DYNAMIC EFFICIENCY
IN THE GIFTS ECONOMY

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ABSTRACT

In the standard analysis of an overlapping generations economy with gifts from children to parents, each generation takes the actions of all other generations as given. The resulting "simultaneous moves" equilibrium is dynamically inefficient. In reality, however, parents precede children in time and realize that children will respond to higher parental saving by reducing their gifts. Incorporating this feature lowers the effective return to saving, resulting in lower steady state capital accumulation. For a broad class of gift economies, we show that the steady state capital stock in the gifts model must be on the efficient side of the golden rule. The analysis therefore overturns the standard presumption of dynamic inefficiency in the gift economy. This result reestablishes the potential relevance of the gift model to the U.S. economy, renders moot an important part of the debate on Ricardian Equivalence, extends the recent literature on the effects of implicit taxation on capital accumulation, and provides a motivation for the presence of a Social Security type system that unconditionally transfers resources from young to old.

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1. Introduction

Barro (1974) demonstrated the potential importance of intergenerational altruism for the analysis of dynamic issues in macroeconomics. The literature since Barro has focused mainly on two patterns of altruism: in the first, parents care about their children and leave them bequests; in the second, children care about their parents and provide them with gifts in their old age. A common result in the latter case is capital overaccumulation: the steady state capital stock is above the golden rule level, and the economy is dynamically inefficient (e.g., Carmichael (1982), Abel (1987), Kimball (1987)).[1]

The dynamic inefficiency of the gift economy follows from models in which all generations act simultaneously, taking the gifts and saving of all other generations as given. In reality, of course, parents are born before children and make a large fraction of their consumption decisions before their children become independent adults. A more natural modeling approach would therefore be to make parents the "leaders" in a sequential game.[2] The point of this paper is that incorporating this element of realism changes the gift economy in a fundamental way: it overturns the presumption that equilibria are dynamically inefficient.

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[1] This result requires that children discount the lifetime utility of their parents relative to their own lifetime utility, a point we discuss further below.

[2] Our analysis pursues a suggestion in Kimball (1987, p.313, footnote 17) to drop the standard Nash assumption of taking others' actions as given. For other recent articles using this Stackelberg approach, see Veall (1986), Laitner (1988), and Hansson and Stuart (1989).

1

Formally, our strategic assumptions result in an extra term in the Euler equation relating an individual's first and second period consumption. Parents know that an extra dollar of accumulated assets will reduce the old-age support they receive from their children. Holding constant the economy-wide interest rate, this lowers the private return to saving. It is perhaps not surprising that in a general equilibrium model, introducing this wedge between the private and social return to saving lowers the aggregate capital stock. We prove a much stronger result, however: for a broad class of preferences (members of the HARA class) and strategies, this effect is strong enough that the steady state interest rate is necessarily greater than the golden rule interest rate. The standard result of capital overaccumulation is therefore reversed.

Our analysis has implications in a variety of contexts. First, empirical evidence that the U.S. economy is dynamically efficient (Abel, et al (1989)) casts doubt on the relevance of any model -- including the standard gifts model -- that implies capital overaccumulation. This is troubling, since we observe many examples of children supporting parents in their old age, a phenomenon that would undoubtedly be even more prevalent in the absence of public support mechanisms like Social Security and Medicare. While these examples lead naturally to the study of altruism on the part of children, the evidence on dynamic efficiency appears to rule out the standard gift analysis. Our dynamic efficiency result reconciles these observations and reestablishes the potential relevance of the gifts model to the U.S. economy.

Second, the gifts model has played an important role in the literature on Ricardian equivalence. The dynamic inefficiency of the

2

standard gifts economy meant that the government could cut taxes, roll over the debt perpetually, and never raise future taxes, i.e. that it could run a rational Ponzi game and rebate the proceeds to the current generation.[3] This suggested a possible counterexample to Barro's claim that Ricardian equivalence would hold in any economy linked by positive intergenerational transfers (cf. Barro (1974, 1976), Feldstein (1976), Carmichael (1982))[4]. Since the possibility of rational Ponzi games depends on dynamic inefficiency, however, our results render this debate moot.

Third, our work contributes to the public finance literature on the effects of direct and indirect taxation of capital. While the incentive effects of direct capital income taxation are well known (e.g. Summers (1981)), recent work has begun to examine the indirect taxation of saving implied by asset-based means-testing in public and private transfer programs. Such testing links the receipt and/or size of income transfers to the value of an individual's accumulated assets. This gives rise to a version of the "Samaritan's dilemma" (Buchanan (1975)), whereby individuals anticipating the receipt of transfers conditional on low net worth rationally reduce their asset accumulation. Evidence on the disincentive effects of asset-based means testing on private saving is presented in Feldstein (1992) for college aid programs, and in Hubbard, Skinner and Zeldes (1992) for Aid to Families with Dependent Children (AFDC) and Medicaid. Our work studies the implicit tax on saving

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[3] See O'Connell and Zeldes (1988, 1991) for further discussion.

[4] Note, however, that O'Connell and Zeldes (1991) argue that Ricardian Equivalence may hold even if the government runs a rational Ponzi game.

that arises within an altruistic family, following related work by Bruce and Waldman (1990), Hansson and Stuart (1989), Laitner (1988), Veall (1986), and others. We find that the intra-family Samaritan's Dilemma, in the form of an implicit tax on saving, has a striking effect on aggregate capital accumulation in an otherwise standard overlapping generations model with altruism.

Finally, while our analysis emphasizes the dynamic efficiency of the gift economy, it does not establish optimality of decentralized gift equilibria. As suggested by Veall (1986) and Hansson and Stuart (1989), a mandatory government program that transfers resources from children to parents unconditionally (i.e., without asset-based means-testing) may improve welfare by overcoming the externality associated with children's concern for their parents. Such programs prevent parents from relying on gifts from their children to finance old-age consumption.[5] Thus the very Samaritan's dilemma that pushes the gift economy to dynamic efficiency in the absence of public intervention may also help explain the existence of institutions like the Social Security system.[6]

Given the impact of our change in strategic assumptions on the gift analysis, it is natural to ask whether the bequest economy would be

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[5] See also Laitner (1988).

[6] Veall (1986) and Hansson and Stuart (1989) investigate the effects of Social Security in a storage economy with a fixed exogenous interest rate. They make assumptions on the relationship between the interest rate, the rate of time preference, and the intergenerational discount factor that rule out a steady state equilibrium in which both gifts and saving are positive. In our treatment, based on the Diamond (1965) neoclassical model, the interest rate adjusts to achieve a steady state equilibrium with positive saving and gifts.

4

similarly affected by dropping the assumption that generations move simultaneously. The answer is no: we show below that incorporating the correct timing of events does not alter the standard results in the bequest literature.

The structure of the paper is as follows. In Section 2, we present the gift model and, for purposes of comparison, an analogous model with bequests. We describe the solution technique and the resulting first order conditions in each case, and compare them to the conditions that emerge in the simultaneous-moves model. In Section 3, we show that the steady state interest rate in the gift economy will be greater than the growth rate if strategies are linear in the previous generation's saving. Section 4 then studies a broad class of preferences that has been extensively used in the macroeconomics literature; it turns out that linearity of gift behavior is self-propagating in this case, in the sense that if one anticipates linear behavior of one's children, one's own optimal gift behavior will be linear. In a steady state equilibrium, we derive a closed form solution for a stationary linear gift function and give examples of steady state equilibria in which gifts are strictly positive. Section 5 concludes the paper.

2. The bequest model and the gift model.

The basic model is the neoclassical growth model of Diamond (1965) with either a gift or bequest motive added.[7] Individuals live for two periods. The population grows at rate n, with the subscript t

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[7] We do not consider economies in which gift and bequest motives exist simultaneously. For examples of this, see Abel (1987) and Kimball (1987).

5

denoting the generation born at time t. We normalize the size of generation 0 to equal 1. Each individual in generation t supplies labor inelastically in period 1 and earns a competitively determined real wage w_t, receiving no wage earnings while old. The young use their saving to purchase capital, which has a net rate of return of r_{t+1} between t and $t+1$. The production function is homogeneous of degree one, so net output per worker, f, is completely exhausted by competitively determined factor payments: $f(k_t) = w_t + r_t k_t$, where k_t is the capital stock per worker in period t. In period 1, there are two generations alive: the initial old (generation 0) and the initial young (generation 1).

Gifts are given by the current young and received in the same period by the current old; bequests are left by parents in the second period of life and received by their children in the same period. We denote the per-capita gift or bequest <u>received</u> by generation t by g_{t+1} and b_{t-1}, respectively; the time subscript refers to the generation <u>giving</u> the gift or bequest. Figure 1 shows the timing of the transfers.

Solving the model requires specifying the strategic behavior of successive generations. We focus here on two simple alternatives: (i) (Nash equilibrium in quantities) generation t takes the saving and transfer done by all other generations as given; (ii) ("Stackelberg") generation t takes all actions prior to date t as given, but assumes that the saving and transfer of its children, which occur in the future, are functions of its own behavior.[8,9]

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[8] Our second alternative is a Markov perfect equilibrium, i.e., a subgame perfect equilibrium in which strategies depend on a single state variable (the bequest given by parents in the bequest case, and the saving of parents in the gift case). While the simplicity of this model has obvious appeal, two caveats are worth mentioning. First, we do not

6

A. The bequest model

To set the stage for our analysis of the gift economy, we begin by showing that the two above approaches are equivalent in the bequest economy. In the bequest model, generation t divides the sum of its bequest received and its first period earnings into consumption when young (c_{1t}) and saving. In the second period, it divides its accumulated assets (including interest) (s_t) into second period consumption (c_{2t}) and a bequest (b_t). We begin with the familiar recursion in which the maximized utility of the generation born at t depends on its own consumption and the discounted maximized utility of its children:

$$V_t(b_{t-1}) = \max_{\{c_{1t}, b_t\}} [u(c_{1t}) + \beta u(c_{2t})] + \theta V_{t+1}(b_t) \qquad (1)$$

$$\text{subject to the budget constraints:}$$

$$\text{(i)} \qquad c_{1t} + s_t/(1+r_{t+1}) = w_t + b_{t-1}$$

$$\text{(ii)} \qquad c_{2t} = s_t - b_t(1+n)$$

$$\text{(iii)} \qquad b_t \geq 0.$$

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consider equilibria that might emerge if strategies were allowed to depend on history in a more complicated manner. Second, a more general model would include many periods and some overlap in the decisions of parents and children.

[9] Since we treat each generation as a representative agent, we do not deal with the strategic issues that arise when $1+n$ children are separately giving gifts to the same parent, or when a parent gives separate bequests to $1+n$ children. See Abel (1987) for an analysis of Nash equilibria in the gift and bequest economies under this assumption.

The parameters β and θ are the intertemporal and intergenerational discount factors, respectively. We assume that both lie between 0 and 1.[10]

To solve (1) using the Stackelberg approach, we substitute (i) and (ii) into the recursion (to eliminate c_{2t} and s_t) and take partial derivatives with respect to c_{1t} and b_t. This yields:

$$u'(c_{1t}) - \beta u'(c_{2t})(1+r_{t+1}) = 0 \qquad (2)$$

$$-\beta u'(c_{2t})(1+n) + \theta V_{t+1}'(b_t) = 0. \qquad (3)$$

Using the envelope condition $\partial V_{t+1}/\partial b_t = u'(c_{1,t+1})$, (3) implies:[11]

$$-\beta u'(c_{2t})(1+n) + \theta u'(c_{1,t+1}) = 0. \qquad (3')$$

Imposing the steady state condition that $c_{1t} = c_{1,t+1}$, we get the familiar result that $1+r = (1+n)/\theta > 1+n$; i.e., that the steady state in the bequest economy is dynamically efficient.

It is straightforward to see that equations (2) and (3)' also emerge using a Nash equilibrium in quantities. Equation (2) still holds since generation t takes the bequest it receives as given. Equation (3)' holds because generation t takes its children's saving as given, thereby

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[10] This assumption is standard. See Abel (1987) for a discussion.

[11] The envelope condition is derived as follows. For generation t, calculate the derivative of the value function with respect to b_{t-1}:
$V'_t(b_{t-1}) = u'(c_{1t}) \cdot (\partial c_{1t}/\partial b_{t-1}) + \beta u'(c_{2t}) [-(1+r_{t+1})(\partial c_{1t}/\partial b_{t-1}) + (1+r_{t+1}) - (1+n) \cdot (\partial b_t/\partial b_{t-1})] + \theta V'_{t+1} \cdot (\partial b_t/\partial b_{t-1}) = \partial c_{1t}/\partial b_{t-1} \cdot [u'(c_{1t}) - \beta u'(c_{2t})(1+r_{t+1})] + \partial b_t/\partial b_{t-1} \cdot [-\beta u'(c_{2t})(1+n) + \theta V'_{t+1}] + \beta u'(c_{2t}) \cdot (1+r_{t+1}) = \partial c_{1t}/\partial b_{t-1} [0] + \partial b_t/\partial b_{t-1} [0] + \beta u'(c_{2t}) \cdot (1+r_{t+1}) = \beta u'(c_{2t}) \cdot (1+r_{t+1}) = u'(c_{1t})$. This holds for generation t+1 as well.

assuming that an increase in its own bequest is consumed by its children in the first period of life.[12]

B. The gift model

In the gift economy, the welfare of the generation born at time t depends on its own consumption and on the welfare of its parents. Each generation anticipates receiving a transfer from its own children, and treats this prospective transfer as a function of its own saving, as well as of the interest rate and wage rate its children will face. The young split their wage between consumption, saving, and a per-capita gift of g_t to parents; consumption when old is the sum of endowment, assets, and the gift received from children. Maximized utility takes the form:

$$V_t(g_{t+1}(s_t)) = \max_{\{s_t, g_t\}} [u(c_{1t}) + \beta u(c_{2t})] + \theta V_{t-1}(g_t), \qquad (4)$$

$$s.t. \quad (i) \qquad c_{1t} = w_t - (g_t/(1+n)) - s_t/(1+r_{t+1})$$

$$(ii) \qquad c_{2t} = s_t + g_{t+1}(s_t)$$

$$(iii) \qquad c_{2,t-1} = s_{t-1} + g_t$$

$$(iv) \qquad g_t \geq 0.$$

We again assume that θ and β are between 0 and 1. We restrict our analysis to the case when children place a lower weight on their

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[12] This equivalence between the Nash and Stackelberg approaches in the bequest economy depends on the assumption that all parental decisions are made before children decide on saving. In a multiperiod model with some overlap in the decisions of parents and children, the strict equivalence would not hold. We conjecture that in such an economy the Nash and Stackelberg equilibria would be similar as long as parental bequest decisions are made before the bulk of the saving decisions of children.

parents' lifetime utility than on their own ($\theta < 1$) not because theory necessarily requires it, but because this leads to the standard dynamic inefficiency result that we are contesting.[13]

Using the "Stackelberg" approach, we assume that individuals treat all variables dated t-1 and earlier (e.g. $c_{1,t-1}$ and s_{t-1}) as given by history, and also take future reaction functions $g_{t+k}(s_{t+k-1})$ as given.[14] Under these assumptions, and given the budget constraints (i)-(iv), the problem can be rewritten as follows:

$$V_t(g_{t+1}) = \max_{\{s_t, g_t\}} [u(c_{1t}) + \beta u(c_{2t})] + \theta [u(c_{1,t-1}) + \beta u(c_{2,t-1})] + \theta^2 V_{t-2}(g_{t-1}). \qquad (5)$$

The first order conditions are

$$u'(c_{1t}) = \beta (1+g'_{t+1})(1+r_{t+1})u'(c_{2t}) \qquad (6)$$

and

$$u'(c_{1t}) \geq \theta\beta(1+n)u'(c_{2t-1}) \qquad \text{(with equality if } g_t > 0), \qquad (7)$$

where g'_{t+1} is the partial derivative of g_{t+1} with respect to s_t. Equation (7) describes the tradeoff between consuming an extra unit when young and giving that unit as a gift to one's parents. Equation (6) describes the tradeoff between consuming an extra unit today and saving the unit in order to consume the proceeds when old. The term $1+g'$ reflects the reaction of children to an increase in parents' saving; since g' will

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[13] With $\theta > 1$, the steady state in the standard gift model is dynamically efficient. For further discussions of the appropriate choice of the discount factor in the gift economy, see Carmichael (1982), Burbidge (1983,1984), Buiter and Carmichael (1984), and the clarifying comments in Abel (1987, p. 1038 footnote 7).

[14] We assume that $g(\cdot)$ is twice differentiable.

10

generally be negative, this reaction lowers the effective return to saving. It is the presence of this term that leads to the key results of this paper.

The Nash approach followed in the literature yields the same pair of first-order conditions, but with one important difference: since parents ignore the reactions of their children, g' is identically zero. This assumption has powerful implications, as can be seen by deriving the requirement for a steady state. From (6) and (7), a constant interest rate and lifetime consumption pattern imply

$$1+r = \theta(1+n)/(1+g').$$ (8)

It follows that in the Nash approach, as noted first by Carmichael (1982), the equilibrium interest rate is below the population growth rate as long as $\theta < 1$. The capital stock is therefore above the Golden Rule level, implying dynamic inefficiency.[15,16] It is worth emphasizing that this inefficiency emerges in spite of the fact that transfers from young to old serve to lower the capital stock relative to what would prevail in an otherwise identical economy without altruism (Abel, 1987). It is therefore not the gift motive per se that is causing the dynamic inefficiency; rather, for the gift motive to be operative (i.e. for gifts to be strictly positive), the corresponding non-altruistic economy must be dynamically inefficient.

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[15] The result of dynamic inefficiency when $\theta < 1$ is not completely general; for example, Kimball (1987) shows that dynamic efficiency can emerge in the gift economy for certain preferences if per-capita income growth is sufficiently large.

[16] One implication of dynamic inefficiency is that the government can run rational Ponzi games. This is explored in depth in O'Connell and Zeldes (1991).

11

In our case, the presence of the term $1+g' < 1$ serves to raise the steady state interest rate.[17] We next examine whether this effect will be strong enough to generate dynamic efficiency.

3. Dynamic efficiency in the gift economy

How large will g' be in equilibrium? As we show in this section, the answer depends in part on the second derivative of the gift function, g".

As long as $g_t > 0$, equations (6) and (7) together imply

$$u'(c_{1t}) = \beta u'(c_{2t})(1+g'_{t+1})(1+r_{t+1}) = \theta \beta u'(c_{2,t-1})(1+n) . \tag{9}$$

Differentiating (9) with respect to s_{t-1}, and dividing the result by (9), we get

$$A(c_{1t}) \cdot \partial c_{1t}/\partial s_{t-1} = \partial s_t/\partial s_{t-1}[A(c_{2t})(1+g'_{t+1}) - (g''_{t+1}/(1+g'_{t+1}))]$$
$$= A(c_{2t-1}) \cdot \partial c_{2,t-1}/\partial s_{t-1}, \tag{10}$$

where $A(c_{1t}) \equiv -u''(c_{1t})/u'(c_{1t})$ is the coefficient of absolute risk aversion. The resource constraint governing consumption of old and young in period t is $c_{2,t-1} + (1+n)c_{1t} + [(1+n)/(1+r_{t+1})]s_t = s_{t-1} + w_{1t}(1+n)$. Differentiating this with respect to s_{t-1} yields

$$\partial c_{2,t-1}/\partial s_{t-1} + (1+n)\partial c_{1t}/\partial s_{t-1} + [(1+n)/(1+r_{t+1})]\partial s_t/\partial s_{t-1} = 1. \tag{11}$$

Using (10) and the fact that $\partial c_{2,t-1}/\partial s_{t-1} = 1+g'_t$, we can substitute for the partial derivatives in equation (11) to derive the following recursion for $1+g_t'$:

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[17] This is directly analogous to the literature on the effects of capital income taxation on the steady state interest rate (e.g., Summers (1981)).

12

$$1+g_t' = \tag{12}$$

$$\left[\frac{(1+n)A(c_{2,t-1})}{A(c_{1t})} + \frac{(1+n)\ A(c_{2,t-1})}{(1+r_{t+1})(1+g_{t+1}')[A(c_{2t}) - \frac{g_{t+1}''}{(1+g_{t+1}')^2}]} + 1 \right]^{-1}$$

Next, we examine the properties of the steady state. Setting $c_{1t}=c_{1t-1}=c_1$ and $c_{2t}=c_{2,t-1}=c_2$, and using equation (8), we get the following expression:

$$1+g' = \theta \cdot \left[\frac{\theta(1+n)\ A(c_2)}{A(c_1)} + \frac{A(c_2)}{A(c_2) - \frac{g''}{(1+g')^2}} + \theta \right]^{-1} \tag{13}$$

This gives us

Proposition 1 A sufficient condition for dynamic efficiency of the steady state equilibrium in the gift economy is that the gift function have zero curvature at the equilibrium gift.

Proof: If $g'' = 0$, (13) becomes

$$1+g' = \theta \cdot [\theta(1+n)(A(c_2)/A(c_1)) + 1 + \theta]^{-1} < \theta, \tag{14}$$

where the inequality is obvious on inspection of the term in square brackets. Substituting $1+g' < \theta$ into (8) then implies $r > n$.[18] Q.E.D.

The dynamic inefficiency ordinarily found in the gifts economy necessarily disappears in steady state as long as the equilibrium gift function has zero curvature at the equilibrium. A sufficient (but not

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[18] Note that $g''=0$ is sufficient but not necessary for our proof; for small g'', the condition will also hold.

necessary) condition for this is that the gift function be linear in the saving of parents. As we show in the next section, a wide class of utility functions is consistent with such a gift function.

4. Gift behavior with HARA utility

In this section, we derive a closed-form linear gift function for the class of hyperbolic absolute risk aversion (HARA) preferences, under the assumption that the restriction $g_t \geq 0$ does not bind for any generation. The HARA utility function is given by

$$u(c) = \frac{\gamma}{(1-\gamma)} \left[\frac{c}{\gamma} - \eta \right]^{1-\gamma} \tag{15}$$

The HARA class includes linear ($\gamma = 0$), quadratic ($\gamma = -1$), constant absolute risk aversion ($\gamma = \infty$ and $\eta = -1$), and constant relative risk aversion ($\gamma > 0$, $\eta = 0$) preferences as special cases (see, for example, Ingersoll (1987, p. 39)).

With HARA preferences, the intertemporal and intergenerational first-order conditions (6) and (7) can be written as follows:

$$c_{2t} = \tau_{t+1} c_{1t} - (\tau_{t+1} - 1)\gamma\eta , \tag{16}$$

$$c_{2,t-1} = \sigma c_{1t} - (\sigma - 1)\gamma\eta , \tag{17}$$

where $\tau_{t+1} \equiv [\beta(1+g'_{t+1})(1+r_{t+1})]^{1/\gamma}$ and $\sigma \equiv [\beta\theta(1+n)]^{1/\gamma}$. The remaining equations are the budget constraints (4)(i)-(iii). Equations (17), (4)(i) and (4)(iii) can be used to solve for s_t as a linear function of g_t and s_{t-1}:

$$s_t = (1+r_{t+1}) \left[(w_t + \frac{(1-\sigma)\gamma\eta}{\sigma}) - (\frac{1}{1+n} + \frac{1}{\sigma})g_t - (\frac{1}{\sigma})s_{t-1} \right] . \tag{18}$$

14

To get an equation for g_t, first substitute (4)(i) into (16) and then eliminate c_{2t} in the resulting expression using (4)(ii). This yields

$$g_t = (1+n)\left[(w_t + \frac{(1 - r_{t+1})\gamma\eta}{r_{t+1}}) - (\frac{1}{1+r_{t+1}} + \frac{1}{r_{t+1}})s_t - (\frac{1}{r_{t+1}})g_{t+1}(s_t)\right] . \qquad (19)$$

Given a conjecture about the form of the reaction function $g_{t+1}(s_t)$, equations (18) and (19) define an implicit relationship between g_t and s_{t-1}. Although this relationship is in general complicated and nonlinear, an extremely simple case emerges when g_{t+1} is linear:

Proposition 2 If utility is HARA and the reaction function of generation $t+1$ is linear, then the reaction function of generation t is linear.[19]

Proof: Substitute for g_{t+1} in (19) using $g_{t+1}(s_t) = a_{t+1} - h_{t+1}s_t$, and then eliminate s_t using (18). This yields an optimal gift function of the form $g_t(s_{t-1}) = a_t - h_t s_{t-1}$, where a_t and h_t are functions of a_{t+1} and h_{t+1}. The algebra appears in Appendix 1.

Proposition 2 states that linearity of reaction functions is preserved if utility is HARA. It still leaves us without a boundary condition, however, to tie down the sequence of linear gift functions. Our next step is to focus on steady states. In Appendix 1, we show that both h_t and a_t are constant in a steady state, so that the gift function is stationary, i.e., identical for all generations. As shown in the Appendix, the stationary steady state linear gift function takes the form:

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[19] This proposition assumes that the restriction $g_t \geq 0$ is never binding.

15

$$g_t = a - \left[\frac{\sigma + \theta(1+n)}{\sigma + \theta(1+n) + \sigma\theta} \right] \cdot s_{t-1}, \qquad a > 0. \qquad (20)$$

Substituting (20) into (8), the steady state interest rate is given by[20]

$$1 + r = (1 + n)\left[1 + \theta + \frac{\theta(1+n)}{\sigma}\right] > 1 + n. \qquad (21)$$

We therefore have

Proposition 3 A stationary linear gift function exists for HARA, and the corresponding steady state equilibrium is dynamically efficient.[21]

Proof: Appendix 1.

Proposition 3 leaves open the question of whether the steady states in question are genuine gift equilibria in the sense that gifts are nonnegative for all generations. In the next section, we provide an example in which we can derive a specific set of parameters for which gifts will be positive in the steady state.[22]

Up to this point, we have been assuming that parents conjecture a linear gift function for their children. Under this assumption, the derived gift function for parents is also linear and the resulting steady state is dynamically efficient. We conjecture that a more general result

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[20] It is interesting to note that the steady state interest rate in the gift economy with linear conjectures depends only on the parameter γ (the coefficient of relative risk aversion in the constant relative risk aversion case) and not on η.

[21] Again, we ignore the restriction $g_t \geq 0$.

[22] We do not examine the behavior of the economy outside of the steady state.

is true for HARA preferences and differentiable gift functions: that no nonlinear gift function is consistent with steady state, and thus that any steady state gift equilibrium must be dynamically efficient. While we have not been able to prove this conjecture, we prove a related, more limited result in Appendix 2: that it is impossible to have a stationary nonlinear polynomial gift function with HARA utility.

4.1 Examples of gift equilibria

To verify the existence of gift equilibria with HARA preferences, we need to establish that the steady state gift can be positive. Since all of saving is in the form of capital (there is no government debt), $k_{t+1} = s_t/(1+n)(1+r_{t+1})$, or, imposing steady state, $s = (1+n)(1+r)k$. It then follows from equation (20) that the condition for a non-negative steady state gift is

$$g_t = \frac{(1+n) \cdot (\sigma w + (1-\sigma)\gamma\eta)}{\sigma + (1+n)} - \left[\frac{(\sigma + \theta(1+n))(1+n)^2}{\sigma}\right] \cdot k \geq 0, \qquad (22)$$

where (recall) $\sigma \equiv [\beta\theta(1+n)]^{1/\gamma}$. Since w and k depend not only on preferences, but also on the production technology, we must specify a production function. We assume a Cobb-Douglas production function: $f(k_t) = k_t^{\rho}$, $\rho \epsilon (0,1)$. In this case, using equation (21), substantial but straightforward algebra leads to the following condition for positive gifts:[23]

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[23] A competitive market for the services of capital implies $r = f'(k)$, which in turn implies $k(r) = (r/\rho)^{-1/(1-\rho)}$. Similarly, $w(r) = (1-\rho)[k(r)]^{\rho} = (1-\rho)[r/\rho]^{-\rho/(1-\rho)}$. Substituting these into equation (22) and then using equation (21) to eliminate r gives equation (23).

17

$$g_t = \left[\frac{1+n}{\sigma+(1+n)}\right]\left[\sigma(1-\rho)\rho^{\frac{\rho}{1-\rho}}\left\{(1+n)(1+\theta+\frac{\theta(1+n)}{\sigma})-1\right\}^{-\frac{\rho}{1-\rho}}+(1-\sigma)\gamma\eta\right]$$

$$-\frac{(\sigma+\theta(1+n))(1+n)^2}{\sigma}\rho^{\frac{1}{1-\rho}}\left[(1+n)(1+\theta+\frac{\theta(1+n)}{\sigma})-1\right]^{-\frac{1}{1-\rho}} \geq 0$$

$$(23)$$

Table 1 gives examples of parameter values for β, θ, n, γ, η, and ρ such that steady state gifts are positive.[24]

5. Conclusions

Since parents precede children in time, their optimal behavior must allow for the response of their children to decisions they make. In the bequest economy, incorporating this observation produces results that do not differ from those obtained in a model in which generations move simultaneously. In the gift economy, however, the same is not true; children will respond to higher parental saving by reducing their gifts. This lowers the effective return to saving whenever gifts are positive, resulting in lower steady state capital accumulation. For a broad class of preferences (HARA utility), we show that the steady state capital stock in the gifts model must be on the efficient side of the golden rule if conjectured gift functions are linear. We derive a

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[24] Using the Nash equilibrium approach, Abel (1987) and Weil (1987) derive conditions on the economy without altruism that must prevail in order for the gift or bequest motive to be operative. Although our framework is more complicated, it may be possible to derive analogous conditions for (23) to hold.

stationary linear gift function for the HARA class of utility functions and calculate explicit gift equilibria for a broad range of parameters when the production function is Cobb-Douglas. The analysis overturns the standard presumption of dynamic inefficiency in the gift economy.

As described in the introduction, our main result has a variety of implications. First, given the empirical evidence on dynamic efficiency in the U.S., it reestablishes the potential relevance of the gift model to the U.S. economy. Second, it renders moot the debate on Ricardian Equivalence and Ponzi games in the gift economy. Third, it extends the literature on the effects of implicit taxation on capital accumulation.
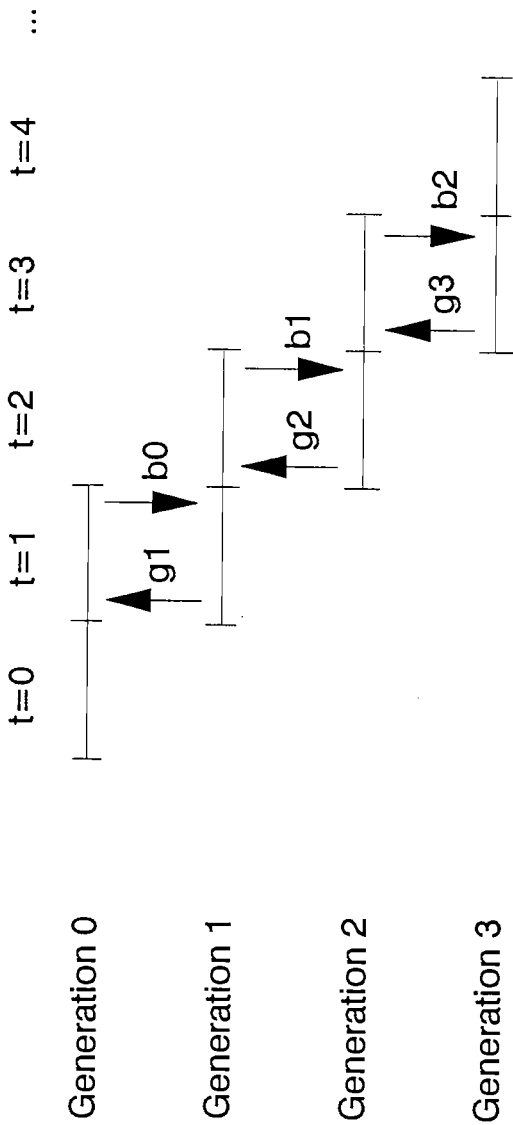
Figure 1

Table 1: Examples of steady states with a positive gift

β	θ	n	γ	η	ρ	g	s	k	r	1+g'
.74	.80	.81	1	0	.25	.036	.206	.020	4.70	.746
.74	.80	.81	3	0	.25	.030	.204	.019	4.82	.751
.50	.80	0	1	0	.20	.007	.140	.037	2.80	.790
.80	.80	.90	.5	0	.30	.025	.222	.022	4.37	.717
.30	.40	.30	1	.13	.33	.001	.123	.017	5.15	.916

Notes: The right side of the table displays calculated values of
g (equilibrium gift), s (equilibrium assets), k (capital per worker), r
(real interest rate), and 1+g', given the values of parameters on the left
side of the table: β (intertemporal discount factor), θ (discount factor
applied to parents' utility), n (population growth rate), γ and η (utility
function parameters), and ρ (capital share in output). For convenience
we reproduce here the underlying relationships:

$$V_t(g_{t+1}(s_t)) = \underset{\{s_t, g_t\}}{\text{Max}} [u(c_{1t}) + \beta u(c_{2t})] + \theta V_{t-1}(g_t) \qquad \text{(basic recursion)}$$

$$u(c) = \frac{\gamma}{(1-\gamma)} \left[\frac{c}{\gamma} - \eta \right]^{1-\gamma} \qquad \text{(period utility function)}$$

$$f(k_t) = k_t^\rho. \qquad \text{(per capita production function)}$$

The parameters are chosen to roughly correspond to a period length of
30 years. Thus, for example, in row 2, n = 0.81 corresponds to an
annual population growth rate of 2%. The interest rate of 4.82
corresponds to an annual rate of about 6%.

21

Appendix 1: Proofs of Propositions 2 and 3

To prove Proposition 2, let the conjectured linear gift function take the form

$$g_{t+1}(s_t) = a_{t+1} - h_{t+1}s_t , \qquad (A1.1)$$

for arbitrary constants a_{t+1} and h_{t+1}. Equations (18), (19) and (A1.1) then imply

$$g_t = \sigma \cdot \left[\frac{\phi_{t+1}[w_t + (\xi_2/\sigma)] + a_{t+1} - [\xi_{1,t+1} - \tau_{t+1}(\xi_2/\sigma)]}{\tau_{t+1} + \phi_{t+1} + (\sigma\phi_{t+1}/(1+n))} \right] \qquad (A1.2)$$
$$- \left[\frac{\tau_{t+1} + \phi_{t+1}}{\tau_{t+1} + \phi_{t+1} + (\sigma\phi_{t+1}/(1+n))} \right] s_{t-1} ,$$

where $\xi_2 \equiv (1 - \sigma)\gamma\eta$, $\xi_{1,t+1} \equiv (1 - \tau_{t+1})\gamma\eta$, $\phi_{t+1} \equiv (1+r_{t+1})(1-h_{t+1})$, and (recall) $\sigma \equiv [\beta\theta(1+n)]^{1/\gamma}$ and $\tau_{t+1} \equiv [\beta(1+g'_{t+1})(1+r_{t+1})]^{1/\gamma}$. Equation (A1.2) is of the form $g_t = a_t - h_t s_{t-1}$. Q.E.D.

To prove Proposition 3, equate coefficients between equation (A1.2) and the function $g_t = a_t - h_t s_{t-1}$ to get a pair of dynamic equations of the form $a_t = a_t(a_{t+1}, h_{t+1})$ and $h_t = h_t(h_{t+1})$. In a steady state, $\tau_{t+1} = \sigma$, $\xi_{1,t+1} = \xi_2$, $\phi_{t+1} = \theta(1+n)$; we therefore have

$$h_t = h = \frac{\sigma + \theta(1+n)}{\sigma + \theta(1+n) + \sigma\theta} \in (0,1] , \qquad (A1.3)$$

from which we see that there is a unique steady state interest rate, r, obtained by substituting (A1.3) into equation (8) in the text (with g' = -h). The dynamic equation for a_t can then be written in the form

$$a_t = (\frac{1+n}{1+r})[a_{t+1} + \theta(1+n)(w_t + \frac{\xi_2}{\sigma})] . \qquad (A1.4)$$

Since saving, consumption, the wage and the interest rate are all constant in a steady state, the budget constraints imply that the gift must also be constant. Time-invariance of h_t (h_t = h) therefore implies a_t =

22

a. It follows that the unique steady state solution for a_t is the stationary solution to (A1.4):

$$a_t = a = \frac{\theta(1+n)^2[w + (\xi_2/\sigma)]}{r - n} > 0. \qquad \text{Q.E.D.} \qquad \text{(A1.5)}$$

23

Appendix 2. Uniqueness of the linear gift function

To prove that the only stationary polynomial gift function is linear, consider a conjectured gift function that is a polynomial of degree n:

$$g_{t+1} = \sum_{i=0}^{n} \delta_i [s_t]^i. \tag{A2.1}$$

Substitute $g_{t+1}(s_t) = \Sigma \delta_i s_t^i$ into (19), and substitute out for s_t using (18). This yields an equation involving two polynomials of degree n, one in g_t and the other in s_{t-1}. Equating the coefficients in this expression with those of (A2.1) allows one to solve for δ_0 through δ_n. It is easy to show that $\delta_i = 0$ for $i \geq 1$, and that the unique choices for δ_0 and δ_1 are given by $\delta_0 = a$ and $\delta_1 = -h$. The unique stationary polynomial gift function therefore takes the linear form $g = a - h \cdot s$. Q.E.D.

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