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ECONOMIC GROWTH AND DECLINE WITH ENDOGENOUS PROPERTY RIGHTS

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ABSTRACT

This paper introduces endogenous property rights into a neoclassical growth model. It identifies a mechanism that generates growth rates which are increasing at low levels of capital, and decreasing at high levels of capital. The driving force behind changes in property rights is the attempt of each rent-seeking group to secure exclusive access to a greater share of capital by excluding others. We characterize an equilibrium in which there is a shift from common to private property, followed by a switch back to common property.

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I. Introduction

Postwar data indicate that among rich countries growth rates decline with the level of income per capita. However, this fails to hold among poor countries, some of which remain in low-growth traps. If anything, a comparison between African and Southeast Asian countries suggests that at low levels of income per capita the growth rate is increasing in the level of income.[1] With respect to long run trends, an extensive literature, dating back at least to Vico (1723), analyzes the rise and decline that leading countries have experienced, relative to other countries. For instance the Roman Empire in the fifth century, the Dutch Republic in the seventeenth century and Britain in the nineteen century[2].

To explain differences in growth rates, the economic growth literature has focused on the production side.[3] Another approach, which is complementary, is to consider institutional factors, such as property rights. It is often claimed that over the long-run economies with ill defined property rights tend to have low growth rates. This is the case of poor economies which have a high physical return to capital but which lack a system to impartially enforce contracts, or of economies which possess such a system, but suffer from an overwhelming level of redistributional activities on the part of interest groups. These rent-seeking activities may be reflected in high government expenditures and high inflation as in some Latinamerican countries, or in the ungovernability observed in Britain when the Heath government fell after the miners' strike. Similarly, one can interpret the coefficients exhibited by African and Latin American dummies, in cross-country growth regressions, as a proxy for the absence of well defined or well enforced property rights (e.g., Barro (1991)).

Tracing back differences in growth rates to differences in tax rates or security of property

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[1]See Azariadis and Drazen (1990), Barro (1991), Barro and Sala-i-Martin (1992), Baumol (1986).

[2]For example: Kennedy (1987), Landes (1969), Maddison (1982), North (1981), Olson (1982) and Spengler (1932).

[3]See Lucas (1988), Romer (1986) and Solow (1957).

is like replacing exogenous technical change with exogenous institutional change. Arguments like the ones above do not explain why property rights are different across countries, nor why they change over time. In this paper we present a model of endogenous property rights, in which the same forces that induce the adoption of private property rights are the same forces that cause their subsequent erosion. As a result the growth rate increases at low levels of capital, and decreases at high levels of capital. The driving force behind the evolution of property rights is the attempt of each rent-seeking group to secure for itself a greater share of aggregate wealth at the expense of others.

The model's layout is like that of the "Ak" growth model (Rebelo 1991). It contains linear production technology and time-additive preferences. The only difference is that we replace the representative agent by two rent-seeking groups that interact strategically. Each of these groups can at any time displace the other group from its access to the capital stock by incurring a fixed loss. The idea is that when an economy is poor there are no institutions to protect private property rights, and the growth rate is low. There is a shift to Private Property when the economy becomes rich enough so that it is worthwhile to incur the costs of creating institutions to defend private profits. Lastly, as the economy becomes very rich, rent-seeking becomes profitable, leading interest groups to erode these institutions, and the economy comes full circle back to Common Property.

To model this idea we consider three types of property rights regimes: private, common and leader-follower. Under "Private Property" each group has access only to its own capital stock. Under "Common Property" both groups have access to the aggregate capital stock. In the "leader-follower" regime only one group (the leader) has access to the entire capital stock, while the other group (the follower) has access to none.

We assume that initially Common Property prevails and that at any moment any group can become the leader by incurring a one-time loss (think of the cost of building a wall around an estate). If the second group does not match this move, a leader-follower regime sets in. If it does match it, Private Property emerges. However, once Private Property is established, any group can at any time become the leader by once again incurring a one-time loss (think of the cost of destroying the other's wall). If this act is matched by the competing group, there is a shift to Common Property (there are no walls left). One can interpret the first loss as the cost of instituting a system to enforce contracts. In the leader-follower case this system favors one group, while under Private Property it is impartial. The second loss can be interpreted as the cost of creating a rent-seeking organization.

We model groups' interaction as a preemption game. Each group must choose its consumption path and two capital levels at which to switch, basing its decisions on the strategy of the other group. We construct a Markov perfect equilibrium for this game, and compute in closed-form the path of the capital stock. We show that if the marginal product of capital and the elasticity of intertemporal substitution in consumption are sufficiently high, then one can find two capital stock levels: k^{**} and k^{*} for which the following holds:

i. There is a switch from Common to Private Property when k reaches k^{**}.

ii. The switch to Private Property is followed by a reversion to Common Property when k reaches k^{*}.

iii. For $k < k^{**}$ the growth rate is increasing. At k^{**} it jumps up, following a decreasing path thereafter until it converges to a constant as k reaches k^{*}, as shown in (33) and Figure 3.

The key for (i) and (ii) is that under both regimes as k goes up: (a) the payoff to leading grows at a faster rate than the payoff to waiting, and (b) the loss that the leader has to incur is high relative to the initial stock of capital. Thus, initially no one will find it profitable to become the leader. However as k increases there is a point where it becomes profitable for a group to become the leader. Knowing this, the other group will match this action provided the payoff to matching is greater than the follower's payoff. This occurs if the loss of a joint switch is sufficiently smaller than the loss a leader has to incur.

Note that both groups end worse off than if they had not switched from Private to Common Property. However, the switch must occur not because of a lack of coordination, but because of the fact that leading becomes more profitable than waiting.

Since production technology is linear, result (iii) reflects only switches in property rights generated by interest group competition. Although this result is consistent with the stylized facts mentioned earlier, we should note that the actual growth rate of any economy will also reflect very important factors such as those identified in the endogenous growth literature: human capital accumulation, production externalities and technological innovation.

During the first phase, even though Common Property prevails, the growth rate is increasing because a switch to Private Property is forthcoming. As this switch gets closer, groups reduce their level of appropriation of common resources because they discount less the fact that under Private Property each group will have access to only half of the capital stock, and the private rate of return will increase. During the Private Property regime, decreasing growth is caused by the expectation of a switch to Common Property, which induces groups to behave as if in the future they will receive a lottery and the interest rate will fall (each group will attain access to the other's capital stock and the private rate of return will be lower). Therefore, as the switch becomes closer, the appropriation rate increases and the growth rate declines gradually. This result provides support to the observation that redistributive activity increases as an economy becomes richer, with effects such as increasing resistance to innovations, deteriorating labor relations, and higher fiscal deficits.

This model should not be considered as a full description of the real world. It leaves out aspects such as wars, famines and politics.[4] Yet, it introduces endogenous institutional change

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[4]In addition to stories of "decline from within," there are other "barbarians at the gate" theories. In the case of Rome , an increase in the military power of the Barbarians. In the Dutch case the destruction of their commercial supremacy in wars with the British and the French. In the case of the Italian cities, the conquest of Constantinople by the Turks and the discovery of America.

into a neoclassical framework and identifies a mechanism to explain differing growth rates among countries with identical preferences and technology. This mechanism may act parallel to the ones identified by the recent endogenous growth literature.

In section II we present a brief description of the literature. In sections III.1-III.3 we present the model, in section III.4 we summarize the results, and in section IV we present the conclusion.

II. Related Literature

This paper is related to Benhabib and Rustichini (1992), Benhabib and Radner (1992), and Benhabib and Ferri (1987). The first paper considers a common access economy and looks at trigger-strategies in discrete time. It analyzes equilibria where enforceability of cooperation (low appropriation) is possible only at some levels of capital. It finds that with linear technology there exists a switching equilibrium where the appropriation rate starts out high and switches to a lower level as the capital stock reaches a certain threshold. As a consequence the growth rate is increasing in the level of capital. In order to generate the other stylized fact that at high levels of capital the growth rate is decreasing, it replaces the linear technology with a Cobb-Douglas one. The second paper analyzes a similar problem in continuous time using linear preferences. The third paper characterizes a switching equilibrium of a wage bargaining game.

The key difference between these models and ours is that we formulate the problem as a preemption game and allow the accumulation equations to switch between one regime and another, with each switch entailing a one-time loss. It is this specification that allows us to generate two switches (from Common to Private and back to Common Property) for the same utility and production functions.

With regards to the solution method we use techniques of the preemption games literature, and those used in the literature on differential games of joint exploitation of resources.[5] In the first, the payoffs to lead, follow and match are postulated as functions of time only. A novel feature of the present paper is to make these payoff functions depend on a state variable (the capital stock), and to make this state variable the result of an accumulation game which takes into account the underlying preemption game.

With respect to the literature on institutional change, our paper is related to the work of North and Olson. Olson (1982) argues that the decline of leading countries, such as Britain, was caused by the spontaneous formation of interest groups, which were able to overcome the free-rider problem due to long periods of stability. Once these groups were established they engaged in redistributive activities, causing a de facto elimination of private property. This argument is intuitively compelling, but its microfoundations are not clear. In particular, high voracity is not a necessary consequence of the existence of competing interest groups. In fact, groups may find it profitable to limit current voracity and wait until the goose that lays the golden egg fattens. Also, the argument as it is presented only explains the decline of rich economies, but not the rise of some poor economies. In this respect the models of Benhabib et. al. and ours formalize Olson's idea about the erosion of Private Property, and extend it to explain shifts from Common to Private Property.

According to North (1990) during the Middle Ages, law and order existed only within the boundaries of the manor or the town; traders faced the danger of expropriation and pillage. Moreover, there was no impartial third party with the coercive power to enforce agreements. By the end of the fifteenth century, nation-states were created, where a Prince offered security in exchange for revenue. In the case of England and the Dutch Republic, the Prince's access to the possessions of his subjects was limited by a representative body. This did not occur in France and Spain, where the king obtained extensive powers to tax in exchange for privileges

[5] For preemption games see Fudenberg and Tirole (1985), Hendricks and Wilson (1987), and Reinganum (1981). For differential games in common access economies (and fisheries) see Benhabib and Radner (1992), Haurie and Pohjola (1987), Lancaster (1973), and Tornell and Velasco (1992).

conferred to the nobility, like tax farms. During the following centuries, first the Dutch and then the English became the world leaders in terms of economic growth and technological innovation. Meanwhile, Spain declined from being the most powerful nation in the sixteenth century to a second rate power, which constantly suffered fiscal crises.

III. The Model

We consider an "Ak" growth model with the peculiarity that the representative agent is replaced by two infinitely lived agents (i and j) who interact strategically (we shall refer to them as interest groups). They derive utility from consumption of a single good which can be instantaneously transformed into capital. This good is produced with a linear technology using capital as the only input. The capital stock of group h is denoted by k_h, and

$$k \equiv k_i + k_j, \qquad K \equiv (k_i, k_j)$$

There are three types of property rights regimes that describe the ways in which these groups interact. First, we define "Private Property" as the regime that permits each group access only to its own capital stock. The accumulation equations are the same as those in the representative agent case

$$\dot{k}_h(t) = ak_h(t) - c_h(t), \qquad h = i, j. \tag{1}$$

Second, under "Common Property" there are no individual capital stocks, and everyone has access to the aggregate capital stock. The accumulation equation is

$$\dot{k}(t) = ak(t) - c_i(t) - c_j(t) \tag{2}$$

This type of equation has been used to study many different phenomena of joint exploitation of resources such as fisheries (Levhari and Mirman (1980)), labor conflicts (Lancaster (1973)) and macroeconomics (Pohjola (1986)). In the context of labor conflicts c_i is the wage bill and c_j is the amount of profits not reinvested. In the context of fiscal deficits, the c's are transfers to government agencies or rent-seeking groups.

Lastly, under the "leader-follower" regime only one group (the leader) enjoys access to aggregate capital, while the other group (the follower), who does not, will have zero consumption. The accumulation equation is

$$\dot{k}(t) = ak(t) - c_l(t) \quad (3)$$

where subscript "l" stands for leader. Under all regimes the following constraint must be satisfied

$$k(t) \equiv k_i(t) + k_j(t) \geq 0 \quad (4)$$

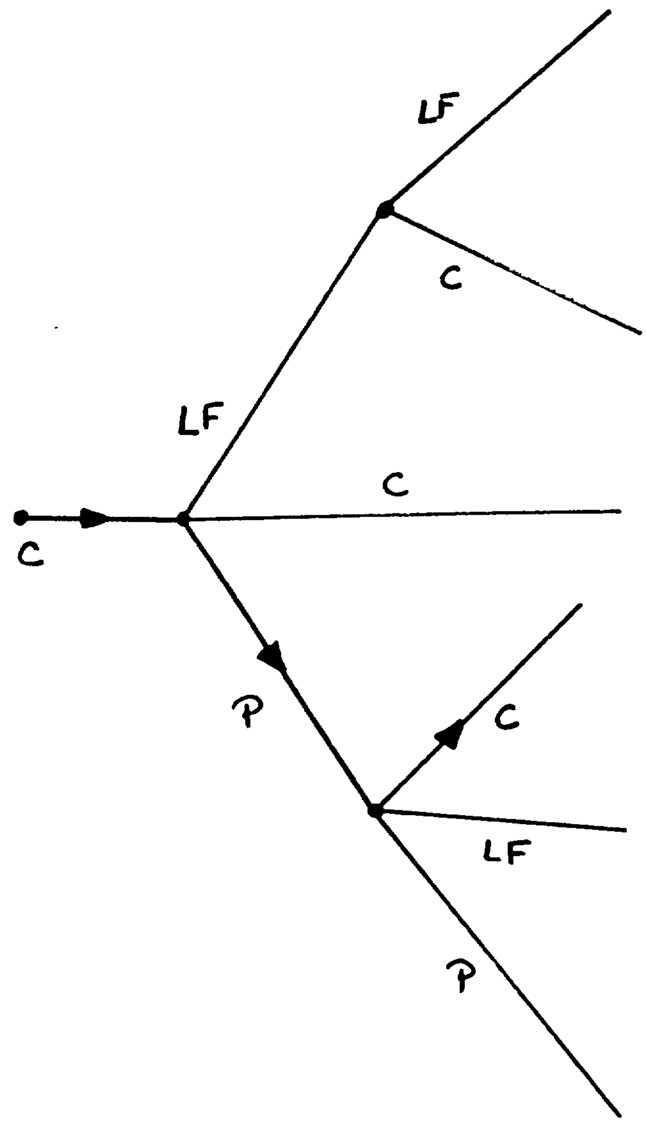
The possible paths that the economy can follow are illustrated in figure 1. Initially Common Property prevails. At any instant a group can displace the other group and secure exclusive access to the entire capital stock by incurring a one-time utility loss q_l. If the other group simultaneously undertakes the same action, each group incurs a loss q_m and attains private access to half of total capital. In the first case there is a switch to the leader-follower regime. In the second case there is a switch to Private Property. The losses q_m and q_l can be interpreted as the costs of creating a legal enforcement system (which might be impartial or favor the leader), or as a direct payment to the other group[6].

[insert Figure 1]

If a switch to Private Property occurs, each group can regain its access to the entire capital stock and displace the other group by incurring another one-time utility loss p_l.

[6]The following paragraph quoted in North (1990), p. 113 is illustrative: "The admission of the right of parliament to legislate, to enquire into abuses, and to share in the guidance of national policy, was practically purchased by the money granted to Edward I and Edward III" (Taken from Stubbs, 1896, The Constitutional History of England, Vol II).

Figure 1



However, if the other group undertakes the same action simultaneously, the loss to each group is p_m and there is a switch to Common Property. In this case, the loss can be interpreted as the cost of forming a political organization, or as the costs of altering the rules of a Private Property economy in order to induce redistribution.

We model this problem as a preemption game where each group has to choose its consumption policy and its switching rule in order to maximize its lifetime utility subject to the strategy of the other group. There are three types of outcomes: a "non-switching" one, where neither group ever switches; a "matching" one, where both groups always switch simultaneously, and a "leader-follower" one, where the end result is that one group becomes the leader. The peculiarity of this game is that the payoffs to follow, lead and match are functions of the capital stocks. Since these stocks depend on the consumption policies, which in turn are functions of the switching rules, the problem is rather complicated. We will not consider all possible cases, just the one where there is a switch from Common to Private Property, followed by a switch back to Common Property.

To keep matters simple we will restrict the number of regime switches to two or less, and we will assume that the follower cannot revolt against the leader and regain its access to any share of capital. Since equilibrium consumption policies are functions of all future switch dates (see (26)), the first assumption will allow us to obtain closed-form solutions. The second one rules out paths along which switches away from the leader-follower regime occur. This simplifies the computation of the payoffs associated with leading and following. Given these assumptions, the valuation function for group i is given by

$$\int_0^\infty \frac{\sigma}{\sigma-1} c_i(t)^{\frac{\sigma-1}{\sigma}} e^{-\delta t}\, dt \;-\; q_i(\tau_i,\tau_j)\; I(t,\tau_i) e^{-\delta\tau_i} \;-\; p_i(T_i,T_j,\tau_i,\tau_j)\; I(t,T_i) e^{-\delta(T_i+\tau_i)} \tag{5}$$

where τ_h and $\tau_h + T_h$ are the first and second switch dates of group h, σ is the elasticity of intertemporal substitution, I is an indicator function: $I(t,x)=1$ if $t=x$ and zero otherwise, and

$$q_i(\tau_i,\tau_j) = \begin{cases} q_l & \text{if } \tau_i<\tau_j \\ q_m & \text{if } \tau_i=\tau_j \\ \infty & \text{if } \tau_i>\tau_j \end{cases} \qquad p_i(T_i,T_j,\tau_i,\tau_j) = \begin{cases} p_l & \text{if } T_i < T_j \text{ and } \tau_i=\tau_j \\ p_m & \text{if } T_i = T_j \text{ and } \tau_i=\tau_j \\ \infty & \text{if } T_i > T_j \text{ or } \tau_i>\tau_j \end{cases} \tag{6}$$

We impose the following restrictions on parameters[7]

$$1 < \sigma < \frac{a}{a-\delta} < 2 \tag{7}$$

It will turn out that the matching outcome will occur in equilibrium for the following values of the losses of switching regime

$$(a)\; p_l > q_l > [\sigma/(\sigma-1)]k_0^{(\sigma-1)/\sigma} z^{-1/\sigma} \qquad (b)\; p_m \le p_l[2-\sigma]^{1/\sigma} \qquad (c)\, q_m \le q_l[2-\sigma]^{1/\sigma} \tag{8}$$

where $z=a(1-\sigma)+\delta\sigma>0$ by (7). According to (8a), the lower bound on the loss of becoming the leader increases with the size of the economy. Condition (8b) restricts p_m to be sufficiently smaller than p_l. This insures that in equilibrium matching is preferred to following. A similar interpretation applies to (8c).

The solution concept we will use is Markov Perfect (Feedback Nash) equilibrium. Following Basar and Olsder (1982) and Simon and Stichombe (1989), we make the following definitions: the state consists of three elements: the capital stocks of each group (K), the

[7]This restriction implies: (i)$\sigma<2$ and $a(1-\sigma)+\delta\sigma>0$, which are necessary for the value functions to be bounded; (ii) $a>2\delta$, which is necessary for the aggregate capital stock to be increasing (in the representative agent case this condition is $a>\delta$); and (iii) $\sigma>1$, which is necessary for the value of leading to become greater than the values of following and waiting at higher levels of k, given that initially it is lower.

prevalent property rights regime (R), and the number of previous regime switches (N). We shall call "node" a realization of the state: (K,R,N), where $K=(k_i,k_j)$ can be any pair of non-negative real numbers, R={P,C,LF}; P, C, and LF refer to Private, Common and leader-follower respectively, and N={0,1,2}. A strategy consists of a consumption policy and a switching rule describing the actions that a group would take at every possible node. A strategy is Markov (feedback) if it depends solely on the realization of the state. A pair of Markov strategies forms a Markov perfect equilibrium if they are best responses to each other starting at every node in the game.[8]

We shall construct an equilibrium where there is a switch from Common to Private Property, followed by another switch from Private back to Common Property. We start by characterizing the last phase, after which we will consider the second and the first phases.

III.1 <u>The Third Phase</u>[9]

In this section we find Markov perfect equilibria of the continuation games starting at any node (K,R,2), with R={P,C,LF}. Since two switches have already occurred, and no more switches are allowed, only one of the three property rights regimes described above will prevail forever. We consider each in turn.

Under the leader-follower regime the leader owns the entire capital stock, while the follower must have zero consumption. It follows that the leader faces a simple control problem of maximizing the first term in (5) subject to the accumulation equation (3) and constraint (4). This is the standard "Ak" model. The solution is standard:

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[8]Markov strategies do not allow groups to precommit to specific switching dates, nor do they allow history dependent strategies, such as trigger strategies.

[9]Subsection III.1 draws on joint work with Andres Velasco.

$$c(K,L,2) = zk, \qquad with \quad z \equiv a(1-\sigma)+\delta\sigma > 0 \tag{9}$$

$$\hat{k}(t;K,L,2) = ke^{\sigma(a-\delta)t} \tag{10}$$

$\hat{k}$(t;K,L,2) is the aggregate capital stock after a period of length t if consumption policy (9) is followed starting at any node (K,L,2). The intuition is as follows: along the optimal path consumption grows at rate σ(a-δ). When the return on capital 'a' is greater than the rate of time preference 'δ', it pays to sacrifice current consumption in exchange for higher future consumption. The higher the elasticity of intertemporal substitution 'σ', the more profitable in utility terms is this substitution. Substituting (8) and (9) into (5), the payoff to the leader is

$$J_f(k) = \frac{\sigma-1}{\sigma}k^{\sigma/(\sigma-1)}z^{-1/\sigma} \tag{11}$$

In order for the integral in (5) to converge it is necessary that z>0, as implied by condition (7). Note that under Private Property each group solves the same problem as the leader with an initial capital of k_h instead of k. Thus, optimal consumption under this regime is $c(k_h,P,2)=zk_h$.

Under Common Property both groups engage in a differential game in which the valuation function of each group is given by the first term in (5) and the accumulation equation is given by (2). Since no more switches will occur, the state is completely described by the value of aggregate capital (k). Thus, it suffices to find a pair of consumption policies that are just functions of k, and that are best responses to each other for any value of k. The solution to this problem is in appendix A. Here we present an heuristic derivation: suppose that at each instant group h appropriates a share β_h of k. It follows that the rate of return to group i is a-β_j.

Thus, if consumption policy c_i is maximizing i's payoff, it is necessary that $\dot{c}_i/c_i = \sigma[(a-\beta_j)-\delta]$[10].

Next, note that c_i must grow at the same rate as k ($\dot{k}/k = a-\beta_i-\beta_j$). Therefore i's best response to $c_j=\beta_j k$ is to set $c_i=[z+(\sigma-1)\beta_j]k$. Similarly, j's best response to $c_i=\beta_i k$ is $c_j=[z+(\sigma-1)\beta_i]k$. The unique solution to these equations is $\beta_i=\beta_j=z/(2-\sigma)$. Thus, the pair of consumption policies (c_i,c_j), with

$$c_h(K,C,2) = [z/(2-\sigma)]k, \qquad h=i,j \tag{12}$$

forms a Markov perfect equilibrium of the game starting at any node (K,C,2). If both groups follow consumption policy (12) for a period of length t, starting from any node (K,C,2), then the resulting aggregate capital stock is

$$\hat{k}(t;K,C,2) = ke^{\frac{\sigma(a-2\delta)}{2-\sigma}t} \tag{13}$$

Even though a group can consume the entire capital stock, it does not do so in equilibrium. Note however that since $\sigma<2$, the marginal propensity to consume is greater than in the representative agent case ($c_i=zk$). The reason is that the rate of return perceived by i is not 'a' but $a-\beta_j$. Thus i's willingness to substitute future for present consumption is reduced, and the tragedy of the commons results (i.e., the growth rate is smaller than in the representative agent case). Substituting (12) and (13) in the first term of (5) we obtain the value of the continuation game starting at any node (K,C,2)

$$J_m(k) = \frac{\sigma-1}{\sigma}k^{\frac{\sigma}{\sigma-1}}\left[\frac{2-\sigma}{z}\right]^{\frac{1}{\sigma}} \tag{14}$$

The subscript 'm' stands for matching. The integral in (5) converges if and only if $z/(2-\sigma)>0$. Comparing (11) and (14) we have (recall that $1<\sigma<2$ and $z>0$)

[10]Recall that in the representative agent case $\dot{c}/c = \sigma(a-\delta)$.

$$J_l(k) > J_m(k) > 0, \qquad J_l'(k) > J_m'(k) > 0 \qquad \textit{for all } k > 0 \tag{15}$$

This result will be the key to obtaining the switching equilibrium.

Summing up, we have shown that starting at any node in which two switches have already occurred, the pair of strategies (ψ_{i2},ψ_{j2}), with

$$\psi_{h2} = \psi(K,R,2) = \begin{cases} I_{h2}=0, & c_{h2}=kz/(2-\sigma) & \text{if } R=C \\ I_{h2}=0, & c_{h2}=kz & \text{if } R=LF \text{ and } h=L \\ I_{h2}=0, & c_{h2}=0 & \text{if } R=LF \text{ and } h=F \\ I_{h2}=0, & c_{h2}=k_h z & \text{if } R=P \end{cases}$$

forms a Markov perfect equilibrium. Recall that I=0 means the group should not switch, and R stands for regime.

III.2 The Second Phase of Growth: Private Property

During this phase Private Property prevails and one switch has already occurred. As shown in figure 1, at any node (K,P,1) there are three possibilities: (a) both groups incur a loss p_m and there is a switch to Common Property; (b) only one group incurs a loss p_l, it becomes the leader, and the other group becomes the follower; or (c) neither group incurs a loss, and both wait. The payoffs are: F=0 for the follower, $L(k)=J_l(k)-p_l$ for the leader, $M_1(k)=J_m(k)-p_m$ if both groups switch, and W_1 if both groups wait. The J's were derived in the last subsection, and W_1 will be derived below.

In this subsection we will characterize the equilibria in which there is a switch from Private to Common Property. For such a switch to occur it is necessary that the strategies of both groups lead them to switch at the same level of aggregate capital. Thus we will consider the following switching rule:

S1: If Private Property prevails switch if $k \geq k^*$, do not switch if $k < k^*$.

where k^* is a certain threshold level. We will proceed as follows: first we will construct a pair of consumption policies that are best responses to each other starting at any node (K,P,1), given that both groups are following rule S1. Second, we will determine the conditions under which each group finds it optimal to follow rule S1 given that the other is doing so.

If both groups are following rule S1 and $k < k^*$, the valuation function of group i at node (K,P,1) is

$$\int_0^T (\sigma/\sigma-1)c_i(t)^{\sigma-1/\sigma}e^{-\delta t}dt + e^{-\delta T}[J_m(k(T))-p_m] \tag{5'}$$

where T is the time to reach k^* starting at $t=0$ with an aggregate capital stock of k. The scrap-value function is given by the payoff associated with having Common Property forever because both groups will switch simultaneously as soon as k reaches k^*. Starting at any node (K,P,1) each group chooses a consumption path to maximize (5') subject to (1), (4) and the strategy of the other group. To solve this game we treat T as a parameter and obtain a pair of optimal consumption policies. Then we show that aggregate capital is strictly increasing and we obtain T as the time that it takes for k to reach k^*.

The solution is in appendix B. Here we present an heuristic derivation: since Private Property prevails, for a consumption policy c_h to maximize h's payoff it is necessary that $\dot{c}_h/c_h = \sigma[a-\delta]$ (as in the representative agent case), and that at $k=k^*$ it equals the consumption level that will prevail just after the switch to Common Property takes place: $c_i(T)=c_j(T)=k^* z/(2-\sigma)$ (this is to satisfy the transversality condition). Substituting these equations in the accumulation equation (1), we have that a consumption policy which satisfies these necessary conditions is

$$c_h(K,P,1) = \frac{zk}{2-\sigma e^{-zT(k,k^*)}}, \qquad h=i,j \tag{16}$$

It may be surprising that even though Private Property prevails, individual consumption is a function of aggregate capital, not of individual capital. The intuition behind this can be seen by rewriting (16) as $c_i(0)=[k_i(0)+e^{-aT}k_j(T)]z/[1-(\sigma-1)e^{-zT}]$ (see appendix B). The first term in brackets is group i's wealth: its own capital plus what i expects j's capital will be at the time of the switch, discounted from the time when capital becomes Common Property to the present at the rate 'a'. The other term is the marginal propensity to consume. Thus, although Private Property prevails, each group behaves as if at a future date T it will win a lottery and the interest rate will fall. As this date gets closer, the marginal propensity to consume increases until it reaches the constant level it will have after the switch to Common Property takes place. It is through this channel, rather than through the production side, that our model generates the time-varying growth rates mentioned in the introduction.

We shall denote by $\hat{k}(t,T;K,P,1)$ the aggregate capital stock at time t that results if both groups follow consumption policy (16) starting at time 0 with initial capital k, and expect a switch to Common Property at time T. Using (A9) we have

$$\hat{k}(t,T;K,P,1) = ke^{at}\frac{2e^{z(T-t)}-\sigma}{2e^{zT}-\sigma}, \qquad t<T \tag{17}$$

We show in appendix B that if $a>2\delta$, as implied by condition (7), then (see (A11)-(A12))

$$\frac{\partial\hat{k}(t,T)}{\partial t} > 0, \qquad \frac{\partial\hat{k}(t,T)}{\partial T} > 0, \qquad \frac{\partial\hat{k}(t,T)}{\partial t\partial T} > 0, \qquad \frac{d\hat{k}(T,T)}{dT} > 0 \tag{18}$$

Since $\hat{k}(T,T;K,P,1)$ is strictly increasing in T, for each level of aggregate capital $k<k^*$ there

exists a unique waiting time T(k,k*) before $\hat{k}$(t,T;K,P,1) hits k*. Thus, by setting t=T in (17) and inverting k* = $\hat{k}$(T,T;K,P,1), we have

$$T(k,k^*) = \left[\log(2) - \log\left\{\frac{k}{k^*}(2-\sigma)+\sigma e^{-aT(k,k^*)}\right\}\right]\frac{1}{\sigma(a-\delta)} \tag{19}$$

Note that T(k,k*) is increasing in k*, decreasing in k and that T=0 if k=k*. Note also that (18) implies that if at t=0 aggregate capital stock is k, then T(k,k*) is the time to reach k* provided that both groups follow consumption policy (16).

We show in appendix B that if a switch to Common Property will occur at time T, and if group i is following consumption policy (16), then the unique consumption policy that maximizes j's payoff on [t,T) is (16). The same holds for group i. Since for any pair (k,k*) the waiting time T(k,k**) is uniquely determined by (19), it follows that starting at any node (K,P,1) the pair of consumption policies defined by (16) and (19) are best responses to each other given that both groups use rule S1.

The value of waiting conditional on both groups following switching rule S1 is (see (A10))

$$W_1(k,k^*) = \frac{\sigma}{\sigma-1}\left[\frac{k}{2-\sigma e^{-zT(k,k^*)}}\right]^{\frac{\sigma-1}{\sigma}}[1-(\sigma-1)e^{-zT(k,k^*)}]z^{-\frac{1}{\sigma}} \tag{20}$$

This is the payoff that is obtained if starting at node (K,P,1) both groups follow consumption policy (16) until k* is reached and then switch to Common Property, and follow consumption policy (12) forever. Note that by setting k=k*, W_1 becomes equal to the payoff function associated with having Common Property forever (which is given by (14)). Note also that since W_1 is increasing in T, and T is increasing in k* we have that (see (A13))

$$W_1(k^*,k^*) = J_m(k^*), \qquad W_1(k,k^{*\prime}) > W_1(k,k^*) \qquad \forall\, k^{*\prime}>k^* \quad (21)$$

That is, for a given k, a path with a greater switching capital (k*) yields a greater payoff than another path with a smaller k*. The intuition is as follows: the higher k*, the longer the period during which Private Property prevails. Since there is less pillage under Private than under Common property, consumption grows at a higher rate for a longer period. Therefore, in the hypothetical case where both groups could commit to not switching, both would choose to remain under Private Property forever if the alternative was switching to Common Property.

The next step is to determine the conditions under which switching rule S1 is a best response to itself. Two conditions must be satisfied for this to be the case. First, switching early to become the leader should not be profitable. This occurs if and only if waiting is preferred to leading, given that a switch to Common Property will take place when k hits k*:

$$W_1(k,k^*) > J_l(k)-p_l \qquad \forall k<k^* \quad (22)$$

We will refer to (22) as the "no-preemption" condition. The second condition is that at k* matching be preferred to following

$$J_m(k^*)-p_m \geq 0 \qquad (23)$$

We will refer to (23) as the "matching" condition. If the no-preemption condition is violated, profitable unilateral deviations exist along the waiting path. If the matching condition is violated, only one group switches at k*.

In the remainder of this subsection we determine the values of k^*, p_l and p_m for which (22)-(23) are satisfied. We will refer to four levels of k: k^{**}, $\bar{k}$, $\bar{k}'$ and $\bar{k}''$; k^{**} is the aggregate capital stock at which the <u>first</u> switch (from Common to Private Property) took place. At $\bar{k}$ the values of leading and matching are equal, and at $\bar{k}'$ and $\bar{k}''$ the values of matching and leading are zero respectively:

$$J_l(\bar{k})-p_l=J_m(\bar{k})-p_m, \qquad J_m(\bar{k}')-p_m=0, \qquad J_l(\bar{k}'')-p_l=0 \tag{24}$$

It follows from (15) that $\bar{k}$, $\bar{k}'$ and $\bar{k}''$ are unique. For future reference we will write the value of $\bar{k}$ explicitly

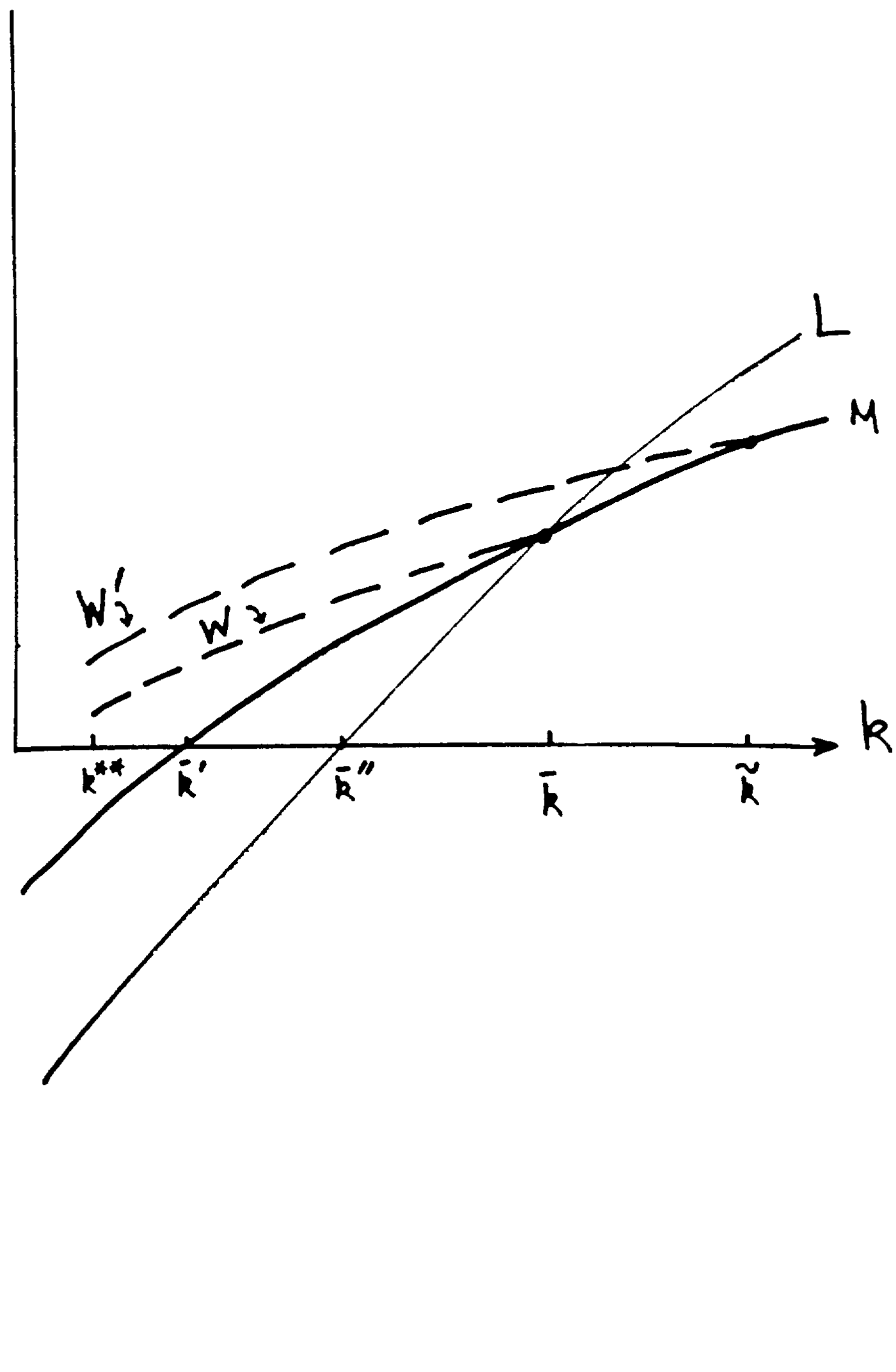
$$\bar{k}(p_l,p_m) = \left[\frac{\sigma-1}{\sigma}\cdot\frac{z^{1/\sigma}(p_l-p_m)}{1-(2-\sigma)^{1/\sigma}}\right]^{\sigma/(\sigma-1)} \tag{24'}$$

To illustrate the argument we use Figure 2. The M-curve represents the value of matching (where Common Property prevails forever), the L-curve represents the value of leading, the horizontal axis represents the value of following, and the W-curve represents the value of waiting (drawn for the case $k^*=\bar{k}$). From (21) we know that the W and the M curves coincide at $k=k^*$, and that for lower k's the W-curve lies above the M-curve. The W'-curve is drawn for the case $k^*=k > \bar{k}$. Figure 2 is drawn under assumption (8): that p_l is sufficiently greater than p_m so that the value of leading is lower than that of matching at k^{**}.

[insert Figure 2]

First we show that the no-preemption condition is satisfied only for $k^* \leq \bar{k}$. Let $k^*=\bar{k}$: since the W-curve lies above the L-curve everywhere, waiting is preferred to leading at any

Figure 2



$k<k^*=\bar{k}$. Therefore, along the waiting path neither group will find it profitable to deviate and become the leader. The same holds for any $k^*<\bar{k}$. Now consider the case $k^*>\bar{k}$. To show that the no-preemption condition is violated let $k^*=\tilde{k}>\bar{k}$. In this case the value of waiting is represented by the W'-curve. At $\tilde{k}$ the value of waiting equals that of matching, which is lower than the value of leading, as can be seen in figure 2. Since the W' and L curves are continuous, it follows that there exists some interval ending in $\tilde{k}$ on which leading is preferred to waiting. Hence, the no-preemption condition is violated for any switching capital greater than $\bar{k}$.[11]

Next, we consider the matching condition (23). For a matching equilibrium to exist it is necessary that (23) holds for some k* for which (22) also holds. Since (22) only holds for $k^*\leq\bar{k}$, we need (23) to hold for some $k^*\leq\bar{k}$. We can see from figure 2 that is the case if and only if the M-curve crosses the horizontal axis at a lower k than the L-curve does. Algebraically, it is necessary that $J_m(\bar{k}'')-p_m\geq 0$. Using (11), (14) and (24), this condition can be written as $p_m \leq (2-\sigma)^{1/\sigma}p_l$, which is condition (8b). Since $1<\sigma<2$, (8b) requires p_m to be lower than p_l. This reflects the idea that a king will face less opposition to obtain taxing

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[11]Algebraically, the no-preemption condition is satisfied for any $k^*\leq\bar{k}$ because $\forall\ k<k^*\ W_1(k,k^*)>J_m(k)-p_m$, *and* $\forall\ k\leq\bar{k}\ J_m(k)-p_m\geq J_l(k)-p_l$. To show that it does not hold for any $k^*>\bar{k}$ suppose that it holds for some $k^*=\tilde{k}>\bar{k}$, then $J_l(\tilde{k})-p_l>J_m(\tilde{k})-p_m=W_1(\tilde{k},\tilde{k})$. Since J_l and W_1 are continuous functions of k, there exists an $\epsilon>0$ such that $J_l(\tilde{k}-\epsilon)-p_l>W_1(\tilde{k}-\epsilon,\tilde{k})$. Therefore, at $\tilde{k}-\epsilon$ each group will find it profitable to deviate unilaterally, contradicting the initial supposition.

powers if he grants similar privileges to the nobility like the "tax farms" which existed in Absolutist France.

If (8b) did not hold, leading would become preferred to waiting at a level of k for which the value of matching would be negative. In this case a switch to Common Property could not take place because at levels of k where $M_1(k)$ was positive, preemption would have had already taken place. If (8b) holds, the matching condition is satisfied for any $k^* \geq \bar{k}'$ because $J'_m(k) > 0$. Note that $\bar{k}'$ need not be greater than k**, unless $p_m > J_m(k^{**})$.

Now we consider the restriction on p_l given in (8a). Since the no-preemption condition only holds for $k^* \leq \bar{k}$, an equilibrium exists where Private Property prevails for a positive amount of time only if $\bar{k} > k^{**}$. Since at $\bar{k}$ the values of matching and leading are equal, there exists a $\bar{k}$ such that $\bar{k}$ is greater than k** if and only if $J_m(k^{**})-p_m > J_l(k^{**})-p_l$. Furthermore, in the next subsection the highest value that k** will take is $\underline{k}$, with $J_l(\underline{k})=q_l$. Therefore, given that (8b) is satisfied, $\bar{k}$ will be greater than any $k^{**} \leq \underline{k}$ if and only if $p_l > q_l$. This is condition (8a)[12]. This restriction reflects the idea that the bigger the economy, the more costly it is for an interest group to change the structure of property rights.

The last step is to bring the two pieces together. We have shown that for a given k*, (16) and (19) define a pair of consumption policies which are best responses to each other,

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[12]We derived condition (8a) as follows: we need that $J_l(k^{**})-J_m(k^{**}) < p_l - p_m$. From (8b) we have that $p_l - p_m \geq [1-(2-\sigma)^{1/\sigma}]p_l$, while from (15) and the definition of $\underline{k}$ we get $J_l(k^{**})-J_m(k^{**}) \leq J_m(\underline{k})-J_l(\underline{k})-[1-(2-\sigma)^{1/\sigma}]q_l$. Condition (8a) follows directly.

conditional on both groups using switching rule S1 (i.e., switching to Common Property at k^*). We have also shown that if the p's satisfy (8), the no-preemption condition is satisfied for any $k^* \leq \bar{k}$, and the matching condition is satisfied for any $k^* \geq \bar{k}'$. Therefore, for any k^* in $[\min(k^{**}, \bar{k}'),\ \bar{k}]$ if group j is following consumption policy (16) and switching rule S1, the best response of group i is to switch when k hits k^*. At any $k < k^*$, by preempting i gets L(k), while by waiting it can insure $W_1(k,k^*) > L(k)$. At k^*, by switching i gets $M_1(k^*) \geq 0$, while by not switching it gets zero. Hence, if (7)-(8) are satisfied, the pair of Markov strategies (ψ_{i1}, ψ_{j1}) with

$$\psi_{h1} = \psi(K,P,1) = \begin{cases} I_h = 0, \quad c_h = zk/[2 - \sigma e^{-zT(k,k^*)}] & \text{if } k < k^* \\ I_h = 1, \quad c_h \text{ as in } \psi_{h2} & \text{otherwise} \end{cases}$$

where $k^* \in [\min(k^{**}, \bar{k}'),\ \bar{k}]$, forms a Markov perfect equilibrium starting at any node (K,P,1). $T(k,k^*)$ and $\bar{k}$ are given by (19) and (24'). For $k < k^*$ this strategy instructs each group not to switch and to follow consumption policy (16). For $k \geq k^*$ it instructs each group to switch immediately.

There exist multiple Markov perfect equilibria indexed by the level of aggregate capital at which the switch occurs (k^*). We have shown in (21) that the higher k^*, the higher the payoff to each group. However, the highest value of k^* for which there is no temptation to preempt is $\bar{k}$. Although each group is better-off under Private Property, at $\bar{k}$ the switch to Common Property occurs not because of a lack of coordination, but because of the fear that each group has of becoming the follower.

III.3 The First Phase of Growth

During the first phase Common Property prevails and no switches have yet occurred. As shown in figure 1, at any node (K,C,0) there are three possibilities: (a)each group imposes private access on half of total capital by incurring a loss q_m, and there is a switch to Private Property; (b)there is a switch to the leader-follower regime. The leader incurs a loss (q_l) and attains access to the entire capital stock, while the follower has to have zero consumption; or (c)neither group incurs a loss, and both wait.

As a first step we derive the payoffs of leading and matching. Under the leader-follower regime, the follower has no access to any capital, and cannot recuperate it. The leader owns the entire capital, and at any moment it can restore the follower's access to aggregate capital. Since the leader will never find it profitable to undertake this action, a switch away from this regime will not occur. Therefore, the payoffs of the continuation game associated with a switch to the leader-follower regime are $J_l(k)-q_l$ for the leader and zero for the follower (J_l is given by (11)). To derive the payoff of matching, note that if both groups switch simultaneously, the economy follows the path characterized in the previous subsection: Private Property will prevail for a period of length $T(k,k^*)$, after which reversion to Common Property will occur. Thus the payoff to matching is $W_1(k,k^*)-q_m$, W_1 is given by (20). From (A13), (11) and (20) we have that

$$J_l(k) > W_1(k,k^*)>0, \qquad dJ_l(k)/dk > dW_1(k,k^*)/dk > 0 \qquad \textit{for } k<k^*<\bar{k} \quad (25)$$

We will construct a Markov perfect equilibrium where a switch from Common to Private Property occurs when k reaches a threshold level k**. We shall proceed as in the previous subsection: first, we will suppose that the switching rules instruct both groups to incur the loss at the same levels of k. Conditional on this, we will derive a pair of consumption policies that are best responses to each other starting at any node (K,C,0). Then we will determine the conditions under which there are no profitable unilateral deviations from the proposed switching rule.

We will consider the following switching rule:

<u>S0</u>: If Common Property prevails, switch if $k \geq k^{**}$, do not switch if $k < k^{**}$.

If both groups use S0, then at any node (K,C,0) the valuation function of group i is

$$\int_0^{\tau} \sigma/(\sigma-1)c_i(t)^{(\sigma-1)/\sigma}e^{-\delta t}dt \ + \ e^{-\delta\tau}[W_1(k(\tau),k^{**})-q_m] \qquad (5'')$$

where $\tau=\tau(k,k^{**})$ is the time to reach k** starting at t=0 with aggregate capital stock k. The scrap-value function is given by the value of the continuation game corresponding to a switch to Private Property because both groups are using rule S0. Thus, both will switch simultaneously. Given that both groups are following rule S0, starting at any node (K,C,0) each group maximizes (5'') subject to (2), (4) and the strategy of the other group. We will solve this differential game in two steps: first we will treat τ as a parameter and obtain a pair of consumption policies which are best responses to each other, and then we will show that aggregate capital is increasing and obtain $\tau(k,k^{**})$ as the time that k reaches k**.

In order to simplify the algebra and obtain the closed-form solutions for c and k in (26)-(27), we will restrict attention to the case where the length of the second phase T(k**,k*) is independent of the specific value k^{**} takes. From (19) we can see that this holds if k^* varies with k^{**} to keep k^*/k^{**} constant[13]. Since k^*/k^{**} will be kept constant, $W_1(k^{**},k^*)$ will be a function of k^{**} only. Thus, it can be expressed as $w_1(k^{**})$. Appendix C shows that (25) continues to hold in this case (see (A13)).

The solution to the first step is in appendix C. Here we present an heuristic derivation: suppose that group h follows a non-stationary consumption policy given by $c_h(t)=\gamma_h(t)k(t)$. This implies that $a-\gamma_j(t)$ is i's rate of return after appropriation by j. Thus if c_i is maximizing i's payoff it is necessary that $\dot{c}/c_i=\sigma[a-\gamma_j(t)-\delta]$ (recall that in the representative agent case, consumption has to grow at a rate $\sigma[a-\delta]$). A second condition is that $\dot{c}/c_i=\dot{\gamma}/\gamma_j+\dot{k}/k$. Analogous conditions apply to c_j. A third pair of necessary conditions is that at the time of the switch, both consumption levels be equal and satisfy transversality condition (A14). Note that these six conditions are satisfied only if $\gamma_i(t)=\gamma_j(t)=\gamma(t)$. Substituting this in the first two conditions, we have that the appropriation rate must satisfy the following differential equation: $\dot{\gamma}(t)/\gamma(t)=[2-\sigma]\gamma(t)-z$. Using transversality condition (A14) to determine the terminal condition $\gamma(\tau)$, it follows that (see (A16)-(A17)):

[13]Since k^* has to be lower than $\bar{k}$, and k^{**} will take values in $[\underline{k}',\underline{k}]$ (see (32)), we need to restrict k^*/k^{**} to be not greater than $\bar{k}/\underline{k}$. Otherwise, k^*/k^{**} could not be kept constant. We use this assumption in the computation of the transversality condition (A14).

$$c_h(t,\tau;K,C,0) = \frac{kz}{2-\sigma + [D-2+\sigma]e^{-z(\tau-t)}}, \qquad D \equiv \frac{[2-\sigma e^{-zT}]^{\sigma-1}}{[1-(\sigma-1)e^{-zT}]^{\sigma}} > 1 \tag{26}$$

$$\hat{k}(t,\tau;K,C,0) = ke^{t\sigma\left(\frac{a-2\delta}{2-\sigma}\right)}\left[\frac{2-\sigma + (D-2+\sigma)e^{-z(\tau-t)}}{2-\sigma + (D-2+\sigma)e^{-z\tau}}\right]^{\frac{2}{2-\sigma}}, \qquad 0 \le t < \tau \tag{27}$$

$\hat{k}(t,\tau;K,C,0)$ is aggregate capital at time t, given that a switch to Private Property will occur when k hits k** at τ(k,k**), followed by a switch back to Common Property when k reaches k* at T+τ.

Since a>2δ, 2>σ, and z>0, it follows from (27) that (see (A19))

$$\frac{\partial \hat{k}(t,\tau)}{\partial t} > 0, \quad \frac{\partial \hat{k}(t,\tau)}{\partial \tau} < 0, \quad \frac{\partial \hat{k}(t,\tau)}{\partial t \partial \tau} < 0, \quad \frac{d\hat{k}(\tau,\tau)}{d\tau} > 0 \tag{28}$$

A reduction in τ increases the growth rate because as the switch date to Private Property becomes closer, the appropriation rate (which is equal to the marginal propensity to consume) falls. Note that despite $\hat{k}(t,\tau)$ being decreasing in τ, $\hat{k}(\tau,\tau)$ is increasing in τ. This property implies that τ(k,k**) is uniquely determined by $\hat{k}(\tau,\tau;k,c,0)$=k**. Inverting this equation we get

$$\tau(k,k^{**}) = \left[\log\left(\frac{k^{**}}{k}\right) - \frac{2}{2-\sigma}\log\left(\frac{2-\sigma+e^{-z\tau(k,k^{**})}(D-2+\sigma)}{D}\right)\right]\frac{2-\sigma}{\sigma(a-2\delta)} \tag{29}$$

Since $\partial\hat{k}(t,\tau)/\partial t > 0$, τ(k,k**) is the first time that $\hat{k}(t,\tau;K,C,0)$ hits k** starting at t=0 with an aggregate capital stock k.

We show in appendix C that if a switch to Private Property will occur at τ, and if group i is following consumption policy (26), then on [t,τ) the unique consumption policy that maximizes j's payoff is (26). The same holds for group i. Since for any pair (k,k**) τ is uniquely determined by (29), it follows that the pair of consumption policies defined by (26) and (29) are best responses to each other starting from any node (K,C,0), given that both groups follow rule S0. Substituting (26) and (27) in (5'') we obtain the value of waiting conditional on both groups following switching rule S0

$$W_0(k,k^{**}) = \frac{[zk]^{\frac{\sigma-1}{\sigma}}\sigma/[\sigma-1]}{[2-\sigma+(D-2+\sigma)e^{-z\tau}]^{\frac{2(\sigma-1)}{\sigma(2-\sigma)}}}\int_0^\tau \frac{[2-\sigma+(D-2+\sigma)e^{-z(\tau-s)}]^{\frac{\sigma-1}{2-\sigma}}}{e^{\frac{z}{2-\sigma}s}}ds + e^{-b\tau}[w_1(k^{**})-q_m] \quad (30)$$

The next step is to determine the conditions under which switching rule S0 is a best response to itself. Note that the structure of the preemption game during this phase is the same as that of the second phase: here the leader is the group that builds a wall first, there it was the one that destroyed the other's wall first. The payoff functions associated with leading and following are also the same, up to a constant. The only difference is that during this phase the payoff of matching is not $J_m(k)$-p_m but $w_1(k)$-q_m. However, in both cases the payoff to leading increases at a faster rate than the payoff to matching as shown in (15) and (25). Consequently, we can take the same steps as in the previous subsection.

A pair of switching rules S0 are best responses to each other if and only if there exist no preemption opportunities along the waiting path, and at k** matching is preferred to following. During this phase, the no-preemption and the matching conditions are

$$(a)\ W_0(k,k^{**}) > J_l(k)-q_l \quad \forall k\in[k_0,k^{**}], \quad and \quad (b)\ w_1(k^{**})-q_m \geq 0 \quad (31)$$

where J_l and w_1 are given by (11) and (20). We will show that these conditions are satisfied for any k** in the interval [k',k], with

$$J_l(\underline{k}) - q_l = 0, \qquad w_1(\underline{k}') - q_m = 0 \tag{32}$$

$\underline{k}'$ and $\underline{k}$ are the levels of k at which the values of matching and leading are equal to zero, respectively.

For Common Property to prevail for a positive amount of time, and for a switch to Private Property to take place it is necessary that the values of q_l and q_m be such that $\underline{k} > k_0$ and $\underline{k}' < \underline{k}$. These conditions are analogous to those of the previous subsection, and insure that the payoffs can be drawn as in figure 2. First, $\underline{k} > k_0$ if and only if $q_l > J_l(k_0)$, as stated in assumption (8a). Second, it follows from (25) that $\underline{k}' < \underline{k}$ if and only if at $\underline{k}$ the value of matching is non-negative: $w_1(\underline{k}) \geq q_m$. Substituting the value of $\underline{k}$ into (30), this condition becomes $q_m \leq q_l[1-(\sigma-1)e^{-zT}][2-\sigma e^{-zT}]^{(1-\sigma)/\sigma}$. Note that condition (8c) requires q_m to be not greater than the infimum of the right hand side in the previuos inequality. Note also that this restriction implies $q_l > q_m$[14]. Given that q_l and q_m satisfy (8), the matching condition (31b) holds for any $k^{**} \geq \underline{k}'$ because $dw_1(k)/dk > 0$, and the no-preemption condition (31a) holds for any $k^{**} \leq \underline{k}$. To show this note that: (i) The value of waiting is positive for any $k^{**} > \underline{k}'$, while the value of leading is negative for $k < \underline{k}$ (by (25), (30) and (32)); (ii) $\underline{k}' < \underline{k}$ (by (8c) and (25)); and (iii) along the path generated by (27), $\tau(k,k^{**})$ is the first time that $\hat{k}(t,\tau;K,C,0)$ hits

[14] $q_l > q_m$ is consistent with the idea that it less costly to introduce a system to enforce contracts which ex-ante treats everyone equally than a system which favors only one group.

k^{**}[15].

The last step is to bring the two pieces together. We have shown that starting at any node (K,C,0), (26) and (29) define a pair of consumption policies which are best responses to each other, conditional on both groups switching to Private Property at k^{**}. We have also shown that if the p's and q's satisfy (8), the matching and no-preemption conditions are simultaneously satisfied for any k^{**} in $[\min(k_0,\underline{k}'), \underline{k}]$. Thus, if group j is using consumption policy (1) and switching rule S0, the best response of group i is to follow rule S0. Hence, if conditions (7) and (8) are satisfied, the pair of Markov strategies (ψ_{i0},ψ_{j0}) where

$$\psi_{h0} = \psi_h(K,C,0) = \begin{cases} I_h=0, & c_h=zk/[2-\sigma+e^{-z\tau(k,k^{**})}(D-2+\sigma)] \qquad \text{if } k<k^{**} \\ I_h=1, & c_h= \text{as in } \psi_{h1} \qquad \text{if } k\geq k^{**} \end{cases}$$

with k^{**} in $[\min(k_0,\underline{k}'), \underline{k}]$ forms a Markov Perfect equilibrium for the game starting at any node (K,C,0). According to strategy ψ_{h0}, if no switches have occurred yet, as long as Common Property prevails and $k<k^{**}$, group h should not switch and should use consumption policy (27). If $k \geq k^{**}$ it should switch and use the consumption policy corresponding to Private Property, provided the other group switches simultaneously; otherwise it should use the leader's consumption policy forever. As in the previuos subsection, there exist multiple equilibria indexed by the switching level of capital k^{**}.

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[15]We did not consider the case $k^{**}\in(\underline{k},\underline{k}'']$ where $\underline{k}''$ is defined by $J_l(\underline{k}'')-q_l=J_m(\underline{k}'')-q_m$ because since (30) is rather complicated, we cannot check analytically whether the no-preemption condition is satisfied in this range. For the case $k^{**}>\underline{k}''$, the no-preemption condition is violated for the same reasons as in the previous subsection.

III.4 Summary

We have constructed a Markov perfect equilibrium in which an economy switches from Common to Private Property when it becomes sufficiently rich ($k>k^{**}$) so that it pays each group to incur the one-time loss necessary to institute Private Property. However, Private Property does not last forever. Once this economy becomes very rich ($k>k^{*}$) it becomes profitable for each group to incur the one-time loss necessary to erode private property rights, and a switch back to Common Property takes place.

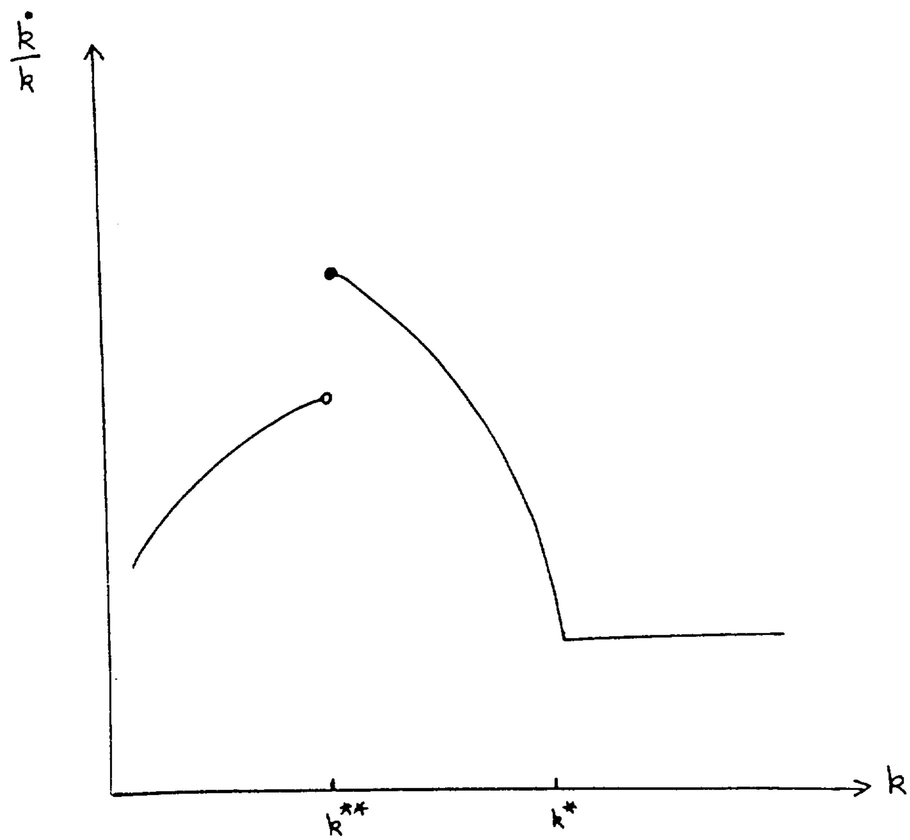
This sequence of institutional changes generates a time-varying growth rate which is depicted in figure 3, and is given by:

$$\frac{\dot{k}(t)}{k(t)} = \begin{cases} a - \dfrac{2z}{2 - \sigma + (D-2+\sigma)e^{-z(\tau-t)}} & \text{if } k<k^{**} \\ a - \dfrac{2z}{2 - \sigma e^{-z(T+\tau-t)}} & \text{if } k\in[k^{**},k^{*}) \\ a - \dfrac{2z}{2 - \sigma} & \text{if } k \geq k^{*} \end{cases} \tag{33}$$

τ is the time at which the switch from Common to Private Property occurs, and $T+\tau$ is the time at which the switch from Private back to Common Property takes place. As shown in figure 3, the growth rate increases during the first phase when Common Property prevails. It jumps up when the shift to Private Property occurs (at k^{**}). Thereafter, the growth rate follows a decreasing path until the second switch back to Common Property occurs (at k^{*}). At this point the growth rate becomes constant.[16]

[16] 'σ' is the elasticity of intertemporal substitution, 'a' the productivity of capital, 'δ' the rate of time preference, $z=a(1-\sigma)+\delta\sigma>0$, and 'D' is a constant defined in (27). $\tau(k_0,k^{**})$ and $T(k^{**},k^{*})$ are given by (19) and (29) respectively. Note that the growth rate is higher in the second phase than in the third because $\sigma<2$ and $a-2z/(2-\sigma e^{-z(T+\tau-t)}) \geq a-2z/(2-\sigma) = \sigma(a-2\delta)/(2-\sigma) > 0$.

Figure 3



Since production technology is linear, the transition dynamics in (33) reflect only switches in property rights generated by interest group competition. However, the actual growth rate of any economy will also reflect very important factors such as those identified in the growth literature: human capital accumulation, production externalities and technological innovation.

Note that in our model interest groups' competition affects growth in two ways: directly, by determining the property rights regime and thus the relevant accumulation equation, and indirectly through the savings rate. Here is an intuitive explanation: at the time of the switch to Private Property, each group experiences a sudden drop in its wealth because it losses access to half of aggregate capital. Moreover, the private rate of return increases because each group acquires exclusive access to its share of capital. Both of these effects reduce consumption at the time of the switch, and the growth rate jumps up. Before this switch, even though Common Property prevails, the growth rate is increasing and is higher than in an economy where Common Property will prevail forever because a switch to Private Property is forthcoming. As this switch gets closer, groups reduce their level of appropriation of common resources because they discount less the fact that each group's wealth will fall and the rate of return will increase. Therefore, the closer the switch date, the closer their behavior to that which prevails under Private Property.

The declining growth rate after the switch to Private Property reflects anticipation of the switch to Common Property. Each group behaves as if in the future it will win a lottery and the interest rate will fall. The lottery is gaining access to the entire capital stock, and the fall in the private rate of return is due to the fact that the other group will be appropriating a share of the common capital stock. The closer the switch, the less this event is discounted, and the savings rate falls.

IV. Concluding Remarks

Differences in property rights are an important factor in explaining why growth rates differ across countries. Similarly, improvement and erosion of property rights may help explain why an economy rises and declines. On the one hand, in the economic growth literature the institutional framework is typically taken as given, and the growth path is the result of optimizing behavior. On the other hand, the literature on institutional change does not offer parsimonious optimizing models. In this paper we developed a minimal growth model where there is endogenous institutional change and agents are optimizing.

In the model switches in property rights regimes are generated by the attempt of each rent-seeking group to secure for itself access to a greater share of the pie. Switches do not occur frequently because it is costly for interest groups to change the regime. We considered an economy where initially there are no institutions to protect private property rights, and the growth rate is low. There is a shift to Private Property when the economy becomes rich enough so that it is worthwhile to incur the costs of creating institutions to defend private profits. Lastly, as the economy becomes very rich rent-seeking becomes profitable, leading interest groups to erode these institutions, and the economy comes back to Common Property.

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APPENDIX

A. The third phase: Common Property

Here we derive a solution for the differential game where each player maximizes the first term in (5) subject to (2) and (4). First we find a pair of Markov strategies ($\ddot{C}_i$(k), $\ddot{C}_j$(k)) which satisfies the necessary conditions for an equilibrium. Afterwards we will show that if group j follows consumption policy $\ddot{C}_i$(k), then j's unique best reply is to set $c_j = \ddot{C}_j$(k). Hence, we will conclude that the pair ($\ddot{C}_i$(k), $\ddot{C}_j$(k)) forms a Markov perfect equilibrium starting at any node (K,C,2). In order to solve the first step let j's consumption policy be $\ddot{C}_j = \beta_j$k (β_j is a constant to be endogenously determined), and let us disregard restrictions (4) (it will turn out that it will be satisfied). It follows that i's Hamiltonian is

$$H_i(k,c_i,\hat{c}_j(k),\mu_i) = [\sigma/(\sigma-1)]c_i^{(\sigma-1)/\sigma}e^{-\delta t} + \mu_i[ak-\hat{c}_j(k)-c_i] \tag{A1}$$

Thus i's optimal policy ($\ddot{C}_i$) must satisfy the following conditions

$$\partial H_i/\partial c_i = \hat{c}_i(t)^{-1/\sigma}e^{-\delta t} - \mu_i(t) = 0 \tag{A2}$$

$$\dot{\mu}_i(t) = -\partial H_i/\partial k = -\mu_i(t)[a-\beta_j] \tag{A3}$$

$$\lim_{t\to\infty} \mu_i(t)\hat{k}(t) = 0 \tag{A4}$$

Now let i's optimal policy take the form $\ddot{C}_i = \beta_i$k. It follows that $\ddot{C}_j$ must satisfy conditions analogous to (A2)-(A4) interchanging j and i. In order for the β's to be best responses to each other it is necessary that they satisfy the previous six conditions plus $\dot{c}_i/c_i = \dot{c}_j/c_j = \dot{k}/k$. Differentiating (A2), and using (2) and (A3) we have

$\dot{k}/k = a-\beta_i-\beta_j = \dot{c}_i/c_i = \sigma(a-\delta-\beta_j) = \dot{c}_j/c_j = \sigma(a-\delta-\beta_i)$. These equations are simulataneously satisfied if and only if $\beta_i = \beta_j = [a(1-\sigma)+\delta\sigma]/(2-\sigma) = z/(2-\sigma)$. Thus, our candidates are $\hat{C}_i = \hat{C}_j = kz/(2-\sigma)$, and the resulting capital stock path is given by (12). Next we check that the transversality condition is satisfied: substituting (12) in (A1) we get $\mu_h(t) = [zk/(2-\sigma)]^{-1/\sigma}\exp(-\delta t + [2\delta - a]t/[2-\sigma])$. Substituting this expression and (13) in (A4) we get

$$\lim_{t\to\infty}\mu_h(t)\hat{k}(t) = [zk/(2-\sigma)]\lim_{t\to\infty}e^{-z/(2-\sigma)t} = 0, \qquad h=i,j$$

The limit is zero because $\sigma<2$ and $z>0$ by (7).

Now we turn to the second step: we set $\hat{C}_j = kz/(2-\sigma)$ and we let group i choose its optimal consumption policy. This is a standard optimal control problem, the Hamiltonian is given by (A1) replacing β_j by $z/(2-\sigma)$. Since the Hamiltonian is strictly concave in (c_i,k) for every t --the instantaneous utility function is strictly concave and the production function is linear, and since constraint (4) is linear in (c_i,k), it follows that if an admissible pair $(\hat{C}_i, \hat{K})$ is such that (A2)-(A4) and (4) are satisfied, then this pair is the unique solution to i's control problem (theorem 4.5 of Seierstad and Sydsaeter (1987)). Thus $\hat{C}_i = kz/(2-\sigma)$ is i's best response to $\hat{C}_j = kz/(2-\sigma)$. Lastly, since the same holds for group j, we conclude that the pair of Markov strategies $\hat{C}_i(k) = \hat{C}_j(k) = kz/(2-\sigma)$ forms a Markov perfect equilibrium starting at any node (K,C,2).

B. The Second Phase

Here we find the optimal consumption policy that maximizes (5') subject to (1), (4) and fixed T. The Hamiltonian of group i is $H_i = (\sigma/\sigma - 1)c_i^{(\sigma-1)/\sigma}e^{-\delta t} + \pi[ak_i - c_i]$. It follows that i's consumption policy must satisfy the following first order conditions

$$c_i(t)^{-1/\sigma}e^{-\delta t} = \pi(t), \qquad \dot{\pi}/\pi = a \tag{A5}$$

$$\pi(T) = \partial(e^{-\delta T}J_m(k(T)))/\partial k_i = e^{-\delta T}\big[[k_i(T)+k_j(T)]z/(2-\sigma)\big]^{-1/\sigma} \tag{A6}$$

From (A5) it follows that $c_i(t) = c_i(0)\exp([a-\delta]\sigma t)$. Substituting this expression in (1) and solving the differential equation we get

$$k_i(t) = e^{at}\big[k_i(0) + c_i(0)[e^{-zt} - 1]/z\big], \qquad 0 \le t \le T \tag{A7}$$

Transversality condition (A6) implies that $c_i(t)$ must satisfy

$$c_i(t) = \big[z[k_i(T)+k_j(T)]/(2-\sigma)\big]e^{-\sigma(a-\delta)(T-t)} \tag{A8}$$

Since group j is following the same switching rule as group i and is solving an identical problem, we have that $k_j(t)$ and $c_j(t)$ are given by (A7) and (A8) interchanging i and j. To obtain consumption policy (16) we set t=T in (A7), and substitute the values of $k_i(T)$ and $k_j(T)$ into (A8). To obtain the alternative expression for consumption which appears in the text, we substitute in (A8) only the value of $k_i(T)$, not the value of $k_j(T)$. Substituitng (16) in (A7) we have that group h's capital is given by

$$\hat{k}_h(t,T;K,p,1) = \frac{e^{at}}{2-\sigma e^{-zT}}\big\{k_h(0)[1+e^{-zt}-\sigma e^{-zT}] + k_{-h}(0)[e^{-zt}-1]\big\} \quad h=i,j \tag{A9}$$

Now we show that the pair of consumption policies (16) are best replies to each other, given that both groups will switch at T. Suppose that group j follows consumption policy (16). Since: (i)$\dot{c}_i$ satisfies necessary conditions (A5)-(A6); (ii)the Hamiltonian is strictly concave in

(c_i, k_i) for each t; (iii)the scrap value function is strictly concave in k_i $- - d^2 J_m / dk^2 = -\sigma^{-1} k^{-(1+\sigma)/\sigma} [(2-\sigma)/z]^{1/\sigma} < 0$, and (iv)constraint (4) is linear, it follows from theorem 3.4 of Seierstad and Sydsaeter (1987) that (16)-(17) is the unique solution to group i's problem. The same holds for group j.

To obtain payoff function (20) we use the fact that consumption grows at a rate σ(a-δ) during the second phase, and at a rate σ(a-2δ)/(2-σ) during the third phase. Thus, from (5') we have

$$W(k,k^*) = \int_0^T \frac{\sigma}{\sigma-1}\left[c_o e^{\sigma(a-\delta)t}\right]^{\frac{\sigma-1}{\sigma}} e^{-\delta t} dt + \int_T^\infty \frac{\sigma}{\sigma-1}\left[c_o e^{\sigma(a-\delta)T} e^{\sigma(a-2\delta)(t-T)/(2-\sigma)}\right]^{\frac{\sigma-1}{\sigma}} e^{-\delta t} dt - q_m e^{-\delta T} \quad \text{(A10)}$$

where $c_o = zk/[2-\sigma\exp(-zT(k*,k))]$. The derivatives of (17) are

$$\frac{\partial \log(\hat{k}(t,T))}{\partial t} = a - \frac{2z}{2-\sigma e^{-\sigma(T-t)}} \geq \frac{\sigma(a-2\delta)}{2-\sigma} > 0, \qquad \frac{\partial^2 \log(\hat{k}(t,T))}{\partial t^2} = \frac{-2\sigma z^2 e^{-z(T-t)}}{[2-\sigma e^{-z(T-t)}]^2} < 0 \quad \text{(A11)}$$

$$\frac{\partial \log(\hat{k}(t,T))}{\partial T} = \frac{2z}{2-\sigma e^{-z(T-t)}} - \frac{2z}{2-\sigma e^{-zT}} > 0 \qquad \frac{\partial^2 \log(\hat{k}(t,T))}{\partial t \partial T} = \frac{2\sigma z^2 e^{-z(T-t)}}{[2-\sigma e^{-z(T-t)}]^2} > 0 \quad \text{(A12)}$$

$$\frac{d\hat{k}(T,T)}{dT} = \frac{\partial \hat{k}(t,T)}{dt}\Big|_{t=T} + \frac{\partial \hat{k}(t,T)}{\partial T} > 0$$

Next, we obtain the derivatives of (20). It will be useful to express $W_1(k,k^*)$ as $X(k,T(k,k^*))$.

$$\frac{\partial W_1}{\partial k*} = \frac{\partial X}{\partial T}\frac{\partial T}{\partial k*}, \qquad \frac{\partial W_1}{\partial k} = \frac{\partial X}{\partial k} + \frac{\partial X}{\partial T}\frac{\partial T}{\partial k}, \qquad \frac{\partial w_1(k^{**})}{\partial k^{**}} = \frac{\partial X}{\partial k}\Big|_{k=k^{**}} \quad \text{(A13)}$$

$$\frac{\partial X}{\partial T} = \frac{\sigma[zk]^{(\sigma-1)/\sigma}[1-e^{-zT}]}{e^{-zT}[2-\sigma e^{-zT}]^{(2\sigma-1)/\sigma}} > 0, \qquad \frac{\partial X}{\partial k} = \frac{E}{[zk]^{1/\sigma}} > 0, \qquad E = \frac{1-(\sigma-1)e^{-zT}}{[2-\sigma e^{-zT}]^{(\sigma-1)/\sigma}} \in (0,1)$$

The signs follow from $z > 0$, $\sigma < 2$, $a > 2\delta$, $dT/dk < 0$, $dT/dk^* > 0$ (see (19)). $E(T) < 1$ because $E(T=0) = [2-\sigma]^{1/\sigma} < 1$, $E(T=\infty) = 2^{(1-\sigma)/\sigma} < 1$ and $dE/dT = z[\sigma-1]e^{-zT}[1-e^{-zT}][2-\sigma e^{-zT}]^{(1-\sigma)/\sigma} > 0$. The ordering in (25) follows from the fact that $dJ_1(k)/dk=[zk]^{-1/\sigma}$, $E<1$ and $dT/dk<0$.

C. The First Phase: Common Property

Here we derive a solution for the differential game where each group maximizes (5'') subject to (2) and (4), and a fixed τ. We will take the same steps as in section A. First we let group h's consumption policy be $\hat{c}_h=\gamma_h(t)k$. It follows that i's Hamiltonian is $H_i=[\sigma/\sigma-1]c_i^{\sigma/\sigma-1}e^{-\delta t}+\nu_i[ak-c_i-\gamma_j(t)k]$. The first order conditions are given by (A2)-(A3) substituting $(\nu_i,\gamma_j(t))$ for (μ_i,β_j), and the transversality condition

$$\nu_i(\tau) = \partial(e^{-\delta\tau}w_1(k(\tau)))/\partial k(\tau) = [k(\tau)z]^{-1/\sigma}e^{-\delta\tau}\frac{1-(\sigma-1)e^{-zT}}{[2-\sigma e^{-zT}]^{(\sigma-1)/\sigma}} \quad \text{(A14)}$$

In (A14) we used $dT(k(\tau),k^*)/dk(\tau)=0$, which follows from the fact that we are restricting attention to cases where $k^*/k(\tau)$ is constant. The first order conditions for group j are analogous to i's interchanging subindexes i and j. Another condition that the γ's must satisfy is $\dot{c}_h/c_h=\dot{k}/k+\dot{\gamma}_h/\gamma_h$, $h=i,j$. Differentiating (A2) and using (2) and (A3) we have

$$\frac{\dot{k}}{k} = a-\gamma_i-\gamma_j = \sigma[a-\delta-\gamma_j]-\frac{\dot{\gamma}_i}{\gamma_i} = \sigma[a-\delta-\gamma_i]-\frac{\dot{\gamma}_j}{\gamma_j} \quad \text{(A15)}$$

These equations imply that $\dot{\gamma}_i/\gamma_i-\dot{\gamma}_j/\gamma_j=\sigma[\gamma_i-\gamma_j]$. Thus, since $\gamma_i(\tau)=\gamma_j(\tau)$ (by (A14)), it follows that $\gamma_i(t)=\gamma_j(t)=\gamma(t)$ $\forall t\leq\tau$. By substituting this in (A15) we have that $\gamma(t)$ must satisfy $\dot{\gamma}=z\gamma+(2-\sigma)\gamma^2$. The general solution to this differential equation is

$$\gamma(t) = z[ze^{zt}/x + (2-\sigma)]^{-1} \quad \text{(A16)}$$

where x is a constant. To determine its value use (A2) and (A14) to get $\hat{c}_i(\tau)$. Since $\gamma(\tau)=c_i(\tau)/k(\tau)=z/D$, we have that $x=z/[e^{-z\tau}(D-2+\sigma)]$. Thus

$$\gamma(t) = z[2-\sigma + e^{-z(\tau-t)}[2-\sigma + D]]^{-1}, \qquad D=[1-(\sigma-1)e^{-zT}]^{-\sigma}[2-\sigma e^{-zT}]^{\sigma-1}>1 \quad \text{(A17)}$$

'D' is greater than one because $D=E^{-\sigma}$ and $0<E<1$.

To obtain (26) substitute (A17) in $c=\gamma(t)k$. To obtain (27) substitute (26) in (2) and define $Y=\log(k)$. It follows that

$$dy(s)/ds = a-\frac{2z}{r + qe^{zs}}; \quad thus, \quad y(s) = as-\frac{2zs}{r}+\frac{2\log(r+qe^{zs})}{r}+x \qquad \text{(A18)}$$

where x is a constant, $r=2-\sigma$ and $q=[D-2+\sigma]\exp(-z\tau)$. By using the initial condition $k(0)=k$ and by setting $k(s)=\exp(y(s))$ in (A18) we obtain (27).

Now we show that the pair of consumption policies defined in (26) are best replies to each other, given that both groups will switch at τ. Set $\hat{C}_j$ as in (26) and let group i choose its optimal consumption policy. Since: (i)$\hat{C}_i$ satisfies necessary conditions (A2), (A3) and (A14); (ii)the Hamiltonian of i's control problem is strictly concave in (c_i,k) for every t; (iii)the scrap value function is strictly concave in k_i (see A13); and (iv)constraint (4) is linear, it follows from theorem 3.4 of Seierstad and Sydsaeter (1987) that (26)-(27) is the unique solution to group i's problem. The same holds for group j. Lastly the derivatives of (27) are

$$\frac{\partial \hat{k}(t,\tau)}{\partial t} = a - \frac{2z}{F} \geq a-\frac{2z}{2-\sigma} > 0 \qquad \frac{\partial^2 \hat{k}(t,T)}{\partial t^2}=\frac{2z^2(D-2+\sigma)e^{-z(\tau-t)}}{F^2} > 0 \qquad \text{(A19)}$$

$$\frac{\partial \hat{k}(t,\tau)}{\partial \tau} = \frac{-2ze^{-z(\tau-t)}(D-2+\sigma)}{(2-\sigma)F} < 0 \qquad \frac{\partial^2 \hat{k}(t,T)}{\partial t \partial \tau}=\frac{-2z^2 e^{-z(\tau-t)}(D-2+\sigma)}{F^2} < 0 \qquad \text{(A20)}$$

$$\frac{d\hat{k}(\tau,\tau)}{d\tau} = a-\frac{2z}{2-\sigma}\cdot\frac{2-\sigma+2(D-2+\sigma)e^{-z(\tau-t)}}{F}>0 \qquad F=2-\sigma+(D-2+\sigma)e^{-z(\tau-t)}>0 \qquad \text{(A21)}$$