NBER WORKING PAPER SERIES

TESTS OF CAPM ON AN INTERNATIONAL PORTFOLIO OF BONDS AND STOCKS

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Working Paper No. 4598

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December, 1993

Helpful comments were supplied by Geert Bekaert, Bernard Dumas, Jeff Frankel and Bill Schwert. I would like to thank Anthony Rodrigues for preparing the bond data for this paper, and for many useful discussions. I would also like to thank John McConnell for excellent research assistance. This paper is part of NBER's research program in International Finance and Macroeconomics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #4598
December 1993

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ABSTRACT

This paper estimates and tests an international version of the Capital Asset Pricing Model. Investors from the U.S., Germany and Japan choose a portfolio that includes bonds and equities from each of these countries to maximize a function of the mean and variance of returns. Investors in each country evaluate returns in terms of their home currency. The CAPM does have some power in explaining ex ante returns. It predicts fairly large risk premia on the equities, but small ones on bonds. The model is rejected, however, when tested against a more general alternative that allows for more investor heterogeneity than the CAPM.

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I. Introduction

Portfolio-balance models of international asset markets have enjoyed little success empirically.[1] These studies frequ ,' investigate a very limited menu of assets, and often impose the assumption of a representative investor.[2] This study takes a step toward dealing with those problems by allowing some investor heterogeneity, and by allowing investors to choose from a menu of assets that includes bonds and stocks in a mean-variance optimizing framework.

The model consists of U.S., German and Japanese residents who can invest in equities and bonds from each of these countries. Investors can be different because they have different degrees of aversion to risk. More importantly, within each country nominal prices paid by consumers (denominated in the home currency) are assumed to be known with certainty. This is the key assumption in Solnik's (1974) capital asset pricing model (CAPM). Investors in each country are concerned with maximizing a function of the mean and variance of the returns on their portfolios, where the returns are expressed in the currency of the investors' residence. Thus, U.S. investors hold the portfolio that is efficient in terms of the mean and variance of dollar returns; Germans in terms of mark returns; and, Japanese in terms of yen returns.

The estimation technique is closely related to the "CASE" (Constrained Asset Share Efficiency) method introduced by Frankel (1982) and elaborated by Engel, Frankel, Froot and Rodrigues (1993). The mean-variance optimizing model expresses equilibrium asset returns as a function of asset supplies and the covariance of returns. Hence, there is a constraint relating the mean of returns and the variance of returns. The CASE method estimates the mean-variance model imposing this constraint. The covariance of returns is modeled to follow a multivariate GARCH process.

One of the difficulties in taking such a model to the data is that there is scanty time-series evidence on the portfolio holdings of investors in each country. We do not know, for example, what proportion of Germans' portfolios are held in Japanese equities, or U.S. bonds.[3] We do have data on the total value of equities and bonds from each country held in the market, but not a

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[1] See Frankel (1989) or Glassman and Riddick (1993) for recent surveys.

[2] Although, notably, Frankel (1982) does allow heterogeneity of investors. Recent papers by Thomas and Wickens (1993) and Clare, O'Brien, Smith and Thomas (1993) test international CAPM with stocks and bonds, but with representative investors.

[3] Tesar and Werner (1993) have a limited collection of such data.

breakdown of who holds these assets. Section II shows how we can estimate all the parameters of the equilibrium model using only the data on asset supplies and data that measures the wealth of residents of the U.S. relative to that of Germans and Japanese. The data used in this paper have been available and used in previous studies. The supplies of bonds from each country is constructed as in Frankel (1982). The supply of nominal dollar assets from the U.S., for example, increases as the government runs budget deficits. These numbers are adjusted for foreign exchange intervention by central banks, and for issues of Treasury bonds denominated in foreign currencies. The international equity data has been used in Engel and Rodrigues (1993). The value of U.S. equities is represented by the total capitalization on the major stock exchanges as calculated by Morgan Stanley's Capital International Perspectives. The shares of wealth are calculated as in Frankel (1982) -- the value of financial assets issued in a country, adjusted by the accumulated current account balance of the country.

The Solnik model implies that investors' portfolios differ only in terms of their holdings of bonds. If we had data on portfolios from different countries, we would undoubtedly reject this implication of the Solnik model. However, we might still hope that the equilibrium model was useful in explaining risk premia. In fact, our test of the equilibrium model rejects CAPM relative to an alternative that allows diversity in equity as well as bond holdings. Probably the greatest advantage of the CASE method is that it allows CAPM to be tested against a variety of plausible alternative models based on asset demand functions. Models need only require that asset demands be functions of expected returns and nest CAPM to serve as alternatives. In section VI, CAPM is tested against several alternatives. CAPM holds up well against alternative models in which investors' portfolios differ only in their holdings of bonds. But, when we build an alternative model based on asset demands which differ across countries in bond and equity shares, CAPM is strongly rejected. While our CAPM model allows investor heterogeneity, apparently it does not allow enough.

There are many severe limitations to the study undertaken here, both theoretical and empirical. While the estimation undertaken here involves some significant advances over previous literature, it still imposes strong restrictions. On the theory side, the model assumes investors look only one period into the future to maximize a function of the mean and variance of their wealth. It is a partial equilibrium model, in the classification of Dumas (1993). Investors in different countries are assumed to face perfect international capital markets with no informational asymmetries. The data used in the study are crude. The measurement of bonds and equities entail some leaps of faith, and the supplies of other assets -- real property, consumer durables, etc. -- are not even considered. Furthermore, there is a high degree of aggregation involved in measuring both the supplies of assets and their returns.

Section II describes the theoretical model, and derives a form of the model that can be estimated. It also contains a brief discussion relating the mean-variance framework to a more general intertemporal approach. Section III discusses the actual empirical implementation of the model. Section IV presents the results of the estimation, and displays time-series of the risk premia implied for the various assets.

The portfolio balance model is an alternative to the popular model of interest parity, in which domestic and foreign assets are considered perfect substitutes. This presents some inherent difficulties of interpretation in the context of our model with heterogeneous investors, which are discussed in Section V. These problems are discussed, and some representations of the risk-neutral model are derived to serve as null hypotheses against the CAPM of risk-averse agents.

Section VI presents the test of CAPM against alternative models of asset demand. The concluding section attempts to summarize what this study accomplishes, and what would be the most fruitful directions to proceed in future research.

II. The theoretical model

The model that is estimated in this paper assumes that investors in each country face nominal consumer prices that are fixed in terms of their home currency. While that may not be a description that accords exactly with reality, Engel (1993) shows that this assumption is a much more justifiable than the alternative assumption that is usually incorporated in international financial models -- that the domestic currency price of any good is equal to the exchange rate times the foreign currency price of that good.

Dumas, in his 1993 survey, refers to this approach as the "Solnik special case", because Solnik (1974) derives his model of international asset pricing under this assumption. Indeed, the presentation in this section is very similar to Dumas' presentation of the Solnik model. The models are not identical because of a slightly differing assumptions about the distribution of asset returns.

There are six assets -- dollar bonds, U.S. equities, deutschemark bonds, German equities, Japanese bonds and Japanese equities. Time is discrete.

Table 1 lists the variables used in the derivations below.

The own currency returns on bonds between time t and time t+1 are assumed to be known with certainty at time t, but the returns on equities are not in the time t information set.

U.S. investors are assumed to have a one-period horizon, and maximize a function of the mean and variance of the real value of their wealth. However, since prices are assumed to be fixed in dollar terms for U.S. residents, this is equivalent to maximizing a function of the dollar value of their wealth.

Let W^u_{t+1} equal dollar wealth of U.S. investors in period t+1. At time t, investors in the U.S. maximize $F^{US}(E_t(W^u_{t+1}),V_t(W^u_{t+1}))$. In this expression, E_t

refers to expectations formed conditional on time t information. V_t is the variance conditional on time t information. We assume the derivative of F^{US} with respect to its first argument, F_1^{US}, is greater than zero, and that the derivative of F^{US} with respect to its second argument, F_2^{US}, is negative.

Following Frankel and Engel (1984), we can write the result of the maximization problem as

$$(1) \quad \lambda_t^u = \rho_{US}^{-1}\Omega_t^{-1}.E_t Z_{t+1}^{US}$$

In equation (1) we have

$$Z_{t+1} = \begin{bmatrix} R_{t+1}^u - (1+i_{t+1}^u) \\ R_{t+1}^g \dfrac{S_{t+1}^g}{S_t^g} - (1+i_{t+1}^u) \\ (1+i_{t+1}^g)\dfrac{S_{t+1}^g}{S_t^g} - (1+i_{t+1}^u) \\ R_{t+1}^j \dfrac{S_{t+1}^j}{S_t^j} - (1+i_{t+1}^u) \\ (1+i_{t+1}^j)\dfrac{S_{t+1}^j}{S_t^j} - (1+i_{t+1}^u) \end{bmatrix}$$

$$\Omega_t \equiv V_t(Z_{t+1}) = E_t[(Z_{t+1}-E_t Z_{t+1})(Z_{t+1}-E_t Z_{t+1})'],$$

$$\rho_{US} = -2F_2^{US}W/F_1^{US},$$

and λ_t^u is the column vector that has in the first position the share of wealth invested by U.S. investors in U.S. equities, the share invested in German equities in the second position, the share in mark bonds in the third position, the share in Japanese equities in the fourth position and the share in Japanese bonds in the fifth position.

We will assume, as in Frankel (1982), that ρ_{US} (and ρ_G and ρ_J defined later) are constant. These correspond to what Dumas (1993) calls "the market average degree of risk aversion", and can be considered a taste parameter. The degree of risk aversion can be different across countries.

Let $r_{t+1} \equiv \ln(R_{t+1})$, so that $R_{t+1} = \exp(r_{t+1})$. Now, we assume that r_{t+1} is distributed normally, conditional on the time t information. So, we have that

$$E_t R_{t+1} = E_t \exp(r_{t+1}) = \exp(E_t r_{t+1} + \sigma_t^u/2)$$

where $\sigma_t^u = V_t(r_{t+1})$.

Then, note that for small values of $E_t r_{t+1}$ and $\sigma_t^u/2$, we can approximate

$$E_t R_{t+1} = \exp(E_t r_{t+1} + \sigma_t/2) \cong 1 + E_t r_{t+1} + \sigma_t^u/2.$$

Using similar approximations, and using lower case letters to denote the natural logs of the variables in upper cases, we have

$$E_t Z_{t+1} \cong E_t z_{t+1} + D_t, \text{ where}$$

$$z_{t+1} \equiv \begin{bmatrix} r^u_{t+1} - i^u_{t+1} \\ r^g_{t+1} + s^g_{t+1} - s^g_t - i^u_{t+1} \\ i^g_{t+1} + s^g_{t+1} - s^g_t - i^u_{t+1} \\ r^j_{t+1} + s^j_{t+1} - s^j_t - i^u_{t+1} \\ i^j_{t+1} + s^j_{t+1} - s^j_t - i^u_{t+1} \end{bmatrix}$$

$$\Omega_t = V_t(Z_{t+1}) \cong V_t(z_{t+1})$$

and

$D_t = \text{diag}(\Omega_t)/2$, where diag() refers to the diagonal elements of a matrix.

So, we can rewrite equation (1) as

$$(2) \quad \lambda_t^u = \rho_{US}^{-1} \Omega_t^{-1} (E_t z_{t+1} + D_t).$$

Now, assume Germans maximize $F^G(E_t(W^g_{t+1}), V_t(W^g_{t+1}))$, where W^g represents the mark value of wealth held by Germans. After a bit of algebraic manipulation, the vector of asset demands by Germans can be expressed as:

$$(3)\quad \lambda_t^g = \rho_G^{-1}\Omega_t^{-1}(E_t z_{t+1}+D_t) + (1-\rho_G^{-1})e_3,$$

where e_j is a vector of length five that has a one in the jth position and zeros elsewhere.

Japanese investors, who maximize a function of wealth expressed in yen terms, have asset demands given by:

$$(4)\quad \lambda_t^J = \rho_J^{-1}\Omega_t^{-1}(E_t z_{t+1}+D_t) + (1-\rho_J^{-1})e_5.$$

Note that in the Solnik model, if the degree of risk aversion is the same across investors, they all hold identical shares of equities. Their portfolios differ only in their holdings of bonds. Even if they have different degrees of risk aversion, there is no bias toward domestic equities in the investors' portfolios. This contradicts the evidence we have on international equity holdings (see Tesar and Werner (1993), for example), so this model is not the most useful one for explaining the portfolio holdings of individuals in each country. Still, it may be useful in explaining the aggregate behavior of asset returns.

Then, taking a weighted average, using the wealth shares as weights, we have

$$(5)\quad \lambda_t = (\mu_t^J\rho_J^{-1}+\mu_t^g\rho_G^{-1}+\mu_t^u\rho_{US}^{-1})\Omega_t^{-1}(E_t z_{t+1}+D_t) + \mu_t^g(1-\rho_G^{-1})e_3 + \mu_t^J(1-\rho_J^{-1})e_5.$$

The vector λ_t contains the aggregate shares of the assets. While we do not have time series data on the shares for each country, we have data on λ_t, and so it is possible to estimate equation (5). This equation can be interpreted as a relation between the aggregate supplies of the assets and their expected returns and variances.

II.' A note on the generality of the mean-variance model

The model that we estimate in this paper is a version of the popular mean-variance optimizing model. This model rests on some assumptions that are not very general. The strongest of the assumptions is that investors' horizons are only one period into the future.

It is interesting to compare our model with that of Campbell (1993), who derives a log-linear approximation for a very general intertemporal asset pricing model. Campbell assumes that all investors evaluate real returns in the same way -- as opposed to our model in which real returns are different for U.S. investors, Japanese investors and German investors.

In order to focus on the effects of assuming a one-period horizon, we shall follow Campbell and examine a version of the model in which all consumers evaluate returns in the same real terms. This would be equivalent to assuming that all investors evaluate returns in terms of the same currency and nominal goods prices are constant in terms of that currency.

So, we will assume investors evaluate returns in dollars. In that case, we can derive from equation (2) that

(6) $E_t z_{t+1} = \rho \Omega_t \lambda_t - D_t$.

Let z_i represent the excess return on the ith asset. The expected return can be written

$$E_t z_{i,t+1} = \rho \Omega_{it} \lambda_t - \text{var}_t(z_{i,t+1})/2.$$

In this equation, var_t refers to the conditional variance, and Ω'_{it} is the ith row of Ω_t.

We can write

$$\Omega_{it}\lambda_t = \sum_{j=1}^{n} \text{Cov}_t(z_{i,t+1}, z_{j,t+1})\lambda_j = \text{Cov}_t(z_{i,t+1}, \sum_{j=1}^{n} z_{j,t+1}\lambda_j)$$

$$= \text{Cov}_t(z_{i,t+1}, z_{m,t+1}).$$

Cov_t refers to the conditional covariance, and $z_{m,t+1}$, which is defined to equal $\sum_{j=1}^{n} z_{j,t+1}\lambda_j$, is the excess return on the market portfolio.

So, we can write

(7) $E_t z_{i,t+1} = \rho \text{Cov}_t(z_{i,t+1}, z_{m,t+1}) - \text{var}_t(z_{i,t+1})/2.$

Compare this to Campbell's equation (25) for the general intertemporal model:

(8) $E_t z_{i,t+1} = \rho \text{Cov}_t(z_{i,t+1}, z_{m,t+1}) - \text{var}_t(z_{i,t+1})/2 + (\rho - 1)V_{ih,t}$,

where

$$V_{ih,t} \equiv Cov_t(z_{i,t+1}, (E_{t+1}-E_t)\sum_{j=1}^{\infty}\beta^j z_{m,t+j+1}).$$

β is the discount factor for consumers' utility. Campbell's equation is derived assuming that Ω_t is constant over time, but Restoy (1992) has shown that equation (8) holds even when variances follow a GARCH process.

Clearly the only difference between the mean-variance model of equation (7) and the intertemporal model is the term $(\rho-1)V_{ih,t}$. This term does not appear in the simple mean-variance model because it involves an evaluation of the distribution of returns more than one period into the future. Extending the empirical model to include the intertemporal term is potentially important, but difficult and left to future research. However, note that Restoy (1992) finds that the mean-variance model is able to "explain the overwhelming majority of the mean and the variability of the equilibrium portfolio weights" in a simulation exercise.[4]

III. The empirical model

The easiest way to understand the CASE method of estimating CAPM is to rewrite equation (5) so that it is expressed as a model that determines expected returns:

$$(9)\quad E_t z_{t+1} = -D_t + (\mu_t^j\rho_J^{-1}+\mu_t^g\rho_G^{-1}+\mu_t^u\rho_{US}^{-1})^{-1}[\Omega_t\lambda_t - \mu_t^g(1-\rho_G^{-1})\Omega_t e_3 - \mu_t^j(1-\rho_J^{-1})\Omega_t e_5]$$

Under rational expectations, the actual value of z_{t+1} is equal to its expected value plus a random error term:

$$z_{t+1} = E_t z_{t+1} + \varepsilon_{t+1}.$$

The CASE method maximizes the likelihood of the observed z_{t+1}. Note that when equation (9) is estimated, the system of five equations incorporates cross-equation constraints between the mean and the variance.

There are four versions of the model estimated here:

MODEL 1

This version estimates all of the parameters of equation (9) -- the three values of ρ, and the parameters of the variance matrix, Ω_t. It is the most general version of the model estimated. It allows investors across countries to differ not only in the currency of denomination that they evaluate returns

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[4] I would like to thank Geert Bekaert for pointing out an error in this section in the version of the paper presented at the conference.

in, but also their degree of risk aversion.

<u>MODEL 2</u>

Here we constrain ρ to be equal across countries. Then, using equation (9), we can write

$$(10)\ E_t z_{t+1} = -D_t + \rho\Omega_t\lambda_t + \mu_t^g(1-\rho)\Omega_t e_3 + \mu_t^j(1-\rho)\Omega_t e_5.$$

<u>MODEL 3</u>

Here we assume μ^i is constant over time for each of the three countries. We do not use data on μ^i, and instead treat the wealth shares as parameters. Since our measures of wealth shares may be unreliable, this is a simple alternative way of "measuring" the shares of wealth. However, in this case, neither the μ^i nor the ρ_i are identified. We can write equation (24) as

$$(11)\ E_t z_{t+1} = -D_t + \alpha\Omega_t\lambda_t - \gamma_1\Omega_t e_3 - \gamma_2\Omega_t e_5.$$

The parameters to be estimated are α, γ_1, γ_2 and the parameters of Ω_t. In the case in which the degree of risk aversion is the same across countries, α is a measure of the degree of risk aversion.

<u>Model 4</u>

The last model we consider abandons the assumption of investor heterogeneity and assumes that all investors are concerned only with dollar returns. So we can use equation (2) to derive the equation determining equilibrium expected returns under these assumptions. We have presented this model in section II' as equation (6), repeated here for convenience:

$$(12)\quad E_t z_{t+1} = \rho\Omega_t\lambda_t - D_t.$$

The mean-variance optimizing framework yields an equilibrium relation between the expected returns and the variance of returns, such as in equation (9). However, the model is not completely closed. While the relation between means and variances is determined, the level of the returns or the variances is not determined within the model. For example, Harvey (1989) posits that the expected returns are linear functions of data in investors' information set. The equilibrium condition for expected returns would then determine the behavior of the covariance matrix of returns. Our approach takes the opposite tack. We specify a model for the covariance matrix, and then the equilibrium condition determines the expected returns.

Since the mean-variance framework does not specify what model of variances is appropriate, we are free to choose among competing models of variances. Bollerslev's (1986) GARCH model appears to describe the behavior of the variances of returns on financial assets remarkably well in a number of settings, so we estimate a version of that model.

Our GARCH model for Ω_t follows the positive-definite specification in Engel and Rodrigues (1989):

$$(13)\ \Omega_t = P'P + G\varepsilon_t\varepsilon_t'G + H\Omega_{t-1}H,$$

In this equation, P is an upper triangular matrix, and G and H are diagonal matrices.

This is an example of a multivariate GARCH(1,1) model: the covariance matrix at time t depends on one lag of the cross-product matrix of error terms and one lag of the covariance matrix. In general, Ω_t could be made to depend on m lags of $\varepsilon\varepsilon'$ and n lags of Ω_t. Furthermore, the dependence on $\varepsilon_t\varepsilon_t'$ and Ω_{t-1} is restrictive. Each element of Ω_t could more generally depend independently on each element of $\varepsilon_t\varepsilon_t'$ and each element of Ω_{t-1}. However, such a model would involve an extremely large number of parameters. The model described in equation (27) involves estimation of 25 parameters -- 15 in the P matrix and 5 each in the G and H matrices.

IV. Results of Estimation

The estimates of the models are presented in Tables 2-5.

The first set of parameters reported in each table are the estimates of the risk aversion parameter. Model 1 allows the degree of risk aversion to be different across countries. The estimates for ρ_{US}, ρ_G and ρ_J reported in Table 2 are not very sensible economically. Two of the estimates are negative. The mean-variance model assumes that higher variance is less desirable, which implies that ρ should be positive.

Furthermore, we can test the hypothesis that the ρ coefficients are equal for all investors against the alternative of Table 2 that they are different. This can be easily done with a likelihood ratio test, since Table 3 estimates the constrained model. The value of the χ^2 test with 2 degrees of freedom is 4.056. The 5% critical value is 5.91, so we cannot reject the null hypothesis of equal values of ρ at this level.

In fact, the likelihood value for Model 1 is not as dependent on the actual values of the ρ's as it is on their relative values. If we let ρ be different across countries, we are unable to reject some extremely implausible values. For example, we cannot reject $\rho_{US} = 1414$, $\rho_G = 126$ and $\rho_J = 1.6$.

Based both on the statistical test and the economic plausibility of the estimates, the restricted model -- Model 2 -- is preferred to Model 1. Table 3 shows that the estimate of ρ in Model 2 is 4.65. This is not an unreasonable estimate for the degree of relative risk aversion of investors. It falls within the range usually considered plausible. It is also consistent with the estimates from Models 3 and 4. Model 3 -- the model which treats the wealth shares as unobserved constants -- estimates the degree of risk aversion to equal 4.03. (Recall when reading Table 4 that the coefficient of risk aversion in Model 3 is the parameter α.) When we assume all investors consider returns in dollar terms -- as in Model 4 -- the estimate of ρ is 4.09, as reported in Table 5.

Inspection of Tables 1-4 shows that the parameters of the variance matrix, Ω_t are not very different across the models. The matrix P from equation (13) is what was actually estimated by the maximum likelihood procedure, but we report P'P in the tables because it is more easily interpreted. P'P is the constant part of Ω_t.

The GARCH specification seems to be plausible in this model. Most of the elements of the H matrix were close to one, which indicates a high degree of persistence in the variance. One way to test GARCH is to perform a likelihood ratio test relative to a more restrictive model of the variance. Table 5 reports the results of testing the GARCH specification against a simple ARCH specification in which the matrix H in equation (13) is constrained to be zero. This imposes 5 restrictions on the GARCH model. As table 5 indicates, the restricted null hypothesis is rejected at the 1% level for each of Models 1-4.

Figures 1 and 2 plot the diagonal elements of the Ω_t matrix for Model 2. The time series of the variances for the other models are very similar to the ones for Model 2. In Figure 1 the variances of the returns on U.S., German and Japanese equities relative to U.S. bonds are plotted. As can be seen, the variance of U.S. equities is much more stable that the variances for the other equities. In the GARCH model, the 1-1 element in both the G and H matrices are small in absolute value. This leads to the fact that the variance does not respond much to past shocks and changes in the variance are not persistent. On the other hand, figure 1 shows us that toward the end of the sample the variance of Japanese equities fluctuated a lot and at times got relatively large. Recall that in measuring returns on Japanese and German equities relative to U.S. bonds a correction for exchange rate changes is made, while that is not needed when measuring the return on U.S. equities relative to U.S. bonds.

The variances of returns on German and Japanese bonds relative to the returns on U.S. bonds are plotted in Figure 2. Interestingly, the variance of Japanese bonds fluctuates much more than the variance of German bonds. The variance is much more unstable near the beginning of the sample period (while the variance of returns on Japanese equities gyrated the most at the end of the sample).

Figures 3 and 4 plot the point estimates of the risk premia. These risk premia are calculated from the point of view of U.S. investors. The risk premia are the difference between the expected returns from equation (9) and the risk neutral expected return for U.S. investors, which is obtained from equation (6) setting ρ equal to zero.

In some cases the risk premia are very large. (The numbers on the graph are the risk premia on a monthly basis. Multiplying them by 1200 gives the risk premia in percentage terms at annual rates.) The risk premia on equities are much larger than the risk premia on bonds. Furthermore, the risk premia vary a great deal over time. Comparing Figure 3 to Figure 1, it is clear that the risk premia track the variance of returns, particularly for the Japanese equity markets. The risk premia reached extremely high levels in 1990 on Japanese equities, which reflects the fact that the estimated variance was large in that year. The average risk premium on Japanese equities (in annualized rates or return) is 6.07 per cent. For U.S. equities it is 5.01 per cent, and 3.36 per cent is the average risk premium for German equities.

The risk premia on equities is always positive, but in a few time periods the risk premia on the bonds are actually negative. The risk premia on bonds in this model are simply the foreign exchange risk premia. They also show much time variation. At times they are fairly large, reaching a maximum of approximately 4 percentage points on the yen in 1990. Note, however, that the average risk premia -- 0.18 per cent for German bonds and 0.79 per cent for Japanese bonds -- are an order of magnitude smaller than the equity risk premia.

However, Figures 3 and 4 present only the point estimates of the risk premia, and do not include confidence intervals. The evidence in Section V suggests that these risk premia are only marginally statistically significant.

V. Tests of the null hypothesis of interest parity

If investors perceive foreign and domestic assets to be perfect substitutes, then a change in the composition of asset supplies (as opposed to a change in the total supply of assets) will have no effect on the asset returns. Suppose investors choose their portfolio only on the basis of expected return. In equilibrium, the assets must have the same expected rate of return. Thus, in equilibrium, investors are indifferent between the assets (the assets are perfect substitutes), and the composition of their optimal portfolio is indeterminate. A change in the composition does not affect their welfare, and does not affect their asset demands. Thus, sterilized intervention in foreign exchange markets, which has the effect of changing the composition of the asset supplies, would have no effect on expected returns.

In our model, investors in general are concerned with both the mean and the variance of returns on their portfolio. The case in which they are concerned only with expected returns is the case in which ρ equals zero. We shall test the null hypothesis that consumers care only about expected return and not risk.

Consider first the version of the model in which all investors have the same degree of risk aversion -- Model 2. That is, ρ is the same across all three countries. Then, the mean-variance equilibrium is given by equation (10). If we constrain ρ to equal zero in that equation, then we have the null hypothesis of

$$(14)\quad E_t z_{t+1} = -D_t + \mu_t^g \Omega_t e_3 + \mu_t^J \Omega_t e_5 .$$

Since the version of the model in which ρ is the same across all countries is a constrained version of the most general mean-variance model, then equation (14) also represents the null hypothesis for the general model (given in equation (9)).

We estimate two other versions of the mean-variance model. Model 3, as mentioned in above, treats the shares of wealth as constant but unobserved. The model is given by equation (11). If ρ is the same for investors in all countries, then $\alpha = \rho$. So, the null hypothesis of risk neutrality can be written as:

$$(15)\quad E_t z_{t+1} = -D_t + \gamma_1 \Omega_t e_3 + \gamma_2 \Omega_t e_5 .$$

The final version of the mean-variance model that we estimate is the one in which all investors evaluate returns in dollar terms -- Model 4. Equation (12) shows the equation for equilibrium expected returns in this case. The null hypothesis then, is simply

$$(16)\quad E_t z_{t+1} = -D_t .$$

So, equation (14) is the null hypothesis for Model 1 and Model 2, equation (15) is the null for Model 3 and equation (16) is the null for Model 4.

However, we have finessed a serious issue for the models in which investors assess asset returns in terms of different currencies. If investors are risk neutral, they require that expected returns expressed in terms of their domestic currency be equal. However, if expected returns are equal in dollar terms, then they will not be equal in yen terms or mark terms unless the exchange rates are constant. This is simply a consequence of Siegel's (1972) paradox (see Engel (1984, 1992) for a discussion).

The derivation of equation (9) does not go through when investors in one or more countries are risk neutral. The derivation proceeded by calculating the asset demands, adding these across countries and equating asset demands to asset supplies. However, when investors are risk neutral, their asset demands are indeterminate. If expected returns on the assets (in terms of their home currency) are different from each other, they would want to take an infinite negative position in assets with lower expected returns and infinite positive position in assets that have higher expected returns. If all assets have the same expected returns, then they are perfect substitutes, so the investor will not care about the composition of his portfolio. Hence, the derivation that uses the determinate asset demands when $\rho \neq 0$ does not work when $\rho = 0$.

If investors in different countries are risk neutral, then there is no equilibrium in the model presented here. Since it is not possible for expected returns to be equalized in more than one currency, then investors in at least one country would end up taking infinite positions.

So, we will consider three separate null hypotheses for our mean-variance model. One is that U.S. residents are risk neutral, so that expected returns are equalized in dollar terms. The other two null hypotheses are that expected returns are equalized in mark terms and in yen terms. The first of three hypotheses is given by equation (16), which was explicitly the null hypothesis when all investors considered returns in dollar terms. The second two null hypotheses can be expressed as:

(17) $E_t z_{t+1} = -D_t + \Omega_t e_3$,

and

(18) $E_t z_{t+1} = -D_t + \Omega_t e_5$.

So, equations (16), (17) and (18) can represent alternative versions of the null hypothesis for Models 1 and 3 (expressed in equations (9) and (11). Model 4 -- the one in which investors consider returns in dollar terms -- admits only equation (16) as a restriction.

The foregoing discussion suggests that the model in which ρ is restricted to be equal across countries will not have an equilibrium in which $\rho = 0$. However, we will still treat equation (14) as the null hypothesis for this model. Note that equation (14) is a weighted average of equations (16), (17) and (18) where the weights are given by the wealth shares. Equation (14) should be considered the limit as ρ goes to zero across investors. It is approximately correct when ρ is approximately zero. The same argument can be used to justify equation (15) as a null hypothesis for the model expressed in equation (11).

To sum up:

Model 1) The general mean-variance model given by equation (9) will be tested against the null hypotheses of equations (14), (16), (17) and (18).

Model 2) The mean-variance model in which ρ is restricted to be equal across countries, equation (10), will be tested against the null hypothesis of equation (14).

Model 3) The version of the mean-variance model in which the wealth shares are treated as constant -- equation (11) -- will be tested against the null hypotheses of equations (15), (16), (17) and (18).

Model 4) The version of the mean-variance model in which investors evaluate assets in dollar terms, given by equation (12), will be tested against the null hypothesis of equation (16).

The results of these tests are reported in Table 7. The null hypothesis of perfect substitutability of assets is not rejected at the 5% level for any model.

All but equation (18) can be rejected as null hypotheses at the 10% level when Model 1 is the alternative hypothesis. The p-value in all cases is close to 0.10, so there is some weak support for Model 1 against the null of risk neutrality.

For Model 2, the p-value is about 0.12. Since we were unable to reject the null that the coefficient of risk aversion was equal across countries, it is not surprising that Models 1 and 2 have about equal strength against the null of risk neutrality.

It is something of a success that the estimated value of ρ is so close to being significant at the 10% level. There are many tests which reject the perfect substitutability, interest parity model. But, none of these tests that reject perfect substitutability are nested in a mean-variance portfolio balance framework. For example, Frankel (1982), who does estimate a mean-variance model, finds that if he restricts his estimate of ρ to be non-negative that the maximum likelihood estimate of ρ is zero. Clearly, then, he would not reject a null hypothesis of $\rho = 0$ at any level of significance.

Our model performs better than Frankel's because we include both equities and bonds, and because we allow a more general model of Ω_t.[5]

Model 3 is unable to reject the null of perfect substitutability at standard levels of significance.

Model 4 rejects perfect substitutability at the 10% level. It might seem interesting to test the assumptions underlying Model 4. That is, does Model 4, which assumes investors assess returns in dollar terms outperform Model 2 which assumes investors evaluate returns in their home currency? Unfortunately, Model 4 is not nested in Model 1 or Model 2, so such a test is not possible.

Model 4 is nested in Model 3, the model which treats the wealth shares as constant and unobserved. Comparing equation (11) to equation (12), the restrictions that Model 4 place on Model 3 are that $\gamma_1 = 0$ and $\gamma_2 = 0$. The LR test statistic for this restriction is distributed χ^2 with 2 degrees of freedom. The value of the test statistic is 0.200, which means that the null hypothesis is not rejected. So, we cannot reject the hypothesis that all investors evaluate returns in dollar terms. However, equation (11) is not a

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[5] Frankel assumes Ω_t is constant.

very strong version of the model in which investors evaluate returns in terms of different currencies. It does not use the data on shares of wealth, and treats those shares as constants. It performs the worst of all the models against the null of perfect substitutability. So, we really cannot decisively evaluate the merits of allowing investor heterogeneity.

VI. Tests of CAPM Against Alternative Models of the Risk Premia

The CASE method of estimating the CAPM is formulated in such a way that it is natural to compare the asset demand functions from CAPM to more general asset demand functions. Unlike many other tests of CAPM, the alternative models have a natural interpretation and can provide some guidance to the nature of the failure of the mean-variance model if CAPM is rejected.

Any asset demand function that nests the asset demand functions derived above -- equations (2), (3) and (4) -- can serve as the alternative model to CAPM. That means, practically speaking, that the only requirement is that asset demands depend on expected returns with time-varying coefficients. Thus, in principle, we could use the CASE method to test CAPM against a wide variety of alternatives -- models based more directly on intertemporal optimization, models based on noise traders, etc.

In practice, because of limitations on the number of observations of returns and asset supplies, it is useful to consider alternative models that do not have too many parameters. This can be accomplished by considering models which are similar in form to CAPM, but do not impose all of the CAPM restrictions.

Thus, initially, we consider models in which the asset demand equations in the three countries take exactly the form of equations (2), (3), and (4), except that the coefficients on expected returns need not be proportional to the variance of returns. We will only test the version of CAPM in which the degree of risk aversion is assumed to be equal across countries. For that version, we can write the alternative model as:

(19) $\lambda_t^u = A_t^{-1}(E_t z_{t+1} + D_t)$

(20) $\lambda_t^g = A_t^{-1}(E_t z_{t+1} + D_t) + a_G e_3$

(21) $\lambda_t^j = A_t^{-1}(E_t z_{t+1} + D_t) + a_J e_5$.

In the alternative model, asset demands are functions of expected returns, but the coefficients, A_t^{-1} are not constrained to be proportional to the inverse of the variance of returns. As with the Solnik model, we assume in the alternative that the portfolios of investors in different countries differ only in their holdings of nominal bonds.

Aggregating across countries gives us:

$$\lambda_t = A_t^{-1}(E_t z_{t+1} + D_t) + \mu_t^g a_G e_3 + \mu_t^j a_J e_5.$$

This can be rewritten as

$$(22)\quad E_t z_{t+1} = -D_t + A_t \lambda_t - \mu_t^g a_G A_t e_3 - \mu_t^j a_J A_t e_5.$$

The matrix of coefficients, A_t, is unconstrained. However, for formal hypothesis testing, it is useful if model (10) is nested in model (22). So, we hypothesize that A_t evolves according to:

$$(23)\quad A_t = Q'Q + J\varepsilon_t\varepsilon_t'J + KA_{t-1}K.$$

We will assume that the variance of the error terms in the alternative model follows a GARCH process as in equation (13).

Thus, the CAPM described in equations (10) and (13) imposes the following restrictions on the alternative model described by equations (22), (23) and (13):

$$\rho^{1/2}P = Q$$
$$\rho^{1/2}G = J$$
$$\rho^{1/2}H = K, \text{ and}$$
$$\rho^{-1}-1 = a_G = a_J.$$

So, CAPM places 26 restrictions on the alternative model.

The alternative model was estimated by maximum likelihood methods. The value of the log of the likelihood is 2481.9026. Thus, comparing this likelihood value with the one given in Table 3, the test statistic for the CAPM is 28.89. This statistic has a chi-square distribution with 26 degrees of freedom. The p-value for this statistic is 0.316, which means we would not reject CAPM at conventional levels of confidence.

We now consider two generalizations of equation (22). In the first, we posit that the asset demands do not depend simply on the expected excess returns, $E_t z_{t+1} + D_t$. Instead, there may be a vector of constant risk premia, c, so that we replace equation (22) with

$$(24)\quad E_t z_{t+1} = c - D_t + A_t \lambda_t - \mu_t^g a_G A_t e_3 - \mu_t^j a_J A_t e_5.$$

CAPM places 31 restrictions on this alternative -- those listed above, and the restriction that $c = 0$. This test of CAPM is directly analogous to the tests for significant "pricing errors" in, for example, Gibbons, Ross and Shanken (1989) and Ferson and Harvey (1993).

This model was also estimated by maximum likelihood methods. The value of the log of the likelihood is 2482.6712. The chi-square statistic with 31 degrees of freedom is 30.428, which has a p-value of 0.495. So, again, we would not reject CAPM.

Another alternative is to retain equation (19) to describe asset demand by U.S. residents, but to replace equations (20) and (21) with

$$(25)\quad \lambda_t^g = A_t^{-1}(E_t z_{t+1} + D_t) + a_G$$

$$(26)\quad \lambda_t^J = A_t^{-1}(E_t z_{t+1} + D_t) + a_J.$$

In these equations, a_G and a_J are vectors. These equations differ from (20) and (21) by allowing more investor heterogeneity across countries. Each of the portfolio shares may differ between investors across countries -- rather than just the bond-holdings as in the Solnik model and in the alternative given by equation (22). Thus, aggregating equations (19), (25) and (26), and rewriting in terms of expected returns, we get:

$$(27)\quad E_t z_{t+1} = -D_t + A_t \lambda_t - \mu_t^g A_t a_G - \mu_t^J A_t a_J.$$

The CAPM model places 34 restrictions on equation (27). The model of equation (27) was estimated using maximum likelihood techniques. When the vectors a_G and a_J were left unconstrained, the point estimates of the portfolio shares were implausible. So, the model was estimated constraining the elements of a_G and a_J to lie between -1 and 1. This restriction is arbitrary, and is not incorporated in the optimization problems of agents, but it yields somewhat more plausible estimates of the optimal portfolio shares.

The value of the log of the likelihood in this case was 2499.8842. This gives us a chi-square statistic (34 d.f.) of 64.854. The p-value for this statistic is .0011. We reject CAPM at the one per cent level.

So, we reject CAPM precisely because the Solnik model does not allow enough diversity across investors in their holdings of equities. However, it would not be correct to conclude that a model that has home country bias in both equities and bonds outperforms the Solnik model. That is because our estimates of a_G and a_J are not consistent with home-country bias.

The vectors a_G and a_J represent the constant difference between the shares held by Americans on the one hand, and Germans and Japanese, respectively, on the other. Our estimate of a_G shows that Germans would hold the fraction 0.23857 more of their portfolio in U.S. equities than Americans. Furthermore, they would hold -1.0 less of German equities, and -1.0 less of Japanese equities than Americans. On the other hand, there would be home bias in bond holdings -- they would hold 1.0 more of German bonds. They would hold -0.31843 less of Japanese bonds, but they would hold 2.07988 more of U.S. bonds. (Recall that U.S. bonds are the residual asset. So, while the estimation constrained the elements of a_G to lie between -1 and 1, the difference between the share of U.S. bonds held by Germans and Americans is not so constrained.)

Likewise, the estimated difference between the Japanese and American portfolio is not indicative of home-country bias in equity holdings. While we do estimate that Japanese hold -1.0 less of U.S. equities than Americans, they also hold less of both German and Japanese equities. The difference between the American and Japanese share of German equities is very small: -0.00047, and of Japanese equities, -0.06301. But Japanese are also estimated to hold smaller shares of German bonds and Japanese bonds, the differences being -0.20343 and -0.12084, respectively. But, Japanese are estimated to hold much more of American bonds. The difference in the portfolio shares is 2.38784.

So, in fact, a general asset demand model that allows for diversity in equity holdings can significantly outperform CAPM. But the failure of CAPM is not due to the well-known problem of home-country bias in equity holdings.

VII. Conclusions

There are three main conclusions to be drawn from this paper.

First, the version of international CAPM presented here performs better than many versions estimated previously. Section V shows that the model has some weak power in predicting excess returns, whereas almost all previous studies have found that international versions of CAPM have little or no power. The models presented here differ from past models by allowing a broader menu of assets -- equities and bonds -- and by allowing some investor heterogeneity.

Second, the version of CAPM estimated here -- the Solnik model -- does not allow for enough investor heterogeneity. Section VI presents a number of tests of CAPM against alternative models of asset demand. The alternative models do not impose the constraint between means of returns and variances or returns that is the hallmark of the CAPM.

Some of these alternative models do not significantly outperform CAPM. Specifically, CAPM cannot be rejected in favor of models which still impose the Solnik result -- that portfolios of investors in different countries differ in their bond shares but not their equity shares. But, the alternative models need not impose the Solnik result. So, when the alternative model is generalized so that it does not impose the CAPM constraint between means and variances, and does not impose the Solnik result, CAPM is rejected.

The third major conclusion regards the usefulness of the CASE approach to testing the CAPM. In the CASE method, the alternative models are all built up explicitly from asset demand functions. In section VI, we considered several different models of asset demands. In each case, we built an equilibrium model from those asset demand functions that served as an alternative to CAPM. In some of the cases, we were not able to reject CAPM. But, when we altered our model of asset demand in a plausible way, we arrived at an equilibrium model which rejects CAPM. The advantage of the CASE approach is that we know very explicitly the economic behavior behind the alternative equilibrium models. When we fail to reject CAPM, we realize that it is not because CAPM is an acceptable model, but because the alternative model is as unacceptable as CAPM. When we reject CAPM, we know precisely the nature of the alternative model that is better able to explain expected asset returns. In this case, we have learned that CAPM must be generalized in a way to allow cross-country investor heterogeneity in equity demand. Perhaps incorporating capital controls or asymmetric information into the CAPM will prove helpful, but this is left for future work.

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Data Appendix

Foreign Exchange Rate

The foreign exchange rates that were used in calculating rates of return, and in converting local currency values into dollar values are taken from the data base at the Federal Reserve Bank of New York. They are the 9 A.M. bid rates from the last day of the month.

Equity Data

The value of outstanding shares in each of the three markets comes from monthly issues of Morgan Stanley's Capital International Perspectives. These figures are provided in domestic currency terms. I thank William Schwert for pointing out that these numbers must be interpreted cautiously because they do not correct adequately for cross-holding of shares, a particular problem in Japan.

The return on equities in local currency terms is taken from the same source. The returns are on the index for each country with dividends reinvested.

Bond Data

The construction of the data on bonds follows Frankel (1982) closely. For each country, the cumulative foreign exchange rate intervention is computed, on a benchmark of foreign exchange holdings in March 1973. That cumulative foreign exchange intervention is added to outstanding government debt while foreign government holdings of the currency are subtracted.

Germany:

dmasst = dmdebt + bbint -ndmcb

dmdebt = German central government debt excluding social security contributions. Bundesbank Monthly Report, Table VII.

dbbint = (DM/\$ exchange rate, IFS lie ae) × (Δforeign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)$_{t-1}$ × (\$/SDR)/(\$/SDR)$_{t-1}$, IFS line sa - ΔSDR allocations, IFS line 1bd × (\$/SDR));

$$\text{bbint} = \sum \text{dbbint} + 32.324 \times (\text{DM}/\$)_{1973:3}$$

ndmcb is derived from the IMF Annual Report

Japan

ynasst = yndebt + bjint - njncb

yndebt = $\sum$ Japanese central government deficit interpolated monthly, Bank of Japan, Economic Statistics Monthly, Table 82.

dbjint = (Yen/\$ exchange rate, IFS lie ae) × (Δforeign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)$_{t-1}$ × (\$/SDR)/(\$/SDR)$_{t-1}$, IFS line sa - ΔSDR allocations, IFS line 1bd × (\$/SDR));

bjint - $\sum$ dbjint + 18.125 × (Yen/\$)$_{1973:3}$

nyncb is derived from the IMF Annual Report

U.S.

doasst = dodebt + fedint - ndolcb

dodebt = Federal debt, month end from Board of Governors Flow of Funds -carter

carter = 1.5952 in 78:12 and 1.3515 in 79:3, the Carter bonds

dfedint = Δforeign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)$_{t-1}$ - ΔSDR allocations, IFS line 1bd × (\$/SDR));

fedint = $\sum$ dfedint + 14.366

ndolcb is derived from the IMF Annual Report

Wealth Data

Outside wealth is measured by adding government debt and the stock market value to the cumulated current account surplus on Frankel's benchmark wealth.
The monthly current account is interpolated from IFS line 77ad

Interest Rates

Interest rates are one month Eurocurrency rates obtained from the Bank for International Settlements tape.

TABLE 1

i^u_{t+1} ≡ the dollar return on dollar bonds between time t and t+1

i^g_{t+1} ≡ the mark return on mark bonds

i^J_{t+1} ≡ the yen return on yen bonds

R^u_{t+1} ≡ the gross dollar return on U.S. equities

R^g_{t+1} ≡ the gross mark return on German equities

R^J_{t+1} ≡ the gross yen return on Japanese equities

S^g_t ≡ the dollar/mark exchange rate at time t

S^J_t ≡ the dollar/yen exchange rate

$\mu^u_t \equiv W^u_t/(S^J_t W^J_t + S^g_t W^g_t + W^u_t)$, share of U.S. wealth in total world wealth

$\mu^g_t \equiv S^g_t W^g_t/(S^J_t W^J_t + S^g_t W^g_t + W^u_t)$, share of German wealth in total world wealth

$\mu^J_t \equiv S^J_t W^J_t/(S^J_t W^J_t + S^g_t W^g_t + W^u_t)$, share of Japanese wealth in total world wealth

TABLE 2

GARCH-CAPM MODEL WITH RHO DIFFERENT ACROSS COUNTRIES (MODEL 1)

RHO (U.S., GERMANY, JAPAN)

-1.3565e-07 -3.2690e-07 3.7562e-08

P'P MATRIX

0.00059049	0.00016524	-1.7010e-05	0.00020655	-2.9160e-05
0.00016524	0.00031849	0.00016684	0.00026570	0.00016344
-1.7010e-05	0.00016684	0.00021890	0.00013559	0.00017095
0.00020655	0.00026570	0.00013559	0.00068145	0.00029210
-2.9160e-05	0.00016344	0.00017095	0.00029210	0.00028625

DIAGONAL ELEMENTS OF G MATRIX

-0.024400 0.14830 0.19370 0.42910 0.35240

DIAGONAL ELEMENTS OF H MATRIX

0.84760 0.95300 0.90130 0.82470 0.82200

LOG-LIKELIHOOD VALUE

2469.47942

TABLE 3

GARCH-CAPM MODEL WITH RHO EQUAL ACROSS COUNTRIES
(MODEL 2)

RHO

4.6540

P'P MATRIX

0.00054260	0.00014092	-2.0086e-05	0.00018704	-3.4940e-05
0.00014092	0.00028210	0.00016825	0.00026609	0.00018164
-2.0086e-05	0.00016825	0.00022854	0.00013989	0.00018902
0.00018704	0.00026609	0.00013989	0.00068100	0.00031419
-3.4940e-05	0.00018164	0.00018902	0.00031419	0.00031561

DIAGONAL ELEMENTS OF G MATRIX

-0.039763	0.087661	0.17620	0.40677	0.37041

DIAGONAL ELEMENTS OF H MATRIX

0.86051	0.96511	0.90048	0.83463	0.80805

LOG-LIKELIHOOD VALUE
2467.45718

TABLE 4

GARCH-CAPM MODEL WITH WEALTH SHARES CONSTANT
(MODEL 3)

ALPHA

4.03400

GAMMA

-1.11955 0.739053

P'P MATRIX

0.000561370	0.000149152	-2.04214e-05	0.000190815	-3.82150e-05
0.000149152	0.000295907	0.000170993	0.000273723	0.000181881
-2.04214e-05	0.000170993	0.000232242	0.000145623	0.000192640
0.000190815	0.000273723	0.000145623	0.000701762	0.000321168
-3.82150e-05	0.000181881	0.000192640	0.000321168	0.000322825

DIAGONAL ELEMENTS OF G MATRIX

-0.0363077	0.105138	0.179205	0.417467	0.366548

DIAGONAL ELEMENTS OF H MATRIX

0.855632	0.961629	0.898721	0.826812	0.806122

LOG-LIKELIHOOD VALUE

2467.61284

TABLE 5

GARCH-CAPM MODEL IN DOLLAR TERMS
(MODEL 4)

RHO

4.09263

P'P MATRIX

0.000559210	0.000147301	-2.12924e-05	0.000190461	-3.82462e-05
0.000147301	0.000292889	0.000170081	0.000272490	0.000181657
-2.12924e-05	0.000170081	0.000231566	0.000144383	0.000192179
0.000190461	0.000272490	0.000144383	0.000698603	0.000320088
-3.82462e-05	0.000181657	0.000192179	0.000320088	0.000322144

DIAGONAL ELEMENTS OF G MATRIX

-0.0374155	0.101296	0.177600	0.416692	0.367108

DIAGONAL ELEMENTS OF H MATRIX

0.856192	0.962377	0.899183	0.827584	0.806167

LOG-LIKELIHOOD VALUE

2467.51288

TABLE 6

Tests of Significance of GARCH Coefficients
(Likelihood Ratio Tests, 5 d.f.)

Model	Chi-Square Statistic
Model 1	26.740
Model 2	24.174
Model 3	24.039
Model 4	24.050

All statistics significant at 1% level

TABLE 7

LR Tests of Null Hypothesis of Perfect Substitutability

Chi - Square Statistics
(P-value in Parentheses)

Model	Null 14	Null 15	Null 16	Null 17	Null 18
Model 1	6.433 (0.092)	-------	6.714 (0.082)	6.719 (0.081)	5.955 (0.114)
Model 2	2.388 (0.122)	-------	-------	-------	-------
Model 3	-------	1.640 (0.200)	2.981 (0.395)	2.986 (0.394)	2.222 (0.528)
Model 4	-------	-------	2.781 (0.095))	-------	-------

FIGURE 1

VARIANCE OF EQUITY RETURNS RELATIVE TO U.S. BONDS.

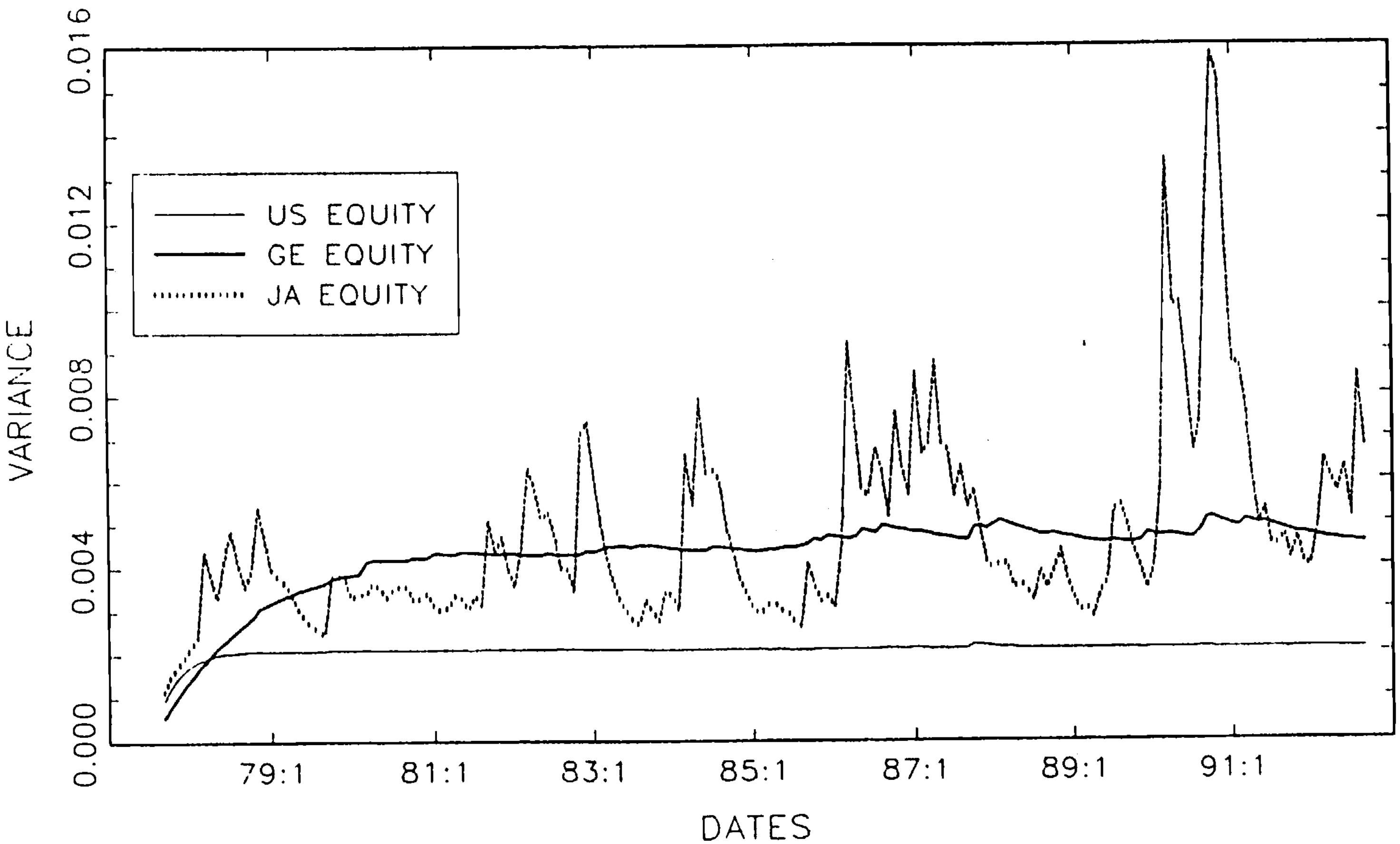

FIGURE 2

VARIANCE OF BOND RETURNS RELATIVE TO U.S. BONDS

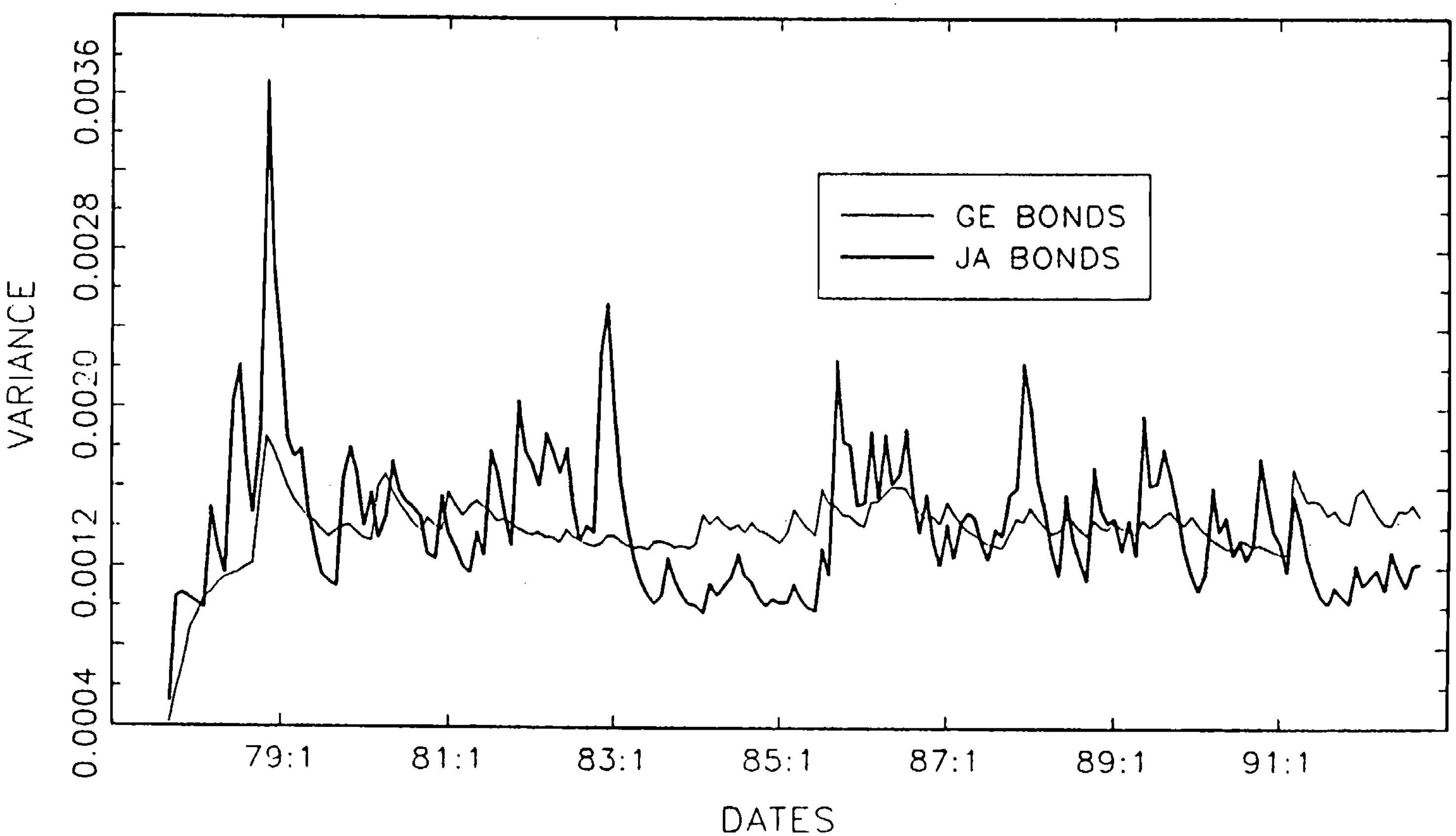

FIGURE 3

RISK PREMIA ON EQUITY RETURNS RELATIVE TO U.S. BONDS

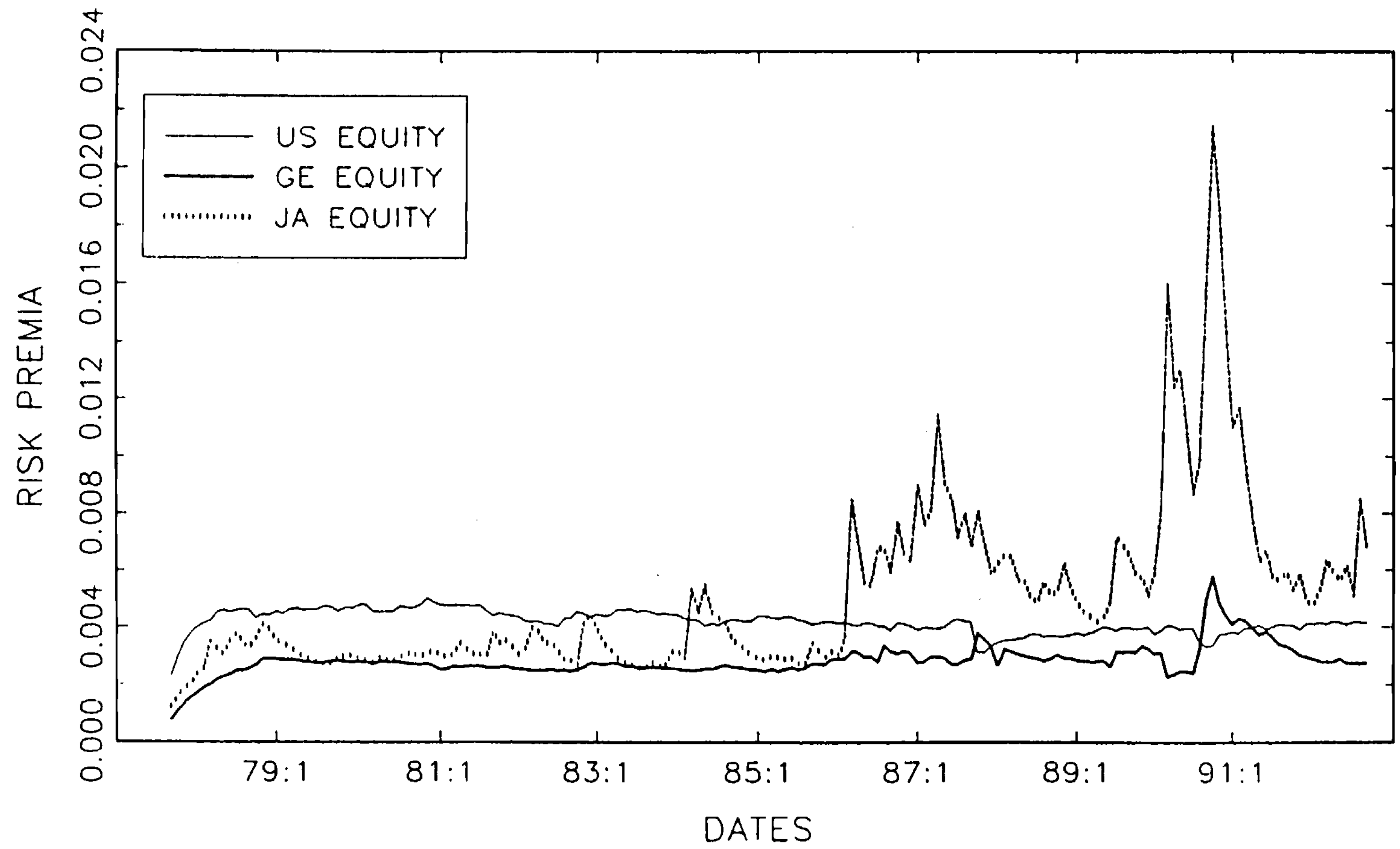

FIGURE 4

RISK PREMIA OF BOND RETURNS RELATIVE TO U.S. BONDS