NBER WORKING PAPER SERIES

# ON THE OPTIMAL TAXATION OF CAPITAL INCOME

Larry E. Jones
Rodolfo E. Manuelli
Peter E. Rossi

Working Paper No. 4525

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November, 1993

We thank V. V. Chari, Ken Judd, Nick Bull and seminar participants at several places for their comments and the National Science Foundation for financial support. Remaining errors are ours, of course. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #4525
November 1993

# ON THE OPTIMAL TAXATION OF CAPITAL INCOME


## ABSTRACT

One of the best known results in modern public finance is the Chamley-Judd result showing that the optimal tax rate on capital income is zero in the long-run. In this paper, we reexamine this result by analyzing a series of generalizations of the Chamley-Judd formulation. We show that in a model with human capital, if the tax code is sufficiently rich and there are no pure profits from accumulating human capital, then all distorting taxes are zero in the long-run under the optimal plan. In this sense, income from physical capital is not special. To gain a better understanding of these two conditions, we study examples in which they are not satisfied and show that the optimal tax rate on income from physical capital does not go to zero.

In those cases where the limiting tax rate is non-zero, we calculate its value for alternative specifications of the marginal welfare cost of taxation. Our results indicate that even for conservative specifications, tax rates of 10% and higher are possible under the optimal code.



Larry E. Jones
MEDS/KGSM
Northwestern University
Evanston, IL 60208
and NBER

Rodolfo E. Manuelli
Department of Economics
University of Wisconsin
Madison, WI 53706
and NBER

Peter E. Rossi
Graduate School of Business
University of Chicago
Chicago, IL 60637

# 1. Introduction

One of the most interesting and relevant topics in public finance concerns the optimal choice of tax rates. This question has a long history in economics beginning with the seminal work of Ramsey (1927). In that paper, Ramsey characterized the optimal levels for a system of excise taxes on consumption goods. He assumed that the government's goal was to choose these taxes to maximize social welfare subject to the constraints it faced. These constraints were assumed to be of two types. First, a given amount of revenue must be raised. Second, Ramsey understood that whatever tax system the government adopted, consumers and firms in the economy would react in their own interest through a system of (assumed competitive) markets. This observation gives rise to a second type of constraint on the behavior of the government--it must take into account the equilibrium reactions by firms and consumers to the chosen tax policies. This gives what has become known as a 'Ramsey Problem': Maximize social welfare through the choice of taxes subject to the constraints that final allocations must be consistent with a competitive equilibrium with distortionary taxes and that the given tax system raises a pre-specified amount of revenue.

Ramsey's insights have been developed extensively in the last few years [see the excellent survey in Auerbach (1985)] as applied to optimal commodity taxation. A parallel literature has concentrated on optimal taxation of factor income in dynamic settings. Contributions to this literature include Atkinson and Sandmo (1980), Chamley (1985) and (1986), Judd (1985) and (1990), Stiglitz (1985), (1987), Barro (1990), King (1990), Lucas (1990), Yuen (1990), Chari, Christiano and Kehoe (1991), Zhu (1991), Bull (1992) and Jones, Manuelli and Rossi (1993). Most of this literature discusses the setting of income taxes so as to maximize the utility of an infinitely lived representative consumer with perfect foresight subject to competitive equilibrium behavior and the need to fund a fixed stream of government expenditures. (Exceptions are the OLG group). These two approaches, commodity taxation and factor income taxation, are complementary. In some simple settings commodity taxation is similar to factor income taxation [see, for example, Atkinson and Stiglitz (1972)]. It seems that each approach has some advantages depending on the nature of the problem. For the purpose of this paper the direct approach that concentrates on factor taxation seems preferable because it makes it easier to impose interesting restrictions on tax rates and allows us to use asymptotic properties of the equilibrium (e.g., steady state relations) instead of more difficult to describe properties of the relevant Slutzky matrices.

The most startling finding of the literature on factor income taxation that studies representative agent infinitely-lived families models is that the optimal setting of the tax rate on capital income is zero in the long run. This was first exposited in Chamley and Judd in the context of simple single sector models of exogenous growth and has been shown to hold in cases with steady state, endogenous growth as well. Refinements to the stochastic case have been explored in King, Chari, Christiano and Kehoe and Zhu.

Our goal is to determine which are the key features of the economy which drive the Chamley/Judd zero capital tax result and, more generally, to explore the sensitivity of the optimal long-run tax rate to changes in both the set of feasible instruments and technology. We first ask why is it that capital income is treated so differently than other sources of income (e.g., labor income) in the Chamley/Judd set-up? When we introduce human capital with a constant returns to scale accumulation technology, we see that there is nothing special about capital income taxes- labor taxes are zero in the long-run as well. In fact, under a restricted but widely used class of preferences, all taxes (capital, labor and consumption) converge to zero in the limit. With constant returns to scale or linear capital accumulation technologies, no-arbitrage conditions insure the absence of profits for the planner to tax in the long-run.

To confirm the role of profits in determining the optimal long-run tax rates, we consider two examples which give rise to profits or rents which the planner can tax. We show how inelastic labor supply or the presence of productive public goods result in positive long-run taxes on capital income.

The qualitative nature of the Chamley/Judd results is changed not only by altering the technological assumptions but also by altering the feasible set of tax policies. All dynamic factor taxation exercises impose constraints on which sources of income can be taxed as well as on the path of permissible tax rates to avoid degenerate lump-sum tax results in which the government confiscates the initial capital stock. Presumably, these restrictions are based on political constraints which are not explicitly modelled. Other plausible sets of restrictions on tax policies can result in optimal non-zero limiting tax rates. In a third example (with no human capital), we show that if the planner cannot distinguish between capital and labor income, then it is optimal to tax both sources of income in the limit. (The example with productive public goods also features a restriction on tax rates.)

Taken as a whole, our findings force a reconsideration of the Chamley/Judd result in two important respects. First, our finding that all taxes converge to zero for plausible preferences and technologies suggests that the fundamental characteristic of optimal dynamic policies is the timing of taxes and not differences among types of factor taxation. Optimal solutions to a broad class of problems have the essential characteristic that the government taxes at a high rate in the initial periods to build up a surplus which it lives off forever. Second, realistic changes in technology or in the constraints on tax policies can easily result in positive long-run tax rates.

The remainder of the paper is organized as follows. In section 2, we give the extension of the Chamley-Judd model which includes human capital. In section 3, we develop a general formulation for studying the limiting behavior of the tax rate on capital income and give the examples described above to show how all three result in a non-zero tax rate on capital income in the long run. In section 4, we present some estimates of the asymptotic tax rates on capital income as a function of the marginal welfare loss of distortionary taxation for the examples in section 3. These show that the limiting tax rates on capital income are substantial even for low values of the cost of taxation. Finally, section 5 discusses extensions and offers some concluding comments.

## 2. Is Physical Capital Special?

We start by describing a generalized version of the model analyzed by Judd (1985) and Chamley (1986). We extend their approach to include both physical and human capital. Human capital is used as an input in the production of effective labor both for market and non-market uses.

This expanded setting allows us to address one of the questions posed in the introduction, namely whether there is something special about capital income. Specifically, we show that if three conditions are satisfied:

(i) there are no profits from accumulating either capital stock,

(ii) the tax code is sufficiently rich, and,

(iii) there is no role for relative prices to reduce the value of fixed sources of income,

then, both capital and labor income taxes are zero in the steady state. Moreover, if preferences satisfy an additional condition, <u>all</u> taxes are asymptotically zero.

We consider the simplest infinitely lived agent model consistent with the presence of both human and physical capital. We assume that there is one representative family that takes prices and tax rates as given. Their utility maximization problem is given by:

$$(P.1) \qquad \max \qquad \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_{1t} - n_{2t})$$

$$\text{s.t.} \qquad \sum_{t=0}^{\infty} [(1 + \tau_t^c)c_t + x_{ht} + (1 + \tau_t^m)x_{mt} + x_{kt} - (1 - \tau_t^n)w_t z_t -$$

$$(1 - \tau_t^k)r_t k_t - (1+\tau_t^c)T_t]p_t \leq 0,$$

$$k_{t+1} \leq (1 - \delta_k)k_t + x_{kt} \ ,$$

$$h_{t+1} \leq (1 - \delta_h)h_t + G(x_{ht}, h_t, n_{1t}),$$

$$z_t \leq M(x_{mt}, h_t, n_{2t}),$$

where $z_t$ is effective labor allocated to the production of market goods, and $T_t$ are transfers received from the government that the household takes as given. We assume that both $G$ and $M$ are homogeneous of degree one in market goods $(x_{jt}, j = h, m)$ and human capital, and $C^2$ with strictly decreasing (but everywhere positive) marginal products of all factors. The household takes as given the price of consumption at time t in terms of numeraire, $p_t$, as well as the tax rates, $\tau_t^j$, $j=k,n,m,c$. The standard non-negativity constraints apply.

Our formulation has some popular specifications as special cases. For example $M = hm(n)$ and $G = hg(n)$ correspond to Lucas (1988) and Bull (1992) as well as a large number of papers in the endogenous growth literature. The idea that the accumulation of human capital is an internal activity that uses market goods as well as human capital and labor appears in Heckman (1976) and it is relatively standard in the labor economics literature. Heckman (1976) assumes $G(x, h, n) = F(x, hn)$ with $F$ homogeneous of degree one.

The necessary conditions for an interior solution of the consumer's maximization problem are given by

(1.a) $$p_t = \beta^t \frac{u_c(t)}{1 + \tau_t^c} \frac{(1 + \tau_0^c)}{u_c(0)} \qquad t = 0, 1, \ldots$$

(1.b) $$u_\ell(t) = (1 - \tau_t^n) w_t M_n(t) \frac{u_c(t)}{1 + \tau_t^c} \qquad t = 0, 1, \ldots$$

(1.c) $$u_\ell(t) = \frac{u_c(t)}{1 + \tau_t^c} \frac{G_n(t)}{G_x(t)} \qquad t = 0, 1, \ldots$$

(1.d) $$p_t = p_{t+1}[1 - \delta_k + (1 - \tau_{t+1}^k) r_{t+1}] \qquad t = 0, 1, \ldots$$

(1.e) $$(1 + \tau_t^m) = (1 - \tau_t^n) w_t M_x(t) \qquad t = 0, 1, \ldots$$

(1.f) $$p_t / G_x(t) = p_{t+1} \left\{ \frac{(1 - \delta_h + G_h(t + 1))}{G_x(t + 1)} + (1 - \tau_{t+1}^n) w_{t+1} M_h(t + 1) \right\} \qquad t = 0, 1, \ldots,$$

in addition to the constraints on problem ($P$.1). Using the first order conditions and the assumption that $G$ and $M$ are homogeneous of degree one it is possible to show that in "equilibrium" the consumer's budget constraint can be greatly simplified.

Specifically, consider the term $\sum_{t=0}^{\infty} p_t[x_{kt} - (1-\tau_t^k) r_t k_t]$. Using the law of motion for $k_t$ we can rewrite this sum as

$$\sum_{t=0}^{\infty} p_t[x_{kt} - (1 - \tau_t^k) r_t k_t] = p_0[(1 - \delta_k) + (1 - \tau_0^k) r_0] k_0$$

$$+ \sum_{t=1}^{\infty} k_t \{p_t[1 - \delta_k + (1 - \tau_t^k) r_t] - p_{t-1}\}.$$

However, the second term on the right hand side is zero given (1.d).

Next, consider the term $\sum_{t=0}^{\infty} p_t[x_{ht} + (1 + \tau_t^m) x_{mt} - (1 - \tau_t^n) w_t M(x_{mt}, h_t,$

$n_{2t})]$. Given the law of motion for the stock of human capital (which holds as an equality) and the assumption that $G$ and $M$ are homogeneous in $(x, h)$ it follows that this term is given by

$$\sum_{t=0}^{\infty} p_t[((1 + \tau_t^m) - (1 - \tau_t^n)w_t M_x(t))x_{mt} +$$

$$\frac{h_{t+1} - (1 - \delta_h + G_h(t))}{G_x(t)} h_t - (1 - \tau_t^n)w_t M_h(t)h_t].$$

Using (1.e) and rearranging we obtain that the infinite sum is given by,

$$-p_0\left[ \frac{1 - \delta_h + G_h(0)}{G_x(0)} + (1 - \tau_0^n)w_0 M_h(0)]h_{0t} \right] +$$

$$\sum_{t=1}^{\infty} h_t \left\{ \frac{p_{t-1}}{G_x(t-1)} - p_t\left[ \frac{(1 - \delta_h + G_h(t))}{G_x(t)} + (1 - \tau_t^n)w_t M_h(t) \right]\right\}.$$

The second term in this expression equals zero from (1.f).

Thus, in equilibrium, the consumer's budget constraint is given by

$$\text{(2)} \qquad \sum_{t=0}^{\infty} p_t[(1 + \tau_t^c)(c_t - T_t)] = p_0\left\{ [1 - \delta_k + \right.$$

$$\left. (1 - \tau_0^k)r_0]k_0 + \left[ \frac{1-\delta_h+G_h(0)}{G_x(0)} + (1 - \tau_0^n)w_0 M_h(0) \right] h_0 \right\}.$$

The right hand side is simply the value of wealth at time zero while the left hand side does not include any terms that depend on $x_{jt}$, $k_t$ or $h_t$. The reason for this is simple since the activity "capital income" and the activity "labor income" display constant returns to scale in reproducible factors, their "profits" cannot enter the budget constraint in equilibrium since, if they are positive, the scale of this activity can be increased without any cost to the consumer and, conversely, if "profits" are negative the activity should be eliminated.

The representative firm rents capital and effective labor and it is subject to no taxes. Profit maximization implies

(3.a) $$r_t = F_k(k_t, z_t)$$

(3.b) $$w_t = F_z(k_t, z_t).$$

The Ramsey problem for this economy can be described as maximizing the welfare of the representative family given feasibility, the government's budget constraint and the first order conditions from both the household's and the firm's maximization problems as well as the household's budget constraint.[2] Using the method described in Lucas and Stokey (1983), this problem can be considerably simplified. The basic idea is that -- whenever it is possible -- the first order conditions should be used as defining prices and tax rates given an allocation. Hence, these conditions, along with the prices and tax rates as choice variables, need not be explicitly included in the planner's problem.

We first assume that taxes at time zero ($\tau_0^c$, $\tau_0^n$, $\tau_0^k$, $\tau_0^m$) are given and equal to zero and normalize $p_0$ to equal one. Next, (1.a) defines $p_t$, (1.b) determines $\tau_t^n$, (1.c) can be used to compute $\tau_t^c$, (1.d) to calculate $\tau_t^k$ and (1.e) to set $\tau_t^m$. This process leaves (1.f) as a condition that must be imposed. The reason for having to include this extra constraint is simple: We restricted the tax code to impose a tax on the output of the effective labor activity ($\tau_t^n$) and this tax affects both the static choice of labor supply ($n_{2t}$) and the dynamic choice of human capital ($h_t$). It is then necessary to guarantee that--given an allocation--the tax $\tau_t^n$ from (1.b) and (1.f) coincide. Imposing this equality is equivalent to requiring $\phi(t) = 0$ where

$$\phi(t) = \phi(v_{t-1}, v_t) = u_c(t-1)G_n(t) - \beta u_c(t)G_n(t-1)$$

$$\left\{ 1 - \delta_h + G_h(t) + G_n(t) \frac{M_h(t)}{M_n(t)} \right\},$$

where $v_t = (c_t, n_{1t}, n_{2t}, x_{ht}, h_t, x_{mt})$. In addition, it is necessary to impose (2).

---

[2]In this case, given that for each sequence of tax rates, the problems faced by consumers and firms are concave, it is appropriate to impose the first order conditions. For nonconvex cases, see Mirrlees (1975) and (1986) and Grossman and Hart (1983).

The planner's problem is then,

$$(P.2) \qquad \max \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_{1t} - n_{2t})$$

$$\text{s.t.} \sum_{t=0}^{\infty} \beta^t \Big[ u_c(t)(c_t - T_t) \Big] - W_0 = 0 \qquad (\lambda)$$

$$F(k_t, M(x_{mt}, h_t, n_{2t})) + (1 - \delta_k)k_t - k_{t+1} - x_{ht} - x_{mt} - c_t - g_t = 0 \qquad (\beta^t \mu_{1t})$$

$$(1 - \delta_h)h_t + G(x_{ht}, h_t, n_{1t}) - h_{t+1} = 0 \qquad (\beta^t \mu_{2t})$$

$$\phi(v_{t-1}, v_t) = 0 \qquad (\beta^t \eta_t),$$

where the first constraint is equation (2) after all prices have been substituted out using (1) and (3), $W_0 = u_c(0) \Big\{ (1 - \delta_k + F_k(0))k_0 + \Big[ \frac{(1-\delta_h+G_h(0))}{G_x(0)} + F_z(0)M_h(0) \Big] h_0 \Big\}$, and the symbols in parentheses indicate the Lagrange multipliers for each constraint.

Note that the first constraint is similar to the objective function in the sense that they are both discounted infinite sums of terms. Thus, given the Lagrange multiplier, $\lambda$, -- which, of course, is endogenous -- it is possible to rewrite $(P.2)$ as:

$$(P.3) \qquad \max \qquad \sum_{t=0}^{\infty} \beta^t W(c_t, n_{1t}, n_{2t}, T_t; \lambda) - W_o$$

subject to the 'flow' constraints from $(P.2)$, where

$$W(c, n_1, n_2, T; \lambda) \equiv u(c, 1-n_1-n_2) + \lambda\, u_c\, (c - T).$$

The first order conditions for this problem evaluated at the steady state are

$$(4.a) \qquad W_c^* = \mu_1^* - \eta^*(\phi_c^* + \beta\, \phi_{\underline{c}}^*)$$

$$(4.b) \qquad W_{n_1}^* = -\mu_2^* G_n - \eta^*(\phi_{n_1}^* + \beta\, \phi_{\underline{n}_1}^*)$$

$$(4.c) \qquad W_{n_2}^* = -\mu_1^* F_z^* M_n^* - \eta^*(\phi_{n_2}^* + \beta\, \phi_{\underline{n}_2}^*)$$

$$(4.d) \qquad 0 = \mu_1^* - \mu_2^* G_x^* - \eta^*(\phi_{x_n}^* + \beta\,\phi_{\underline{x}}^*)$$

$$(4.e) \qquad \mu_1^*(F_z^* M_x^* - 1) + \eta^* \phi_{x_m}^* = 0$$

$$(4.f) \qquad 1 = \beta(1 - \delta_k + F_k^*)$$

$$(4.g) \qquad 1 = \beta(1 - \delta_h + G_h^*) + \beta \frac{\mu_1^*}{\mu_2^*} F_z^* M_h^* + \frac{\beta\eta^*}{\mu_2^*}(\phi_h^* + \beta\,\phi_{\underline{h}}^*)$$

and the constraints from ($P$.2). To simplify notation we use the shorthand notation $\phi_x \equiv \frac{\partial\phi}{\partial x_t}(t)$ and $\phi_{\underline{x}} \equiv \frac{\partial\phi}{\partial x_t}(t+1)$.

The result of Judd and Chamley that capital taxes are zero in the limit follows directly from an evaluation of these first order conditions. Consider (1.d) and (4.f). From (1.d) evaluated at the steady state, it follows that $1 = \beta\,[1 - \delta_k + (1 - \tau_\infty^k)\,F_k^*]$. This condition and (4.f) directly imply that $\tau_\infty^k = 0$. Next we show that labor income taxes are zero as well.

**Proposition 1:** Assume that the solution to ($P$.2) converges to a steady state. Then, $\tau_\infty^n = 0$ and $\tau_\infty^m = 0$.

PROOF: We argue that generically, (4) has a solution when $\eta^* = 0$ and that, at that solution, $\tau_\infty^n = 0$.

Consider first the implications of assuming that $\eta^* = 0$. From (4.e) and (1.e) it follows that $1 + \tau_\infty^m = 1 - \tau_\infty^n$. Hence, if $\tau_\infty^n = 0$, then, $\tau_\infty^m = 0$ as well.

Since $W_{n_1}^* = W_{n_2}^* = W_n$, *and* $\frac{\mu_1^*}{\mu_2^*} = G_x^*$, the first order conditions are:

$$W_n^* + W_c^* F_z^* M_n^* = 0$$

$$G_x^* = \frac{G_n^*}{F_z^* M_n^*}$$

$$1 = F_z^* M_x^*$$
$$1 = \beta(1 - \delta_k + F_k^*)$$
$$1 = \beta(1 - \delta_h + G_h^* + G_x^* F_z^* M_h^*)$$
$$\delta_k k^* = x_k^*$$
$$\delta_h h^* = G(x_h^*, h^*, n_1^*)$$
$$c^* + x_m^* + x_k^* + x_h^* + g = F(k^*, M(x_m^*, h^*, n_2^*)).$$

This system has eight independent equations that can (generically) be solved for the eight endogenous variables ($c^*$, $n_1^*$, $n_2^*$, $x_m^*$, $x_h^*$, $x_k^*$, $k^*$, $h^*$).

From these equations, it follows that in any solution to the planner's problem, it must be the case that, $G_x^* = G_n^* / (F_z^* M_n^*)$. On the other hand, (1.b) and (1.c) imply that

$(1 - \tau_\infty^n) G_x^* = G_n^* / (F_z^* M_n^*)$. These two conditions imply that $\tau_\infty^n = 0$. ■

The previous proposition shows that $\tau_\infty^n = \tau_\infty^m = \tau_\infty^k = 0$. However, in general, this implies $\tau_\infty^c \neq 0$. To see this use (1.b) and the first order conditions in the proof above to get

$$1 + \tau_\infty^c = \frac{u_c^*}{u_\ell^*} F_z^* M_n^*.$$

From the planner's problem $F_z^* M_n^* = -W_n^*/W_c^* = \dfrac{u_\ell^* + \lambda u_{\ell c}^* c^*}{u_c^*(1+\lambda) + \lambda u_{cc}^* c^*}$. Hence,

$$1 + \tau_\infty^c = \frac{u_\ell^* u_c^* + \lambda u_{\ell c}^* u_c c^*}{u_c^* u_\ell^* + \lambda[u_c^* u_\ell^* + u_{cc}^* u_\ell^* c^*]}.$$

It follows that $\tau_\infty^c = 0$ if and only if

(5) $$u_{\ell c}^* u_c^* c^* = u_c^* u_\ell^* + u_{cc}^* u_\ell^* c^*.$$

In general, (5) will not be satisfied. However, there is an interesting class of functions that is consistent with this condition. It is straightforward to verify that if $u(c,\ell)$ is given by

$$u(c, \ell) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} v(\ell) & \text{if } \sigma > 0 \quad \sigma \neq 1, \\ \frac{c^{1-\sigma}}{1-\sigma} + v(\ell) & \text{if } \sigma > 0 \quad \sigma \neq 1, \\ \ln(c) + v(\ell) & (\text{if } \sigma = 1), \end{cases}$$

(5) holds.

Although a narrow class of functions from a theoretical point of view, it includes most of the functional forms used in applied work on optimal taxation as a special case. Additionally, a subset of this class contains the class of functions that are necessary for the economy to have a balanced growth path. It follows that in endogenous growth models that satisfy our technological constant-returns-to-scale assumption and have a balanced growth path all taxes must be zero in the long run.

Note that in this case, since all taxes are zero in the long run, it follows that the government must raise revenue in excess of expenditures in the initial periods. More precisely, since the long run interest rate is $(\beta^{-1} - 1)$ the steady state level of government assets (net claims on private income) is $b$, where $b$ is defined by $(\beta^{-1} - 1)b = g + T$.

As can be seen from the proof, our zero tax results are driven by zero profit conditions. Zero profits follow from the assumption of linearity in the accumulation technologies. In particular, if either G or M violate this assumption, $\tau^n_\infty \neq 0$. (Additionally, if physical capital accumulation is subject to decreasing returns, $\tau^k_\infty \neq 0$.)

In addition, there are two other features of the model that are essential for the result that taxes vanish asymptotically. The reader can verify that if transfers had been fixed in 'before tax' levels of consumption (i.e., $T_t$ enters the budget constraint rather than $(1+\tau^c_t)T_t$), Proposition 1 does not hold unless the limiting value of transfers is zero. The reason for this is simple: If transfers are not fixed in terms of consumption, it is possible for the planner to affect their value at time zero (the planner would like to make transfers as small as possible) by manipulating relative prices. In this example it is possible to show that $\tau_\infty^n \neq 0$. Thus, the first essential condition is that transfers are fixed in terms of after tax consumption. The second essential condition is that the tax code is sufficiently rich. It can be verified that if (5) does not hold and the planner is constrained to set $\tau_t^c = 0$ for all t, *then* $\tau_\infty^n \neq 0$.

To summarize, the model we discussed shows that, under some conditions, there is nothing special about capital income as all taxes are zero in the steady state. It also indicates that some features of the economy and available policies are essential in deriving this result.

## 3. When is the Limiting Tax on Capital Non-Zero?

In this section we discuss the other question raised in the introduction: What are the key features of the single sector model that imply that the limiting tax rate on capital income is zero under the optimal plan? We begin by describing an abstract framework that is useful in determining the asymptotic value of the capital income tax in general and then turn to the development of a series of three examples in which the limiting value of the optimal capital income tax is non-zero.

To keep the presentation as simple as possible, from now on we restrict attention to a one capital good growth model.

Consider a pseudo planner's problem given by:

$$(P.4) \qquad \max \sum_{t=0}^{\infty} \beta^t \{W(c_t, n_t, k_t, \lambda)\} - m(c_0, n_0, k_0, \lambda)$$

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

$$k_{t+1} \leq (1 - \delta)k_t + x_t$$

$$\phi_i(c_t, n_t, k_t, c_{t+1}, n_{t+1}, k_{t+1}) \leq 0 \qquad i = 1, 2, \ldots, I$$

This problem is general enough to have as a special case a one capital good version of (*P.3*). More precisely, consider a standard one sector growth model where preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

and the resource constraints are:

$$c_t + g_t + x_t \leq F(k_t, n_t)$$
$$k_{t+1} \leq (1-\delta)k_t + x_t.$$

Then, following the steps described in the previous section, it is possible to show that the Ramsey problem for this economy when the planner can choose $\tau_t^k$ and $\tau_t^n$, t=1,2,..., can be found as the solution to ($P.4$) where

$$W( c, n, k; \lambda) \equiv u( c, 1 - n) + \lambda[u_c\, c - u_\ell\, n],$$
$$m( c_o, n_o, k_o; \lambda) \equiv \lambda u_c( c_o, 1-n_o) [F_k(k_o,n_o) (1-\tau_o^k) + 1-\delta_k],$$

and the constraints, $\phi_i(t)$ correspond, for example, to bounds on the feasible tax rates.

Letting $\mu_t$ be the Lagrange multiplier corresponding to the resource constraint (once again the law of motion for capital has been substituted in) and $\eta_{it}$ the Lagrange multiplier for the $\phi_i$ constraint in period $t$, the first order conditions for ($P$.3) are:

$$c: \qquad W_c(t) - \mu_t + \sum_{i=1}^{I} \eta_{it}\phi_{ic}(t) + \beta \sum_{i+1}^{I} \eta_{it+1}\phi_{i\underline{c}}(t+1) = 0$$

$$n: \qquad W_n(t) + \mu_t F_n(t) + \sum_{i=1}^{I} \eta_{it}\phi_{in}(t) + \beta \sum_{i=1}^{I} \eta_{it+1}\phi_{i\underline{n}}(t + 1) = 0$$

$$k': \; -\mu_t + \beta\mu_{t+1}[1-\delta + F_k(t+1)] + \beta W_k(t+1) + \beta \sum_{i=1}^{I} \eta_{it}\phi_{ik}(t+1) + \beta^2 \sum_{i=1}^{I}$$

$$\eta_{it+1}\phi_{i\underline{k}}(t+2)=0,$$

where, as before, for any variable $x_t$ we use the following convention:

$$\phi_{ix}(t) = \frac{\partial\phi_i}{\partial x_t} (x_{t-1}, x_t)$$

$$\phi_{i\underline{x}}(t) = \frac{\partial \phi_i}{\partial x_{t-1}} (x_{t-1}, x_t).$$

At the steady state, we can summarize the restrictions that the model imposes in the following set of equations:

$$\text{(6.a)} \qquad W_c^* - \mu^* + \sum_{i=1}^{I} \eta_i^* (\phi_{ic}^* + \beta \phi_{i\underline{c}}^*) = 0$$

$$\text{(6.b)} \qquad W_n^* + \mu^* F_n^* + \sum_{i=1}^{I} \eta_i^* (\phi_{in}^* + \beta \phi_{i\underline{n}}^*) = 0$$

$$\text{(6.c)} \qquad -1 + \beta(1 - \delta + F_k^*) + \frac{\beta W_k^*}{\mu^*} + (\mu^*)^{-1} \beta \sum_{i=1}^{I} \eta_i^* (\phi_{ik}^* + \beta \phi_{i\underline{k}}^*) = 0$$

$$\text{(6.d)} \qquad c^* + g = F(k^*, n^*) - \delta k^*.$$

In the interpretation of (*P.4*) as the Ramsey problem faced by a planner in the standard one sector growth model, it is possible to show that (1.a) holds in any interior equilibrium. The steady state version of this condition is

$$1 = \beta[1 - \delta_k + (1 - \tau_\infty^k) F_k^*],$$

Given the definition of W (in this case), it follows that $W_k^* = 0$. If there are no binding $\phi_i$ constraints (i.e., if the tax bounds are not binding) then (6.c) directly implies $\tau_\infty^k = 0$.[3]

Although for the standard case the formulation in (6) is unnecessarily cumbersome it will prove useful in studying deviations from the basic setup that are both economically interesting and that yield non-zero limiting tax rates on capital income. In terms of this more general setting it follows that $\tau_\infty^k \neq 0$ if and only if

---

[3]Although this formulation takes $\lambda$ as given, in the true problem $\lambda$ is such that the appropriate version of (2) (the budget constraint) is satisfied. Since our arguments do not depend on the value of $\lambda$ (only the fact that it is positive), we can study (P.4) to determine the properties of optimal steady state taxes.

$$\beta W_k^* + \beta \sum_{i=1}^{I} \eta_i^*(\phi_{ik}^* + \beta\phi_{ik}^*) \neq 0.$$

In what follows, we will describe three economic settings that result in this expression being non-zero. Our examples will highlight the fact that rents, restrictive tax codes or profits generated by public goods will result in either $W_k^* \neq 0$ or $\eta_i^* \neq 0$.

## (a) Pure Rents: Inelastic Labor Supply

In this example we study a model that differs from the simple one capital good version of the setting of section 2 in only one dimension: labor is supplied inelastically. The model then resembles the standard one sector growth model studied by Cass [1965] and Koopmans [1965]. We assume that there is a bound on the rate at which labor income (a "pure rent") can be taxed. This bound might arise, for example, due to political or other types of constraints that we do not explicitly model. In the absence of such a constraint it is possible to show that the solution to the problem is similar to that obtained whenever lump sum taxes are available. It is to prevent this uninteresting (from the point of view of this paper) outcome that we impose an exogenously given bound on taxation of a factor in fixed supply. In this case the wage rate will be given by $w_t = F_n(k_t, 1)$. However, the marginal condition that determines the marginal rate of substitution between consumption and leisure as a function of the after-tax real wage - equation (1.b) - no longer applies. It can be shown that, the relevant version of (2) is

$$(7) \qquad \sum_{t=0}^{\infty} \beta^t u_c(t)[c_t - (1 - \tau_t^n)F_n(t)] = u_c(0)[F_k(0)(1 - \tau_0^k) + 1 - \delta]k_0.$$

Since taxation of labor income generates only income effects it is clear that the optimal tax rate is $\tau_t^n = 1$. If this is the bound on labor taxes, it can be shown that the result in section 2 holds--the limiting capital income tax rate is zero. To make the problem more interesting--and to highlight the consequences of less than full taxation of profits--we will make two additional assumptions.

First, that there is an upper bound on the tax rate on labor income given by $\bar{\tau}^n < 1$. Second, we assume that the present value of labor income evaluated using the prices $\{p_t\}$ induced by the solution to the *unconstrained* planner's problem (described in Proposition 2) and the initial revenue from capital taxation falls short of the value of government expenditure. With these two assumptions it follows that the government will choose $\tau_t^n = \bar{\tau}^n$.

For this problem, it follows that the relevant pseudo utility function $W$ is

$$W(c, n, k, \lambda) = u(c) + \lambda[u_c(c)(c - (1 - \bar{\tau}^n)F_n(k, 1))]$$

and $W_k = -\lambda u_c(c)(1 - \bar{\tau}^n)F_{kn}$ is different from zero if $F_{kn} \neq 0$.

The first order conditions for this problem in the steady state are then given by (6) with the $\eta_i^* = 0$, i = 1, ..., I. The relevant conditions are,

$$W_c^* = \mu^*$$

$$1 = \beta[F_k^* + 1 - \delta] + \frac{\beta W_k^*}{\mu^*}$$

Since $\mu^* > 0$ (this is the multiplier corresponding to the resource constraint) it suffices to show that $W_k^* < 0$ to prove that $\tau_\infty^k > 0$. Note that given $F_{kn} > 0$ (from our assumption that $F$ is concave and homogeneous of degree one and $F_{kn} \neq 0$), $W_k^* < 0$ if and only if $\lambda > 0$. We now show that $\lambda > 0$.

**Proposition 2:** Let $(\hat{c}, \hat{x}, \hat{k})$ be the solution to the unconstrained planner's problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.} \quad c_t + k_{t+1} + g_t \leq F(k_t, 1) + (1 - \delta)k_t$$

$$k_0 > 0 \text{ given.}$$

If

$$(8) \qquad \hat{u}_c(0)\tau_0^k F_k(\hat{k}_0, 1)k_0 + \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)\bar{\tau}^n \hat{F}_n(t) < \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t) g_t,$$

then the solution to the planner's problem is such that $\lambda > 0$.

Before we present the proof of the claim it is useful to interpret (8). The left hand side is revenue from taxation of capital at time zero plus the present value of labor income taxation. Since both $k_0$ and labor are in fixed supply, taxation of these two factors is equivalent to lump sum taxation. The right hand side is the present value of government spending. Thus, (8) says that, at the first best allocation, revenue from lump sum taxes falls short of expenditures. Without this assumption the problem becomes uninteresting as no distortionary taxes are used.

PROOF: Consider the following less restrictive version of the Ramsey problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t + g_t + k_{t+1} \leq F(k_t, 1) + (1 - \delta)k_t$$

$$\text{s.t. } \sum_{t=0}^{\infty} \beta^t u_c(t)[c_t - (1 - \bar{\tau}^n)F_n(t)] - u_c(0)[F_k(0)(1 - \tau_0^k) + 1 - \delta]k_0 \geq 0.$$

and let $\lambda$ be the Lagrange multiplier associated with the second constraint. Suppose first that $\lambda = 0$. In this case the solution is "first best" and given by $(\hat{c}, \hat{k})$.

Note that,

$$\sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)(\hat{c}_t - (1 - \bar{\tau}^n)\hat{F}_n(t)) = \sum_{t=0}^{\infty} \beta^t \hat{u}^c(t)[(\hat{F}_k(t) + 1 - \delta)\hat{k}_t +$$

$$\bar{\tau}^n F_n(t) - \hat{k}_{t+1} - g_t]$$

$$= \hat{u}_c(0)(\hat{F}_k(0) + 1 - \delta)k_0 + \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)\bar{\tau}^n F_n(t) - \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t) g_t,$$

where we used the property that $\hat{u}_c(t) = \hat{u}_c(t+1)\beta[\hat{F}_k(t + 1) + 1 - \delta]$ for $t \geq 0$.

Using this equality in the second constraint of the less restrictive problem we get

$$\hat{u}_c(0)(\hat{F}_k(0) + 1 - \delta)k_0 + \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t)\bar{\tau}^n \hat{F}_n(t) - \sum_{t=0}^{\infty} \beta^t \hat{u}_c(t) g_t -$$

$$\hat{u}_c(0)[(1 - \tau_0^k)\hat{F}_k(0) + 1 - \delta]k_0 \geq 0$$

or,

$$\hat{u}_c(0)\tau_0^k\hat{F}_k(0)k_0 + \sum_{t=0}^{\infty} \beta^t\hat{u}_c(t)\bar{\tau}^n\hat{F}_n(t) \geq \sum_{t=0}^{\infty} \beta^t\hat{u}_c(t)g_t.$$

This contradicts (8) and hence it shows that $\lambda > 0$. ■

An alternative, more intuitive argument that shows that $\lambda > 0$ can be constructed as follows: Denote by $V^*$ the maximized value of the planner's objective function, and note that the change in $V^*$ due to an increase in $\tau_0^k$ is given by $\lambda u_c^*(0)F_k^*(0)k_0$. Since increases in $\tau_0^k$ are equivalent to increases in lump sum taxes and since the higher the level of lump sum taxes the higher the value of utility, it follows that $\lambda$ must be positive.

Our findings for the case of factor income taxation coincide with the results of Diamond and Mirrlees (1971) and Stiglitz and Dasgupta (1971) that also show that the existence of pure profits affects the optimal commodity tax schedule.

## (b) Equal Taxes on Capital and Labor

The previous example discussed a particular form of violation of the assumptions that give rise to the Chamley-Judd result: in terms of the pseudo-utility function $W$ of the planner's problem, the term $W_k$ is not zero; however, we did not consider the alternative possibility that the constraints $\phi_i$ are binding in the steady state. A simple example of such a case is when the planner cannot distinguish between capital and labor income and, hence, it is constrained to set $\tau_t^k = \tau_t^n$, $t = 1, 2, ....$ This is the case studied in this section.

For this setting, the appropriate versions of (1.b) and (1.d) with $p_t$ substituted out imply that $\tau_t^n = \tau_t^k$ if and only if:

$$\phi \equiv \left[ \frac{u_c(t-1)}{\beta u_c(t)} - (1 - \delta) \right] \Big/ F_k(t) - \frac{u_l(t)}{u_c(t)F_n(t)} = 0$$

Note that this condition defines a function $\phi$ that depends on $(c_{t-1}, n_{t-1}, k_t, c_t, n_t)$.

In this case, it can be easily shown that the appropriate $W$ function is given by

$$W(c, n, \lambda) = u(c, 1 - n) + \lambda[u_c(c, 1 - n)c - u_\ell(c, 1 - n)n].$$

Letting the Lagrange multiplier on the resource constraint be $\mu_t$, and the Lagrange multiplier corresponding to the constraint $\phi$ be $\eta_t$ we have that, in the steady state, the system of equations described in (6) is:

$$W_c^* = \mu^* - \eta^*(\phi_c^* + \beta\phi_{\underline{c}}^*)$$

$$W_n^* = -\mu^* F_n^* - \eta^*(\phi_n^* + \beta\phi_{\underline{n}}^*)$$

$$1 = \beta[F_k^* + 1 - \delta] + \beta\,\frac{\eta^*}{\mu^*}\,\phi_k^*$$

We now prove that it is not possible to satisfy the steady state version of the first order conditions for the planner's problem if it is assumed that $\tau_\infty^n = \tau_\infty^k = 0$.

**Proposition 3:** Assume that either $W_c$ or $W_n$ depends on $\lambda$ and that the solution to the planner's problem is continuous in $\tau_0^k$. Then the steady state tax rates on capital and labor income are non-zero.

PROOF: We argue by contradiction. As before, a necessary and sufficient condition for $\tau_\infty^k = 0$ is that $1 = \beta[(1 - \delta) + F_k^*]$. In this context this condition is equivalent to either $\eta^* = 0$ or $\phi_k^* = 0$. We next show that, in general, neither of these two equalities can hold.

Consider first $\phi_k^* = 0$. From the equation defining $\phi$ it follows that

$$\phi_k^* = \frac{u_\ell^*}{u_c^* F_n^*}\left[-\frac{F_{kk}^*}{F_k^*} + \frac{F_{kn}^*}{F_n^*}\right]$$

However, if $\tau_\infty^k = 0$, the appropriate version of equation (1.b) implies $u_\ell^* = u_c^* F_n^*$. Thus, under the assumption $\tau_\infty^n = \tau_\infty^k = 0$, the steady state value of $\phi_k^*$ is

$$\phi_k^* = -\frac{F_{kk}^*}{F_k^*} + \frac{F_{kn}^*}{F_n^*},$$

and under our assumptions this is strictly positive.

Next we argue that $\eta^* \neq 0$. Suppose to the contrary that it is equal to zero. Then (6) and (1.b) imply

$$(9)\begin{cases} \beta[F_k^* + 1 - \delta] = 1, \\ F_n^* u_c^* - u_\ell^* = 0, \\ F_n^* W_c^* + W_n^* = 0, \\ c^* + g = n^*[F_k^*(k^*/n^*) + F_n^* - \delta k^*/n^*], \end{cases}$$

Formally, the system of equations in (9) is a system of four equations in the four unknowns ($k^*$, $n^*$, $c^*$, $\lambda$). Generically, it has a locally unique solution. Since this system does not depend on $\tau_0^k$ it follows that $\lambda$ (as well as ($k^*$, $n^*$, $c^*$)) does not depend on $\tau_0^k$. This, however, cannot be the case in general. In fact, we can show that $\lambda$ must vary as $\tau_0^k$ varies. In particular, consider any situation in which $\lambda > 0$ and increase $\tau_0^k$ up to the point in which at the first best allocation (the " ^ " allocation described in the proof of Proposition 2) the present value of the sequence $\{g_t\}$ equals $\tau_0^k F_k(0)k_0$. In this case the government in effect has access to lump sum taxes and it is optimal for it to choose $\tau_t^k = \tau_t^n = 0$ $t \geq 1$. However, since the allocation is first best it follows that $\lambda = 0$ contradicting the result that $\lambda$--as well as the steady state values $k^*$, $n^*$, $c^*$--is determined by (9). Therefore, $\eta^* \neq 0$.[4] ■

[4]This result depends on our assumption that at least one of $W_c$ and $W_n$ depend on $\lambda$. This need not be true for arbitrary utility functions. Consider, for example, the function $u$ given by

$$u(c, 1 - n) = ln(c) - \phi ln\,(n).$$

In this case it can be readily verified that $W_c = u_c$ *and* $-W_n = u_\ell$ and, hence, Although the previous claim rules out asymptotically zero taxes it does not show under what conditions the asymptotic tax rates are positive. It is possible to extend the analysis and show that indeed $\tau_\infty^k = \tau_\infty^n > 0$. Although we suspect this result to hold quite generally, we have been able to formally prove it in the case of a "large" discount factor ($\beta$ close to one), or in the case of separable utility and $\beta$ "not too small." Formally, we summarize our results in

**Proposition 4:** Assume either that $u_{c\ell} > 0$ and $\beta \geq \underline{\beta}$ for some $\underline{\beta}$ to be specified or that $u_{c\ell} = 0$ and $\delta - (1 - \beta^{-1})^2 \geq 0$. Then $\tau_\infty^k = \tau_\infty^n > 0$.

PROOF: Assume to the contrary that $\tau_\infty^k = \tau_\infty^n < 0$. Then, it follows from the steady state equations that $\eta^* > 0$ and, from the consumer's first order condition that $u_\ell^* > u_c^* F_n^*$. Next, multiply the first equation in the specific version of (6) by $F_n^*$ and add it to the second to obtain,

$$W_n^* + W_c^* F_n^* = -\eta^* [F_n^*(\phi_c^* + \beta^{-1}\phi_{c'}^*) + (\phi_n^* + \beta^{-1}\phi_{n'}^*)]$$

Simple calculations show that,

$$W_n^* + W_c^* F_n^* = (1 + \lambda)(u_c^* F_n^* - u_\ell^*) + \lambda[u_{\ell\ell}^* n^* + u_{cc}^* c^* F_n^* - u_{c\ell}^* (c^* + n^* F_n^*)]$$

is negative. Thus, since $\eta^* > 0$ it must be that

$$F_n^*(\phi_c^* + \beta^{-1}\phi_{c'}^*) + (\phi_n^* + \beta^{-1}\phi_{n'}^*) > 0.$$

However, using the definition of $\phi$ and imposing the steady state assumption it follows that

$$\phi_c^* + \beta^{-1}\phi_{c'}^* = \frac{u_{cc}^*}{\beta u_c^* F_k^*}(\delta - (1 - \beta^{-1})^2) - \frac{u_{c\ell}^*}{u_c^* F_n^*},$$

(9) contains only three independent equations in three unknowns ($c^*$, $k^*$, $n^*$). In fact, the system of equations simply describe the steady state under lump sum taxation. This is another example in which $\tau_\infty^k = \tau_\infty^n = 0$ and thus that the level of long-run debt can be determined as in section 2.

$$\phi_n^* + \beta^{-1}\phi_{n'}^* = \frac{u_{c\ell}^*}{\beta u_c^* F_k^*}[\beta^{-1} + \beta(1 - \delta) - 2] + \frac{u_\ell^*}{u_c^* F_n^*}\left[\frac{u_{\ell\ell}^*}{u_\ell^*} + \frac{F_{nn}^*}{F_n^*}\right].$$

Then, note that the solution is continuous in $\beta$ and for $\beta = 1$, $\phi_c^* + \beta^{-1}\phi_{c'}^* < 0$ and $\phi_n^* + \beta^{-1}\phi_{n'}^* < 0$ contradicting the necessary condition. Thus, by continuity, there exists $\underline{\beta} < 1$ such that the condition is also violated for all $\underline{\beta} < \beta < 1$.

Alternatively, assume $u_{c\ell} = 0$. In this case, $\phi_n^* + \beta^{-1}\phi_{n'}^* < 0$ and, if $\delta > (1 - \beta^{-1})^2$, $\phi_c^* + \beta^{-1}\phi_{c'}^* < 0$ as well. Thus, this results in a contradiction. ■

Our result has a second or third best flavor. Starting from a relatively generous (in terms of available taxes) tax regime a restriction on the ability to independently tax goods and factors results in a distortion. This result also has an analog in the literature on commodity taxation in the sense that even in cases where relatively sharp results are available (e.g., constant tax rates) they depend on the "right" number of taxes being available [See Auerbach (1985)]. Moreover, in some cases restrictions on tax rates can result in productive inefficiencies.

A natural question to ask is whether restrictions on tax rates that do *not* involve the tax rate on capital have similar consequences. There are two examples of this that are interesting. The first includes multiple consumption goods with the restriction that they all have to be taxed at the same rate. The second allows for different types of labor with the restriction that the planner must use the same tax rate for all types. In both cases it can be shown that $\tau_\infty^k > 0$ although there are no restrictions imposed directly on the tax rate of capital income.[5] Another example that illustrates how the value of one tax rate depends on the existing menu of taxes can be constructed from the model in section 2 by assuming that there are no consumption or intermediate goods taxes available ($\tau^c_t = \tau^m_t = 0$). In that case it is straightforward to show that although $\tau^k_\infty$ is still zero it is not possible for $\tau^n_\infty$ to be zero as well.

### (c) Productive Public Goods

The previous two examples illustrated how departures from the basic model in the form of the existence of "pure rents" (as in the case of an inelastic labor supply) or restrictions across tax rates (as in the equality of capital and labor income taxes) can result in positive limiting tax rates on capital.

To this point, we have followed the Ramsey program and ignored the presence of public goods. We now generalize the model to include public goods that are productive. Since many public goods are of this form, this is an important special case quantitatively. Here, the fact that limiting capital taxes are non-zero results due to a combination of the "pure rents" case and the "restrictions across tax rates" outlined above. The specification which we study is an interesting example in which profits are generated in equilibrium.

We model the case in which public expenditures have an impact on the overall productive capacity of the economy. We interpret $g$ as expenditure in public capital (roads, infrastructure) and services (administration of justice

---

[5]Stiglitz (1987) analyzes the two-types of labor case in the context of heterogeneous agents. He concludes that the result that $\tau^k_\infty > 0$ whenever there is only one tax rate on labor income depends on both income distribution and technological substitution possibilities. Using the single agent framework of section 2 and assuming that the ratio of the marginal productivity of the two types of labor *is not* independent of the capital stock it is possible to show that the limiting tax rate on capital is non-zero. It seems then that the essential ingredient is that the planner must use the same tax rates for both types of labor and not that the planner puts arbitrary weights on the utility of different individuals.

and crime and fire prevention) that, for some reason, cannot be sold directly to firms. To simplify we assume that $g$ enters in the aggregate production function.[6] This results in pure profits. For simplicity, we do not distinguish the separate effects of fixed factors and government expenditures and consider a per capita production function at the firm level given by

$$y \leq F(k, n, N_f g),$$

where $N_f$ is the number of firms. We assume that $F$ is homogeneous of degree one and concave in all three variables.

Given our assumptions, profits are given by,[7]

$$\pi_t = F_g(t)\, g_t N_f \,.$$

The existence of pure profits brings up the issue of why are they not "taxed away" by the government. It seems reasonable to assume that it is not possible to distinguish pure profits (the return to $g$) from capital income. In this case we restrict the government to use only one tax on both pure profits and capital income.

The relevant version of the budget constraint (2) is

$$\sum_{t=1}^{\infty} \beta^t \left\{ u_c(t)c_t - [u_c(t-1) - \beta(1-\delta)u_c(t)] \frac{F_g(t)}{F_k(t)} \frac{N_f\, g_t}{\beta} - u_\ell(t)n_t \right\}$$

$$+ u_c(0)c_0 - (1-\tau_0^k) \frac{F_g(0)}{F_k(0)} \frac{N_f\, g_0}{\beta} = u_c(0)\{[(1-\tau_0^k)F_k(0) + 1 - \delta]k_0\}$$

---

[6]For evidence of the effect of some forms of government expenditures on the marginal product of capital see Aschauer (1989).

[7]Consider the more general setting given by $F(k, n, z, N_f\, g) = F^1(k, n, z)F^2(N_f\, g)$, where $F^1$ is constant returns to scale in $(k, n, z)$ and $F^2$ is an arbitrary increasing function. For this alternative interpretation profits are,

$$\pi = F_z^1 z F^2(N_f\, g).$$

If this formulation is used, instead of the simpler specification in the text, the result that $\tau_\infty^k > 0$ depends on $F_{zk}^1 z F^2(N_f\, g) > 0$.

where, as above, we take $\tau_0^k$ as given. Note that, as in the previous case, the tax rate on pure profits generated by the public good is given by

$$1 - \tau_t^k = \frac{u_c(t-1) - \beta(1-\delta)u_c(t)}{\beta u_c(t) F_k(t)} .$$

The $W$ function corresponding to this setting is

$W(c, n, k, g, c_-, n_-, \lambda) = u(c, 1-n) + \lambda$

$$\left[ u_c c - (u_{c_-} - \beta(1-\delta)u_c) \frac{F_g}{F_k} \frac{N_f g}{\beta} - u_\ell n \right],$$

where $(c_-, n_-)$ denote values of $(c, n)$ lagged one period. In this case the pseudo utility function displays a form of "habit persistence" in the sense that previous values of consumption and labor supply appear in the instantaneous utility function. It is straightforward to show that the relevant steady state first order necessary conditions are

$$W_c^* + \beta W_{c_-}^* - \mu^* = 0$$

$$W_n^* + \beta W_{n_-}^* + \mu^* F_n^* = 0$$

$$1 = \beta[1 - \delta + F_k^*] + \frac{\beta W_k^*}{\mu^*} ,$$

where

$$W_k^* = -\lambda u_c^* N_f (1 - \beta(1-\delta)) \frac{F_{gk}^* F_k^* - F_{kk}^* F_g}{\beta (F_k^*)^2} .$$

Thus, if $F_{gk}^* \geq 0$, $\tau_\infty^k > 0$, as $\lambda > 0$.

Our formulation of the impact of public goods on productivity is only one of many possible candidates. Glomm and Ravikumar (1992) and Judd (1992) assume that the production function depends on private inputs chosen by firm $i$ ($x_i$), the economy-wide average value of those same inputs ($\overline{x}$), and the stock of the public good $Ng$. They assume that the production function is homogeneous of degree one with respect to $x_i$ (resulting therefore in zero profits), and with respect to $x_i$, $\bar{x}$ and $Ng$ when it is evaluated at $x_i = \bar{x}$. In this interpretation some of the elements in $\bar{x}$ generate *negative* externalities and their effects *exactly* offset that of the public goods. In this setting, Judd shows that $\tau_\infty^k = 0$. Zhu's (1991) formulation assumes that the production function is of the form $F(x_i)L(Ng)$ with $F$ homogeneous of degree one giving rise to increasing returns in all factors. In this case private profits are zero and $\tau_\infty^k = 0$.

Our analysis suggest that for public goods to result in positive capital income tax rates asymptotically it is necessary that the presence of public goods results in private economic profits.

## 4. Parameterized Examples

To study the magnitude of tax rates implied by the examples presented in section 3, we present numerical estimates of $\tau_\infty^k$ in this section. In all cases we study the steady state of the system and we use versions of (6) taking $\lambda$ -- the marginal welfare cost of taxation -- as given.

In the context of the models we studied, $\lambda$ measures the amount that the consumer would be willing to pay to have its distortionary taxes decreased by a dollar (measured in time zero consumption) at the same time that lump sum taxes are increased by a dollar. Unfortunately, there are no readily available estimates of $\lambda$ that are consistent with our model. Chamley (1981) finds $\lambda = .5$ when a version of a suboptimal tax code is implemented in a neighborhood of the unconstrained steady state. Judd (1987) studied the marginal welfare cost of taxation at a tax distorted steady state. His estimates range from $\lambda = 1$ to $\lambda = 2$. Both papers studied the effect of a suboptimal tax regime. In our own work on optimal taxation in endogenous growth models (see Jones, Manuelli and Rossi (1993)) we found $\lambda$ to vary between .18 and .58 for models that are close to those described in section 2 and 3 (a) and (b). However, when productive public goods were introduced, the resultant estimates of $\lambda$ were as high as one. For the purpose of deriving a conservative estimate of the limiting tax rates on capital, we chose relatively low values of $\lambda$. Specifically, we consider values of $\lambda$ between 0 and .40 for the cases described in sections 3 (a) and (b). For the case of productive public goods we extend the range to $\lambda$=.60.

In all of our calculations we use a utility function given by $u(c, \ell) = (c\ell^{\eta})^{1-\sigma}/(1 - \sigma)$ and consider different values of sigma. In the first example of section 3, $\eta = 0$, while for the case $\tau^n = \tau^k$, we use $\eta = 1.8$, which is consistent with the observed labor supply. For the model with productive public goods, we use $\eta = 2.29$. We describe the steady state as imposing a tax on labor equal to 30% and a tax on capital equal to 20%. In the first two examples, we assume a Cobb-Douglas production function given by $Ak^{\alpha}n^{1-\alpha}$, with $\alpha = .36$ which corresponds to capital's share in GNP. The value of $A$ is chosen to make the model consistent with a steady state capital stock of one. In all cases we assume government consumption to be 20% of GNP.

For the inelastic labor supply case, we must determine $\bar{\tau}^n$. We assume $\bar{\tau}^n$ =.2 although increasing this value to .3 results in only minor changes in our estimates.

Table I presents the limiting tax rate on capital for the inelastic labor supply case (see section 3 (a)) for $\sigma = 2$ and various values of $\lambda$.

**Table I: Limiting Tax Rate on Capital Income**
**Inelastic Labor Supply**

| $\lambda$ | $\tau^k_\infty$ (in %) |
|---|---|
| 0.00 | 0 |
| 0.05 | 3.0 |
| 0.10 | 6.2 |
| 0.15 | 9.7 |
| 0.20 | 13.3 |
| 0.25 | 17.1 |
| 0.30 | 21.1 |

| | |
|---|---|
| 0.35 | 25.2 |
| 0.40 | 29.4 |

Basic Parameters: $\sigma = 2$, $\eta = 0$,
$g/GNP = .20$, $A = 1.15$, $\delta = .10$, $\beta = .95$, $\alpha = .36$

Even for relatively low values of $\lambda$ the model implies relatively high values of $\tau^k_\infty$. For smaller values of $\sigma$, the implied tax rates are smaller. For example, for $\lambda = .20$ and $\sigma = .5$, $\tau^k_\infty$ is 10.3% instead of 13.3%. On the other hand, larger values of $\sigma$ result in higher values for the limiting tax rate on capital income. In the case $\lambda = .20$ and $\sigma = 3$ the resulting $\tau^k_\infty$ is 16.5%.

Table II contains the implied values of $\tau^k_\infty$ in the case in which the government is constrained to choosing $\tau^k_t = \tau^n_t$. The introduction of an elastic labor supply and the constraint on the equality of tax rates has a surprisingly small effect (given the overall uncertainty about the parameters) on the limiting tax rates. The values are relatively large even for conservative estimates of the cost of distortionary taxes. As in the example with inelastic labor supply, the limiting tax rates are monotone increasing in $\sigma$ although they are much less sensitive. For the case $\lambda = .20$ we obtained $\tau^k_\infty = 11.5$ for $\sigma = .5$ and $\tau^k_\infty = 12.3$ for $\sigma = 3$.

**Table II: Limiting Tax Rate on Capital Income $\tau^k_t = \tau^n_t$**

| $\lambda$ | $\tau^k_\infty = \tau^n_\infty$ (in %) |
|---|---|
| 0.00 | 0.0 |
| 0.05 | 3.3 |
| 0.10 | 6.4 |
| 0.15 | 9.2 |
| 0.20 | 11.9 |
| 0.25 | 14.6 |
| 0.30 | 17.1 |

| | |
|---|---|
| 0.35 | 19.6 |
| 0.40 | 22.0 |

Parameters: $\sigma = 2$, $\eta = 1.8$,
$g/GNP = .20$, $A = .95$, $\delta = .10$, $\beta = .95$, $\alpha = .36$

Finally, we also computed the limiting tax rates on capital income as a function of the marginal welfare cost of taxation for an example of the case in which there are productive public goods. We assume a Cobb-Douglas production function given by $A\, k^{\alpha_1}\, n^{\alpha_2}\, g_p^{\,1-\alpha_1-\alpha_2}$, where $g_p$ is productive government spending. To calibrate the model, we chose $\alpha_2$=.64 and $\alpha_1$=.28. This implies that the optimally chosen $g_p$/GNP ratio is .08 which is close to the U.S. average.

As in the other cases, the estimated limiting values of the tax rate on capital income are non-zero (see Table III). For a given value of $\lambda$, they are about half of our estimates for the previous cases. However, as noted earlier, there is good reason to believe that $\lambda$ should be higher in the case of productive public goods.

**Table III: Productive Public Goods**

| $\lambda$ | $\tau^k_\infty$ (in %) |
|---|---|
| 0.00 | 0.0 |
| 0.05 | 1.4 |
| 0.10 | 2.8 |
| 0.15 | 4.1 |
| 0.20 | 5.4 |
| 0.25 | 6.8 |
| 0.30 | 8.2 |

| | |
|---|---|
| 0.40 | 11.0 |
| 0.50 | 14.2 |
| 0.60 | 17.7 |

Parameters: $\sigma = 2$, $\eta = 2.29$, $g_T/GNP = .08$
$g/GNP = .20$, $A = 1.8$, $\delta = .10$, $\beta = .95$, $\alpha_1 = .28$, $\alpha_2 = .64$

Overall, our numerical results indicate that even for reasonably low values of the marginal welfare cost of distortionary taxation the implied limiting tax rate on capital income is substantial. Moreover, the numerical estimates are relatively insensitive to alternative environments that result in $\tau^k_\infty > 0$ and to different parameter values.

## 5. Extensions and Conclusions

Our analysis of optimal factor taxation in dynamic models suggests that simple intuitive arguments purporting to explain why the tax rate on capital income is zero in the limit (e.g. due to infinite supply elasticity) are not very useful. By useful we mean a type of economic intuition that is robust to relative minor changes in the structure of the model. All the different environments discussed in this paper are very close, yet, our conclusions about limiting behavior of tax rates are quite different.

A more promising route to understanding the basic forces that drive the asymptotic behavior of optimally chosen tax rates is to delineate features of the economy which account for the result. In this paper we showed that, in the context of a model with both human and physical capital, if there are constant returns to scale in the reproducible factors (no profits), a sufficiently rich tax code and no possibilities for relative prices to affect wealth then limiting tax rates on both capital and labor income are zero. Each of these three conditions is essential.

Consider the linearity or "no profits" condition first. In both the model in which public goods result in pure profits and that of an inelastic labor supply (another example of pure profits), the presence of rents result in positive limiting tax rates on capital. Our interpretation is that by distorting the choice of capital the planner can "tax" the pure rents.

The second element is the richness of the tax code. As the discussion in section 3 (b) shows (as well as the model in section 2 in the absence of consumption taxes), the presence of restrictions across tax rates results in non-zero taxes on capital income. More generally, a limited ability to set sufficiently many taxes independently gives the same result. We interpret this as a standard "second best" argument: the imposition of an additional restriction (e.g. a restriction across tax rates) calls for a change in how the unaffected policy instruments are chosen. In this case, the restrictions force a switch from zero to some positive level of capital income taxation in the long run.

The third element that seems essential is the absence of a role for a change in relative prices as a form of extracting rents from the private sector. In this respect, the model in section 2 in which transfers (a pure rent from the point of view of both the consumers and the planner) are not fixed in terms of consumption results in non-zero taxes on labor income.

The reader may wonder if the three features that we have identified overturn the zero limiting tax result only in the Chamley-Judd environment. Although it is impossible to give an exhaustive answer, we have explored other environments in which both the Chamley-Judd result obtains and the violation of one of the three conditions that we identified results in non-zero limiting tax rates. These more general environments allow for heterogeneity (see also Judd (1985)), multiple consumption goods and types of labor and multi-sector settings in which the price of capital in the steady state is endogenous.

Finally, the model in section 2 suggests that given the linearity assumption and a rich tax code, the Ramsey problem has very strong implications about both the timing of tax revenues and the structure of the tax system: Under the optimal scheme, revenue is 'front-loaded' and all factors are treated symmetrically in the limit. This revenue front-loading is a disturbing but essential feature of the optimal tax code. Reasonable restrictions on the time path of deficits (e.g., period by period budget balance) can be shown to undo the zero limiting tax result. In essence, this is another type of restriction on tax codes not unlike that described in section 3 (b). This example highlights the interdependence of the initial behavior of the optimal tax code and its limiting properties.

Because of the delicacy of the mapping between the features of the economy and the structure of optimal tax codes, further progress will necessitate a detailed analysis of the entire time path of taxes. Even for relatively simple examples, this will require a reliance on numerical methods.

## REFERENCES

Aiyagari, S. Rao and Dan Peled (1991), "Optimal Capital Income Taxation and Long Run Debt with Borrowing Constraints," Federal Reserve Bank of Minneapolis Working Paper 474, (January).

Aschauer, David A. (1989), "Does Public Capital Crowd Out Private Capital?," *Journal of Monetary Economics*, Vol. 24, (September), pp. 171--188.

Atkinson, A. B. and Joseph E. Stiglitz (1972), "The Structure of Indirect Taxation and Economic Efficiency," *Journal of Public Economics*, 1, pp. 97--119.

Atkinson, Anthony B. and Agnar Sandmo (1980), "Welfare Implications of the Taxation of Savings," *The Economic Journal*, Vol. 90, (September), pp. 529--549.

Auerbach, Alan J. (1985), "The Theory of Excess Burden and Optimal Taxation," in A.J. Auerbach and M. Feldstein (eds) *Handbook of Public Economics*, Vol. II, Elsevier Science Publishers B. V. (North Holland), Amsterdam.

Barro, Robert J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, Vol. 98, part 2, (October) pp. S103-S125.

Bull, Nicholas (1992), "When the Optimal Dynamic Tax is None," Working Paper.

Chamley, Christophe (1985), "Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium," *International Economic Review*, Vol. 26, No. 2, (June), pp. 451--468.

_______ (1986), "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, No. 3, (May), pp. 607--622.

Chari, V. V., Lawrence J. Christiano and Patrick J. Kehoe (1991), "Optimal Fiscal Policy in a Business Cycle Model," Federal Reserve Bank of Minneapolis Working Paper No. 465 (January).

Diamond, Peter A. and James Mirrlees (1971), "Optimal Taxation and Public

Production I: Production Efficiency and II: Tax Rules," *American Economic Review*, 61, 8--27 and 261--278.

Grossman, Sanford J. and Oliver D. Hart (1983), "An Analysis of the Principal Agent Problem," *Econometrica*, Vol. 51, No. 1 (January), pp. 7--45.

Heckman, James J. (1976), "A Life-Cycle Model of Earnings, Learning, and Consumption," *Journal of Political Economy*, Vol. 84, (August), pp. S11--S44.

Jones, Larry E., Rodolfo E. Manuelli and Peter E. Rossi (1993), "Optimal Taxation in Models of Endogenous Growth," forthcoming, *Journal of Political Economy*.

Judd, Kenneth L. (1985), "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, Vol. 28, (October), pp. 59--83.

_______ (1987), "The Welfare Cost of Factor Taxation in a Perfect Foresight Model," *Journal of Political Economy*, Vol. 95, No. 4, pp. 675--709.

_______ (1990), "Optimal Taxation in Dynamic Stochastic Economies," Working Paper.

King, Robert G. (1990), "Observable Implications of Dynamically Optimal Taxation," University of Rochester Working Paper, (August).

Lucas, Robert E. Jr (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics*, Vol. 22 (January), pp. 3--42.

_______ (1990), "Supply Side Economics: An Analytical Review," *Oxford Economic Papers*, Vol. 42 (April), pp. 293--316.

Lucas, Robert E. Jr and Nancy L. Stokey (1983), "Optimal Fiscal and Monetary Policy in an Economy Without Capital," *Journal of Monetary Economics*, Vol. 12 (July), pp. 55--93.

Mirrlees, James A. (1975) "The Theory of Moral Hazard and Unobservable Behavior--Part I" Nuffield College, Oxford, mimeo.

_______ (1986), "The Theory of Optimal Taxation" in K. J. Arrow and M. D. Intrilligator (eds.) *Handbook of Mathematical Economics*, Elsevier

Science Publishers B. V. (North Holland), Amsterdam.

Ramsey, Frank P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal*, Vol. 37, (March), pp. 47--61.

Stiglitz, Joseph and Partha S. Dasgupta. (1971), "Differential Taxation, Public Goods and Economic Efficiency," *Review of Economic Studies* 38, pp. 151--174.

Stiglitz, Joseph E. (1985), "Inequality and Capital Taxation," IMSSS Technical Report.

______ (1987), "Pareto Efficient and Optimal Taxation and the New Welfare Economics," in A. J. Auerbach and M. Feldstein (eds.) *Handbook of Public Economics*, Vol. II, Elsevier Science Publishers B.V. (North Holland), Amsterdam.

Yuen, Chi-Wa (1990), "Taxation, Human Capital Accumulation and Economic Growth," Working Paper.

Zhu, Xiadong (1991), *Optimal Fiscal Policy in a Stochastic Growth Model*, Ph.D. Thesis, University of Chicago.