

NBER WORKING PAPER SERIES

THE TERM STRUCTURE OF FORWARD  
EXCHANGE PREMIA AND THE  
FORECASTABILITY OF SPOT EXCHANGE  
RATES: CORRECTING THE ERRORS

Richard H. Clarida

Mark P. Taylor

Working Paper No. 4442

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
August 1993

This project was started during Clarida's stay as a visiting scholar at the IMF in June 1992. We would like to thank Neil Ericsson for sharing with us his insights on the topic of cointegration in multivariate systems. Any errors that may reside on these pages are those committed by the authors. This paper is part of NBER's research program in International Finance and Macroeconomics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #4442  
August 1993

THE TERM STRUCTURE OF FORWARD  
EXCHANGE PREMIA AND THE  
FORECASTABILITY OF SPOT EXCHANGE  
RATES: CORRECTING THE ERRORS

ABSTRACT

We present theory and evidence that challenges the view that forward premia contain little information regarding subsequent spot rate movements. Using weekly dollar-mark and dollar-sterling data, we find that spot and forward exchange rates together are well represented by a vector error correction model; that there exists exactly the number of cointegrating relationships predicted by a simple theoretical framework and that a basis for this cointegrating space is the vector of forward premia. Dynamic forecasts indicate that the information in the forward premia can be used to reduce the root mean squared forecast error for the spot rate (relative to a random walk forecast) by at least 33 percent at a 6-month horizon and by some 50 to 90 percent at a 1-year horizon.

Richard H. Clarida  
Department of Economics  
International Affairs Building  
Room 1020  
Columbia University  
420 West 118th Street  
New York, NY 10027  
and NBER

Mark P. Taylor  
International Monetary Fund  
700 19th Street, NW  
Washington, DC 20431  
and CEPR

## I. Introduction

This paper revisits one of the oldest questions in international finance: does the forward exchange rate contain useful information about the future path of the spot exchange rate? Professional thinking on this subject has undergone a significant shift over the past twenty years. Combining the uncovered interest parity theorem (Fisher (1930)) with the covered interest parity theorem (Keynes (1923)) and the efficient markets hypothesis, the equilibrium forward exchange rate established at date  $t$  for delivery of foreign exchange at date  $t + n$ ,  $F_{n,t}$ , should be the best available predictor of the level of the spot exchange rate realized at date  $t + n$ ,  $S_{t+n}$ . <sup>1/</sup> In an influential paper, Frenkel (1981) tested this hypothesis using data for the 1970s. Running log-linear regressions of the form:

$$s_{t+1} = \alpha + \beta f_{1,t} + \gamma z_t + \epsilon_{t+n}; \quad (1)$$

Where lower-case letters denote variables in logarithmic form, he found that he could not reject the hypothesis that  $\beta = 1$  and  $\gamma = 0$ , where  $z_t$  is a vector of information variables known at time  $t$ . These results were taken to be supportive of the efficient markets - interest parity hypothesis that:

$$E(s_{t+n} | \Omega_t) = f_{n,t}; \quad (2)$$

---

<sup>1/</sup> See Hodrick (1987), MacDonald and Taylor (1992) and Froot and Thaler (1990) for general discussions of the empirical evidence on international parity conditions and on the efficiency of foreign exchange markets. Isard (1993) and Taylor (1993) discuss uncovered and covered interest parity in detail.

where  $\Omega_t$  is the set of information available to the market at time  $t$ .

In the early 1980s, researchers such as Hansen and Hodrick (1980), Cumby and Obstfeld (1980, 1984), Meese and Singleton (1982), Fama (1984), Huang (1984) and Meese (1986) began to recognize that a potential problem with regressions such as (1) is that if--as appears to be the case-- $s_{t+n}$  and  $f_{n,t}$  are nonstationary variables, the usual asymptotic theory invoked to construct hypothesis tests becomes inapplicable. For this reason, researchers interested in uncovering the information contained in the forward exchange rate have in recent years estimated regressions of the variety:

$$s_{t+n} - s_t = \alpha + \beta(f_{n,t} - s_t) + \epsilon_{t+n}. \quad (3)$$

In distinct contrast to the findings reported for the levels regressions run by Frenkel (1981) and others, Fama (1984) and Cumby and Obstfeld (1984) and others (most recently McCallum (1992)) find that the forward premium mispredicts the direction of the subsequent change in the spot rate. <sup>1/</sup> That is, when foreign exchange is selling at a forward premium, the dollar tends on average to appreciate over the length of the forward contract, not depreciate as would be implied by interest parity.

<sup>2/</sup>

---

<sup>1/</sup> See also Bilson (1981), Longworth (1981), Huang (1984), Gregory and McCurdy (1984). Froot (1990) reports that the average estimated value of  $\beta$  as in (3), across 75 published estimates, is -0.88.

<sup>2/</sup> Our choice of the term "forward premium" to describe the gap between the current spot and forward rates is quite arbitrary. We could equally as well use the term "forward discount" (as in, e.g., Froot and Frankel (1989)), since a discount is just a negative premium.

Equivalently, via the covered interest arbitrage condition:

$$i_{n,t}^{us} - i_{n,t}^* = f_{n,t} - s_t ; \quad (4)$$

(where  $i_{n,t}^{us}$  and  $i_{n,t}^*$  represent U.S. and foreign nominal interest rates on identical assets of an n-period maturity), these findings indicate that, when U.S. interest rates exceed foreign interest rates, the dollar tends on average to appreciate over the holding period, not to depreciate in order to offset on average the interest differential in favor of the U.S. 1/

Not only do these results indicate that interest parity is violated, the inability of projection equations such as (3) to account for much of the observed variance in actual exchange rate changes has led most, if not in fact virtually all, researchers to conclude that:

... forward premia contain little information regarding subsequent exchange rate changes. As emphasized by Dornbusch (1980), Mussa (1979), and Frenkel (1981), exchange rate changes over the recent period of floating seem to have been largely unanticipated (Cumby and Obstfeld (1984), p. 139).

In this paper, we present a theoretical framework and provide evidence that challenges the view--a view we shared until completing this project--that forward premia contain little information regarding subsequent changes in the spot exchange rate. Our theoretical framework--which draws upon a similar framework developed recently by Hall, Anderson, and Granger (1992) to study the term structure of treasury bill yields--predicts that in a

---

1/ See Froot and Thaler (1990) for a general discussion of this issue.

$(j + 1)$ -variable system of  $j$  forward rates and one spot exchange rate, there should exist  $j$  cointegrating vectors and exactly one common trend which propels the nonstationary component of each of the  $j$  forward rates and the one spot exchange rate. In fact, the theoretical framework predicts that a basis which spans the space of cointegrating relationships is just the vector of the  $j$  forward exchange rate premia.

Using weekly data on the spot exchange rate and 1, 3, 6, and 12 month forward exchange rates for Germany and the United Kingdom, we find for each country that, when we estimate equations similar to (3), the estimated slope coefficient is, in every case, negative and significantly different from unity. Thus, these results concur with the broad stylized facts of this literature. Further analysis, however, indicates that, as predicted by our theoretical framework and the Granger Representation Theorem (Granger and Engle (1987)), the term structure of forward exchange rates together with the spot exchange rate comprise a system that is well represented by a vector error correction model. Employing Johansen's (1991) maximum likelihood approach, we test and confirm for each country the existence of  $j - 4$  cointegrating relationships as predicted by the theory. We then test and confirm for each country the joint hypothesis that a basis for this cointegrating space is the vector of four forward premia. We next test, and reject for each country, the hypothesis that the spot exchange rate is exogenous with respect to the term structure of forward rates. Out-of-sample dynamic forecasting exercises indicate that the information contained in the term structure of forward premia can be used to reduce the root mean

If  $\phi_{h(j),t}$  is a stationary stochastic process, then the forward exchange rate at horizon  $h(j)$  and the spot rate share a common stochastic trend  $z_t$  and are cointegrated such that the forward premium at horizon  $h(j)$  is a stationary stochastic process:

$$f_{h(j),t} - s_t = h(j)\mu + E(v_{t,j} - v_t | \Omega_t) + \phi_{h(j),t}. \quad (10)$$

It follows that, among the  $j$  forward rates and the spot exchange rate contained in  $y_t$ , there will exist at least  $j$  cointegrating vectors that are defined by the  $j$  forward premia  $f_{h(1),t} - s_t, f_{h(2),t} - s_t, \dots, f_{h(j),t} - s_t$ , so long as the departures from interest parity at all horizons are stationary stochastic processes. However, since (6) and (9) imply that all  $j + 1$  variables in  $y_t$  share a common stochastic trend  $z_t$ , we know from the results of Stock and Watson (1988) that there will exist exactly  $j$  independent cointegrating vectors among the  $j + 1$  variables in  $y_t$ . Thus, this theoretical framework has the following empirical implications.

First, a vector comprised of the spot exchange rate and  $j$  forward exchange rates should be well represented by a vector error-correction model. This follows from (6), (9), (10) and the Granger Representation Theorem. Second, there should exist a unique common trend and thus exactly  $j$  cointegrating vectors in such a system, an implication that follows from (6) and (9) and the Stock-Watson Common Trends Representation Theorem (Stock and Watson (1988)). Third, a basis for this space of  $j$  cointegrating vectors should be defined by the  $j$  forward premia in this system  $f_{h(1),t} - s_t, f_{h(2),t} - s_t, \dots, f_{h(j),t} - s_t$ . This follows from (10). We also note that the results reported in Phillips (1990) imply that it must be

squared error in forecasting the spot rate by more than 33 percent at a 6-month horizon and by 50 to 90 percent at a 1-year horizon.

## II. Theoretical Framework

Consider a vector  $y_t$  comprised of the logarithm of the spot exchange rate  $s_t$  and the logarithm of  $j$  forward exchange rates at horizons  $h(1), \dots, h(j)$ :

$$y_t = [s_t, f_{h(1),t}, f_{h(2),t}, \dots, f_{h(j),t}]'. \quad (5)$$

We suppose, and confirm empirically below, that the spot exchange rate possesses a unit root and evolves according to:

$$s_t = z_t + v_t ; \quad (6)$$

where  $v_t$  is a zero-mean stationary stochastic process and  $z_t$  is a random walk:

$$z_t = \mu + z_{t-1} + e_t . \quad (7)$$

Using equation (2), we define the deviation from interest rate parity at horizon  $h(j)$ :

$$\phi_{h(j),t} = f_{h(j),t} - E(s_{t+h(j)} | \Omega_t). \quad (8)$$

Combining (6) and (8) we obtain an expression for the forward exchange rate at horizon  $h(j)$ :

$$f_{h(j),t} = h(j)\mu + z_t + E(v_{t+j} | \Omega_t) + \phi_{h(j),t}. \quad (9)$$



possible to select a triangular representation of the cointegration space of this system such that each of  $j$  variables in the system is cointegrated with the one remaining "right-hand-side" variable not included among these  $j$  variables.

If exchange rate changes  $\Delta s_{t+1}$  are not Granger-caused by any other available information--that is to say, agents have no information useful for forecasting  $\Delta s_{t+1}$  beyond the history of that variable, so that  $E(\Delta s_{t+1} | \Omega_t) = E(\Delta s_{t+1} | \Delta s_t, \Delta s_{t-1}, \dots)$ --our theoretical framework implies that the term structure of forward premia should not contain any information that helps to improve a forecast of the spot exchange rate given the history of the spot exchange rate itself. Thus, merely establishing that spot and forward exchange rates are cointegrated does not guarantee that the term structure of forward premia contains useful information about the future path of the spot exchange rate.

Conversely, if exchange changes  $\Delta s_{t+1}$  are Granger-caused by available information other than the history of the spot exchange rate, our theoretical framework implies that the term structure of forward premia should contain information that helps to improve a forecast of the spot exchange rate given the history of the spot exchange rate itself. To see this more clearly, rearrange (8):

$$f_{h(j),t} - s_t = E \left[ \sum_{i=1}^{h(j)} \Delta s_{t+i} \mid \Omega_t \right] + \phi_{h(j),t} \quad (11)$$

From (11) we see that the theoretical framework implies that the forward premium is the optimal forecast of the sum of  $h(j)$  future values of  $\Delta s_t$ , plus  $\phi_{h(j),t}$ . If information other than the history of  $\Delta s_t$  is relevant for forecasting  $\Delta s_t$ , then (11) implies that the forward premium must contain information useful in forecasting  $\Delta s_t$ . If  $\Delta s_t$  is not Granger-caused by other information, then (11) implies that the forward premium must be a linear combination of current and lagged  $\Delta s_t$ , plus  $\phi_{h(j),t}$ . In effect, previous researchers have tested equation (11) under the assumption that  $\phi_{h(j),t}$  is constant; they have then concluded that the forward premia contain little information with respect to future exchange rate changes (see the references cited above). It is the contention of this paper that, in so doing, they threw the baby out with the bath water.

Note that we have deliberately avoided use of the term 'risk premium' to describe  $\phi_{h(j),t}$ . This is because we remain agnostic as to the causes of  $\phi_{h(j),t}$ . From equation (8), we see that  $\phi_{h(j),t}$  is defined as any departure from the simple efficient markets hypothesis (equation (2)). While a number of researchers have interpreted this as due to risk, Froot and Frankel (1989) have used survey data to show that the bias in the premium as a spot rate forecast is not due entirely to risk. Indeed, these authors cannot reject the hypothesis that all of the bias is due to systematic expectational errors. <sup>1/</sup> This is not inconsistent with our

---

<sup>1/</sup> Systematic expectational errors might be generated, for example, by the influence of 'chartist' or 'technical' analysts on market traders (Frankel and Froot (1990), Taylor and Allen (1992), Allen and Taylor (1993), Froot et al (1992)), and/or because of learning by some traders (Lewis (1989), Cutler, Poterba and Summers (1990)). See Froot and Thaler (1990) for a general discussion.

framework:  $\phi_{h(j),t}$  can be interpreted as the deviation of agents' expectations from full market rationality. It is our contention that, whatever the determinants of  $\phi_{h(j),t}$ , the forward rate contains valuable information, which may be extracted, with respect to future spot rates.

Before moving on to the empirical results, we should comment on an alternative theoretical framework that can be employed to interpret the joint behavior of the spot exchange rate and term structure of forward exchange premia. Our framework imposes the testable restriction that  $\phi_{h(j),t}$ , the departure from interest rate parity, is a realization of a stationary stochastic process. If instead  $\phi_{h(j),t}$  possesses a unit root, we should be able to reject the hypothesis that each of the  $j$  forward premia  $f_{h(1),t} - s_t, f_{h(2),t} - s_t, \dots, f_{h(j),t} - s_t$  is stationary. To see this point, one which has been made recently by Evans and Lewis (1992), suppose that:

$$\phi_{h(j),t} = \phi(j)x_t + w_{jt}; \quad (12)$$

where  $w_t$  is a zero mean, stationary stochastic process, and  $x_t$  is a random walk. Using (9) we see that:

$$f_{h(j),t} = h(j)\mu + z_t + \phi(j)x_t + E_t v_{t+j} + w_{jt}. \quad (13)$$

From (6), we see that  $f_{h(j),t} - s_t$  inherits the unit root present in  $x_t$ . Thus, if this alternative interpretation of the data is correct, we should be able to reject the hypothesis that each of the  $j$  forward premia is stationary. Moreover, unless  $x_t$  is proportional to  $z_t$ , this alternative interpretation of the data also implies that among the  $j + 1$  variables in

the system, there are two common trends and thus  $j - 1$  cointegrating vectors. Thus, a finding of  $j - 1$  or fewer cointegrating vectors in a system comprised of  $j$  forward exchange rates and the spot exchange rate is evidence in favor of this alternative interpretation.

### III. The Data and Empirical Preliminaries

We investigate weekly data on spot and 4, 13, 26, and 52 week forward exchange rates for West Germany and the United Kingdom, obtained from the Harris Bank data base maintained by Richard Levich. The sample runs from 1977:1 through 1990:26. The choice of starting date reflects the view, first expressed by Hansen and Hodrick (1982) in their classic study of the forecastability of excess returns in the foreign exchange market, that during the early years of floating and until the Rambouillet Agreement of February 1976, market participants may very well have believed that a return to fixed parities was imminent. If this was in fact the case, then departures from interest parity during these years would have reflected not only a risk premium, but also an extra component incorporating the effect of a return to fixed parities on expected payoffs to foreign exchange speculation.

To see this, suppose that in the absence of a return to fixed exchange rates, the equilibrium spot rate is governed by:

$$s_t = z_t. \tag{14}$$

If the probability of a return to fixed rates is constant and equal to  $1 - \pi$ , interest parity implies:

$$f_{1,t} = \pi z_t + (1 - \pi) E \underline{s}_{t+1}; \quad (15)$$

where  $\underline{s}_{t+1}$  is the spot rate next period if floating exchange rates are abandoned. Note that even if  $\underline{s}_t = \underline{s}$ , the forward premium will not be stationary during a sample in which a return to fixed exchange rates never occurs, since from (14) and (15) we have:

$$f_{1,t} - s_t = (\pi - 1)z_t + (1 - \pi)\underline{s}. \quad (16)$$

If  $\underline{s}_{t+1} = x_{t+1}$  with  $x_{t+1}$  a unit root process, then during a sample in which a return to fixed exchange rates never occurs, forward and spot exchange rates will not even be cointegrated and at most only  $j - 1$  cointegrating relationships can exist among a set of  $j$  forward exchange rates and a spot exchange rate. This is of course just one example of a "peso problem" (Rogoff (1977), Krasker (1980)). As demonstrated by Evans and Lewis (1992), it will often be the case that a peso problem introduces an extra common trend into a system of spot and forward asset prices.

Table 1 reports the results of Dickey-Fuller tests of the null hypotheses that the time series for spot and forward dollar exchange rates for the United Kingdom and Germany contain a unit root. Three tests are reported, a modified Dickey-Fuller t-test  $Z(\tau_r)$  in which a trend and a constant is included in the regression, a modified Dickey-Fuller t-test  $Z(\tau_\mu)$  in which only a constant is included, and a modified Dickey-Fuller

F-test  $Z(\Phi_3)$  of the hypothesis that the change in each spot and forward rate is stationary about a constant drift. We present Phillips and Perron (1986) modified statistics which make a nonparametric correction for serial correlation and potential heteroskedasticity.

The results in Table 1 confirm the findings, reported in many earlier studies, that spot and forward mark and sterling exchange rates appear to possess a unit root. In no instance can the hypothesis of a unit root in the level of a spot or forward rate be rejected at even the 15 percent level, while in all instances can the hypothesis that the change in a spot or forward exchange rate is nonstationary be rejected at the 1 percent level.

#### IV. Testing the Simple Efficiency Hypothesis

In this section, we apply standard efficiency tests to our data, and demonstrate that this generates results consistent with those found in the literature. In particular, we apply tests of foreign exchange market efficiency under the assumption that  $\phi_h(j)_t$  is a constant. We term this the simple efficiency hypothesis.

Foreign exchange market efficiency tests have typically taken one of two forms. One form of test has been undertaken in the regression framework of equation (3), where the null hypothesis is  $H_0: \beta = 1$  (e.g. Hansen and

Hodrick (1980), Fama (1984), Cumby and Obstfeld (1984), McCallum (1992)).

1/ The second kind of test derives and tests the cross-equation restrictions implied by (11) for a vector autoregression (VAR) in spot rate changes and the forward premium (e.g. Hakkio (1981)).

In this section, we apply only single-equation efficiency tests, for two reasons. First, since as we shall see, these tests easily reject the simple efficiency hypothesis, there is no need to employ a second test which exploits information on the time series properties of the data (there is no apparent lack of test power). Secondly, the single-equation tests are simpler and yield a regression coefficient which can be interpreted intuitively as a measure of the quality of the forward premium as a predictor of the rate of depreciation.

We do, however, employ a VAR in the forward premium and the spot rate change in order to generate the empirical distributions of our single-equation efficiency tests. The procedure is as follows: we estimate a VAR in the spot rate change and the forward premium. From this, we generate the expected value of  $(s_{t+h(j)} - s_t)$  conditional on  $H_t$ , where  $H_t$  is an information set consisting of current and lagged values of the spot rate change and the forward premium:

$$H_t = \left\{ \Delta s_t, \Delta s_{t-1}, \dots, (f_{h(j),t} - s_t), (f_{h(j),t-1} - s_{t-1}) \dots \right\}$$

---

1/ A variant of this test is to test an exclusion restriction on other information known at time  $t$  in (3) (e.g., Hansen and Hodrick (1980)). Huang (1984) uses Bayesian techniques and considers specific alternatives.

$$H_t \subseteq \Omega_t$$

Projecting both sides of (11) onto  $H_t$ , holding  $\phi_{h(j),t}$  constant, we have

$$\left( f_{t(j),t} \cdot s_t \right)^* = E \left[ \sum_{i=1}^{h(j)} \Delta s_{t+i} \mid H_t \right] \quad (17)$$

The right hand side of (17) is the VAR forecast of  $(s_{t+h(j)} - s_t)$ , while since the current forward premium is an element of  $H_t$ , the "theoretical forward premium",  $(f_{t(j)} - s_t)^*$ , should be equal to the actual forward premium (less a constant term). Thus, we can generate artificial data which has the same sample moments as the actual data but which satisfy the simple efficiency restrictions by applying Monte Carlo methods, using the estimated VAR coefficients and residual covariance matrix, and generating the theoretical spread from (17) using dynamic forecasts from the VAR. We then regress the artificial  $(s_{t,h(j)} - s_t)$  onto  $(f_{h(j),t} - s_t)^*$  and a constant intercept and generate a test-statistic as the square of the ratio of the slope coefficient minus one to the estimated standard error of the slope coefficient. This experiment is replicated one thousand times and the percentage of times that the absolute value of the test statistic exceeds the absolute value of the statistic obtained with the real data is the empirical significance level of the test statistic. The asymptotic significance level of the test statistic is also available since, under the null hypothesis, it is asymptotically distributed as chi-square with one degree of freedom.



The results of these simple efficiency tests applied to the dollar-sterling and dollar-mark data for the four horizons is given in Table 2. In every case, the point estimate of the slope coefficient is negative. Moreover, the hypothesis that the slope coefficient is unity is in every case rejected with asymptotic and empirical significance levels less than 2 percent.

These results are thus in accordance with those of previous researchers in this area.

#### V. A Vector Error Correction Model

Based upon the theoretical framework developed in Section II and our finding that the variables under study are integrated of order one, we investigate a dynamic vector error correction model (VECM) (Engle and Granger (1987); Johansen (1991)) for the spot exchange rate and the term structure of forward exchange rates in the United Kingdom and Germany. Letting  $y_t = [s_t, f_{4,t}, f_{13,t}, f_{26,t}, f_{52,t}]'$  denote the  $j + 1 = 5$  by 1 vector of the system's variables for a particular currency, the vector error correction model can be written:

$$\Delta y_t = \mu + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{k-1} \Delta y_{t-k+1} + \Pi y_{t-k} + \zeta_t \quad (18)$$

If the matrix  $\Pi$  is of full rank  $r = 5$ , the VECM reduces to the usual VAR in the levels of stationary variables. If  $\Pi$  is the null matrix so that  $r = 0$ , the VECM represents a VAR in first-differences. The VECM differs from the

usual VAR in that it allows for the existence of long-run "equilibrium" relationships among a system's variables. If the matrix  $\Pi$  is of reduced rank  $r < 5$ , it can be factored into the product of two 5 by  $r$  matrices  $\alpha$  and  $\beta$  such that:

$$\Pi = \alpha\beta'; \quad (19)$$

where  $\beta'$  is the  $r$  by 5 matrix of the system's  $r$  cointegrating vectors, and  $\alpha$  is the 5 by  $r$  matrix of  $r$  adjustment coefficients for each of the system's 5 equations.

Each cointegrating relationship defines a long-run equilibrium to which the system ultimately returns after a shock. The parameters in the  $\alpha$  matrix determine the rates at which each of the system's variables adjust in response to lagged deviations from the  $r$  cointegrating relationships. Stock and Watson (1988) prove that the long-run behavior of a system of  $n$  variables with  $r < n$  cointegrating relationships is governed by  $n - r$  common stochastic trends. Thus a test for the cointegration rank  $r$  is also a test for the number of common trends.

Table 3 presents the results of two tests developed by Johansen (1991) to investigate the hypothesis that the number of cointegrating vectors in a system of  $n$  variables is less than or equal to  $r$ . Note that the Stock and Watson results cited above imply that this is also a test of the hypothesis that the number of stochastic trends in the  $n$ -variable system is greater than or equal to  $n - r$ . According to both the trace and the  $\lambda$ -max statistics, we cannot reject for either the mark or sterling the hypothesis

that  $r \leq 4$ , but, we can reject at the 5 percent level the hypothesis that  $r \leq 3$ . Thus, for both sterling and the deutsche-mark, these findings are consistent with the predictions of the theoretical framework that, in a system comprised of a spot exchange rate and  $j$  forward exchange rates, exactly one common trend and  $j$  cointegrating relations are needed to account for the dynamic behavior of the system.

Another prediction of the theoretical framework is that a basis for the space of cointegrating relationships is defined by the vector of  $j = 4$  forward premia  $[f_{4,t} - s_t, f_{13,t} - s_t, f_{26,t} - s_t, f_{52,t} - s_t]'$ . A likelihood ratio statistic is employed to test this hypothesis. Conditional on there being four cointegrating vectors in the system, the likelihood ratio test statistic for this hypothesis is distributed as  $\chi^2(4)$  under the null. The results of this test are reported in Table 4. As can be seen from the table, for neither sterling nor the mark is it possible to reject the hypothesis that the vector of forward premia defines a basis for the space of  $j = 4$  linearly independent cointegrating relationships implied by the estimated VECMs for the United Kingdom and Germany.

We conclude from this evidence that the theoretical framework outlined in Section II is well supported by the data. In particular, both mark and sterling systems of the spot exchange rate and the term structure of forward rates are well modeled by a VECM. In both systems, exactly one common trend and thus four cointegrating vectors are required to explain the dynamic behavior of spot and forward exchange rates. These four cointegrating relations are, as predicted by the analysis, defined by the vector of four

forward premia for each currency. We now investigate whether or not the term structure of forward premia contains incremental predictive content for the time path of the spot exchange rate.

Tables 5 and 6 present FIML estimates of the five-equation VECMs for the sterling and mark systems, respectively. Of particular interest are the results for the  $\Delta s_t$  equation reported in the first two columns of the tables. As can be seen in Table 5, the spot dollar-sterling exchange rate is not exogenous with respect to lagged information contained in the term structure of forward premia. Indeed, the lagged 13, 26, and 52 week forward premia contain statistically significant information about the future path of the dollar-sterling spot exchange rate that is not contained in the lagged change in the spot rate. Similarly, Table 5 reports that the spot dollar-mark exchange rate is not exogenous with respect to lagged information contained in the term structure of forward premia. The entire lagged term structure of forward premia contains statistically significant information about the future path of the dollar-mark spot exchange rate that is not contained in the lagged change in the spot dollar-DM rate. These results suggest that, at least for dollar-mark and dollar-sterling spot and forward exchange rates since 1977, the answer to the question that began this paper is "yes".

In order to assess the usefulness of the information in the term structure of forward exchange rates, the following out-of-sample forecasting exercise was conducted. The full VECM for each currency was estimated through 1989:26 and a forecast of the spot exchange rate for 1989:27 through

1990:26 was computed using only information available through the estimation period 1977:1-1989:26. The model was then re-estimated through 1989:27, and a dynamic forecast of the spot exchange rate in each week from 1989:28-1990:26 was computed. This process was continued until the model was re-estimated through 1990:25 and a single one step-ahead forecast was computed. Figure 1 depicts at each horizon the ratio of the root-mean-squared error (RMSE) of these forecasts to the RMSE obtained from a naive random walk forecast  $E_t s_{t+j} - s_t$ , the metric in which exchange rate forecasts have been judged since the original work of Meese and Rogoff (1983), while Table 7 gives detailed results for selected horizons. As can be seen in the figure, the term structure of forward premia contains information that can substantially improve forecasts of the dollar-mark and dollar sterling spot exchange rates.

At a 10-week horizon, the forecast obtained from the estimated VECM for the dollar-mark system has a RMSE that is 14 percent smaller than that derived from the random walk forecast. At a horizon of 26 weeks, the VECM forecast has a 33 percent smaller RMSE than does the random walk forecast. At horizons of 40 weeks and longer, the VECM forecast for the dollar-mark exchange rate has nearly a 50 percent smaller RMSE than the random walk forecast.

At forecast horizons under 20 weeks, the VECM for the dollar-sterling exchange rate is dominated by the random walk forecast. However, at longer horizons, the VECM forecast substantially improves upon the random walk forecast. At a 26-week horizon, the forecast obtained from the estimated

VECM for the dollar-sterling system has a RMSE that is some 33 percent smaller than that derived from the random walk forecast. This forecasting advantage is maintained at successively longer horizons. At horizons in excess of 47 weeks, the forecast for the dollar-sterling exchange rate obtained from the VECM bests the random walk forecast by more than 50 percent, with the RMSE declining to some 10 percent of the random walk RMSE at the 52-week horizon.

These results are all the more impressive when one recalls that the forecasts are entirely dynamic, with no extraneous information dated later than the date of the forecast. This contrasts with, for example, model-based forecasting exercises such as Meese and Rogoff (1983) which use information on exogenous variables dated later than the forecast date and are still unable to beat the random walk convincingly.

## VI. Conclusion

In this paper we have developed a theoretical framework and presented econometric evidence which challenges the now common view that forward foreign exchange premia contain little information regarding subsequent movements in the spot foreign exchange rate.

Using a weekly data base on spot and forward dollar-mark and dollar-sterling exchange rates, we were able to reject the simple efficient markets (constant risk premium) hypothesis. It is evidence such as this which has led previous researchers to ignore the information in the term structure

with regard to future spot rate movements. The theoretical framework developed in this paper, however, is able to accommodate rejection of the simple efficiency hypothesis whilst still allowing forward premia to contain information pertinent to future spot rate changes, and directly implies that the appropriate way in which to extract this information is through the estimation of vector error correction models in spot and forward rates, rather than through single-equation methods.

The data strongly support this theoretical framework in terms of satisfying the implied restrictions on the cointegration space and in admitting VECM representations which provide dynamic, out-of-sample forecasts which are extremely efficient when examined in the usual metric (i.e., relative to random-walk forecasts).

While we remain agnostic as to the causes of departures from the simple efficient markets hypothesis, the results of this paper demonstrate that there is valuable information concerning the future path of spot exchange rates which can be extracted from the term structure of forward premia.

Table 1. Unit Root Tests

	$Z(\tau_{\mu})$	$Z(\tau_{\tau})$	$Z(\Phi_3)$
<u>Dollar-Sterling</u>			
$s_t$	-1.27	-1.37	0.95
$\Delta s_t$	-26.32*	-26.31*	346.19*
$f_{4,t}$	-1.27	-1.39	0.98
$\Delta f_{4,t}$	-26.32*	-26.30*	346.13*
$f_{13,t}$	-1.28	-1.45	1.06
$\Delta f_{13,t}$	-26.39*	-26.38*	347.97*
$f_{26,t}$	-1.31	1.55	1.20
$\Delta f_{26,t}$	-26.35*	-26.34*	347.05*
$f_{52,t}$	-1.37	-1.74	1.55
$\Delta f_{52,t}$	-26.78*	-26.77*	358.50*
<u>Dollar-Mark</u>			
$s_t$	-0.88	-1.01	0.72
$\Delta s_t$	-25.72*	-25.72*	330.68*
$f_{4,t}$	-0.89	-1.01	0.71
$\Delta f_{4,t}$	-25.71*	-25.70*	330.36*
$f_{13,t}$	-0.91	-1.02	0.69
$\Delta f_{13,t}$	-25.86*	-25.86*	334.31*
$f_{26,t}$	-0.96	-1.06	0.69
$\Delta f_{26,t}$	-25.82*	-25.81*	333.22*
$f_{52,t}$	-1.08	-1.14	0.73
$\Delta f_{52,t}$	-26.40*	-26.39*	348.16*

Note: The sample is 1977:1-1990:26. The Phillips-Perron Statistics were constructed using a lag truncation parameter of 13 and a Newey-West (1987) lag window. \* indicates significance at the 1 percent level; otherwise, not significant at the 15 percent level.



Table 2. Test of the Simple Efficiency Hypothesis

$$(s_{t+h(j)} - s_t) = \alpha + \beta (f_{h(j),t} - s_t) + \epsilon_{t+h(j)}$$

$$H_0: \beta = 1$$

Dollar-Sterling

h(1) = 4		h(2) = 13		h(3) = 26		h(4) = 52	
$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$
-3.473	27.29	-3.790	18.90	-3.704	29.09	-3.128	72.44
(0.856)	(0.00)	(1.102)	(0.00)	(0.872)	(0.00)	(0.485)	(0.00)
	[0.00]		[0.001]		[0.001]		[0.00]

Dollar-Mark

h(1) = 4		h(2) = 13		h(3) = 26		h(4) = 52	
$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$	$\hat{\beta}$	$\chi^2(1)$
-3.746	10.26	-3.176	8.18	-4.086	9.46	-2.834	5.94
(1.481)	(0.001)	(1.460)	(0.004)	(1.654)	(0.002)	(1.573)	(0.015)
	[0.002]		[0.006]		[0.008]		[0.018]

Notes: Estimates obtained by least squares with a method of moments correction to the covariance matrix to allow for moving average errors up to order  $(h(j)-1)$  (Hansen (1982)). Figures in parentheses below coefficient estimates are estimated standard errors. Test statistics are the squared "t-ratio" for  $H_0: \beta = 1$ . Figures in parentheses below test statistics are marginal significance levels from the  $\chi^2(1)$  distribution; those in brackets are the empirical significance levels generated by Monte Carlo methods.

Table 3. Tests of Cointegrating Rank of  $y_t = \{s_t, f_{4,t}, f_{13,t}, f_{26,t}, f_{52,t}\}'$

	$\lambda$ -max Statistic	5 Percent Critical Value	Trace Statistic	5 Percent Critical Value
<u>Dollar-Sterling</u>				
Ho: $r \leq 4$	3.729	3.762	3.729	3.762
Ho: $r \leq 3$	16.129	14.069	19.858	15.410
<u>Dollar-Mark</u>				
Ho: $r \leq 4$	0.196	3.762	0.196	3.762
Ho: $r \leq 3$	16.245	14.069	16.441	15.410

Note: Critical values are from Osterwald-Lenum (1990) Table 2. These values are correct if  $\mu > 0$ . If, in truth  $\mu = 0$ , the appropriate critical values are those reported in Osterwald-Lenum Table 3. Using these more conservative values and the  $\lambda$ -max statistic, we still reject at the 5 percent level for both currencies  $r \leq 3$  in favor of  $r = 4$ . For the trace statistic, we reject at the 5 percent level for sterling and at the 10 percent level for the DM the hypothesis  $r \leq 3$  in favor of  $r = 4$ .

Table 4. Tests of the Null Hypothesis that Four  
 Linearly Independent Forward Premiums Comprise a Basis  
 for the Cointegration Space

	$\chi^2(4)$	Marginal Significance Level
Dollar-Sterling	2.88	58 percent
Dollar-Mark	5.32	26 percent

Note: The test is conditional on there being four linearly independent co-integrating vectors.

Table 5. FIML Error Correction Model for the Five-Variable System: Dollar-Sterling

Explanatory Variable	Model for $\Delta s_t$		Model for $\Delta f_{4,t}$		Model for $\Delta f_{13,t}$		Model for $\Delta f_{26,t}$		Model for $\Delta f_{52,t}$	
	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
$\Delta s_{t-1}$	-1.701	1.026	-1.749	1.023	-2.044	1.023	-2.374	1.034	-3.295	1.074
$\Delta f_{4,t-1}$	1.847	1.030	1.917	1.027	2.432	1.029	2.827	1.042	3.472	1.078
$\Delta f_{13,t-1}$	--	--	-0.058	0.031	-0.407	0.061	-0.467	0.069	--	--
$\Delta f_{26,t-1}$	-0.153	0.041	-0.130	0.032	--	--	--	--	--	--
$\Delta f_{52,t-1}$	-0.011	0.007	--	--	--	--	--	--	-0.189	0.030
$(s-f_4)_{t-1}$	--	--	-0.143	0.045	-0.643	0.115	-0.807	0.193	-0.723	0.324
$(s-f_{13})_{t-1}$	0.203	0.071	-0.921	0.436	-0.981	0.306	0.197	0.072	0.300	0.076
$(s-f_{26})_{t-1}$	-0.178	0.059	0.178	0.059	-0.184	0.093	-0.123	0.060	-0.248	0.062
$(s-f_{52})_{t-1}$	0.521	0.175	0.519	0.174	0.485	0.174	0.339	0.176	-0.734	-0.182
Constant	-0.002	0.001	-0.002	0.001	0.002	0.001	0.003	0.001	0.004	0.001
	Q(77) = 74.22 (0.57)		Q(77) = 73.16 (0.60)		Q(77) = 75.91 (0.51)		Q(77) = 72.22 (0.63)		Q(77) = 80.19 (0.38)	
	H(325) = 343.33 (0.24)									
	REST(18) = 9.61 (0.94)									

Notes: A "--" indicates that the coefficient was found to be insignificant in the reduction process; the Q-Statistics are Ljung-Box Statistics computed at 77 autocorrelations of the residual series; H is Hosking's (1980) multivariate portmanteau statistic computed at 13 autocorrelations; REST is a likelihood ratio statistic for the exclusion restrictions. All statistics are distributed as central chi-square under the null hypothesis, with the degrees of freedom indicated; figures in parentheses are marginal significance levels.

Table 6. FIML Error Correction Model for Five-Variable System: Dollar-Mark

Explanatory Variable	Model for $\Delta s_t$		Model for $\Delta f_{4,t}$		Model for $\Delta f_{13,t}$		Model for $\Delta f_{26,t}$		Model for $\Delta f_{52,t}$	
	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
$\Delta s_{t-1}$	-1.789	0.710	-1.734	0.705	-1.876	0.700	-2.284	0.702	-2.344	0.739
$\Delta f_{4,t}$	3.267	0.746	3.075	0.736	2.104	0.722	2.589	0.415	2.604	0.740
$\Delta f_{13,t}$	-1.799	0.229	-1.6433	0.212	-1.596	0.182	-0.985	0.143	-0.252	0.033
$\Delta f_{26,t}$	0.328	0.067	0.310	0.057	0.277	0.040	--	--	--	--
$\Delta f_{52,t}$	--	--	--	--	--	--	--	--	--	--
$(s-f_4)_{t-1}$	-0.981	0.213	-0.939	-0.242	0.601	0.171	0.597	0.171	0.596	0.173
$(s-f_{13})_{t-1}$	-0.743	0.202	-0.361	-0.128	-0.261	0.088	-0.283	0.087	-0.167	0.089
$(s-f_{26})_{t-1}$	-0.376	0.143	--	--	-0.224	0.024	--	--	-0.18	0.010
$(s-f_{52})_{t-1}$	-0.061	0.018	0.408	0.135	0.412	0.135	0.366	0.135	-0.712	0.130
Constant	0.006	0.002	0.005	0.002	0.005	0.002	0.006	0.002	0.008	0.002
	Q(77) = 72.21 (0.63)		Q(77) = 73.16 (0.60)		Q(77) = 74.91 (0.55)		Q(77) = 78.65 (0.43)		Q(77) = 81.22 (0.35)	
	H(325) = 337.36 (0.31)									
	REST(18) = 5.8 (0.99)									

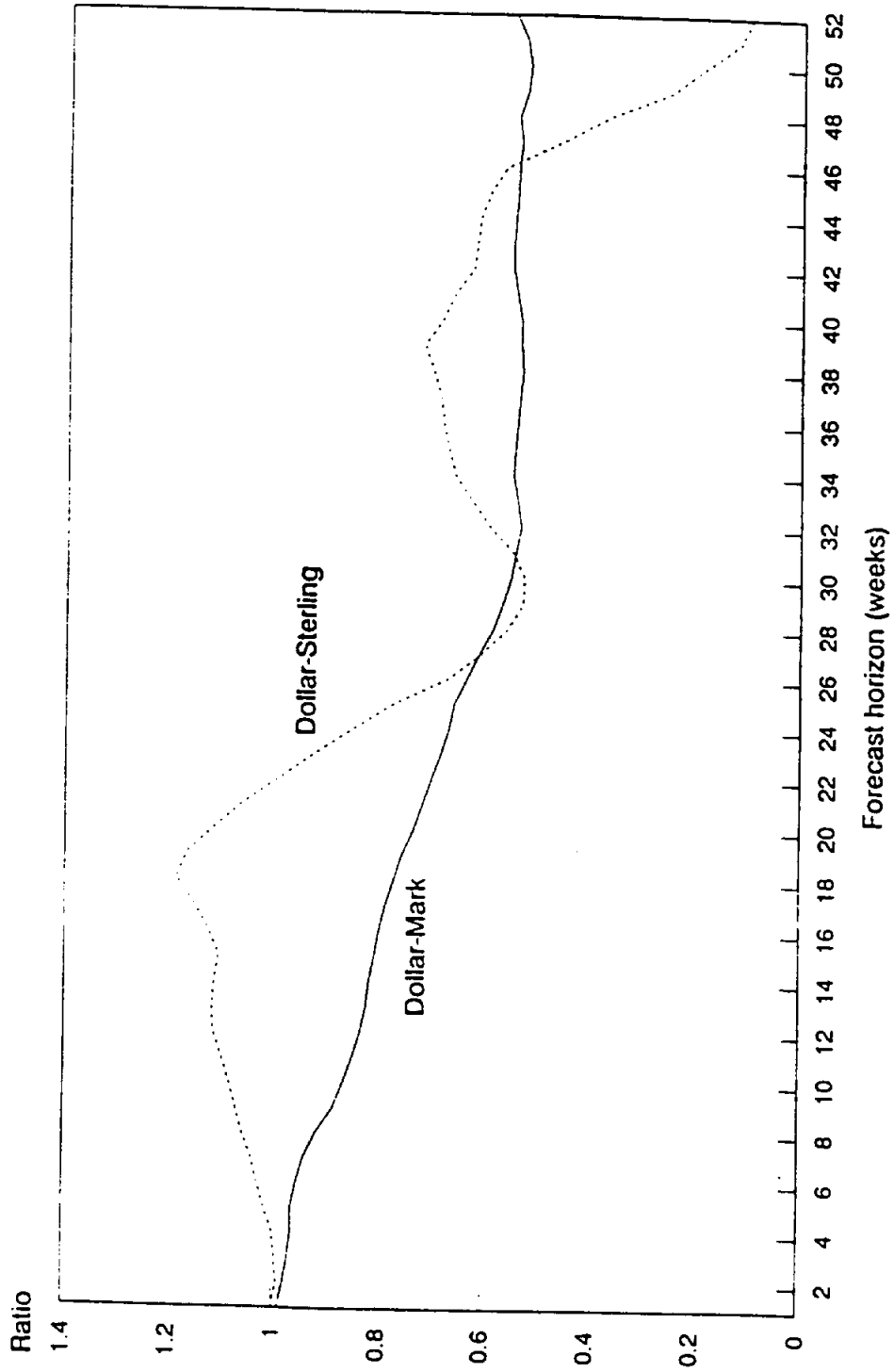
Notes: A "--" indicates that the coefficient was found to be insignificant in the reduction process; the Q-Statistics are Ljung-Box Statistics computed at 77 autocorrelations of the residual series; H is Hosking's (1980) multivariate portmanteau statistic computed at 13 autocorrelations; REST is a likelihood ratio statistic for the exclusion restrictions. All statistics are distributed as central chi-square under the null hypothesis, with the degrees of freedom indicated; figures in parentheses are marginal significance levels.

Table 7. Rolling Estimation Forecast Results 1/

Weeks ahead	RMSE from VECM system	Ratio of RMSE from VECM system to VECM from random walk
<u>Dollar-Sterling</u>		
1	0.014	1.000
2	0.021	0.990
3	0.025	0.994
13	0.044	1.119
26	0.035	0.673
39	0.044	0.721
52	0.098	0.100
<u>Dollar-Mark</u>		
1	0.014	0.989
2	0.020	0.978
3	0.024	0.969
13	0.050	0.825
26	0.071	0.636
39	0.076	0.536
52	0.084	0.552

1/ RMSE stands for root mean square error of the forecast.

FIGURE 1. Ratio of RMSE of VECM Dynamic Forecasts to  
RMSE of Random Walk Forecasts



References

- Allen, H., and M. P. Taylor, "Chartist Analysis," in P. Newman, J. Eatwell, and M. Milgate (eds.), The New Palgrave Dictionary of Money and Finance, (London: MacMillan, 1993).
- Bilson, J.F.O., "The Speculative Efficiency Hypothesis," Journal of Business, Vol. 54 (October 1981), pp. 435-51.
- Campbell, J., and R. Clarida, "The Term Structure of Euromarket Interest Rates: An Empirical Investigation," Journal of Monetary Economics, Vol. 19 (1987).
- Campbell, J., and R. Shiller, "Cointegration and Tests of Present Value Models," Journal of Political Economy, Vol. 95 (1987).
- Cumby, R., and M. Obstfeld, "A Note on Exchange Rate Expectations and Nominal Interest Differentials," Journal of Finance, Vol. 36 (June, 1980).
- Cumby, R., "International Interest Rate and Price Level Linkages," in J. Bilson and R. Marston (eds.), Exchange Rate Theory and Practice (Chicago: University of Chicago Press, 1984).
- Cutler, D.M., J. M. Poterba, and L.H. Summers, "Speculative Dynamics and the Role of Feedback Traders," American Economic Review, Vol. 80 (1990), 63-68.
- Dornbusch, R., "Exchange Rate Economics: Where Do We Stand?" Brookings Papers on Economic Activity, 11 (1980).
- Engle, R., and C. Granger, "Cointegration and Error Correction," Econometrica, Vol. 55 (1987).
- Evans, M., and K. Lewis, "Trends in Expected Returns in Currency and Bond Markets," NBER Working Paper No. 4116 (1992).
- Fama, E., "Forward and Spot Exchange Rates," Journal of Monetary Economics, Vol. 14 (1984).
- Fisher, I., The Theory of Interest, (New York: MacMillan, 1930).
- Frankel, J. A., and K. A. Froot, "Chartists, Fundamentalists, and the Demand for Dollars," in A. S. Courakis and M. P. Taylor (eds.), Private Behaviour and Government Policy in Interdependent Economies, (Oxford: Oxford University Press, 1990).
- Frenkel, J. A., "Flexible Exchange Rates, Prices, and the Role of News," Journal of Political Economy, Vol. 89 (1981).



- Froot, K.A., "Short Rates and Expected Asset Returns", NBER Working Paper No. 3247, January 1990.
- Froot, K.A., and J.A. Frankel, "Forward Discount Bias: Is It An Exchange Risk Premium?", Quarterly Journal of Economics, Vol. 104 (1989), 139-161.
- Froot, K.A., D.S. Scharfstein, and J.C. Stein, "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation," Journal of Finance, Vol. 47 (1992), 1461-84.
- Froot, K.A., and R.H. Thaler, "Anomalies: Foreign Exchange," Journal of Economic Perspectives, Vol. 4 (1990), 179-192.
- Gregory, A., and T. H. McCurdy, "Testing the Unbiasedness Hypothesis in the Forward Foreign Exchange Market: A Specification Analysis," Journal of International Money and Finance, Vol. 3 (December 1984), pp. 357-68.
- Hakkio, C.S., "The Term Structure of the Forward Premium", Journal of Monetary Economics, 8 (1981), pp. 41-58.
- Hall, A., H. Anderson, and C. Granger, "A Cointegration Analysis of Treasury Bill Yields," Review of Economics and Statistics, Vol.44 (1992).
- Hansen, L.P., and R. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Exchange Rates," Journal of Political Economy, Vol. 88 (1980).
- Hansen, L.P., and R. Hodrick, "Risk Averse Speculation in the Forward Foreign Exchange Market," in J. Frenkel (ed.), Exchange Rates and International Macroeconomics (Chicago: University of Chicago Press, 1982).
- Hansen, L.P., "Large Sample Properties of Generalized Method of Moments Estimators", Econometrica, 50 (1982), pp. 1029-54.
- Hodrick, R.J., The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets, (Chur, Switzerland: Harwood Academic Publishers, 1987).
- Hosking, J.R.M., "The Multivariate Portmanteau Statistic," Journal of the American Statistical Association, Vol. 75 (1980), pp. 602-08.
- Huang, R.D., "Some Alternative Tests of Forward Exchange Rates as Predictors of Future Spot Rates," Journal of International Money and Finance, Vol. 3 (1984), 153-167.
- Isard, P., "Uncovered Interest Rate Parity," in P. Newman, J. Eatwell, and M. Milgate (eds.), The New Palgrave Dictionary of Money and Finance (London: MacMillan, 1992).

- Johansen, S., "Estimation and Hypothesis Testing of Cointegration Vectors," Econometrica, Vol. 59 (1991).
- Keynes, J.M., A Tract on Monetary Reform (London: MacMillan, 1923).
- Krasker, W. S., "The 'Peso Problem' in Testing the Efficiency of Forward Exchange Markets," Journal of Monetary Economics, Vol. 6 (1980), pp. 269-76.
- Lewis, K.K., "Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange", American Economic Review, Vol. 79 (1989), pp. 269-76.
- Longworth, D., "Testing the Efficiency of the Canadian-U.S. Exchange Market Under the Assumption of No Risk Premium," Journal of Finance, Vol. 36 (March 1981), pp. 43-49.
- MacDonald, R., and M. P. Taylor, "Exchange Rate Economics: A Survey," International Monetary Fund (Washington), Staff Papers, Vol. 39 (1992), pp. 1-57.
- McCallum, B.T., "A Reconsideration of the Uncovered Interest Parity Relationship", NBER Working Paper No. 4113, 1992.
- Meese, R., "Empirical Assessment of Foreign Currency Risk Premiums," in Financial Risk: Theory, Evidence, and Implications, Federal Reserve Bank of St. Louis (1986).
- Meese, R., and K. Rogoff, "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?," Journal of International Economics, Vol. 14 (1983), pp. 1029-35.
- Meese, R., and K. Singleton, "A Note on Unit Roots and the Empirical Modeling of Exchange Rates," Journal of Finance, Vol.37 (1982).
- Mussa, M., "Empirical Regularities in the Behavior of Exchange Rates," Carnegie-Rochester Conference on Public Policy, Vol. 11 (1979).
- Osterwald-Lenum, M., "Fractiles of the Asymptotic Distribution of the Cointegration Rank Test Statistic," Institute of Economics, Copenhagen (1990).
- Phillips, P., "Optimal Inference in Cointegrated Systems," Cowles Foundation Discussion Paper, No. 866 (1990).
- Phillips, and P. Perron, "Testing for Unit Roots in Time Series Regression," Cowles Foundation Discussion Paper (1986).
- Rogoff, K., Essays on Expectations and Exchange Rate Volatility, unpublished Ph.D. Dissertation, MIT, 1980.

- Stock, J., and M. Watson, "Testing for Common Trends," Journal of the American Statistical Association, Vol. 83 (1988).
- Taylor, M.P., "Covered Interest Rate Parity," in P. Newman, J. Eatwell, and M. Milgate (eds.), The New Palgrave Dictionary of Money and Finance (London: MacMillan, 1993).
- Taylor, M.P., and H. Allen, "The Use of Technical Analysis in the Foreign Exchange Market," Journal of International Money and Finance, Vol. 11 (1992), 304-314.