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MEASURING NOISE IN INVENTORY MODELS

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ABSTRACT

This paper has two purposes. One is to assess different models of inventory behavior in terms of their ability to well approximate the realized data on inventories. We do this initially for the pure production smoothing model and then for a sequence of generalizations of the model. Our analysis both performs specification tests as well as measures the deviations of the data from each null model, which we refer to as model noise. This involves the introduction of a noise ratio which provides a metric for measuring the magnitude of the noise component of the data. A second purpose is to explore whether observed cost shocks, including in particular carefully measured series on raw materials prices, can be helpful in explaining inventory movements. We find that the basic production level smoothing model of inventories, augmented by buffer stock motives, observed cost shocks, properly measured, and to a lesser extent stockout avoidance motives, appears to well approximate monthly inventory data.

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1. Introduction

A large empirical literature has explored various rational expectations models of inventory accumulation.[1] This literature has taken as a baseline and has typically estimated rational expectations formulations of a pure production smoothing model of inventories, where firms accumulate inventories in order to smooth production in the face of fluctuating demand. A number of authors have shown that the pure rational expectations versions of the model are inconsistent with the data, at least as measured by model specification tests. These initial rejections of the model have led researchers to analyze empirically augmented forms of the models. Blanchard [1983], West [1986] and Kahn [1992] have added a stockout avoidance motive, Maccini and Rossana [1984], Blinder [1986] and Miron and Zeldes [1988] have added cost shocks in the form of real input prices; Eichenbaum [1989] and Kollintzas [1992] has added cost shocks in the form of unobservable technology shocks; Ramey [1991] has added nonconvexities in production; West [1988] has added backlogs of unfilled orders.[2] These authors have typically concluded that such modifications improve the fit of the model but do not fully explain the rejections of model specification. One problem with these different studies is that it is difficult to assess whether the specification rejections are economically interesting-i.e, whether a given model is a useful method of explaining the bulk of realized inventory volatility. At the same time, it is difficult to understand the degree to which generalizations of the model are quantitatively important in helping us to understand movements in inventories.

One purpose of the current paper is to assess different models of inventory

[1]See Blinder and Maccini [1991a, b] for a survey of the literature and an assessment of recent research.

[2]Other empirical studies in recent years that test the production smoothing model, generally with data other that the aggregate Commerce Department data, include Fair [1989], Haltiwanger and Maccini [1989], Ghali [1987], Kashyap and Wilcox [1992], Krane and Braun [1991], Rossana [1990], and Schuh [1992].

behavior in terms of their ability to well approximate the realized data on inventories. We utilize signal extraction methods developed by Durlauf and Hall [1990] which thus far have not been exploited in inventory research or previously applied to Euler equation estimation. We do this initially for the pure production smoothing model and then for a sequence of generalizations of the model by treating the data behavior described by the Euler equation as the sum of two unobserved components. The first component is that combination of the data that can be explained by the Euler equation implied by a specified null model; the second component is model noise. We then perform a signal extraction exercise on the data which provides estimates of the variances of the unobserved components. These estimates have the desirable property that the variance of the estimated noise component is a lower bound on the variance of the true noise component. We then compute a noise ratio by dividing the noise variance by the variance of the sum of the components in order to interpret the empirical performance of Euler equations. The noise ratio is closely related to the J-statistic which is conventionally used to test the orthogonality restrictions implied by an Euler equation, and will be proportional when the Euler equation defines a sequence of identically distributed white noise variables. An advantage of the noise ratio is that it provides a way of assessing the degree to which an Euler equation well approximates observed data behavior, *i.e.* whether the noise component is large. It applies even when the model is rejected by formal hypothesis testing procedures. While we apply the noise ratio here to a sequence of inventory equations, we feel that it is useful in other work with Euler equations.

Another purpose of this paper is to determine whether observed cost shocks can be helpful in explaining inventory movements. An important innovation in the analysis is that we use a carefully measured series on raw materials prices as one of our cost shocks. In constructing the series, special attention has been paid to measure the real price to a particular industry of purchasing materials and supplies from outside the industry. To the extent that cost shocks are relevant for short run fluctuations, real materials prices are likely to be important in that they are much more volatile over the cycle than other cost factors such as real wage rates. Our approach improves on the previous literature that has explored cost shocks in that Maccini and Rossana [1984], Miron and Zeldes [1988] and Ramey [1991] use rather imprecise measures of materials prices and obtain rather poor results, while Eichenbaum [1989] who relies on an unobservable cost shock to capture the full range of cost shocks must impose additional restrictions to identify the cost shock. An important finding of the present paper is that observed cost shocks, properly measured, contribute significantly to the explanation of inventory movements.

The paper is organized as follows. Section 2 outlines the theoretical model we consider. Section 3 describes the econometric methodology. Section 4 provides estimates of model noise under different assumptions about the the timing of information, the presence of a stockout avoidance motive, and the inclusion of observed cost shocks. Section 5 explores some alternative specifications of the underlying inventory model. Summary and conclusions follow in Section 6.

2. Inventory models

The model to be used is the standard buffer stock-production smoothing model of inventory behavior. The model contains two key features: (1) variable demand and rising marginal cost of production--which generates production smoothing, and (2) stochastic demand, which generates buffer stock behavior. In this paper, we modify the basic model to allow for a stockout avoidance motive and for cost shocks in the form of real input prices, forces which counter the firm's incentive to smooth production. In future work, we plan to consider technology shocks, nonconvexities in production, and order backlogs. We formulate the model in a linear quadratic framework. The specific model we employ is essentially that of Eichenbaum [1989], but is very closely related to models used by Blanchard [1983], Blinder [1986], Blinder and Maccini [1991a], Ramey [1991] and West [1986], among others.

Production costs are given by

$$C(Y_{t+s}) = (c_1+\Gamma_{t+s})Y_{t+s}+\frac{1}{2c}Y_{t+s}^2 \quad c > 0 \tag{2.1}$$

where Y_{t+s} is the level of output produced by the firm and Γ_{t+s} is a cost shock.

Inventory holding costs take the form

$$B(N_{t+s}, X_{t+s}) = b_1 N_{t+s}+ \frac{b}{2}(N_{t+s}-\alpha X_{t+s})^2 \quad b, \alpha > 0 \tag{2.2}$$

where N_{t+s} is the stock of finished goods inventories at the end of period $t+s$ and X_{t+s} is real sales. Inventory holdings costs are quadratic in inventories, which balances two forces. The linear term (as well as the quadratic term over some range of values) rises with the stock of inventories, capturing increased storage costs, insurance costs and the like. At the same time, over some range of inventory values, the quadratic term falls with inventories, reflecting the notion that higher levels of inventories for a given level of expected sales reduce the likelihood that the firm will stock out and lose sales.

Inventory accumulation is governed by the identity

$$N_{t+s} - N_{t+s-1} = Y_{t+s} - X_{t+s} \tag{2.3}$$

With cost minimization, the firm is assumed to face an exogenous sales process and its objective is to

$$Min \;\; E_t \sum_{s=0}^{\infty} \beta^s TC_{t+s} \tag{2.4}$$

where

$$TC_{t+s} = C(Y_{t+s}) + B(N_{t+s}, X_{t+s}) \tag{2.5}$$

which using the inventory identity

$$= C(N_{t+s} - N_{t+s-1} + X_{t+s}) + B(N_{t+s}, X_{t+s})$$

$$= (c_1 + \Gamma_{t+s})(N_{t+s} - N_{t+s-1} + X_{t+s}) + \frac{1}{2c}(N_{t+s} - N_{t+s-1} + X_{t+s})^2$$

$$+ b_1 N_{t+s} + \frac{b}{2}(N_{t+s} - \alpha X_{t+s})^2 \tag{2.6}$$

and $\beta = \frac{1}{1+r}$ is the discount factor implied by a constant real rate of interest r. Production decisions are initially assumed to be made before demand is revealed at t, so that inventories alone serve to buffer demand shocks. The model contains an incentive for the firm to smooth production in the face of variable demand ($c > 0$), and, since demand is stochastic, an incentive to use inventories to buffer demand shocks. In addition, the model contains forces, in particular cost shocks (Γ_t) and an allowance for the possibility of stockouts ($\alpha > 0$), which serve to increase production variability.

Let lower case letters represent the expected values of corresponding upper case letters, i.e. $x_{t+s} = E(X_{t+s} \mid \mathfrak{F}_t)$, where $\mathfrak{F}_t$ denotes a common information set available to all firms at t. The Euler equation for the cost minimization problem is

$$\beta n_{t+s+1} - [1 + \beta + bc] n_{t+s} + n_{t+s-1}$$

$$= (1 - \alpha bc) x_{t+s} - \beta x_{t+s+1} + c\gamma_{t+s} - \beta c \gamma_{t+s+1} + \tilde{c} \tag{2.7}$$

where $\tilde{c}$ is a constant. Solving this equation for the optimal level of inventories yields

$$n_{t+s} = \lambda n_{t+s-1} + [1 - (1 - \alpha bc)\lambda] \sum_{i=0}^{\infty} (\beta\lambda)^i x_{t+s+i}$$

$$- x_{t+s} + c[1-\lambda] \sum_{i=0}^{\infty} (\beta\lambda)^i \gamma_{t+s+i} - c\gamma_{t+s} + \tilde{c} \tag{2.8}$$

$s = 0, 1, 2...$, where λ is the stable root of the relevant characteristic equation and is given by

$$\lambda = 1+\frac{1}{2}(r+\frac{bc}{\beta})-[(r+\frac{bc}{\beta})^2+4\frac{bc}{\beta}]^{1/2} \tag{2.9}$$

Observe that the parametric assumptions regarding a firm's cost structure predict that $0<\lambda<1$. Further, observe that $1-\alpha bc>1$ only if α, b, or c is negative, which we rule out by assumption in this paper. Hence our model also predicts that $1-\alpha bc<1$.

A relationship for *actual* inventory accumulation, i.e. for calendar date t, may be derived as follows. Set $s = 0$ in (2.8), recognize that $n_{t-1} = N_{t-1}$, and rearrange the resulting relationship to get

$$n_t - N_t = (1-\lambda)N_{t-1} + [1-(1-\alpha bc)\lambda]\sum_{i=0}^{\infty}(\beta\lambda)^i x_{t+i}$$

$$-x_t + c[1-\lambda]\sum_{i=0}^{\infty}(\beta\lambda)^i\gamma_{t+i} - c\gamma_t + \tilde{c} \tag{2.10}$$

Then, take expected values of (2.3) for $s = 0$ and observe that under our assumptions, $\mathrm{E}Y_t = Y_t$, which implies

$$\mathrm{E}(N_t - N_{t-1}) = n_t - N_{t-1} = \mathrm{E}Y_t - \mathrm{E}X_t = Y_t - x_t \tag{2.11}$$

Now, use the inventory identity evaluated at $s = 0$ and (2.11) for

$$N_t - N_{t-1} = Y_t - X_t = Y_t - x_t + (x_t - X_t)$$

$$= n_t - N_{t-1} + (x_t - X_t) \tag{2.12}$$

Substituting (2.10) into (2.12) and rearranging terms yields

$$N_t = \lambda N_{t-1} - X_t + [1-(1-\alpha bc)\lambda]x_t$$

$$+ [1-(1-\alpha bc)\lambda]\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i}$$

$$-c\lambda\gamma_t + c[1-\lambda]\sum_{i=1}^{\infty}(\beta\lambda)^i\gamma_{t+i} + \bar{c} \quad (2.13)$$

This relationship governs inventory accumulation in the model.

3. Econometric methodology

Current sales and cost shocks observable

In order to capture the testable implications of this model, we exploit the expectations based relationships within the model. Our approach identifies parameters by estimating the equivalent of the Euler equation associated with optimal inventory accumulation for different specifications of the objective function and information sets of agents. Unlike other work in this literature, however, we complement specification testing of the model by attempting to assess the ability of the model to successfully approximate the data. We therefore provide a noise ratio calculation which is a natural metric for measuring the accuracy of the null model in approximating the data even if the model is rejected by formal hypothesis testing procedures. Durlauf and Hall [1990] advocate this calculation as a way of assessing the reasonableness of a particular model in explaining data movements.

In order to provide a rationale for the noise ratio, we derive our estimating equation in a somewhat different fashion from the direct use of an Euler equation. Define the following variable

$$\Delta_t = N_t - \lambda N_{t-1} + (1-\alpha bc)\lambda X_t + c\lambda\Gamma_t. \quad (3.1)$$

Under the null hypothesis that agents observe X_t and Γ_t at the time that inventories at t are chosen, $x_t = X_t$ and $\gamma_t = \Gamma_t$, we can use (2.13) to rewrite this variable as

$$\Delta_t = (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i} + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\gamma_{t+i} + \bar{c}. \tag{3.2}$$

This is a pure expectations-based variable. Since this variable equals a conditional expectation, this allows us to define a perfect foresight analog.

$$\Delta_t^* = (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i X_{t+i} + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\Gamma_{t+i} + \bar{c}$$

$$= (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i} + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\gamma_{t+i} + \eta_t \tag{3.3}$$

where η_t is a forecast error related to current and future sales and cost shocks. The variable Δ_t^* may be directly constructed from the sales and cost series if β, α, b, c and λ are known. As shown in Durlauf and Hall [1990], all testable implications of the null model can be summarized in the orthogonality of $\Delta_t - \Delta_t^*$ to any information measurable at t.

Observe that $\Delta_t - \Delta_t^*$ is not in general white noise as it depends on current as well as future realizations of different variables. In order to efficiently estimate the underlying parameters in $\Delta_t - \Delta_t^*$, it is useful to filter the variable by $(1-\beta\lambda L^{-1})$ to generate

$$g_t = (1-\beta\lambda L^{-1})(\Delta_t - \Delta_t^*)$$

$$= -\beta\lambda\Delta_{t+1} + \Delta_t - (1-\lambda(1-\alpha bc))\beta\lambda X_{t+1} - c(1-\lambda)\beta\lambda\Gamma_{t+1} + \hat{c} \tag{3.4}$$

where $\hat{c} = -(1-\beta\lambda)\bar{c}$. This filter produces a left hand side variable that is proportional to the Euler equation. Under the null hypothesis, g_t is white noise. Again, the parameters of this process are not observed. However, using (3.1), (3.4) can be written as

$$g_t = -\beta\lambda N_{t+1} + (1+\beta\lambda^2)N_t - \lambda N_{t-1} - \beta\lambda X_{t+1}$$

$$+ (1-\alpha bc)\lambda X_t - c\beta\lambda\Gamma_{t+1} + c\lambda\Gamma_t + \widehat{c} \tag{3.5}$$

to which generalized methods of moments (GMM) can be applied. Following the literature, we impose a value for β equal to .995. Defining $\mu = 1-\alpha bc$, we may estimate λ, μ and c by GMM. The idea is to choose these parameters to force orthogonality of the implicitly estimated g_t to some set of instruments.

We shall refer to g_t and its subsequent variants as Euler equation forecast errors, as the variable captures, under the null hypothesis, an exact linear combination of inventories, sales and cost shocks which represents the optimal adjustment to information received between $t-1$ and t, i.e. $\mathfrak{F}_t - \mathfrak{F}_{t-1}$. Under the null hypothesis, g_t equals some combination of the information which lies in $\mathfrak{F}_t - \mathfrak{F}_{t-1}$. We denote the value of this particular combination of new information as ν_t.

$$\text{H}_0\text{:}\ \ g_t = \nu_t \tag{3.6}$$

Under any alternative H_1, the variable g_t may be defined as the sum of ν_t and a variable S_t which represents the component of g_t which deviates from the null.

$$\text{H}_1\text{:}\ \ g_t = \nu_t + S_t \tag{3.7}$$

We shall refer to S_t as model noise.

Letting L_t denote the econometrician's information set, and Z_t denote an operator which linearly projects a variable onto this information set, we can construct an estimate of model noise, $S_{t\,|\,t}$, through[3]

$$S_{t\,|\,t} = g_t Z_t. \tag{3.8}$$

since ν_t is by definition orthogonal to L_t. As shown in Durlauf and Hall [1990] and Durlauf and Hooker [1993], the estimate of model noise is the solution to a signal extraction problem which attempts to identify the unobservable S_t from the available data. Those papers verify that an important property of the estimated model noise is that the variance of estimated model noise is a lower bound on the variance of actual model noise, i.e. for each g_t,

$$\mathrm{Var}(S_{t\,|\,t}) \leq Var(S_t). \tag{3.9}$$

The variance of model noise is interesting as it provides a metric of the extent to which a model well approximates the data. Standard specification tests ask whether model noise possesses a zero variance – i.e whether the Euler equation fully characterizes the interactions of a set of time series. This requirement may be too extreme to assess the utility of macroeconomic models, where the combination of data measurement problems and many auxiliary assumptions employed for the sake of analytical convenience suggest that model noise is unlikely to equal zero. The variance of model noise provides a way of determining whether a model is a useful way of thinking about the behavior of a particular time series. In other words, low variance in estimated model noise means that the joint behavior of inventories with other variables is close to that predicted by the optimal inventory model.

Our analysis focuses on the noise embedded in the Euler equation rather than the inventory series itself for two reasons. First, since our interest is in the relevance of the optimal inventory model in explaining inventory movements, the Euler equation provides the context for assessing these movements relative to those variables which are supposed to affect them under the null model. Second, our experience suggests that estimates of

[3] By definition, $S_{t\,|\,k}$ equals the projection of S_t onto L_k.

noise based on the Euler equation possess relatively good finite sample properties.

In assessing the magnitude of the variance of model noise it is useful to consider an estimated noise ratio NR, which we define as

$$NR = \frac{Var(S_{t\mid t})}{Var(g_t)}. \tag{3.10}$$

This ratio normalizes the estimated noise variance bound. Although S_t and ν_t are not orthogonal, $S_{t\mid t}$ and ν_t are orthogonal since $S_{t\mid t}$ is constructed to lie in L_t. This allows us to interpret the ratio NR as a lower bound on the percentage of $Var(g_t)$ which is attributable to noise. We use the noise ratio for the first time to assess the performance of Euler equations.

There are many ways to construct measures of $Var(S_{t\mid t})$ given L_t and the null model, each one of which corresponds to a different way of estimating the parameters of the null model in order to construct an observable version of the series g_t. In the subsequent analysis, we follow the following procedure. GMM parameter estimates are used to construct g_t. This g_t series is then linearly projected against the instrument set used to estimate the model parameters in order to compute $S_{t\mid t}$ and its associated variance. This means that the noise ratio measures the implied model noise associated with the set of model parameter estimates which minimize the $J-$statistic proposed by Hansen [1982] to test the orthogonality restrictions in the GMM procedure. Our procedures therefore measure the noise ratio implicitly associated with each specification test we employ. When the Euler equation innovations are white noise, our estimated model noise variance ratio will represent the lowest admissible for all possible noise/model decompositions which are consistent with the data. When these innovations are not white noise, our noise ratio estimate is the lowest admissible value for the ratio compatible with those model parameter estimates which minimize deviations between the null model and the data as measured by the J-statistic. In fact, for white noise errors one can show that the noise ratio will equal the J-statistic divided by the number of observations in the data set; for non-white noise errors this equivalence will not hold.[4] An advantage of the noise ratio is that it provides a metric for assessing the degree to which the Euler equation approximates the data, even when the model is rejected by specification tests such as the J-statistic.

Finally, observe that if current sales and cost shocks are known when decisions are made at time t, then by (2.5) end-of-period inventories at t are also known and are thus contained in the information set.

Current sales and cost shocks unobservable

If agents at t cannot observe sales and cost shocks at the time that inventories are determined, then $x_t \neq X_t$ and $\gamma_t \neq \Gamma_t$. This is an important alternative case in that in most inventory models current sales and costs are assumed to be unknown to the firm at the time it makes decisions so that inventories serve to buffer current sales and cost surprises. The previous equations should then be modified as follows. Using (2.13), under this formulation of the null hypothesis,

$$\Delta_t' = N_t - \lambda N_{t-1} + X_t$$

$$= (1-(1-\alpha bc)\lambda)x_t + (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i}$$

$$- c\lambda\gamma_t + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\gamma_{t+i} + \tilde{c} \tag{3.11}$$

[4]When the Euler equation implicitly defines a non-$i.i.d$ sequence, the noise ratio will not be proportional to the J-statistic because of a need in the latter case to correct the covariance matrix of the equation's innovations in computing whether the model's coefficients obey the overidentifying restrictions. Such a correction is unnecessary in computing the variance of the model noise. However, the two are still closely related asymptotically as the noise ratio will converge to zero if the J-statistic converges to a bounded value. We thank Spencer Krane for these observations.

which defines a new perfect foresight series

$$\Delta_t'^* = (1-(1-\alpha bc)\lambda)X_t + (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i X_{t+i}$$

$$- c\lambda\Gamma_t + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\Gamma_{t+i} + \bar{c}$$

$$= (1-(1-\alpha bc)\lambda)x_t + (1-(1-\alpha bc)\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i}$$

$$- c\lambda\gamma_t + c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i\gamma_{t+i} + \xi_t \tag{3.12}$$

where ξ_t is a forecast error.

All testable implications of the null model can be summarized in the orthogonality of the newly defined $\Delta_t' - \Delta_t'^*$ to all information measurable at $t-1$, since the new forecast error contains an event measurable at t. Notice that g_t' now equals

$$g_t' = (1-\beta\lambda L^{-1})(\Delta_t' - \Delta_t'^*)$$

$$= -\beta\lambda\Delta_{t+1}' + \Delta_t' - (1-(1-\alpha bc)\lambda)X_t + c\lambda\Gamma_t - c\beta\lambda\Gamma_{t+1} - \widehat{c}$$

$$= -\beta\lambda N_{t+1} + (1+\beta\lambda^2)N_t - \lambda N_{t-1} - \beta\lambda X_{t+1}$$

$$+ (1-\alpha bc)\lambda X_t - c\beta\lambda\Gamma_{t+1} + c\lambda\Gamma_t - \widehat{c} \tag{3.13}$$

which is exactly the same form as g_t. Hence, the estimating equation is independent of whether sales and cost shocks are observable. We again assume that $\beta = .995$ and use GMM to estimate λ, $\mu = 1-\alpha bc$ and c.

The difference in estimating the model when sales and cost shocks are unobservable is that the information set available to the econometrician does not include current sales and cost shocks. Note that since current sales and cost shocks are unobservable at t, inventories at t are also unknown and are thus excluded as well from the information set. Letting L_{t-1} denote the econometrician's information set, and Z_{t-1} the projection operator for this information, our estimate of model noise is now

$$S_{t\,|\,t-1} = g_t' Z_{t-1} \tag{3.14}$$

which may be used to estimate $NR' = \dfrac{Var(S_{t\,|\,t-1})}{Var(g_t')}$.

From the perspective of specification testing, the change in the dating of the econometrician's information set means that g'_t will now be MA(1).

Multiple cost shocks

Cost shocks typically derive from multiple sources. For example, production costs can be affected by fluctuations in materials prices M_{t+s}, wages W_{t+s} and energy prices V_{t+s}. In order to account for the impact of these different input prices on firm production decisions, it is necessary to choose a particular functional form for the cost function.

We relate the cost function to the input prices through the relation

$$C(Y_{t+s}) = [c_1 + \theta_1 W_{t+s} + \theta_2 M_{t+s} + \theta_3 V_{t+s}]\, Y_{t+s} + \frac{1}{2c} Y_{t+s}^2 \tag{3.15}$$

so that the θ_i is the marginal cost response to a change in input price i. Notice that if

$$\Gamma_{t+s} = \theta_1 W_{t+s} + \theta_2 M_{t+s} + \theta_3 V_{t+s}, \tag{3.16}$$

then we will recover our cost function (2.7).

The solution for actual inventories (2.13) is then

$$N_t = \lambda N_{t-1} - X_t + [1-(1-\alpha bc)\lambda]x_t$$

$$+ [1-(1-\alpha bc)\lambda]\sum_{i=1}^{\infty}(\beta\lambda)^i x_{t+i}$$

$$+ c(1-\lambda)\sum_{i=1}^{\infty}(\beta\lambda)^i[\theta_1 w_{t+i} + \theta_2 m_{t+i} + \theta_3 v_{t+i}], \tag{3.17}$$

where the lower case letters again denote expected values. This implies that our estimating equation becomes

$$g_t = -\beta\lambda N_{t+1} + (1+\beta\lambda^2)N_t - \lambda N_{t-1} - \beta\lambda X_{t+1}$$

$$+ (1-\alpha bc)\lambda X_t - \beta\lambda\Psi_1 W_{t+1} + \lambda\Psi_1 W_t - \beta\lambda\Psi_2 M_{t+1} + \lambda\Psi_2 M_t$$

$$- \beta\lambda\Psi_3 V_{t+1} + \lambda\Psi_3 V_t + \hat{c}, \tag{3.18}$$

where $\Psi_1 = \theta_1 c$, $\Psi_2 = \theta_2 c$ and $\Psi_3 = \theta_3 c$, with $\Psi_i > 0$. This equation forms the basis for our empirical work with cost shocks. Notice that since both future sales and cost shocks are discounted by the same factor, $(\beta\lambda)^i$, this equation will have the same statistical properties as (3.5).

4. Econometric results

The data used for inventories and sales are the constant dollar finished goods inventory and shipments data published by the Bureau of Economic Analysis. They are monthly, seasonally adjusted, measured in 1982 dollars and cover the period 1959:1 to 1990:6. The data were adjusted to place inventory stocks and shipments in comparable units.[5] To measure cost shocks, we use real materials prices, real wage rates and real energy prices. The nominal materials prices are weighted averages of producer price indexes used by a particular industry, where the weights are the value of each material input expressed as a fraction of the value of shipments of materials to the industry. We work with series for both crude materials prices which measure the prices of unprocessed materials and supplies and intermediate materials prices which measure the prices of partially processed materials and supplies. Whenever necessary, all series were deseasonalized and detrended with the use of a quadratic time trend. In this paper, since the model is strictly applicable only to industries that produce to stock, we focus on the nondurables aggregate and selected SIC two-digit industries – Food, Chemicals, Petroleum and Coal, and Rubber and Plastics – which do not carry unfilled orders and are presumed largely to produce to stock.

Tables 1 through 4 report our empirical results with different formulations of the production smoothing buffer stock model. We estimate the implied Euler equation forecast error for inventories – the g_t equation – with two different information sets, which essentially differ on whether current sales, end-of-period inventories and cost shocks are included.[6] Recall that this difference reflects whether inventories buffer current sales and cost shocks. Estimation is carried out with Generalized Methods of Moments. Associated with the parameter estimates of each model are J-statistics which test the orthogonality of g_t to different information sets. Under the null hypothesis of no model noise, these statistics are distributed as χ^2_q, where q equals the number of overidentifying

[5]This adjustment is needed because inventories are measured at cost whereas shipment are measured at market. See West [1983] for the appropriate adjustment.

[6]Certain authors, e.g. Kashyap and Wilcox [1992], West and Wilcox [1992], and Fuhrer, Moore, and Schuh [1992], have pointed out that estimates of parameters of Euler equations can be sensitive to the choice of a left hand side variable or in effect of a normalization. This problem appears to be more severe with the estimation of underlying cost parameters than with the estimation of combinations of parameters such as λ, which evidently can be estimated more tightly. Nevertheless, the equation we estimated, (3.18), does embody a particular normalization. As a check, we explored alternative normalizations by, for example, dividing (3.18) by $-\beta\lambda$ and estimating the resulting equation. This produced results very close to those presented.

restrictions in the model, i.e. the number of instruments less the number of parameters to be estimated. Finally, we report estimates of model noise bounds, NR, which allow us to assess how well the model approximates the data.

Table 1 reports coefficient estimates and associated standard errors for the pure production smoothing model with the two different information sets. Pure production smoothing arises when there is neither a stockout avoidance motive ($\alpha = 0$ and $\mu = 1$) nor cost shocks ($\gamma = 0$). The first row for each industry reports results with X_t and N_t observable; cost shocks are excluded entirely from the information set since this version of the model disallows cost shocks. Perhaps not surprisingly, in view of the literature on empirical work with inventory models, the model performs poorly. In particular, the noise ratios, which place a lower bound on the ability of the model to explain the data are very high, typically in excess of 50%. The J-statistics overwhelmingly reject the overidentifying restrictions of the model. All 5 test statistics in the Table are statistically significant at the 5% level.

The second row reports results with X_t and N_t excluded from the information sets, which captures the idea that inventories buffer sales surprises. The pure production smoothing model now performs much better in that the noise ratios are much lower, on the order of 13% or less and the J-statistics drop substantially. All 5 test statistics are, however, still statistically significant. An additional major caveat is that the estimates of λ are generally quite high, on the order of .7. As is well known, the inventory accumulation equation (2.13) can be rewritten in stock adjustment form where $1 - \lambda$ represents the speed at which inventories adjust towards desired levels. The point estimates of λ imply that inventories are adjusting towards optimal levels at only about 30% per month, which seems implausibly low. Hence, allowing inventories to buffer current sales surprises improves considerably the fit of the pure production smoothing model. Nevertheless, substantial room for improvement remains.

Table 2 allows for a stockout avoidance motive. Compared to Table 1, this relaxes the assumption that $\alpha = 0$. We therefore estimate $\mu = 1 - \alpha bc$. When X_t and N_t are observable, this model is equivalent to the model analyzed by Eichenbaum [1989] as the production-level smoothing model. When a stockout avoidance motive is considered and X_t and N_t are observable, the noise ratios and J-statistics fall slightly, but the model is still decisively rejected. On the other hand, in contrast to Eichenbaum [1989], allowing for a stockout avoidance motive in a model where X_t and N_t are not observable further reduces the noise ratios and J-statistics so that the model fits the data reasonably well. The model is in fact accepted in the cases of food and petroleum. Further, the noise ratios are now quite small, suggesting that the rejections of the model are not associated with large departures from the data. Two caveats arise, however. The estimates of λ are a bit larger than in Table 1, implying somewhat slower adjustment speeds. Further, the estimates of μ are quite close to unity; in fact, many are insignificant from one.[7]

Table 3 tests the model with cost shocks as well as a stockout avoidance motive. In Table 3, the material price component of the cost shocks is measured by crude materials prices. Even when values of X_t, N_t, M_t, W_t and V_t are included in the information set, the model fits the data quite well. The noise ratios are all below 20% and the J-statistics accept the model for food and rubber. The model fits especially well when X_t, N_t, M_t, W_t and V_t are excluded from the information set as the noise ratios and the J-statistics are substantially lower. All noise ratios are now below 8% and the model is accepted for all industries. This further highlights the support for treating inventories as a buffer for current sales and cost shocks. Moreover, the estimates of λ are now generally more plausible, implying adjustment speeds, $1-\lambda$, in the range of $.4-.5$. The estimate of the parameters associated with cost shocks from the instrument set that excludes current values are generally positive and frequently significant despite a good deal of collinearity among the measures of input prices. This is quite contrary to the

[7]Estimates of μ near unity and thus possibly estimates of α near zero are not necessarily evidence against a stockout avoidance motive. Kahn [1992] has argued that a better approximation to a stockout avoidance motive is to allow inventory holding costs, (2.2), to depend on $N_{t+s-1}+Y_{t+s}-\widehat{\alpha}X_{t+s}$, which using the inventory identity implies that the argument of the holding cost function is $N_{t+s}-(\widehat{\alpha}-1)X_{t+s}$. Then estimates of μ near unity and α near zero are evidence that $\alpha-1$ is near unity, which is very much evidence of a stockout avoidance motive.

body of previous studies which finds that observed cost shocks often have the wrong sign and are invariably insignificant. Estimates of μ are more mixed. Again, in some cases they are insignificantly different from unity.

To explain inventory movements, these results provide strong support for a model which augments production smoothing motives by allowing for observed cost shocks, buffer stock motives, and to a lesser extent stockout avoidance motives. The model contrasts with that put forward by Eichenbaum [1989] and Kollintzas [1992] in two respects. First, these authors focus on a model with unobservable cost shocks and provide evidence that the unobservable noise in the inventory Euler equation behave as an AR(1) process. However, the empirical analysis assumed that this noise was a cost shock. By using actual cost shock data, our approach does not require any identifying assumptions concerning the source of model noise and thus directly shows how cost shocks can reconcile production smoothing models with the data. Second, Eichenbaum at least implicitly ignored a buffer stock motive for current stocks by assuming that firms know current sales and cost shocks when making decisions. We have shown that allowing for a buffer stock motive significantly improves the ability of the model to explain inventory movements.

Table 4 reports results with intermediate materials prices used as a measure of materials prices. The results represent a deterioration from those of Table 3. The noise ratios generally rise, and even with instrument set 4, the overidentifying restrictions of the model are rejected at the 5% significance level in some industries. Further, the estimates of the parameters associated with the materials price cost shock frequently have the wrong sign. In view of the poorer performance of the results for intermediate input prices and the inevitable difficulties in ensuring that intermediate materials prices are exogenous, the results for crude materials prices are clearly preferred.

5. Additional results

Target Inventories

To capture the stockout avoidance motive, a target inventory equation must be specified. Following Eichenbaum [1989], we assumed that the target depends on current sales. An alternative assumption is that it depends on next period's sales, as in Blanchard [1983], Ramey [1991], and West [1986] among others. This is perhaps a more plausible assumption in that N_{t+s} in (2.2) is end-of-period inventories. To see whether this specification makes a difference, assume that (2.2) takes the form

$$B(N_{t+s}, X_{t+s}) = b_1 N_{t+s} + \frac{b}{2}(N_{t+s} - \alpha X_{t+s+1})^2. \qquad (2.2')$$
$$b, \alpha > 0$$

Then, straightforward manipulation results in a modified estimating equation, which is

$$g_t' = -\beta\lambda N_{t+1} + (1+\beta\lambda^2)N_t - \lambda N_{t-1} - \beta\lambda[1 + (\frac{\alpha bc}{\beta})]X_{t+1} + \lambda X_t$$
$$- \beta\lambda\Psi_1 W_{t+1} + \lambda\Psi_1 W_t - \beta\lambda\Psi_2 M_{t+1} + \lambda\Psi_2 M_t - \beta\lambda\Psi_3 V_{t+1} + \lambda\Psi_3 V_t + c. \quad (3.18')$$

We now estimate λ, $\mu' = 1 + (\frac{\alpha bc}{\beta})$, and Ψ_i for $i = 1, 2, 3$. The model predicts μ' to be greater than unity.

The results are presented in Table 5. We only present the results for a model where the instrument set excludes current values and where crude materials prices are used as the materials price series. These are the conditions that produced the best results in prior tests. The results are essentially the same as in Table 3. Again, the noise ratios are all under 8% and the J-statistics accept the model for all industries. The adjustment speeds are extremely close to those in Table 3. Finally, μ' is generally estimated to be above unity, as the model now predicts, although it is statistically significantly greater than zero only for chemicals. We thus conclude that the specification of the target stock makes little difference in the analysis.

Costs of Changing Output

Frequently, researchers estimate an inventory model which allows for costs of changing output. See e.g. Blanchard [1983], Eichenbaum [1989], Ramey [1991] and West [1986]. Such costs may be due to costs of changing underlying inputs such as labor. To incorporate costs of changing output into the model, assume now that production costs are given by

$$C(Y_{t+s}, Y_{t+s-1}) = (c_1+\Gamma_{t+s})Y_{t+s}+\frac{1}{2c}Y_{t+s}^2+\frac{1}{2m}(Y_{t+s}-Y_{t+s-1})^2 \quad c, m > 0 \quad (2.1')$$

where $C(Y_{t+s}, Y_{t+s-1})$ denotes the cost of producing Y_{t+s} at $t+s$ given that output the previous period is Y_{t+s-1}; Γ_{t+s} is again given by (3.15). Strictly convex costs of changing output create an additional incentive for the firm to smooth output beyond that already contained in the rising marginal cost of the level of output. In this case, the estimating equation becomes

$$\begin{aligned} g_t'' = \beta^2\lambda N_{t+2} - \beta\lambda[1+\beta\delta+(2+\beta)\delta]N_{t+1} \\ +[1+\beta^2\lambda+\beta\lambda(2+\beta)\delta+\lambda\delta(1+2\beta)]N_t - \lambda[1+\delta+(1+2\beta)\delta]N_{t-1} \\ +\lambda\delta N_{t-2}+\beta^2\lambda\delta X_{t+2}-\beta\lambda[1+(2+\beta)\delta]X_{t+1}+\lambda[(1+2\beta)\delta+\mu]X_t-\lambda\delta X_{t-1} \\ -\beta\lambda\Psi_1 W_{t+1}+\lambda\Psi_1 W_t-\beta\lambda\Psi_2 M_{t+1}+\lambda\Psi_2 M_t-\beta\lambda\Psi_3 V_{t+1}+\lambda\Psi_3 V_t+c \quad (3.18'') \end{aligned}$$

where now λ, $\mu = 1-\alpha bc$, Ψ_i and $\delta = (c/m)$ are estimated. The model predicts that $\delta > 0$.

The results are reported in Table 6, and again we only report results for the case where the instrument set excludes current values and materials prices are measured by crude materials prices. This modification of the model again leaves the results of Table 3 essentially unchanged. The estimates of δ are invariably of the wrong sign and generally are insignificant, although the estimate for nondurables is marginally significant. Not surprisingly, the point estimates and standard errors of λ, μ and Ψ_i are very close to their values in Table 3. Further, little change takes place in the noise ratios and the J-statistics. We thus conclude that modifying the model to incorporate costs of changing output does nothing to improve the fit of the model, and that once cost shocks, buffer stock motives and stockout avoidance motives are controlled for, costs of changing output are not evident in the data.

Underlying cost parameters

So far, we have estimated structural parameters, namely λ, μ, and the Ψ_i's, which are combinations of the underlying cost parameters. This was done for two reasons. First, we structured our analysis to follow in parallel to Eichenbaum [1989] in order to contrast our results using observed cost shocks with those he obtained with unobserved cost shocks. Second, certain combinations of parameters, such as the stable root of the characteristic equation, can be estimated more precisely than the underlying cost parameters, as is evident from the wide confidence intervals that estimates of cost parameters have produced in the literature. However, since many studies have in fact focused on these parameters, most notably West [1986] and Ramey [1991] who both worked with aggregate BEA data,[8] it is useful to derive the underlying cost parameters that are implied by our estimates.

To derive cost parameters, observe that (2.7) implies that the roots must satisfy

$$\beta\lambda + \frac{1}{\lambda} = 1 + \beta + bc \tag{5.1}$$

or

[8]See Blanchard [1983], Kashyap and Wilcox [1992], Krane and Braun [1991], and Schuh [1992] for studies that have estimated structural cost parameters with different data sets.

$$\frac{1}{bc} = [\beta\lambda + \frac{1}{\lambda} - (1+\beta)]^{-1} \qquad (5.2)$$

which is ratio of the slope of the marginal cost of production ($\frac{1}{c}$) to the slope of marginal inventory holding costs (b). As is well known, only relative convexities, not individual cost parameters, are identified. Further, from $\mu = 1 - \alpha bc$, we have that

$$\alpha = \frac{1-\mu}{bc} \qquad (5.3)$$

which is the target-inventory sales ratio, which is identified. Finally, from $\Psi_i = \theta_i c$, we have that

$$\frac{\theta_i}{b} = \frac{\Psi_i}{bc} \qquad (5.4)$$

which is the response of the marginal production cost to a change in the real input price relative to marginal inventory holding costs.

The cost parameters together with standard errors (in parentheses) are presented in Table 7. The table uses the results of Table 3 with instrument set 4, which represents our preferred set of results, and Table 6, to derive underlying cost parameters. Recall that Tables 3 and 6 report respectively estimates of models excluding and including adjustment costs to changing output. As the table indicates, the marginal cost of production relative to marginal inventory holding costs is positive and quantitatively similar in all industries. Furthermore, not surprisingly, given the literature that has estimated cost function parameters, the confidence intervals for the estimates of the slope of relative marginal production cost are considerably wider than those for λ, which highlights an advantage of estimating certain combinations of underlying cost parameters, such as the stable root of the characteristic equation. Still, the estimates of the relative marginal production cost, $\frac{1}{bc}$, are generally significant at the 5% level for the parameters derived from Table 3, which are our preferred estimates, and at least at the 10% level for those derived from Table 6. Similarly, the target inventory-sales ratios are plausible, indicating that firms in most industries shoot for target inventories of just under 1 month's sales, with exceptions for rubber and perhaps petroleum. Finally, the relative marginal cost of a change in the i'th real input price is generally positive and frequently significant, though the quantitative magnitudes vary a good deal across industries.

Of particular interest is the sign of the marginal cost of production, which indicates whether the industry production function exhibits increasing or decreasing marginal costs. The quantitative magnitude of marginal production cost depends on the particular normalization used.[9] Consider first the normalization $b=1$, which was considered by Ramey [1991]. It is immediately evident from Table 7, whether one uses Table 3 or Table 6 as the source of underlying cost parameters, that marginal cost is positive and generally significant in all industries, contrary to Ramey's original results. It is also useful to recognize that in a model with adjustment costs the change in total cost due to a change in current production has slope $(1+\beta)\frac{1}{bm}+\frac{1}{bc}$. As the table indicates, under the normalization that $b=1$, this slope is also positive and generally significant at the 10% level in all industries. An alternative normalization, advocated by West [1986], is that $(1+\beta)\frac{1}{m}+\frac{1}{c}=1$. The results for this case are presented in the last 2 columns of Table 7. In the model estimated in Table 3, which ignores adjustment costs to changing output, we assumed that $\frac{1}{m}=0$, which under the West normalization implies that $\frac{1}{c}=1$. To better gauge the impact of this normalization, we use the model and the results of Table 6, which are presented in the bottom row of each industry of Table 7. As the column for $\frac{1}{c}$ in the table indicates, the marginal cost of production is still positive and often significant in all industries. Overall, then, we conclude that our results are

[9]Note that we are not presenting estimates of cost function parameters estimated under different normalizations. Rather, we are taking our estimates of the slopes of relative marginal cost derived from our estimates of λ, μ, δ, and the Ψ_i's and exploring the implications for our parameter estimates of different normalizations that have been used in the literature.

evidence in favor of rising marginal cost.

5. Summary and conclusions

In this paper, we have attempted to identify the sources and magnitude of misspecification or noise in various forms of the inventory production smoothing model. We do this by considering the impact of a buffer stock motive, a stockout avoidance motive and observable cost shocks on the goodness of fit of the model. An innovation of the paper is that we assess goodness of fit both in terms of a noise ratio which measures the contribution of model noise to the interactions of inventories, sales and costs shocks, as well as through specification tests of the model.

Our conclusions are fourfold. First, we have identified substantial noise in the pure production smoothing model where firms do not use inventories to buffer current shocks, avoid stockouts or smooth cost shocks to accumulate inventories. Over 50% of the variance of estimated Euler equation forecast errors can be attributed to model noise in this case. When an incentive to buffer current shocks is incorporated into the model, the amount of model noise drops dramatically, to 10%-15% of the total variance. The model, however, is still rejected in formal hypothesis testing. Second, we found some evidence that the stockout avoidance motive provides additional power in explaining model noise. The combination of buffer stock and stockout avoidance motives eliminated some of the dramatic rejections reported by Eichenbaum [1989] and others in the literature. Third, we have shown that the remaining noise in the model can be well explained by observable cost shocks. Cost shocks based on crude material prices, wage rates and energy prices in conjunction with the buffer stock and stockout avoidance motives eliminated all rejections of the model and generated very low point estimates of model noise. Finally, and more generally, the paper illustrates the usefulness of the noise ratio as a metric for assessing the degree to which Euler equations approximate the observed data, and should be useful in other studies as well.

Collectively, these results suggest that a generalized rational expectations production smoothing model can well explain observed inventory movements.

Table 1
Pure Production Smoothing Model

Industry	λ	NR	J-Stat[1]
Nondurables			
Inst. Set 1	.547 (.037)	.490	96.48*
Inst. Set 2	.658 (.052)	.129	43.51*
Chemicals			
Inst. Set 1	.634 (.034)	.426	81.50*
Inst. Set 2	.700 (.042)	.120	37.82*
Food			
Inst. Set 1	.489 (.040)	.577	74.22*
Inst. Set 2	.681 (.080)	.072	17.90*
Petroleum			
Inst. Set 1	.448 (.040)	.591	90.99*
Inst. Set 2	.666 (.066)	.037	11.27*
Rubber			
Inst. Set 1	.547 (.034)	.520	88.82*
Inst. Set 2	.692 (.073)	.115	28.55*

Inst. Set 1: N_t, N_{t-1}, N_{t-2}, N_{t-3}, X_t, X_{t-1}, X_{t-2}, X_{t-3}, constant
Inst. Set 2: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, constant

[1]The asterisk denotes that the overidentifying restrictions of the model are rejected at the .05 level of significance.

Table 2
Stockout Avoidance Model

Industry	λ	μ	NR	J-Stat[1]
Nondurables				
Inst. Set 1	1.920	.855	.524	85.21*
	(.142)	(.033)		
Inst. Set 2	.720	.944	.078	25.78*
	(.054)	(.024)		
Chemicals				
Inst. Set 1	.697	.923	.339	53.83*
	(.035)	(.023)		
Inst. Set 2	.743	.922	.090	17.96*
	(.044)	(.025)		
Food				
Inst. Set 1	1.923	.729	.432	60.87*
	(.133)	(.049)		
Inst. Set 2	.742	.879	.025	6.53
	(.082)	(.056)		
Petroleum				
Inst. Set 1	.603	.991	.396	66.16*
	(.051)	(.027)		
Inst. Set 2	.732	.975	.021	6.75
	(.089)	(.026)		
Rubber				
Inst. Set 1	.638	.950	.345	70.40*
	(.035)	(.026)		
Inst. Set 2	.755	.964	.058	17.36*
	(.080)	(.025)		

Inst. Set 1: N_t, N_{t-1}, N_{t-2}, N_{t-3}, X_t, X_{t-1}, X_{t-2}, X_{t-3}, constant
Inst. Set 2: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, constant

[1]The asterisk denotes that the overidentifying restrictions of the model are rejected at the .05 level of significance.

Table 3
Model with Cost Shocks
Crude Material Prices

Industry	λ	μ	Φ_1	Φ_2	Φ_3	NR	J-Stat[1]
Nondurables							
Inst. Set 3	.466 (.073)	.714 (.099)	2767 (1145)	79.53 (67.90)	82.86 (49.23)	.115	32.12*
Inst. Set 4	.530 (.083)	.771 (.099)	2131 (1032)	69.25 (57.08)	61.19 (40.44)	.039	15.11
Chemicals							
Inst. Set 3	.433 (.054)	.572 (.144)	685.2 (251.1)	13.05 (4.57)	20.42 (9.58)	.188	52.44*
Inst. Set 4	.563 (.062)	.737 (.101)	480.4 (190.1)	12.50 (3.69)	-.227 (7.23)	.079	16.42
Food							
Inst. Set 3	.434 (.053)	.292 (.210)	690.3 (409.3)	-49.65 (20.27)	-12.62 (24.00)	.126	20.05
Inst. Set 4	.653 (.066)	.822 (.101)	179.4 (211.7)	1.78 (6.11)	6.27 (11.41)	.064	12.39
Petroleum							
Inst. Set 3	.305 (.083)	.965 (.085)	353.9 (172.0)	13.09 (14.84)	-26.90 (14.79)	.136	32.20*
Inst. Set 4	.559 (.081)	.947 (.031)	42.83 (46.37)	8.68 (5.86)	4.77 (6.84)	.027	9.11
Rubber							
Inst. Set 3	.497 (.099)	1.008 (.169)	-260.5 (351.7)	18.34 (15.48)	31.42 (14.90)	.099	22.02
Inst. Set 4	.631 (.105)	1.016 (.096)	-164.7 (202.5)	7.722 (8.768)	12.03 (7.16)	.062	16.49

Inst. Set 3: N_t, N_{t-1}, N_{t-2}, N_{t-3}, X_t, X_{t-1}, X_{t-2}, X_{t-3}, CM_t, CM_{t-1}, CM_{t-2}, CM_{t-3}, W_t, W_{t-1}, W_{t-2}, W_{t-3}, V_t, V_{t-1}, V_{t-2}, V_{t-3}, constant

Inst. Set 4: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, CM_{t-1}, CM_{t-2}, CM_{t-3}, W_{t-1}, W_{t-2}, W_{t-3}, V_{t-1}, V_{t-2}, V_{t-3}, constant

[1] The asterisk denotes that the overidentifying restrictions of the model are rejected at the .05 level of significance.

Table 4
Model with Cost Shocks
Intermediate Materials Prices

Industry	λ	μ	Φ_1	Φ_2	Φ_3	NR	J-Stat[1]
Nondurables							
Inst. Set 3	.392	.577	3551	−175.4	8.99	.170	49.38*
	(.067)	(.154)	(1315)	(90.1)	(28.58)		
Inst. Set 4	.519	.770	1841	−78.61	23.10	.069	25.50*
	(.080)	(.103)	(822.5)	(56.33)	(19.44)		
Chemicals							
Inst. Set 3	.502	.714	385.3	−10.39	18.05	.281	57.11*
	(.045)	(.079)	(134.1)	(32.34)	(6.49)		
Inst. Set 4	.614	.843	106.3	14.86	10.26	.141	30.51*
	(.047)	(.057)	(96.3)	(24.37)	(5.35)		
Food							
Inst. Set 3	.448	.418	1008	−43.34	−2.51	.191	28.44*
	(.040)	(.121)	(347)	(26.37)	(15.29)		
Inst. Set 4	.632	.813	203.3	−3.12	4.96	.071	13.06
	(.061)	(.090)	(205.4)	(17.15)	(10.11)		
Petroleum							
Inst. Set 3	.287	1.035	482.3	−152.9	−119.9	.089	25.02*
	(.103)	(.120)	(258.0)	(91.4)	(61.6)		
Inst. Set 4	.491	.966	101.8	−7.46	1.35	.022	6.36
	(.080)	(.035)	(54.8)	(20.77)	(15.37)		
Rubber							
Inst. Set 3	.499	.750	54.85	−32.73	29.88	.144	35.38*
	(.063)	(.096)	(155.5)	(15.88)	(9.06)		
Inst. Set 4	.661	.905	38.4	−17.76	8.59	.072	18.88
	(.068)	(.053)	(85.3)	(9.48)	(3.60)		

Inst. Set 3: N_t, N_{t-1}, N_{t-2}, N_{t-3}, X_t, X_{t-1}, X_{t-2}, X_{t-3}, IM_t, IM_{t-1}, IM_{t-2}, IM_{t-3}, W_t, W_{t-1}, W_{t-2}, W_{t-3}, V_t, V_{t-1}, V_{t-2}, V_{t-3}, constant

Inst. Set 4: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, IM_{t-1}, IM_{t-2}, IM_{t-3}, W_t, W_{t-1}, W_{t-2}, W_{t-3}, V_t, V_{t-1}, V_{t-2}, V_{t-3}, constant

Table 5
Additional Tests

Industry	λ	μ'	Φ_1	Φ_2	Φ_3	NR	J-Stat[1]
Nondurables	.510 (.095)	1.254 (.132)	2365 (1326)	77.1 (68.3)	69.7 (48.9)	.043	15.54
Chemicals	.552 (.072)	1.281 (.129)	518.9 (240.4)	10.92 (4.70)	.822 (7.883)	.083	16.30
Food	.656 (.072)	1.159 (.116)	143.1 (236.9)	3.51 (6.32)	10.44 (12.39)	.057	12.08
Petroleum	.549 (.081)	1.051 (.035)	47.03 (48.48)	8.84 (5.22)	4.66 (7.16)	.028	9.35
Rubber	.614 (.118)	.932 (.133)	−273.6 (284.1)	11.59 (12.00)	12.63 (8.46)	.047	14.17

Inst. Set: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, CM_{t-1}, CM_{t-2}, CM_{t-3}, W_{t-1}, W_{t-2}, W_{t-3}, V_{t-1}, V_{t-2}, V_{t-3}, constant

[1]An asterisk denotes that the overidentifying restrictions of the model are rejected at the .05 level of significance.

Table 6
Costs of Changing Output

Industry	λ	μ	δ	Φ_1	Φ_2	Φ_3	NR	J-Stat[1]
Nondurables	.585 (.084)	.816 (.073)	-.270 (.122)	1617 (795)	35.3 (44.9)	51.79 (30.47)	.040	13.04
Chemicals	.572 (.069)	.747 (.108)	-.033 (.168)	463 (199)	12.46 (4.82)	-.55 (7.27)	.079	16.72
Food	.661 (.076)	.829 (.100)	-.033 (.102)	149 (211)	.76 (6.45)	6.14 (11.34)	.071	12.81
Petroleum	.603 (.083)	.954 (.022)	-.130 (.090)	36.88 (33.91)	4.24 (4.64)	5.08 (4.74)	.033	9.25
Rubber	.656 (.128)	1.040 (.114)	-.288 (.165)	-247 (270)	10.54 (11.54)	12.64 (8.71)	.039	8.82

Inst. Set: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, CM_{t-1}, CM_{t-2}, CM_{t-3}, W_{t-1}, W_{t-2}, W_{t-3}, V_{t-1}, V_{t-2}, V_{t-3}, constant

[1] An asterisk denotes that the overidentifying restrictions of the model are rejected at the .05 level of significance.

Table 7
Underlying Cost Parameters

Industry	$\frac{1}{bc}$	α	$\frac{\theta_1}{b}$	$\frac{\theta_2}{b}$	$\frac{\theta_3}{b}$	$\frac{1}{mb}$	$(1+\beta)\frac{1}{bm} + \frac{1}{bc}$	:	$\frac{1}{c}$	$\frac{1}{m}$
								:Norm: $(1+\beta)\frac{1}{m} + \frac{1}{c} = 1$		
Nondurables										
Table 3	2.39	.547	5093	166	146	–	–	:	–	–
	(1.20)	(.137)	(963)	(84)	(63)	–	–	:	–	–
Table 6	3.37	.620	5453	119	175	-.910	1.55	:	2.16	-.58
	(1.84)	(.184)	(1128)	(106)	(67)	(.614)	(.90)	:	(1.14)	(.12)
Chemicals										
Table 3	2.93	.771	1408	37	-.66	–	–	:	–	–
	(1.15)	(.131)	(280)	(14)	(21.1)	–	–	:	–	–
Table 6	3.10	.780	1435	39	-1.71	-.102	2.90	:	1.07	-.035
	(1.36)	(.136)	(284)	(14)	(22.3)	(.540)	(1.42)	:	(.38)	(.168)
Food										
Table 3	5.37	.956	963	9.7	34	–	–	:	–	–
	(2.55)	(.370)	(981)	(34.6)	(66)	–	–	:	–	–
Table 6	5.70	.975	849	4.3	35	-.188	5.32	:	1.07	-.035
	(3.18)	(.428)	(1019)	(37.1)	(68)	(.640)	(3.03)	:	(.23)	(.102)
Petroleum										
Table 3	2.86	.152	122	25	14	–	–	:	–	–
	(1.46)	(.122)	(95)	(21)	(22)	–	–	:	–	–
Table 6	3.80	.175	140	16	19	-.494	2.81	:	1.35	-.175
	(2.11)	(.130)	(88)	(19)	(20)	(.502)	(1.28)	:	(.33)	(.092)
Rubber										
Table 3	4.59	-.073	-756	35	55	–	–	:	–	–
	(3.37)	(.410)	(621)	(24)	(22)	–	–	:	–	–
Table 6	5.49	-.220	-1360	58	69	-1.58	2.34	:	2.35	-.676
	(5.14)	(.510)	(887)	(33)	(30)	(1.21)	(1.60)	:	(1.82)	(.165)

Inst. Set: N_{t-1}, N_{t-2}, N_{t-3}, X_{t-1}, X_{t-2}, X_{t-3}, CM_{t-1}, CM_{t-2}, CM_{t-3}, W_{t-1}, W_{t-2}, W_{t-3}, V_{t-1}, V_{t-2}, V_{t-3}, constant

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