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FLEXIBILITY: A PARTIAL ORDERING

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ABSTRACT

We use an approach developed by Krishna and Young to examine the ability of economies to adjust to exogenous shocks. While, in general, economies cannot be ranked in terms of their flexibility, we provide a partial ordering for certain types of economies. In particular, properties of the revenue function are used to show that placing restrictions on factor mobility and on production in certain sectors reduces the flexibility of a small open economy with respect to all price, endowment, and technology shocks of small enough magnitude. Since one can think of these restrictions as distortions, we would expect them to reduce the level of GNP in the economy. The insight provided by the analysis of flexibility is that, not only is the level of GNP affected, but also the intrinsic ability of the economy to adjust to shocks is reduced.

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1. Introduction

The term "flexibility" in the Oxford English dictionary is defined as adaptability to various purposes. In economic usages it carries connotations of an ability to adjust. The flexibility of an economy is often thought of as a desirable characteristic as a flexible economy is thought of as being able to adapt its resource use, and change its production patterns, in response to exogenous shocks more than an inflexible one. The presumption is that this responsiveness to exogenous shocks would then result in greater national income and could foster growth. For example, Young (1989) argues that it was the ability of Hong Kong to adapt to changing opportunities in the world market that enabled it to grow much faster than other similar countries.

Yet, the term is relatively badly defined in the literature. In common usage, there is a tendency to confuse actual adjustment of an economy, as reflected in changes in its resource allocations and thereby in its production composition, with the ability to adjust. For example, Young (1989) measures flexibility by computing an index of change in value of output from different sectors. In labor economics, a measure of actual adjustment often used is an index of gross job creation and destruction as in Davis and Haltiwanger (1992). While such measures are related to actual adjustment, it says little about the ability to adjust. The adjustment may have occurred because the economy was subjected to shocks. Another economy with less actual adjustment may really be more intrinsically flexible if it was not subject to significant shocks and hence did not need to adjust. Moreover, one economy may respond more than another to a given shock, given one set of base conditions, and less at some other set of base conditions. Thus, flexibility must, in general, be defined relative to a given shock and at some given existing situation.

This distinction between ex-ante and ex-post measures is essential. We shall refer to measures of the former as measures of flexibility, and the latter as measures of adjustment. In this paper we are concerned with measures of flexibility.

We adopt an approach to flexibility recently developed by Krishna and Young (1992) in our analysis of the ability of different types of economies to adjust to external shocks. Their approach is a dual one in which flexibility can only be defined relative to given shocks and for given initial conditions. Their definition is based on a metric which defines the distance from a baseline economy which is either perfectly flexible or perfectly inflexible. It provides a microeconomically well based definition of flexibility. It is appealing because it relates flexibility to the responsiveness of the economy, as measured by the derivatives of the revenue function, to exogenous shocks.[1] They develop measures of flexibility in response to price, factor endowment, and technology shocks, and they point out that an economy can be flexible in response to one kind of shock and not another. For example, they show that the standard Heckscher Ohlin 2 x 2 x 2 model with fixed coefficients in production and incomplete specialization provides an example of an economy which is perfectly inflexible in its response to price shocks, but perfectly flexible in response to endowment shocks. It is empirically implementable by estimating the revenue function, and so provides a basis for empirical estimates of flexibility that are easily interpreted. The approach is particularly fitting for this paper since Chipman (1972) was one of the early papers examining the propertities of the revenue function.

However, since Krishna and Young's measure is dependent on the shock itself as well as the initial conditions, an implication is that economies cannot in general be ordered in terms of their flexibility. We build on their work and show that some <u>kinds</u> of economies can be so ordered. We thus provide a <u>partial</u> ordering of such economies in terms of their flexibility. In particular, we argue that permitting capital mobility in a baseline economy enhances the flexibility of an economy in response to all price, endowment, or technology shocks, which are "small enough" in magnitude. We then argue that command economies, that target production levels in some, but not in all, sectors similarly reduce the flexibility of the baseline economy.

It is worth emphasizing at the outset that the approach used here and in Krishna and Young (1992) is static and atemporal. The measure of flexibility does not capture another common usage of the term, namely that a flexible economy adjusts faster to any given shock than an inflexible one. This interpretation of flexibility is certainly a valid one, but to our knowledge is not well based as yet in terms of the technology underlying it.

It is certainly possible to capture slow or incomplete adjustment to shocks by reduced form estimates such as in Hammermesh (1988). However, it is not quite clear what the estimated parameters measure and how they can be interpreted in terms of a general underlying technology. In contrast, the measures of Krishna and Young (1992) are related to the matrix of the relevant own and cross elasticities. For example, flexibility in response to price shocks is shown to depend on price elasticities of supply and so is intuitively appealing.

It is worth noting that some temporal aspects can be incorporated into the static approach used here. Revenue functions in the short run, when some factors of production are fixed, as well as revenue functions in the long run, when all factors of production are mobile, can both be estimated using the appropriate restrictions in the estimation process. This would permit measures of short run and long run flexibility in response to given shocks.

There is also a large literature relating flexibility to adjustment costs which is microeconomically well based. Fixed adjustment cost models lead to s, S type models where adjustment occurs once pressures are sufficient to warrant the cost of adjustment. Variable adjustment models, which have the property that adjustment costs are bounded away from zero at the origin also lead to such regions of inaction.[2] Variable adjustment cost models, which use quadratic adjustment costs, have zero adjustment costs at the origin and result in slow adjustment over time. Increasing hazard models, such as Cabalero and Engel (1992) relate the probability of adjustment to the level of pressure for adjustment.

There are even models which aggregate to the macro level in remarkably clever ways, and provide insights into macroeconomic behavior such as Caplin (1985) and Caplin and Leahy (1991) among others. However, the cost of such aggregation is the special assumptions that need to be made to permit it. For this reason, they are not well suited as a base for empirical work as yet.

Another strand in the literature focuses on the value of flexibility for a firm in the face of uncertainty. Stigler (1939) in a classic paper describes one plant as being more flexible than another if it has a flatter average cost curve. He points out that with uncertainty, a firm may well choose a technology which is not efficient for any given level of output, but is optimal for the uncertain environment it faces. Jones and Ostroy (1984) provide a masterful summary of the history of flexibility as an economic concept and formalize the notion of flexibility in a sequential decision context and relate its value to the amount of information an agent expects to receive. Fuss and McFadden (1978) argue that neglecting uncertainty in estimating revenue or cost functions can seriously bias the resulting estimates. They distinguish between ex-ante and expost technological possibilities and provide econometric specifications which permit this distinction to be made in practice. Their work is of great importance in refining and developing ways of implementing the definitions of Krishna and Young (1992) which neglect uncertainty.

The next section develops the results on capital mobility. Section 3 contains the main results for a command economy, while Section 4 has some concluding remarks and directions for future research.

2. Capital Mobility and Flexibility

It turns out to be convenient to use the dual approach and describe technology using the revenue function. We will first define the revenue functions we use and then derive some properties we will rely on in the following sections:[3]

The revenue function is denoted by $r(P,V)$ where P and V denote exogenously given vectors of prices and endowments, respectively. There are n goods and m factors.

The revenue function is defined to be:[4]

$$r(P,V) = \max_{X} P'X \text{ subject to } (X,V) \text{ being in the feasible set.} \tag{1}$$

The reader is reminded that, given constant returns to scale, the revenue function is homogeneous of degree one in both prices and endowments. It is convex in prices and concave in endowments. Its derivative with respect to prices equals the vector of equilibrium outputs, $r_P(P,V) = X(P,V)$, while its derivative with respect to endowments equals the shadow prices (i.e. equilibrium returns) of the factors of production, $r_V(P,V) = W(P,V)$. Of course, these derivatives are homogeneous of degree zero in both prices and endowments.

The revenue function can also be defined as the value function for the program that minimizes factor payments subject to the constraint that price weakly falls short of costs:

$$r(P,V) = \min_{W} W'V \text{ such that } P \leq c(W).^{5} \tag{2}$$

In Theorems 1 and 2 below, we examine the effect of permitting capital mobility on flexibility. In this we build on the work of Neary (1985). For this purpose we will divide the factor endowment vector V into two component vectors, K and L, of dimensions $m_K x1$ and $m_L x1$, respectively. The m_K factors in K are mobile internationally and so can be acquired at given world factor prices denoted by the vector ρ. In this case factors can be thought of as negative traded intermediate goods.

Define:

$$R(P,\rho,L) = \underset{X,\,K}{\text{Max}}\; P'X - \rho'K \text{ subject to } (X,K,L) \text{ being feasible.} \tag{3}$$

It is convenient to think of this as a two stage process where X is chosen first and K is chosen afterwards. Let $X(\cdot)$ denote the solution to the first stage, that is, for a given K. Note that $P'X(\cdot) = r(P,K,L)$. At the second stage, K is chosen. Thus:

$$R(P,\rho,L) = \underset{K}{\text{Max}}\; r(P,K,L) - \rho'K\,. \tag{4}$$

Let $K(P,\rho,L)$ solve this problem. The work of Neary (1985) shows that when $m_L \geq n$, these choices are well behaved functions of their arguments.[6] National Income therefore equals:

$$G(P,\rho,K,L) = r(P,K(\cdot),L) - \rho'[K(\cdot) - K] = R(P,\rho,L) + \rho'K \tag{5}$$

where $K(\cdot)$ denotes the choice of K in (3) and K denotes the vector of domestic owned factors which are mobile. The first equality shows that GNP equals GDP less factor payments for net imports of mobile factors.

We will call $K(\cdot)$ the virtual quantity vector for the mobile capital since having a capital stock of $K(\cdot)$ will generate the same output choices as having availability to factor markets at the factor price vector ρ or $X(P,K(\cdot),L) = X(P,\rho,L)$.

The measure of flexibility with respect to price changes in Krishna and Young (1992) is related to the matrix of second order derivatives of the GNP function with and without capital mobility. Our results will be valid for comparisons of the same baseline economy, with and without capital flexibility. Thus, we wish to relate the matrix of second order derivatives with and without capital mobility <u>at</u> the capital stock, K, given by the virtual quantity $K(P,\rho,L)$. At this point, GNP equals:

$$G(P,\rho,K,L) = r(P,K(\cdot),L). \tag{6}$$

Therefore:

$$G_P(P,\rho,K,L) = R_P(P,\rho,L), \text{ and } G_{PP}(P,\rho,K,L) = R_{PP}(P,\rho,L) \tag{7}$$

However, by definition:

$$R(P,\rho,L) \equiv r(P,K(P,\rho,L),L) - \rho' K(P,\rho,L) \tag{8}$$

so that:

$$\begin{aligned} R_P(P,\rho,L) &\equiv r_P(P,K(P,\rho,L),L) + r_K(\cdot)' K_P(\cdot) - \rho' K_P(\cdot) \\ &= r_P(P,K(P,\rho,L),L) \end{aligned} \tag{9}$$

and

$$R_{PP}(P,\rho,L) = r_{PP}(P,K(\cdot),L) - r_{PK}(P,K(\cdot),L)K_P(\cdot) \tag{10}$$

Moreover, since by definition of $K(\cdot)$, $r_K(P,K(P,\rho,L),L) = \rho$, so that $r_{KP}(P,K(P,\rho,L),L) + r_{KK}(P,K(P,\rho,L),L)K_P(\cdot) = 0$. Hence:

$$K_P(\cdot) = -r_{KK}^{-1}(P,K(\cdot),L)r_{KP}(P,K(\cdot),L). \tag{11}$$

Substituting back gives:

$$R_{PP}(P,\rho,L) - r_{PP}(P,K(\cdot),L) = -r_{PK}(\cdot)r_{KK}^{-1}(\cdot)r_{KP}(\cdot). \tag{12}$$

This is the key result relied on in Theorem 1. This result is not new by any means. It is identical to that obtained in Neary (1985) and reproduced here for completeness. Neary points out that since (12) defines a positive definite matrix[7], it implies that the baseline economy with capital mobility has a greater own supply elasticity than the one without capital mobility. In this way it is more flexible than the other.

Below, we go further and use this result in conjunction with the definition of Krishna and Young (1992) to generalize this insight of Neary's.

Krishna and Young (1992) define flexibility in response to price changes as:

$$I(P^0) = \frac{r(P^0, V) - r(P^1, V) - (P^0 - P^1)' \, r_P(P^1, V)}{r(P^0, V)} \tag{13}$$

The index is the actual change in nominal national income less the change in income that would have occurred if the economy had kept on producing base period outputs (that is, if it had been totally inflexible), divided by first period income. The numerator can be interpreted as the error from a first order approximation of the revenue function. By the mean value theorem, the numerator of this index is also equal to the second order terms evaluated at some point, P^{**}, between the two prices:

$$I(P^0) = \frac{(P^0 - P^1)' r_{PP}(P^{**}, V)\,(P^0 - P^1)}{2r(P^0, V)} \tag{14}$$

$$\text{for } P^{**} \in (P^0, P^1)$$

Their measure of flexibility is thus fully defined by the price changes and the matrix of second order derivatives with respect to price of the revenue function at some point between the two prices.

Using this definition of flexibility in conjunction with the definitions of national income above gives the indices of flexibility with and without capital mobility to be:

$$I^m(P^0) = [(P^0 - P^1)'\{R_{PP}(P^*, \rho, L)\}(P^0 - P^1)]/2G(P^0, \rho, K, L) \tag{15}$$

and

$$I(P^0) = [(P^0 - P^1)'\{r_{PP}(P^{**}, K, L)\}(P^0 - P^1)]/2r(P^0, K, L) \tag{16}$$

respectively at $K = K(\cdot)$, the virtual quantity, and P^* and P^{**} somewhere in the open interval (P^0, P^1).

Using (6) and (12) gives the difference in the two, $\Delta(P^*, P^{**})$, evaluated at $P^* = P^{**} = P^0$ to be:

$$\Delta(P^0, P^0) = [(P^0 - P^1)'\{-r_{PK}(\cdot)r_{KK}^{-1}(\cdot)r_{KP}(\cdot)\}(P^0 - P^1)]/2r(P^0,K,L). \tag{17}$$

This expression is positive since $r_{KK}(\cdot)$ is negative definite. Since P^* and P^{**} lie in the open interval (P^0, P^1), by continuity, $\Delta(P^*,P^{**}) > 0$ for P^1 close to P^0. This results in Theorem 1.

Theorem 1.

International factor mobility increases the flexibility of the baseline economy with respect to all small price changes.

Similarly, flexibility with respect to factor endowment changes will be related to the matrix of second order derivatives with respect to factor availability of the baseline economy, with and without factor mobility. Again we use the identity given in (8). Differentiating gives:

$$\begin{aligned} R_L(P,\rho,L) &\equiv r_L(P,K(P,\rho,L),L) + r_K(\cdot)'K_L(\cdot) - \rho'K_L(\cdot) \\ &= r_L(P,K(P,\rho,L),L) \end{aligned} \tag{18}$$

and

$$R_{LL}(P,\rho,L) = r_{LL}(P,K(\cdot),L) - r_{LK}(P,K(\cdot),L)K_L(\cdot)\,. \tag{19}$$

Again by definition of $K(\cdot)$, $r_K(P,K(P,\rho,L),L) = \rho$, so that $r_{KL}(P,K(P,\rho,L),L)$ $+ r_{KK}(P,K(P,\rho,L),L)K_L(\cdot) = 0$. Hence:

$$K_L(\cdot) = -r_{KK}^{-1}(P,K(\cdot),L)r_{KL}(P,K(\cdot),L) \tag{20}$$

Substituting back gives:

$$R_{LL}(P,\rho,L) - r_{LL}(P,K(\cdot),L) = -r_{LK}(\cdot)r_{KK}^{-1}(\cdot)r_{KL}(\cdot). \tag{21}$$

This is the key fact used in Theorem 2. Again, this result is due to Neary (1985) and provided for completeness.

Krishna and Young (1992) define the flexibility with respect to factor endowment changes in terms of the ability of the economy to absorb additional factors without the need for factor price adjustments. Consider the function:

$$I(V^0) = \frac{-[r(P, V^0) - r(P, V^1) - (V^0 - V^1)' r_V(P, V^1)]}{r(P, V^0)} \tag{22}$$

This function can be interpreted as the difference in the response of an economy which is perfectly flexible (that is, an economy which can absorb factors at given wages so that the change in national income is the change in endowments times original wages), and that of an economy which can only fully absorb factors of production if wages change. Note that it is not an index of structural flexibility, but of the loss due to imperfect flexibility. It is easily seen that it is the error from a first order expansion of the revenue function, i.e. it equals the negative of the second order terms evaluated at some endowment, V^*, between V^1 and V^0 and hence corresponds to a positive definite quadratic form:

$$I(V^0) = -[\frac{1}{2} \frac{(V^0 - V^1)' r_{VV} (P, V^*) (V^0 - V^1)}{r (P, V^0)}] \geq 0 \tag{23}$$

The indices of inflexibility with respect to endowment shocks, with and without capital mobility, are thus given by:

$$I^m(V^0) = -[(L^0 - L^1)'[R_{LL}(P,\rho,L^*)\}(L^0 - L^1)]/2G(P^0,\rho,K,L^0) \tag{24}$$

and

$$I(V^0) = -[(L^0 - L^1)'\{r_{LL}(P,K,L^{**})\}(L^0 - L^1)]/2r(P^0,K,L^0) \tag{25}$$

respectively at $K = K(\cdot)$, the virtual quantity, and L^* and L^{**} somewhere in the open interval (L^0, L^1).

Using (6) and (21) gives the difference in the two, $\Delta(L^*, L^{**})$, evaluated at $L^* = L^{**} = L^0$ to be:

$$\Delta(L^0, L^0) = [(L^0 - L^1)'\{r_{LK}(\cdot)r_{KK}^{-1}(\cdot)r_{KL}(\cdot)\}(L^0 - L^1)]/2r(P,K,L^0). \tag{26}$$

This expression is negative since $r_{KK}(\cdot)$ is negative definite. Since L^* and L^{**} lie in the open interval (L^0, L^1), by continuity, $\Delta(L^*,L^{**}) < 0$ for L^1 close to L^0. This results in Theorem 2.

Theorem 2.

International factor mobility reduces the loss from inflexibility of the baseline economy with respect to all small endowment changes.

Note that Theorems 1 and 2 can also be interpreted as dealing with technical change as the exogenous shock as in Krishna and Young (1992). The effects of technology changes which are Hicks Neutral are equivalent to those of price changes, while factor augmenting technical change is equivalent to factor endowment changes.

Caution should be exercised in interpreting the above results. They deal with comparisons between economies at a given baseline. At this baseline, there would be <u>no</u> capital inflows or outflows actually occurring. All we get is the pure effect on flexibility which is related to the curvature of the revenue function.

The intuition behind our results is easily understood by looking at them in terms of the Le Chatelier principle. Note first that in our model of a small country, the absence of capital mobility is the only distortion. Thus, removing the distortion must raise GNP. Formally, GNP without capital mobility is given by $r(P,K,L)$. Also, approximating this about $K = K(P,\rho,L)$, the virtual quantity defined above gives:

$$r(P,K,L) = r(P,K(\cdot),L) + (K-K(\cdot))'r_K(P,K(\cdot),L) + (1/2)(K-K(\cdot))'r_{KK}(P,K^*,L)(K-K(\cdot)) \tag{27}$$

$$= r(P,K(\cdot),L) + (K-K(\cdot))'\rho + (1/2)(K-K(\cdot))'r_{KK}(P,K^*,L)(K-K(\cdot))$$

for K^* in the interval $(K, K(\cdot))$. Thus:

$$G(P,\rho,K,L) - r(P,K,L) = -(1/2)(K-K(\cdot))'r_{KK}(P,K^*,L)(K-K(\cdot)) > 0 \tag{28}$$

Hence, GNP must rise when capital mobility is permitted. Since it is the same when $K=(\cdot)$, the GNP function without capital mobility is tangent to that with capital mobility at this point and lies below it everywhere else. The GNP function is convex in prices and concave in endowments. This results in the curvature relations depicted in Figure 1(a) and (b) which are at the heart of our results.

3. Command Economies and Flexibility

In this section, we examine the effect of targeting the level of production in some, but not all, sectors of the economy. To do this we divide the output vector, X, into two component vectors, Y and Z of size n_y and n_z, respectively. Competitive behavior determines the levels of outputs in Y, while the levels of output in Z are exogenously determined and are denoted by $\bar{Z}$. How or why these levels are determined is unimportant for our purposes, but it is natural to think of their being set to satisfy development goals or more general political objectives.

Before considering the command economy, it is useful to rewrite the revenue function when all outputs are competitively determined as:

$$r(P, Q, V) = \underset{Y, Z}{\text{Max}}\ P'Y + Q'Z \text{ subject to } (Y, Z, V) \text{ being feasible} \tag{29}$$

where P and Q denote exogenously given vectors of prices for Y and Z, respectively. As usual this function is convex in prices and concave in endowments. Its derivatives with respect to prices, $r_P(P, Q, V) = Y(P, Q, V)$ and $r_Q(P, Q, V) = Z(P, Q, V)$, give equilibrium output choices, and these derivatives are homogeneous of degree zero in prices and endowments.

We define the revenue function for the command economy as:

$$R(P, Q, \bar{Z}, V) = \underset{Y}{\text{Max}}\ P'Y + Q'\bar{Z} \text{ subject to } (Y, \bar{Z}, V) \text{ being feasible} \tag{30}$$

where production targets determine outputs in $\bar{Z}$. In this case, $R_P(P, Q, \bar{Z}, V) = Y(P, Q, \bar{Z}, V)$ gives equilibrium outputs in Y and $R_Q(P, Q, \bar{Z}, V) = \bar{Z}$.

We want to examine the economy's flexibility with and without production targets, and, as before, our measures of flexibility are those defined in Krishna and Young (1992). Thus we need to relate $R(\cdot)$ to $r(P,\bar{Z},V)$, where $Q(P,\bar{Z},V)$ denotes virtual prices for Z. $Q(P, \bar{Z}, V)$, is implicitly defined as the solution in Q to $\bar{Z} = Z(P, Q, V)$. It is called the virtual price vector for the command economy since the competitive economy would choose to produce the targeted levels of Z at prices given by P and $Q(P,\bar{Z},V)$.[8]

Using the definition of virtual prices, (29) can be written as

$$R(P,Q, Z^*, V) = P'Y(P, Q(P, \bar{Z},V), V) + Q'Z(P, Q(P, \bar{Z},V), V). \tag{31}$$

Adding and subtracting $Q(P, \bar{Z}, V)'Z(P, Q(\cdot), V)$ and using the fact that $Z(P,Q(\cdot),V) = \bar{Z}$, this expression becomes:

$$R(P, \bar{Z}, V) = r(P, Q(\cdot), V) + (Q - Q(\cdot))'\bar{Z}\ . \tag{32}$$

Thus for $Q = Q(\cdot)$, GNP is the same in the competitive and command economies:

$$R(P, \bar{Z}, V) = r(P, Q(\cdot), V). \tag{33}$$

As in the preceding section, our results on flexibility rely on the second order derivatives of the revenue function, with and without targeted production, at this point. Theorem 3 states that targeting production in some, but not all, sectors reduces the economy's flexibility in response to price changes in the nontargeted sector. The result relies on :

$$R_{PP}(P, \bar{Z}, V) \; - r_{PP}(P,Q(\cdot), V) = -r_{PQ}(\cdot)\, r_{QQ}^{-1}(\cdot)\, r_{QP}(\cdot) \tag{34}$$

which follows from differentiating (32) and using the definition of the virtual price vector. That is,

$$\begin{aligned} R_P(P, \bar{Z}, V) &= r_P(P, Q(\cdot), V) + r_Q(P, Q(\cdot), V)'\, Q_P(.) - \bar{Z}' Q_P(\cdot) \\ &= r_P(P, Q(\cdot), V) \end{aligned} \tag{35}$$

since $r_Q = Z(P, Q(\cdot), V) = \bar{Z}$. Therefore,

$$R_{PP}(P, \bar{Z}, V) = r_{PP}(P, Q(\cdot), V) + r_{PQ}(\cdot)\, Q_P(\cdot) \tag{36}$$

But $Q(P, \bar{Z}, V)$ is the solution to $\bar{Z} = Z(P, Q, V)$, so that

$$\begin{aligned} Q_P(\cdot) &= - Z_P(\cdot)/Z_Q(\cdot) \\ &= - r_{QP}(\cdot)\, r_{QQ}^{-1}(\cdot)\,. \end{aligned} \tag{37}$$

Substituting gives $R_{PP}(P, \bar{Z}, V) = r_{PP}(P, Q(\cdot), V) - r_{PQ}(\cdot)\, r_{QQ}^{-1}(\cdot)\, r_{QP}(\cdot)$ and hence (34).

We use these results in conjunction with our earlier definitions of price and endowment flexibility. With and without production targets, these indices are

$$I^c(P^0) = [(P^0-P^1)' \{R_{PP}(P^*, \bar{Z}, V)\} (P^0-P^1)]/2R(P, \bar{Z}, V) \tag{38}$$

and

$$I(P^0) = [(P^0-P^1)' \{r_{PP}(P^{**}, Q(\cdot), V)\} (P^0-P^1)]/2r(P, Q(\cdot), V) \tag{39}$$

respectively, where P^* and P^{**} are both in the open interval (P^0, P^1).

Using (33) and (34) gives the difference in the two, $\Delta(P^*, P^{**})$, evaluated at $P^* = P^{**} = P^0$ to be:

$$\Delta(P^0, P^0) = [(P^0-P^1)' \{-r_{PQ}(\cdot)\, r_{QQ}^{-1}(\cdot) r_{QP}(\cdot)\}(P^0-P^1)]/2r(P^0, Q(\cdot), V). \tag{40}$$

This expression is negative since $r_{QQ}(\cdot)$ is positive definite. Since P^* and P^{**} lie in the open interval (P^0, P^1), by continuity, $\Delta(P^*, P^{**}) < 0$ for P^1 close to P^0. This results in Theorem 3.

Theorem 3.

Setting production targets in some sectors decreases the flexibility of the baseline economy with respect to all small price changes.

Theorem 4 states that the command economy is also less flexible with respect to endowment changes. Again the result relies on differentiating (32). Differentiating with respect to V gives:

$$\begin{aligned} R_V(P, \bar{Z}, V) &= r_V(P, Q(\cdot), V) + r_Q(P, Q(\cdot), V)' Q_V(\cdot) - \bar{Z}' Q_V(\cdot) \\ &= r_V(P, Q(\cdot), V), \end{aligned} \tag{41}$$

and

$$R_{VV}(P, \bar{Z}, V) = r_{VV}(P, Q(\cdot), V) + r_{QV}(\cdot)\, Q_V(\cdot). \tag{42}$$

But $Q_V(\cdot) = -Z_V(\cdot)/Z_Q(\cdot) = -r_{QV}(\cdot)\, r_{QQ}^{-1}(\cdot)$.

Substituting gives

$$R_{VV}(P, \bar{Z}, V) - r_{VV}(P, Q(\cdot), V) = -r_{VQ}(\cdot)\, r_{QQ}^{-1}(\cdot)\, r_{QV}. \tag{43}$$

The indices of inflexibility with respect to factor endowment changes, with and without production targets are given by

$$I^c(V^0) = -[(L^0-L^1)' \{R_{VV}(P, \bar{Z}, L^*)\} (L^0-L^1)]/2R(P, \bar{Z}, L^0) \tag{44}$$

and

$$I(V^0) = -[(L^0-L^1)' \{r_{VV}(P, Q(\cdot), L^{**})\} (L^0-L^1)]/2r(P, Q(\cdot), L^0) \tag{45}$$

respectively for $Q=Q(\cdot)$ and L^* and L^{**} in the open interval (L^0, L^1).

Using (33) and (43) gives the difference in the two, $\Delta(L^*, L^{**})$, evaluated at $L^* = L^{**} = L^0$ to be:

$$\Delta(L^0, L^0) = (L^0-L^1)' \{r_{QV}(\cdot)\, r_{QQ}^{-1}(\cdot)\, r_{QV}(\cdot)\} (L^0-L^1)]/2r(P, Q(\cdot), V) \tag{46}$$

This expression is positive since $r_{QQ}(\cdot)$ is positive definite. Since L^* and L^{**} lie in the open interval (L^0, L^1), by continuity, $\Delta(L^*, L^{**}) > 0$ for L^1 close to L^0. This results in Theorem 4.

Theorem 4.

Setting production targets in some sectors increases the loss from inflexibility of the baseline economy with respect to all endowment changes.

Theorems 3 and 4 are subject to the same types of qualifications and interpretation as Theorems 1 and 2. In particular, our comparisons are made at a given baseline, where the economy faces prices for Z given by the virtual price vector. Thus we are comparing the curvature of the revenue function with and without production targets in certain sectors. Introducing production targets places additional constraints on the economy's resource allocation problem. In terms of the Le Chatelier principle, introducing production targets cannot increase GNP. Formally, GNP without targets is given by $r(P, Q, V)$. Approximating this about $Q(P, \bar{Z}, V)$, the virtual price vector, gives:

$$r(P,Q,V) = r(P,Q(\cdot),V) + (Q - Q(\cdot))'\bar{Z} + \frac{1}{2}(Q - Q(\cdot))' r_{QQ}(P,Q^*,V)(Q - Q(\cdot))$$

for $Q^* \in (Q, Q(\cdot))$. Thus

$$R(P,Q,\bar{Z},V) - r(P,Q,V) = -\frac{1}{2}(Q - Q(\cdot))' r_{QQ}(P,Q^*,V)(Q - Q(\cdot)) < 0.$$

Since $r(P, Q, V) = R(P, Q, \bar{Z}, V)$ at $Q(P, \bar{Z}, V)$, the GNP function for the command economy is tangent to $r(\cdot)$ at that point and lies below it elsewhere.

4. Concluding Remarks

In general, economies cannot be ranked in terms of their ability to adjust to exogenous shocks. Nonetheless, properties of the revenue function can be used to provide a partial ordering in which certain types of economies are compared. In this paper we showed that placing restrictions on factor mobility and on production in certain sectors reduces the flexibility of a small open economy with respect to all price, endowment, and technology shocks of small enough magnitude. Since one can think of these restrictions as distortions, we would expect them to reduce the level of GNP in the economy. The insight provided by the analysis of flexibility is that, not only is the level of GNP affected, but also the intrinsic ability of the economy to adjust to shocks is reduced.

Our results are straightforward applications of the le Chatelier Principle in that we examine the impact of restrictions when the constraints were just binding. They are more general than the usual le Chatelier applications since they deal with small but noninfinitesimal changes. In future work we plan to examine other Le Chatelier applications, such as the effect of quotas, which have more complex effects than the simple restrictions considered here. We also plan to examine flexibility in non Le Chatelier contexts, that is, when the relevant distortions are substantial (as, for example, in the literature on piecemeal policy reform)[9]. In the applications above, the effects on GNP and the effects on flexibility are in the same direction. An additional question of interest is under what conditions these effects are opposing. Finally, we have assumed differentiability throughout, and it is of interest to know if this assumption can be relaxed.

ENDNOTES

1 Another strand of literature examines response of domestic prices to external price shocks. For example, Jones and Pervis (1983) relate the divergence of equilibrium exchange rates from purchasing power parity to the ability of an economy to adjust to external shocks, Chipman (1981) examines the responses of domestic prices to external price shocks for the West German economy.

2 Kemp and Wan (1974) provide an early analysis.

3 A good reference source for those unfamiliar with the general approach is Dixit and Norman (1980).

4 Note that this specification is valid only when there is no joint production.

5 Throughout we define vectors as column vectors, and the notation ′ denotes transpose.

6 Throughout we assume the revenue function is differentiable, so that we invoke the required conditions on the number of goods and factors (mobile and immobile, constrained and unconstrained).

7 Note that $[\bar{Z}R_{PK}]R_{KK}^{-1}[R_{KP}Z] = -Z'R_{PK}R_{KK}^{-1}R_{KK}R_{KK}^{-1}R_{KP}Z > 0$.

8 In this we owe an intellectual debt to Neary and Roberts (1980) who used the concept of virtual prices in their analysis of consumer behavior in the face of quantity constraints.

9 See Migrom and Roberts (1992) for a general approach to comparing equilibria of models when changes are discrete.

References

Caballero, R. J. and E.M.R.A. Engel, 1992. "Microeconomic Adjustment Hazards and Aggregate Dynamics." Mimeo, MIT.

Caplin, A., 1985. "The Variability of Aggregate Demand with (S,s) Inventory Policies." *Econometrica* 53: 1395-1410.

______ and J. Leahy, 1991: State-Dependent Pricing and the Dynamics of Money and Output." *Quarterly Journal of Economics* 106:

Chipman, J.S., 1972. "The Theory of Exploitative Trade and Investment Policies: A Reformulation and Synthesis." In L.E. Di Marco (ed.), *International Economics and Development*. New York: Academic Press; 209-242.

Chipman, ____, 1981. "Internal-External Price Relationships in the West German Economy, 1958-79," in R. Richter and W. F. Stolper (eds), *Zeitschrift Fü die gesamte Staata wissenschaft* 137: 612-637.

Davis, S. J. and J. Haltiwanger, 1992. "Gross Job Creation, Gross Job Destruction, and Employment Reallocation." *Quarterly Journal of Economics* 107: 819-863.

Dixit, A. and V. Norman, 1980. *Theory of International Trade*. Cambridge: Cambridge University Press.

Fuss, M. and D. McFadden, 1978. "Flexibility Versus Efficiency in Exante Plant Design," in Fuss and McFadden (eds.), *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1, New York: North-Holland Publishing, 311-364.

Hammermesh, D., 1988. "Labor Demand and the Structure of Adjustment Costs." N.B.E.R. Working Paper #2572.

Jones, R.A. and J. M. Ostroy, 1984. "Flexibility and Uncertainty." *Review of Economic Studies* 51: 13-32.

Jones, R. W. and D. D. Purvis, 1983. "International Differences in Response to Common External Shocks: The Role of Purchasing Power Parity," in E. Claassen and P. Salir (eds.), *The Theory of Flexible Exchange Rates*. Amsterdam: North Holland, 33-55.

Kemp, M. C. and H. Y. Wan, Jr., 1974. "Hysteresis of Long-Run Equilibrium from Realistic Adjustment Costs," in G. Horwich and P. A. Samuelson (eds.), *Trade Stability, and Macroeconomics*. New York: Academic Press, 221-242.

Krishna, K. and A. Young, 1992. "Conceptually Based Measures of Structural Adaptability." NBER Working Paper #4039.

Milgrom, P. and J. Roberts, 1992. "Comparing Equilibria," Mimeo, Stanford University.

Stigler, G., 1939. "Production and Distribution in the Short Run." *Journal of Political Economy* 47: 305-357.

Neary, J.P., 1985. "International Factor Mobility, Minimum Wage Rates, and Factor Price Equalization: A Synthesis." *Quarterly Journal of Economics* 100: 551-569.

______ and K.W.S. Roberts. "The Theory of Household Behavior Under Rationing." *European Economic Review* 13: 25-42.

Young, A., 1989. "Hong Kong and the Art of Landing on One's Feet: A Case Study of a Structurally Flexible Economy." Unpublished Ph.D. Thesis. Fletcher School of Diplomacy.

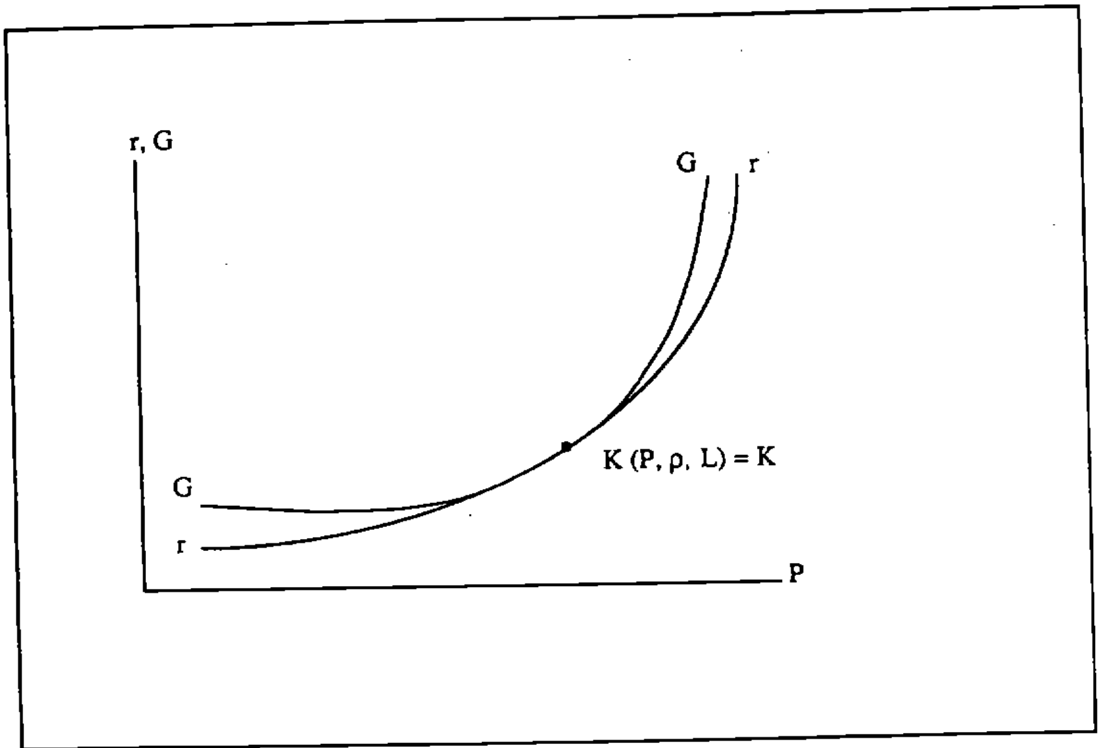

Figure 1 (a).

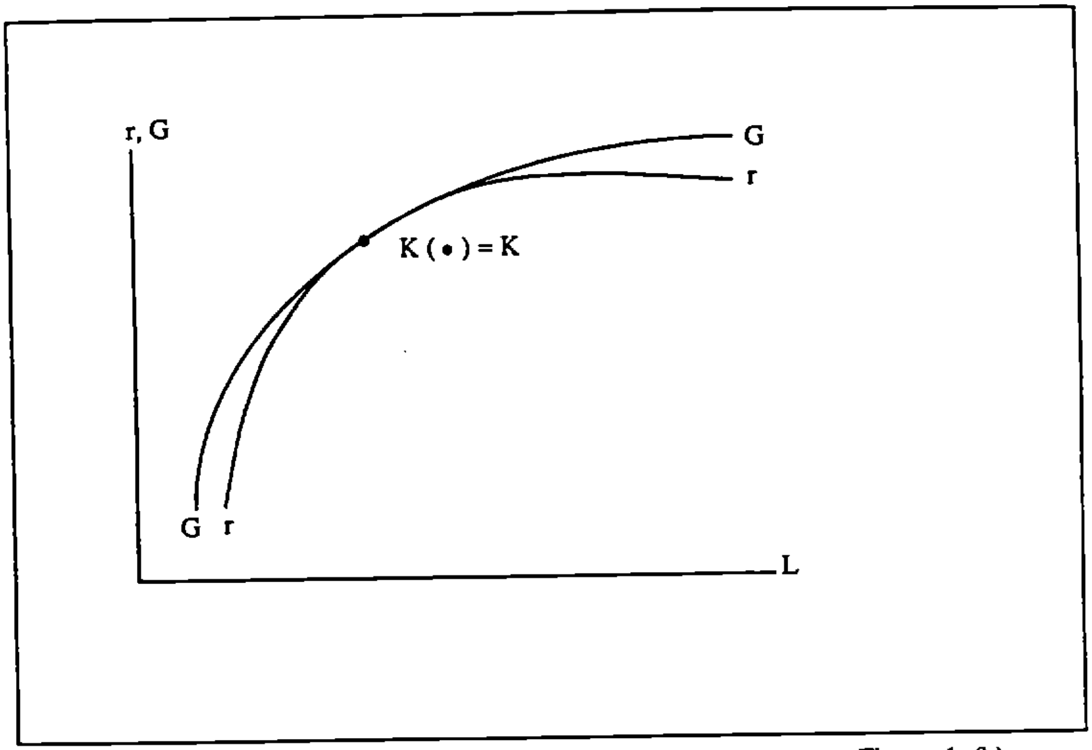

Figure 1 (b).