terms. These requirements create complex and generally unknown boundaries to the set of feasible parameters in the parameter space. The model, and particularly the policy behavior, is specified to make the boundaries as simple as possible. But without implausibly simple policy rules and a dichotomy between the real and nominal sectors of the economy, it is difficult to characterize the boundaries analytically.

The economics of the model implies that when parameters fall outside the feasible boundaries, either no equilibrium exists (too many unstable roots) or the equilibrium is underdetermined (too few unstable roots).[10] In either case, the likelihood function is not defined. This puts the numerical optimization problem into somewhat uncharted waters because it is neither an unconstrained maximization nor a constrained maximization with a separate routine that checks if the constraint is satisfied. Penalty function methods associated with constrained optimization typically assume the objective function is defined in "bad" regions of the parameter space and tack on to that function a smooth, continuously differentiable function that penalizes the objective function when the algorithm tries parameter values that violate the constraint. If, as in our case, "bad" parameter values imply that the

[10]More precisely, when there are too many unstable roots, an equilibrium exists only if the exogenous stochastic processes are linearly related, which violates the maintained assumption that they are uncorrelated. When there are too few unstable roots the price level is not determined. We can sometimes pick an equilibrium in this case by treating the largest stable root as if it were unstable. Then we can use penalty-function methods by using this equilibrium to calculate the implied distribution of the data, adding on to the likelihood a penalty term that depends monotonely on the degree to which the largest stable root falls below zero.

likelihood function is not defined, the usual penalty function methods cannot be applied.

As an alternative, our likelihood-evaluation algorithm checks whether the candidate parameter vector implies nonsensical steady state values or the wrong number of unstable roots and returns a large negative likelihood value in these cases. This approach introduces discontinuities in the likelihood function, which can create difficulties for some gradient-based algorithms. There are two kinds of difficulties. Some algorithms may try to take a gradient of the objective function at points where the function value is worse than that at the beginning of the iteration. If such a point happens to be in the region where the objective function is flat at a large negative value because of non-existence of equilibrium, this obviously creates a problem. Other routines attempt sophisticated line searches that interpolate polynomials across function values obtained in the line search. When the function is discontinuous, these methods may not only fail to be useful, they may fail to find an improved value even when a less sophisticated line search would easily find it. Our routine that avoids these difficulties is available as a Matlab m-file.

Our procedure is to linearize the model's first-order conditions and constraints about the steady state and to use the resulting model to generate first and second moments for the full data matrix. With 134 (when we use filtered data) observations and 3-10 variables, the covariance matrix we are using is of order 402-1340. Such a matrix cannot even be stored on PC's with the most common memory endowments. We are nonetheless able to compute the likelihood by using recursive methods to factor and invert the covariance matrix, therefore never having to store the entire matrix.

At early stages of our work, numerical difficulties in evaluating the likelihood led us to focus attention on fitting to quasi-differences, i.e. $Y(t)-\rho Y(t-1)$ for some ρ in (0,1), where these difficulties lessened. This allows deviation of initial levels from steady-state to influence the fit, though the influence is diminished relative to use of levels data.[11] Note that this does not mean we are modeling in

[11] We have greatly reduced our model's numerical difficulties by four main techniques. We have found a way to double the accuracy of Matlab's Ricatti equation solver lyap.m by essentially applying it twice. We have truncated the triangular orthogonalization generated by the block Levinson algorithm at a fixed lag length in every function

quasi-differences, constraining the model to imply nonstationarity as would be the case if we fit a VAR in quasi-differences. We are modeling in levels, using the result to generate implications about the second-order moments of the time series of quasi-differences. It is true that as ρ approaches unity, using the differenced data will make the fit emphasize higher-frequency characteristics of the data, and in particular will pay less attention to the distance of the initial value from the steady-state.

Our method for solving the linearized model, based on the QZ decomposition, is somewhat non-standard and probably competitive or superior in efficiency with standard methods. It makes no use of the fact that the model comes from optimization problems, working only off boundary conditions limiting the size of unstable roots. It handles singularities in the matrix of coefficients on derivatives automatically and does not require explicit casting of the model into state-space form. (The algorithm must be told how many unstable roots to squash, however.) Though we can make the code available, this part of our code is model-specific in its current form.

Results

In judging the model's goodness of fit, we are not simply testing the model to see whether it is true. We do use statistical measures of fit and compare, with likelihood-based test statistics, our model to others, including naive no-change predictions and VAR's. Like many RBC researchers, we are not ready to cast our model aside as soon as we find a VAR that clearly fits better. On the other hand, we are also not ready to make excuses for our model that imply we are ready to rely on it for policy conclusions even though it clearly misses or contradicts patterns of behavior observed in the data. Our model has been formulated with enough sources of random disturbance and enough free parameters that it is not implausible that it could match the observed time-series variation in the 10 variables we aim at explaining. While the model does yet fit quite as well as an unrestricted VAR with a

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evaluation. At some stages of our work we used a Bayeisan hill-climbing algorithm that can be adjusted to make it insensitive to modest levels of rounding error in likelihood evaluation. And finally we have used the likelihood conditioned on the data being at "steady state" at a distant past point, as described in the text. A separate paper or papers describing these numerical innovations is in preparation.

similar number of parameters, it comes close enough to suggest that with a little more work and a few judicious modifications of the model structure a fit as good as a VAR fit may be attainable.

The Three-Variable Data Set

First we report the parameter estimates implied by fitting the flexible-price model to quasi-differences of time series on consumption, hours, and investment. Table 1 reports the estimated values for the 33 free parameters associated with the version of the model that is formulated in the levels of nominal variables.[12] Because at this stage we are not using data on 7 of the 10 variables and policy shocks are neutral in the flexible-price model, the values for the remaining 12 parameters from the policy rules have no effect on the likelihood value and are not reported. Most of the estimated parameters look reasonable. The risk aversion parameter, γ, is estimated to be 8.82, which is in line with some previous estimates. There is some slight convexity to the technology that transforms output goods into consumption and investment goods (μ = 1.025) and the elasticity of substitution between labor and capital is estimated to be quite low (σ = -.32). Point estimates for two important parameters -- the discount rate, β, and the depreciation rate, δ -- are less reasonable, but so imprecisely estimated that they are within one standard error of plausible values. Most of the exogenous processes have roots away from the unit circle: only the shock to total factor productivity has a root above .95.

Table 2 compares the forecast errors from the estimated model with those implied by naively assuming no change in the data and by fitting an unrestricted VAR to the three quasi-differenced time series. The VAR is estimated with 2 lags of each variable and a constant term. The number of free parameters in the VAR, then, (including the 6 free parameters in the covariance matrix of the innovations) is 27.

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[12] We actually maximize the likelihood concentrated with respect to a scale factor for the variances. Thus we can fix the variance of one of the exogenous processes as a normalization, and the standard errors we compute on the variances are actually standard errors on their ratios to the pegged variance parameter. This follows from the fact that the concentrated likelihood is almost exactly the same as the likelihood integrated over the same parameter, so that the concentrated likelihood can be interpreted as an approximate marginal posterior p.d.f. The standard errors of the estimates are computed using a BFGS-update of the inverse of the Hessian matrix.

By the log determinant criterion, the model fits the data about 22% better than does the assumption of no change and it fits only 1.8% worse than the unrestricted VAR.[13] The model also comes close to matching the contemporaneous correlations among the VAR innovations.

On the other hand, the log determinant criterion is proportional to the maximized likelihood for the VAR and for the naive no-change model for which the covariance matrix of first differences is the innovation covariance matrix. For our structural model, the log determinant criterion is not proportional to the maximized likelihood. For direct comparison of likelihood, we should compare -.5×134×(log det covariance matrix)-.5*134*3 for the two naive models to our fitted log likelihood of 1545.48. The two naive models have, by this calculation, log likelihoods of 1469.04 and 1563.71, respectively. The "likelihood" reported in the table for the model is computed from the log det covariance matrix just as it was for the two naive models. The model's actual likelihood is slightly lower. This is partly because unlike the reduced-form models, it does not leave the covariances among the innovations unrestricted. However the model fits the data unconditionally, while the VAR is fitting only conditional on the initial observations, so the two likelihoods are not strictly comparable.[14]

For convenience, the table also reports Choleski decompositions of the covariance matrices, which make comparisons by variable across models easier. The model improves on a random walk for consumption, hours, and investment. In the case of investment the model produces an error variance below that of the VAR.

The estimated parameters imply a steady state that matches the means of the data in some but not all respects. Except for consumption, the steady state values of the three series are near their means, as are the ratios of hours and investment to

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[13]To see this, divide one half of the difference in the relevant log determinants by the number of variables. For the likelihood values, the same sort of estimate of "average percentage difference in standard errors of forecast" is the difference in likelihoods divided by sample size times number of variables.

[14]It is also true that the model implies that the covariance matrix of innovations varies over time, so that in computing its likelihood, errors at different dates are weighted differently. But it appears that this effect is not strong here compared to the effect of restrictions on the covariances at a point in time.

consumption. The model also implies a labor share of income equal to .90, which is higher than the two-thirds typically cited.

Figure 1 shows time series charts that compare the model's output with the innovations from the VAR. The shaded areas correspond to NBER business cycle peaks and troughs. For consumption, the model keeps pace with the VAR in the middle of the sample and during most recessions, but performs less well than the VAR in the 1960s and outperforms the VAR in the 1980s. In the second panel, the model performs remarkably well in predicting hours fluctuations, though it has a slight tendency to exaggerate declines in hours during recessions. The model also predicts investment as well as the VAR.

Figure 2 contrasts the reduced-form moving average representations over 20 quarters from the VAR with those from the model. The covariance matrices are orthogonalized in the order consumption, hours, and investment. Estimating the VAR in quasi-differences eliminates most of the dynamics in the response functions, with the responses of the three series dying out quickly following their own disturbance. The model appears to reproduce the contemporaneous correlations among innovations well, but implies more persistent responses in consumption to consumption and hours innovations than observed in the data.

Once estimated, the model can be used to evaluate the underlying exogenous sources of fluctuations in consumption, hours, and investment over the sample period. It turns out that shocks to only three of the 11 exogenous processes in the model account for all the fluctuations in the three endogenous series. The three important disturbances are π, a shock to consumption's share in utility, θ, a shock to investment in the cost-of-adjustment technology, and α, a shock to the marginal product of capital in the production function.

Table 3 reports the percentage of each variable's forecast error variance due to the three shocks in the short and medium runs. In the short run, all three shocks contribute to consumption fluctuations, with π and θ accounting for over one-third of the variance each and α accounting for one-quarter. As the forecast horizon extends, however, θ becomes the dominant source of fluctuations in consumption. Two-thirds of the error variance of hours is due to the preference shock and one-third is due to the cost-of-adjustment shock, regardless of the forecast horizon. Finally, fluctuations in investment arise almost entirely from shocks to the technology that transforms output goods into consumption and investment goods.

For the model's implications to be credible, the estimates must produce sensible dynamic responses to the exogenous shocks. Figure 3 reports the responses of consumption, hours, and investment to the three important structural disturbances.[15] The first row shows that a transitory preference shock that makes consumption more desirable raises consumption and lowers leisure contemporaneously. Investment rises gradually, reaching a peak after about one year before declining smoothly. Responses to a shock to θ appear in the second row. Higher θ, making investment goods relatively more expensive, can be thought of as a decline in the productivity of the capital goods sector. Because θ is serially uncorrelated (ρ_θ = -3191 implying almost no persistence), the shock lowers the one-period return to investment, causing investment to fall precipitously and return immediately to its normal level. Hours worked fall with the caital stock, but then rise to compensate for the decline in capital. Consumption drops to a permanently lower level consistent with the one-time decline in the capital stock. The shock to the marginal product of capital, α, raises the rental price of capital and drives down investment and hours worked initially. The serial correlation of the shock (estimated root of .72) generates a smooth decline in consumption, which bottoms out after about two years. As noted in Table 3, disturbances to α are unimportant sources of fluctuations in hours and investment, but relatively important for short-run movements in consumption.

<u>The 10-Variable Data Set</u>

We had substantial numerical difficulties with the 10-variable neo-classical version of the model, and did not achieve a respectable fit with it. The fit remained particularly bad for the price variables. We do not take these difficulties as proof that this version of the model cannot fit the data -- we had substantial numerical difficulties with each version of the model at one point or another, so

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[15]The responses are plotted over a five-year period as monthly samplings of the underlying continuous response function, not as responses of the time-aggregated actual data.

that the fact that we do not have results to display for this version of the model at the deadline for this manuscript could be simple misfortune. But the difficulty with the neo-classical model fit to price data does accord with speculation by one of the authors (Sims [1989]), and may yet turn out to be a robust result.

For the sticky-price model we obtained apparently converged results. The likelihood value at the best fit is 5348.91, quite a bit lower than the 5742.26 obtainable with a first-order VAR containing constant terms and with an unrestricted covariance matrix of disturbances. The VAR in this case, however, has 165 parameters compared to 49 for the model. The difference in likelihoods is still large relative to degrees of freedom, but again we must recognize that we cannot draw firm conclusions from comparing conditional and unconditional likelihoods. The log determinants of the residual covariance matrices are -95.705 for the VAR and -92.651 for the model, and these are on a comparable sample. The difference corresponds to an average improvement by the VAR over the model of 15% of the standard error of forecast. The model has almost the same log determinant of the covariance matrix of errors as does the naive no-change forecast. Table 7 shows that, in constrast to the 3-variable data set, here the model's fit is closer to that of a naive no-change forecast than to that of a VAR.

While the fit achieved here leaves plenty of room for improvement, it is still interesting to explore what sort of economic interpretation this model supplies. Tables 5 and 6 give most of the story. The model is converged to a parameter value in the purely Ricardian region of the parameter space. Shocks to the tax equation (the Fpolicy column) have no effect on any of the 10 variables other than taxes themselves. Monetary policy looks weak, judged by the size of the entries in column 1. However this reflects a low estimate of the variance of shocks to monetary policy, and the tendency of larger shocks to dominate variance decompositions, where squared responses matter. In Table 6, which shows cumulative per cent responses to sustained one-time shifts of one "standard error unit"[16], we see that responses of C,

[16]Since this is a continuous time model, a sustained shift in the disturbance (which is modeled as white noise) is even more atypical of realizations of the model than it would be in a discrete model. The sizes of the responses look large because the typical disturbance is so little sustained that it never builds nearly this much cumulative response.

L, and I to a monetary contraction are substantial and of reasonable signs. Because the model is so tightly parameterized, it does not produce fancy dynamics in the impulse responses. Figures 4-6 show four typical response shapes. In all three graphs the x axis is in monthly units. In Figures 5 and 6 the responses have not yet begun to die away after three years.

Inflation responds to nothing but the price adjustment shock, while wages respond to nothing but the wage adjustment shock. This is a model in which most fluctuations in prices are persistent, generating no expectation of inflation or deflation, and in which the rest of the economy has little impact on price movements. In this sense the fitted model is showing extremely sticky prices, with none of the expecational instability we have noted is in principle possible in these models.

An open question is whether in this fitted model the extreme price stickiness and strong influence of real shocks is dependent on the particular monetary and fiscal policy rules that are estimated here. We could simulate the model with alternative policy rules to check this.

Table 1. Three-Variable Data Set: Estimated Parameters for Flexible Price Model
(Standard errors in parentheses)

Preferences

$\bar{\pi}$ = 0.341 (.060)	ρ_π = -2.912 (.194)	σ_π^2 = 3.961e-4 (2.730e-1)	
$\bar{\beta}$ = 0.165 (.288)	ρ_β = -0.635 (.153)	β_π = -4.019 (.206)	σ_β^2 = 5.277e-6 (3.209e-2)
$\bar{\gamma}$ = 8.882 (.145)			

Technologies

$\bar{\theta}$ = 0.528 (.082)	ρ_θ = -3.191e+3 (.240)		
$\bar{\alpha}$ = 0.352 (.092)	ρ_α = -0.324 (.126)	α_θ = 1.186 (.088)	
$\bar{A}$ = 96.794 (.105)	ρ_A = -8.715e-5 (.094)	A_α = 2.520 (.147)	A_θ = 0.559 (.205)
σ_θ^2 = 2.137e-2 (.369)	σ_α^2 = 1.995e-5 (0.363)	σ_A^2 = 6.847e-6 (.216)	
$\bar{\phi}$ = 2.934e-4 (.182)	ρ_ϕ = -0.673 (.105)	σ_ϕ^2 = 1.962e-2 (3.194e-2)	
$\bar{\delta}$ = 0.250 (.135)	ρ_δ = -1.821 (.232)	σ_δ^2 = 8.169e-2 (fixed)	
μ = 1.025 (.324)	σ = -0.315 (.037)		

Government Spending and Population

$\bar{g}/Y$ = 7.731e-2 (.292)	ρ_g = -5.428e-2 (0.114)	σ_g^2 = 4.823e-6 (3.165e-2)
$\bar{n}$ = 1.400e-2 (.292)	ρ_n = -5.930 (.047)	σ_n^2 = 4.894e-6 (3.186e-2)

Log Likelihood Value = 1545.48

ρ_x is the first-order AR coefficient on the continuous-time process x, y_x is the coefficient determining how y depends on x, and σ_x^2 is the variance of the process.Policy parameters do not affect the likelihood and were fixed arbitrarily to ensure a unique equilibrium exists.

Table 2. Three-Variable Data Set: Fit of the Flexible Price Model

Covariance/Correlation Matrices

First Difference of Data

Log Det = -24.926 Likelihood = 1469.04

Choleski decomposition

	C	L	I
C	.0169		
L	.0026	.0106	
I	.0072	.0048	.0215

VAR Innovations (2 lags plus constant, 27 free parameters; estimated using quasi-differenced data)

Log Det = -26.339 Likelihood = 1563.71

Choleski decomposition

	C	L	I
C	.0124		
L	.0016	.0089	
I	.0080	.0048	.0173

Model Residuals (33 free parameters)

Log Det = -26.229 "Likelihood" = 1556.34

Choleski decomposition

	C	L	I
C	.0128		
L	.0015	.0092	
I	.0087	.0045	.0171

The Model's Steady State Versus Means of the Data

	U.S. Data Means	Model Steady State
C	13.96	20.48
L	0.353	0.350
I	2.292	3.110
L/C	0.025	0.017
I/C	0.164	0.152

Table 3. Three-Variable Data Set: Variance Decomposition

Percentage of Forecast Error Variance Attributable

	After 6 months to			After 3 years to		
	π	θ	α	π	θ	α
C	35	42	22	8	83	8
L	68	32	0	67	32	0
I	1	98	1	10	85	2

Table 4

Estimated Parameters for 10-variable Sticky Price Model

1	$\overline{g/Y}$	0.0681	2	ρ_g	-0.00147
3	$\bar{n}$	0.0149	4	ρ_n	-9.63
5	$\bar{\pi}$	0.614	6	ρ_π	-0.000285
7	$\bar{\beta}$	0.06	8	ρ_β	-0.000488
9	$\beta\pi$	-1.56	10	$\bar{\theta}$	1.81
11	ρ_θ	-1.28	12	$\bar{\alpha}$	0.624
13	ρ_α	-0.000754	14	$\alpha\theta$	0.412
15	$\bar{A}$	107	16	ρ_A	-0.000306
17	$A\alpha$	-0.612	18	$A\theta$	-1.089
19	$\bar{\phi}$	0.00532	20	ρ_θ	-0.00165
21	$\bar{\delta}$	0.465	22	ρ_δ	-0.0107
23	μ	25.8	24	σ	-25.0
25	γ	5.65	26	a_{inf}	0.00748
27	a_r	-1.06	28	a_L	1.66
29	b_τ	-5.47	30	b_L	1.36
31	b_x	2.41	32	b_{inf}	1.82
33	$\bar{P}$	0.563	34	$\overline{BGNP}$	0.118
35	χ_P	0.00804	36	χ_w	0.00019

Equation Variances

37	mpol	0.0188	38	fpol	0.0209
39	Padj	0.0283	40	wadj	0.00166
41	g	0.0175	42	n	0.00372
43	π	0.00196	44	β	0.00107
45	θ	0.00786	46	α	0.00722
47	A	0.00301	48	ϕ	0.0986
49	δ	0.401			

Note: The parameters are as they appear in the text, except that bars over the parameters indicate steady-state values and the double-greek-letter parameters refer to responses in the exogenous dynamics. For example, $A\alpha$ is the coefficient on the level of α in the differential equation with $\dot{A}$ on the left.

Table 5

Variance Decomposition, 10-Variable Sticky Price Model

	Mpolicy	Fpolicy	Padj	Wadj	g	n	pi	beta	theta	alpha	A	phi	delta
C	0.01	0.00	0.00	0.00	0.00	0.00	0.26	0.06	0.42	0.12	0.13	0.00	0.00
L	0.02	0.00	0.00	0.00	0.00	0.00	0.80	0.18	0.00	0.00	0.00	0.00	0.00
I	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.02	0.05	0.02	0.02	0.00	0.79
tax/C	0.00	0.52	0.31	0.00	0.01	0.00	0.04	0.01	0.04	0.01	0.01	0.03	0.00
Infl.	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.00	0.00	0.98	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00
M/P	0.03	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.22	0.06	0.07	0.56	0.00
r	0.20	0.00	0.00	0.00	0.00	0.00	0.65	0.14	0.00	0.00	0.00	0.00	0.00
g	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
n	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6
Cumulative Responses (x 100)

	Mpolicy	Fpolicy	Padj	Wadj	g	n	pi	beta	theta	alpha	A	phi	delta
C	-16	0	-0	-0	-7	0	67	31	77	41	42	0	0
L	-15	0	-0	-0	0	0	63	29	1	0	0	0	0
I	-16	0	-0	-0	0	2	80	37	47	27	27	-0	141
tax/C	0	24	25	-0	1	0	7	3	-5	-3	-3	-4	0
Infl.	0	0	121	0	0	0	0	0	-0	-0	-0	0	0
W	-0	0	0	29	-0	0	1	1	0	0	0	0	0
M/P	-45	0	-0	-0	0	0	47	22	71	38	39	110	0
r	39	0	0	-0	0	0	20	9	0	0	0	0	0
g	0	0	0	0	94	0	0	0	0	0	0	0	0
n	0	0	0	0	0	8	0	0	0	0	0	0	0

Table 7
Standard deviations of errors

	Model	Diff.'s	VAR
C	0.0130	0.0119	0.0114
L	0.0101	0.0102	0.0084
I	0.0234	0.0238	0.0195
tx/Y	0.0119	0.0108	0.0096
infl	0.0201	0.0174	0.0143
W	0.0047	0.0048	0.0045
M/P	0.0146	0.0111	0.0126
r	0.0092	0.0090	0.0084
g	0.0161	0.0154	0.0146
n	0.0029	0.0029	0.0026

Appendix: The First-Order Conditions

The first-order conditions for the firm are

$$X = \left(\frac{Y}{C+g}\right)^{1-\mu} \tag{A1}$$

$$Q = \theta\left(\frac{Y}{I^{*}}\right)^{1-\mu} \tag{A2}$$

$$r = A^{\sigma}\alpha\left(\frac{Y}{K}\right)^{1-\sigma} \tag{A3}$$

$$w = A^{\sigma}\left(\frac{Y}{L}\right)^{1-\sigma} . \tag{A4}$$

For the agent they are

∂C^{*}: $$\pi Z^{-\gamma}\left(\frac{1-L}{C^{*}}\right)^{1-\pi} = \nu X \tag{A5}$$

∂L: $$(1-\pi)Z^{-\gamma}\left(\frac{C^{*}}{1-L}\right)^{\pi} = \xi w \tag{A6}$$

(where $Z=C^{*\pi}(1-L)^{1-\pi}$)

∂I: $$Q\lambda = \omega \tag{A7}$$

∂C: $$\lambda=\nu \tag{A8}$$

∂M: $$-\frac{\dot{\lambda}}{\lambda} = -\frac{P^2}{M^2}\frac{Y\psi}{\lambda} - \beta - \frac{\dot{P}}{P} \tag{A9}$$

∂B: $$-\frac{\dot{\lambda}}{\lambda} = i - \beta - \frac{\dot{P}}{P} \tag{A10}$$

∂Y: $$\lambda + \frac{\psi P}{M} = \xi + \phi V \nu \tag{A11}$$

∂V: $$\phi Y \nu = -\psi \tag{A12}$$

∂K: $$-\frac{\dot{\omega}}{\omega} = r\frac{\xi}{\omega} - \delta - \beta \tag{A13}$$

Here are the agent FOC's manipulated to get rid of Lagrange Multipliers:

$$(1-2\phi V)\frac{w}{X} = \frac{1-\pi}{\pi}\frac{C^*}{(1-L)} \tag{A14}$$

$$i = \phi V^2 \tag{A15}$$

$$\left(1 - \pi(1-\gamma)\right)\frac{\dot{C}^*}{C^*} + (1-\gamma)(1-\pi)\frac{\dot{L}}{1-L} = i - \beta - \frac{\dot{X}}{X} - \frac{\dot{P}}{P}$$
$$+ \frac{\dot{\pi}}{\pi} + \dot{\pi}\,(1-\gamma)\,\log\left(\frac{C^*}{1-L}\right) \tag{A16}$$

$$(1-2\phi V)\frac{r}{Q} + \frac{\dot{Q}}{Q} - \delta = i - \frac{\dot{P}}{P} \tag{A17}$$

Data Appendix

Consumption: Personal Consumption Expenditures, NIPA, deflated by the PCE deflator.

Investment: Residential plus Non-Residential Fixed Investment, NIPA, deflated by their respective deflators, NIPA.

Employment: Civilian employment times weekly hours index for total goods production, scaled to lie in the unit interval.

Real wages: Index of compensation per hour in the nonfarm business sector, deflated by the GDP deflator.

Price level: GDP deflator, NIPA.

Government purchases: Total federal plus state and local purchases, NIPA, deflated by the GDP deflator.

Government tax revenues: Total federal plus state and local tax receipts, less transfer payments, with federal grants to state and local governments netted out, NIPA, deflated by the GNP deflator.

Money: The Federal Reserve Board's monetary base, not adjusted for reserve requirement changes.

Interest rate: Three-month Treasury bill rate, secondary market, in basis points.

Population: Quarterly growth rate of the civilian non-institutional population, at annual rates.

Per capita series are obatined by deflating by the population. All series seasonally adjusted except the interest rate and population. Monthly series are time-averaged to quarterly frequencies.

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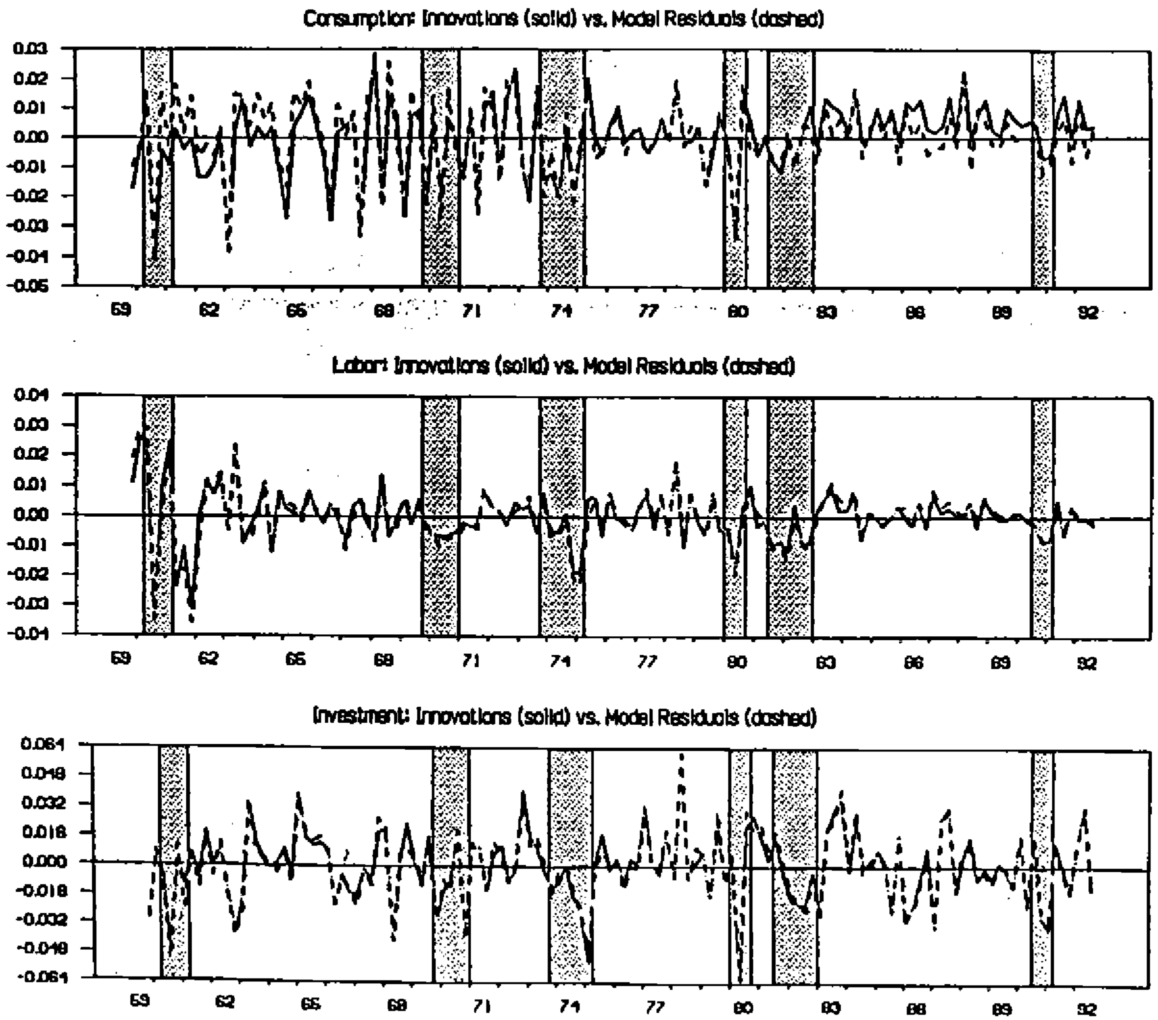
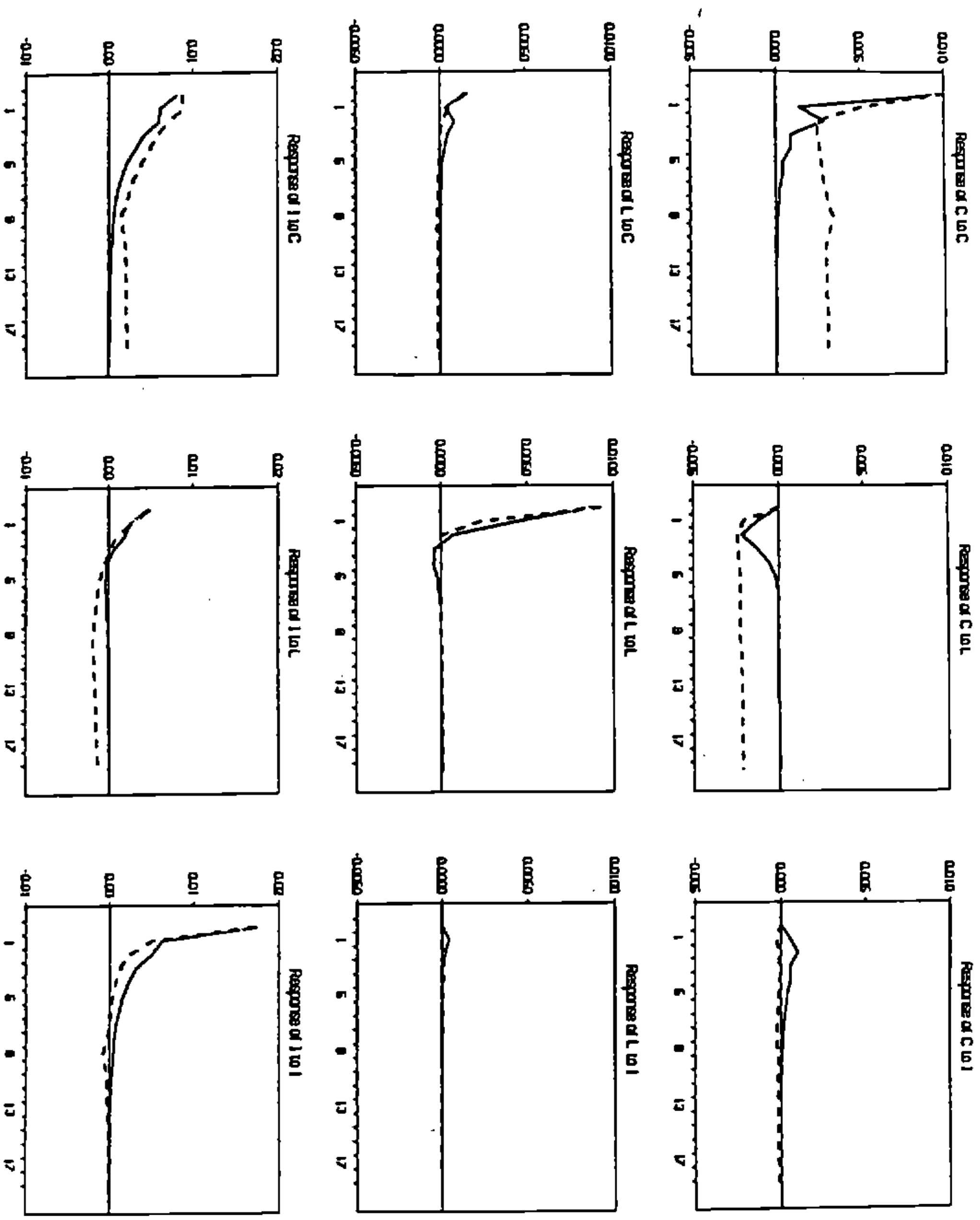
FIGURE 1



Consumption: Innovations (solid) vs. Model Residuals (dashed)
Labor: Innovations (solid) vs. Model Residuals (dashed)
Investment: Innovations (solid) vs. Model Residuals (dashed)
59
62
65
68
71
74
77
80
83
86
89
92

FIGURE 2



Response of C to C
Response of C to L
Response of C to I
Response of L to C
Response of L to L
Response of L to I
Response of I to C
Response of I to L
Response of I to I

FIGURE 3

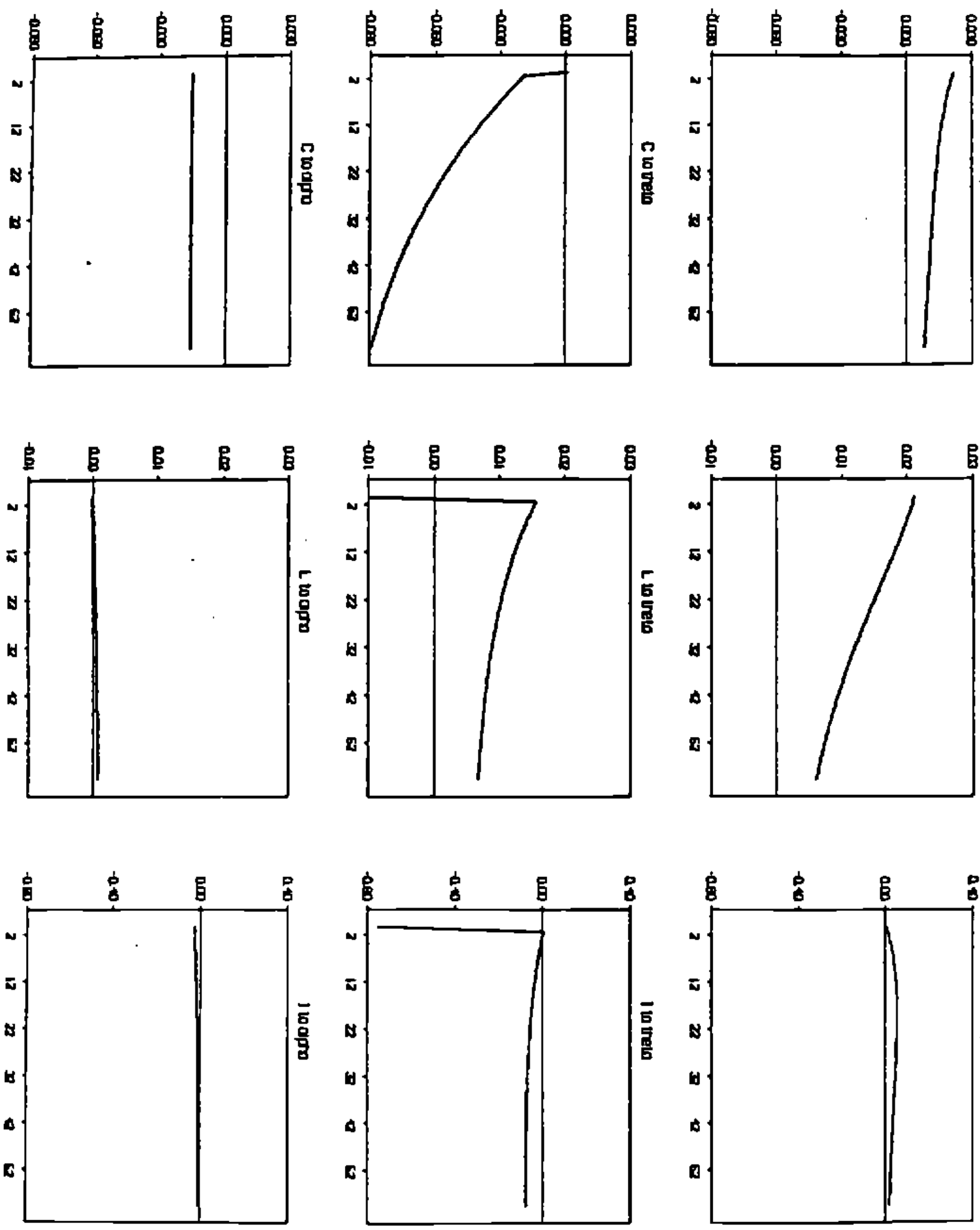
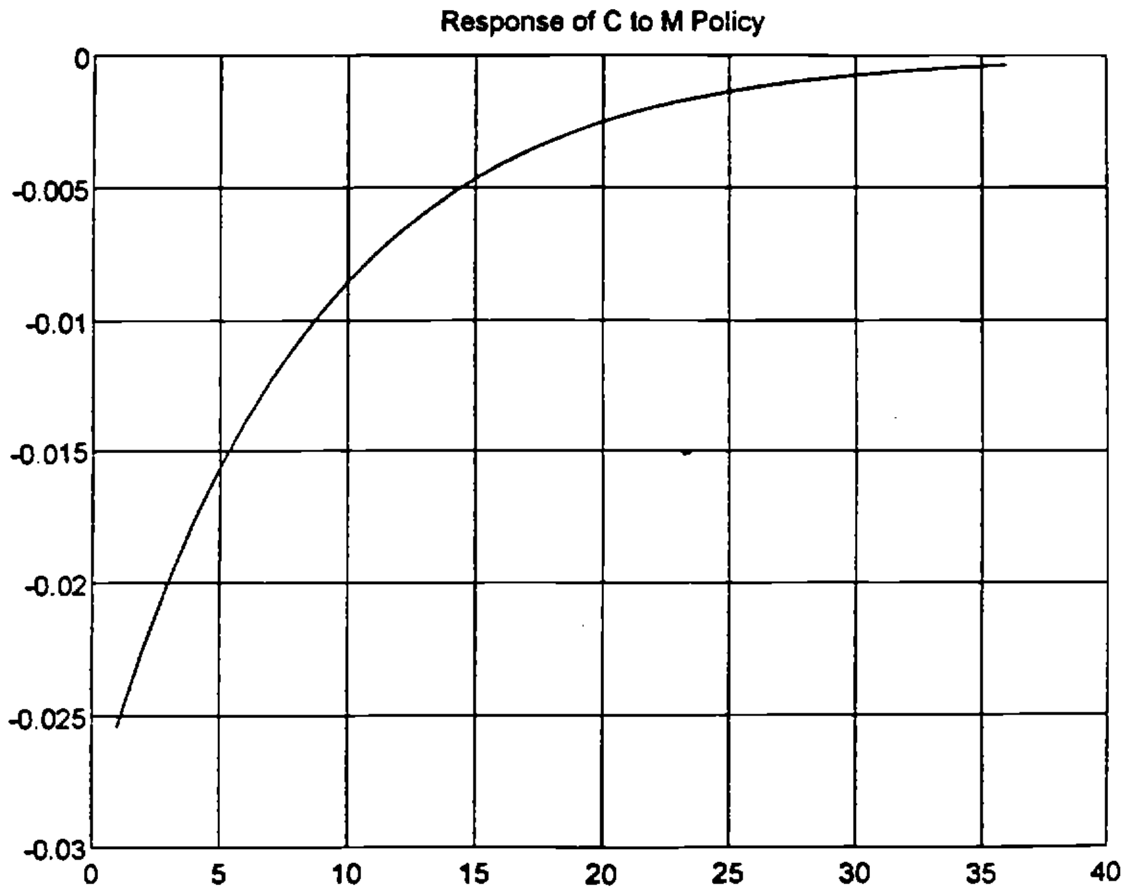
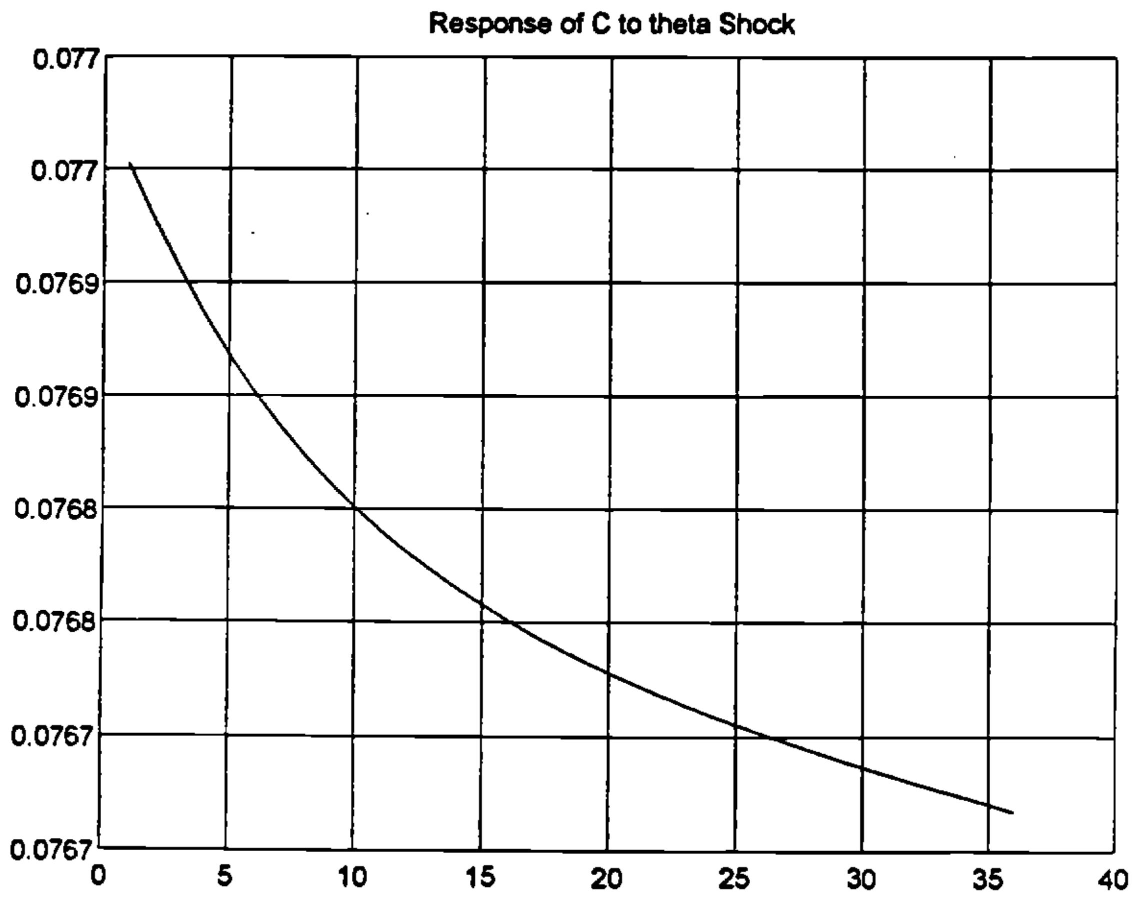


FIGURE 4



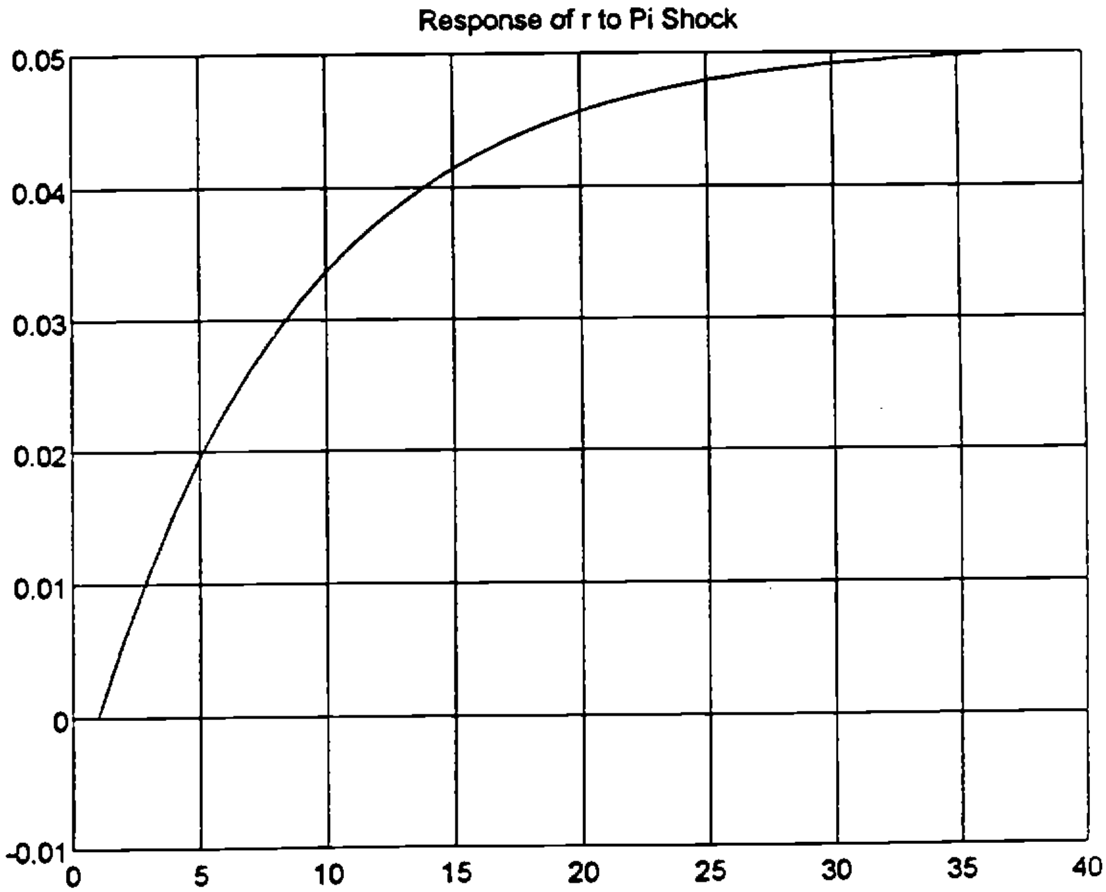
Response of C to M Policy
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
0
5
10
15
20
25
30
35
40

FIGURE 5



Response of C to theta Shock
0.077
0.077
0.0769
0.0769
0.0768
0.0768
0.0767
0.0767
0
5
10
15
20
25
30
35
40

FIGURE 6



Response of r to Pi Shock
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
5
10
15
20
25
30
35
40