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ABSTRACT

To forecast future option prices, autoregressive models of implied volatility derived from observed option prices are commonly employed [see Day and Lewis (1990), and Harvey and Whaley (1992)]. In contrast, the ARCH model proposed by Engle (1982) models the dynamic behavior in volatility, forecasting future volatility using only the return series of an asset. We assess the performance of these two volatility prediction models from S&P 500 index options market data over the period from September 1986 to December 1991 by employing two agents who trade straddles, each using one of the two different methods of forecast. Straddle trading is employed since a straddle does not need to be hedged. Each agent prices options according to her chosen method of forecast, buying (selling) straddles when her forecast price for tomorrow is higher (lower) than today's market closing price, and at the end of each day the rates of return are computed. We find that the agent using the GARCH forecast method earns greater profit than the agent who uses the implied volatility regression (IVR) forecast model. In particular, the agent using the GARCH forecast method earns a profit in excess of a cost of $0.25 per straddle with the near-the-money straddle trading.

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1 Introduction

In efficient capital markets, when the volatility of the rate of return on an asset is constant over time, the asset's volatility is reflected in its option price. Schmalensee and Trippi (1978) and later many others have shown that call-option implied volatilities yield better forecasts of standard deviation than do simple estimates using the standard deviation estimated from past returns.

Market volatility, however, can change dramatically when it is affected by stock market crashes or monetary policy. For example, the standard deviation of S&P 500 daily returns increased from about 1.2% to 9.4% in the ten days surrounding the stock market crash of October 1987. Changes in volatility of this magnitude may have large effects on the level of stock prices and thus the option prices. Under changing volatility, it is still possible to compute the expected implied volatility over the life of an efficiently priced option if unexpected changes in volatility over the life of the option are non-systematic (see Wiggins (1987) and Hull and White (1987)). However, such a circumstance is not guaranteed, particularly when the asset in question is a well-diversified portfolio, and. with time-varying volatilities, we cannot even take for granted that the option market is efficient with respect to any volatility forecast.

The autoregressive conditional heteroskedasticity (ARCH) model proposed by Engle (1982) allows the conditional variance to change over time as a function of past errors, leaving the unconditional variance constant. Many authors, Bollerslev (1986), Coulson and Robins (1985), Engle and Lilien (1987), and Domowitz and Hakkio (1985), have successfully applied ARCH models to financial and macro-economic data to explain changing risk premiums and inflation volatilities. ARCH models capture the dynamic behavior of market volatility using specific volatility equations of the asset return series without presupposing any option pricing formula, or using observed option prices. Hence, option price estimates derived from ARCH models can be used to test whether observed market options are efficiently priced.

The usual way to measure the performance of a volatility prediction model is to assess its ability to predict future volatilities. However, since volatility is unobservable, there is no natural metric for measuring the accuracy of any particular model. But realized rates of return allow us to test the efficacy of variance driven option prices and provide a test for market efficiency with respect to volatility forecasts.

Engle, Hong, Kane and Noh (1993a) proposed a framework to assess profits from options trading for competing algorithms to forecast the volatility of an asset in a simulated market. They evaluated four separate volatility forecast models by comparing cumulative profits from options trading and found that profits earned by the GARCH forecast model dominate those earned by any of the other three alternatives. Engle, Kane and Noh (1993b) extended the analysis of one-day options trading to long-term options trading and assessed profits for the GARCH estimation using the S&P 500 option data in European style options trading. Recently, Harvey and Whaley (1992) conducted an S&P 100 index option market efficiency analysis using an implied volatility measure, and found that the implied volatility method fails to make significantly positive profits after transaction costs. They used one pair of call and put near-the-money options each day and delta-hedged using an S&P 500 futures contract.

In this paper, we test the efficiency of the S&P 500 index option market based on the performance of these two volatility prediction models. We use one type of straddle, which has maturity above 15 days and is nearest to the money, each day rather than trade different types of call and put options separately. There are two advantages of using straddle trading instead of using call and put option trading separately. The first advantage of using straddle trading is that we don't need to delta-hedge the straddle; optimal hedge ratios can not be easily computed with time varying volatilities. The second advantage is that under straddle trading the Black-Scholes (1973) option straddle prices are relatively insensitive to dividend adjustment.

We find that, in some cases, both the agent using a GARCH forecast model and the agent using an Implied Volatility Regression (IVR) forecast model generate abnormally high positive rates of return in the S&P 500 index option market. Overall, the agent using the GARCH method performs better than the agent using the IVR method.

The paper is organized as follows. In section 2, we discuss two different specifications for volatility prediction, the GARCH forecasting method and the IVR forecasting method, and a Hull and White (1987) type option valuation method with changing volatility. Section 3 describes the option data, option trading and the rate of return calculation. Section 4 reports an economic analysis of the options trading results.

2 Volatility Estimation and Option Valuation Methods

Estimates of the annualized standard deviation of monthly stock returns reported in French, Schwert and Stambaugh (1987), and Schwert (1990) range from a low of 2% in the early 1960's to a high of 20% in the early 1930's. Daily volatility also fluctuates and can change very rapidly; for example we estimate that the annualized standard deviation of daily returns increased from about 1.2% to 9.4% in a ten day period around the stock market crash of October 1987. Changes in volatility of this magnitude may have important effects on stock returns and thus on option prices. This volatility feedback effect has been emphasized by Pindyck (1984) and French, Schwert

Table 1: Mean of the S&P 500 index returns and the option implied volatilities.
(April 21, 1986 through December 31, 1991)

	Overall	Monday	Tuesday	Wednesday	Thursday	Friday
			Full sample			
Daily returns	0.00035	-0.00088	0.00105	0.00141	-0.00041	0.00045
Annualized Std.(%)	23.35335	33.63851	19.73830	18.75554	20.04322	22.50050
Annualized call IV(%)	17.22740	17.40196	17.69595	17.22947	17.14915	16.64077
Annualized put IV(%)	21.37534	21.87024	21.54610	21.09352	21.03273	21.38205
Change in call IV	0.01464	0.83808	0.27036	-0.38630	0.11644	-0.72130
Change in put IV	0.02140	0.55290	-0.17295	-0.40292	0.04884	0.14675
Number of observations	1350	250	281	280	276	263
			Excluding Oct. 15–30, 1987			
Daily returns	0.00051	0.00027	0.00078	0.00109	-0.00018	0.00054
Annualized Std.(%)	19.21636	20.56689	18.68657	15.75358	19.42364	21.51048
Annualized call IV(%)	16.95233	17.04146	17.36266	16.98105	16.99871	16.34738
Annualized put IV(%)	20.96582	21.28109	21.02004	20.74570	20.84406	20.97059
Change in call IV	-0.00700	0.68152	0.25735	-0.29965	0.09115	-0.73796
Change in put IV	0.01023	0.33460	-0.17423	-0.22242	0.04442	0.11148
Number of observations	1339	248	279	278	274	260

Notes: Daily standard deviation and annaulaized standard deviation of the S&P 500 index returns have only one observation over the sample period. Implied volatility is estimated as explained in section 2.2. The change in volatility is the first difference of the implied volatility series.

and Stambaugh (1987).

Table 1 shows the mean and standard deviation of S&P 500 index returns and the implied volatilities of S&P 500 index options , computed in section 2.2, over the period of April 21, 1986 thorough December 31, 1991. It shows that the volatility of S&P 500 index returns increases on Monday and decreases on Tuesday compared to that of the overall period. The effect of the market crash on October 19, 1987 is dramatic. Including the market crash of 1987, the average volatility on Monday is increased by 64% and it is 44% higher that that of the overall period. The table also shows that the put implied volatilities of S&P 500 index options are higher than those of call implied volatilities, and the volatility of S&P 500 index returns generally lies between these two. Changes of call and put implied volatilities show that the market crash on October 19, 1987 induced a large increase in both call and put implied volatilities i.e., the change of average Monday call option implied volatility increased from .33 to .55, and the change of put option implied volatility increased from .68 to .84 when we included the market crash. Since, in the following, we use two different volatility estimation methods, one using the S&P 500 index return series and the other using the S&P 500 index option implied volatilities, we employ the following facts from Table 1. First, if we use the return series to estimate and forecast future volatilities, then we need to specify a model which increases volatility on Monday and decreases it on Tuesday. Second, if we use implied volatility series to estimate and forecast future volatilities, then we need to use call and put implied volatility series separately.

2.1 GARCH Method

Many authors (Chou (1988), French, Schwert and Stambaugh (1987), Pagan and Schwert (1989), Engle and Gonzalez-Rivera (1989), Nelson (1991)) have used the GARCH model of Engle (1982) and Bollerslev(1986) to estimate stock market volatility. More recently, to capture some of the implications of volatility feedback, Engle, Lilien and Robins (1987) developed a GARCH-M specification, and Campbell and Hentschel (1992) modified the GARCH model to explain the asymmetric effects of shocks to stock returns.

In this paper, we use the following GARCH specification for the S&P 500 index return series.

$$r_t = a_0 + a_1 r_{t-1} + \epsilon_t \tag{1}$$

$$h_t = d_t^{\delta}\{b_0 + d_{t-1}^{-\delta}(b_1\epsilon_{t-1}^2 + b_2 h_{t-1})\}, \tag{2}$$

where r_t denotes the S&P 500 index return at time t and d_t denotes number of days elapsed from the last trading date. French and Roll (1986) have shown that the variance rate slows down significantly in days when the market is closed. Inclusion of the dummy variable d_t helps to capture this phenomenon since d_t raised to the power δ measures the average speed of the variance rate over the d_t calendar days. If date t is a Monday, then $d_t = 3$, and h_{t-1} is increased by d_t^{δ} and if date t is a Tuesday, then h_{t-1} is diminished by d_{t-1}^{δ}. Table 1 supports this claim since the standard deviation of the S&P 500 returns on Monday is higher than that of the sample period while the standard deviation of the S&P 500 returns on Tuesday is lower than that of the sample period.

We use the past 1,000 rolling observations of the S&P 500 index return series to update estimates of the above specification and then compute multi-day volatility forecasts as follows,

$$h_{t,t+1} = d_{t+1}^{\delta}\{b_0 + d_t^{-\delta}(b_1\epsilon_t^2 + b_2 h_t)\} \tag{3}$$

$$h_{t,t+i} = d_{t+i}^{\delta}\{b_0 + d_{t+i-1}^{-\delta}(b_1 E[\epsilon_{t+i-1}^2|\Omega_t] + b_2 h_{t,t+i-1})\} \tag{4}$$

$$= d_{t+i}^{\delta}\{b_0 + d_{t+i-1}^{-\delta}(b_1 + b_2)h_{t,t+i-1}\} \quad i = 2,3,\cdots,\tau, \tag{5}$$

where $\epsilon_{t,t+i}$ and $h_{t,t+i}$ denote the predictions of ϵ_{t+i} and h_{t+i} at time t. Then forecasts of call and put option prices at the next trading day , based on the above conditional volatility forecasts and market closing index, are calculated using the Black-Scholes option pricing formula:

$$C_{t+1,t+1+\tau} = S_t N(d_1) - Ke^{-r_t\tau}N(d_2) \tag{6}$$

$$P_{t+1,t+1+\tau} = S_t[N(d_1) - 1] + Ke^{-r_t\tau}[1 - N(d_2)] \tag{7}$$

$$d_1 = \frac{ln(S_t/K) + (r_t + 1/2\sigma_{t+1,t+1+\tau}^2)\tau}{\sigma_{t+1,t+1+\tau}\sqrt{\tau}}$$

$$d_2 = d1 - \sigma_{t+1,t+1+\tau}\sqrt{\tau},$$

where $C_{t+1,t+1+\tau}$, $P_{t+1,t+1+\tau}$ are Black-Scholes call and put option prices forecasts at time $t+1$ until the maturity date, and S_t is the market closing price of the S&P 500 index as a substitute for S_{t+1}, and K is the exercise price, r_t is the risk-free rate, τ is the time to the maturity date, and $\sigma_{t+1,t+1+\tau} = (1/\tau)\sum_{i=2}^{\tau+1} h_{t,t+i}$ is the volatility prediction at time $t+1$ until the maturity date.

The above option valuation formula uses a deterministic volatility assumption even though we get time varying volatility forecast series from the GARCH specification.

A discrete-time option valuation under stochastic volatilities can be devised, following Hull and White (1987), using Monte Carlo simulation via

$$C_{t+1,t+1+\tau} = \frac{1}{N}\sum_{j=1}^{N} BS_j(S_{t+1}, K, \bar{\sigma}_j),$$

where $BS_j(\cdot)$ represents the Black-Scholes call option price formula of equation (6), $\bar{\sigma}_j = (1/\tau)\sum_{i=2}^{t+1+\tau} h^j_{t,t+i}$, and N is the number of replications for random variables $\eta_{t+i,j} = \epsilon_{t+i,j}/\sqrt{h_{t+i,j}}$, $i = 1, \cdots, \tau - 1$ and $j = 1, \cdots, N$, which are drawn from the standard normal distribution with mean 0 and standard deviation of 1.[1]

2.2 Implied Volatility Regression Method

As a second specification for conditional volatility forecasts, we use a time series regression of option implied volatilities of the S&P 500 index. Since Schmalensee and Trippi (1978) used option implied volatility as an estimate for conditional volatility, many others followed their method to provide future volatilities over different time horizons. Day and Lewis (1988) found that implied volatility has incremental information regarding weekly S&P 100 index returns. They also compared the ability of implied volatilities and GARCH-based alternatives to provide out-of-sample forecasts of future volatility for the S&P 100 index. Harvey and Whaley (1992) used time series regressions of option implied volatilities to forecast the one-day ahead volatility of S&P 100 index options. We estimated implied volatilities using a GLS version of Whaley's (1982) nonlinear regression procedure which was used by Day and Lewis (1988). With this approach, each observation is weighted in proportion to the day's total trading volume in this contract as

[1] From equation (2) we get the following relationship: $h_{t+i} = b_0 + (b_1\eta^2_{t+i-1} + b_2)h_{t+i-1}$, $i = 1, 2, \cdots, \tau$, where η_i follows standard normal distribution with mean 0 and standard deviation 1.

a percentage of the total trading volume of its expiration series. This weighting scheme effectively places the greatest weight on those options that are either at the money or one strike price (five dollars) out of the money. Consequently, it gives the greatest weight to the option prices that are most sensitive to the volatility of the underlying stock index. In addition, since thinly traded options and far-out-of-the-money options receive relatively little weight, this scheme tends to minimize the extent to which the estimates of implied volatility are affected by noise caused by either non-synchronous trading or the size of the bid-ask spread. Let $C_k(v_0(\tau))$ denote the theoretical price of an option having a strike price indexed by k and expiration at τ, given an estimate $v_0(\tau)$ of the volatility of the return on the stock over the time to expiration. The actual price of option k with expiration at τ is denoted by $C_{k,\tau}$. Define

$$Y_k = C_{k,\tau} - C_k(v_0(\tau)), \tag{8}$$

and let N denote the number of strike prices with expiration at τ. Given an initial estimate of volatility implicit in the prices of options with expiration at τ, a new estimate of the implied volatility, $v(\tau)$, is chosen so as to minimize

$$\sum_{k=1}^{N} (\delta_k Y_k)^2, \tag{9}$$

where δ_k is the proportion of trading volume in options expiring at τ represented by trading in contract k. At each iteration of this process, the new estimate of the implied volatility is given by

$$v(\tau) = v_0(\tau) + [(DX)'(DX)]^{-1}(DX)'(DY), \tag{10}$$

where D is an $N \times N$ diagonal weighting matrix whose diagonal elements equal the percentage of the trading volume in a given expiration series represented by trading in that contract, X is an $N \times 1$ vector whose elements are the partial derivatives of the call options expiring at τ with respect to the underlying volatility, evaluated at $v_0(\tau)$, and Y is an $N \times 1$ vector whose elements are defined by equation (8). If the estimate converges to within a given tolerance level, then the estimate $v(\tau)$ is taken as acceptable. If the estimate is not within the desired tolerance, the procedure is repeated using $v(\tau)$ in place of $v_0(\tau)$. Finally, the implied volatility of each day is computed as an average of implied volatilities with different maturities weighted by their proportion of trading volume. This approach is implemented each day for every option in the sample.

Using the volatilities computed as above, we run the following time-series regression of implied volatilities to predict one-step ahead implied volatility which is used to price options

$$\begin{aligned}\Delta v_{c,t} &= c_0 + a_1 d_{1,t} + a_2 d_{2,t} + a_3 r_{t-1} + a_4 \Delta v_{c,t-1} \\ &\quad + a_5 \Delta v_{c,t-2} + a_6 \Delta v_{p,t-1} + a_7 \Delta v_{p,t-2} + \epsilon_{1t} \end{aligned} \tag{11}$$

$$\begin{aligned}\Delta v_{p,t} &= c_0 + a_1 d_{1,t} + a_2 d_{2,t} + a_3 r_{t-1} + a_4 \Delta v_{c,t-1} \\ &\quad + a_5 \Delta v_{c,t-2} + a_6 \Delta v_{p,t-1} + a_7 \Delta v_{p,t-2} + \epsilon_{2t}, \end{aligned} \tag{12}$$

where $v_{c,t}$ and $v_{p,t}$ denote call and put option implied volatility at time t, respectively, and $d_{1,t}$ and $d_{2,t}$ denote Monday and Friday dummies at time t, respectively. Call and put option forecast prices are calculated using the option pricing formula (6) and (7) , where $\sigma_{t+1,t+\tau+1}$ is replaced by the forecasts $v_{c,t+1}$ and $v_{p,t+1}$, respectively.

2.3 Estimation Results

The GARCH method and the IVR method are distinctive in that they use different data sets. The GARCH volatility forecasts are computed using the past realized return series of the asset and are not affected by the realized option prices. On the other hand, the implied volatilities are estimated from the past realized option prices series.

Table 2 and Table 3 show the estimation results of GARCH and IV regressions over the sample period of April 21, 1986 thorough December 31, 1991. The left panel of Table 2 presents results of GARCH regressions for the full sample and the right panel presents results excluding the market crash. A comparison shows that the coefficient of the AR term in the variance equation decreases by 9% and the coefficient of the squared error term in the variance equation increases by 187% when we include the market crash. Thus it becomes more responsive to innovations. It also reveals that weekend volatility effects on Monday increase from 1.13 to 1.32. Theses numbers are in the range of the ratio of weekend volatility relative to weekday volatility reported in French and Roll (1986).

Table 3 presents results of IV regressions. The Monday dummy variable is important for the put volatility regression, since it has a significantly positive coefficient for both sample periods. For call volatility regressions, the Monday dummy has a significantly positive effect only when we include the crash period. The Friday dummy variable has a significantly negative effect only in the call volatility regression for both sample periods, which may be consistent with a large number of traders closing out positions before the weekend. For a leverage effect, we use the lagged S&P 500 return and it has a significantly negative effect on put volatility changes and negatively enters call regression when we include the crash period. Both panels show that the call volatility change and

Table 2: GARCH Regression for volatility estimation.
(April 21, 1986 through December 31, 1991)

Variables	Full sample (1,350 observations) Coefficient	t-ratio	Excluding Oct. 15-30, 1987 (1,339 observations) Coefficient	t-ratio
a_0	0.0007125	3.833	0.0006089	3.278
a_1	0.0005311	2.589	0.0005305	2.577
b_0	0.0000048	2.054	0.0000014	2.125
b_1	0.0823935	1.327	0.0286867	3.098
b_2	0.8671057	12.522	0.9550874	78.375
δ	0.2535461	5.638	0.1096219	2.313
Log L.	-3315.53		-3224.84	
Correct Prediction	55%		55%	

Notes: All the t-ratios are heteroskedasticity-consistent as in Bollerslev and Wooldridge (1988). In the out-of-sample analysis, GARCH model is estimated using the past 1,000 observations of returns on the S&P 500. A forecast is obtained for November 3, 1986 and then the model's parameters are re-estimated using another past 1,000 observations. This procedure is then repeated 1251 times for the full sample and 1240 times for the sample that excludes the observations around the October 1987 stock market crash. Correct prediction is the percent of times that the trading decision is correct based on volatility estimates.

Table 3: Regressions of the change in implied volatility on the S&P 500 index option.
(April 21, 1986 through December 31, 1991)

Variables	Full sample (1350 observations)				Excluding Oct. 15-30, 1987 (1339 observations)			
	Change in call volatility		Change in put volatility		Change in call volatility		Change in put volatility	
	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio
INTERCEP	0.149	1.623	-0.045	-0.391	0.117	1.363	-0.075	-0.889
$MONDAY_t$	0.375	1.935	0.689	2.851	0.256	1.409	0.487	2.741
$FRIDAY_t$	-0.862	-4.600	0.075	0.321	-0.819	-4.655	0.146	0.845
$RETURN_{t-1}$	-33.698	-4.737	-98.635	-11.071	-6.150	-0.806	-42.452	-5.696
$\Delta CALL_{t-1}$	-0.597	-18.787	-0.068	-1.741	-0.580	-19.279	-0.020	-0.698
$\Delta CALL_{t-2}$	-0.237	-8.315	0.076	2.147	-0.233	-8.518	0.111	4.151
ΔPUT_{t-1}	0.182	7.893	-0.386	-13.349	0.139	5.392	-0.523	-20.593
ΔPUT_{t-2}	0.045	1.890	-0.271	-9.201	0.054	1.993	-0.289	-10.904
R-SQUARE	0.254		0.182		0.2465		0.268	
ROOT MSE	2.636		3.303		2.465		2.416	
Correct Prediction	53%				53%			

Notes: All the t-ratios are heteroskedasticity-consistent. In the out-of-sample analysis, volatilities are estimated using the first 100 observations of returns on the S&P 500. A forecast is obtained for Nov 3, 1986 and then the model's parameters are re-estimated. This procedure is then repeated 1251 times for the full sample and 1240 times for the sample that excludes the observations around the October 1987 stock market crash. Correct prediction is the percent of times that the trading decision is correct based on volatility estimates.

the put volatility change are mean-reverting series as shown by the significantly negative coefficients on lagged call volatility change in the call regression, and the significantly negative coefficients on the lagged put volatility in the put regression. This can be explained by the reasoning of Harvey and Whaley (1992). If good news about the market arrives late in the trading day then, because the index option market is so active, it is likely that the information is quickly incorporated in option prices. If the information is not fully reflected in the index level by the close of trading, the observed index level is lower than it should be, and the implied volatility of the call is higher than it should be. On the following day, when all stocks in the index have traded in reaction to the previous day's news, the observed index level catches up and the implied volatility of the call is reduced. On the other hand, if bad news arrives late in the trading day, the price of index puts is quickly bid up. If not all stocks in the index are traded by the close of trading, the observed closing index level is higher than it should be. The implied volatility of puts is higher than it should be, and on the following day the implied volatility of puts is reduced.

3 Application to the S&P 500 Index Option

3.1 Data

S&P 500 index option data were obtained from the Chicago Board of Options Exchange for the period from October 1985 through February 1992. The data record contains call/put indicator, exercise price, expiration date, last sale price and the number of contracts traded.

Index option trading volume increased substantially over the period of 1986 through

Table 4: S&P 500 index options trading days and volumes. (November 21, 1985 - December 31, 1991)

Year	Trading Days	Call Volume	Put Volume	Total Volume	Daily Average Volume
1985	64	669	572	1241	19.4
1986	244	893005	751032	1644037	6737.9
1987	252	2889457	2684717	5574174	22119.7
1988	253	2227054	1990206	4217260	16669.0
1989	251	2954205	2692315	5646520	22496.1
1990	253	5233105	5464571	10597676	41888.0
1991	253	5409209	5610283	11019492	43555.3

1991, except for a sharp drop after the October 1987 crash. Daily average volume of 6,738 in 1986 increased to 43,555 in 1991. For the period of April 1986 through December 1991, the annualized average rate of return of the S&P 500 index was 13% with a standard deviation of 23%.

From the above data, we collect straddle data which have at least fifteen days to expiration and whose volumes are greater than 100 per day. Then we select a straddle at each day whose exercise price is closest to the index level. As a proxy for the risk-free rate of interest, we use the one-month Treasury bill rate from the CRSP tape.

There are two issues to be mentioned with regard to option pricing. One is the dividend yield adjustment in option valuation and the other is non-synchronousness between index closing price and index option closing price.

First, to derive the implied dividend yield from the observed option prices, we use the following put-call parity for option prices with dividend yield adjustment:

$$P = C + \{Ke^{-r\tau} - Se^{-\gamma\tau}\} \tag{13}$$

where C, P are Black-Scholes call and put option prices with dividend yield adjustment and γ is the compounded dividend yield rate.[2] We produced an implied dividend yield series from equation (13) using near the money option prices, and the mean of the implied dividend yield was 4% with a standard deviation of 3%.[3] These magnitudes of dividend yield may affect option prices enough to make an agent change her decision to sell/buy those options when she compares her forecast prices with the market prices. In the next section we compute trading profits both with and without implied dividend yield and show that trading decisions for straddles are not sensitive to dividends.

Second, the index option market stays open until 3:15 p.m. and the stock market closes at 3:00 p.m. and this gives an option holder a wild-card option opportunity. Harvey and Whaley used the S&P 500 futures index as a proxy for the expected 3:15 p.m. S&P 100 index level. In this paper, we use 3:00 p.m. stock market closing prices and do not allow a wild card option opportunity. Since we use two different volatility forecasting methods to price options, and then execute a trading strategy based on deviations of the market price from the model prices, a trading strategy which produces profits cannot be attributable to mis-pricing options.

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[2] Let V_d be the present value of all known dividends paid during the option's time remaining until expiration, and define $\gamma = -(1/\tau)ln(1 - V_d/S)$. Then the Black-Scholes call option and put option prices with dividend yield adjustment are computed from the following formulas,

$$C = Se^{-\gamma\tau}N(d_1') - Ke^{-r_t\tau}N(d_2) \quad (14)$$

$$P = -Se^{-\gamma\tau}N(-d_1') + Ke^{-r_t\tau}N(-d_2') \quad (15)$$

$$d_1' = d_1 - \frac{\gamma\sqrt{\tau}}{\sigma}$$

$$d_2' = d_2 - \frac{\gamma\sqrt{\tau}}{\sigma}$$

where d_1 and d_2 are as defined in equations (6) and (7).

[3] Using the put-call parity equation (13), we get the following relationship, $\gamma = -(1/\tau)ln((C - P + Ke^{-r\tau})/S)$. Since the dividend yield should be positive, we drop all observations for which $\{P - (C + Ke^{-r\tau}) - S)\}$ is negative.

3.2 Rate of Return Calculation

During the sample period, on each day, each agent applies her forecasting method to get a volatility estimate and forecasts the straddle price of tomorrow. She compares her straddle price forecast with the market straddle price, and if her straddle price is greater than the market straddle price, then she buys the straddle, and if her straddle price is less than the market straddle price, then she sells the straddle. Each agent picks one type of straddle each day and invests $100. When she sells straddle, we allow her to invest the money in a risk-free asset. Later we apply a filtering strategy where each agent trades only when the price change is expected to be greater than $.25 or $.75. In these cases, the number of trading days are reduced and we allow her to invest her money in a risk-free asset when she does not trade straddle. In this way, we can compare the performance of each agent for different forecasting algorithms and for different trading filters.

When the agent buys straddles, the rate of return is computed as follows,

$$RT_t = \frac{100}{(C_t + P_t)}[(C_t + P_t - C_{t-1} - P_{t-1})], \tag{16}$$

where C_t and P_t are call option price and put option price, respectively.

When the agent sells straddles, the rate of return is computed as,

$$RT_t = \frac{100}{(C_t + P_t)}[-(C_t + P_t - C_{t-1} - P_{t-1})] + r_f. \tag{17}$$

The net rate of return from straddles trading after transaction costs of $.25 per straddle is computed as

$$NRT_t = RT_t - \frac{100}{(C_t + P_t)} * \frac{1}{4}. \qquad (18)$$

4 Empirical Results

The forecasting experiment and options trading for both the GARCH and the implied volatility methods begins at November 3, 1986 and ends at December 30, 1991 providing options trading opportunities on 1,251 days. The GARCH forecasting method provides one common conditional volatility estimate for both call and put options prices while the implied volatility forecasting method provides different volatility estimates for call and put options prices.

Since we are using both in-the-money options and out-of-the-money options data and we are using two separate volatilities, call and put, to forecast option prices, we need to check whether the two different kinds of option data give consistent estimates for volatilities, and also whether one option pricing formula is consistent for both call and put option data. To do this, we select in-the-money call and put options whose exercise price is closest to the index level, and select out of the money call and put options whose exercise price is closest to the index level. Table 5 shows that correlation coefficients between implied volatilities form in-the-money options and out-of-the-money options are .92 and .93 for calls and puts, respectively, during the entire sample period. The correlation between IV from in (out of)-the-money options for calls and in (out of)-the-money options for puts is .84 (.88). However, the correlation between IV from in-the-money options for calls and out-of-the-money options for puts is .80. This may be explained by the fact that the asymmetric effects of positive and negative returns on agents expectations give higher premia for put options. For the sample period excluding

Table 5: Correlations among the closest in-the-money call and put implied volatilities and the closest out-of-the-money call and put implied volatilities.
(April 21, 1986 – December 31, 1991)

Full sample

	VCALIN	VCALOUT	VPUTIN	VPUTOUT
VCALIN	1.00	0.92	0.84	0.80
VCALOUT		1.00	0.90	0.88
VPUTIN			1.00	0.93
VPUTOUT				1.00

Excluding Oct. 15–30, 1987

	VCALIN	VCALOUT	VPUTIN	VPUTOUT
VCALIN	1.00	0.91	0.83	0.80
VCALOUT		1.00	0.90	0.86
VPUTIN			1.00	0.95
VPUTOUT				1.00

Table 6: Correlations between annualized GARCH volatility forecasts and options implied volatility forecasts. (November 3, 1986 – December 31, 1991)

	Full sample			Excluding Oct. 15–30, 1987		
	Obs.	Mean	Std.	Obs.	Mean	Std.
GARCH	1251	17.87	6.44	1240	17.42	3.79
Call	1251	17.41	6.28	1240	17.12	5.35
Put	1251	21.48	7.81	1240	21.05	5.79
	Correlation Coefficient			Correlation Coefficient		
	GARCH	Call	Put	GARCH	Call	Put
GARCH	1.00	0.75	0.80	1.00	0.68	0.65
Call		1.00	0.94		1.00	0.93
Put			1.00			1.00

the fifteen days around the market crash, the correlation coefficients mentioned above remain almost the same.

Table 6 shows that the average of the GARCH volatility forecasts is greater than that of the call option implied volatility forecasts, but is less than that of the put option implied volatility forecasts for both sample periods. Including the fifteen days around the market crash on October 1987 increases all the three volatilities about .4 percetage points. Additionally, the standard deviation of the GARCH volatility increases nearly two times.

The correlation coefficient between the call option implied volatility forecasts and the put option implied volatility forecasts is .94 for the full sample and is .93 for the subsample, which suggests that the two volatilities are moving together. However, the correlation coefficient between the GARCH volatility forecast and the option implied volatilities is about .78 for the full sample and is about .67 for the subsample. To decide how close these two volatility forecasts are, we use the correlation coefficients of Table 5 as a benchmark. Based on those correlation coefficients, we can infer that the two forecasting methods may be quite different at some periods. This is partly supported by the fact that the correlation between the dummy variable which indicates positions (buying or selling) of the agent using the GARCH forecasts and those of the agent using the implied volatility forecasts is .35 i.e., both agents agree to buy or sell their straddles only for 437 days out of 1251 days.

Table 7 shows the actual S&P 500 index options prices and the options price forecasts of the GARCH and the implied volatility methods. The GARCH forecasts for straddle prices are lower than the actual straddle prices of the S&P 500 index options, and the implied volatility forecasts for straddle prices are higher than the actual straddle prices. The correlation coefficients among the three prices are very high, from the low .84 between the GARCH prices and the actual prices to the highest .98 between the implied volatility forecast prices and the actual prices.

The correlation coefficient between the expected return from the GARCH forecasting method and that from the implied volatility forecasting method is .52, which is fairly low compared to the correlation coefficient between the volatility estimates of the two methods.

4.1 Near-the-money Straddle Trading

Table 8 shows the daily rate of return of near-the-money straddles trading during the full sample period before and after transaction cost. The average daily rate of return from options trading before transaction costs is 1.35% for the GARCH forecasting method

Table 7: Average option prices of S&P 500 index, GARCH forecasts and implied volatility forecasts.(November 3, 1986 – December 31, 1991)

		Full sample			Excluding Oct. 15–30, 1987		
	Type	Obs.	Mean	Std.	Obs.	Mean	Std.
S&P 500	Call	1251	7.49	3.59	1240	7.38	3.19
	Put	1251	6.49	4.16	1240	6.26	2.84
	Straddle	1251	13.98	6.14	1240	13.64	4.74
GARCH	Call	1251	7.44	3.57	1240	7.29	2.88
	Put	1251	5.63	3.08	1240	5.45	2.07
	Straddle	1251	13.08	4.99	1240	12.74	4.42
IVR	Call	1251	7.21	3.23	1240	7.10	2.84
	Put	1251	6.80	3.33	1240	6.63	2.47
	Straddle	1251	14.01	5.77	1240	13.73	4.77

and .68% for the IVR forecasting method, respectively. These profits are far from certain in the sense that standard deviations corresponding to these are 10.74% and 10.80%. respectively. However, t-ratios of 4.44 and 2.22 indicate that they are significantly greater than zero.[4] This argument is supported by Figure 1 which shows the cumulative rate of return from options trading of agents using the GARCH and the IVR forecasting methods, respectively.

The more realistic trading strategy will be to trade options when their profits are predicted to exceed trading costs. For this purpose straddles are traded only if the absolute price deviation is greater than \$.25 or \$.50. Under these filters, the number of days of straddle transactions are reduced more dramatically for the IVR method than the GARCH method. For example under a \$.50 filter, straddles are traded 69% of the

[4] Since rates of return from straddle trading for each day are assumed to be independent, the t-ratio is computed as a ratio of mean to standard deviation divided by the square root of number of observation.

total days for the GARCH method and 25% of the total days for the IVR method.

Since the two methods show different numbers of trading days and partly non-overlapping trading dates for filters greater than zero, we need to compare their performances over the entire sample period allowing agents to invest in the risk-free asset when they don't trade options. For the sample period, daily average rates of return from both options trading and the risk-free investment using the GARCH forecasting method is greater than that of the IVR method for all three types of filters. For example, with a \$.25 filter, an agent using the GARCH method makes 1.62% average daily return over 1,048 days and the agent using the IVR method makes 1.04% average daily return over 655 days. The total rate of return from both options trading and risk-free investment yields the agent using the GARCH method 1.36% and the agent using the IVR method only .56%. This suggests that the GARCH agent makes much higher profits than the IVR agent.

After transaction costs of \$.25, and under a zero filter, the agent using the IVR method has negative profits while the agent using the GARCH method has positive profits, which are not significant. When we apply the \$.50 filter, the agent using the GARCH forecasts makes a .84% daily rate of return from straddles trading on 863 days with t-ratio of 2.04 and the agent using the IVR forecasts makes -.25% daily rate of return over 313 days with t-ratio of -.26. Altogether the GARCH agent makes a .58% total return with a t-ratio of 2.06 and the IVR agent makes -.05% total return with a t-ratio of -.18.

When we compute the daily rate of return of near-the-money straddles trading during the sample period using the implied dividend, the results change only slightly compared to Table 8.

Over the sample period, out of 1251 observations, the agent using the GARCH method short-sells straddles for 793 days and makes 1.52% return and buys straddles for 458 days and also makes 1.05% return. This implies that even if, on average, the GARCH method under-predicts market volatilities, the GARCH method predicts volatilities well enough to make the right decision to make profits. For the IVR method, the agent using the IVR method makes 1.28% return when she short-sells straddles for 614 days, while she makes .10% return when she buys straddles for 637 days.

Table 9 reports the rate of return for the subsample period which excludes the market crash on October 1987. The rate of return of the GARCH method is decreased and that of IVR is increased when we exclude the crash period. However, the rate of return of the GARCH method is still higher than that of the IVR method. For example, the GARCH method makes 1.05% return and the IVR method makes .48% with a \$.5 filter before transaction costs. After transaction costs of \$.25 per straddle, both methods make significantly positive rates of return after using a \$1 filter. The GARCH method makes .28% return and the IVR method makes .16% return.

5 Conclusion

We examined two volatility forecast methods, the GARCH forecasting method and the IVR forecasting method to investigate whether they produce future volatility forecasts which can generate positive profits from straddle trading in the S&P 500 index option market. We find that the agent using the GARCH volatility forecast can earn significantly positive profits with near-the-money straddle trading against the market even after transaction costs of \$.25 per straddle, and that the agent using the GARCH forecasting method performs better than the agent using the IVR forecasting method.

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Figure 1: Cumulative Rate of Return from Straddles Trading.

(November 3, 1986 - December 31, 1991)

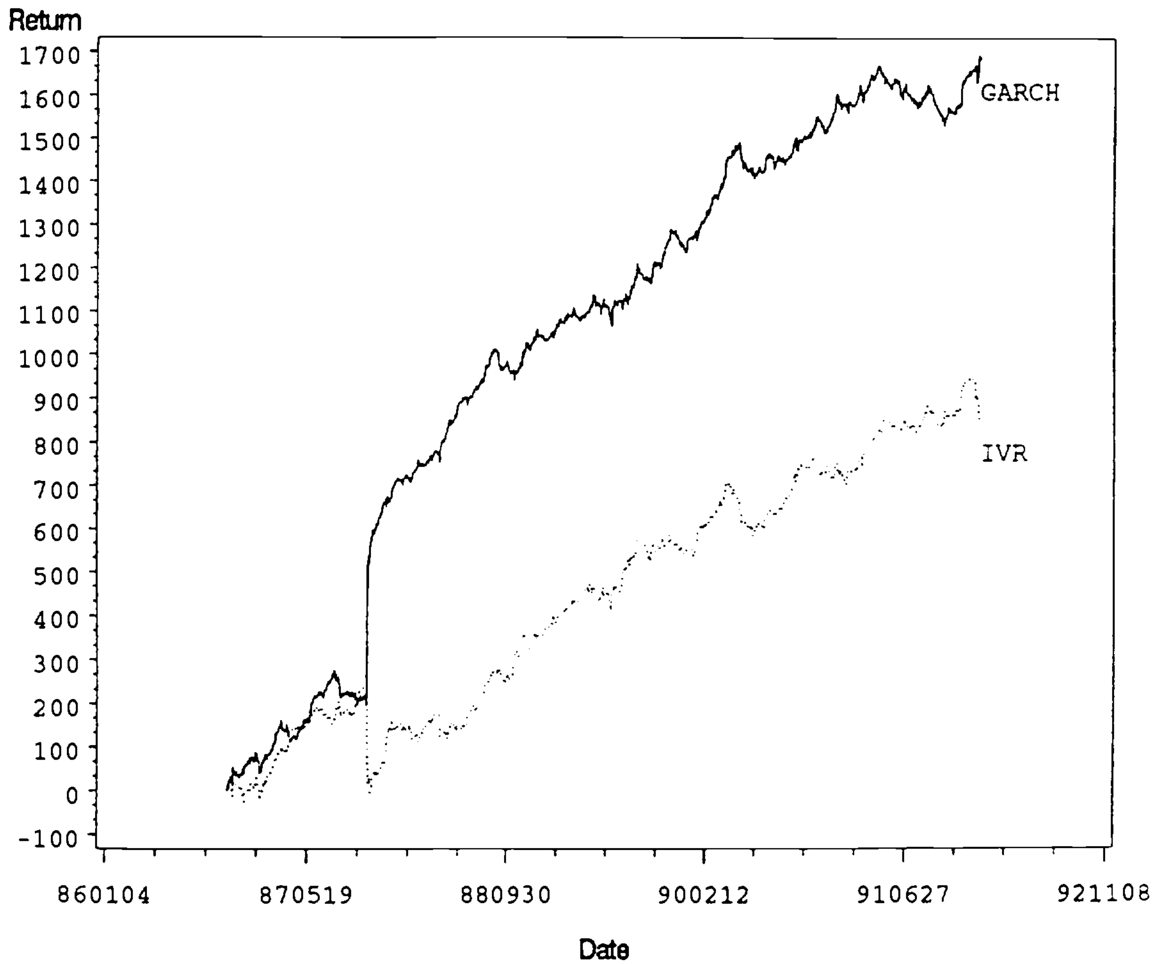

Table 8: Rate of Return from Transaction of $100 worth Straddles per Day.
(November 3, 1986 - December 31, 1991)

Before Transaction Cost

		GARCH				IVR			
filter	Type	Obs.	Mean	Std.	t-ratio	Obs.	Mean	Std.	t-ratio
0.00	STRADDLE	1251	1.348	10.736	4.441	1251	0.680	10.798	2.227
	TOTAL	1251	1.348	10.736	4.441	1251	0.680	10.798	2.227
0.25	STRADDLE	1048	1.620	11.298	4.642	655	1.041	13.421	1.985
	TOTAL	1251	1.361	10.357	4.649	1251	0.557	9.721	2.026
0.50	STRADDLE	863	1.950	12.013	4.770	313	0.955	17.576	0.961
	TOTAL	1251	1.354	10.016	4.780	1251	0.257	8.790	1.035

After Transaction Cost : $.25

		GARCH				IVR			
filter	Type	Obs.	Mean	Std.	t-ratio	Obs.	Mean	Std.	t-ratio
0.00	STRADDLE	1251	0.166	10.793	0.544	1251	-0.605	10.820	-1.976
	TOTAL	1251	0.166	10.793	0.544	1251	-0.605	10.820	-1.976
0.25	STRADDLE	1048	0.465	11.353	1.327	655	-0.210	13.444	-0.400
	TOTAL	1251	0.394	10.392	1.341	1251	-0.098	9.725	-0.358
0.50	STRADDLE	863	0.835	12.050	2.036	313	-0.255	17.554	-0.257
	TOTAL	1251	0.584	10.014	2.064	1251	-0.045	8.771	-0.183

Table 9: Rate of Return from Transaction of $100 worth Straddles per Day.
(November 3, 1986 - October 14, 1987 and November 1, 1987 - December 31, 1991)

Before Transaction Cost

		GARCH				IVR			
filter	Type	Obs.	Mean	Std.	t-ratio	Obs.	Mean	Std.	t-ratio
0.00	STRADDLE	1240	1.042	7.188	5.107	1240	0.865	7.211	4.226
	TOTAL	1240	1.042	7.188	5.107	1240	0.865	7.211	4.226
0.25	STRADDLE	1038	1.256	7.200	5.620	644	1.405	7.509	4.747
	TOTAL	1240	1.056	6.603	5.630	1240	0.741	5.453	4.788
0.50	STRADDLE	853	1.511	7.247	6.091	303	1.871	7.436	4.380
	TOTAL	1240	1.048	6.048	6.100	1240	0.476	3.756	4.461
1.00	STRADDLE	530	1.735	7.247	5.511	82	3.284	8.491	3.503
	TOTAL	1240	0.756	4.810	5.535	1240	0.240	2.317	3.646

After Transaction Cost : $.25

		GARCH				IVR			
filter	Type	Obs.	Mean	Std.	t-ratio	Obs.	Mean	Std.	t-ratio
0.00	STRADDLE	1240	-0.146	7.266	-0.706	1240	-0.427	7.248	-2.075
	TOTAL	1240	-0.146	7.266	-0.706	1240	-0.427	7.248	-2.075
0.25	STRADDLE	1038	0.094	7.280	0.418	644	0.138	7.558	0.464
	TOTAL	1240	0.083	6.660	0.440	1240	0.084	5.445	0.541
0.50	STRADDLE	853	0.388	7.302	1.553	303	0.630	7.390	1.484
	TOTAL	1240	0.275	6.057	1.600	1240	0.173	3.658	1.661
1.00	STRADDLE	530	0.611	7.288	1.930	82	2.130	8.617	2.239
	TOTAL	1240	0.276	4.771	2.035	1240	0.164	2.265	2.545