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ABSTRACT

Recent estimates of the potential growth effects of tax reform vary widely, ranging from zero (Lucas 1990) to eight percentage points (Jones, Manuelli, and Rossi 1993). Using an endogenous growth model, we assess which model features and parameter values are important for determining the quantitative impact of tax reform. We find that the critical parameters are factor shares, depreciation rates, the elasticity of intertemporal substitution, and the elasticity of labor supply. The elasticities of substitution in production, on the other hand, are relatively unimportant. The quantitative estimates in several recent papers are compared with each other and with some of the evidence from U.S. experience.

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Growth models with endogenously determined rates of technical change provide a useful framework for studying the effects of fiscal policy on the long-run growth rate. Recent papers by Jones and Manuelli (1990), King and Rebelo (1990), Lucas (1990), Rebelo (1991), Yuen (1991), Kim (1992), Jones, Manuelli, and Rossi (1993), and others have used endogenous growth models to look at both the positive and normative effects of taxation. Unfortunately, although all of these authors use a similar basic framework and calibrate their models to U.S. data, their quantitative conclusions differ wildly.

For example, Lucas (1990) calculates that eliminating the capital tax and raising the labor tax in a revenue-neutral way would have a trivial effect on the U.S. growth rate, raising it by only three hundredths of a percentage point, while Jones, Manuelli, and Rossi (1993) conclude that eliminating all distorting taxes could raise it by as much as eight percentage points! King and Rebelo (1990) and Kim (1992) conclude that tax reform would raise the growth rate by modest, but nontrivial amounts.

The goals of the present paper are to trace the sources of these conflicting results by examining the features of the preferences, technology, and tax policy that are critical for conclusions about the long-run growth effects of taxation, and to compare their predictions with some of the evidence offered by U.S. experience. We look at flat-rate taxes levied on the income from physical and human capital, the proceeds of which are rebated in lump-sum form, and we compare steady state growth rates.

As shown in Rebelo (1991), if all income is taxed at a common rate τ, then, compared with the values in an untaxed economy, the steady-state input ratios and factor shares in all industries are unchanged, and the interest rate is reduced by the factor $(1 - \tau)$. The change in the growth rate is then equal to the change in the interest rate multiplied by the elasticity of intertemporal substitution. These effects are independent of the properties of the production functions (beyond linear homogeneity) and preferences (beyond a constant rate of time preference and constant elasticity of intertemporal substitution).

If income from different sources is taxed at different rates, however, the situation is substantially more complicated. For determining growth effects, the important features of the economy include two types of parameters. The first group consists of those, like factor shares and depreciation rates, that can be calibrated in a straightforward way on the basis of easily observable quantities. The second consists of parameters, like elasticities of substitution in production and the elasticity of labor supply, that cannot be calibrated on the basis of observations from an economy growing along a balanced path. Price variation is needed to estimate these elasticities, and, by definition, prices do not vary along balanced growth paths. One of our goals here is to assess the sensitivity of growth effects to the values of these hard-to-observe parameters.

First we consider the technology parameters. We assume that the preferences and technologies are of the CES variety and compare economies that have different elasticities of substitution but that, in the absence of taxes, have identical interest rates, factor shares, and factor price ratios. That is, we compare economies that are observationally equivalent in the untaxed steady state (or when both factors are taxed at the same rate), but respond differently when factors are taxed asymmetrically. Under the assumption of inelastic labor supply, we show that while the quantitative responses of the steady-state interest rate and growth rate to fiscal reform are quite sensitive to the factor share parameters in the input-producing sectors, they are very insensitive to the substitution elasticities. Surprisingly, steady-state revenues are also quite insensitive to the elasticity parameters.

Elastic labor supply is then considered. First we show that if leisure time is quality adjusted in the same way work time is, then the analysis of the preceeding sections remains unchanged: only a reinterpretation of the results is needed. We then show that if leisure time is measured in "raw hours," the effect of taxation on the interest rate depends on the elasticity of labor supply, and we derive a quantitative relationship for that effect.

We then proceed to a numerical comparison of some of the tax models mentioned earlier. We find that the very small growth effect found by Lucas is due to the technology he uses for the sector producing human capital, and the very large effect found by Jones, Manuelli, and Rossi is due to their assumption about the elasticity of labor supply. We also show that the calculated growth effects of taxation are very sensitive to assumptions about the rate of depreciation and the tax treatment of depreciation, and we argue that some that some commonly used assumptions overstate the potential growth effects of tax reform.

The rest of the paper is organized as follows. The basic model is described in section 1, and general properties of the steady state are discussed in section 2. The technology parameters are studied in sections 3 and 4, and elastic labor supply is discussed in section 5. Numerical comparisons are carried out in section 6, and conclusions are drawn in section 7.

1. The basic model

Time is continuous, all markets are perfectly competitive, and there is an infinitely-lived representative household. At each date t, existing stocks of physical and human capital, k_t and h_t, are used as inputs into production. These stocks depreciate at the rates δ_1 and δ_2, respectively. The household owns the stocks, which it supplies, inelastically, to firms. The economy has three types of firms, which produce new physical capital, I_{1t}, new human capital, I_{2t}, and consumption goods c_t. The household uses its income to buy goods of all three types. Let q_{it} and p_{it}, $i = 1, 2$, be the net-of-tax returns and purchase prices for the two factors, measured in terms of contemporaneous consumption goods. The revenue from all taxes is rebated to the household in lump-sum form, T_t, at each date t.

The household's problem is, given (δ_1, δ_2), (k_0, h_0), and $\{p_{1t}, p_{2t}, q_{1t}, q_{2t}, T_t, t \geq 0\}$, to choose $\{c_t, I_{1t}, I_{2t}, k_t, h_t, \quad t \geq 0\}$ to

$$\max \int_0^\infty e^{-\rho t}(c_t^{1-\sigma} - 1)/(1-\sigma)dt$$

(1a) $$s.t. \quad \dot{k}_t = I_{1t} - \delta_1 k_t,$$

(1b) $$\dot{h}_t = I_{2t} - \delta_2 h_t,$$

(1c) $$p_{1t}I_{1t} + p_{2t}I_{2t} + c_t - q_{1t}k_t - q_{2t}h_t - T_t \leq 0,$$

where $\rho, \sigma > 0$.

The factor returns q_{it} are determined as follows. Let θ_{it} and $\ell_{it}, i = 1,2,3$, be the proportions of the physical and human capital stocks employed in the production of physical capital, human capital, and consumption goods, respectively; let G, H and F be CRS production functions for the three sectors; and let τ_{ij} be the flat-rate tax – constant over time – on income earned by factor $i = 1,2$, employed in sector $j = 1,2,3$. Profit maximization and competition imply that factors are paid their marginal products and that the net-of-tax return on factor i must be equal in all sectors where it is employed. Hence

(2) $$\begin{aligned} q_{it} &= (1-\tau_{i1})p_{1t}G_i(\theta_{1t}k_t, \ell_{1t}h_t) + \omega_i\tau_{i1}p_{it}\delta_i \\ &= (1-\tau_{i2})p_{2t}H_i(\theta_{2t}k_t, \ell_{2t}h_t) + \omega_i\tau_{i2}p_{it}\delta_i \\ &= (1-\tau_{i3})F_i(\theta_{3t}k_t, \ell_{3t}h_t) + \omega_i\tau_{i3}p_{it}\delta_i, \qquad i = 1,2, \end{aligned}$$

where $\omega_i \in [0,1]$ is the extent to which depreciation on factor i is tax deductible.

To characterize a competitive equilibrium, the conditions for utility maximization and profit maximization must be combined with those for budget balance for the government and market clearing. Only balanced growth paths will be considered.[1]

[1] As shown in Bond, Wang, and Yip (1992), the model here is globally asymptotically stable, so balanced growth paths represent the long run of the economy. See Caballé and Santos (1991) and Faig (1991) for other analyses of stability, and see Mulligan and Sala-i-Martin (1993) for a discussion of transitional dynamics.

Along any such path, consumption and both kinds of capital grow at a common, constant rate g, and the interest rate, the growth rate, and the sectoral allocation of factors are constant. Let $\hat{c} = c/k, z = h/k$, and $z_i = \ell_i h/\theta_i k, \quad i = 1, 2, 3$.

It is shown in the Appendix that if income is taxed gross of depreciation ($\omega_1 = \omega_2 = 0$), then the balanced growth path satisfies

(3a) $$r = \rho + \sigma g,$$

(3b) $$(1 - \tau_{11})G_1(1, z_1) - \delta_1 = r,$$

(3c) $$(1 - \tau_{22})H_2(1, z_2) - \delta_2 = r,$$

(3d) $$\frac{(1 - \tau_{11})G_1(1, z_1)}{(1 - \tau_{21})G_2(1, z_1)} = \frac{(1 - \tau_{12})H_1(1, z_2)}{(1 - \tau_{22})H_2(1, z_2)},$$

(3e) $$\frac{(1 - \tau_{11})G_1(1, z_1)}{(1 - \tau_{21})G_2(1, z_1)} = \frac{(1 - \tau_{13})F_1(1, z_3)}{(1 - \tau_{23})F_2(1, z_3)},$$

(3f) $$g + \delta_1 = \theta_1 G(1, z_1),$$

(3g) $$g + \delta_2 = \frac{\theta_2}{z} H(1, z_2),$$

(3h) $$\hat{c} = \theta_3 F(1, z_3),$$

(3i) $$\sum_{i=1}^{3} \theta_i = 1,$$

$$\text{(3j)} \qquad \sum_{i=1}^{3} z_i \theta_i = z.$$

Equation (3a) relates the consumption growth rate to the interest rate; (3b) and (3c) equate the real rate of return on each factor, net of taxes and depreciation, to the interest rate; (3d) and (3e) follow from the equality of factor returns in all sectors; (3f)-(3h) are market-clearing conditions for the three outputs; and (3i) and (3j) are resource constraints for the two factor inputs. These equations can be used to solve for $r, g, \hat{c}, z$, and (θ_i, z_i), $\quad i = 1,2,3$, all of which are constant along the balanced growth path.

Notice that the system is block-recursive: (3b)-(3d) can be solved independently for (r, z_1, z_2) and then (3a) solved for g. Notice, too, that if tax rates vary only by factor $(\tau_{ij} = \tau_i, \quad j = 1,2,3)$ or only by sector $(\tau_{ij} = \tau_j, \quad i = 1,2)$, then (3b)-(3d) take the simpler form [2]

$$\text{(4a)} \qquad (1-\tau_1)G_1(1,z_1) - \delta_1 = r,$$

$$\text{(4b)} \qquad (1-\tau_2)H_2(1,z_2) - \delta_2 = r,$$

$$\text{(4c)} \qquad \frac{G_1(1,z_1)}{G_2(1,z_1)} = \frac{H_1(1,z_2)}{H_2(1,z_2)}.$$

[2] The assumption that tax rates vary only by sector is probably not too bad for the U.S., where in the education sector physical capital and the (substantial) fraction of the labor input representing students' time are both untaxed, and where in the goods sector both physical and human capital are taxed at similar rates.

Alternatively, it is shown in the Appendix that if income is taxed net of depreciation and tax rates vary only by factor, then along the balanced growth path

(4a′) $$(1-\tau_1)[G_1(1,z_1)-\delta_1]=r,$$

(4b′) $$(1-\tau_2)[H_2(1,z_2)-\delta_2]=r,$$

(4c′) $$\frac{G_1(1,z_1)}{G_2(1,z_1)}=\frac{H_1(1,z_2)}{H_2(1,z_2)},$$

where $\tau_i,\quad i=1,2$, is the tax rate for factor i.

Notice that the solution to (4a)-(4c) or to (4a′)-(4c′) depends only on the technologies, depreciation rates, and tax rates in the input-producing sectors. The preference parameters ρ and σ affect only how the interest rate is translated into a growth rate, the relationship in (3a). Hence we can first focus our attention on the interest rate, and then later use the preference parameters to calculate the growth rate.

2. General properties of the steady state

Suppose that income is taxed net of depreciation and tax rates vary only by factor. If income from both factors is taxed at a common rate τ, then it is clear from (4a′)-(4c′) that the steady-state input ratios are unaffected and the interest rate is reduced by the factor $(1-\tau)$. These conclusions are the same for any CRS production functions. Moreover, it is clear that a tax levied on factors employed in the consumption goods sector – which in this setting is equivalent to a consumption tax – is completely nondistorting.[3]

If the returns to physical and human capital are taxed at different rates, the situation is more complicated. Figure 1 shows the consequences when income from physical capital is taxed and income from human capital is not. The solid lines show the determination of the steady-state interest rate and input ratios when there are no taxes. Equation (4a′) is graphed in the northeast quadrant, (4b′) in the northwest, and the ratios of the marginal products in the southern quadrants. Each of the curves must be roughly as shown: $[G_1(1, z_1) - \delta_1]$ is increasing in z_1; $[H_2(1, z_2) - \delta_2]$ is decreasing in z_2; $G_1(1, z_1)/G_2(1, z_1)$ is increasing in z_1 ; and $H_1(1, z_2)/H_2(1, z_2)$ is increasing in z_2. The steady state corresponds to the set of values lying in a rectangle, as shown.

If a flat-rate tax of τ_1 is imposed on income from physical capital, then the curve in the northeast quadrant is shifted down by the factor $(1 - \tau_1)$, as shown. The new steady state interest rate, call it R, must lie between the values $A = (1 - \tau_1)r$ and r, where r is the interest rate in the untaxed economy. Hence, the new steady state has a lower interest rate and higher ratios of human to physical capital in both industries: z_1 and z_2 both rise. Analogous arguments hold if both factors are taxed but the income from physical capital is taxed more heavily, or if the relative magnitudes of the tax rates are reversed.

3 See Rebelo (1991).

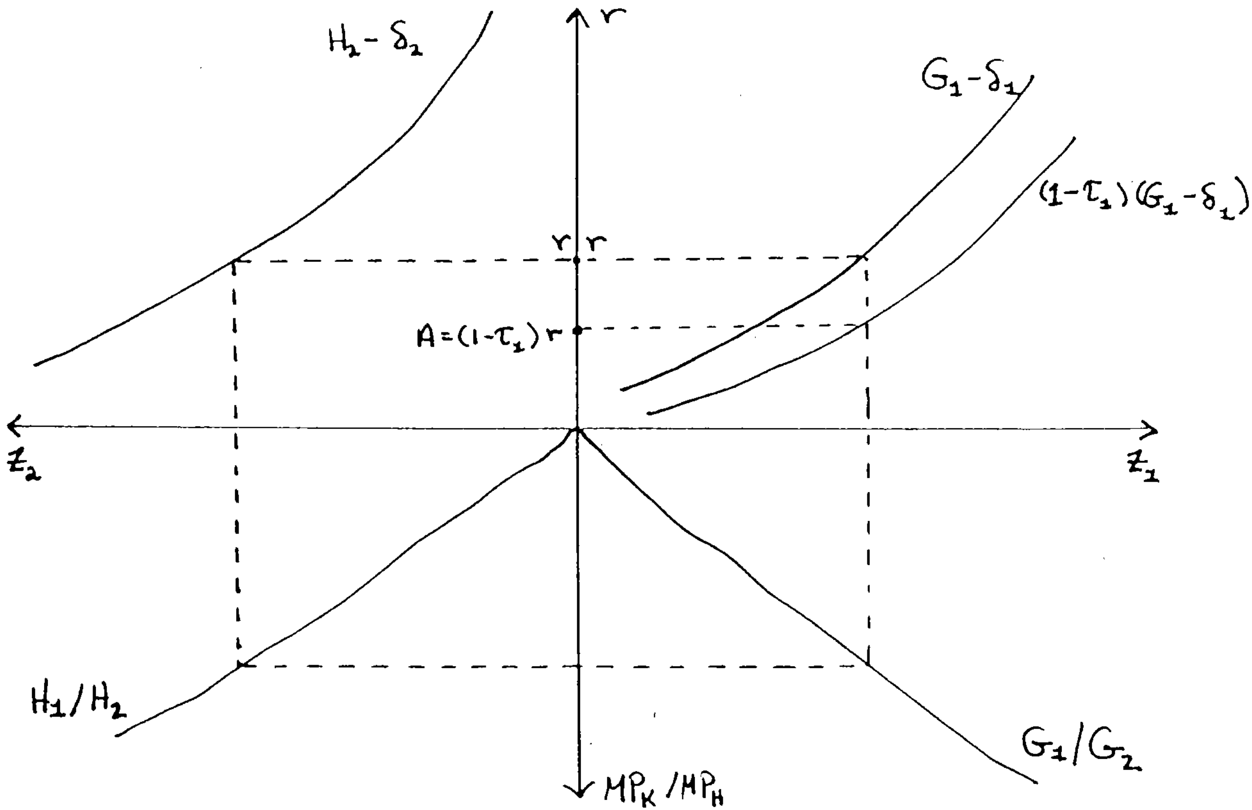


Figure 1

Quantitatively, the changes in the interest rate and the input ratios depend on properties of the production functions in both input-producing sectors. An obvious conjecture is that the degree of substitutability between the two factors is crucial. If they are highly substitutable, then one might expect that the economy adjusts by substituting away from the more heavily taxed factor, and that other variables – including the interest rate – adjust very little. Thus, one might expect a large impact on input ratios and factor shares, but a relatively small impact on the interest rate. The reverse might be expected if the substitution possibilities are poor. As we will see below, these conjectures about input ratios and factor shares are correct, but the conjecture about the interest rate is not. The response of the interest rate – and hence of the growth rate – is very insensitive to the elasticities of substitution in the production technologies.

An interesting special case is the one studied in Lucas (1990). Lucas assumes that human capital is produced using human capital as the only input, so $H(k_2, h_2) = Bh_2$. It then follows immediately from (4b′) that the interest rate is $r = B - \delta_2$ in the untaxed economy and $R = (1 - \tau_2)(B - \delta_2)$ in the taxed economy. That is, the steady state interest rate is completely determined by B, appropriately adjusted for taxes and depreciation. In this case the curve in the northwest quadrant in Figure 1 is horizontal.

In the next two sections, we will see how the quantitative responses of the interest rate and the input ratios depend on the elasticities of substitution in the two input-producing sectors.

3. Common technology for the production of both inputs: G = H

Suppose that physical and human capital are produced with the same technology, $G = H$; this is the assumption used in the benchmark case of King-Rebelo (1990) and in model 1 of Jones-Manuelli-Rossi (1993). In addition assume that $\delta_1 = \delta_2 = 0$. We will compare economies that have different CES technologies but that, in the absence of taxes (or when income from all sources is taxed at a common rate), have identical steady-state input ratios, factor shares, and interest rates.

Fix z and r. For each $\alpha \leq 1$, there is a unique CES function, call it $G(\cdot, \cdot; \alpha)$, with elasticity parameter α and with scale and weight parameters that depend on α, z, and r, such that (4a) and (4b) hold for the input ratio z and interest rate r. As shown in the Appendix, that function is

$$G(k, h; \alpha) = \begin{cases} r[w^{1-\alpha}k^{\alpha} + (1-w)^{1-\alpha}h^{\alpha}]^{1/\alpha}, & \alpha \neq 0, \\ r(k/w)^{w}[h/(1-w)]^{1-w}, & \alpha = 0, \end{cases} \tag{5}$$

where $w = 1/(1+z)$. For fixed z and r, (5) defines a family of economies indexed by α. Every economy in this family has a steady state with interest rate r, input ratio z, and factor share w for capital, when income is untaxed (or taxed at a common rate for both factors). The economies in this family have different elasticities of substitution $\eta = 1/(1-\alpha)$, but that parameter could never be identified by observing an economy growing along a balanced path.

If income from different sources is taxed differentially, these economies have different steady-state interest rates, input ratios, and factor shares. Let $m = (m_1, m_2)$, where $m_i = (1 - \tau_i), \quad i = 1, 2$, and let $Z(m, \alpha), R(m, \alpha)$, and $S(m, \alpha)$ denote the steady-state input ratio, (net-of-tax) interest rate, and (gross-of-tax) factor share ratio in the economy with tax policy m and technology α. It follows from (4a)-(4c) that

$$m_1 G_1[1, Z(m,\alpha);\alpha] = m_2 G_2[1, Z(m,\alpha);\alpha] = R(m,\alpha),$$

and by definition the factor share ratio is

$$S(m,\alpha) = Z(m,\alpha)\frac{G_2[1, Z(m,\alpha);\alpha]}{G_1[1, Z(m,\alpha);\alpha]}.$$

Evaluating the derivatives of G and substituting, we find that

$$\text{(6a)} \qquad \frac{Z(m,\alpha)}{z} = \left(\frac{m_2}{m_1}\right)^{\eta}, \quad \text{all } \eta,$$

$$\text{(6b)} \qquad \frac{S(m,\alpha)}{s} = (m_2/m_1)^{\eta-1}, \quad \text{all } \eta,$$

$$\text{(6c)} \qquad \frac{R(m,\alpha)}{rm_1} = \begin{cases} [w + (1-w)(m_2/m_1)^{\eta-1}]^{1/(\eta-1)}, & \eta \neq 1, \\ (m_2/m_1)^{1-w} & \eta = 1, \end{cases}$$

where $s = (1-w)/w$ is the factor share ratio in the untaxed economy. Since physical and human capital are produced with the same technology, if their returns are taxed at different rates, the ratio of their rental rates must be m_2/m_1, to offset the tax differential. Then (6a) and (6b) are simply the standard formulas relating the changes in input and factor share ratios to changes in the factor price ratio for CES production functions. Notice that the impact of a given tax policy on these ratios increases with the elasticity of substitution and does not depend on w.

From (6c) we see that the effect of any tax policy (m_1, m_2) on the interest rate can be thought of as follows. If all income were taxed at the same rate, then the interest rate would be $R = rm_1$. If the tax rates differ, then the right side of this equation must be multiplied by the term on the right side of (6c), a term that depends only on the ratio m_2/m_1. Thus, the effect of fiscal policy on the interest rate is homogeneous of degree one in the pair (m_1, m_2).

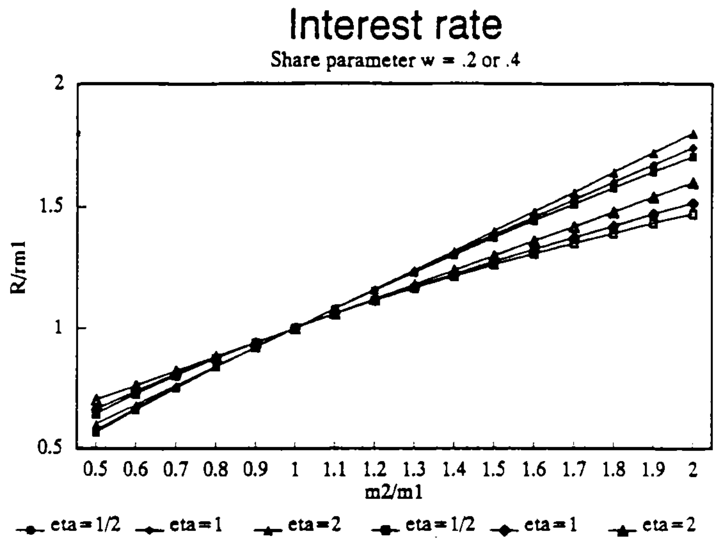
To a first-order approximation, the response of the interest rate to changes in tax policy when $m_2/m_1 \approx 1$ is independent of η. To see this, differentiate (6c) and find that

$$\frac{d\ell n(R/rm_1)}{d\ell n(m_2/m_1)}\Big|_{m_1=m_2} = 1 - w.$$

This conclusion is a direct consequence of the fact that for fixed (r, z), all members of the family of production functions defined in (5) have identical marginal rates of substitution when $Z = z$, and that the latter holds if $m_1 = m_2$.

To study the quantitative importance of the elasticity of substitution when m_2/m_1 is not close to one, we can simply compute the ratios in (6a)-(6c) as functions of the parameters η and w and the ratio m_2/m_1. Figure 2 depicts results for such computations. (Since Z/z and S/s are so closely related, only the latter is reported.)

The ratios R/rm and S/s are plotted as functions of the ratio m_2/m_1, which varies from 0.5 to 2.0. The first panel, which displays R/rm_1, has six curves. The three with solid symbols (with open symbols) correspond to economies with a share parameter of $w = .20$ (of $w = .40$), and with substitution elasticities of 1/2, 1, and


Interest rate
Share parameter w = .2 or .4
R/rm1
m2/m1
eta=1/2 eta=1 eta=2 eta=1/2 eta=1 eta=2

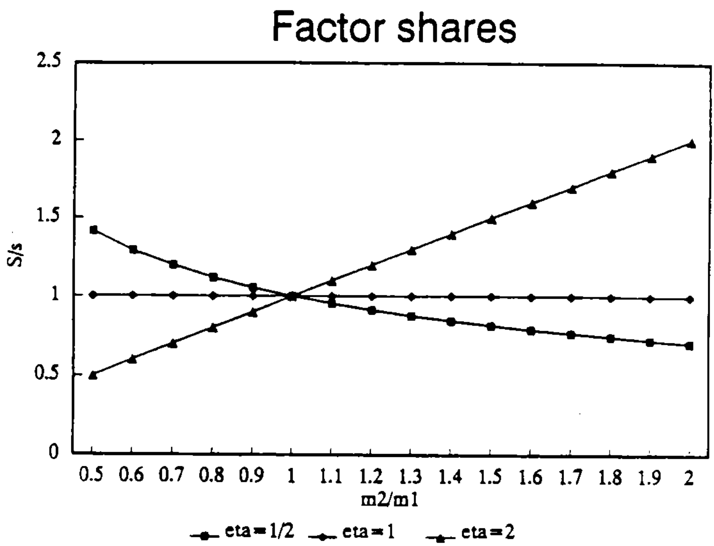

Factor shares
S/s
m2/m1
eta=1/2 eta=1 eta=2

Figure 2

2. As noted above, the ratio S/s is independent of w, so the second panel contains only three curves, corresponding to the three substitution elasticities.

The results are quite striking. The response of the interest rate is quite sensitive to the share parameter w but is extremely insensitive to the elasticity parameter η. The response of the factor share ratio, on the other hand, does not depend at all on w, but is extremely sensitive to η.

Thus, to accurately predict the impact of a given tax policy on the long-run interest rate (and growth rate), it is important to know the share parameter w for the input-producing industry. The elasticity parameter η, however, is of minor importance. The policy's impact on input ratios and factor shares, on the other hand, are sensitive to η but not to w. Since w is observable in the untaxed steady state but η is not, it may be easier to predict the impact of fiscal reform on the interest rate than its impact on factor ratios and factor shares.

4. Different technologies for producing the two inputs: $G \neq H$

In this section we will extend the analysis to allow the two input-producing sectors to have different technologies. To study the effects of taxing the two factors at different rates, we will fix parameter values (z_1, z_2, r, q) and compare economies with CES technologies for producing physical and human capital that, when untaxed, have the specified input ratios, interest rate, and rental ratio $q = q_1/q_2$. The method for doing this is exactly as in the previous section, except that scale and weight parameters must now be found for each sector. For fixed (z_1, z_2, r, q), the resulting technologies are

$$G(k_1, h_1; \alpha) = r\left[w^{1-\alpha}k_1^{\alpha} + (1-w)^{1-\alpha}\left(\frac{h_1}{q}\right)^{\alpha}\right]^{1/\alpha}, \tag{7a}$$

$$H(k_2, h_2; \beta) = qr\left[v^{1-\beta}k_2^{\beta} + (1-v)^{1-\beta}\left(\frac{h_2}{q}\right)^{\beta}\right]^{1/\beta}, \tag{7b}$$

where $w = q/(q+z_1)$ and $v = q/(q+z_2)$ are factor shares. For fixed (z_1, z_2, r, q), these equations define a two-dimensional family of economies, indexed by the pair (α, β). Every economy is this family has a steady state with input ratios z_1 and z_2, interest rate r, rental ratio q, and factor shares w and v for capital.

Given a tax policy $m = (m_1, m_2)$, the rental ratio, input ratios, and interest rate – call them $Z_1(m, \alpha, \beta)$, $Z_2(m, \alpha, \beta)$, $R(m, \alpha, \beta)$, and $Q(m, \alpha, \beta)$ – will depend on the elasticity parameters α and β. To compute them, notice that (4a)-(4c) imply that Z_1, Z_2, R, and Q satisfy

$$\frac{G_1(1, Z_1)}{G_2(1, Z_1)} = \frac{H_1(1, Z_2)}{H_2(1, Z_2)} = Q, \tag{8a}$$

$$m_1 G_1(1, Z_1) = m_2 H_2(1, Z_2) = R, \quad \text{all } m, \alpha, \beta. \tag{8b}$$

Evaluating the derivatives of G and H and substituting, we find that the changes in the input and factor share ratios are

$$\frac{Z_1(m, \alpha, \beta)}{z_1} = \left[\frac{Q(m, \alpha, \beta)}{q}\right]^{\eta}, \tag{9a}$$

(9b) $$\frac{Z_2(m,\alpha,\beta)}{z_2} = \left[\frac{Q(m,\alpha,\beta)}{q}\right]^{\gamma},$$

(9c) $$\frac{S_1(m,\alpha,\beta)}{s_1} = \left[\frac{Q(m,\alpha,\beta)}{q}\right]^{\eta-1},$$

(9d) $$\frac{S_2(m,\alpha,\beta)}{s_2} = \left[\frac{Q(m,\alpha,\beta)}{q}\right]^{\gamma-1},$$

where $\eta = 1/(1-\alpha)$ and $\gamma = 1/(1-\beta)$ are the elasticities of substitution in the two sectors, and where $s_1 = (1-w)/w$ and $s_2 = (1-v)/v$ are the factor share ratios in the untaxed economy. As before, the first two equations are the familiar expressions that, for CES technologies, relate changes in the input ratios to a change in the rental ratio.

Next, using (9a) and (9b) to evaluate G_1 and H_2, we find that

$$G_1(1, Z_1; \alpha) = \begin{cases} r[w + (1-w)(Q/q)^{\eta-1}]^{1/(\eta-1)}, & \eta \neq 1, \\ r(Q/q)^{1-w}, & \eta = 1, \end{cases}$$

$$H_2(1, Z_2; \beta) = \begin{cases} (Q/q)^{-1} r[v + (1-v)(Q/q)^{\gamma-1}]^{1/(\gamma-1)}, & \gamma \neq 1, \\ r(Q/q)^{-v}, & \gamma = 1. \end{cases}$$

We can substitute from these expressions into (8b) to solve for the ratios Q/q and R/r. If both elasticities of substitution are unity, $\eta = \gamma = 1$, we find that

(10a) $$\frac{Q(m,0,0)}{q} = \left(\frac{m_2}{m_1}\right)^{1/(1-w+v)},$$

(10b) $$\frac{R(m,0,0)}{rm_1} = \left(\frac{m_2}{m_1}\right)^{(1-w)/(1-w+v)}.$$

If both elasticities differ from unity, $\eta, \gamma \neq 1$, we find that

(11a) $$\frac{Q(m,\alpha,\beta)}{q} \frac{\{w + (1-w)[Q(m,\alpha,\beta)/q]^{\eta-1}\}^{1/(\eta-1)}}{\{v + (1-v)[Q(m,\alpha,\beta)/q]^{\gamma-1}\}^{1/(\gamma-1)}} = \frac{m_2}{m_1},$$

(11b) $$\frac{R(m,\alpha,\beta)}{rm_1} = \left[w + (1-w)\left[\frac{Q(m,\alpha,\beta)}{q}\right]^{\eta-1}\right]^{1/(\eta-1)}$$

$$= \frac{m_2}{m_1}\left[v + (1-v)\left[\frac{Q(m,\alpha,\beta)}{q}\right]^{\gamma-1}\right]^{1/(\gamma-1)} \left[\frac{Q(m,\alpha,\beta)}{q}\right]^{-1}.$$

If physical capital is taxed more heavily, $m_2/m_1 > 1$, then $Q/q > 1$, so the rental rate for physical capital rises relative to the wage rate. The reverse occurs if human capital is taxed more heavily. The intuition behind this is clear: a relatively higher rental rate on the more heavily taxed factor is needed to equate the net-of-tax returns on the two factors.

As before, it follows from the way the family of technologies is defined that, for small differences in the two tax rates (i.e., for m_2/m_1 close to unity), the response of the factor price ratio and the interest rate are, to a first order approximation, independent of the elasticities of substitution. To see this, note that $m_2/m_1 = 1$ implies $Q/q = 1$, and differentiate (11a) and (11b) to find that

$$\frac{d\ell n(Q/q)}{d\ell n(m_2/m_1)}\Big|_{m_2/m_1=1} = \frac{1}{1-w+v},$$

$$\frac{d\ell n(R/rm_1)}{d\ell n(Q/q)}\Big|_{Q/q=1} = 1-w.$$

It is clear from (9c) and (9d), however, that the behavior of the input and factor share ratios in each sector are quite sensitive to that sector's elasticity parameter.

As before, we can explore the economy's responses for large differentials in the tax rates by simply computing the values in (9)-(11). Figures 3a-3d display $R/rm_1, Q/q, S_1/s_1$, and S_2/s_2 as functions of m_2/m_1. The factor shares $w = 0.4$ and $v = 0.2$ are used throughout, and the elasticity parameters take values of $0.5, 1.0$, and 2. Each figure has six curves, drawn for six different elasticity pairs (η, γ), as indicated. The results are qualitatively very similar to those in the previous section. The effects of a given tax policy m_2/m_1 on the steady-state interest rate ratio R/rm_1 and rental ratio Q/q are very insensitive to the elasticity parameters. The effect on the factor share ratio in each industry is very sensitive to that industry's own elasticity, however. Experiments with other parameter values indicate that these conclusions are robust.

Next consider the share parameters. Figure 4 shows the response of the interest rate for Cobb-Douglas technologies with different pairs of share parameters (w, v), where $w = .2$ and $.4$, and $v = .05, .2$, and $.3$, as indicated. The shares, unlike the elasticities, are quantitatively important in determining the interest rate.

The share of tax revenue in total income in the steady state is also quite insensitive to the elasticity parameters. Figure 5 plots the ratio of revenue to total income, including all income generated in the sector producing human capital, as a function of m_2, for $m_1 = .65$ and for share parameters $w = .40$ and $v = .20$. (The computational procedure is described in the Appendix.) The four curves are drawn

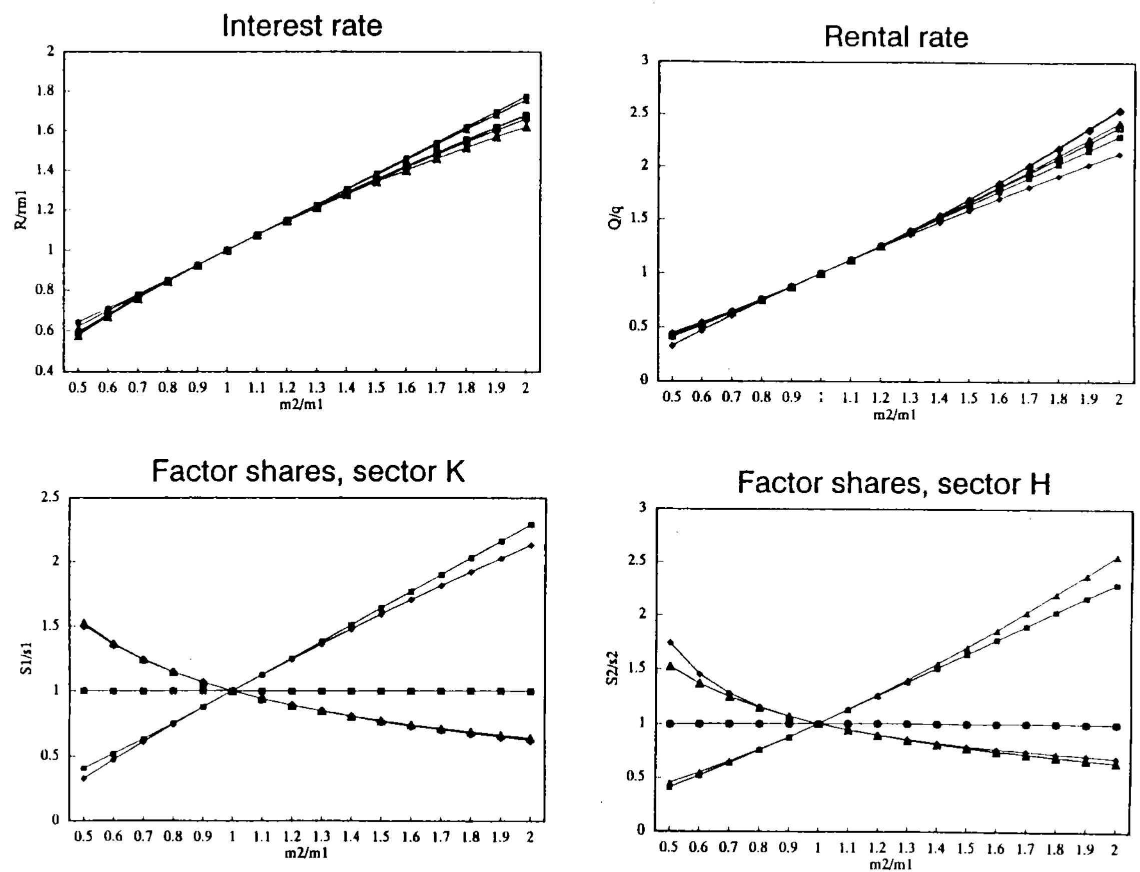


Figure 3

Factor shares (w,v) = (.40,.20), various elasticity pairs (eta, gamma) (2,2) (2,.5) (1,2) (1,1) (.5,1) (.5,.5)

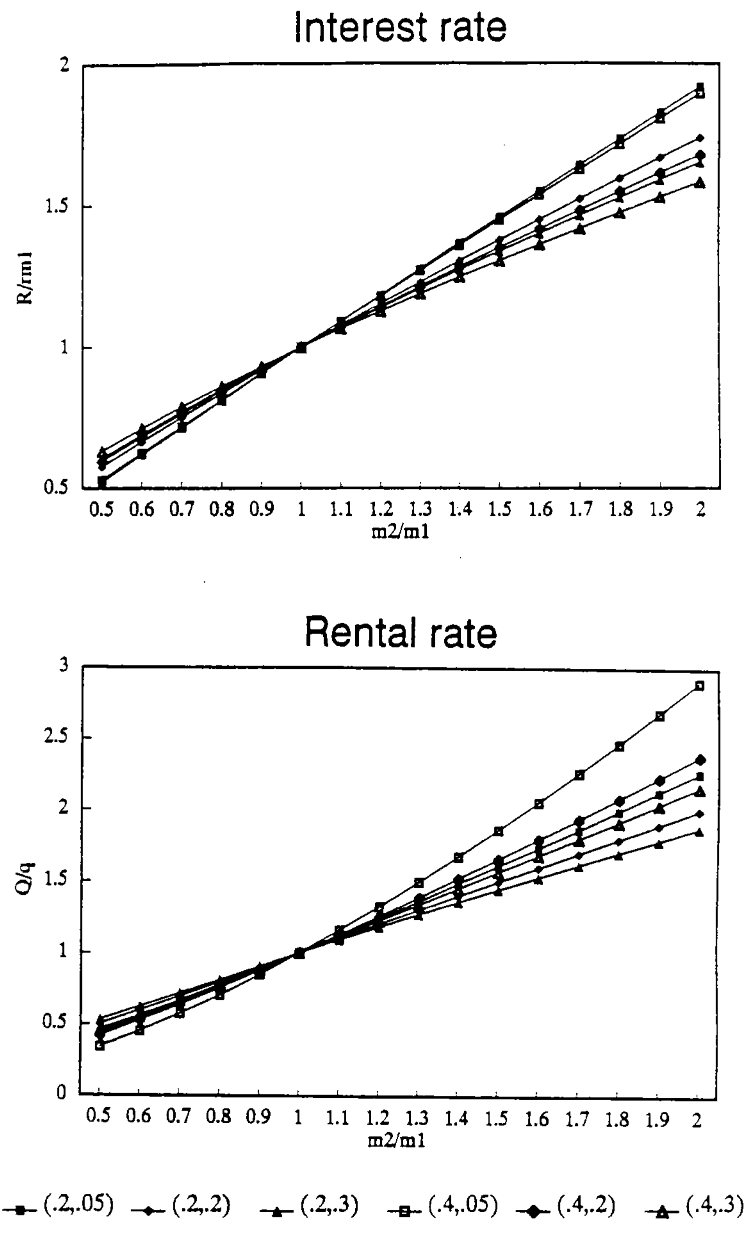


Figure 4

Cobb–Douglas technologies, various factor shares (w,v)

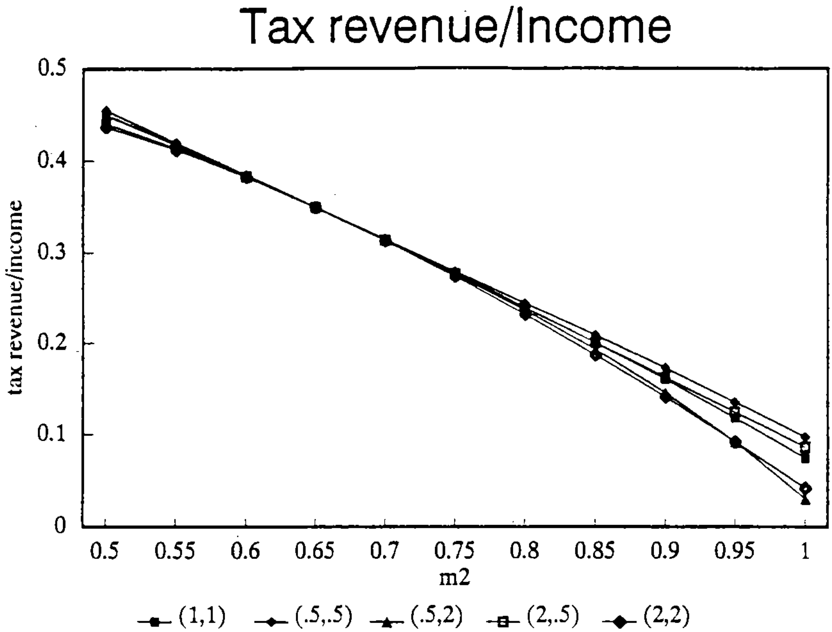


Figure 5

Factor shares (w,v) = (.40,.20), various elasticity pairs (eta, gamma) for different elasticity pairs (η, γ), where each parameter takes values of 0.5 or 2.0. The elasticity parameters have very little effect on the revenue ratio. In addition, the figure looks very similar if income is defined narrowly, excluding output in the sector producing human capital.

To a first order approximation, transitional dynamics in the neighborhood of the untaxed steady state are also independent of the elasticity parameters. To see this observe that, by construction, all members of a family of technologies defined by (7a) or (7b) have identical steady state marginal products. That is, for fixed (z_1, z_2, r, q), the derivatives $G_i(1, z_1; \alpha)$ and $H_i(1, z_2; \beta)$, $i = 1, 2$, are independent of α and β. Hence for any (k_i, h_i) with k_i/h_i near z_i, the level of output is also approximately independent of the elasticity parameter. For example, choose (k_1, h_1) such that $k_1/h_1 \approx z_1$, and let (k_1^s, h_1^s) be any nearby point satisfying $h_1^s/k_1^s = z_1$. Then

$$\begin{aligned} G(k_1, h_1; \alpha) &\approx G(k_1^s, h_1^s; \alpha) + (k_1 - k_1^s)G_1(1, z_1; \alpha) + (h_1 - h_1^s)G_2(1, z_1; \alpha) \\ &= k_1 G_1(1, z_1; \alpha) + h_1 G_2(1, z_1; \alpha), \end{aligned}$$

where the first line uses a first order Taylor series approximation and the second uses Euler's theorem for homogeneous functions. As we have already noted, the derivatives in the second line are independent of α. An analogous argument holds for the technology in the human capital sector.

5. Elastic labor supply

In this section we will briefly discuss two ways of incorporating elastic labor supply into the basic model. Throughout this section, we will let ℓ_4 denote the proportion of time devoted to leisure.

First, suppose that leisure time is quality adjusted in the same way work time is,[4] and suppose instantaneous preferences have the form

$$u(c, \ell_4 h) = \frac{V(c, \ell_4 h)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0,$$

where $V\colon \mathbf{R}_+ \times [0,1] \to \mathbf{R}_+$ is strictly increasing, strictly concave, continuously differentiable, and homogeneous of degree one. To incorporate this change into the household's decision problem, we must modify the objective function and put $(1-\ell_{4t})h_t$ in place of h_t in the budget constraint (1c). The new control variable ℓ_{4t} adds a new first order condition. In addition, we must modify the government's budget constraint and the resource constraint for human capital. As shown in the Appendix, in the steady state, these changes imply that (3j) is replaced by

$$\text{(3j}')\qquad \sum_{i=1}^{3} z_i\theta_i = (1-\ell_4)z,$$

and a new condition determining the steady state mix of consumption and leisure is added. In the case where taxes are levied by sector, gross of depreciation, the new condition is

$$\text{(3k}')\qquad \frac{V_2(c, \ell_4 z)}{V_1(c, \ell_4 z)} = (1-\tau_3)F_2(1, z_3).$$

[4] See Becker (1981, Ch. 1 and 2) for an interpretation of these preferences in terms of home production.

Equations (3a)-(3d) are unaltered, however, so the interest rate and growth rate are unaffected. A consumption tax now distorts the labor-leisure mix, but still has no effect on the steady-state growth rate.

Alternatively, suppose that utility depends on pure leisure time, unadjusted for the level of human capital, and suppose that instantaneous preferences have the form

$$u(c, \ell_4) = \frac{[cv(\ell_4)]^{1-\sigma}}{(1-\sigma)}, \quad \sigma > 0,$$

where $v: [0,1] \rightarrow \mathbf{R}_+$ is strictly increasing, strictly concave, and twice continuously differentiable, and where concavity of u requires that $-\sigma v''(\ell_4)v(\ell_4) > (1 - 2\sigma)[v'(\ell_4)]^2$. As before, the household's objective function and budget constraint must be modified, as well as the government's budget constraint and the resource constraint for human capital. Also as before, the new control variable adds a new first order condition.

It is shown in the Appendix that if income is taxed gross of depreciation and tax rates vary only by sector, then the steady state equations (3c) and (3j) are altered, and a new condition determining the labor-leisure tradeoff is added. The new conditions are

(3c′) $$(1-\tau_2)H_2(1, z_2)(1-\ell_4) - \delta_2 = r,$$

(3j′) $$\sum_{i=1}^{3} z_i\theta_i = (1-\ell_4)z,$$

(3k′) $$\frac{\hat{c}v'(\ell_4)}{v(\ell_4)} = (1-\tau_3)zF_2(1,z_2).$$

Notice that the system of equations is no longer block-recursive: the real interest rate and factor intensities can no longer be determined in isolation. For this reason it is more difficult to explore analytically the properties of the steady state. Nevertheless, it can be shown that a consumption tax still has no effect on the steady state growth rate, provided that the revenue is *not* rebated to households.

Moreover, an argument exactly analogous to the one in section 4 can be used to calculate the steady state effects of flat-rate income taxes on the interest rate as functions of the elasticity of labor supply. Simply substitute $(1-\ell_4)m_2$ for m_2 in (8b) and notice that the rest of the analysis is as before, except that ℓ_4 is unknown. In particular, the second line of (11b) holds with $(1-\ell_4)m_2$ replacing m_2. Then, differentiate as before to find that

$$\frac{d\ell n(R/r)}{d\ell n(m_1)}\Big|_{m_2/m_1=1} = \frac{1}{1+x}\left[x + \frac{d\ell n(1-\ell_4)}{d\ell n(m_1)}\right],$$

$$\frac{d\ell n(R/r)}{d\ell n(m_2)}\Big|_{m_2/m_1=1} = \frac{1}{1+x}\left[1 + \frac{d\ell n(1-\ell_4)}{d\ell n(m_2)}\right],$$

where $x = v/(1-w)$. Recall that v is the share of physical capital in the sector producing human capital, and $1-w$ is the share of human capital in the sector producing physical capital, so $v/(1-w)$ is the ratio of the alien factor shares. In Lucas's (1990) model $v = 0$, so $x = 0$. As the first expression shows, in this case a tax on the sector producing physical capital has steady state growth effects *only* if the supply of labor is elastic.

Notice that the importance of elastic labor supply in producing growth effects is greater the smaller are v and w. In the next section we will see that the large growth effects found by Jones, Manuelli, and Rossi (1993) result from low values for v and w, together with a large labor supply elasticity.

6. Quantitative Comparisons

In this section we will use the framework developed above to compare the results in Lucas (1990), King and Rebelo (1990), Kim (1992), and Jones, Manuelli and Rossi (1993) on the potential effect of tax reform on the long-run growth rate of the U.S. economy. As noted above, the quantitative conclusions in these papers differ dramatically. We will see below that these sharp differences in the conclusions arise from differences in the assumptions about the share parameter in the sector producing human capital, the depreciation rate for human capital, and the elasticity of labor supply.

Two features appear in some of the models we are comparing that do not fit into the framework used here. The first feature is a production function for human capital that displays diminishing point-in-time returns.[5] Technologies of this sort dampen the impact of changes in the rate of return on incentives to invest. The second feature is a utility function that has pure leisure (unadjusted for quality)

[5] See Heckman (1976) and Rosen (1976) for a further discussion.

as an argument. As we saw in the last section, elastic labor supply of this form magnifies the impact of changes in tax policy, since it provides a second avenue, in addition to adjustments to consumption, by which consumers can respond to changes in the rate of return on investment. We will see below that diminishing point-in-time returns do not seem to be quantitatively important in any of the analyses, but elastic labor supply does have a quantitatively significant effect in the Jones, Manuelli and Rossi model.

In this section we allow tax rates to vary by both factor and sector, and we assume that income is taxed gross of depreciation. First we simplify each of the models to conform to our setup and calibrate it using the author's parameter values. We then carry out the tax experiment(s) performed by him, to see whether altering the specification has changed the model's responses significantly. The details of these numerical computations are contained in the Appendix. Then, for each calibration, we perform the exercise of eliminating all income taxes.

Cobb-Douglas production functions are used throughout, so (3a)-(3d) take the form

(12a) $$r = \rho + \sigma g,$$

(12b) $$(1 - \tau_{11}) A w z_1^{1-w} - \delta_1 = r,$$

(12c) $$(1 - \tau_{22}) B (1 - v) z_2^{-v} - \delta_2 = r,$$

$$(12d) \qquad \frac{(1-\tau_{21})(1-w)}{(1-\tau_{11})wz_1} = \frac{(1-\tau_{22})(1-v)}{(1-\tau_{12})vz_2},$$

where τ_{ij} is the tax rate on factor i employed in sector j.

6.1 The Lucas model

First consider the model in section 4 of Lucas (1990). It includes diminishing point-in-time returns in human capital accumulation, elastic labor supply, and an elasticity of substitution of 0.60 (rather than unity) in the goods production technology. In addition, Lucas ignores depreciation, which can be interpreted as an assumption that the production functions are defined net of depreciation and returns are taxed net of depreciation. Lucas's baseline parameters are displayed in Table 1. The key assumptions are that human capital is produced using human capital only, and that the human capital sector is untaxed.

The experiment Lucas runs is cutting the capital tax to zero while raising the labor tax to 0.46. As noted above in section 2, in our version of Lucas's model, this change has no effect on the growth rate. The nonzero effect Lucas obtains comes from the fact that labor supply in his model is (very slightly) elastic. Similarly, in our version of Lucas's model, eliminating all taxes has no effect on the growth rate.

TABLE 1

	Lucas (1990)	
Additional features	- diminishing point-in-time returns to human capital accumulation - elastic labor supply - CES production function with 0.6 elasticity of substitution - taxes levied net of depreciation	
Baseline parameters	$\delta_k = 0, \ \delta_h = 0, \ w = .24, \ v = .00$ $\tau = (.26,.40,.00,.00), \ \sigma = 2, \ \rho = 0.0340$	
Benchmark indicators	$g = 0.0150, \quad r = 0.0640$	
Experiments	change in the growth rate	
	Lucas's model	our model
- reduce capital tax to 0% and raise labor tax to 46%	-0.0003	0.0000
- eliminate all taxes	n.a.	0.0000

6.2 The King and Rebelo model

Next consider the model in Section III of King and Rebelo (1990). Except for diminishing point-in-time returns in human capital accumulation, their model is identical to ours: labor supply is inelastic, technologies are Cobb-Douglas, and returns are taxed gross of depreciation. Their baseline parameters are displayed in Table 2.

King and Rebelo look at two experiments, increasing all tax rates by .10 and raising the tax rate on physical capital only by .10. Each experiment is performed for two sets of parameter values. In each case, our modified version of their model gives results very similar to the original. Thus, introducing diminishing point-in-time returns in the sector producing human capital does not make much difference quantitatively.

Eliminating all taxes in the modified model raises the growth rate by 0.0330 for King and Rebelo's baseline parameters, or when a lower value ($v = .05$ instead of .33) is used for capital's share in the sector producing human capital. If a lower depreciation rate for human capital is used, $\delta_h = 0.012$ instead of 0.100, eliminating all taxes raises the growth rate by only 0.0143.

6.3 The Kim model

Kim (1992) begins with a much more detailed description of the U.S. tax system, but is able to aggregate to obtain effective net tax rates on the two factors. After the aggregation, his model is identical to ours except for the presence of

TABLE 2

	King and Rebelo (1990)	
Additional features	- diminishing point-in-time returns to human capital accumulation	
Baseline parameters	$\delta_k = .1,\ \delta_h = .1,\ w = .33,\ v = .33$ $\tau = (.20,.20,.20,.20),\ \sigma = 1,\ \rho = 0.0120$	
Benchmark indicators	$g = 0.0200,\ \ r = 0.0320$	
Experiments	change in the growth rate	
	King-Rebelo model	our model
- increase all taxes by .10	-0.0152	-0.0167
- raise capital tax by .10	-0.0052	-0.0058
- raise capital tax by .10 with v = .05	-0.0011	-0.0012
- increase all taxes by .10 with δ_h=.012	-0.0067	-0.0071
- eliminate all taxes	n.a.	0.0330
- eliminate all taxes with v = .05	n.a.	0.0330
- eliminate all taxes with δ_h = .012	n.a.	0.0143

an inflation tax and the partial deductibility of depreciation on physical capital. His baseline parameters, which are displayed in Table 3, are similar to King and Rebelo's except that the depreciation rates are substantially lower and σ is substantially higher. Kim's experiment is to eliminate all taxes. This change raises the growth rate by 0.0085 in his model and by 0.0091 in our version of it.

6.4 The Jones, Manuelli, and Rossi model

Finally, consider Model II in Jones, Manuelli and Rossi (1993). They assume that labor supply is elastic, and their production function for human capital has human capital and market goods as inputs. This is important because of the tax treatment of various factors: human capital employed directly in the sector producing human capital is not taxed, but all factors employed in producing the market goods used by that sector are taxed. In other respects their model conforms with our set-up: the technologies are Cobb-Douglas and taxes are levied gross of depreciation. Modifying our structure to incorporate their tax structure is not hard, so we will do so.

Let the technology for the sector producing human capital be $J(I_{2t}, \ell_{2t} h_t) = \Phi I_{2t}^{\psi}(\ell_{2t} h_t)^{1-\psi}$ where I_{2t} is the input of market goods, ℓ_{2t} is the proportion of time allocated as direct labor input, and $\Phi > 0$. The constraints (1b) and (1c) for the consumer's problem become

$$\dot{h}_t = \Phi I_{2t}^{\psi}(\ell_{2t} h_t)^{1-\psi} - \delta_2 h_t, \qquad \text{(1b}'\text{)}$$

TABLE 3

	Kim (1992)	
Additional features	- inflation tax - partial deduction of depreciation on physical capital	
Baseline parameters	$\delta_k = .05$, $\delta_h = .01$, $w = v = .34$ $\sigma = 1.94$, $\rho = .01$ $\tau = (.34,.17,.34,.17)$	
Benchmark indicators	$g = 0.0150$, $r = 0.0391$	
Experiments	change in the growth rate	
	Kim's model	our model
- eliminate all taxes	0.0085	0.0091

(1c′) $$I_{1t} + I_{2t} + c_t - q_{1t}k_t - q_{2t}(1 - \ell_{2t} - \overline{\ell}_4)h_t - T_t \leq 0,$$

where $\overline{\ell}_4$ is the (fixed) proportion of time allocated to leisure and $(1 - \ell_2 - \overline{\ell}_4)$ the proportion allocated to goods production, and where prices are unity because all market goods are produced with the same technology. As shown in the Appendix, the steady state conditions are

(13a) $$r = \rho + \sigma g,$$

(13b) $$(1 - \tau_{11})Awz_1^{1-w} - \delta_1 = r,$$

(13c) $$(1 - \tau_{21})A(1 - w)z_1^{-w} = \frac{1 - \psi}{\psi\mu},$$

(13d) $$(1 - \overline{\ell}_4)(1 - \psi)\Phi\mu^{-\psi} - \delta_2 = r,$$

(13e) $$g + \delta_2 = \ell_2\Phi\mu^{-\psi},$$

where $\mu = \ell_2 h/I_2$ is the input ratio in the human capital sector. The baseline parameter values use by Jones, Manuelli, and Rossi are displayed in Table 4. Notice that $v = \psi w = .17$ is the (implicit) share of physical capital in the sector producing human capital.

The experiment Jones, Manuelli and Rossi run is to eliminate all taxes. They do this for a number of alternative values for the parameter σ (cf. their Table III). Comparisons with our version of their model are reported in Table 4 for $\sigma = 2$ and

TABLE 4

	Jones, Manuelli, and Rossi (1993)	
Additional features	- elastic labor supply - market goods used in producing human capital	
Baseline parameters	$\delta_k = .1$, $\delta_h = .1$, $w = .36$, $\psi = .48$ $\tau = (.21,.31,.00,.00)$, $\sigma = 1.5$, $\rho = 0.0200$ $1 - \bar{\ell}_4 = .29$, $\ell_2 = .12$	
Benchmark indicators	$g = 0.0200$, $r = 0.0500$	
Experiments	change in the growth rate	
	JMR model	our model
- eliminate all taxes	0.0350	0.0211
- eliminate all taxes with $\sigma = 1.1$	0.0830	0.0333

$\sigma = 1.1$. In both cases their growth effects are much larger than ours, increases of 0.0350 and 0.830 instead of with 0.0211 and 0.0333. The reason is the very high elasticity of labor supply in their model. For $\sigma = 2$, their model predicts that the total supply of labor to non-leisure activities $(1 - \ell_4)$ increases by 16% in the new steady state, and for $\sigma = 1$ it predicts that labor supply increases by an astounding 48%!

6.5 Comparing the experiments

The seven experiments reducing all tax rates to zero produce growth effects between 0.0 and 3.3 percentage points. As noted above, the low value is an immediate consequence when (untaxed) human capital is the only input in the production of human capital. The high value occurs when the elasticity of intertemporal substitution and the depreciation rates are high. Moreover, elastic labor supply can raise that figure even higher. Can anything be said about which range of values is most plausible?

U.S. experience provides what is almost a natural experiment in tax reform. Before the Sixteenth Amendment was approved in 1913, the U.S. Constitution severely restricted the ability of the federal government to levy taxes on income. Even after approval of the Amendment, income tax revenues were, until World War II, a negligible fraction of GNP. That fraction increased dramatically in the early 1940's, from 2% to 15% of GNP.

Figure 6 shows income tax revenue as a fraction of GNP and the growth rate of per capita real GNP for the period 1889 – 1989.[6] In line (1), revenue consists of revenue from federal, state, and local individual income taxes. In line (2) it includes, in addition, revenue from social security and retirement taxes and from the federal corporate profits tax.

While there are other aspects of government policy that changed after World War II, we would expect, on the basis of some of the models above, that such a dramatic increase in income taxation would generate a noticeable negative effect on the growth rate. It does not. The growth rate of per capita real GNP, while it displays substantial variation both before and after 1942, displays no clear break in its average value.

We performed three statistical tests, all of which confirm the visual impression that the average growth rate is the same before and after 1942. The first was a t-test for the difference in means (allowing for different variances). The average growth rate is 2.31 before 1942 and 1.22 after, and the t-value for the difference is 0.92. For the second, we estimated the mean rate of growth for the two periods by regressing the logarithm of per capita real GNP on a constant and a time trend, and performed a Chow test. The least squares growth rates are 1.37 and 1.61 in the two subperiods, and the $F_{1,97}$ -value is 3.57. The final test was non-parametric. The median growth rates are 2.10 and 2.11 in the two subperiods, and the p-values are

[6] The tax data is from the *Survey of Current Business*. The GNP and population data are from *One Hundred Years of Economic Statistics* (for 1889 – 1928) and *The Economic Report of the President* (for 1929 – 1991).

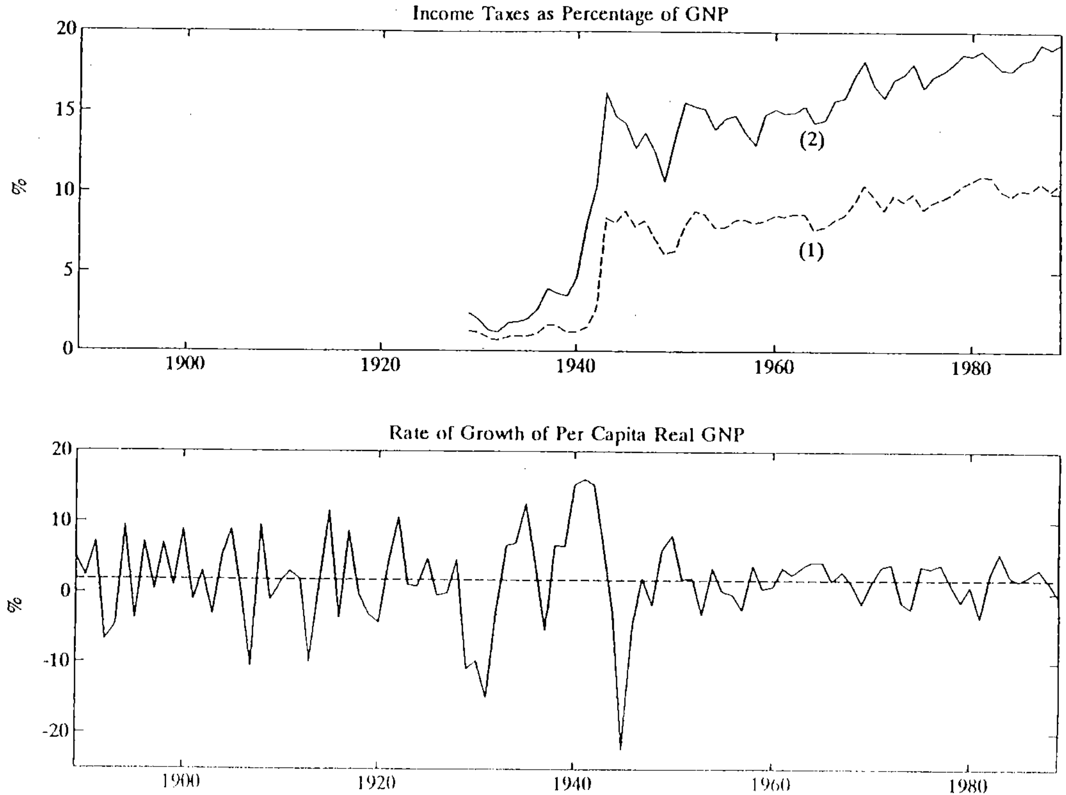

Income Taxes as Percentage of GNP
%
20
15
10
5
0
1900
1920
1940
1960
1980
(2)
(1)
Rate of Growth of Per Capita Real GNP
%
20
10
0
-10
-20
1900
1920
1940
1960
1980

Figure 6

0.38 and 0.46.[7] In each case, we could not reject, at the 5% level, the hypothesis that the average growth rate was the same in the two subperiods.

We view this evidence as suggesting that the growth effects implied by some of the calibrations above are implausibly large. Among the endogenous growth models, the U.S. experience over the last century seems to accord best with Lucas's calibration, or, more generally, with any in which the share of human capital in producing human capital is close to one. (It also accords well with the Solow model, in which growth is driven by exogenous technical change.) Several modifications of the other models substantially dampen the growth effects they produce, however.

First, the assumption that income is taxed gross of depreciation overstates the impact of tax reduction. To see this, suppose that all income is taxed at the same rate, so factor proportions in both industries are independent of the tax rate, and let r be the interest rate when there are no taxes. Then it follows from (4a)-(4c) and (4a′)-(4c′) that raising the tax rate by $\Delta\tau$ reduces the (after-tax) interest rate by $\Delta\tau(r + \delta)$ if income is taxed gross of depreciation, but by only $\Delta\tau r$ if income is taxed net of depreciation. If the interest and depreciation rates are about equal, the former effect is about twice as large as the latter. In the U.S., depreciation of physical capital is at least partly deductable, so assuming that income is taxed gross of depreciation exaggerates the potential effects of tax reform.

[7] The second test was motivated by Watson's (1992) Monte Carlo study, which shows that the least squares estimate of the growth rate is more robust than the geometric mean to differences in the serial correlation properties of the series. The third test is described in Gibbons (1985, pp. 131-140).

Second, a depreciation rate of 10% for physical capital is too high. Calculations based on the capital consumption allowance and estimates of the aggregate capital stock produce an average depreciation rate of a little over 6%.[8]

Third, a depreciation rate of 10% for human capital is probably also too high. Estimates of depreciation at the individual level range from 0.2% (Heckman 1975), to 1.2% (Mincer 1974), to 3-4% (Haley 1976). If working lifetimes are about 40 years, then, ignoring population growth, about 2.5% of the workforce retires each year. Retirees have more experience but less education than younger workers, and average wages peak well before retirement for most workers. If retirees embody 2.5% to 4.0% of the total stock of human capital, then summing individual depreciation and retirement effects gives a range of 2.7% to 8.0%. The latter figure involves

[8] The depreciation rate is equal to the ratio (capital consumption allowance /output) /(capital/output). From the 1991 *Economic Report of the President* we find that for 1989, $CCA/Y = 554/5201 = 0.107$. Christiano (1988, pp. 260-262) calculates the capital-output ratio to be 2.65, of which 10% is is consumer durables, 33% producer structures and equipment, 33% government and private residential, and 24% government non-residential. Since the capital consumption allowance figures exclude consumer durables and government, the relevant capital-output ratio for our purposes is $K/Y = 2.7 * [1 - .10 - (.025 * .33) - .24] = 1.73$, where we have used the information in Young and Musgrave (1980, Tables 1.A.2 and 1.A.6) to estimate the proportion of residential capital owned by the government. Combining these two figures, we obtain a depreciation rate of $0.107/1.73 = 0.062$. This figure agrees well with calculations for the period 1950–1975 based on the capital stock estimates in Young and Musgrave. Using their figures (Table 1.A.2) for the capital stock (excluding government capital, consumer durables, and inventories), and figures from the *Economic Report of the President* (1991, Tables B-1, B-3, and B-16) for output and the capital consumption allowance, and making use of the formula $\delta = (CCA/Y)/(K/Y)$, we obtain depreciation rates for 1950–1975 of $\delta_{50} = .0885/1.44 = 0.613$, $\delta_{55} = .0887/1.44 = .0620$, $\delta_{60} = .0939/1.52 = .620$, $\delta_{65} = .0880/1.46 = .0600$, $\delta_{70} = .0951/1.52 = .0620$, and $\delta_{75} = .1065/1.61 = .0660$.

substantial double counting, however: if individual depreciation rates are high, then retirees account for a correspondingly smaller proportion of the total stock of human capital.

Fourth, there is little evidence of the very strong labor supply effects postulated by Jones, Manuelli and Rossi. Average weekly hours per employed person have fallen very steadily and dramatically over the last century, from 53.5 hours in 1889 to 34.6 hours in 1989. Labor force participation has risen over the same period, with the proportion of the population that is employed growing from 35.0% in 1889 to 47.4% in 1989.[9] The trend in hours slightly outweighs the trend in employment, with weekly hours per head of population falling from 18.7 to 16.4.

If the King and Rebelo model is recalibrated with $\delta_k = \delta_k = .06$, eliminating all taxes raises the long run growth rate by 2.5 percentage points rather than 3.3. If depreciation on physical capital is taken to be fully deductible, the growth effect is further reduced, to 1.8 percentage points. And as Tables 2 and 3 show, even smaller growth effects are obtained if a smaller value is used for the depreciation rate on human capital, δ_h, or for the elasticity of intertemporal substitution, $1/\sigma$.

[9] The ratio employment/population (in millions) is $21.6/61.8 = .350$ for 1889 and $117.3/247.3 = .474$ for 1989. For the earlier year, the figure for population is from Maddison (1982, Table B2), and those for employment and average weekly hours are from Kendrick (1961, Tables A-VII and A-IX). For the later year, all the figures are from the 1993 *Economic Report of the President* (Tables B-29, B-30, and B-42).

7. Conclusions

The U.S. economy over the last century conforms very well to the description of a balanced growth path, with stable values for the capital-output ratio, capital's share in income, leisure's share in total time, and the interest rate. Hence those data can have very little information about some of the elasticities that are relevant here. One goal of the present paper has been to study how sensitive quantitative conclusions about growth effects are to these badly estimated parameters.

On the technology side, we found that the elasticities of substitution in production are not critical for growth or revenue effects. Thus, assuming Cobb-Douglas production functions in all sectors is harmless: within a wide range, conclusions about growth and revenue effects are very insensitive to this assumption. (Elasticities may be important for welfare conclusions, however, since they are critical in determining the size of the distortion in input ratios resulting from asymmetric taxation of factor incomes. For example, see Lucas (1990) and Davies and Whalley (1991).)

By contrast, share parameters are quite important for growth effects. Excellent information about factor shares in the goods-producing sector is readily available, but a better estimate of capital's share in the sector producing human capital – a parameter about which information should be available – would be very useful.

On the preference side, two elasticities are important. Differences in estimates of the elasticity of intertemporal substitution can easily account for differences by a factor of two or three in estimates of growth effects. Similarly, differences in estimates of the long-run elasticity of labor supply can account for differences of several percentage points in beliefs about interest rate effects. For the reasons discussed above, it seems unlikely that aggregate U.S. time series can be used to improve our estimates of either parameter. Other data sources, cross-section or cross-country, are needed.

The depreciation rates for both types of capital, the tax treatment of depreciation, and the tax treatment of inputs in the sector producing human capital are also critical for determining growth effects. Among these, the depreciation rate for human capital is the most problematic. Although depreciation rates for individual human capital have been estimated from age-earnings profiles, those estimates are inappropriate in the current context, where the largest source of depreciation comes from the fact that lifetimes are finite. An overlapping generations model would allow a more satisfactory treatment of this issue, but at the cost of raising a new and equally difficult problem: how human capital is transmitted from one generation to the next.

We should emphasize that even if the growth effects of tax reform are small, the welfare effects may be large, as shown in Lucas (1990) and Davies and Whalley (1991). In particular, capital taxation can lead to a fairly large bias in the composition of the capital stock between its physical and human components, a bias that can have serious welfare consequences. The analysis here suggests that studying these consequences in models in which growth is exogenous (or absent) may be harmless. There is, as yet, no theoretical presumption or empirical evidence of substantial growth effects from factor taxation.

APPENDIX

A.1 Balanced growth conditions

First we derive (3a)-(3j). Let λ_1, λ_2 and ν be the multipliers associated with the constraints (1a)-(1c) in the household's problem. Then the conditions for a maximum are

$$c_t^{-\sigma} = \nu_t, \tag{A1}$$

$$\lambda_{it} = \nu_t p_{it}, \qquad i = 1, 2, \tag{A2}$$

$$\dot{\lambda}_{it} = (\rho + \delta_i)\lambda_{it} - \nu_t q_{it}, \qquad i = 1, 2, \tag{A3}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{1t}k_t = \lim_{t\to\infty} e^{-\rho t}\lambda_{2t}h_t = 0, \tag{A4}$$

and (1a)-(1c). Use (2) and (A2) to substitute into (A3) for q_{it} and ν_t to get

$$\dot{\lambda}_{1t}/\lambda_{1t} = \rho + (1 - \omega_1\tau_{11})\delta_1 - (1 - \tau_{11})G_1(1, z_{1t}), \tag{A5}$$

$$\dot{\lambda}_{2t}/\lambda_{2t} = \rho + (1 - \omega_2\tau_{22})\delta_2 - (1 - \tau_{22})H_2(1, z_{2t}). \tag{A6}$$

Budget balance for the government implies that

$$\begin{aligned} T_t = {} & \left[\tau_{11}\theta_{1t}p_{1t}G_1 + \tau_{12}\theta_{2t}p_{2t}H_1 + \tau_{13}\theta_{3t}F_1\right] k_t \\ & + \left[\tau_{21}\ell_{1t}p_{1t}G_2 + \tau_{22}\ell_{2t}p_{2t}H_2 + \tau_{23}\ell_{3t}F_2\right] h_t \\ & - \omega_1\delta_1 p_{1t}\left[\tau_{11}\theta_{1t} + \tau_{12}\theta_{2t} + \tau_{13}\theta_{3t}\right] k_t \\ & - \omega_2\delta_2 p_{2t}\left[\tau_{21}\ell_{1t} + \tau_{22}\ell_{2t} + \tau_{23}\ell_{3t}\right] h_t. \end{aligned}$$

Let $\hat{c} = c_t/k_t$ be the ratio of consumption to the capital stock, let $z_t = h_t/k_t$ be the economy-wide input ratio, and let $z_{it} = \ell_{it}h_t/\theta_{it}k_t$ be the input ratio in sector i. Clearing in the output markets implies

$$I_{1t} = \theta_{1t}k_tG(1, z_{1t}),$$

$$I_{2t} = \theta_{2t}k_tH(1, z_{2t}),$$

$$c_t = \theta_{3t}k_tF(1, z_{3t}),$$

and clearing in the factor markets implies (3i) and (3j).

Along the balanced growth path, consumption and both kinds of capital grow at a common, constant rate g,

$$\text{(A7)} \qquad \dot{c}_t/c_t = \dot{k}_t/k_t = \dot{h}_t/h_t = g.$$

Then, since factor prices are constant along the balanced growth path, it follows (A1) and (A2) that

$$\text{(A8)} \qquad \dot{\nu}_t/\nu_t = \dot{\lambda}_{1t}/\lambda_{1t} = \dot{\lambda}_{2t}/\lambda_{2t} = -\sigma g.$$

Therefore, the transversality condition (A4) holds if $\rho > (1 - \sigma)g$, exactly the condition needed to ensure that total utility is bounded if consumption grows at the rate g. It will be assumed throughout that this is the case.

To describe the balanced growth path, use the market-clearing conditions to eliminate I_1 and I_2 from (1a) and (1b); use Walras' law to drop (1c); and substitute from the steady-state conditions (A7) and (A8) into the laws of motion (1a), (1b), (A5), and (A6). If income is taxed gross of depreciation, then $\omega_i = 0, i = 1, 2$, so (3d) and (3e) follow from (2).

If income is taxed net of depreciation but tax rates vary only by factor, then $\omega_1 = \omega_2 = 1$, and $\tau_{i1} = \tau_{i2} = \tau_{i3} = \tau_i, i = 1, 2$. In this case (4a′) and (4b′) follow from (A7) and (A8), and (4c′) follows from (2).

A.2 The family of CES production functions

Next we will show how the families of CES functions used in sections 3 and 4 are constructed. Any CES production function can be written as

$$G(k,h) = \begin{cases} A[\theta k^\alpha + (1-\theta)h^\alpha]^{1/\alpha}, & \alpha \neq 0, \\ Ak^\theta h^{1-\theta}, & \alpha = 0, \end{cases}$$

where $0 < A, 0 \leq \theta \leq 1$, and $\alpha \leq 1$. Fix z and r. For each $\alpha \leq 1$, we want to define a CES function, call it $G(\cdot, \cdot; \alpha)$, with elasticity parameter α and with scale and weight parameters A and θ (that depend on α, z, and r) such that (4a) and (4b) hold for the input ratio z and interest rate r. That is, we require that

$$\frac{G_1(1, z; \alpha)}{G_2(1, z; \alpha)} = \frac{z^{1-\alpha}\theta}{1-\theta} = 1,$$

$$G_1(1, z; \alpha) = \theta A[\theta + (1-\theta)z^\alpha]^{(1-\alpha)/\alpha} = r.$$

Solving these two equations for A and θ as functions of α, z, and r, using them to write $G(\cdot, \cdot; \alpha)$, and rearranging terms, we obtain (5). The method used in section 4 is the same.

A.3 Tax revenue

To compute the ratio of tax revenue to output, assume that tax rates vary only by factor, and let τ_1 and τ_2 be the tax rates on income from physical and human capital. Using (2) we find that

$$\frac{\text{Tax revenue}}{\text{Income}} = \frac{\tau_1 k_t p_1 G_1 + \tau_2 h_t p_1 G_2}{k_t p_1 G_1 + h_t p_1 G_2}$$

$$= \frac{\tau_1 (G_1/G_2) + \tau_2 Z}{(G_1/G_2) + Z}$$

$$= \frac{\tau_1 Q + \tau_2 Z}{Q + Z},$$

where $Z \equiv h_t/k_t$, and where the last line uses (8a). To express Z in terms of Q, first use (3a) and (3g) to find that

$$\theta_2 = \frac{Z(R-\rho)}{H(1, z_2)\sigma}.$$

Assume that consumption goods and physical capital are produced with the same technology, $F = G$. Then $Z_1 = Z_3$, so (3j) implies that

$$Z = Z_1 + (Z_2 - Z_1)\theta_2.$$

Substituting for θ_2 we obtain

$$Z = \frac{Z_1 H(1, Z_2)}{H(1, Z_2) - (Z_2 - Z_1)[(R-\rho)/\sigma]}.$$

Then, substituting from (9) and (10b) or (11b), we can write Z_1, Z_2, and R in terms of Q/q. The latter can be computed by using (10a) or (11a).

A.4 Elastic labor supply

Next consider elastic labor supply. If leisure time is quality adjusted, the first order condition (A1) for the household's problem is replaced by a pair of conditions and the law of motion (A3) for λ_2 is slightly altered. The new conditions are

$$V(c_t, \ell_{4t}h_t)^{-\sigma} V_1(c_t, \ell_{4t}h_t) = \nu_t,$$

$$V(c_t, \ell_{4t}h_t)^{-\sigma} V_2(c_t, \ell_{4t}h_t) = \nu_t q_{2t}$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - V^{-\sigma}(c_t, \ell_{4t}h_t) V_2(c_t, \ell_{4t}h_t)\ell_{4t} - \nu_t q_{it}(1 - \ell_{4t}).$$

Combining the first two equations to eliminate ν_t, substituting from (2) for q_{2t}, and normalizing by k_t gives (3k′). Substituting from the second into the third reproduces (A3).

If leisure time is not quality adjusted, then the new conditions for the household's problem are

$$v(\ell_{4t})^{1-\sigma} c_t^{-\sigma} = \nu_t,$$

$$c_t^{1-\sigma} v(\ell_{4t})^{-\sigma} v'(\ell_4 t) = \nu_t q_{2t} h_t,$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - \nu_t q_{it}(1 - \ell_4 t).$$

Combining the first two to eliminate ν_t, substituting from (2) for q_{2t}, and normalizing by k_t gives (3k″). Substituting from (A2), (A8), and (2) into the third gives (3c″).

A.5 The Jones, Manuelli, and Rossi model

To derive our version of the Jones, Manuelli, and Rossi model, replace the constraints (1b) and (1c) in the household's problem with

$$\dot{h}_t = J(I_{2t}, \ell_{2t}h_t) - \delta_2 h_t,$$

$$I_{1t} + I_{2t} + c_t - q_{1t}k_t - q_{2t}h_t(1 - \ell_{2t} - \bar{\ell}_4) - T_t \leq 0,$$

where $\bar{\ell}_4$ is the fixed proportion of time devoted to leisure. The conditions for a maximum are (A1) and

$$\lambda_{1t} = \nu_t,$$

$$\lambda_{2t}J_1(I_{2t}, \ell_{2t}h_t) = \nu_t,$$

$$\lambda_{2t}J_2(I_{2t}, \ell_{2t}h_t) = \nu_t q_{2t},$$

$$\dot{\lambda}_{1t} = (\rho + \delta_1)\lambda_{1t} - \nu_t q_{1t},$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - \lambda_{2t}\ell_{2t}J_2(I_{2t}, \ell_{2t}h_t) - \nu_t q_{2t}(1 - \ell_{2t} - \bar{\ell}_4).$$

Eliminating ν_t, using (A7) and (A8), and substituting from (2), we find that the conditions for a steady state include

$$g + \delta_2 = \ell_2 J(1/\mu, 1),$$

$$J_2(1, \mu)/J_1(1, \mu) = (1 - \tau_{21})G_2(1, z_1),$$

$$-\sigma g = \rho + \delta_1 - (1 - \tau_{11})G_1(1, z_1),$$

$$-\sigma g = \rho + \delta_2 - \ell_2 J_2(1, \mu) - J_1(1, \mu)(1 - \tau_{21})G_2(1, z_1)(1 - \ell_2 - \bar{\ell}_4),$$

$$r = \rho + \sigma g.$$

Substituting for G and J and their derivatives gives (13a)-(13e).

A.6 Calibration

Tables A1-A3 show our calibrations of the King-Rebelo, Kim, and Jones-Manuelli-Rossi models. Each table displays the baseline calibration values, the computed values for all endogenous variables in response to tax experiments, changes in the baseline values when the model is recalibrated, and the results of tax experiments for each new calibration. For Lucas's model, the interest rate is computed by taking – for Lucas's values – capital's share, adjusted for the capital tax, divided by the capital/output ratio: $r = (.24)(1 - .36)/2.4$.

Table A1

King and Rebelo (1990)		
	Calibrated	Uncalibrated
Exogenous	$\delta_k = \delta_h = .1$, $w = v = .33$ $\sigma = 1$, $A = 1$ $\tau = (.20,.20,.20,.20)$	$\rho = .0120$, $B = .1750$
Endogenous	$r = .0320$, $g = .0200$	$z_1 = z_2 = .3554$
Tax experiment	values for endogenous variables	
$\tau = (.30,.30,.30,.30)$ $\tau = (.30,.20,.30,.20)$ $\tau = (.00,.00,.00,.00)$	$g = .0035$, $r = .0155$, $z_1 = z_2 = .3554$ $g = .0142$, $r = .0262$, $z_1 = z_2 = .4056$ $g = .0530$, $r = .0650$, $z_1 = z_2 = .3554$	
Recalibration $v = .05$ $\tau = (.30,.20,.30,.20)$ $\tau = (.00,.00,.00,.00)$	$\rho = .0120$, $B = .1846$, $z_1 = .3554$, $z_2 = 3.376$ $g = .0188$, $r = .0308$, $z_1 = .4276$, $z_2 = 4.062$ $g = .0530$, $r = .0650$, $z_1 = .3554$, $z_2 = 3.376$	
Recalibration $\delta_h = .012$ $\tau = (.30,.30,.30,.30)$ $\tau = (.00,.00,.00,.00)$	$\rho = .0120$, $B = .05835$, $z_1 = z_2 = .3554$ $g = .0129$, $r = .0249$, $z_1 = z_2 = .3990$ $g = .0343$, $r = .0463$, $z_1 = z_2 = .2971$	

Table A2

Kim (1992)		
	Calibrated	Uncalibrated
Exogenous	$\delta_k = .05$, $\delta_h = .01$, $w = v = .34$, $A = 1$ $\sigma = 1.94$, $\rho = .010$ $\tau = (.34,.17,.34,.17)$	$B = .05569$
Endogenous	$g = .0150$	$z_1 = z_2 = .2467$ $r = .0391$
Tax experiment	values for endogenous variables	
$\tau = (.00,.00,.00,.00)$	$g = .0241$, $r = .0567$, $z_1 = z_2 = .1729$	

Table A3

Jones, Manuelli, and Rossi (1993)		
	Calibrated	Uncalibrated
Exogenous	$\delta_k = \delta_h = .1$, $w = .36$ $1 - \bar{\ell}_4 = .29$, $\ell_2 = .12$ $\Phi = 1.0$, $\sigma = 1.5$, $\rho = .0200$ $\tau = (.21,.31,.00,.00)$	$A = 1.401$ $\psi = .4828$ $(v = w\psi = .1738)$
Endogenous	$g = .0200$	$r = .0500$, $\mu = 1.00$ $z_1 = .2174$
Tax experiment	values for endogenous variables	
$\tau = (.0,.0,.0,.0)$	$g = .0411$, $r = .0816$, $\ell_2 = .117$ $\mu = .6728$, $z_1 = .2070$	
Recalibration $\sigma = 1.1$	$A = 1.280$, $\psi = .5103$, $(v = \psi w = .1837)$ $\mu = 1.000$, $r = .0420$, $z_1 = .2298$	
$\tau = (.0,.0,.0,.0)$	$g = .0533$, $r = .0733$, $\ell_2 = .1255$ $\mu = .6758$, $z_1 = .2171$	

REFERENCES

Becker, Gary S. *A Treatise on the Family.* Harvard University Press. Cambridge, MA: 1981.

Bond, Eric W., Ping Wang, and Chong K. Yip. "A General Two Sector Model of Endogenous Growth with Human and Physical Capital." Working paper, Pennsylvania State University, 1992.

Caballé, Jordi, and Manuel S. Santos. "On Endogenous Growth with Physical and Human Capital." Working paper, Universitat Autonoma de Barcelona, 1991.

Christiano, Lawrence J. "Why Does Inventory Investment Fluctuate So Much?" *Journal of Monetary Economics* 21 (March 1988): 247-280.

Council of Economic Advisors. *Economic Report of the President.* United States Government Printing Office. Washington, DC: 1991 and 1993.

Davies, James, and John Whalley. "Taxes and Capital Formation: How Important is Human Capital?" In B. Douglas Bernheim and John B. Shoven, eds., *National Saving and Economic Performance*, University of Chicago Press. Chicago, IL: 1991, pp. 163-197.

The Economist. *One Hundred Years of Economic Statistics.*

Faig, Miquel. "A Simple Economy with Human Capital: Transitional Dynamics, Technology Shocks, and Fiscal Policies." Working paper no. 9121, Department of Economics, University of Toronto, 1991.

Gibbons, Jean D. *Non-Parametric Statistical Inference.* Dekker. New York, NY: 1985.

Haley, William J. "Estimation of the Earnings Profile from Optimal Human Capital Accumulation," *Econometrica* 44 (November 1976): 1223-38.

Heckman, James J. "A Life-Cycle Model of Earnings, Learning, and Consumption." *Journal of Political Economy* 84 (August 1976): S11-S44.

Jones, Larry E., and Rodolfo E. Manuelli. "A Convex Model of Equilibrium Growth: Theory and Policy Implications." *Journal of Political Economy* 98 (October 1990): 1008-1038.

Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi. "Optimal Taxation in Models of Endogenous Growth." *Journal of Political Economy* 101 (forthcoming, 1993).

Kendrick, John W. *Productivity Trends in the United States.* Princeton University Press. Princeton, NJ: 1961.

Kim, Se-Jik. "Taxes, Growth and Welfare in an Endgogenous Growth Model." Ph.D. dissertation, University of Chicago, 1992.

King, Robert G., and Sergio Rebelo. "Public Policy and Economic Growth: Developing Neoclassical Implications." *Journal of Political Economy* 98 (October 1990): S126-S150.

Lucas, Robert E., Jr. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22 (July 1988): 3-42.

Lucas, Robert E., Jr. "Supply-Side Economics: An Analytical Review." *Oxford Economic Papers* 42 (1990): 293-316.

Maddison, Angus. *Phases of Capitalist Development.* Oxford University Press. Oxford: 1982.

Mincer, Jacob. *Schooling, Experience and Earnings.* Columbia University Press. New York, NY: 1974.

Mulligan, Casey B., and Xavier Sala-i-Martin. "Transitional Dynamics in Two-Sector Models of Endogenous Growth." *Quarterly Journal of Economics* 108 (forthcoming, 1993).

Rebelo, Sergio. "Long Run Policy Analysis and Long Run Growth." *Journal of Political Economy* 99 (June 1991): 500-521.

Rosen, Sherwin. "A Theory of Life Earnings." *Journal of Political Economy* 84 (August 1976): S45-S67.

U.S. Department of Commerce. *Survey of Current Business.*

Watson, Mark. "A Note on Estimating Deterministic Trends in the Presence of Serially Correlated Errors." Working paper, Department of Economics, Northwestern University, 1992.

Yuen, Chi-Wa. "Taxation, Human Capital Accumulation and Economic Growth." Ph.D. dissertation, University of Chicago, 1991.

Young, Allan H. and John C. Musgrave. "Estimation of Capital Stock in the United States." In Dan Usher, ed., *The Measurement of Capital.* University of Chicago Press. Chicago, IL: 1980, pp. 23-58.