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# A SEMI-CLASSICAL MODEL OF PRICE LEVEL ADJUSTMENT

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## ABSTRACT

This paper investigates the theoretical and empirical properties of a model of aggregate supply behavior that was introduced in the 1970s but has received inadequate attention. The model postulates that price changes occur so as to gradually eliminate discrepancies between actual and market-clearing values and to reflect expected changes in market-clearing values. Its implications are more "classical" than most alternative formulations that reflect gradual price adjustment. Empirical results, which utilize a proxy for market-clearing output that is a function of fixed capital and the real price of oil, are moderately encouraging but not entirely supportive.

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## I. Introduction

The purpose of this paper is to investigate the theoretical and empirical properties of one particular model of aggregate supply behavior that has not, in my opinion, attracted the attention that it deserves. The model in question features price level stickiness--i.e., gradual adjustment in response to shocks--but nevertheless has several "classical" characteristics. Its specification was first proposed by Grossman (1974) but was more prominently introduced by Barro and Grossman (1976). Shortly thereafter it was independently conceived and more formally justified by Mussa (1977, 1981a, 1981b, 1982), and was used as the centerpiece of a number of papers by myself.[1] This model--which I will call the P-bar model--has never been the subject of extensive empirical study, however, and has almost disappeared from the literature in recent years.

In the following sections it will be suggested that recent neglect of the P-bar model is unwarranted. In particular, I will argue that its theoretical properties are more satisfactory than those of some leading operational models of aggregate supply--here attention will be focused especially on the formulations of Taylor (1979, 1980, 1993) and Fuhrer and Moore (1993a, 1993b)--and that its empirical performance is reasonably satisfactory. There is a problem relating to the implied time series properties of capacity output, but essentially the same flaw pertains to the other models under discussion, as well. Throughout, the perspective of the analysis will be that of a macroeconomic researcher whose concern is the development of a compact quarterly econometric model that has reasonable theoretical properties and is empirically consistent with the postwar U.S. data.

The outline of the paper is a follows. The P-bar model is introduced and briefly discussed in Section II. Then its basic theoretical properties are considered in Section III, after which some discussion of alternative models is presented, with Section IV being devoted primarily to the model of Fuhrer and Moore. Empirical analysis with quarterly U.S. data for 1954-1990 is developed in Section V. A few theoretical issues are then discussed in Section VI and a brief conclusion follows.

## II. Basic Description of the P-Bar Model

The P-bar model of aggregate supply shares the assumption of most econometric models that prices adjust to shocks incompletely, within each period, with output being determined by the quantity demanded at the resulting price level. This particular model's distinguishing characteristic is that the determination of each period's price level depends upon movements of the hypothetical price level that would, given prevailing conditions, make output equal to its capacity value. Let $p_t$ and $y_t$ denote logarithms of the price level and output, respectively, in the aggregate or for a representative producer. Also, let $\bar{y}_t$ denote the "capacity" or "natural rate" level of $y_t$, the value that the latter would assume if prices were perfectly flexible. Then $\bar{p}_t$ is defined as the "market clearing" value of $p_t$ that would induce $y_t$ to equal $\bar{y}_t$, given current conditions. In terms of these variables, the P-bar price adjustment equation can be written as

(1) $$p_t - p_{t-1} = \gamma(\bar{p}_{t-1} - p_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}),$$

with $1 \geq \gamma > 0$. Here $E_t(\cdot)$ represents the expectation of the indicated variable conditional upon information available at time t, with this information henceforth assumed to include all relevant variables realized in t or in previous periods. Thus price adjustments are specified to occur in proportion to the previous period's discrepancy between $p_t$ and its market-clearing value, provided that no changes in the market clearing value itself are expected. But if they are, then the expected change in $\bar{p}_t$ is also a component of the realized change in $p_t$.

An alternative formulation, employed in my papers and by Obstfeld and Rogoff (1984), reflects the fact that $p_t - \bar{p}_t$ will by construction be related to $y_t - \bar{y}_t$. Indeed, if the model is linear, these variables will be proportional--with a negative coefficient, of course--and equation (1) can equivalently be written as

$$(2) \qquad p_t - p_{t-1} = \gamma_1(y_{t-1} - \bar{y}_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}),$$

with $\gamma_1>0$. The constant of proportionality relating $\gamma_1$ and $\gamma$ will depend, obviously, on the properties of the model's aggregate demand relations.

Which of these two formulations, (1) or (2), should be considered more nearly structural? The answer to that question depends upon the analysis used to justify the implied type of price setting behavior. Mussa (1981b) bases his argument on profit maximization calculations made in the face of lump-sum price adjustment costs with averaging across individual firms, an approach which gives rise to (1).[2] My own preferred rationale presumes that prices must be set at the start of the period in which they will apply and that production will equal whatever quantity is demanded.[3] One basic assumption is that it is costly, in terms of real resources, for producers to make between-period changes in output relative to capacity.[4] But it is also costly, of course, for output to differ from its capacity level. Thus if these two cost components are quadratic, the producer will set a price that is expected to yield a magnitude of demand (and output) that is dependent both on $\bar{y}_t$ and $y_{t-1} - \bar{y}_{t-1}$, the extent of dependence on the latter being higher for higher adjustment costs. From the perspective of this approach, formulation (2) is the more basic and more nearly structural.[5]

In terms of its superficial appearance, the adjustment specification (2) looks much like a typical expectational Phillips relation of the 1970s. There are two significant differences, however, that should be noted. First, the "expected inflation" term pertains to changes in the market-clearing price $\bar{p}_t$, not to changes in $p_t$ itself. Second, the output gap term pertains to the previous period's discrepancy between $y_t$ and $\bar{y}_t$, not the value for the current period. As a consequence, the mode of behavior represented by (2) makes $p_t$ an entirely predetermined variable. That raises some issues that will be addressed momentarily, but first it should be noted that the interaction of these two unusual features is itself quite significant. Specifically, a version of (2) in which the third term is the expectation of $\Delta p_t$, rather than $\Delta\bar{p}_t$, makes no sense if expectations are rational. To see that, replace $E_{t-1}\Delta\bar{p}_t$ in (2) with $E_{t-1}\Delta p_t$ and apply the conditional expectation operator to the equation. The resulting expression is

(3) $$0 = \gamma_1(y_{t-1} - \bar{y}_{t-1}),$$

which implies that $y_t = \bar{y}_t$ for all t, i.e., that output is always equal to its capacity value. But that means that the specification with $E_{t-1}\Delta p_t$ cannot be used, given the assumption of rational expectations, to yield a model in which output fluctuates relative to capacity.[6]

What about the property of $p_t$ being entirely predetermined? In practice, one would presumably want to add a stochastic disturbance term to (2), to reflect the effects of the many small influences that are omitted in any tractable model. But that would not alter the important properties of the specification to any appreciable extent. A more significant modification would, however, be possible. Specifically, one could regard the magnitude of $p_t$ determined by (2) as a <u>planned</u> value, which might be altered <u>within</u> period t in response to the occurrence of conditions significantly different than those previously expected. Indeed, one might interpret the non-dynamic "price stickiness" models of Mankiw (1985) or Ball and Romer (1990) as pertaining to this latter form of adjustment. Recognizing such adjustments would, interestingly, make the model one with a <u>reduced</u> degree of price stickiness relative to the P-bar formulation (2). Since a major theme of our discussion will be the rather "classical" nature of the P-bar model--i.e., its similarity in several respects to models with perfectly flexible prices--it will strengthen the argument[7] to exclude within-period adjustments of the Mankiw-Ball-Romer type. Accordingly, all of the discussion below will be based on equation (2) rather than an extended model that includes such adjustments.

A closely related issue involves the role of inventories, briefly mentioned in fn. 4. A producer that holds a stock of finished goods has available a third way of responding to shocks, in addition to output and price adjustments. Incorporating inventory holdings into our model would therefore serve to make it more flexible. But since we are emphasizing the classical properties of systems including (2), our argument will again be strengthened by abstracting from these extra features.[8]

From its definition, it is clear that one can obtain an explicit expression for the market clearing price $\tilde{p}_t$ only after adoption of a specification regarding aggregate demand. In my previous work, the demand specification typically used was[9]

(4) $$y_t = \beta_0 + \beta_1(m_t - p_t) + \beta_2 E_{t-1}(p_{t+1} - p_t) + \beta_3 y_{t-1} + v_t,$$

where $m_t$ is the log of the money stock and $v_t$ is a stochastic disturbance term, which we shall here take to be a random walk (so that $\xi_t = v_t - v_{t-1}$ is white noise). This expression can be obtained by writing IS and LM functions of the form

(5) $$y_t = b_0 + b_1[r_t - E_{t-1}(p_{t+1} - p_t)] + b_2 y_{t-1} + v_{1t}$$

and

(6) $$m_t - p_t = c_0 + c_1 y_t + c_2 r_t + v_{2t},$$

and solving out the endogenous variable $r_t$.[10] There are two problems with this specification, however, even if one accepts the general spirit of an IS-LM type of model. One concerns the expectation operator in (5); it would seem more appropriate if specified as $E_t(\cdot)$. And the second pertains to the real interest rate appearing in the IS function; many analysts would think that a long-term real rate such as $R_t^N - E_t(p_{t+N} - p_t)$, with N an integer greater than 1, should appear rather than the one-period real rate.[11] Thus (4) can be used only as a source of examples, not general conclusions regarding $\bar{p}_t$ or the properties of a model that incorporates the P-bar relation (2).

## III. Properties of the P-Bar Model

The just-mentioned problems are quite important with respect to arguments concerning the famous or infamous "policy ineffectiveness proposition" that played a prominent role in my 1979 and 1980 papers. It is not difficult to show[12] that if (4) represents aggregate demand, then the time series process for $y_t - \bar{y}_t$ will be given as

$$(7) \qquad y_t - \bar{y}_t = \beta_1(m_t - E_{t-1}m_t) + [1-\gamma_1(\beta_1 + \beta_2)](y_{t-1} - \bar{y}_{t-1}) + (v_t - E_{t-1}v_t) - (\bar{y}_t - E_{t-1}\bar{y}_t)$$

so that it is only the surprise component of monetary policy that affects $y_t - \bar{y}_t$, for any policy feedback rule that bases the systematic part of $m_t$ entirely on variables from period t-1 and before. Thus the policy ineffectiveness proposition will hold in this case. But it will not hold if the demand schedule includes $E_tp_{t+1}$ or any other variable $z_t$ for which the behavior of $z_t - E_{t-1}z_t$ is affected by the policy rule.[13] So this proposition should not be regarded as a general implication of the P-bar model.

For illustrative purposes, however--and possibly as a useful approximation--let us provisionally adopt (4) as an AD specification.[14] Then we can write

$$(8) \qquad p_t = \frac{1}{\beta_1+\beta_2}[\beta_0 + \beta_1m_t + \beta_2E_{t-1}p_{t+1} + \beta_3y_{t-1} + v_t - y_t],$$

which implies that

$$(9) \qquad \bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_0 + \beta_1m_t + \beta_2E_{t-1}p_{t+1} + \beta_3y_{t-1} + v_t - \bar{y}_t].$$

From these two expressions we see that

(10) $$p_t - E_{t-1}\bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_1(m_t - E_{t-1}m_t) + v_t - E_{t-1}v_t - (y_t - E_{t-1}\bar{y}_t)]$$

and that

(11) $$p_{t-1} - \bar{p}_{t-1} = \frac{(-1)}{\beta_1+\beta_2}[y_{t-1} - \bar{y}_{t-1}].$$

Substitution of these into the P-bar equation (2), followed by some rearrangement, yields

(12) $$y_t - \bar{y}_t = \beta_1(m_t - E_{t-1}m_t) + [1-\gamma_1(\beta_1 + \beta_2)]\ (y_{t-1} - \bar{y}_{t-1}) + \xi_t - u_t$$

where $\xi_t$ and $u_t \equiv \bar{y}_t - E_{t-1}\bar{y}_t$ are the (exogenous) unexpected components of $v_t$ and $\bar{y}_t$.

In (12), we see that $y_t - \bar{y}_t$ is explained by its own lagged value and three surprise terms. Consequently, with (4) taken to represent AD, the policy ineffectiveness property holds, as stated previously, and the model implies that $y_t - \bar{y}_t$ is generated by a first-order autoregressive process [denoted AR (1)] with coefficient $1 - \gamma_1(\beta_1 + \beta_2)$.[15] The compatibility of that implication with the U.S. quarterly data will be considered below in Section V.

Continuing with the analysis of the model (2) (4), we wish next to obtain a solution expression for $\Delta p_t$, the inflation rate. A crucial step is to evaluate $E_{t-1}(\bar{p}_t - \bar{p}_{t-1})$, which we do by differencing (9) and applying $E_{t-1}(\cdot)$:

(13) $$E_{t-1}\Delta\bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_1 E_{t-1}\Delta m_t + \beta_2(E_{t-1}p_{t+1} - E_{t-2}p_t) + \beta_3\Delta y_{t-1} - E_{t-1}\Delta\bar{y}_t],$$

Thus we see that to determine the time series properties of $\Delta p_t$, it will be necessary to adopt some assumption about the processes generating $\Delta m_t$ and $\bar{y}_t$. Anticipating evidence discussed in Section V, let us take $\Delta m_t$ to obey an AR(1) process and $\Delta\bar{y}_t$ a MA(1) process:[16]

(14) $$\Delta m_t = \mu_0 + \mu_1\Delta m_{t-1} + e_t$$

(15) $$\Delta\bar{y}_t = u_t + \theta u_{t-1} \qquad \theta>0.$$

And for the sake of discussion, let us suppose that it is permissible to neglect the term in (13) involving $E_{t-1}p_{t+1} - E_{t-2}p_t$. Then we have

$$E_{t-1}\Delta\bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_1(\mu_0 + \mu_1\Delta m_{t-1}) + \beta_3\Delta y_{t-1} - \theta u_{t-1}] \tag{16}$$

so that inflation is given by

$$\Delta p_t = \gamma_1(y_{t-1} - \bar{y}_{t-1}) + \frac{1}{\beta_1+\beta_2}[\beta_1(\mu_0 + \mu_1\Delta m_{t-1}) + \beta_3\Delta y_{t-1} - \theta u_{t-1}]. \tag{17}$$

The latter expression would be operational--i.e., subject to estimation--if $\bar{y}_t$ were observable. Possible proxies will be considered in Section V.

Also of considerable interest is the corresponding univariate expression that we can obtain when $\beta_3 = 0$ by writing (12) as

$$y_t - \bar{y}_t = \phi(y_{t-1} - \bar{y}_{t-1}) + \psi_t, \tag{18}$$

where $\psi_t = \beta_1 e_t + \xi_t - u_t$, implying $y_t - \bar{y}_t = (1 - \phi L)^{-1}\psi_t$. Then defining $\mu_1' = \mu_1/(\beta_1 + \beta_2)$ and $\theta' \equiv \theta/(\beta_1 + \beta_2)$, we have

$$\Delta p_t = \gamma_1(1 - \phi L)^{-1}\psi_{t-1} + \mu_1'(1-\mu_1 L)e_{t-1} + \theta' u_{t-1}, \tag{19}$$

where the constant term is suppressed, or

$$(1-\phi L)(1-\mu_1 L)\Delta p_t = \gamma_1(1-\mu_1 L)\psi_{t-1} + \mu_1'(1-\phi L)e_{t-1} + \theta'(1-\mu_1 L)(1-\phi L)u_{t-1}. \tag{20}$$

Granger's Lemma shows the right hand side of (20) to be a MA(2) process, so we have found the inflation rate to be an ARMA(2,2) process from the univariate perspective. Clearly, therefore, there are many parameter values for $\phi$ and $\mu_1$ that will imply a great deal of inflation persistence.

Next it will be appropriate to demonstrate explicitly that--as mentioned above--the policy ineffectiveness proposition does not apply when $E_t p_{t+1}$ appears in the AD function. For this demonstration let us write the system as

$$p_t - p_{t-1} = \gamma_1(y_{t-1} - \bar{y}_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}) \tag{21a}$$

$$y_t = \beta_1(m_t - p_t) + \beta_2 E_t(p_{t+1} - p_t) + v_t \tag{21b}$$

$$\bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_1 m_t + \beta_2 E_t p_{t+1} + v_t - \bar{y}_t] \tag{21c}$$

$$m_t = \mu_1 m_{t-1} + e_t \tag{21d}$$

In this case, expressions analogous to (10) and (11) above are

$$(22)\qquad p_t - E_{t-1}\bar{p}_t = \frac{1}{\beta_1+\beta_2}[\beta_1 e_t + \beta_2(E_t p_{t+1} - E_{t-1}p_{t+1}) + \xi_t - (y_t - E_{t-1}\bar{y}_t)]$$

and

$$(23)\qquad p_{t-1} - \bar{p}_{t-1} = \frac{(-1)}{\beta_1+\beta_2}[y_{t-1} - \bar{y}_{t-1}].$$

Consequently, substitution into (21a) plus rearrangement yields

$$(24)\qquad y_t - \bar{y}_t = \beta_1(m_t - E_{t-1}m_t) + \beta_2(E_t p_{t+1} - E_{t-1}p_{t+1}) + [1 - \gamma_1(\beta_1 + \beta_2)](y_{t-1} - \bar{y}_{t-1}) + \xi_t - u_t,$$

instead of (12). But the minimal state variable solution to the model implies that $p_t$ is of the form

$$(25)\qquad p_t = \phi_{10} + \phi_{11}m_{t-1} + \phi_{12}e_t + \phi_{13}v_{t-1} + \phi_{14}\xi_t + \phi_{15}y_{t-1} + \phi_{16}\bar{y}_{t-1} + \phi_{17}u_t$$

and thus that

$$(26)\qquad E_t p_{t+1} - E_{t-1}p_{t+1} = [(\phi_{11} + \phi_{15}\beta_1)e_t + (\phi_{13} + \phi_{15})\xi_t + (\phi_{16} - \phi_{15})u_t]/(1-\phi_{15}\beta_2),$$

where we have used (24) to eliminate $y_t - E_{t-1}y_t$. Then tedious algebra reveals that the composite parameters of the latter involve the policy parameter $\mu_1$.[17] Thus the unconditional variance of $y_t - \bar{y}_t$ will depend upon $\mu_1$, which implies failure of the ineffectiveness proposition.

By contrast, the natural rate hypothesis--as defined by Lucas (1972)--does hold in the P-bar model. The unconditional mean of the output gap $E(y_t - \bar{y}_t)$ cannot, that is, be affected by any aspect of the monetary policy rule. Proof of this proposition is readily obtained by application of the unconditional expectation operator $E(\cdot)$ to (24), which yields

$$(27)\qquad E(y_t - \bar{y}_t) = [1 - \gamma_1(\beta_1 + \beta_2)]\, E(y_{t-1} - \bar{y}_{t-1}).$$

But the latter implies that $E(y_t - \bar{y}_t)$ is a stable process that converges toward zero. And the steps in the derivation of (27) have made no reference to the process generating $m_t$, so the foregoing conclusion is quite general.[18]

This property, of conformance to the natural rate hypothesis, is (I would contend) extremely attractive. For it seems implausible, as Lucas said, that output could be kept <u>permanently</u> high (relative to its natural rate or capacity value) by <u>any</u> pattern of behavior of the monetary authorities. And yet it is the case that almost all empirical models of aggregate supply--i.e., wage-price specifications in econometric models--fail to satisfy the natural rate hypothesis. That this is true for specifications involving the concept of a NAIRU (non-accelerating inflation rate of unemployment) can be seen immediately: the existence of a stable relationship between unemployment and the acceleration magnitude[19] implies that the unemployment rate can be permanently lowered by permanently generating a higher acceleration magnitude. In addition, a similar result pertains to a specification of the form

(28) $$p_t - p_{t-1} = \delta(\bar{p}_t - p_{t-1}) \qquad 1>\delta>0,$$

which is basically the same as the price-adjustment equation of the MPS model.[20] To see this, write (28) as

(29) $$p_t - p_{t-1} = \delta(\bar{p}_t - p_t) + \delta(p_t - p_{t-1})$$

and note that with a sustained inflation, such that $\Delta p_t = \Delta p_{t-1}$, a high value of $\Delta p_t$ will keep $\bar{p}_t - p_t$ and--therefore $y_t - \bar{y}_t$--high permanently.

Of more interest, perhaps, is the situation with regard to the wage-price specification, involving overlapping nominal contracts, of John Taylor ((1979, 1980, 1993). This specification is well-known not only from Taylor's own work, but also from its use in econometric models developed and utilized by other researchers (including Gagnon and Tryon (1993) and Masson, Symansky, and Meredith (1990) in the MX3 and Multimod models). In fact, I think it is fair to say that it is currently the leading model of aggregate supply among researchers using estimated econometric models.

A two-period version of Taylor's setup can be used to develop the points at issue. Let $p_t$ be the aggregate price (or wage) index--in log terms--which is an average of the (log) contract prices negotiated in the current and most recent periods, $x_t$ and $x_{t-1}$:

$$(30) \qquad p_t = 0.5\,(x_t + x_{t-1}).$$

Contract prices are set by half of the sellers in period t to keep in step with prices pertaining to the other half of the sellers, with an adjustment added to reflect the effects of excess demand (current and expected):

$$(31) \qquad x_t = 0.5\,(p_t + E_t p_{t+1}) + 0.5\delta E_t(\tilde{y}_t + \tilde{y}_{t+1})$$

where $\tilde{y}_t \equiv y_t - \bar{y}_t$.[21] Together, (30) and (31) imply that

$$(32) \qquad x_t = 0.5\,(x_{t-1} + E_t x_{t+1}) + \delta E_t(\tilde{y}_t + \tilde{y}_{t+1})$$

and thus that

$$(33) \qquad 0 = 0.5\,(E_t \Delta x_{t+1} - \Delta x_t) + \delta E_t(\tilde{y}_t + \tilde{y}_{t+1}).$$

But from the latter we can see that a policy that keeps $\Delta x_{t+1}$ greater than $\Delta x_t$ will keep $y_t - \bar{y}_t$ below zero, on average, if expectations are rational. Thus the Taylor model does not have the natural rate property. It predicts, however, that an accelerating inflation will keep output low--not high, as with the NAIRU model.

Quite recently, Fuhrer and Moore (1993a, 1993b) have developed an aggregate supply specification that can be viewed as a revised version of the Taylor model. Their work suggests that this specification has both theoretical and empirical properties that make it preferable to its predecessor. The Fuhrer-Moore analysis of aggregate supply is developed, however, in the context of a small econometric model whose framework differs from the one presented above, and which does so in an interesting way. Their papers also present the empirical properties of their model in an unorthodox yet interesting fashion. Accordingly, it will be desirable to devote an entire section to the discussion of the Fuhrer-Moore analysis. This constitutes something of a digression, but is useful in preparing the scene for our empirical analysis of the P-bar model by emphasizing the importance of inflation-rate persistence and the use of autocorrelation functions to summarize a model's empirical performance. Readers who dislike digressions could, nevertheless, skip directly to Section V.

## IV. The Fuhrer and Moore Model

A slightly simplified version of the Fuhrer-Moore (F&M) macroeconometric model can be written as follows:

(34) $\rho_t - d(E_t\rho_{t+1} - \rho_t) = r_t - E_t\Delta p_{t+1}$

(35) $\tilde{y}_t = b_0 + b_1\, \rho_{t-1} + b_2\, \tilde{y}_{t-1} + \varepsilon_t$

(36a) $p_t = 0.5(x_t + x_{t-1})$

(36b) $\nu_t = 0.5(x_t - p_t) + 0.5(x_{t-1} - p_{t-1})$

(36c) $x_t - p_t = 0.5(\nu_t + E_t\nu_{t+1}) + 0.5\delta E_t(\tilde{y}_t + \tilde{y}_{t+1})$

(37) $r_t = r_{t-1} + \mu_1\Delta p_{t-1} + \mu_2\tilde{y}_t + e_t.$

Here (34) is a term structure equation that relates the long-term real interest rate $\rho_t$ to the one-period real rate, d>0 being a "duration" parameter (such as 40 for a 10-year real rate in a quarterly model). Equation (35) is F&M's IS specification, with a second lagged $\tilde{y}_t$ term here deleted for clarity. Equation (37) is the model's monetary policy rule, in which it is presumed that the short-term interest rate is used as the policy instrument.

And, finally, equations (36) describe the aggregate supply sector that constitutes a variation on Taylor.[22] The authors' apparent rationale is that Taylor's formula (31) describing contract price determination is replaced with one in which the nominal price of each sector's output is set so as to equate its relative price, $x_t - p_t$, to the average of the <u>other</u> sector's expected relative prices over the life of the contract,[23] with an adjustment (as in Taylor) for current and expected values of $\tilde{y}_t$. In Taylor's setup, by contrast, it is nominal sectoral prices that are related in this fashion.

There is some hint in their exposition that this modification would make better sense theoretically, but the principal advantage claimed by F&M is that it makes the inflation rate, rather than the price level, sticky. One way to see that is to note that in my simplified version

(38) $$x_t - p_t = 0.5\Delta x_t,$$

from (36a), so that $\nu_t = 0.25\ (\Delta x_t + \Delta x_{t-1})$. Putting these in (36c) gives

(39) $$\Delta x_t = 0.5\ \Delta x_{t-1} + 0.5\ E_t\Delta x_{t+1} + 2\delta E_t(\tilde{y}_t + \tilde{y}_{t+1}),$$

which shows that contract price inflation is an average of its own past and expected future values, plus a cyclical adjustment term that will be small empirically, and is therefore sticky. In Taylor's setup, by contrast, we have a similar expression but with contract prices ($x_t$) in place of inflation rates ($\Delta x_t$). Furthermore, with sticky inflation, the F&M model can be shown to match some important features of the U.S. data with much greater accuracy, as we shall see shortly.

My first impression was that the F&M specification also had the advantage of conforming to the natural rate hypothesis. And it is true that $\tilde{y}_t$ cannot be kept permanently away from zero by a constant acceleration, as in the Taylor case discussed above. This can be seen by inspection of (39). But that relation can be rearranged as follows, with $E_t(\cdot)$ operators deleted:

(40) $$0 = 0.5\ (\Delta\Delta x_{t+1} - \Delta\Delta x_t) + 2\delta(\tilde{y}_t + \tilde{y}_{t+1}).$$

From the latter it is clear that an ever-increasing acceleration of inflation, with $\Delta\Delta x_t$ growing over time, will keep the average value of $\tilde{y}_t$ permanently low. So my first impression was incorrect; the F&M specification actually does not satisfy the strict, Lucas (1972) version of the natural rate hypothesis--and this remains true in the slightly more complicated version that is actually used in their papers.

Nevertheless, I find the F&M specification quite interesting and worthy of additional consideration. The same might be said, moreover, for their empirical analysis and other aspects of their model. Let us continue, accordingly, with our discussion of system (34) - (37).

It is striking that this model has no monetary sector, i.e., no money demand function and no mention of a money stock variable. Given that feature, the way that monetary policy works is as follows. The monetary authority's actions in (37) are changes in short-term interest rates.[24] These affect the long-term real rate via (34) and its value feeds into the determination of real aggregate demand in (35). Then demand-induced movements in output affect price determination in the block (36a) - (36c). That description is overly simplified, because it exaggerates the model's recursiveness, but is not seriously misleading. A crucial point is that the system hangs together and permits monetary policy actions to affect $\tilde{y}_t$ and $p_t$ only if the parameter $\delta$ in (36c) is non-zero. If $\delta$ equaled zero, then the three equations (36) would form a self-contained system in the variables $p_t$, $x_t$, and $\nu_t$. In other words, prices would be exogenous. In light of that property of the model, it is rather disturbing that the estimated value of $\delta$ in F&M (1993b) is only 0.007 with a standard error of 0.004.

Be that as it may, let us consider the fact that no money demand equation is included. Of course one--similar to our (4), for example--could be added to the model, in which case it would determine the quantity of money that the monetary authority has to (elastically) supply to conduct its interest-rate-centered policy. But the properties of that money demand function would have no consequences for the behavior of any of the basic variables $\tilde{y}_t$, $p_t$, $r_t$, $\rho_t$, $x_t$, or $\nu_t$. F&M seem to view this property of their setup as a virtue, since it is widely believed that money demand behavior has featured considerable "instability" in recent years. But is this feature theoretically plausible? Strictly speaking, it is not <u>even if</u> one accepts the IS-LM framework for aggregate demand analysis, for properly specified IS functions include wealth terms and real money balances are a component of wealth.[25] Thus a term involving $m_t - p_t$ or $m_{t-1} - p_t$ properly belongs in equation (35), which alters this special property of the F&M model. Their response, presumably, would be that real balance terms are of minor importance quantitatively. But I am not certain that this argument is correct, in the context of their model. For even if the coefficient on $m_t - p_t$ in (34) is small in comparison with the coefficient on $\rho_{t-1}$, it may be the case that $\rho_t$ reacts weakly to policy changes in $r_t$.[26] And if variations in $r_t$ have appreciable effects on $m_t$, via the money demand function, then the impact on aggregate demand via the real balance variable could be of the same order of magnitude as the impact by way of the long term real rate of interest.

The most impressive part of the F&M analysis is the extent to which the pattern of autocorrelations (own and across variables) implied by their estimated model matches those of the U.S. data (or, to be more precise, those of an unconstrained vector autoregression system). These autocorrelation functions are plotted for the three main variables ($\Delta p_t$, $r_t$, and $\tilde{y}_t$) in Figures 1 and 2. There it can be seen that the general qualitative description of the model's implied autocorrelation functions matches those of the unconstrained VAR with impressive accuracy.[27] For the sake of comparison, analogous functions implied by the same framework, except with Taylor-style nominal contracts in place of (36), is shown in Figure 3.[28]

There are a few aspects of the F&M empirical analysis, nevertheless, that are somewhat troubling. One, already mentioned, is the magnitude of the crucial $\delta$ parameter in (36c). Another concerns their choice of variables to be treated as stationary in their VAR system. The results described above are based on a system in which $\Delta p_t$, $r_t$, and $\tilde{y}_t$ are viewed as stationary, although their Dickey-Fuller and Johansen tests for unit roots are somewhat

Figure 1

Autocorrelation Functions, Unconstrained VAR of Fuhrer and Moore (1993a)

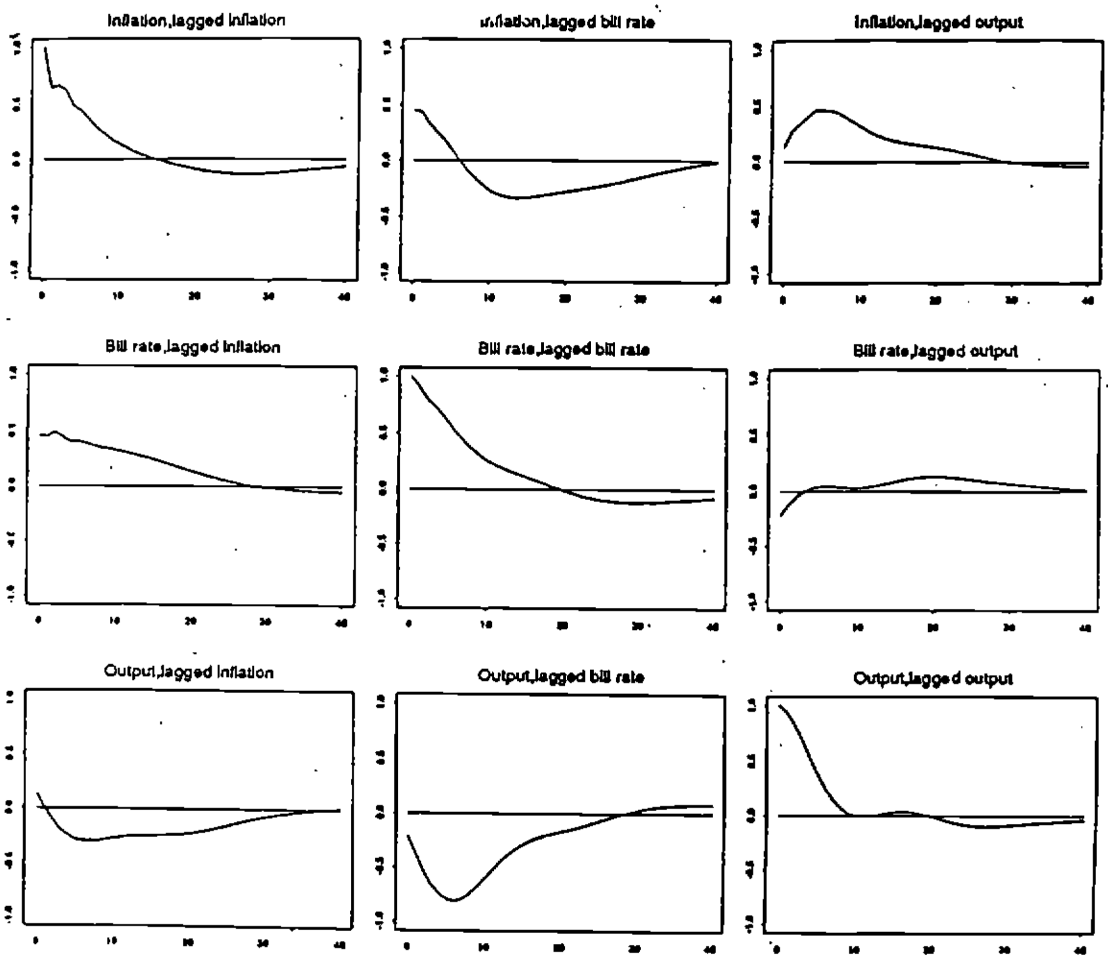


Figure 2

Autocorrelation Functions, Fuhrer-Moore Model

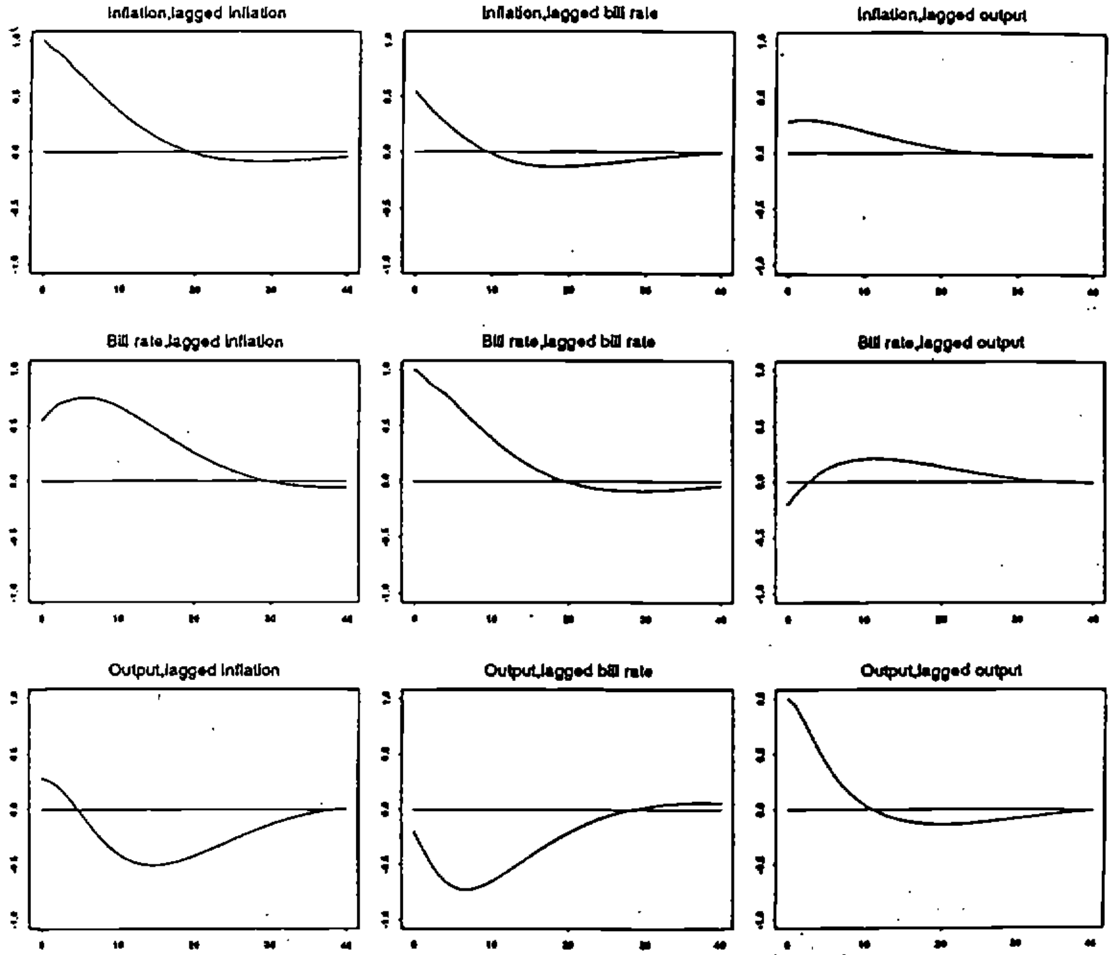
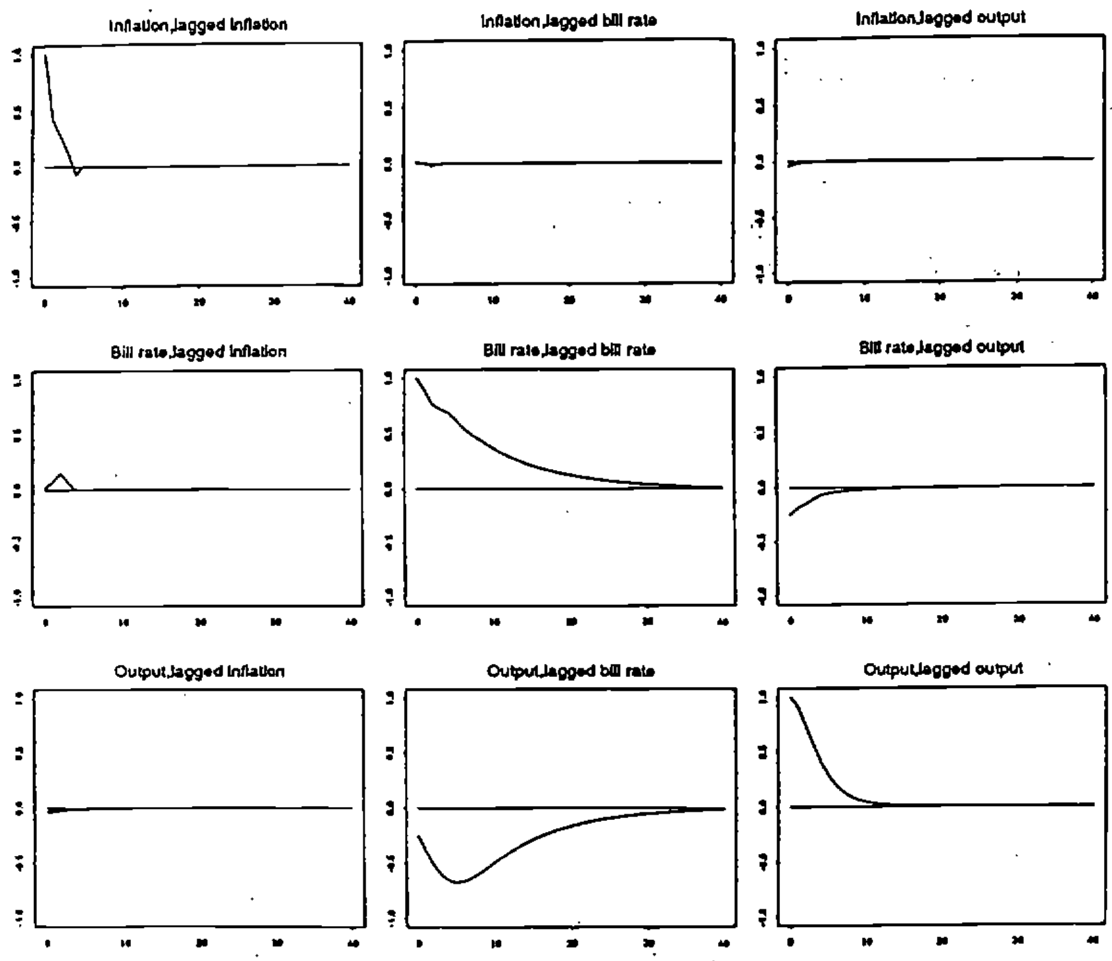


Figure 3

Autocorrelation Functions, Fuhrer-Moore Version of Taylor Model



inconsistent with that specification. I am sympathetic to Cochrane's (1991) argument--that a priori reasoning and general knowledge are more useful than formal tests for unit roots--however, and would accordingly support the F&M stationarity assumption. But I am bothered by their specification of the "normal output" measure--analogous to $\bar{y}_t$--used in constructing $\tilde{y}_t$. In particular, their implied $\bar{y}_t$ is simply a deterministic trend fitted to the $y_t$ data for 1959-1990, i.e., $\tilde{y}_t$ is the deviation from this trend. But any reasonable concept of "normal," or "capacity," or "natural rate" values of $y_t$ would have to recognize that their evolution over time is a consequence of capital accumulation, population growth, and technological progress. All of these processes are ones, however, for which a sizable "permanent" component would be expected, a priori, which would suggest a process for $\bar{y}_t$ that contains a unit root component. This issue will receive additional attention in the next section, where we turn to the empirical properties of the P-bar model.[29]

## V. Empirical Analysis

We now turn to an attempt to evaluate the empirical support, or lack of support, for the P-bar model. An outstanding source of difficulty in this regard is the fact that two crucial variables, $\bar{p}_t$ and $\bar{y}_t$, are unobservable. Of course they are related to each other by equation (9), or its counterpart, so that in principal there is only one variable missing. But that is of little consolation to the researcher, especially since the proper specification of (9) is debatable and its estimation difficult. Another important difficulty is that accurate implementation requires accurate specification of aggregate demand behavior.

The absence of observations on $\bar{y}_t$, or an alternative capacity variable, is also a serious problem for the models of Taylor, F&M, and most others in which price adjustments are sluggish. The problem can typically be glossed over in presentations, because $\bar{y}_t$ does not figure as prominently in the discussion as $\bar{p}_t$ and $\bar{y}_t$ do in ours. But these other models could be quite different empirically if the implicit definition of $\bar{y}_t$ were changed--from a deterministic trend to some unit root specification, for example.

An implication of the P-bar model, or at least the version emphasized in Section II, is that $\tilde{y}_t = y_t - \bar{y}_t$ is a stationary first-order AR process. Let us now consider whether it is possible to reconcile that implication with the presumption--stated above--that $\bar{y}_t$ is generated by a unit root process and also the observed time-series properties of $y_t$. With the latter represented by (the log of) U.S. quarterly data on real GNP, seasonally adjusted, we know that the process is close to

(41) $$(1-0.3L)\Delta y_t = \text{const} + w_t$$

or the nearly-identical trend-stationary model

(42) $$(1-0.35L + 0.30L^2)y_t = \text{trend} + w_t,$$

where $w_t$ denotes a white-noise variate. Actual estimates for 1954.1 - 1990.4 and 1965.1 - 1990.4 are given in Table 1.

Now suppose that the process for $\bar{y}_t$ is $\Delta\bar{y}_t = (1 + 0.65L)u_t$ and that $\tilde{y}_t = 0.95\tilde{y}_{t-1} + \psi_t$. Then for $y_t$ we would have

(43) $$y_t = \bar{y}_t + (1 - 0.95L)^{-1}u_t = \frac{(1 + 0.65L)}{1 - L}u_t + \frac{\psi_t}{1- 0.95L}.$$

Multiplying by $(1 - L)(1 - 0.3L)$ we obtain

(44) $$(1 - 0.3L)(1 - L)y_t = (1 - 0.3L)(1 + 0.65L)u_t + \frac{(1 - 0.3L)(1 - L)}{1 - 0.95L}\psi_t$$

But if we drop all terms in L of higher than first order, the right hand side of the latter becomes

(45) $$(1 + 0.35L)u_t + (1 - 0.35L)\psi_t.$$

Thus if $u_t$ and $\psi_t$ were uncorrelated and had the same variance, (44) would be of the same form as (41). This will not be exactly the case, of course--it cannot be since $u_t$ is a component of $\psi_t$--but it would seem plausible that the specification under discussion would provide a reasonably satisfactory

Table 1

ARMA Models for Real GNP

| AR(1) for $\Delta y_t$ | Sample Period 1954.1 - 1990.4 | 1965.1 - 1990.4 |
|---|---|---|
| Constant | 0.0049 | 0.0050 |
| | (.001) | (.001) |
| AR coefficient | 0.3281 | 0.2837 |
| | (.078) | (.095) |
| $R^2$ | 0.1084 | 0.0796 |
| SE | 0.0095 | 0.0095 |
| DW | 2.07 | 2.05 |

| AR(2) for $y_t$ | | |
|---|---|---|
| Constant | 0.3167 | 0.8840 |
| | (.147) | (.263) |
| Time coeff. | 0.00032 | 0.00078 |
| | (.00016) | (.00024) |
| 1st AR coeff. | 1.3064 | 1.1998 |
| | (.078) | (.094) |
| 2nd AR coeff. | -0.3505 | -0.3220 |
| | (.078) | (.092) |
| $R^2$ | 0.9992 | 0.9979 |
| SE | 0.0094 | 0.0091 |
| DW | 2.10 | 2.14 |

rationalization of the conditions mentioned at the start of this paragraph.

In an attempt to construct an empirical counterpart of the capacity variable, $\bar{y}_t$, I have obtained a quarterly time series for net private nonresidential fixed capital. An annual series on that variable has been published by Musgrave (1992); our quarterly version was constructed by allocating each year's growth in capital to its four quarters in proportion to that quarter's share of the year's gross private nonresidential investment (1987 prices). Values are reported in Appendix A. Let the log of that variable be denoted $k_t$. Then one possible measure of $\bar{y}_t$ would be the fitted value given by the following regression relating $y_t$ to $k_t$:[30]

$$(46) \qquad y_t = \underset{(.060)}{1.539} + \underset{(.0076)}{0.800}k_{t-1} \qquad SE = 0.0371$$

(Here, as in all that follows, the sample period is 1954. - 1990.4.) The time series properties of the implied $\bar{y}_t$ measure are not, however, consistent with the specification used in the previous paragraphs; instead, an AR(2) in $\Delta\bar{y}_t$ is indicated.

A second attempt included $po_t$, the real price of imported oil,[31] as a second explanatory variable.[32] The estimated relation is

$$(47) \qquad y_t = \underset{(.068)}{1.012} + \underset{(.0081)}{0.860}k_{t-1} - \underset{(.0054)}{0.055}po_{t-1} \qquad SE = 0.0282$$

In this case, the implied $\bar{y}_t$ has an ARMA process that may be represented as follows:

$$(48) \qquad \Delta\bar{y}_t = \underset{(.00040)}{0.0072} + u_t + \underset{(.074)}{0.607}u_{t-1}$$

$$R^2 = 0.242 \qquad SE = 0.0049 \qquad DW = 1.87$$

Thus it transpires that a MA(1) process with MA coefficient of 0.607 fits the data quite well. And this provides a good match to the specification assumed in the construction of equations (43) - (45). The implied $\tilde{y}_t$ measure, labelled YGAPKO, is plotted in Figure 4 (together with the measure YGAP54 implied by a linear deterministic trend for $\bar{y}_t$).

Does the YGAPKO measure of $\tilde{y}_t = y_t - \bar{y}_t$ have reasonable properties for use with our P-bar model? One positive characteristic is that the Dickey-Fuller test statistic (constant, no trend, 2 lags) is -3.39, easily adequate to reject a unit-root null hypothesis at the 0.05 significance level. (The MacKinnon critical value is -2.88.) A 0.05 rejection can barely be obtained for the measure implied by (45), whose t-value is -2.92, and the $\tilde{y}_t$ measure implied by a fitted deterministic trend for $\bar{y}_t$ does not permit rejection (t value = -2.38) at all. On the other hand, the implied ARMA process for YGAPKO is not an AR(1). Instead, the following is implied:

(49) $$\underset{}{\tilde{y}_t} = \underset{(.0008)}{0.00012} + \underset{(.078)}{1.245}\tilde{y}_{t-1} - \underset{(.078)}{0.350}\tilde{y}_{t-2}$$

$$R^2 = 0.870 \qquad SE = 0.010 \qquad DW = 2.01$$

Thus the YGAPKO specification is not fully consistent with the P-bar model. Whether that is because the capacity measure is unsatisfactory or because the model is flawed cannot be determined from this one mismatch. In any event, the mismatch is not extremely severe. In what follows, accordingly, the YGAPKO measure of $\tilde{y}_t$ will be utilized, for a lack of anything better, wherever such a measure is required.

Next we turn our attention to the aggregate demand portion of the model. Estimates for a few alternative specifications in first-differenced form are presented in Table 2. In all of these, the (St. Louis, adjusted) monetary base is used as the money stock variable. Both OLS and TSLS estimates are presented, the instruments for the latter including two lagged values of $\Delta y_t$, $\Delta m_t$, and $\Delta p_t$. For some reason, the parameter estimates for both $\beta_1$ and $\beta_2$ are larger when the TSLS procedure is used, and most t statistics are increased even though overall explanatory power is (as it must be) lower. The various estimates of $\beta_3$ are all close to 0.3 and those of $\beta_1$ are all in the general range of 0.25 - 0.55. The magnitude of $\beta_2$ is not pinned down at

Figure 4

Constructed Measure of Market-Clearing Output

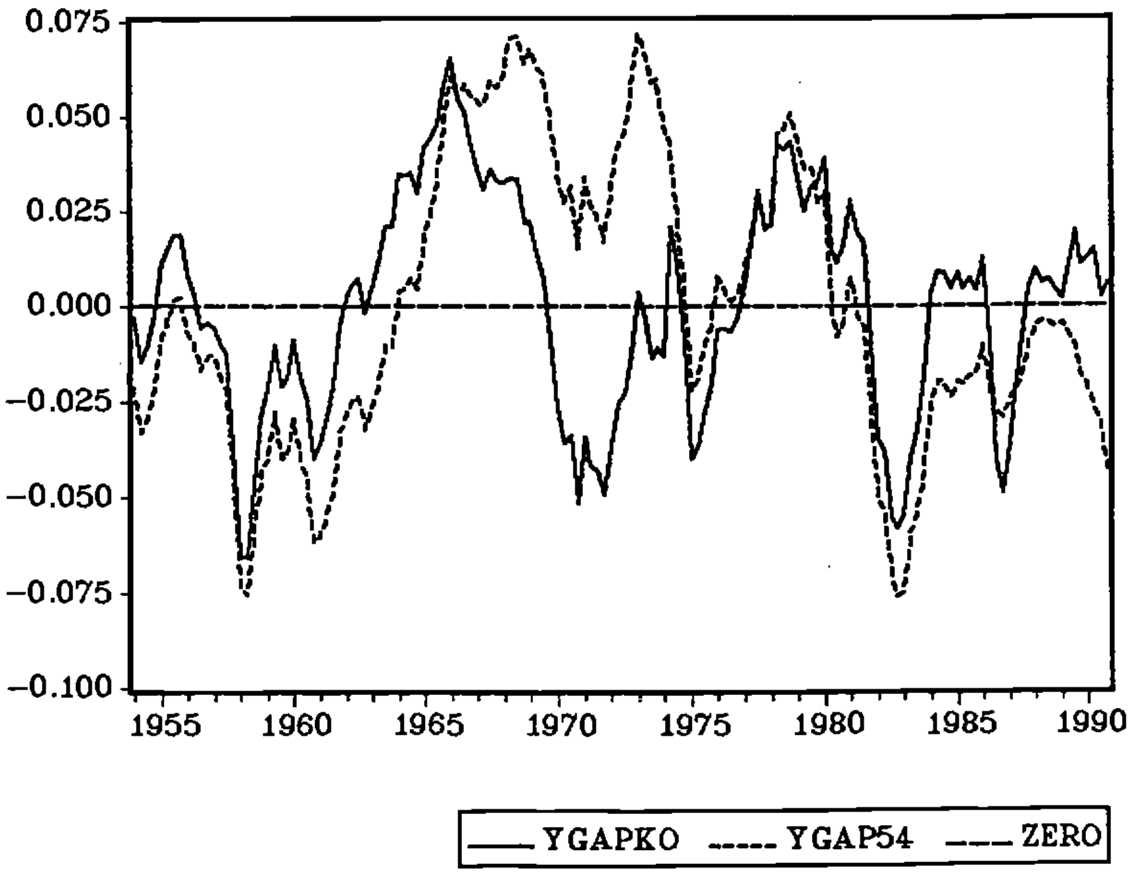


Table 2

Estimates of Aggregate Demand Relation (4)

Dependent Variable is $\Delta y_t$, Sample Period 1954.1 - 1990.4

| Estimates (std. errors) attached to | OLS | TSLS[a] | OLS | TSLS[a] |
|---|---|---|---|---|
| Constant | 0.0042 | 0.0036 | 0.0043 | 0.0038 |
| | (.0010) | (.0010) | (.0010) | (.0011) |
| $\Delta m_t - \Delta p_t$ | 0.294 | 0.552 | 0.276 | 0.505 |
| | (.097) | (.168) | (.106) | (.205) |
| $\Delta(p_{t+1} - p_t)$ | | | 0.077 | 0.812 |
| | | | (.179) | (1.59) |
| $\Delta y_{t-1}$ | 0.301 | 0.278 | 0.302 | 0.279 |
| | (.076) | (.079) | (.077) | (.086) |
| **Statistics** | | | | |
| $R^2$ | 0.161 | 0.121 | 0.163 | -0.035 |
| SE | 0.0092 | 0.0094 | 0.0092 | 0.0103 |
| DW | 2.10 | 2.01 | 2.11 | 2.17 |

[a] Instruments based on constant, $\Delta y_{t-1}$, $\Delta y_{t-2}$, $\Delta m_{t-1}$, $\Delta m_{t-2}$, $\Delta p_{t-1}$, $\Delta p_{t-2}$.

all, however, by the results in Table 2.[33]

An alternative method of estimating $\beta_2$ is to recognize that equations (4)-(6) imply that $\beta_2 = -a_1c_2/(a_1c_1 + c_2)$ and $\beta_1 = a_1/(a_1c_1 + c_2)$. Therefore $\beta_2/\beta_1 = -c_2$, where $c_2$ is the semi-elasticity of money demand with respect to the short-term rate of interest. Let us then estimate a money-demand function, taking care to express $r_t$ in units comparable to $\Delta \log p_t$--i.e., as annualized percentage points divided by 400. The result is as follows:

$$(50) \qquad \Delta m_t - \Delta p_t = \underset{(.0007)}{0.00052} + \underset{(.0591)}{0.119}\Delta y_t - \underset{(.269)}{0.564}r_t + \underset{(.071)}{0.547}(\Delta m_{t-1} - \Delta p_{t-1})$$

$$R^2 = 0.351 \qquad SE = 0.0064 \qquad DW = 2.25$$

Thus the suggested value for $\beta_2/\beta_1$ is about 0.564. If $m_t$ is measured by M1 rather than the monetary base, however, the estimated value of $\beta_2/\beta_1$ jumps to 1.007. So, this approach suggests that $\beta_2$ is of the same order of magnitude as $\beta_1$--say, about 0.3 - 0.4--but probably somewhat smaller.

It would be entirely reasonable to wonder about the neglect of fiscal policy variables in the aggregate demand relation (4). Indeed, the rate of government purchases would be expected to enter in the IS relation (5) and therefore in (4) even if the economy were one in which Ricardian equivalence prevailed. Accordingly, estimates were obtained for versions of the relations appearing in columns one and three of Table 2 but with $\Delta g_t$ and $\Delta g_{t-1}$ also included, $g_t$ being the log of state, local, and federal government purchases. In column one, the estimated parameter values (standard errors) are 0.094 (.054) and -0.097(.055) and for column three the corresponding figures are almost exactly the same. In both cases, the hypothesis that the $\Delta g_t$ terms jointly provide no explanatory power cannot be rejected at the 0.05 significance level. Consequently, it seems unlikely tht our omission of the government spending variables, which was adopted in Section II to keep the model clean and simple, is a significant flaw.

Another behavioral relation that is needed to assemble a quantitative version of the P-bar model is a policy rule for the generation of $\Delta m_t$. For our sample period, the following AR(3) model matches the data rather well:

$$\Delta m_t = \underset{(.008)}{0.00214} + \underset{(.079)}{0.562}\Delta m_{t-1} + \underset{(.092)}{0.0009}\Delta m_{t-2} + \underset{(.079)}{0.297}\Delta m_{t-3} \tag{51}$$

$$R^2 = 0.633 \qquad SE = 0.0047 \qquad DW = 2.00$$

Moreover, lagged values of $\Delta p_t$ and $\tilde{y}_t$ provide no explanatory power and do not upset the estimates in (51). From this equation, accordingly, we can calculate the expected value $E_{t-1}\Delta m_t$.

It is now finally time to turn to the price adjustment equation that is our principle object of concern. For empirical purposes we shall focus on a version, analogous to equation (17), which may be written as follows:

$$\Delta p_t = \gamma_1\tilde{y}_{t-1} + \frac{1}{\beta_1 + \beta_2}[\beta_1 E_{t-1}\Delta m_t + \beta_3\Delta y_{t-1} - E_{t-1}\Delta\bar{y}_t]. \tag{52}$$

In estimating (52), we use predicted values from (51) and (48) to represent the indicated expectations. Results are as follows:

$$\Delta p_t = \underset{(.0016)}{0.0080} + \underset{(.016)}{0.0505}\tilde{y}_{t-1} + \underset{(.074)}{0.556}E_{t-1}\Delta m_t - \underset{(.046)}{0.148}\Delta y_{t-1} - \underset{(.162)}{0.500}E_{t-1}\Delta\bar{y}_t \tag{53}$$

$$R^2 = 0.368 \qquad SE = 0.0053 \qquad DW = 0.87$$

In terms of support provided for the P-bar model, these figures must be regarded as mixed. Unsatisfactory elements are the residual autocorrelation indicated by DW and the wrong-signed coefficient on $\Delta y_{t-1}$. Quite favorable, however, are the magnitudes of the coefficients attached to the key variables, $\tilde{y}_{t-1}$ and $E_{t-1}\Delta m_t$. With regard to the former, the implication is that departures of $y_t$ from $\bar{y}_t$ lead to price level adjustments that come about slowly but surely; the parameter estimate is over three times the size of its standard error. And regarding $E_{t-1}\Delta m_t$, the estimated parameter value agrees nicely with the magnitude implied by our estimated value of $\beta_2/\beta_1$: with the latter ranging from 0.56 to 1.0, the implied value for $\beta_1/(\beta_1 + \beta_2)$ ranges from 0.64 to 0.50.

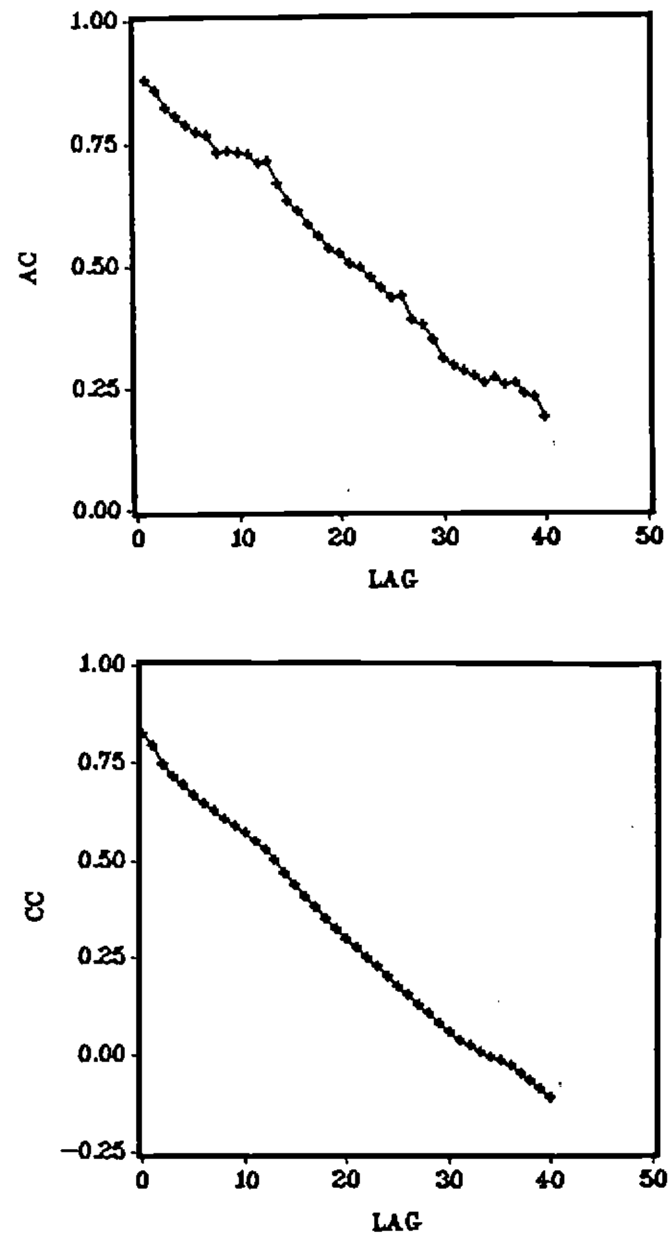
Simply for the sake of comparison, consider also a version of (53) estimated with $\tilde{y}_{t-1}$ defined relative to a deterministic trend. In this case, the coefficient on that variable rises to 0.066 (.012) and the one attached to $E_{t-1}\Delta m_t$ falls to 0.456 (.074). In addition, when an equation explaining $\Delta p_t$ with four lagged values of itself and our preferred measure of $\tilde{y}_{t-1}$ is estimated, the coefficient on the latter remains significantly positive and residual autocorrelation is eliminated.[34]

In principle, it would be interesting to use the estimated model to explore the system's autocorrelation structure in the manner of Fuhrer and Moore. A direct comparison with the F&M results would not be appropriate, however, because of three differences between their estimated system and ours. These are: (i) Different monetary policy instruments are used. (ii) The F&M policy rule, unlike ours, features policy responses designed to stabilize output and inflation--to keep $\tilde{y}_t$ and $\Delta p_t$ reasonably close to target values. (iii) The F&M aggregate demand sector, unlike ours, is specified in terms of $\tilde{y}_t$ rather than $y_t$. That seems inappropriate theoretically and makes it easier to generate autocorrelation functions relating to $\tilde{y}_t$ that are close to those present in the actual data.

Nevertheless, one simulation was conducted using a system composed of equation (52) and column 1 of Table 2. With $m_t$ and $\bar{y}_t$ given by their historical values, this system was simulated, starting with actual values for 1954.1, to generate time paths for $p_t$ and $y_t$. From the artificial data thusly obtained, it is possible to calculate implied autocorrelation functions. The most interesting ones, given the three properties (i) (ii) (iii) listed above, pertain to autocorrelations of $\Delta p_t$ (inflation) with past values of $\tilde{y}_t$ and of itself. The resulting patterns are shown in Figure 5. From the latter it may be seen that the inflation rate's correlations with $\tilde{y}_{t-j}$ have much the same general pattern as in Figure 2, except that the

Figure 5

Inflation Autocorrelation Functions for P-bar Model




AC denotes the autocorrelation coefficients, at lags 1-40, of the inflation rate $\Delta p_t$ while CC denotes cross correlations of $\Delta p_t$ with $\tilde{y}_{t-j}$ for $j$ = 0-40.

magnitude peaks in the initial quarter. Second, the inflation rate's correlations with past values of itself die out more slowly than in Figure 2. Our estimated system exhibits, then, even more inflation persistence than that of F&M.

## VI. Theoretical Issues

Before concluding, we need to briefly consider a few theoretical issues concerning the specification of the P-bar model. The first of these pertains to an apparent distinction between the Barro-Grossman (1976) and Mussa (1981a, 1981b) versions of the P-bar variable--a distinction that is the principal topic of a paper by Obstfeld and Rogoff (1984). Using $\tilde{p}$ to denote the Barro-Grossman concept, Obstfeld and Rogoff state that "The difference between $\bar{p}$ and $\tilde{p}$ deserves emphasis. $\tilde{p}$ is the output price that would prevail in a hypothetical Walrasian general equilibrium with fully flexible prices [whereas] $\bar{p}$ is the output price that would clear the goods market given current levels of the sticky-price system's endogenous variables" (1984, p. 164). By contrast, McCallum (1979, p. 1; 1980, p. 733) evidently sees no important difference in the two specifications. His reasoning is, presumably, as follows.

In a typical application, the only feature of the model that keeps it from being one of the Walrasian type, with fully flexible prices, is the sticky-price adjustment mechanism under consideration. The system's endogenous variables will differ from their Walrasian equilibrium values, then, only to the extent that the prevailing price level $p_t$ differs from $\tilde{p}_t$. In a linear system, furthermore, the difference $z_t - \tilde{z}_t$ between the prevailing and Walrasian values of any variable z will be proportionate to $p_t - \tilde{p}_t$. But $p_t - \bar{p}_t$ will therefore be some linear combination of variables each of which is proportional to $p_t - \tilde{p}_t$, so $p_t - \bar{p}_t$ will itself be proportional to $p_t - \tilde{p}_t$. In this sense, then, there is no significant operational distinction between $\bar{p}_t$ and $\tilde{p}_t$ if the model being utilized is linear.

The foregoing line of reasoning is consistent, it should be noted, with the main result derived by Obstfeld and Rogoff (1984), who utilize a linear system. Specifically, the Obstfeld-Rogoff theorem involving their equation (13) asserts that the Barro-Grossman and Mussa schemes "yield structurally equivalent...models" when the slope in the Barro-Grossman adjustment rule is not too large (1984, p. 165). But, furthermore, when the Barro-Grossman price adjustment mechanism "is interpreted properly," that slope condition always holds and the "apparent convergence problem disappears" (1984, p. 166).

The second theoretical issue to be considered pertains to the specification of $\bar{p}_t$. Mussa (1981a, 1981b) and Obstfeld-Rogoff (1984) both work in continuous-time settings, and the latter authors use[35] a definition of $\bar{p}_t$ that would, in our (2)(4) setup of Section II, imply that

(54) $$\bar{p}_t = \frac{1}{\beta_1} [\beta_0 + \beta_1 m_t + \beta_2 E_{t-1}(p_{t+1} - p_t) + \beta_3 y_{t-1} + v_t - \bar{y}_t]$$

rather than (9).[36] Here the expectation of $p_{t+1} - p_t$ appears on the right-hand side (rhs), instead of the expectation of $p_{t+1}$, and the rhs denominator is correspondingly affected. The idea, evidently, is that the inflation rate is a variable that is independent of the current price level. But while the price level and the inflation rate are certainly conceptually distinct, it is not clear that they should be treated as statistically independent. The issue seems to be whether or not $E_{t-1}(p_{t+1} - p_t)$ is affected by shocks that affect $p_t$. (In the present model, these shocks are dated t-1 but that is beside the point.) And while there are models in which $E_{t-1}\, p_{t+1}$ moves in tandem with $E_{t-1}p_t$, they are rather special models. Consequently, in the present paper (9) has been used rather than (54) in the discussion of Section II.

It may be useful, in this regard, to note that with definition (54) for $\bar{p}_t$, and under the special assumptions used to develop (16), we would have

(55) $$E_{t-1}\Delta\bar{p}_t = \mu_0 + \mu_1\Delta m_{t-1} + (\theta/\beta_1)\, u_{t-1}$$

and inflation would be given as

(56) $$\Delta p_t = \gamma_1(y_{t-1} - \bar{y}_{t-1}) + \mu_0 + \mu_1\Delta m_{t-1} + \beta_3\Delta y_{t-1} + (\theta/\beta_1)\, u_{t-1}.$$

The main point of interest, in a comparison between (56) and (17), is that the coefficient attached to $\Delta m_{t-1}$ may be considerably smaller in the latter, depending on the relative magnitudes of $\beta_1$ and $\beta_2$. In particular, $E_{t-1}\Delta m_t$ does not enter with a unit coefficient when the $\bar{p}_t$ definition of Section II is utilized.

A third issue concerns the fact that monetary policy is actually implemented, in the U.S. and elsewhere, by manipulation of a short-term interest rate such as $r_t$ in equations (5) and (6), rather than some monetary aggregate such as $m_t$. The question, then, is whether this fact makes expressions such as (4), (7), (8), (9), and especially (12) inappropriate. For the macroeconomic model discussed above can be thought of as including (2)(4)(6)(9) and a policy rule for $r_t$ instead of (2)(4)(5)(9) and a policy rule for $m_t$. But either way, the system determines values for $y_t$, $p_t$, $\bar{p}_t$, $r_t$, and $m_t$--and it can be seen that the private sector behavioral equations are equivalent in these two cases. Furthermore, derivation of the solution equation (12) for $y_t$ does not depend on the specification of the policy process for $m_t$ in the analysis of Section II. Thus the only difference is that it cannot be assumed that $m_t - E_{t-1}m_t$ is independent of private sector disturbances when $r_t$ is the instrument; instead, $m_t - E_{t-1}m_t$ will reflect such disturbances as well as the unexpected component of the policy rule that in that case pertains to $r_t$.

## VI. Conclusions

In the foregoing sections we have explored the P-bar model of price level adjustment, which postulates that price changes occur so as to (i) gradually eliminate discrepancies between actual and market-clearing values and to (ii) reflect expected changes in market-clearing values themselves. This model has implications that are more "classical" than most alternative formulations that reflect gradual price adjustment; in particular, it satisfies the natural rate hypothesis. With some informational structures it will also satisfy the policy ineffectiveness proposition, but such a result does not hold in general and can not be presumed.

Empirical implementation is hampered by the absence of any reliable measure of the economy's market clearing or natural-rate value of output. Also, accurate implementation requires accurate specification and estimation of the economy's aggregate demand behavior. A set of results is developed, nevertheless, utilizing a proxy for market-clearing output that is a log-linear function of a measure of fixed capital and the real price of oil. The results are moderately encouraging but not entirely supportive. Two highly positive aspects of the results are that they (i) indicate a plausible rate of price level adjustment in response to recent output levels and (ii) imply an effect of expected money growth that is consistent with the model.

The P-bar model's properties are compared with those of a specification recently proposed by Fuhrer and Moore (1993a, 1993b). The latter does not satisfy the natural rate hypothesis but appears to perform very well empirically. That last conclusion rests, however, on analysis that questionably treats the market-clearing output rate as a deterministic trend.

Appendix A

Net Private Nonresidential Fixed Capital

End of Quarter, $Billion, 1987 Prices

| obs | CAP | | | |
|---|---|---|---|---|
| 1948 | 1027.525 | 1041.450 | 1055.409 | 1069.800 |
| 1949 | 1080.364 | 1090.339 | 1099.785 | 1109.000 |
| 1950 | 1119.414 | 1130.792 | 1143.118 | 1155.500 |
| 1951 | 1167.697 | 1180.266 | 1193.074 | 1205.700 |
| 1952 | 1217.019 | 1228.406 | 1238.883 | 1250.100 |
| 1953 | 1262.464 | 1274.845 | 1287.440 | 1300.000 |
| 1954 | 1310.636 | 1321.203 | 1331.917 | 1342.500 |
| 1955 | 1354.487 | 1367.195 | 1380.503 | 1394.300 |
| 1956 | 1409.297 | 1424.454 | 1439.762 | 1454.900 |
| 1957 | 1469.000 | 1483.012 | 1497.331 | 1511.300 |
| 1958 | 1520.001 | 1528.296 | 1536.376 | 1544.700 |
| 1959 | 1555.171 | 1565.983 | 1577.091 | 1588.200 |
| 1960 | 1600.552 | 1612.926 | 1625.017 | 1637.100 |
| 1961 | 1648.265 | 1659.552 | 1670.910 | 1682.600 |
| 1962 | 1696.094 | 1710.060 | 1724.198 | 1738.100 |
| 1963 | 1752.229 | 1766.809 | 1781.816 | 1797.200 |
| 1964 | 1815.692 | 1834.759 | 1854.381 | 1874.500 |
| 1965 | 1899.707 | 1926.018 | 1953.151 | 1981.400 |
| 1966 | 2011.370 | 2041.738 | 2072.313 | 2102.500 |
| 1967 | 2129.295 | 2156.068 | 2182.743 | 2209.899 |
| 1968 | 2237.624 | 2264.731 | 2292.065 | 2320.300 |
| 1969 | 2349.641 | 2379.193 | 2409.395 | 2439.300 |
| 1970 | 2465.560 | 2491.662 | 2518.021 | 2543.600 |
| 1971 | 2566.515 | 2589.510 | 2612.416 | 2635.500 |
| 1972 | 2659.722 | 2684.334 | 2709.280 | 2735.700 |
| 1973 | 2768.853 | 2803.335 | 2838.459 | 2873.900 |
| 1974 | 2906.255 | 2938.641 | 2970.159 | 3000.700 |
| 1975 | 3020.270 | 3039.298 | 3058.463 | 3077.800 |
| 1976 | 3096.010 | 3114.343 | 3133.058 | 3152.100 |
| 1977 | 3176.085 | 3200.756 | 3225.997 | 3251.900 |
| 1978 | 3283.682 | 3317.720 | 3352.301 | 3387.500 |
| 1979 | 3425.806 | 3464.231 | 3503.586 | 3543.000 |
| 1980 | 3578.239 | 3611.281 | 3644.101 | 3677.400 |
| 1981 | 3709.902 | 3743.212 | 3777.146 | 3810.600 |
| 1982 | 3834.346 | 3857.018 | 3878.984 | 3900.600 |
| 1983 | 3917.057 | 3933.861 | 3951.392 | 3970.200 |
| 1984 | 4000.073 | 4031.486 | 4063.719 | 4096.800 |
| 1985 | 4134.108 | 4172.216 | 4209.434 | 4247.800 |
| 1986 | 4276.876 | 4305.318 | 4333.253 | 4361.500 |
| 1987 | 4384.419 | 4408.003 | 4432.533 | 4457.100 |
| 1988 | 4482.644 | 4509.134 | 4535.732 | 4561.899 |
| 1989 | 4585.375 | 4609.249 | 4633.490 | 4657.500 |
| 1990 | 4678.690 | 4699.626 | 4721.011 | 4742.399 |

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## Footnotes

[1]These include McCallum (1979a, 1980, 1982) and others. The 1980 JMCB piece includes a derivation/justification that is quite different from Mussa's. It will be briefly described below.

[2]The logic of Mussa's formulation has been sharply questioned by Rotemberg (1982).

[3]The presence of inventory holdings would lead to a relaxation of this assumption, which would become that demand is fully satisfied by production or inventory draw-down. That modification would complicate the analysis without altering its essential features. For a theoretical study that uses a special version of the P-bar model with explicit recognition of inventories, see Flood and Hodrick (1986).

[4]Adjustment costs are taken to depend on output relative to capacity, rather than output alone, to reflect the presumption that such costs would not be incurred in response to technological improvements that increase output attainable with given quantities of labor.

[5]For an explicit algebraic treatment of this argument, see McCallum (1980, pp. 773-4). A useful special case obtains when between-priced charges are not costly. Then $\gamma_1 = 0$ and $p_t = E_{t-1}\bar{p}_t$, i.e., prices are set at levels that are expected to be market-clearing. This case has been used by Flood (1981) and McCallum (1989, Ch. 10).

[6]The unsatisfactory feature of this specification would not be eliminated, moreover, by the addition of a stochastic disturbance term to the adjustment equation.

[7]By considering a case that works against the conclusions being reached.

[8]Allan Meltzer has suggested that non-recognition of inventory fluctuations might be significantly detrimental to the model's empirical performance. That is certainly possible but there is no apparent reason why it should be more relevant for this model of price adjustment than for any other.

[9]Actually, a slightly more restrictive case with $b_2 = \beta_3 = 0$ was used in my previous papers.

[10]It would be theoretically appropriate to include a government spending variable in (5), which would then also show up in (4). This possibility will be investigated empirically in Section V.

[11]It is also the case that additional terms, such as $m_t - p_t$ in (5) or its lagged value in (6), might be expected to appear. But they would not have major effects on the properties of the model.

[12]See the next paragraph.

[13]This will be demonstrated momentarily, in equations (21)-(29).

[14]Thus we are implicitly omitting any "real-balance" term from the IS function for simplicity. It will, however, be argued below in Section IV that such an omission is theoretically inappropriate.

[15]The second property will not obtain, however, if monetary policy shocks include both permanent and transitory components, which private agents are unable to observe separately. This point was emphasized by Brunner, Cukierman, and Meltzer (1983).

[16]Here MA(1) means first-order moving average. Note that the implied assumption is not that $\bar{y}_t$ can be expressed as the sum of two components, one purely permanent (a random walk) and one purely transitory (white noise), for this requires that $\theta<0$. The reason for specifying $\theta>0$ is the empirical evidence to be discussed in Section V.

[17]Even in the simplest conceivable case, with $\bar{y}_t = \bar{y}$ and $v_t = \xi_t = u_t = 0$, we find that $E_t p_{t+1} - E_{t-1} p_{t+1} = e_t \beta_1 [\mu_1(\beta_1 + \beta_2) - (1-\gamma_1(\beta_1+\beta_2)) \beta_1(1+\beta_2)]/(\beta_1 + \beta_2)[\beta_1 + \beta_2(1-u_1)]$.

[18]It would not be negated by changes in the specification of aggregate demand behavior or the stochastic properties of shocks.

[19]In other words, the change in the inflation rate. The present point was argued in McCallum (1982).

[20]The difference is that the MPS model uses a variable hereby denoted $p_t^*$ in place of $\bar{p}_t$, with $p_t^*$ given by a markup over unit production cost at "normal" capacity levels. For more information, see McCallum (1979b).

[21]From here on, the tilde symbol will be used with this meaning, not the one mentioned in the first paragraph of this section.

[22]Here I have simplified F&M's specification by using two-period (rather than four-period) contracts and by setting the weights on $x_t$ and $x_{t-1}$ in (39a) at 0.5, rather than estimating such weights empirically.

[23]That is, $x_t - p_t = 0.5\ [0.5(x_t - p_t + x_{t-1} - p_{t-1}) + 0.5\ (x_{t+1} - p_{t+1} + x_t - p_t)]$ implies $0.5\ (x_t - p_t) = 0.25\ (x_{t-1} - p_{t-1}) + 0.25\ (x_{t+1} - p_{t+1})$.

[24]The term "interest rates" will refer to nominal rates unless the word "real" is also included.

[25]In a Sidrauski-style model with explicit optimization on the part of infinite-lived households, a household's choices in t depend upon its asset holdings at the end of t-1.

[26]The F&M specification implies that the long-term nominal rate $R_t$ is related to the short-term nominal rate by $R_t = (1+d)^{-1}\ r_t + (1+d)^{-1}\ E_t R_{t+1}$, where d is a number such as 40.

[27] It should perhaps be noted explicitly that the autocorrelation functions emphasized by F&M are conceptually quite different from VAR-based impulse response functions, such as those featured by Cochrane (1994) and many others.

[28] It may be the case that Taylor's own version of his model provides a much better match. In any event, this discussion should not be interpreted as disrespective of Taylor's work, which has been extremely valuable.

[29] One more item needs to be mentioned before continuing. That is the empirical relationship between the short-term nominal rate of interest and the long-term real rate of interest that is the central topic in Fuhrer and Moore (1993b). What they find is that these two variables have moved together quite closely over the 1965-1990 time period, a correlation that they note is in theory dependent upon the manner in which policy is conducted. The relationship reported is $\rho_t = 0.006 + 0.23r_t$. This is evidently an OLS regression estimate. No standard error, DW, or $R^2$ statistics are reported, but the plot of $\rho_t$ and its fitted value suggest that $R^2$ would be quite high and DW very low.

[30] Test results are consistent with this being a cointegrating relationship.

[31] For observations since 1966, the nominal price is obtained as the ratio of nominal to real imports of petroleum products as reported in the National Income and Product Accounts. For 1954-1966, the Producer Price Index (i.e., WPI) series for crude petroleum was spliced on. Then quarterly averages of monthly observations were calculated. The complete series was finally divided by the GNP deflator (1982 = 100). All series are seasonally adjusted.

[32] The basic idea, of course, is that with the United States being a net importer of oil, total output (net of goods traded to obtain oil used as an input) will be smaller the higher is the real price of oil.

[33] OLS estimation will not provide consistent estimates of $\beta_2$, even in principle.

[34] At the time of the conference, Jeff Fuhrer pointed out that the estimated version (54) of the P-bar price adjustment equation does not satisfy the NRH. That is true, as can be seen by setting $\Delta y_{t-1} = 0$ and $\Delta \bar{y}_t = 0$ in (54), but only because the estimated equation (17) omits the term $\beta_2$ $(E_{t-1}\ p_{t+1} - E_{t-2}p_t)$ that is part of the model. The reason for this omission is that identification of the term seems almost hopeless, given the different information sets relevant to the two expectations. Thus the omission must be judged a weakness of the paper's empirical work, but does not reflect any discredit on the P-bar theory.

[35] See equation (12) of Obstfeld and Rogoff (1984).

[36] It is unclear to me which of these definitions Mussa intends to adopt.