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KEEPING PEOPLE OUT: INCOME DISTRIBUTION, ZONING AND THE QUALITY OF PUBLIC EDUCATION

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ABSTRACT

This paper examines the effect of community zoning regulations on allocations and welfare in a two-community model. Each community uses a local property tax to finance public education. Tax rates are determined by majority vote within each community, and individuals choose in which community to reside. We study exogenously imposed zoning regulations as well as the case where the regulator is endogenously determined by majority vote. Our analysis indicates that a number of outcomes are theoretically possible. Several interesting results emerge from simulations of the model. Although zoning tends to make the rich community more exclusive, this need not increase the quality of education in the rich community relative to the poor community. Welfare effects are not monotone in income; some lower income individuals benefit and some higher income individuals are made worse off when zoning is introduced.

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1. Introduction

A striking feature of US public education, as illustrated in Table 1A, is the large disparity that exists across communities in spending per student. Any explanation of this outcome and any attempt to change it must contend with two factors. First, US public education is financed to a large degree by local property taxes.[1] Second, the process by which individuals choose where to reside results in great differences in average income across communities, even within the same metropolitan area. The goal of this paper is to examine the role that one pervasive institutional feature in the US--community zoning regulation--plays in producing this outcome via its interaction both with the property tax and with the creation of communities that differ with respect to average income.

Our analysis starts by providing a benchmark of a two-community model without zoning in which each community taxes the housing property of its residents and uses the proceeds to fund public education. Tax rates within each community are chosen by majority voting. Individuals differ only in income and derive utility from housing, the consumption of a private good, and the quality of public education they obtain. Individuals decide in which community to reside. All equilibria of this model in which the quality of education varies across communities are stratified, i.e., they are characterized by the existence of a rich community with a high tax rate and high spending per student, and a poor community with a lower tax rate and

[1]On average 45% of spending on primary and secondary public education is financed by local taxes.

lower spending per student. All individuals in the rich community have income at least as great as those in the poor community.

We then examine the effects on equilibrium allocations and welfare that result when the rich community imposes a zoning regulation. The only zoning regulation we examine is one that requires individuals to purchase a minimum level of housing as a condition for residence in the community. We analyze both an exogenously imposed zoning regulation as well as one endogenously determined by a process of majority vote.

At a general level the analysis suggests a wide range of possible outcomes, so we explore some of them via simulation. As might be expected, in all of our simulations we find that the introduction of zoning results in the rich community becoming more exclusive (i.e. smaller and richer) and the poorer community becoming larger. The outflow of the least wealthy individuals of the rich community from that community into the poor one necessarily increases average income in both communities.

Several interesting results are associated with the change in equilibrium community composition. We begin with the implications for welfare. Whereas there may be a general presumption that zoning benefits the rich at the expense of the poor, the actual welfare effects tend to be more subtle than this. While indeed zoning tends to make the poorest individuals worse off and the richest indviduals better off, the highest income individuals that reside in the poor community (in the no-zoning equilibrium) are also made better off, and the lowest income individuals that reside in the rich community (in the no-zoning equilibrium) are in fact made worse off. Of the individuals that actually move as a result of zoning, those with the highest income are made worse off; those with the lowest income are made better off. Thus, it is the welfare effects of zoning on those individuals that are somewhere in the "middle" of the distribution that are the most complex. This is due to the effect on individual welfare of the interaction of two factors (i) the increase, at each tax rate, of each community's property base, and (ii) the existence of a new, wealthier median voter in each community. In general, the effect of zoning on the absolute gap in spending per student between communities is ambiguous.

Finally, zoning allows the rich community to be characterized both by higher spending per student and by lower tax rates, in contrast with the no-zoning model which required both equilibrium spending per student and the tax rate to be higher in the rich community in equilibrium. This is significant since in reality there is not a perfect correlation between average income and tax rates across communities.

Endogenizing the level of zoning significantly complicates the analysis since it is not possible to identify the outcome of majority vote (if it exists) with the level of zoning preferred by the individual with median income. We provide two examples where equilibrium exists but the individual with median income is no longer the decisive voter in the rich community. The effects of the endogenously chosen zoning level on equilibria follow naturally from the comparative statics exercises of the exogenous zoning model.

Although various aspects of zoning have been studied extensively, there has been little work done on zoning in the context of publicly provided goods in multi-community models.[2] In Hamilton (1975), communities use property

[2]See Pagodzinski and Sass (1990,1991) for reviews of the theoretical and empirical literatures on zoning.

taxes to finance locally publicly provided goods. In this model, communities are formed costlessly.[3] Hence, endogenous zoning expressed as a minimum housing requirement allows individuals to perfectly separate themselves out by income and thus delivers, not surprisingly, an efficient allocation of resources. Durlauf (1992) presents a dynamic community model in which communities provide a local public good--education--and impose minimum income restrictions as a requirement for residence in a community (there is no housing market). His analysis, however, is concerned exclusively with the long-run properties of the income distribution in a framework with local and global peer effects. Henderson (1980) and Epple, Romer and Filimon (1988) analyze the endogenous choice of zoning regulations in multi-community models. Their focus, however, is on implications for the pattern of land use given the existence of residents concerned about the characteristics of potential entrants into the community. There are no publicly provided goods in this model.

The paper is organized as follows. Section 2 studies a multicommunity model with no zoning. Section 3 introduces zoning into the rich community and analzyes the comparative statics implications of different exogenous levels of zoning. Section 4 endogenizes the zoning restriction and section 5 concludes.

2. The Model

In this section we study a model of two communities with no zoning which will serve as a benchmark for the subsequent analysis in which zoning is

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[3] In our model we do not allow new communities to be formed (e.g. there is an infinite cost to forming new communities). Reality undoubtedly lies between these two extremes.

introduced. Multi-community models have been analyzed by several authors. Westhoff (1978) and Epple, Filimon and Romer (1984) examine the existence of equilibrium in multi-community models with local publicly provided goods. Epple and Romer (1991) analyze redistribution by local governments in a multi-community setting. Fernandez and Rogerson (1992) examine educational policies in a two-community model in which education is publicly provided at the community level. de Bartolome (1991) and Benabou (1991) study different versions of a multi-community model with peer effects.

There is a continuum of individuals with identical preferences given by

$$u(c,h) + v(q) \tag{1}$$

where c is consumption of a private good, h is the amount of housing purchased by the individual, and q is the quality of public education. We assume that $u(c,h)$ is twice continuously differentiable, concave, with $u(0,h)=-\infty\ \forall h$, and that c and h are normal goods. Individuals differ with respect to their endowed income y whose distribution is characterized by a continuous density function $g(y)$ with support $[\underline{y},\bar{y}]$, $\underline{y}>0$. We normalize the mass of individuals to equal one.

There are two communities (indexed by $j=1,2$). Each community is characterized by a proportional tax rate t on the value of housing and by a quality of public education. We let c be the numeraire and p be the relative (pre-tax) price of housing. For simplicity we assume that p is constant and equal in both communities.[4] Each individual must choose a community in which to live. The budget constraint faced by an individual in community j is:

[4]A variable p would introduce additional complications without increasing the insights available from the analysis.

$$c + \pi_j h = y \tag{2}$$

where $\pi_j = p(1+t_j)$.

Given residence in community j, the indirect utility of an individual with income y is

$$V(\pi_j, q_j, y) = u(y - \pi_j h(\pi_j, y), h(\pi_j, y)) + v(q_j) \tag{3}$$

where $h(\pi_j, y)$ is the individual's housing demand function.

The quality of education is assumed to equal the amount of spending per resident within the community.[5] Thus,

$$q_j = t_j p \bar{h}_j \tag{4}$$

where $\bar{h}_j$ is the average amount of housing consumed in community j (henceforth denoted C_j), i.e.

$$\bar{h}_j = \frac{H_j}{N_j} = \frac{\int_j h(\pi_j, y)\,dy}{N_j} \tag{5}$$

N_j equals the mass of individuals located in C_j, H_j is the total amount of housing purchased in C_j, and $\int_j$ indicates the integral over those individuals that reside in C_j.

We assume that the game among individuals and between communities is played in the following fashion. In the first stage individuals

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[5]Linearity of q is chosen solely for simplicity of exposition; it can easily be extended to increasing concave functions of expenditures per student. The relation between expenditures and quality has been the subject of much controversy. See Card and Krueger (1992) for a review of this debate and evidence in favor of a positive effect of spending per student on quality.

simultaneously choose a community in which to live. Once this choice is made, they are unable to move in any subsequent stage. In the second stage, each community's tax rate is (simultaneously) decided upon by majority vote within that community and individuals consume the private good, purchase housing, and obtain education.

Denote the equilibrium tax and quality outcomes in each community by (t_1^*, q_1^*) and (t_2^*, q_2^*). Taking these outcomes as given, therefore, in any subgame-perfect equilibrium of the above game each individual must reside in the community in which her utility is highest. Note that, when voting, individuals are aware of the effect of different choices of tax rates on the gross price of housing π_j and on the quality of education (via the community's budget constraint as expressed in equations 4 and 5).[6]

It is difficult to characterize the equilibria to this game without imposing an additional assumption on preferences. This is done in Assumption 1.[7]

Assumption 1: $S = u_{cc}h(1-\pi h_y) + u_{ch}hh_y + u_c h_y < 0$

The significance of this assumption is easily understood by noting that the slope of an individual's indifference curve in q-t space is:

$$\frac{v'}{u_c ph} \qquad (6)$$

[6]This is in contrast with Rose-Ackerman (1979) and Epple, Filimon, and Romer (1984) who assume that voters do not take into account the effect of tax changes on the aggregate housing stock demanded in a community.

[7]Westhoff (1977) provides the first use of this kind of (single-crossing) condition to characterize equilibria in a multi-community model. Different versions of this condition have been employed by Roberts (1977), Epple, Filimon and Romer (1984), and Epple and Romer (1991).

Thus, Assumption 1 guarantees that the slope of an individual's indifference curve (in q-t space) is increasing in initial income (i.e. $u_c ph$ is decreasing in y). The power of this assumption to characterize equilibria is apparent in Proposition 1. First, though, we establish that in equilibrium no community can be empty.

Lemma 1: In equilibrium no community is empty.

Proof: If a community were empty then an individual with $y=\bar{y}$ could, by moving into that community, obtain the same quality of education there at a lower tax rate since the that community would possess a strictly higher average housing demand than the other community. Hence, this individual would be made strictly better off than before. Consequently, the initial allocation could not have been an equilibrium.||

Proposition 1: If in equilibrium communities have different qualities of education, then:

(i) $(q_1^*, t_1^*) >> (q_2^*, t_2^*)$.

(ii) The income of every individual in C_1 is at least as great as that of any individual in C_2,

where C_1 is arbitrarily defined as the community with the larger q^*.

Proof: (i) If $q_1^* > q_2^*$ then necessarily $t_1^* > t_2^*$ otherwise all individuals prefer C_1 to C_2. (ii) By Assumption 1, if an individual with income $\tilde{y}$ prefers (q_1^*, t_1^*) to (q_2^*, t_2^*) then so does every individual with income $y \geq \tilde{y}$.||

Thus these equilibria have individuals stratified into communities by initial income. C_1 will be characterized by a higher tax rate, a higher quality of education and by higher income residents than C_2. We refer to an equilibrium with these properties as a <u>stratified equilibrium</u>. Hereafter we will focus only on those equilibria in which the qualities of education are different across communities.

<u>Proposition 2</u>: Majority vote within a community results in t^* equal to a tax rate preferred by the voter whose income is median within the community.

<u>Proof</u>: This follows directly from Assumption 1. The argument is illustrated in Figure 1. Let $\hat{y}$ be the median income in the community. Consider the tax-quality possibility frontier faced by the median voter. Let $z^*=(q^*,t^*)$ be the (q,t) pair preferred by the median voter. Any other (q,t) pair that lies in the area A (that is to the left of the median voter's indifference curve through z^* and with $t>t^*$) will be rejected in favor of z^* by at least 50% of the voters (i.e. at least by all voters with $y\leq\hat{y}$). Similarly any (q,t) pair that lies in area B (that is to the left of the median voter's indifference curve and with $t<t^*$) will be rejected in favor of z^* by at least 50% of the voters (i.e. at least by all voters with $y\geq\hat{y}$). Any point in C must lie outside the tax-quality frontier. Hence z^* is the majority vote outcome.[8] ||

All potential stratified equilibria can be parametrized by the level of income of the "boundary" individual, y_b, where the latter defines a partition of the population into the two communities such that all $y>y_b$ reside in C_1 and all $y<y_b$ reside in C_2. We define $W_j(y_b)$ to equal the utility of an individual with income y_b that resides in C_j given the partition defined by y_b and the tax rate and quality that result from majority vote given such a partition.

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[8]Epple and Romer (1991) use this reasoning very nicely to show that even if individuals were free to move between communities in response to different tax-quality outcomes, majority voting must result in the preferred tax rate of the median voter.

If a stratified equilibrium exists there must be an individual with $y_b = y_b^*$ who is indifferent between the two communities, i.e.

$$W_1(y_b^*) = W_2(y_b^*) \quad (7)$$

To check whether the allocation specified by $y_b = y_b^*$ is an equilibrium, an additional necessary (and sufficient given (7)) condition is $t_1^* > t_2^*$ since Assumption 1 then ensures that all $y > y_b$ prefer C_1 over C_2 and the opposite for $y < y_b$.

To trace out the utility of the boundary individual as a function of y_b we differentiate $W_j(y_b)$ with respect to y_b. This yields

$$\frac{dW_j}{dy_b} = u_c[1 - ph(\tilde{\pi}_j, y_b)(d\tilde{t}_j/dy_b)] + v'[(\partial q_j/\partial t_j)(d\tilde{t}_j/dy_b) + (\partial q_j/\partial y_b)] \quad (8)$$

where $\tilde{t}$ indicates the tax rate chosen by majority vote within the community with an allocation of the population as specified by y_b, i.e. $\tilde{t}_j(y_b)$ is the preferred tax rate of the median voter in community j given a boundary individual y_b. Note that $q = q(t, y_b)$.

Thus, to establish the properties of the W_j curves we need to determine how $\tilde{t}_j$ and q_j react to changes in y_b. There are two important effects at work when y_b increases: (i) The income distribution in both communities is shifted rightward so that the median voter corresponds to an individual with greater income than previously and, (ii) The average housing consumed in each community increases at each π. Consequently, evaluating the effect of a marginal increase in y_b corresponds to (i) evaluating how the preferred tax rate (and hence quality) changes with an increase in the income of the median voter, ceteris paribus, and (ii) evaluating how a given median voter's preferred tax rate (and hence quality) changes with an increase in average housing consumption.

Given a distribution of income within a community, the preferred tax rate of an individual with income y is found by maximizing (3) with respect to t subject to the community's budget constraint as specified in (4). The first order condition for an interior maximum is:

$$-u_c ph(\pi,y) + v'(\partial q/\partial t) = 0 \qquad (9)$$

where the community subscript is omitted.

Recalling that the identity of the median voter is a function of y_b, the equation that implicitly defines the median voter's $(\hat{y})$ preferred tax rate $\tilde{t}$ as a function of y_b is thus:

$$R(y_b,\tilde{t}\ ) = -u_c(\hat{y}(y_b)-\tilde{\pi}h(\tilde{\pi},\hat{y}(y_b)),h(\tilde{\pi},\hat{y}(y_b)))ph(\tilde{\pi},\hat{y}(y_b)) + v'(\partial q/\partial t) = 0 \quad (10)$$

Using the implicit function rule on (10) to solve for the effect of a marginal increase in y_b on $\tilde{t}$ yields:

$$\frac{d\tilde{t}}{dy_b} = \frac{-pS(d\hat{y}/dy_b) + \{v''\tilde{t}p(\partial\bar{h}/\partial y_b)(\partial q/\partial t) + v'p[(\partial\bar{h}/\partial y_b)+\tilde{t}(\partial^2\bar{h}/\partial t\partial y_b)]\}}{-\ \partial R/_{\partial\tilde{t}}} \qquad (11)$$

where S is defined in Assumption 1 and $\partial R/\partial\tilde{t}=[u_{cc}h(h+\pi h_\pi)-u_{ch}hh_\pi-u_c h_\pi]p^2 + v''(\partial q/\partial t)^2+v'(\partial^2 q/\partial t^2)$. In order for the second order condition for an interior maximum to be satisfied $\partial R/d\tilde{t}$ must be negative. The following two assumptions are sufficient to ensure that this condition is satisfied.

<u>Assumption 2</u>: u(c,h) is concave in π, i.e., $u_{cc}h(h+\pi h_\pi)-u_{ch}hh_\pi-u_c h_\pi \le 0$.

Assumption 3: $q(t,y_b)$ is concave in t.

Hence the denominator in (11) is positive The numerator is more difficult to sign. As argued previously, it is possible to decompose (11) into two distinct reactions to the marginal increase in y_b. For a given median voter, the latter's reaction to an increase in average housing demand (brought on by the marginal increase in y_b) is denoted by $(\partial\tilde{t}/\partial y_b)|_{\hat{y}}$ and is equal to

$$\frac{\partial\tilde{t}}{\partial y_b}\bigg|_{\hat{y}} = -\frac{v''(\partial q/\partial y_b)(\partial q/\partial t) + v'\partial(\partial q/\partial t)/\partial y_b}{\partial R/\partial\tilde{t}} \qquad (12)$$

Noting that $\partial q/\partial y_b=\tilde{t}p\partial\bar{h}/\partial y_b$ and that $\partial(\partial q/\partial t)/\partial y_b = p[(\partial\bar{h}/\partial y_b) + \tilde{t}(\partial^2\bar{h}/\partial t\partial y_b)]$, it is easy to see that the numerator of this expression corresponds to the terms that appear within the curly brackets in (11). Furthermore, $\partial\bar{h}/\partial y_b>0$ and $\partial q/\partial t=p\bar{h}+\tilde{t}p^2\bar{h}_\pi$ also is strictly positive since otherwise the median voter (and indeed all individuals) could be made better off by a marginal decrease of the tax rate. After some further manipulation the numerator in (12) can be expressed as

$$p[v''q+v'](\partial\bar{h}/\partial y_b) + v''\tilde{t}^2p^3\bar{h}_\pi(\partial\bar{h}/\partial y_b) + v'\tilde{t}p(\partial^2\bar{h}/\partial t\partial y_b) , \qquad (13)$$

with $\partial^2\bar{h}_1/\partial t\partial y_b=p[\bar{h}_\pi-h_\pi(\pi,y_b)]g(y_b)/N_1$ and $\partial^2\bar{h}_2/\partial t\partial y_b = -p[\bar{h}_\pi-h_\pi(\pi,y_b)]g(y_b)/N_2$. Note that these last two expressions are positive if $h_{\pi y}>0$. Hence, if in addition $-v''q/v'\le 1$, then the numerator in (12) is strictly positive and $\partial\tilde{t}/\partial y_b>0$. That is, a given median voter's preferred tax rate increases with y_b.[9]

The second effect, i.e. the effect on a community's tax rate due to a new median voter with a higher income than previously (for the same given average housing demand at each π as before) is, by Assumption 1, to increase the tax rate. This effect is captured by the first term in expression (11) and is strictly positive.

Lastly, the effect of a marginal increase in y_b on the quality of education is:

$$\frac{dq}{dy_b} = \frac{\partial q}{\partial t}\frac{d\tilde{t}}{dy_b} + \frac{\partial q}{\partial y_b} = [p\bar{h}+\tilde{t}p(\partial\bar{h}/\partial t)](d\tilde{t}/dy_b) + \tilde{t}p(\partial\bar{h}/\partial y_b) \qquad (14)$$

If $d\tilde{t}/dy_b>0$ then necessarily q must increase. If, however, $d\tilde{t}/dy_b<0$ then the effect on quality is ambiguous.[10]

Returning to expression (8), it is possible to rewrite dW/dy_b as

$$\frac{dW_j}{dy_b} = [-u_c ph(\tilde{\pi}_j, y_b)+v'(\partial q_j/\partial t)](d\tilde{t}_j/dy_b)] + u_c + v'(\partial q_j/\partial y_b) \qquad (15)$$

The last two terms in this expression are always positive. In addition, note that $-u_c ph+v'(\partial q/\partial t)$ (evaluated at $y=y_b$) is, by Assumption 1, negative for j=1 (since $y_b<\hat{y}_1$) and positive for j=2 (since $y_b>\hat{y}_2$). Thus, if $d\tilde{t}/dy_b$ is positive, expression (15) is positive for j=2 and ambiguous for j=1. Of

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[9] If u(c,h) is homothetic then $\partial\tilde{t}/\partial y_b$ (for a given $\hat{y}$) is positive (negative) if $-v''q/v'$ is less (greater) than one. Furthermore, the dq/dy_b (for a given median voter) is positive regardless of the sign of $\partial\tilde{t}/\partial y_b$.

[10] If u is homothetic then dq/dy_b is positive regardless of the sign of $d\tilde{t}/dy_b$.

course, if $d\tilde{t}/dy_b<0$ and $d\tilde{q}/dy_b>0$ then (15) is necessarily positive for $j=1,2$.

In general, as our preceding discussion indicates, it is impossible to sign (15) without imposing further restrictions on preferences. In all our simulations, however, we found dW_j/dy_b to be positive. Furthermore, without additional assumptions it is impossible to guarantee existence or uniqueness of equilibrium.[11] In our simulations we restrict our choice of utility functions and parameter values such that equilibrium exists and is unique.

Figure 2 depicts a possible configuration of the $W_j(y_b)$ curves. Equilibrium is given by $y_b=y_b^*$.

3. The Model with Exogenous Zoning

This section studies the effects on welfare and allocations of an exogenously imposed zoning regulation in C_1. The regulation requires all individuals residing in C_1 to purchase at least $M>0$ units of housing. Section 4 then studies the case where the level of required housing purchases is endogenously determined. There are two reasons for considering an exogenously imposed M. First, once lot sizes have been determined and houses built it may be difficult to change a zoning regulation. Hence, zoning regulations in place today may reflect decisions that were made previously. Second, the analysis of exogenously imposed zoning and the effect on various variables of changes in its level provides insights for the analysis of an endogenous M.

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[11]The introduction of a local housing supply function that endogenizes p_j--the price of a unit of housing in C_j--generates simple conditions that ensure the existence of a stable stratified equilibrium (see Fernandez and Rogerson (1993)).

Zoning regulations are only introduced in C_1, the wealthy community. This asymmetry serves to highlight the possible effects of zoning regulations on allocations across communities. A natural interpretation of this situation is that of a central city and its suburbs, where C_1 corresponds to the suburb and C_2 represents the central city.

3.1 General Properties

In order to examine the effects of zoning on the equilibrium described in section 2, we must first establish that our propositions regarding stratification and the median voter continue to hold. The following result and two additional assumptions allow us to establish both.

Proposition 3: For a given allocation of individuals in a community and a given M, $dV(\pi,q,y;M)/dt$ is continuous and increasing in y for $y>\pi M$.

Proof: For individuals for whom the housing constraint is binding, i.e. $h(\pi,y)<M$, (but with strictly postive consumption of the private good), $dV/dt=-u_c pM+v'\partial q/\partial t$ which is increasing in y. For unconstrained individuals, $dV/dt=-u_c ph+v'\partial q/\partial t$ which, by Assumption 1, is strictly increasing in y. Note that these last two expressions are identical at the y level at which an individual is just constrained (i.e. for y such that $h(\pi,y)=M$). Hence dV/dt is a continuous increasing function of y.||

If $y/\pi<M$ then an individual with that level of income cannot afford to purchase M units of housing at the given (after tax) price level. It is assumed that in such a case the individual is forced to spend her entire income on housing. This yields her $V=-\infty$ and thus $dV/dt=0$ for all y and π in this range.

Propostion 3 allows us, for any given M, to identify the tax rate chosen in C_1 with the preferred tax rate of the individual whose income is median in the community. A statement and a proof of this is provided in the next propositon. It is assumed throughout that an individual faced with a choice between two tax rates each of which (because of the zoning constraint) exhausts her entire income on housing, votes for the lowest of the two tax rates.

Proposition 4: Majority voting in C_1, at any given level of M, generates a tax rate preferred by the median voter.

Proof: A tax rate t' greater than that preferred by the median voter ($\tilde{t}$) will be blocked by those individuals with $y \le \hat{y}$. To see this note that if the median voter prefers $\tilde{t}$ to t' then $\int(dV(\pi,q,\hat{y};M)/dt)dt$ (integrated from $\tilde{t}$ to t') is negative. By Proposition 3, then, $\int(dV(\pi,q,y;M)/dt)dt$ is also negative for all y such that $\tilde{\pi}M<y<\hat{y}$ and is zero for those individuals with $y \le \tilde{\pi}M$. Similarly, any tax rate t'smaller than $\tilde{t}$ will be rejected in favor of $\tilde{t}$ since if $\int(dV(\pi,q,\hat{y};M)/dt)dt$ (integrated from t' to $\tilde{t}$) is positive, then (by Proposition 4) $\int(dV(\pi,q,y;M)/dt)dt$ is necessarily positive for all $y>\hat{y}$.||

Two things should be noted from the preceding propositions. First, in equilibrium $y_b>\pi_1 M$ since otherwise, by our assumption on preferences, y_b would prefer C_2 to C_1 if residing in the latter implied zero consumption of the private good. Second, note that the proof of Proposition 4 does not rely on the indirect utility function being single-peaked with respect to t. This is important since, as we show further on, V(•) is not necessarily single-peaked.

In order to establish stratification in this model we need to impose two additional conditions on preferences:

Assumption 4: $du_c(y-\pi h,h)/d\pi > 0$.

Assumption 5: $u_{ch} \geq 0$.[12]

The significance of these assumptions will be made clear in the proof of the next proposition.

Proposition 5: If in equilibrium both communities are non-empty and $q_1^*>q_2^*$, then the equilibrium must be stratified, i.e. all individuals with income above some cutoff level reside in C_1 and the remainder reside in C_2.

Proof: This argument is less straightforward than that employed in the proof of Proposition 1 since now individuals may be constrained in C_1. Suppose that in equilibrium an individual with income y' is indifferent between both communities. If that individual is unconstrained, normality of h implies that no other individual in C_1 is constrained and the result then follows from Proposition 1. If, on the other hand, the indifferent individual is constrained, there are two subcases to examine depending on whether t_1^* is greater or smaller than t_2^* (note that y' must have strictly positive consumption of the private good in C_1 in both cases). We take up each of these possibilities in turn.

(i). Suppose $t_1^*>t_2^*$. We want to show that individuals with income greater than y' prefer C_1 to C_2. Note that the change in utility due to the increase in quality incurred by moving from C_2 to C_1 is the same for all individuals. Consequently, we want

$$\partial u(y-\pi_1 M,M)/\partial y|_{y'} > \partial u(y-\pi_2 h,h)/\partial y|_{y'} \qquad \text{i.e,}$$

$$u_c(y'-\pi_1 M,M) > u_c(y'-\pi_2 h,h)$$

[12]Note that Assumptions 2 and 3 combined imply that πh is an increasing function of π.

Since $\pi_1>\pi_2$ and $M>h(\pi_1,y')$, Assumptions 4 and 5 imply that the above condition is met. Furthermore, since $V(\pi_1,q_1,y;M)-V(\pi_2,q_2,y)=\int u_c(y-\pi_1M,M)dy-\int u_c(y-\pi_2h,h)dy>0$, where the integral is from y' to y" and y" is defined as the level of y such that $h(\pi_1,y)=M)$, these assumptions guarantee that all constrained individuals with y>y' prefer C_1 to C_2. Note that in particular Assumption 4 implies $V(\pi_1,q_1,y";M)>V(\pi_2,q_2,y")$. Assumption 1 then ensures that all individuals with y>y" also prefer C_1.

(ii) Given zoning, an equilibrium in which C_1 possesses both a greater quality and a <u>lower</u> tax rate is feasible. Since, absent the zoning constraint, all individuals would prefer C_1 to C_2, it must be that y' is constrained in equilibrium. In order for other constrained individuals with y>y' to prefer C_1 to C_2, we require (as before) $u_c(y'-\pi_1M,M)>u_c(y'-\pi_2h,h)$. Note that $\pi_1<\pi_2$ implies $M>h(\pi_2,y')$. Thus, for y' to be indifferent between the two communities we must have $c_1=y'-\pi_1M<c_2=y'-\pi_2h(\pi_2,y')$. Assumption 5 then guarantees $u_c(y'-\pi_1M,M)>u_c(y'-\pi_2h,h)$ and similarly that $V(\pi_1,q_1,y;M)-V(\pi_2,q_2,y)=\int u_c(y-\pi_1M,M)dy-\int u_c(y-\pi_2h,h)dy>0$ for any y such that y'<y<y". Thus, all constrained individuals (with y>y') prefer C_1 to C_2 as do, of course, all unconstrained individuals.||

3.2 Preliminary Analysis

In order to examine how zoning and changes in the level of zoning affect the equilibrium, we first turn to an analysis of the effect on the W_j curves of changes in M. Note first that only the W_1 curve is affected by M and that if, for a given allocation of individuals between communities, the housing/consumption choice of any individual in C_1 is affected by zoning, then so is y_b's. Furthermore, in order for the equilibrium with zoning to differ from the no-zoning equilibrium, the restriction must be sufficiently large that y_b^* is constrained by M.

We now define $W_1(y_b;M)$ as the utility of the y_b individual in C_1 given a zoning level of M in C_1. $W_2(y_b)$ is defined as before. Differentiating $W_1(y_b;M)$ with respect to M for the range of values such that $y_b>\pi M$ and $h(\pi,y_b)<M$ yields:

$$\frac{\partial W_1}{\partial M} = -u_c(\pi+pM(\partial\tilde{t}/\partial M)) + u_h + v'[(\partial q/\partial t)(\partial\tilde{t}/\partial M)+\partial q/\partial M] \qquad (16)$$

where the preferred tax rate of the individual with median income is now indicated by $\tilde{t}(y_b;M)$ and quality is written as $q(t,y_b;M)$. Individual housing demand, $h(\pi,y;M)$, now also depends on M and is equal to $h(\pi,y)$ if the latter is greater than or equal to M and is equal to M otherwise.

It is difficult to sign expression (16). To do so it is necessary to examine the effect of changes in M on the median voter's preferred tax rate and quality for a given value of y_b. Algebraically, $\tilde{t}(y_b;M)$ solves:

$$\max_t \ u(y-\pi h,h) + v(q) \qquad (17)$$

When M=0, Assumptions 2 and 3 guarantee that this is a concave maximization problem, but when M>0 they are no longer sufficient since q is no longer a concave function of t. In particular, $q(t,y_b;M)$ has an inflection point at the tax rate that separates two regions: one of low taxes in which no individual is constrained and one of higher taxes such that at least some individual is constrained. Thus, depending on the utility function, there is no guarantee that the maximization program is concave.

The first order condition for the preferred tax rate of the individual with median income is

$$-u_c p(h+\pi h_\pi) + u_h p h_\pi + v' \partial q/\partial t = 0 \tag{18}$$

Note that if h>M, then $-u_c\pi+u_h=0$, whereas if h=M then $h_\pi=0$.

When the median voter chooses a tax rate such that she is unconstrained, use of the implicit function rule on (18) yields:[13]

$$\frac{d\tilde{t}}{dM} = -\frac{(p/N_1)[v'A+v''B]}{\partial R/\partial t} \tag{19}$$

where $R(t,M,y)=-u_c ph+v'(\partial q/\partial t)$ and the second order condition implies $\partial R/\partial t<0$. Furthermore,

$$A = \partial^2 q/\partial M\partial t = \int_{Y_b}^{Y_c} g(y)\,dy - tph_\pi(y_c,\pi)g(y_c)(\partial y_c/\partial M) > 0$$

$$B = (\partial q/\partial M)(\partial q/\partial t) = tp\int_{Y_b}^{Y_c} g(y)\,dy.[p\bar{h}+tp^2\bar{h}_\pi] > 0$$

and

$$\bar{h}_\pi = \frac{1}{N}\int_{Y_c}^{\bar{Y}} h_\pi(\pi,y)\,dy < 0.$$

Note that $y_c(\pi,M)$ is defined as the income level such that $h(y_c,\pi)=M$.

As should be apparent from the definitions of A and B above, the sign of $\partial\tilde{t}/\partial M$ is ambiguous without further assumptions. It is straightforward to show however, the implied quality increases with M independently of the effect of M on $\tilde{t}$ as long as q is locally concave in t at $\tilde{t}$. Clearly, if $\tilde{t}$

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[13] The above discussion implies that the preferred tax rate may have a point of discontinuity. The comparative statics results that follow are obviously not valid at such a point.

increases then so does q. If t decreases Assumption 2 implies that $u_c h$ decreases, whereas local concavity of q and the increase in M both imply that $\partial q/\partial t$ increases. Were q to decrease this would imply an increase in v', in which case (18) cannot be satisfied. Thus, q must increase.

A similar exercise for a median voter that chooses t such that she is constrained yields:

$$\frac{\partial \tilde{t}}{\partial M} = -\frac{[u_{cc}\pi - u_{ch}]pM - u_c p + (p/N_1)[v'A + v''B]}{\partial R/\partial t} \qquad (20)$$

As before, the second order condition implies $\partial R/\partial t<0$. Note that the additional terms in the numerator of this expression as compared with (19) are all negative. Thus it is quite likely that the effect of a marginal increase in M is to increase the preferred tax rate of the median voter when the latter is unconstrained and to decrease it when the median voter is constrained.

To illustrate the effect of zoning on $\tilde{t}$ and q we simulate the model for the following functional forms: $u(c,h)=[(c^{\alpha}-1)+(h^{\alpha}-1)]/(3\alpha)$, $v(q)=(q^{\gamma}-1)/(3\gamma)$, and $g(y)=a_0+a_1 y$, $g(\bar{y})=0$ $\underline{y}=1$ and p=1. For this utility function $u(0,h)=-\infty$, $u_{ch}=0$, and Assumptions 1 and 4 reduce to $\alpha<0$.

Figure 3 examines the median voter's utility as a function of t for given values of M (keeping y_b constant) for two different sets of parameter values of the utility function. In Example 1, $\alpha=-10$ and $\gamma=1\text{x}10^{-4}$ whereas in Example 2, $\alpha=-1\text{x}10^{-4}$ and $\gamma=.5$. In both cases $\bar{y}=20$, $a_0=.1108$, and $a_1=-.0055$. We will make continued use of these particular examples to illustrate various properties of this model and hereafter refer to them as Example 1 and 2 respectively. For both examples in Figure 3, at the lowest value of M portrayed all individuals are unconstrained for all values of t indicated. For all other values of M, the $V(\pi,q,\hat{y};M)$ curves coincide with the unconstrained curve at sufficiently low tax rates and diverge from it as soon as any individual becomes constrained.

Note that the patterns portrayed in Example 1 and 2 are different. All the $V(\bullet)$ curves in Example 2 are double peaked. One peak corresponds to the tax rate that maximizes utility for the unconstrained curve. The other occurs where a large number of individuals (including the median voter) are constrained. In this example the preferred tax rate is discontinuous at the point at which the maximum switches from the first peak to the second peak. In Example 1, on the other hand, the curves appear to be single peaked and thus there is no discontinuity in $\tilde{t}$. In both cases, for small values of M, $\tilde{t}$ is equal to the tax rate that maximizes utility for the unconstrained curve. For M sufficiently large, though, the optimum is attained at a lower value of t but one sufficiently high that the median voter is constrained.

Figure 3A summarizes for Examples 1 and 2 $\tilde{t}(y_b;M)$ and $q(\tilde{t},y_b;M)$ as a function of M. As Example 1 shows, $\tilde{t}$ and q need not follow the same pattern: $\tilde{t}$ can decrease even when the median voter is unconstrained while q continues to increase. Example 2 has $\tilde{t}$ and q moving in sync.

It should be clear that for sufficiently small values of M, $\partial\tilde{t}/\partial M=0$ since the value of t that maximizes the median voter's utility remains unchanged by small changes in M if no individual was constrained at the previous preferred tax rate. As M continues to increase, it takes progressively smaller tax rates for the median voter to be constrained and this will shift the preferred value of t. To explain this behavior, note that for small values of M it takes far too large a t to completely eliminate the free rider effect (i.e. to get all individuals with $y<\hat{y}$ to consume M units of housing). Hence the preferred t in this range will either be the unconstrained value of t or one that has only some individuals with $y<\hat{y}$ constrained. Increases in M allow the median voter to completely eliminate the free rider problem at "reasonable" values of t (and, of course, also to obtain greater quality at lower tax rates). Note also that if t did not fall once the median voter were constrained, higher values of M would impose progressively greater disutility on that individual as consumption of the private good would continue to decrease. Hence it is quite understandable that once the median voter is constrained, further M increases tend to have the effect of decreasing the preferred tax rate.

3.3 Equilibrium

The main questions of interest are: 1. How does zoning affect the allocation of individuals between communities? 2. How does zoning affect the equilibrium qualities of education in both communities? 3. How are equilibrium taxes affected by zoning? And, most importantly, 4. How is individual welfare affected by zoning?

Not surprisingly given our previous discussion, the effect of an increase in M on $W_1(y_b;M)$ is ambiguous. In all our simulations, however, $W_1(y_b;M)$ fell for a marginal increase in M in the vicinity of y_b^*.[14] Hence, in all our simulations an increase in M is associated with an increase in $y_b^*(M)$.

[14]An example where $W(y_b;M)$ increases for an interval of y_b that does not include y_b^* is for the parameter values of Example 2 for M=7.

Figure 4 shows, for two examples (parameters are indicated on the Figure), how the equilibrium value of y_b is affected by changes in M. Each W_1 curve shown corresponds to a different value of M and is labeled accordingly. Also shown is the W_2 curve which, of course, is independent of M. For both of these examples, increases in M have the effect of shifting the W_1 curve downwards, resulting in each case in progressively higher equilibrium values of y_b^* (i.e. $y_b^*(M)$ is an increasing function of M). Thus the effect of zoning in these examples is to increase the number of individuals that live in C_2 (and hence decrease the number of individuals in C_1). Despite the ambiguous nature of our theoretical result, this is not surprising. If y_b^* is to differ as a result of zoning, it must be the case that at the $t^*(M)$ chosen by the median voter, y_b is constrained. Ceteris paribus, this has the effect of making y_b worse off.

Table 1 presents, for the same two examples as Figure 4, the equilibrium tax rates, qualities of public education, y_b^*, and the mean and median incomes across communities as a function of different levels of M imposed in C_1.

A few things should be noted from Table 1. First, as suggested by our previous discussion of the effect on $\tilde{t}(y_b;M)$ and $q(\tilde{t},y_b;M)$ of an increase in M, equilibrium tax rates and quality in C_1 are not generally monotonic functions of M. Quality first increases and then decreases with M and the tax rate behaves similarly (though not necessarily in sync). In C_2, the tax rate and quality are both increasing functions of M.[15] Again, the intuition behind these results is easy to understand: The effect of a median voter with

[15]Theoretically it is possible for both taxes and quality to decrease. For the utility function and the parameter values chosen, however, they must increase as indicated in the footnote that follows equation (13).

greater income is, ceteris paribus, to increase the tax rate (and hence quality). The effect of greater average housing (brought about by the increase in y_b^*) for a given median voter, however, has ambiguous effects on the tax rate and consequently on quality. It is interesting to note that in both examples the ratio of q_1^* to q_2^* decreases as M increases.

Lastly, note that as indicated by the last row in both examples, it is possible for an equilibrium to result in C_1 possessing not only a greater quality of education but also a lower tax rate. A sufficiently high level of zoning ensures that lower income individuals keep out due to the large sacrifice in consumption of the private good that their residence in C_1 would entail.

The effect of zoning on individual welfare is quite interesting. Figures 5(a,b) and 6(a,b,c,d,e) show, at various levels of detail, the effect of different values of M imposed in C_1 on the welfare of individuals in both communities for the same two examples as in Figure 4. At each M, for each income level, individual utility has been calculated at the new equilibrium allocations of individuals, given the new equilibrium quality and tax rate in each community. In all cases the new level of y_b^* brought about by a change in M can be discerned in the figures by the sharp increase in the slope of the corresponding W_1 curve.

In both of our examples, the effect of an M increase is to make the poorest individuals worse off. Why is this? Note that in all cases the effect of higher M is to increase t_2^* and q_2^*. If any residents in C_2 are to be made worse off as a result of this, they must be the poorest ones since these are, by Assumption 1, the least happy with a tax increase for a given quality increase. This is seen clearly in Figures 5(a) and 6(b). This interval of poor individuals that reside in C_2 and that prefer no zoning to any level of zoning is followed by another income interval, likewise residents of C_2 for all levels of M, that prefer some level of zoning to no zoning. These are individuals that have benefited from either one or both of the following elements: (i) a new median voter that is closer to them in income than was the case previously and, (ii) a greater average housing consumption at each after-tax price than previously. Note, however, that within this interval individual's ranking of which level of M they prefer may differ since these two effects will vary in strength according to the level of M imposed. Next is an interval of individuals (of length depending on M) that reside in C_1 when M=0 but with zoning reside in C_2. Individuals in the lower part of this interval prefer some zoning to no zoning, whereas individuals in the upper part of this interval are worse off and prefer no zoning. This is to be expected. If any individual that moves from C_1 to C_2 as a result of an M increase is to be made worse off, it should be those with the highest income level since these are the ones whose q,t tradeoff is most different from that of the median voter in C_2. Finally there is an interval of individuals (again of varying length) who reside in C_1 both before and after zoning is imposed. For M sufficiently large, all of these individuals may be worse off, but for smaller values of M the lower income individuals are worse off with zoning and the higher income individuals are better off with zoning. The lowest income individuals who remain in C_1 benefit from a possibly higher q but suffer from low consumption because of the zoning constraint. Of course, all individuals in C_1 must purchase M units of housing, but the disutility from so doing is greater for the lower income individuals in C_1.

In summary, the introduction of zoning tends to make the rich community more exclusive, lowers utility of the poorest individuals and raises the utility of the richest individuals. For individuals in the "middle" of the income distribution, however, welfare changes are not monotone in income. In particular, a group of individuals that leave C_1 are made better off at the same time that a group of individuals that move to C_2 and a group that remain in C_1 are made worse off.

4. Endogenous Zoning

The previous section treated M as an exogenous parameter and studied its effect on the equilibrium. This section allows the zoning restriction to be endogenously determined and illustrates the properties of equilibrium through some examples. The game played by communities and individuals is accordingly modified. As before, in the first stage, all individuals simultaneously choose a community in which to reside. Once this choice is made, individuals are unable to move in any subsequent stage. In the second stage, individuals in C_1 determine a level of required housing M through a process of majority voting. In the third stage, individuals in both communities choose tax rates, also by majority voting, and individuals make their housing and consumption purchases and obtain education.

4.1 Preliminary Analysis:

In order to shed light on the general equilibrium analysis that follows, it is useful to start by analyzing the problem of endogenous zoning within a simple, partial equilibrium context that allows us to highlight some of its main properties.

Table 2 presents information for the choices made by an individual with income $\hat{y}$=12.9289 when this individual is able to dictate the choice of tax rate and zoning level. In this one community example $\underline{y}$=10, the density has support on [1,20], and NZ indicates no zoning choice allowed, whereas Z indicates the $\hat{y}$ is able to zone. The subscripts indicate various intervals of the income distribution: I (Identical), B (Bottom), TR (Top Truncated), T (Top), and A (All) which correspond to the intervals $[\hat{y},\hat{y}]$, $[\underline{y},\hat{y}]$, $[\hat{y},15]$, $[\hat{y},20]$ and $[\underline{y},20]$ respectively.

As a first step, assume that all individuals are identical and have income $\hat{y}$. Note that, as illustrated in the second row of Table 2, were these individuals able to choose a zoning level, they would choose M such that $h(\pi,\hat{y})<M$. A binding level of M would be chosen since, in its absence, the equilibrium would be inefficient given that each individual's contribution to the average housing demand is infinitesimal and hence housing demand is too small.

If individuals are not identical, however, an additional consideration is introduced. In particular, in the absence of zoning an individual with income $\hat{y}$ is making an implicit transfer to all individuals with income lower than $\hat{y}$ since these purchase less housing. Thus, as shown by the third and fourth rows of Table 2, $\hat{y}$ would choose to impose the same zoning level as before and thus achieve the same allocation as with identical individuals.

If $\hat{y}$ is faced with individuals with income greater than $\hat{y}$, yet another consideration is introduced. The desire for $\hat{y}$ to enjoy an implicit transfer from individuals with income strictly greater than $\hat{y}$. This, however, is accomplished via the tax rate chosen, not the level of zoning, although of course, as shown in the last four rows of the table, there is an interaction between the tax rate and the level of zoning chosen. Note that in the case TR the $\hat{y}$ individual prefers some zoning even though they are at the bottom of the income distribution and that the highest values of M result in cases I and B. Another common feature is that when a binding zoning constraint is chosen, the preferred tax rate also falls.

4.2 Equilibrium

As before, an equilibrium can be depicted graphically as the intersection of two curves, $W_1(y_b)$ and $W_2(y_b)$, where $W_2(y_b)$ is unchanged and $W_1(y_b)$ is now the utility obtained by an individual with income y_b when residing in C_1 given that M, t and q are chosen according to the two-stage procedure outlined above. Note that Proposition 5 implies that any equilibrium in which quality differs across communities must be stratified. Furthermore, Proposition 4 implies that the third stage voting over tax rates results in a preferred tax rate of the individual with median income. What differs from the previous analysis is the addition of a stage in which individuals vote over the zoning level M. In this stage individuals take as given the function $\tilde{t}(y_b;M)$ which will determine tax rates in the third stage conditional on the level of M chosen. The preferred level of M of an individual with income y is defined by:

$$\max_{M} V(\pi(\tilde{t}(y_b;M)), q(\tilde{t}(y_b;M), y, M) \qquad (21)$$

Assuming that π and q are differentiable at the preferred point, the first order condition for this problem is:

$$- u_c(y-\pi h,h)h\frac{\partial\pi}{\partial M} + v'(q)\frac{\partial q}{\partial M} \quad \text{if } h>M$$

$$-u_c(y-\pi M,M)[M\frac{\partial\pi}{\partial M}+\pi] + u_h(y-\pi M,M) + v'(q)\frac{\partial q}{\partial M} \quad \text{if } h<M \tag{22}$$

Figure 7 displays implied preferences over M for different levels of y as well as $\tilde{t}$ and the implied quality as a function of M for the case of α=-2, γ=.0001 (as in the second panel of Table 1) when y_b=13.9. This example illustrates two features. First, the preferences over M are not single peaked and, second, the preferred value of M is not a monotone function of y. The two highest incomes indicated on the diagram have preferred values of M that lie to the left of the preferred value of M for y=17. As illustrated by the tax and quality curves in the bottom panel, this preference for a lower M by individuals with higher y indicates their preference for a higher tax rate and higher quality.

Recall that preferences over tax rates for any given M are not generally single peaked. Nonetheless, it was shown that a majority voting equilibrium always exists and that the individual with median income was decisive. A similar result is not available for the case of voting over M; it is easy to provide examples in which the individual with median income is no longer the decisive voter. In general there may be cases where a majority vote equilibrium does not exist. Below we provide two examples where a majority voting equilibrium does exist (though the decisive voter is not the one with median income).

In particular, for each of the two specifications in Table 1 we have computed the subgame perfect equilibrium for the three-stage game described above. The results appear in Table 3. In both cases voting over M results in a value M^* such that half the individuals have a preferred value of M no greater than M^* and half the individuals have a preferred value of M no less than M^*. In both examples M^* is smaller than the level of M preferred by the individual whose income is median in C_1.

A few points are worth noting about Table 3. In each case the equilibrium has both higher tax rates and higher quality of public education in C_1, as is true in the equilibrium without zoning. Table 3 also indicates the degree to which the zoning restriction is binding for the boundary individual and the individual with median income. In both examples a majority of individuals are constrained. A comparison with Table 1 indicates how the outcomes with endogenous zoning compare with those in which there is no zoning. In particular, in each case the rich community becomes more exclusive, and both the tax rate and the quality of public education increase in both communities. The welfare analysis in the previous section indicates the pattern of those whose utilities increase and decrease as a result of the introduction of zoning.

5. Conclusion

It is important to understand the role played by various factors in generating the large disparities across communities in per student spending on public education. Community zoning regulations undoubtedly affect this outcome although just how zoning regulations interact with income distribution, community composition, tax rates, and the provision of local public goods is far from clear ex ante. This paper aims to provide some insights into these interactions by studying the effects of zoning in a two-community model in which each community uses a local property tax to finance public education. Tax rates are chosen by majority vote and individuals decide in which community they wish to reside. We examine both the case of an exogenously imposed zoning regulation as well as one which is determined endogenously through majority vote at the community level.

Zoning affects outcomes through several channels. First, the imposition of zoning changes the allocation of individuals across communities, thereby affecting each community's distribution of income. Second, for a given allocation of individuals, zoning affects the tax base available to that community. Third, zoning affects the property tax chosen, via majority vote, within a community.

Theoretical analysis indicates a wide range of possible effects. Simulations revealed several interesting results. First, the rich community becomes more exclusive, increasing mean income in both communities. Second, the tax rate and quality increase in both communities under endogenous zoning but not necessarily under exogenous zoning. Third, welfare effects are not monotone in income. Although the richest people are made better off and the poorest people are made worse off, high income individuals in the poor community are made better off and low income individuals in the rich community are made worse off. Fourth, zoning leads to greater spending per student in the poor community, and may either increase or decrease the differences in the quality of education across communities. Fifth, an individual's preferred level of zoning is not monotone in her income. Thus majority voting over zoning need not result in the preferred level of the individual with median income.

Many questions remain open to analysis. It would be of great interest to change the form of the game analyzed so that individuals were able to make use of zoning (and taxes) directly as an instrument to attract certain segments of the population or keep others out.[16] The introduction of private schooling, while it would complicate the analysis considerably, would also add greater realism and thus prove to be of interest. Additional insights would be gained by allowing the number of communities to be determined endogenously but not costlessly.

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[16] For a paper that uses this alternative extensive form, see Epple and Romer (1991).

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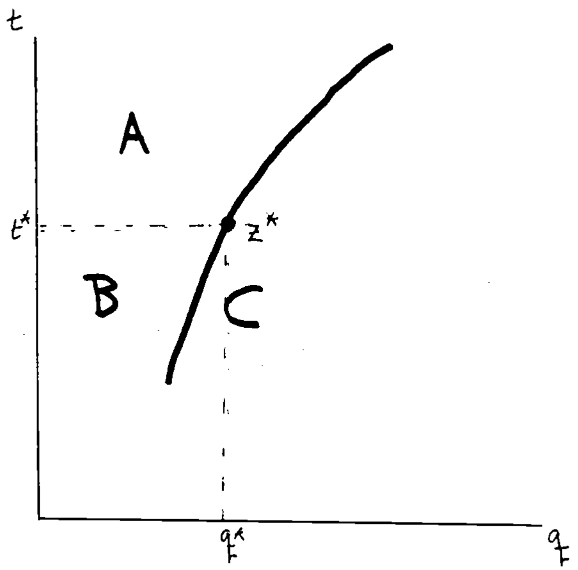


FIGURE 1

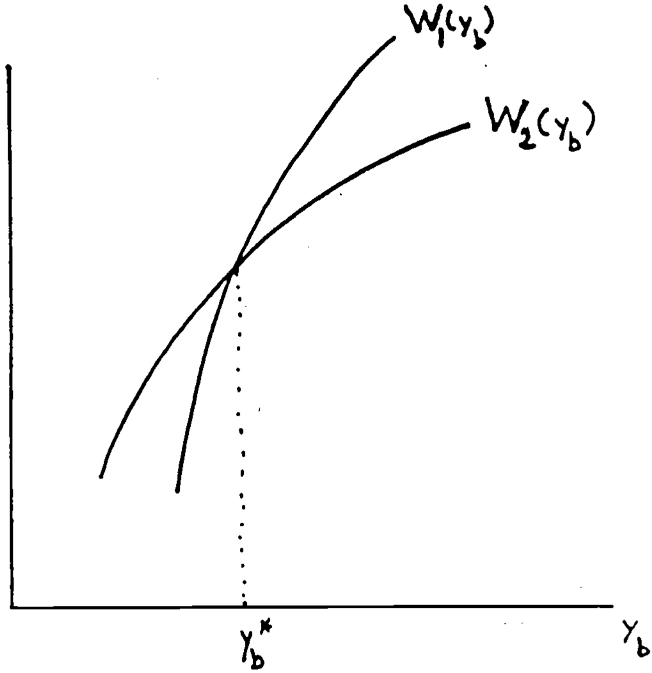
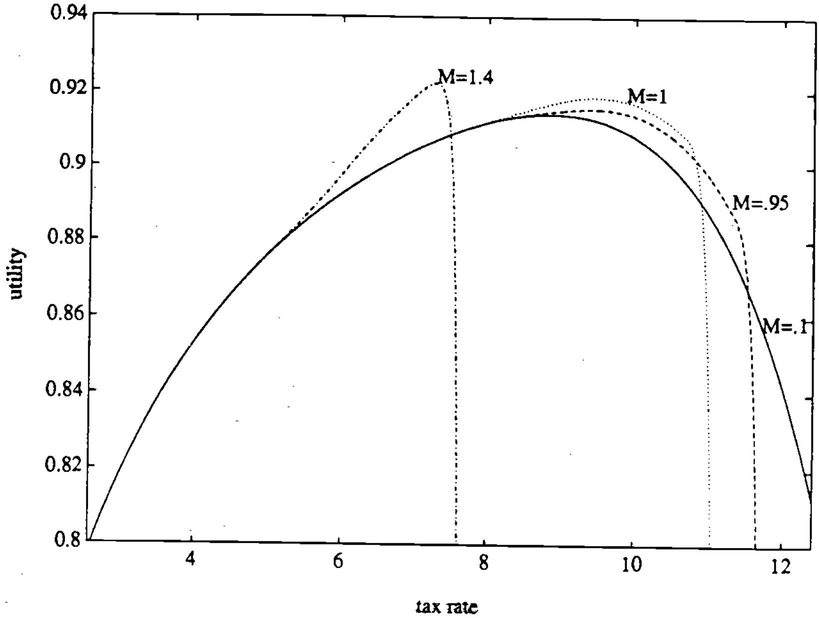


FIGURE 2

FIGURE 3

EXAMPLE 1: $\alpha=-10$ $\gamma=1X10^{-4}$



EXAMPLE 2: $\alpha=-1X10^{-4}$ $\gamma=.5$

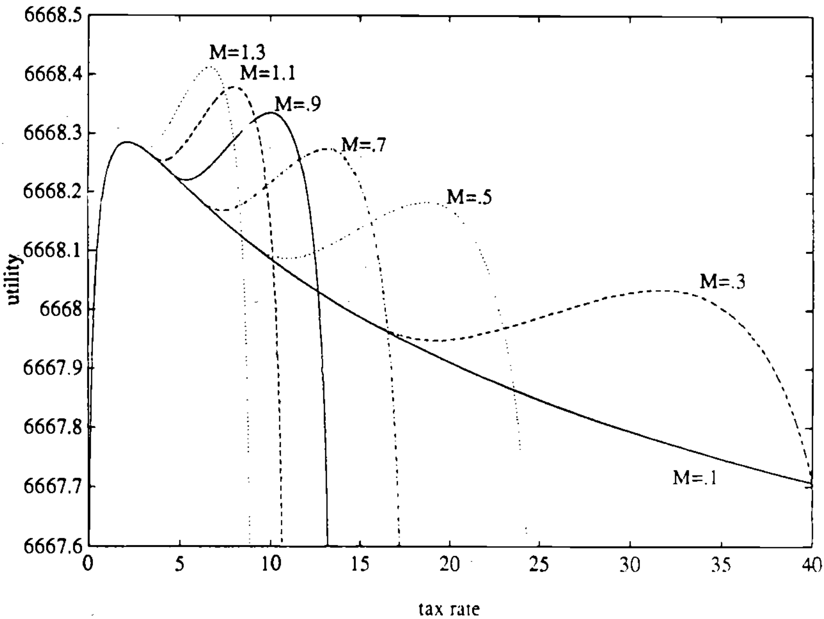
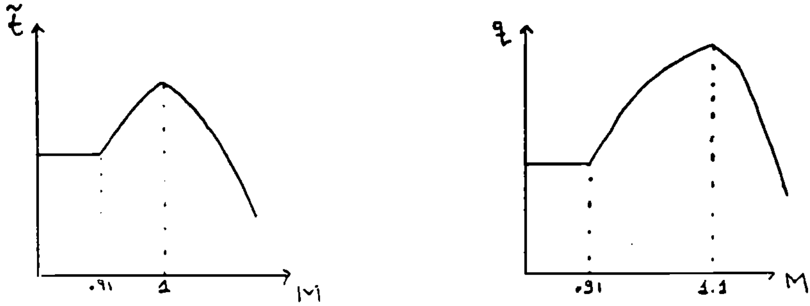


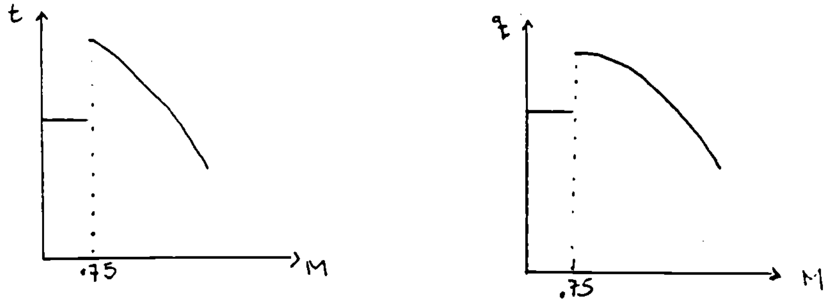
FIGURE 3A

1. $\alpha=-10$ $\gamma=1 \times 10^{-4}$



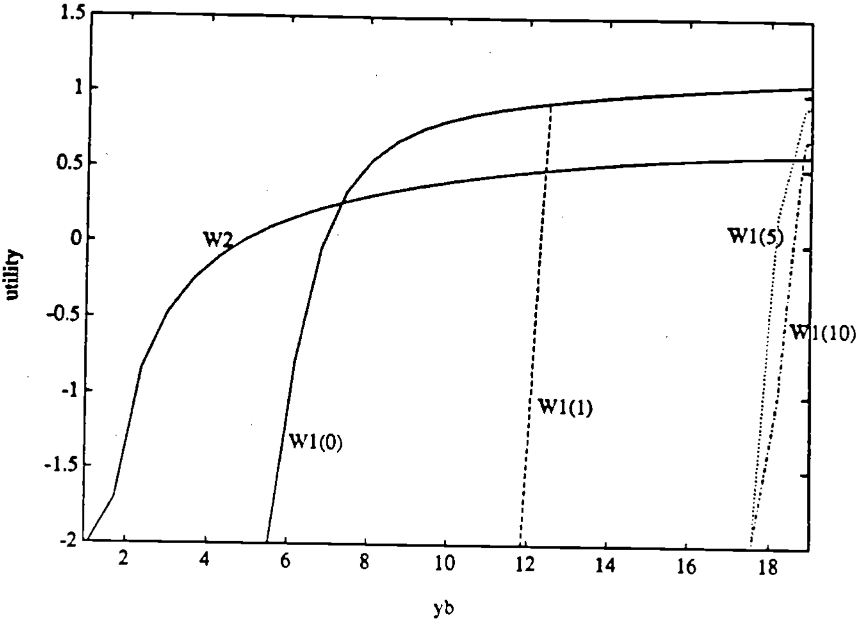
ŷ is constrained for M≥1.1

2. $\alpha=-1\times10^{-4}$ $\gamma=.5$

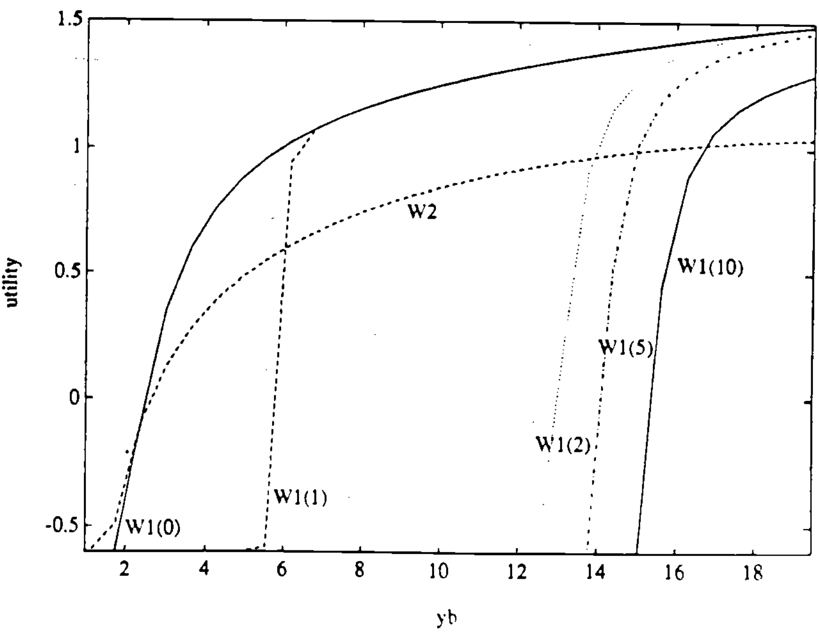


ŷ is constrained for M>.75

FIGURE 4



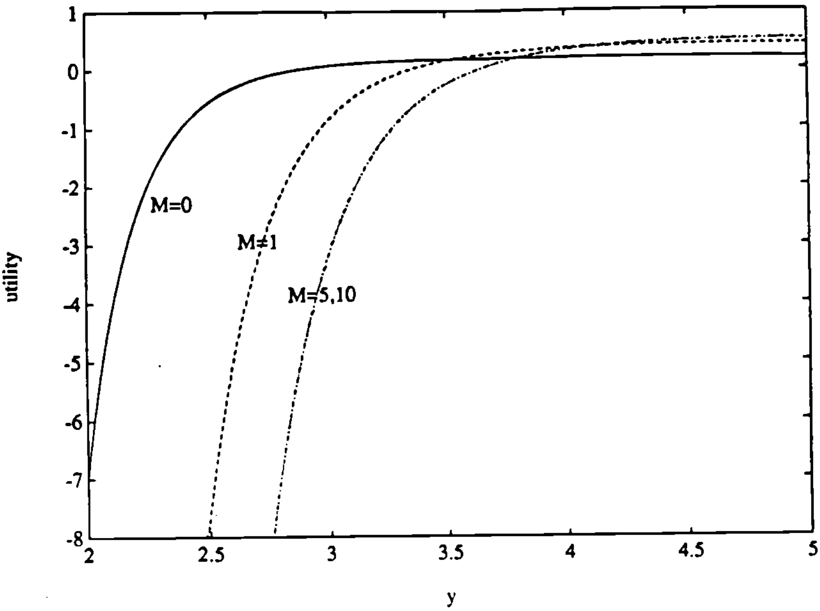
(i) $\alpha=-10$ $\gamma=1\times10^{-4}$



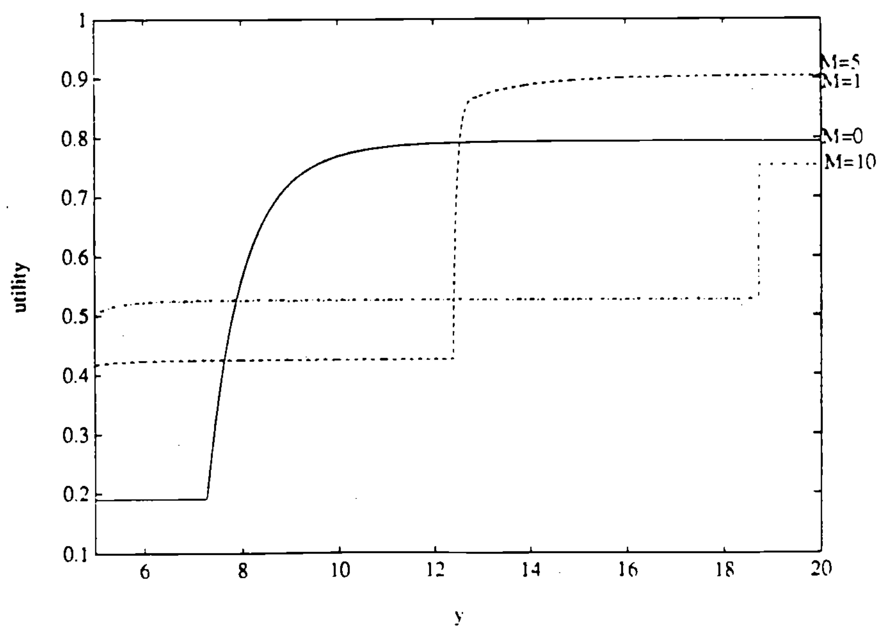
(ii) $\alpha=-2$ $\gamma=1\times10^{-4}$

FIGURE 5

$\alpha = -10 \quad \gamma = 1\times10^{-4}$



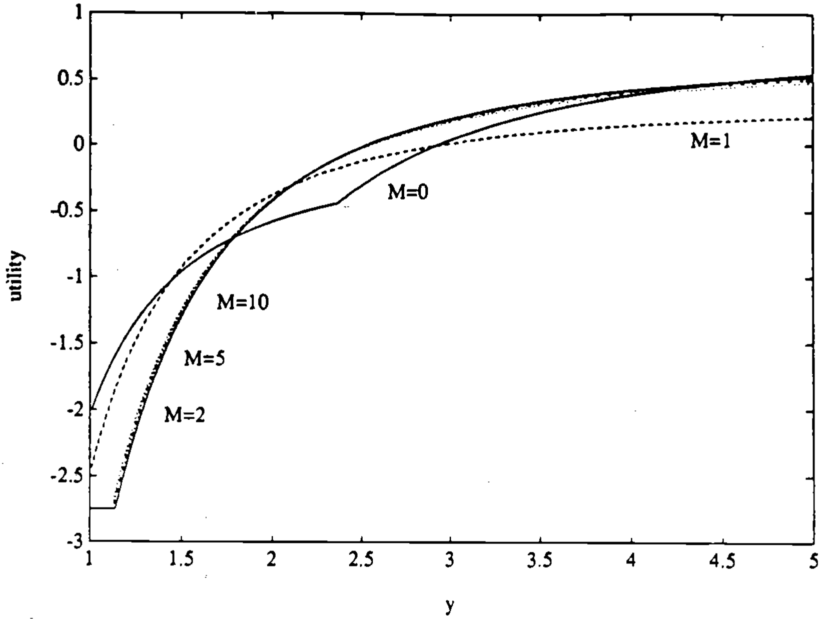
5(a)



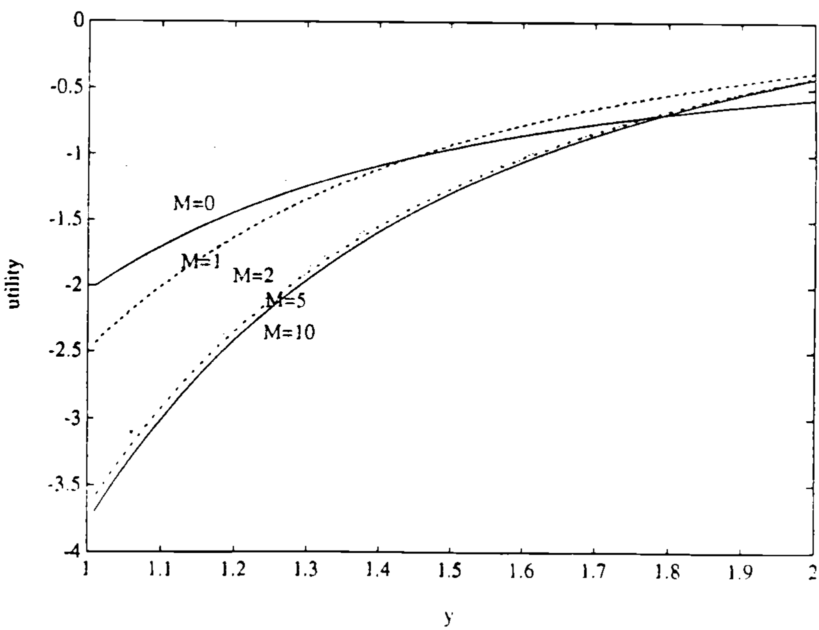
5(b)

FIGURE 6

$\alpha = -2 \ \gamma = 1X10^{-4}$

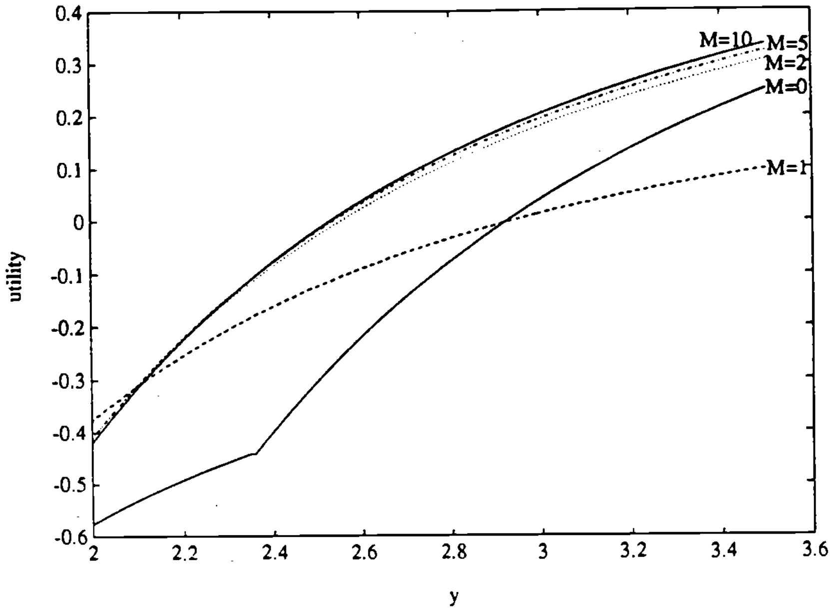


6(a)

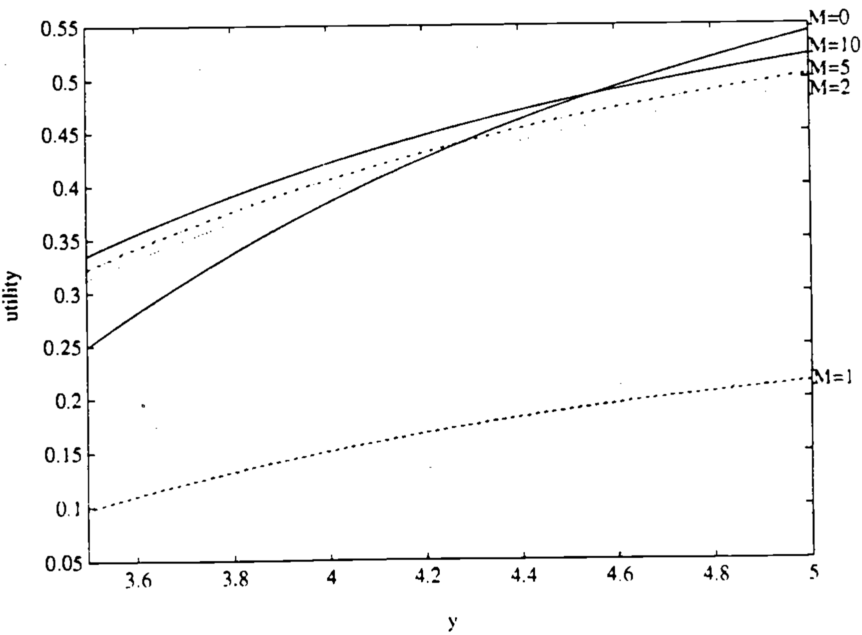


6(b)

FIGURE 6 (CONT)

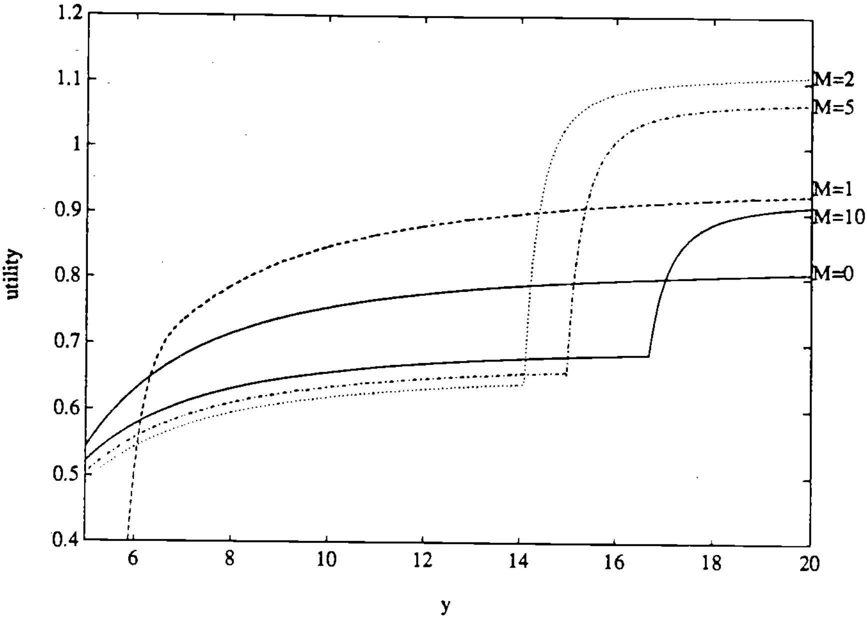


6(c)



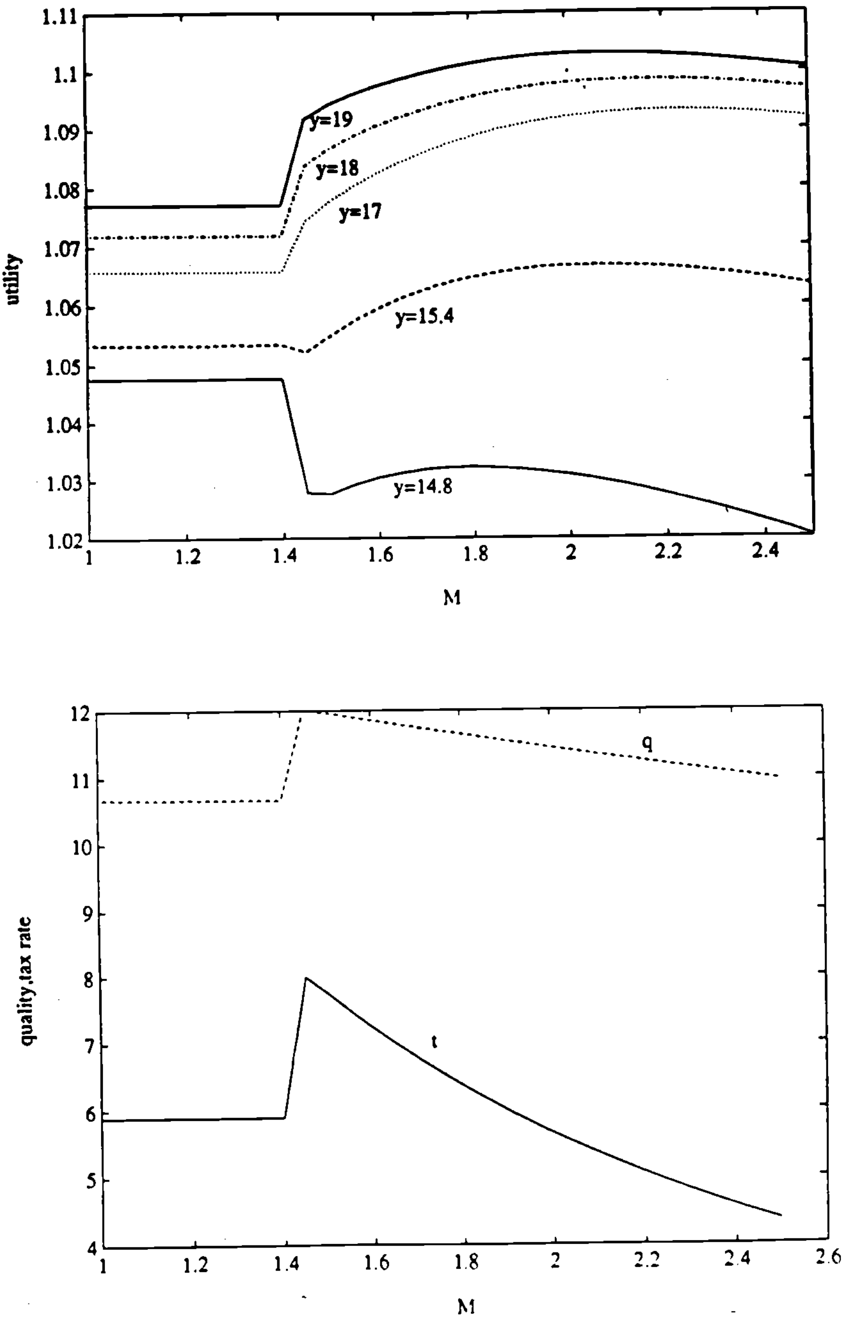
6(d)

FIGURE 6 (CONT)



6(e)

FIGURE 7



1.11
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y=17
y=15.4
y=14.8
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TABLE 1A

Total Spending ($) Per Student for 1986-87 Academic Year
(Primary and Secondary Schooling)

Boston Area		Detroit Area		New York Area	
Quincy	3,693	Dearborn	2,684	Mount Vernon	6,328
Lynn	3,788	Highland Park	3,105	New York City	6,433
Somerville	4,693	East Detroit	3,740	Levittown	7,210
Malden	4,820	Detroit	3,854	Baldwin	7,251
Waltham	5,207	Pontiac	4,553	Hempstead	7,462
Newton	5,515	Royal Oak	5,172	New Rochelle	7,970
Brookline	5,887	Grosse Pointe	5,705	Syosset	9,125
Boston	6,773	Birmingham	6,668	White Plains	11,045
Cambridge	7,244	Bloomfield Hills	6,976	Great Neck	12,868

Source: 1987 Census of Governments

TABLE 1

(i) α—10 γ-1X10^{-4}

	y_b^*	q_1^*	t_1^*	q_2^*	t_2^*	μ_1^*	$\hat{y}_1^*$	μ_2^*	$\hat{y}_2^*$
M-0	7.28	8.86	7.37	1.45	1.21	12.40	11.94	4.48	3.59
M-1	12.36	12.33	10.63	2.92	2.09	14.89	14.58	5.86	4.33
M-5	18.35	12.60	2.52	3.95	2.55	18.90	18.83	7.25	4.90
M-10	18.70	7.85	.79	3.97	2.55	19.15	19.10	7.28	4.91

(ii) α—2 γ-1X10^{-4}

	y_b^*	q_1^*	t_1^*	q_2^*	$t2^*$	μ_1^*	$\hat{y}_1^*$	μ_2^*	$\hat{y}_2^*$
M-0	2.36	4.38	2.92	.28	.43	8.27	7.56	1.70	1.61
M-1	5.80	6.45	4.23	.98	.96	10.53	9.96	3.28	2.82
M-2	13.87	11.42	5.64	2.64	1.71	15.91	15.67	6.34	4.55
M-5	15.00	9.35	1.87	2.81	1.76	16.67	16.46	6.69	4.68
M-10	16.70	5.05	.59	3.02	1.82	17.80	17.67	7.01	4.82

TABLE 2

$\alpha=1$ $\gamma=1$

	$\tilde{t}$	q	M	$h(\pi,\hat{y})$	$V(\pi,q,\hat{y},M)$
NZ_I	1.11	4.02	–	3.64	1.4288
Z_I	.99	4.30	4.32	3.79	1.4346
NZ_B	1.19	3.68	–	3.53	1.4178
Z_B	.99	4.30	4.32	3.79	1.4346
NZ_{TR}	1.07	4.23	–	3.69	1.4347
Z_{TR}	1.04	4.27	4.08	3.73	1.4349
NZ_T	1.01	4.50	–	3.78	1.4420
Z_T	1.01	4.50	0	3.78	1.4420
NZ_A	1.09	4.11	–	3.66	1.4314
Z_A	1.02	4.38	4.12	3.76	1.4376

TABLE 3

	Example 1	Example 2
y_b^*	18.08	13.90
M^*	1.23	2.09
t_1^*	13.07	5.37
t_2^*	2.54	1.71
q_1^*	16.19	11.36
q_2^*	3.93	2.64
$h(\pi_1^*, y_b^*)$	1.17	1.69
$h(\pi_1^*, \hat{y}_1^*)$	1.21	1.91